Heat Exchanger Network Design/Retrofit with Partitioning Technique for Linearization of Specific Heat Capacity-Temperature Relation

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ABSTRACT

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Heat Exchanger Network (HEN) design has been studied for 40 years starting from heuristic technique; pinch analysis and design method to mathematical programming. The stage-wise superstructure by mathematical programming from Yee and Grossmann (1990) was one of the famous models suitable for industrial HEN design under assumption of constant average specific heat capacity (Cp) which generally should be fitted by empirical form of cubic equation. Zhu and Asante (1999) and Ayotte-Sauvé *et al.* (2017) used piece-wise linearization called stream segment model, to approximate Cp. Their Cp approximation techniques used high number of piece-wises to calculate Cp accurately. This paper presents novel technique called partitioning technique; using less number of piece-wise to approximate temperature-dependent Cp more accurately along with stage-wise superstructure model. The concept is to linearize polynomial cubic equation of Cp as a function of temperature with different Cp approximation techniques from Zhu and Asante (1999) and Ayotte-Sauvé *et al.* (2017). Cp is approximated as a linear equation for each partition of temperature range. Our technique in weighted average Cp calculation at stage in stage-wise superstructure model is novel, giving more accurate Cp approximation and HEN synthesis at validation step. Our model synthesizes HEN with less total annualized cost (TAC) and exchanger area calculation error between our model and Pro/II simulated HEN compared to other models; constant Cp model and cubic equation technique from Kim and Bagajewicz (2017) represented by four examples. First, crude preheat train example from Pro/II library is used to validate new concept. Normally, constant specific heat capacity directly affects outlet temperature which increase error of area calculation. It shows that constant heat capacity flow rate can reach error of heat exchanger area 30 % but using the new model can reduce the error to less than 1 %. Next, two examples from Kim and Bagajewicz (2017) get better result when compared to previous solution of cubic equation technique. Forth example is to show new model in retrofit case. Therefore, new model has many advantages that it increases accuracy of HEN design and better solution can be obtained from variable of specific heat capacity.

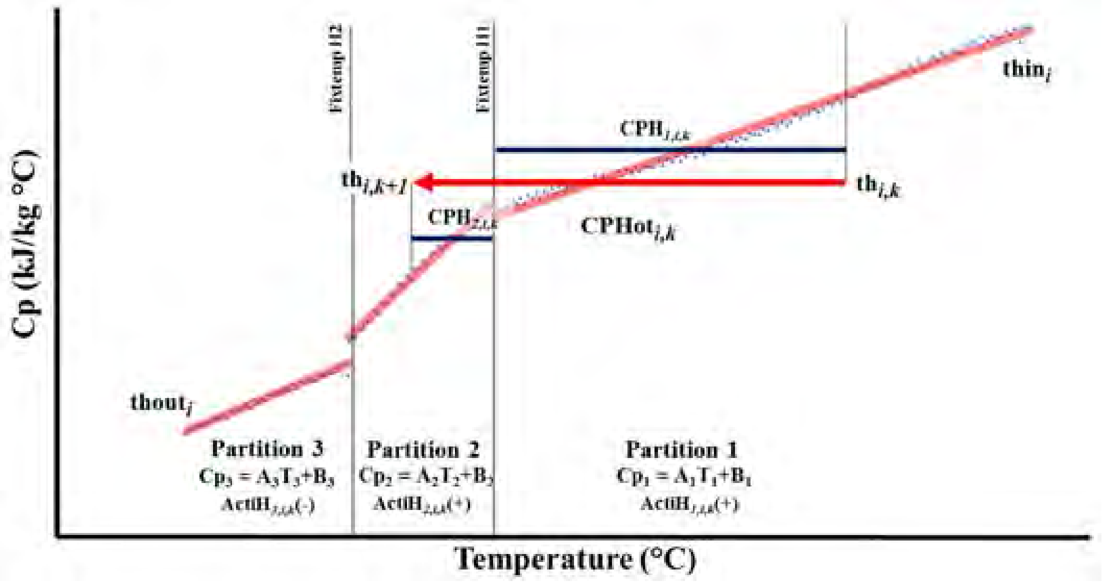
บทคัดย่อ

นายศิวัช วลีเกียรติกุล : การออกแบบและการพัฒนาระบบแลกเปลี่ยนความร้อน (Heat Exchanger Network Design/Retrofit) อ.ที่ปรึกษา : ผศ. ดร. กิตติพัฒน์ สีมานนท์: ๑๔๐ หน้า

การออกแบบระบบแลกเปลี่ยนความร้อนมีการพัฒนาเป็นระยะเวลา 40 ปีโดยมีจุดเริ่มต้นจากเทคนิคการศึกษาสำนึกคือการวิเคราะห์และมีการพัฒนาต่อมาถึงการออกแบบวิธีการเขียนโปรแกรมทางคณิตศาสตร์ วิธีการทางคณิตศาสตร์แบบการแบ่งระบบโครงสร้างของ Yee and Grossmann (1990) เป็นระบบที่ใช้กันอย่างแพร่หลายและเหมาะสมสำหรับอุตสาหกรรมการวิเคราะห์และออกแบบระบบแลกเปลี่ยนความร้อนภายใต้เงื่อนไขสมมุติฐานว่าค่าความจุความร้อนเป็นค่าคงที่แต่โดยความเป็นจริงควรใช้กับข้อมูลผลการทำลองที่ต้องใช้สมการกำลังสาม Zhu and Asante (1999) และ Ayotte-Sauvé *et al.* (2017) ริเริ่มการใช้หลักการแบ่งเป็นเส้นตรงที่เรียกว่าเซ็กเมนต์สตรีมโมเดลเพื่อประมาณค่าความจุความร้อน โดยค่าความจุความร้อนนั้นจำเป็นต้องใช้จำนวนช่วงมากๆเพื่อความแม่นยำ บทความนี้จึงได้นำเสนอเทคนิคใหม่ที่เรียกว่าเทคนิคการแบ่งส่วนโดยมีการใช้จำนวนการแบ่งช่วงที่น้อยกว่าสำหรับการประมาณค่าความจุความร้อนที่แปรพันธ์ตามอุณหภูมิ และมีค่าความแม่นยำที่มากกว่า สำหรับวิธีการทางคณิตศาสตร์แบบการแบ่งระบบโครงสร้าง ค่าความจำความร้อนจะประมาณจากสมการเส้นตรงของแต่ละส่วนที่ถูกแบ่งตามอุณหภูมิ โดยโมเดลของบทความนี้จะใช้เทคนิคการประมาณค่าเฉลี่ยแบบถ่วงน้ำหนักในวิธีการทางคณิตศาสตร์แบบการแบ่งระบบโครงสร้าง และถือว่าเป็นเทคนิคใหม่ที่มีความแม่นยำสูง โมเดลใหม่นี้ได้ออกแบบระบบแลกเปลี่ยนความร้อนที่มีค่าเงินรวมต่อปีที่น้อยกว่า และการคำนวณเปอร์เซ็นความผิดพลาดของการออกแบบพื้นที่การแลกเปลี่ยนความร้อนของเครื่องแลกเปลี่ยนความร้อนที่น้อยกว่าโดยการเปรียบเทียบระหว่างโมเดลใหม่กับโปรแกรมจำลอง Pro/II การเปรียบเทียบนี้จะเปรียบเทียบระหว่างค่าความจำความร้อนที่เป็นค่าคงที่และค่าความจุความร้อนในรูปสมการกำลังสามจาก Kim and Bagajewicz (2017) สำหรับตัวอย่างแรกเป็นตัวอย่างจากโปรแกรมจำลอง Pro/II เพื่อจุดประสงค์สำหรับการตรวจสอบความถูกต้องของตัวโมเดล โดยปกติแล้วค่าความจำความร้อนที่เป็นค่าคงที่จะส่งผลโดยตรงต่อการคำนวณ อุณหภูมิขาออกของเครื่องแลกเปลี่ยนความร้อนเป็นผลให้การคำนวณพื้นที่แลกเปลี่ยนความร้อนมีค่าที่ผิดพลาด ผลลัพธ์ได้แสดงให้เห็นว่าค่าความจำความร้อนที่เป็นค่าคงที่มีเปอร์เซ็นความผิดพลาดของการคำนวณพื้นที่การแลกเปลี่ยนความร้องของเครื่อง

แลกเปลี่ยนความร้อนมากถึงร้อยละ 30 แต่สำหรับโมเดลใหม่สามารถลดความผิดพลาดจากการคำนวณนี้ให้น้อยกว่าร้อยละ 1 ได้ และสำหรับตัวอย่างอีกสองตัวอย่างของ Kim and Bagajewicz (2017) ผลลัพธ์ก็แสดงให้เห็นว่าสามารถใช้โมเดลใหม่และได้คำตอบที่ดีกว่า เมื่อเปรียบเทียบระหว่างคำตอบใหม่กับคำตอบเก่าแบบเทคนิคสมการกำลังสาม สำหรับตัวอย่างสุดท้ายมีไว้เพื่อแสดงให้เห็นว่าโมเดลใหม่นี้สามารถใช้สำหรับการพัฒนาระบบการแลกเปลี่ยนความร้อนได้ ดังนั้นโมเดลใหม่จึงมีข้อดีหลายประการทั้งการเพิ่มความแม่นยำของการออกแบบระบบแลกเปลี่ยนความ้อนและสามารถหาวิธีแก้ปัญหาที่ดีกว่าจากการทำให้ค่าความจุความร้อนแปรพันธ์ตามอุณหภูมิ

GRAPHICAL ABSTRACT



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NOMENCLATURE

Indices

i	Index for hot process stream
j	Index for cold process stream
k	Index for stages (1, …, *NOK*)
n	Number of partitions (1, …, *NOP*)
y	Index for number of years

Sets

CP	Cold process stream
CU	Cold utility
HP	Hot process stream
HU	Hot utility
PT	Partition in the Cp profile (1, …, NOP)
ST	Stage in the superstructure (1, …, NOK)

Parameters

$AC_{n,j}$	Cold stream slope coefficient of Cp at partition *n*, kJ/(kg °C^2)
$AH_{n,i}$	Hot stream slope coefficient of Cp at partition *n*, kJ/(kg °C^2)
$AreaCold_i$	Existing area of cold utility, m^2
$AreaHold_j$	Existing area of hot utility, m^2
$Areaold_{ijk}$	Existing area of process exchanger, m^2
$B_{i,j}$	Area cost coefficient of process stream
$B_{i,CU}$	Area cost coefficient of cold utility
$B_{j,HU}$	Area cost coefficient of hot utility
$BC_{n,j}$	Cold stream y-intercept of Cp at partition *n*, kJ/(kg °C)
$BH_{n,i}$	Hot stream y-intercept of Cp at partition *n*, kJ/(kg °C)

C_{ij}	Area cost of process exchanger, \$/(y m^2)
$C_{i,CU}$	Area cost of cold utility, \$/(y m^2)
$C_{j,HU}$	Area cost of hot utility, \$/(y m^2)
CCU	Cold utility per unit cost, \$/(kW y)
CHU	Hot utility per unit cost, \$/(kW y)
CF_{ij}	Fixed cost of process exchanger, \$/y
$CF_{i,CU}$	Fixed cost of cold utility, \$/y
$CF_{j,HU}$	Fixed cost of hot utility, \$/y
CPC_{avg_j}	Overall average Cp of cold stream *j*, kJ/(kg °C)
CPH_{avg_i}	Overall average Cp of hot stream *i*, kJ/(kg °C)
FC_j	Flow rate of cold stream *j*, kg/s
FH_i	Flow rate of hot stream *i*, kg/s
$FixtempC_{n,j}$	Fix temperature of cold stream for partition *n* and *n+1*, °C
$FixtempH_{n,i}$	Fix temperature of hot stream for partition *n* and *n+1*, °C
$qcuold$	Base case cold utility consumption, kW
$qhuold_j$	Base case hot utility consumption, kW
$thin_i$	Temperature inlet of hot stream *i*, °C
$thout_i$	Temperature outlet of hot stream *i*, °C
$tcin_j$	Temperature inlet of cold stream *j*, °C
$tcout_j$	Temperature outlet of cold stream *j*, °C
$zhuold_j$	Existing exchanger of hot utility
$zcuold_i$	Existing exchanger of cold utility
$zold_{ijk}$	Existing exchanger of process stream

Variables

$ActiC_{n,j,k}$	Activated variable of cold stream for partition *n*, °C
$ActiCF_{n,j,1}$	Activated variable of hot utility for partition *n*, °C
$ActiH_{n,i,k}$	Activated variable of hot stream for partition *n*, °C
$ActiHL_{n,i,NOK+1}$	Activated variable of cold utility for partition *n*, °C

$CPC_{n,j,k}$	Cp calculation of cold stream for partition *n*, kJ/(kg °C)
$CPCF_{n,j,1}$	Cp calculation of hot utility for partition *n*, kJ/(kg °C)
$CPColdF_{j,1}$	Average Cp calculation of hot utility, kJ/(kg °C)
$CPCold_{j,k}$	Average Cp calculation of cold stream *j* at stage *k*, kJ/(kg °C)
$CPH_{n,i,k}$	Cp calculation of cold stream for partition *n*, kJ/(kg °C)
$CPHL_{n,i,NOK+1}$	Cp calculation of cold utility for partition *n*, kJ/(kg °C)
$CPHot_{i,k}$	Average Cp calculation of hot stream *i* at stage *k*, kJ/(kg °C)
$CPHotL_{i,NOK+1}$	Average Cp calculation of cold utility, kJ/(kg °C)
$TCcal_{n,j,k}$	Average mean temperature difference calculation of cold stream for partition *n*, °C
$TCcalF_{n,j,1}$	Average mean temperature difference calculation of hot utility for partition *n*, °C
$THcal_{n,i,k}$	Average mean temperature difference calculation of hot stream for partition *n*, °C
$THcalL_{n,i,NOK+1}$	Average mean temperature difference calculation of cold utility for partition *n*, °C
qhu_j	Heat exchange between cold stream *j* and hot utility, kW
qcu_i	Heat exchange between hot stream *i* and cold utility, kW
$q_{i,j,k}$	Heat exchange between hot stream *i* and cold stream *j* at stage *k*, kW
$tc_{j,1}$	Temperature of cold stream *j* at first stage, °C
$tc_{j,k}$	Temperature of cold stream *j* at stage *k*, °C
$tc_{j,k+1}$	Temperature of hot stream j at stage *k+1*, °C
$th_{i,NOK+1}$	Temperature of hot stream *i* at last stage, °C
$th_{i,k}$	Temperature of hot stream *i* at stage *k*, °C
$th_{i,k+1}$	Temperature of hot stream *i* at stage *k+1*, °C

CHAPTER I
INTRODUCTION

Chemical plants are designed over the year by focusing on economic issue which require a development of the process. One of the methods is heat integration technique which improves energy efficiency. Heat integration techniques focus on heat exchanger network (HEN) and energy efficiency improvement by rearranging HEN which is called retrofit. The retrofit idea is modified from HEN synthesis (the method to design HEN). At the beginning, HEN synthesis is designed and relied on heuristic method, that had been studying for 40 years, called pinch analysis. However, pinch analysis has many limitations on the design steps which is heavily based on previous decision and it will lose the effective design scenario missing from non-simultaneous procedure.

Mathematical models are proposed for HEN synthesis and retrofitting instead of using pinch analysis. Many mathematical models are invented such as transportation model, transshipment model, superstructure model and stage-wise superstructure model, all of them have limitations on their own. They must be selected carefully and embedded in commercial optimization program. Therefore, stage-wise superstructure is selected because of low computational time and high effectiveness. General algebraic modeling system (GAMS) is selected for this study because its powerful solvers on MILP and MINLP.

In this study, stage-wise superstructure model by Yee T.F. and Grossmann I.E. (1990) is used for HEN synthesis which dividing process temperature into stages under assumption of constant specific heat capacity. Only objective function is non-linear. To make model more realistic, the model is developed under temperature-dependent specific heat capacity. Thus, the propose of this research is to design HEN by using mathematical programming model of stage-wise superstructure model through GAMS program using partitioning technique for linearization of specific heat capacity-temperature relation. The temperature-dependent specific heat capacity equations are now embedded and stage-wise superstructure for HEN synthesis will be modified for retrofitting.

CHAPTER II
LITERATURE REVIEW

Heat Exchanger Network (HEN) design has been studied for about 40 years. It is classified into two main categories, Pinch Analysis methods and Mathematical Programming methods. The pinch analysis methods were first introduced by Linnhoff and Flower (1978a). They proposed temperature interval method (TI method) to minimize utility energy consumption based on thermodynamically orientated method. Linnhoff and Flower (1978b) also introduced Paths and Loops techniques to reduce the number of exchangers. In 1983, Linnhoff and Hindmarsh (1983) suggested the pinch design method based on heuristic rule starting from minimizing utility consumption, number of heat exchanger and area of heat exchanger. At the targeting step, minimum temperature difference (ΔT_{min}) is specified to determine minimum utility usage. Smith (2005) conclude pinch design method until improving to mathematical programming. Nowadays, mathematical programming is applied for HEN synthesis and retrofitting. There are many mathematical programming models for HEN design; transshipment model by Papoulias and Grossmann (1983), superstructure model by Floudas *et al.* (1986), hyperstructure model by Ciric and Floudas (1991) and stage-wise superstructure model by Yee and Grossmann (1990) which were studied and compared by Escobar and Trierweiler (2013). They showed that stage-wise superstructure model gave lower total annual cost (TAC) and computation time. Thus, stage-wise superstructure that invented by Yee and Grossmann (1990) was chosen to study and develop by many authors. They distribute their work into three main parts, first area & energy targeting (Yee *et al.* (1990a)), second HEN synthesis (Yee and Grossmann (1990)), and the last process & HEN optimization (Yee *et al.* (1990b)). The main concept of stage-wise superstructure is to set temperature as variable and minimize TAC by trade-off between number of heat exchanger, area of heat exchanger and utility consumption. It can be applied to heat integration of organic Rankine cycles from Hipólito-Valencia *et al.* (2013) for minimizing TAC by input excess heat from HEN to organic Ranking cycles. However, stage-wise superstructure has two main assumptions, isothermal mixing and constant heat capacity flow rate.

HEN retrofit is practical HEN design for the process with existing HEN to recovery utility consumption. Bagajewicz *et al.* (2013) studied HEN comparison between mathematical programming and pinch design method using crude preheat train example. The mathematical model called "Heat integration transportation model (HIT)" has gave more profitable model. Liu *et al.* (2016) shows retrofit of HEN from stage-wise superstructure by including many binary variables to locate existing heat exchanger and solve complex mixed integer non-linear programming (MINLP) model by their hybrid genetic algorithm.

Many authors develop the stage-wise superstructure under temperature dependent specific heat capacity condition. Zhu and Asante (1999) present hybrid method of pinch design and mathematical programming for retrofitting. In their work, stream segment model is introduced together with segmented stream for heat capacity calculation. The design task consists of diagnostic stage, evaluation stage and cost optimization stage which can solve complex HEN with less computational time. Smith *et al.* (2010) improved HEN retrofit model from Zhu and Asante (1999) by including thermal properties of streams (e.g. heat capacity) as a function of temperature in terms of polynomial equation and stream splitting constraints which become effective design effect on lower TAC. Sreepathi and Rangaiah (2015) proposed nodal model similar to HEN retrofit model from Smith *et al.* (2010) with cubic equation of specific heat capacity. They introduced single (sum of operating cost and investment cost) and multi-objective (separate operating cost and investment cost) optimization to calculate solution in new flowchart algorithm and lower TAC can be obtained. Hasan *et al.* (2010) replaced isothermal mixing assumption with phase change process based on stage-wise superstructure. They decompose stream into multizone streams to express phase change zone and cubic equation is specified for enthalpy variable. LNG plant is used as an example and feasible solution can be generated. However, better solution can be obtained but it is next step challenge of their work. Li *et al.* (2012) simplified stage-wise superstructure with variable heat capacity by one linear equation for each stream combine with genetic algorithm and they showed that practical HEN can be designed. Kim *et al.* (2017) and Kim and Bagajewicz (2017) used their new model developed from Floudas *et al.* (1986) combine with cubic equation of specific heat capacity flow rate for HEN synthesis. Due to the complexity of their model, they developed bound contraction procedure called RYSIA to get global solution. The global solutions are shown from their example with consume much computational time.

Removing of constant heat capacity flow rate become challenging topic that many methods solution are proposed by many authors. The simplest one is adding cubic equation of specific heat capacity in terms of temperature variable but it will become highly non-linear equation. Another technique is piece-wise linearization of enthalpy profile from Zhu and Asante (1999) and Ayotte-Sauvé *et al.* (2017) but their techniques give high accurate obtained from increasement number of segments. Therefore, stage-wise superstructure which has lower TAC and computational time than other mathematical programming models is preferred to remove this assumption by our new technique called partitioning technique. It is more simple technique than ordinary non-linear cubic equation added, higher accuracy than previous piece-wise linearization and it will be described here in this journal.

CHAPTER III
METHODOLOGY

This work aims to design HEN with variable specific heat capacity by mathematical programming based on stage-wise superstructure from Yee and Grossmann (1990). The concept of stage-wise superstructure is to divide hot stream i and cold stream j into stage k which shows in Figure 3.1. The location of a heat exchanger match (ijk) is free based on objective function.

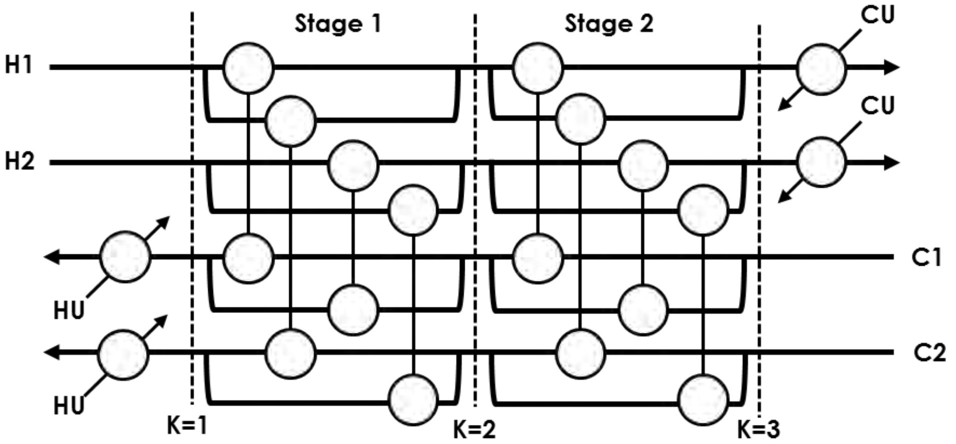


Figure 3.1 Stage-wise superstructure from Yee and Grossmann (1990).

3.1 Stage-Wise Superstructure Model

3.1.1 Overall Heat Balance for Each Stream

Overall heat balance for each stream is equal to overall heat transfer by heat exchanger at each stage including with heat from utility exchanger (Eqs. (3.1-3.2)). It calculates entire heat surplus for hot streams that can be transfer to cold streams and vice versa.

$$(TIN_i \quad - TOUT_i)\ F_i = \sum_{k\in ST} \sum_{j\in CP} q_{ijk} + qcu_i, \qquad i \in HP \qquad (3.1)$$

$$(TOUT_j - TIN_j) \quad F_j = \sum_{k\in ST} \sum_{i\in HP} q_{ijk} + qhu_j, \qquad j \in CP \qquad (3.2)$$

3.1.2 Heat Balance at Each Stage

Heat balance at each stage is used for identifying temperature location at each stage. Eqs. (3.3) and Eqs. (3.4) calculate capacity of heat transfer at each stage. The temperature of adjacent stages is involved that outlet temperature of hot stream at first stage will be inlet temperature of second stage. In the same manner for cold stream, inlet temperature at stage *k* come from outlet temperature at stage *k-1*.

$$\left(t_{i,k} - t_{i,k+1}\right) F_i = \sum_{j \in CP} q_{ijk}, \qquad i \in HP, k \in ST \qquad (3.3)$$

$$\left(t_{j,k} - t_{j,k+1}\right) F_i = \sum_{i \in HP} q_{ijk}, \qquad j \in CP, k \in ST \qquad (3.4)$$

3.1.3 Assignment of Superstructure Inlet Temperatures

Inlet temperature of hot stream is assigned by Eqs. (3.5) and Eqs. (3.6) that inlet temperature at stage *1* and inlet temperature of cold stream is assigned to inlet temperature at stage *NOK+1*. Note, stages are divided into *k=1* to *k=NOK* while temperature at each stage is located at *k=1* to *k=NOK+1*.

$$TIN_i = t_{i,1}, \qquad i \in HP \qquad (3.5)$$

$$TIN_j = t_{j,NOK+1}, \qquad j \in CP \qquad (3.6)$$

3.1.4 Feasibility of Temperatures

The stage-wise superstructure defines that at left-hand side is the highest temperature of each stream and it is exchange energy along stream at each stage by exchanger. Outlet temperature of hot stream will be at right-hand side which is the lowest temperature. Thus, the temperature will define as stage *k* is hotter than stage *k+1* and so on (Eqs. (3.7) and Eqs. (3.8)). Likewise, temperature outlet of hot stream must be the lowest in that line and temperature outlet of cold stream must be the highest temperature (Eqs. (3.9) and Eqs. (3.10)).

$$t_{i,k} \geq t_{i,k+1}, \qquad i \in HP, k \in ST \qquad (3.7)$$

$$t_{j,k} \geq t_{j,k+1}, \qquad j \in CP, k \in ST \qquad (3.8)$$

$$TOUT_i \leq t_{i,NOK+1}, \qquad i \in HP \qquad (3.9)$$

$$TOUT_j \geq t_{j,1}, \qquad j \in CP \qquad (3.10)$$

3.1.5 Hot and Cold Utility Load

As define stage separation, the temperature of stage *1* or *NOK+1* can be relaxed because cold and hot utility loading will replace the deficit or surplus energy of that stream, respectively. The utility loading is calculated by difference temperature between final stage of each stream and target temperature multiply by heat capacity flow rate as shown in Eqs. (3.11) and Eqs. (3.12).

$$(t_{i,NOK+1} - TOUT_i)F_i = qcu_i, \qquad i \in HP \qquad (3.11)$$
$$(TOUT_j - t_{j,1})F_j = qhu_j, \qquad j \in CP \qquad (3.12)$$

3.1.6 Logical Constraints

To define heat exchanger matching, logical constraints equations are used and represent by Eqs. (3.13-3.15). The binary variable of z_{ijk} represent for existing exchanger matching of hot stream *i* and cold stream *j* at stage *k*. zcu_i represent for cold utility matching and zhu_j represent for hot utility matching. The logical constraints equation is described by; if heat is transferred between hot stream *i* to cold stream *j* at stage *k*, an integer value (integer value = 1) will demonstrate for *z* binary variable. The equation of inequality will become true because Ω is upper bound of heat loading and it always true only if z binary variable become one. In contrast, *z* binary variable can be integer value of one or zero if heat is not transfer between each stream but minimum of total annual cost will force *z* binary variable to zero.

$$q_{ijk} - \Omega z_{ijk} \leq 0, \qquad i \in HP, j \in CP, k \in ST \qquad (3.13)$$
$$qcu_i - \Omega zcu_i \leq 0, \qquad i \in HP \qquad (3.14)$$
$$qhu_j - \Omega zhu_j \leq 0, \qquad j \in CP \qquad (3.15)$$

3.1.7 Calculation of Approach Temperatures

The area calculation is calculated by temperature inlet and outlet of hot and cold streams. For calculating area properly, approach temperature is determined as illustrated in Eqs. (3.16-3.19). The approach temperature equation will activate when z binary variable equal one which force value inside the bracket to zero. Temperature difference will less than or equal to temperature difference of hot stream and cold stream. The approach temperature is set at two pairs of exchanger matching.

$$dt_{ijk} \leq t_{i,k} - t_{j,k} + \Gamma(1 - z_{ijk}), \qquad i \in HP, j \in CP, k \in ST \tag{3.16}$$

$$dt_{ijk+1} \leq t_{i,k+1} - t_{j,k+1} + \Gamma(1 - z_{ijk}), \qquad i \in HP, j \in CP, k \in ST \tag{3.17}$$

$$dtcu_i \leq t_{i,NOK+1} - TOUT_{CU} + \Gamma(1 - zcu_i), \qquad i \in HP \tag{3.18}$$

$$dthu_j \leq TOUT_{HU} - t_{j,1} + \Gamma(1 - zhu_j), \qquad j \in CP \tag{3.19}$$

3.1.8 Exchanger Minimum Approach Temperature

Normally, temperature approach is set as higher or equal to exchanger minimum approach temperature (EMAT) for reasonable area calculation by Eqs. (3.20). Frequently, it is set EMAT as 10 °C for counter current heat exchanger type and less for compact heat exchanger.

$$dt_{ijk} \geq EMAT, \qquad i \in HP, j \in CP, k \in ST \tag{3.20}$$

3.1.9 Log-Mean Temperature Difference

Log-mean temperature is an approximate temperature calculation between four points of exchanger temperature. Chen approximation is used (Eqs. (3.21-3.23)) instead of normal log-mean temperature difference because it can calculate when either dt_{ijk} or dt_{ijk+1} equals zero, the driving force will be approximated to zero. Note that Chen approximation is underestimate the driving force results to overestimate the area calculation.

$$LMTD_{ijk} = \left[(dt_{ijk})(dt_{ijk+1})\frac{dt_{ijk}+dt_{ijk+1}}{2}\right]^{1/3}, \qquad i \in HP, j \in CP, k \in ST \quad (3.21)$$

$$LMTD_{i} = \left[(dtcu_{i})(TOUT_{i} - TIN_{CU})\frac{(dtcu_{i})+(TOUT_{i}-TIN_{CU})}{2}\right]^{1/3}, \qquad i \in HP \quad (3.22)$$

$$LMTD_{j} = \left[(dthu_{i})(TIN_{HU} - TOUT_{j})\frac{(dthu_{i})+(TIN_{HU}-TOUT_{j})}{2}\right]^{1/3}, \qquad j \in CP \quad (3.23)$$

3.1.10 Area Calculation

The area calculation is considered when heat exchanger exists. It is calculated by heat transfer divided by overall heat transfer coefficient and log-mean temperature difference as shown in Eqs. (3.24-3.26).

$$Area_{ijk} = \sum_{i \in HP} \sum_{j \in CP} \sum_{k \in ST} \left[q_{ijk} / \left(U_{ij} LMTD_{ijk}\right)\right], \qquad i \in HP, j \in CP, k \in ST \quad (3.24)$$

$$Area_{i} = \left[\frac{qcu_{i}}{(U_{i,CU} LMTD_{i})}\right], \qquad i \in HP \quad (3.25)$$

$$Area_{j} = \left[\frac{qhu_{j}}{(U_{j,HU} LMTD_{j})}\right], \qquad j \in CP \quad (3.26)$$

3.1.11 Objective Function

The objective function of HEN synthesis is to minimize TAC which calculated by exchanger matching, area calculation and utility usage illustrate in Eqs. (3.27). The objective function is divided to cost of utility usage multiply by per unit of utility cost, fixed cost for exchanger matching and area cost. Specially for area cost, it is expedited by B exponent which affect the decreasing of TAC when it less than one. The reason behind this is; more area added in one shell is cheaper when compared for using two shell which same area of heat transfer.

$$
\begin{aligned}
&Min\,TAC = \textstyle\sum_{i \in HP} CCU\, qcu_i + \sum_{j \in CP} CHU\, qhu_j \\
&+ \textstyle\sum_{i \in HP} \sum_{j \in CP} \sum_{k \in ST} CF_{ij} z_{ijk} + \sum_{j \in HP} CF_{i,CU} zcu_i + \sum_{j \in HP} CF_{j,HU} zhu_j \\
&+ \textstyle\sum_{i \in HP} \sum_{j \in CP} \sum_{k \in ST} C_{ij} Area_{ijk}^{B_{ij}} \\
&+ \textstyle\sum_{i \in HP} C_{i,CU} Area_i^{B_{i,CU}} \\
&+ \textstyle\sum_{j \in CP} C_{j,HU} Area_j^{B_{j,HU}} \qquad (3.27)
\end{aligned}
$$

3.1.12 Control Number of Splitting Streams

stage-wise superstructure will split stream in k stage when heat capacity flow rate significantly high. It is usually happening for pre-heat train of crude oil distillation unit when one large heat capacity flow rate of cold stream exchange heat with small heat capacity flow rate of hot product streams. Nonetheless, some of splitting is not in reality that splitting and combine when stage has end. Thus, control number of splitting stream equation is required and important based on each situation.

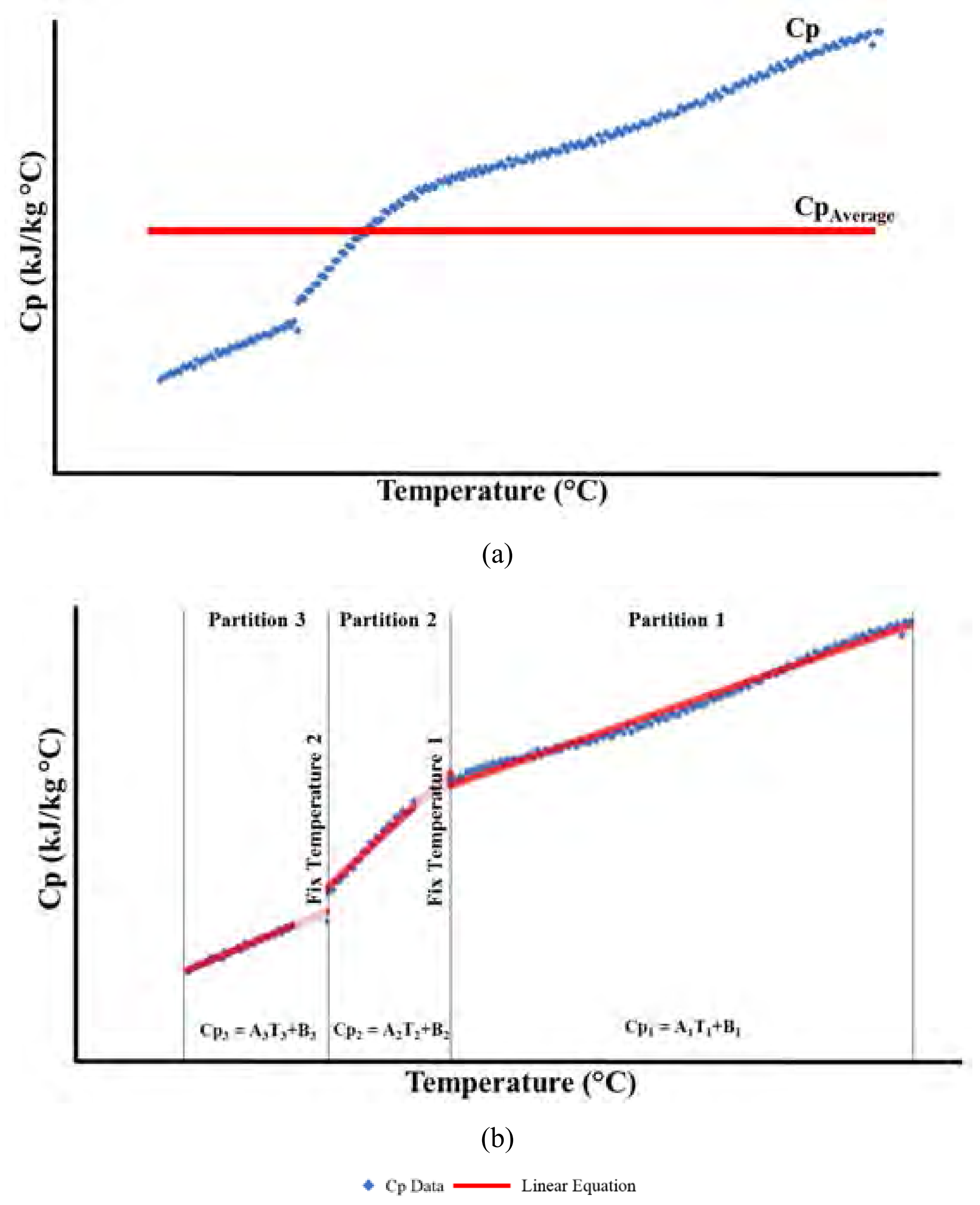
$$\textstyle\sum_j z_{ijk} \leq number\ of\ exchanger, \qquad i \in HP, k \in ST \qquad (3.28)$$

$$\textstyle\sum_i z_{ijk} \leq number\ of\ exchanger, \qquad j \in CP, k \in ST \qquad (3.29)$$

Generally, specific heat capacity is an empirical correlation in form of cubic equation as a function of temperature calculated from fitting experimental data. Unfortunately, third order term of cubic equation makes stage-wise superstructure model more non-linear and difficult to be solved by MINLP solver. One linear equation of Cp is developed to simplify the complexity of third order but it usually increases error of the variable concurrently from unfitting Cp data. To generate accurate data, linearization of cubic equation with partitioning technique is proposed. For Example, the empirical correlation between Cp and temperature is $Cp = aT^3 + bT^2 + cT^2 + d$ where a, b, c and d are specific heat capacity constant. Figure 3.2,a shows the plot between Cp and temperature of crude oil from Pro/II library. The linearization of this cubic equation using partitioning technique divides the plot into three partitions which are represented by three linear approximating equations as shown Figure 3.2,b. Partition 1 is approximated by linear equation; $Cp_1 = A_1T + B_1$.

Partition 2 is approximated by linear equation; $Cp_2 = A_2T + B_2$.

Partition 3 is approximated by linear equation; $Cp_3 = A_3T + B_3$.



(a)

(b)

Figure 3.2 Temperature-dependent Cp and linearization concept; (a) Constant and temperature-dependent Cp (b) partitioning technique concept of temperature-dependent Cp.

Three partition is selected and curative of graph is separated based on how much linear equation fit with this non-linear curve (recommended separate partition at boiling point temperature because jumping step is usually observed at this point). Note that specific heat capacity energy of reality data (area of non-linear graph) must be equal to new linear partition energy (area of linear graph). The concept of partitioning technique and equations that added into stage-wise superstructure will describe below.

3.2 Partitioning Technique of Temperature-Dependent Specific Heat Capacity

3.2.1 Overall Energy Balance

When Cp is variable, overall energy balance equation must be redefined as Eqs. (3.30) and (3.31). Temperature inlet and temperature outlet of stream is not changed but constant heat capacity flow rate is re-writing as average specific heat capacity and flow rate. Average specific heat capacity can be calculated carefully from area under curve of temperature-dependent Cp in the range between inlet and outlet temperature.

$$(thin_i - thout_i) \times FH_i \times CPH_{avg_i} = \sum_i \sum_k q_{i,j,k} + qcu_i,\ i \in HP \tag{3.30}$$

$$\left(tcout_j - tcin_j\right) \times FC_j \times CPC_{avg_j} = \sum_j \sum_k q_{i,j,k} + qhu_j,\ j \in CP \tag{3.31}$$

3.2.2 Stage Energy Balance

It actually is classified into many steps before obtaining average Cp ($\mathrm{CPHot}_{i,k}$ or $\mathrm{CPCold}_{j,k}$) for each stage. Temperature is normally defined as variable for stage-wise superstructure but specific heat capacity is varied with temperature. They are in function of each other. However, concept of new technique is really simple that will describe one by one below but it must be noted that average Cp is calculated from all of these new equations. Hence, stage energy balance can be defined as Eqs. (3.32-3.35).

3.2.2.1 Process stream equations

$$(th_{i,k} - th_{i,k+1}) \times FH_i \times CPH_{i,k} = \sum_j q_{i,j,k}, \qquad i \in HP, k \in ST \qquad (3.32)$$

$$(tc_{j,k} - tc_{j,k+1}) \times FC_j \times CPC_{j,k} = \sum_i q_{i,j,k}, \qquad j \in CP, k \in ST \qquad (3.33)$$

3.2.2.2 Utility streams equations

$$(th_{i,NOK+1} - thout_i) \times FH_i \times CPHL_{i,k} = qcu_i, \qquad i \in HP \qquad (3.34)$$

$$(tcout_j - tc_{j,1}) \times FC_j \times CPCF_{j,k} = qhu_j, \qquad j \in CP \qquad (3.35)$$

All of these equations below are developed to calculate average Cp for each stage. They are categorized into 2 groups; process stream equation for process stream Cp calculation and utility streams equation for hot/cold utility streams Cp calculation. Process stream equation and utility stream equation are related that energy remain from HEN (process stream) are sufficient by utility. This energy must be estimated by group of utility stream equation. Hence, this journal will classify them into process stream Cp and utility stream Cp which actually similar concept.

3.2.3 Activate Cp Equation

As temperature is continuous variable, it can be located in range of inlet/outlet temperature of that stream but Cp is also in function of temperature. It should be note that temperature variable ($th_{i,k}$ and $tc_{j,k}$) presents in stage called stage temperature and partitioning technique. To define used partition, "Activated Cp Equation" is formulated to Eqs. (3.36-3.47). They represent the used partition by showing positive number when that partition is in used, else negative.

3.2.3.1 Process stream equations

$$ActiH_{1,i,k} = th_{i,k} - \max(th_{i,k+1}, FixtempH_{1,i}),$$
$$i \in HP, k \in ST \qquad (3.36)$$

$$ActiH_{n,i,k} = \min(th_{i,k}, FixtempH_{n-1,i}) - \max(th_{i,k+1}, FixtempH_{n,i}),$$
$$i \in HP, k \in ST, n \in PT \qquad (3.37)$$

$$ActiH_{NOP,i,k} = \min(th_{i,k}, FixtempH_{NOP-1,i}) - th_{i,k+1},$$

$$i \in HP, k \in ST \tag{3.38}$$

$$ActiC_{1,j,k} = tc_{j,k} - \max(tc_{j,k+1}, FixtempC_{1,j}),$$
$$j \in CP, k \in ST \tag{3.39}$$

$$ActiC_{n,j,k} = \min(tc_{j,k}, FixtempC_{n-1,j}) - \max(tc_{j,k+1}, FixtempC_{n,j}),$$
$$j \in CP, k \in ST, n \in PT \tag{3.40}$$

$$ActiC_{NOP,j,k} = \min(tc_{j,k}, FixtempC_{NOP-1,j}) - tc_{j,k+1},$$
$$j \in CP, k \in ST \tag{3.41}$$

3.2.3.2 Utility streams equations

$$ActiHL_{1,i,NOK+1} = th_{i,NOK+1} - FixtempH_{1,i},$$
$$i \in HP \tag{3.42}$$

$$ActiHL_{n,i,NOK+1} = \min(th_{i,NOK+1}, FixtempH_{n-1,i}) - FixtempH_{n,i},$$
$$i \in HP, n \in PT \tag{3.43}$$

$$ActiHL_{NOP,i,NOK+1} = \min(th_{i,NOK+1}, FixtempH_{NOP-1,i}) - thout_i,$$
$$i \in HP \tag{3.44}$$

$$ActiCF_{1,j,1} = tcout_j - \max(tc_{j,1}, FixtempC_{1,j}),$$
$$j \in CP \tag{3.45}$$

$$ActiCF_{n,j,1} = FixtempC_{n-1,j} - \max(tc_{j,1}, FixtempC_{n,j}),$$
$$j \in CP, n \in PT \tag{3.46}$$

$$ActiCF_{NOP,j,1} = FixtempC_{NOP-1,j} - tc_{j,1}, \quad j \in CP \tag{3.47}$$

3.2.4 Temperature Calculation

These equations calculate average mean temperature for each partition at stage *k*. They have three equations represent for first partition to final partition (*NOP*) (Eqs. (3.48-3.59)). New parameters (*FixTemp*$_n$) are specified logically to divide non-linear cubic equation into linear equation and they are used to calculate in all of these equations. It may be observed that temperature calculation may be negative value but activate variable (ActiH$_{i,k}$ or ActiC$_{j,k}$) from activate equation (Eqs. (3.36-3.47)) will set the used value.

3.2.4.1 Process stream equations

$$THcal_{1,i,k} = \frac{th_{i,k} + \max\left(th_{i,k+1}, FixtempH_{1,i}\right)}{2}, \qquad i \in HP, k \in ST \tag{3.48}$$

$$THcal_{n,i,k} = \frac{\min\left(th_{i,k}, FixtempH_{n-1,i}\right) + \max\left(th_{i,k+1}, FixtempH_{n,i}\right)}{2}, \qquad i \in HP, k \in ST, n \in PT \tag{3.49}$$

$$THcal_{NOP,i,k} = \frac{\min\left(th_{i,k}, FixtempH_{NOP-1,i}\right) + th_{i,k+1}}{2}, \qquad i \in HP, k \in ST \tag{3.50}$$

$$TCcal_{1,j,k} = \frac{tc_{j,k} + \max(tc_{j,k+1}, FixtempC_{1,j})}{2}, \qquad j \in CP, k \in ST \tag{3.51}$$

$$TCcal_{n,j,k} = \frac{\min\left(tc_{j,k}, FixtempC_{n-1,j}\right) + \max\left(tc_{j,k+1}, FixtempC_{n,j}\right)}{2}, \qquad j \in CP, k \in ST, n \in PT \tag{3.52}$$

$$TCcal_{NOP,j,k} = \frac{\min\left(tc_{j,k}, FixtempC_{NOP-1,j}\right) + tc_{j,k+1}}{2}, \qquad j \in CP, k \in ST \tag{3.53}$$

3.2.4.2 Utility streams equations

$$THcalL_{1,i,NOK+1} = \frac{th_{i,NOK+1} + FixtempH_{1,i}}{2}, \qquad i \in HP \tag{3.54}$$

$$THcalL_{n,i,NOK+1} = \frac{\min\left(th_{i,NOK+1}, FixtempH_{n-1,i}\right) + FixtempH_{n,i}}{2}, \qquad i \in HP, n \in PT \tag{3.55}$$

$$THcalL_{NOP,i,NOK+1} = \frac{\min\left(th_{i,NOK+1}, FixtempH_{NOP-1,i}\right) + thout_i}{2}, \qquad i \in HP \tag{3.56}$$

$$TCcalF_{1,j,1} = \frac{tcout_j + \max(tc_{j,1}, FixtempC_{1,j})}{2}, \qquad j \in CP \tag{3.57}$$

$$TCcalF_{n,j,1} = \frac{FixtempC_{n-1,j} + \max(tc_{j,1}, FixtempC_{n,j})}{2}, \qquad j \in CP, n \in PT \tag{3.58}$$

$$TCcalF_{NOP,j,1} = \frac{FixtempC_{NOP-1,j} + tc_{j,1}}{2}, \qquad j \in CP \tag{3.59}$$

3.2.5 Cp Calculation for each partition

Cp is normally calculated by average mean temperature calculated from previous section and they are set to calculate by linear relationship from each partition (Eqs. (3.60-3.63)).

3.2.5.1 Process stream equations

$$CPH_{n,i,k} = AH_{n,i} \times THcal_{n,i,k} + BH_{n,i}, \qquad i \in HP, k \in ST, n \in PT \tag{3.60}$$

$$CPC_{n,j,k} = AC_{n,j} \times TCcal_{n,j,k} + BC_{n,j}, \qquad j \in CP, k \in ST, n \in PT \tag{3.61}$$

3.2.5.2 Utility streams equations

$$CPHL_{n,i,NOK+1} = AH_{n,i} \times THcalL_{n,i,NOK+1} + BH_{n,i}, \qquad i \in HP \tag{3.62}$$

$$CPCF_{n,j,1} = AC_{n,j} \times TCcalF_{n,j,1} + BC_{n,j}, \qquad j \in CP \tag{3.63}$$

3.2.6 Average Cp Calculation for stage

All of new equation above will be used in these equations (Eqs. (3.64-3.67)). Objective of these equations is to assign average Cp for using in each stage by concept of weighted average calculation. For more understanding, example of partitioning technique for Cp calculation is described below.

3.2.6.1 Process stream equations

$$CPHot_{i,k} = \sum_{n=1}^{NOP}\left[CPH_{n,i,k} \times \max\left(0, ActiH_{n,i,k}\right)\right] / \sum_{n=1}^{NOP}\left[\max\left(0, ActiH_{n,i,k}\right)\right] \qquad i \in HP, k \in ST \tag{3.64}$$

$$CPCold_{j,k} = \sum_{n=1}^{NOP}\left[CPC_{n,j,k} \times \max(0, ActiC_{n,j,k})\right] / \sum_{n=1}^{NOP}\left[\max\left(0, ActiC_{n,j,k}\right)\right] \qquad j \in CP, k \in ST \tag{3.65}$$

3.2.6.2 Utility streams equations

$$CPHotL_{i,NOK+1} = \sum_{n=1}^{NOP}\left[CPHL_{n,i,NOK+1} \times \max(0, ActiHL_{n,i,NOK+1}\right] / \sum_{n=1}^{NOP}\left[\max\left(0, ActiHL_{n,i,NOK+1}\right)\right] \quad i \in HP \qquad (3.66)$$

$$CPColdF_{j,1} = \sum_{n=1}^{NOP}\left[CPCF_{n,j,1} \times \max(0, ActiCF_{n,j,1})\right] / \sum_{n=1}^{NOP}\left[\max\left(0, ActiCF_{n,j,1}\right)\right] \quad j \in CP \qquad (3.67)$$

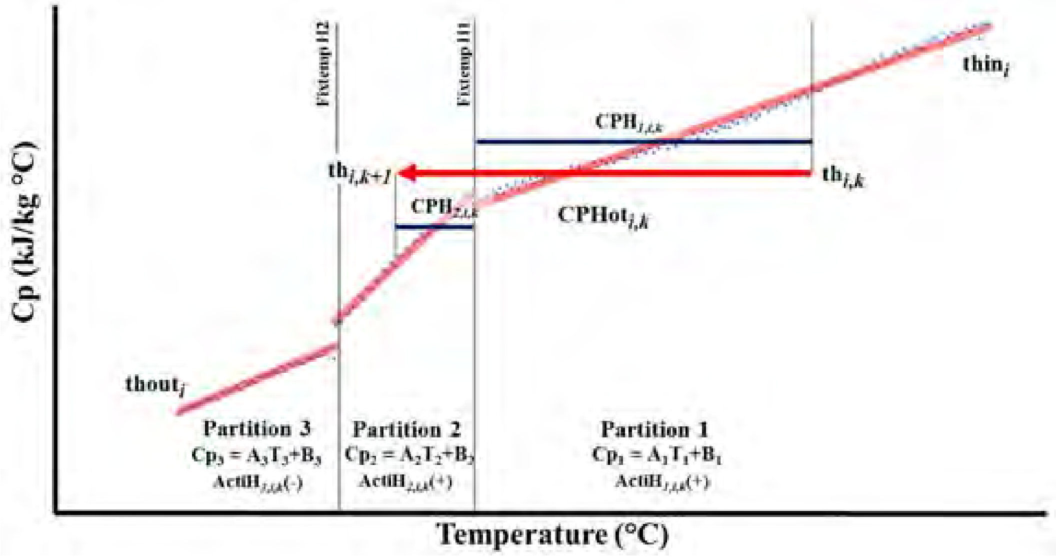


Figure 3.3 Example of partitioning technique for hot process stream.

Figure 3.3 shows example of Cp calculation for hot process stream *i* in stage *k* by partitioning technique. Suppose that a hot process stream has supply temperature ($th_{i,k}$) and target temperature ($th_{i,k+1}$) varied in the temperature range from partition 1 to partition 2, as shown in Figure 3.3. First, $ActiH_{n,i,k}$ of each process temperature ($th_{i,k}$ to $th_{i,k+1}$) occupied partition becomes positive value and temperature difference of the partition is calculated. They show that enthalpy; $ActiH_{1,i,k}$ ($FixtempH_1$ - $th_{i,k+1}$), and enthalpy; $ActiH_{2,i,k}$ ($th_{i,k}$ - $FixtempH_1$), are positive except enthalpy; $ActiH_{3,i,k}$ ($FixtempH_2$ - $th_{i,k+1}$), is negative value. Maximum operation force negative value to zero value (enthalpy of partition 3). Next, $CPH_{n,i,k}$ are determined by average mean temperature calculation between $th_{i,k}$ and $th_{i,k+1}$. Finally, $CPHot_{i,k}$ is calculated from $CPH_{1,i,k}$ and $CPH_{2,i,k}$ by weighted average calculation concept as shown in Figure 3.3. The advantages of our technique are linear equation fitting very well with data and using small number of partitions (only 3 partition is enough for this case study). Moreover, average Cp calculation from each stage have high accuracy and it is free to calculate without any fixed Cp data from user.

3.3 Retrofit

Objective of heat exchanger synthesis is to minimize TAC by trading between number of heat exchanger, area of heat exchanger and utility consumption but retrofit objective is to maximize net present value (NPV) by adding new heat exchanger or increasing area of heat exchanger to reduce energy consumption. Thus, only objective function is changed in stage-wise superstructure and new concept of temperature-dependent Cp can be used. The model will design HEN and new parameter of exiting heat exchanger, area of existing heat exchanger and based utility consumption must be specified. Maximum operation is used to indicate new heat exchanger and exiting heat exchanger. Only new exchanger will calculate fixed cost and adding area of heat exchanger is calculated for old heat exchanger. New objective function is saving energy consumption subtracted by capital cost as shown in Eqs. (3.68).

$$
\begin{aligned}
Max\ NPV = \sum_{y=1}^{n} & \frac{\sum_i CCU(qcuold_i - qcu_i) + \sum_j CHU(qhuold_j - qhu_j)_y}{(1+i)^y} \\
& - \sum_i \sum_j \sum_k CF_{ij} \times \max\left(0, \left(z_{ijk} - zold_{ijk}\right)\right) \\
& - \sum_i CF_{i,CU} \times \max(0, (zcu_i - zcuold_i)) \\
& - \sum_j CF_{j,HU} \times \max(0, (zhu_j - zhuold_j)) \\
& - \sum_i \sum_j \sum_k C_{ij} \max(0, (Area_{ijk} - Areaold_{ijk}))^{B_{ij}} \\
& - \sum_i C_{i,CU} \max(0, (Area_i - AreaCold_i))^{B_{i,CU}} \\
& - \sum_j C_{j,HU} \max(0, (Area_j - AreaHold_j))^{B_{j,HU}} \qquad (3.68)
\end{aligned}
$$

It must be noted that to avoid any inappropriate matching, the lower bound of existing exchanger heat transfer is specified. It is used to force retrofit model to design new HEN based on previous network.

CHAPTER IV

RESULTS AND DISCUSSION

Four cases are optimized by general algebraic modeling system program (GAMS) version 24.2 using CPLEX as mixed integer linear programming (MILP) solver, CONOPT as non-linear programming (NLP) solver and DICOPT as mixed integer non-linear programming (MINLP) testing in desktop computer (Intel® Core™ i7-4720HQ CPU at 2.6 GHz, 16 GB of RAM, 64-bit Windows 8). To validate new technique, Pro/II (version 9.1) library model is used to show its performance of HEN synthesis and HEN retrofit. Two examples from Kim and Bagajewicz (2017) using ordinary non-linear cubic equation technique are applied for comparison with our model using partitioning technique.

4.1 Case Study 1

This example aims to validate crude preheat train designed by new model with commercial software program (Pro/II simulation). It contains five hot product streams, one cold crude oil stream and stage model of five stages to design HEN. The constraints to design HEN for this case are; allowing three or less stream splitting per stage on cold stream and prohibiting stream splitting on every hot stream. EMAT is set at 10 °C. Cp of each stream in crude preheat train is from Pro/II simulation which is separated into constant Cp and variable Cp as shown in Figure 3.2. Diamond blue dot represents experimental data of Cp from Pro/II and red line represent linear line fitting experimental data of Cp. All process streams data including supply and target temperature are shown in Table 4.1.

Table 4.1 Process streams data of case study 1.

Streams	T_{in} (°C)	T_{out} (°C)	F (kg/s)	h (kW/m^2 °C)	Cost ($/kW y)
H1	43.33	25	37.38	1	-
H2	200.04	25	21.88	1	-
H3	272.79	25	21.76	1	-

H4	342.72	50	26.24	1	-
H5	370.72	50	87.06	1	-
C1	50	376.80	194.24	1	-
HU	500 (steam)	500 (condensate)	-	1	60
CU	10	15	-	1	5

Annual investment cost ($/y) = 3,460 + 300 × (Area; m^2) for all exchangers (Pan *et al.* (2013))

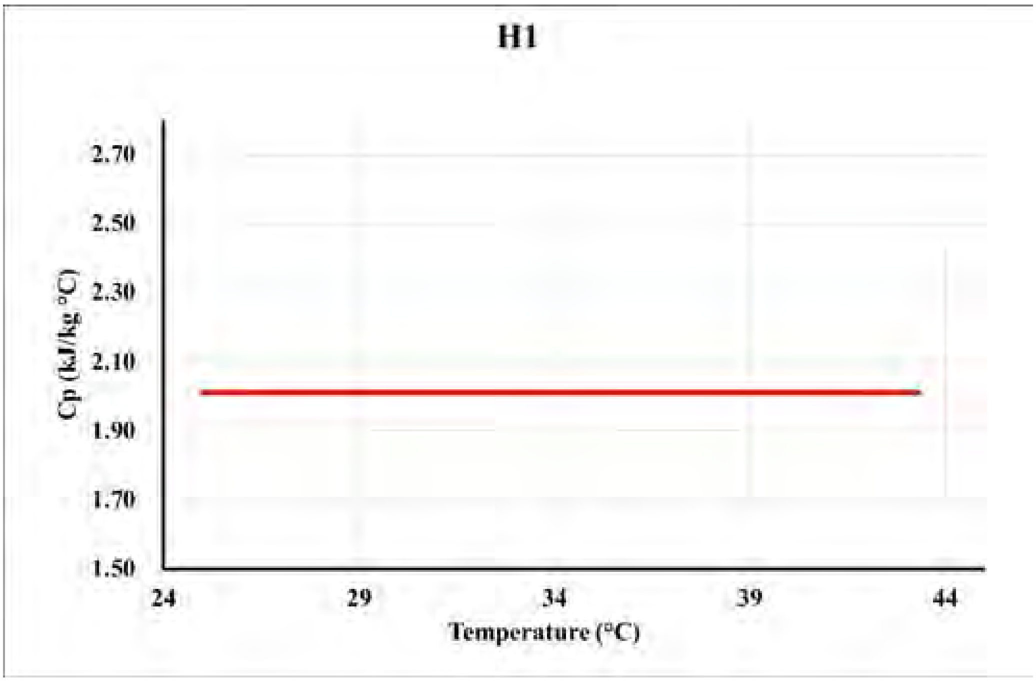
The validation steps are divided into Constant Cp, 1 partition divided, 2 partitions divided, 3 partitions divided and 5 partitions divided which show the performance of new model and percent accuracy when increasing number of partitions. For Cp linearization, Cp data as function of temperature is partitioned into five or fewer temperature intervals based on each case that will show further and linear equation with parameters; A_n and B_n, fitting Cp data at each temperature interval of each stream. GAMS result; heat duty of each heat exchanger and HEN are set into Pro/II simulation to validate partitioning technique.

4.1.1 Constant Cp

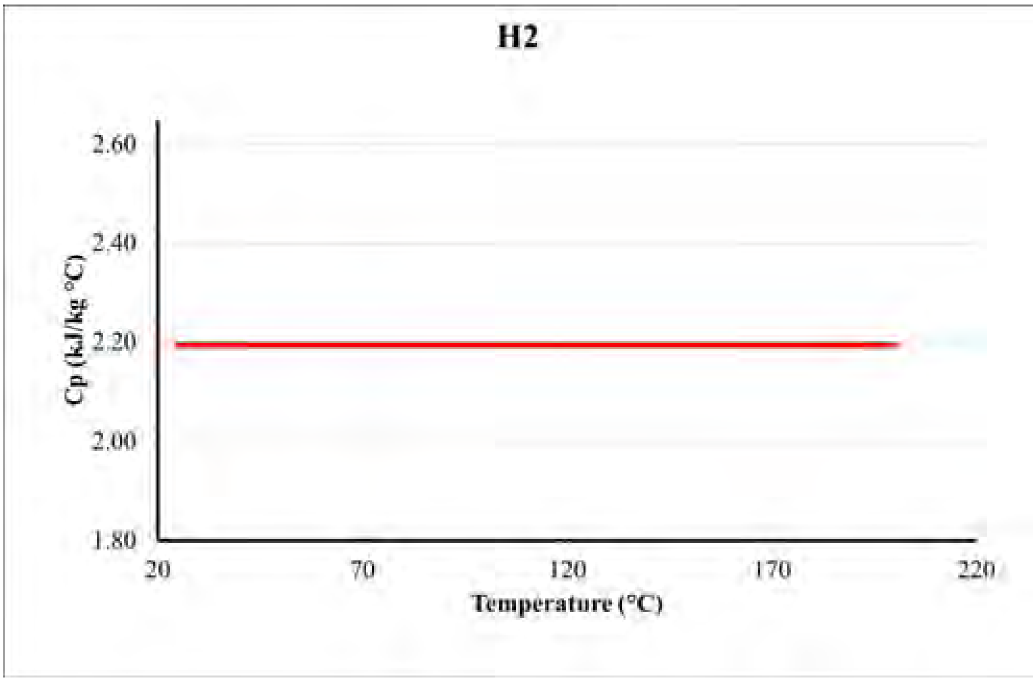
First case is constant Cp that represent for normal stage-wise superstructure. The Cp value is set as constant and shown in Table 4.2. Cp graph is shown in Figure 4.1 and it can be observed that Cp is constant along the supply and target temperature. The base case of crude preheat train by Pro/II software is shown in Figure 4.2.

Table 4.2 Specific heat capacity of constant Cp.

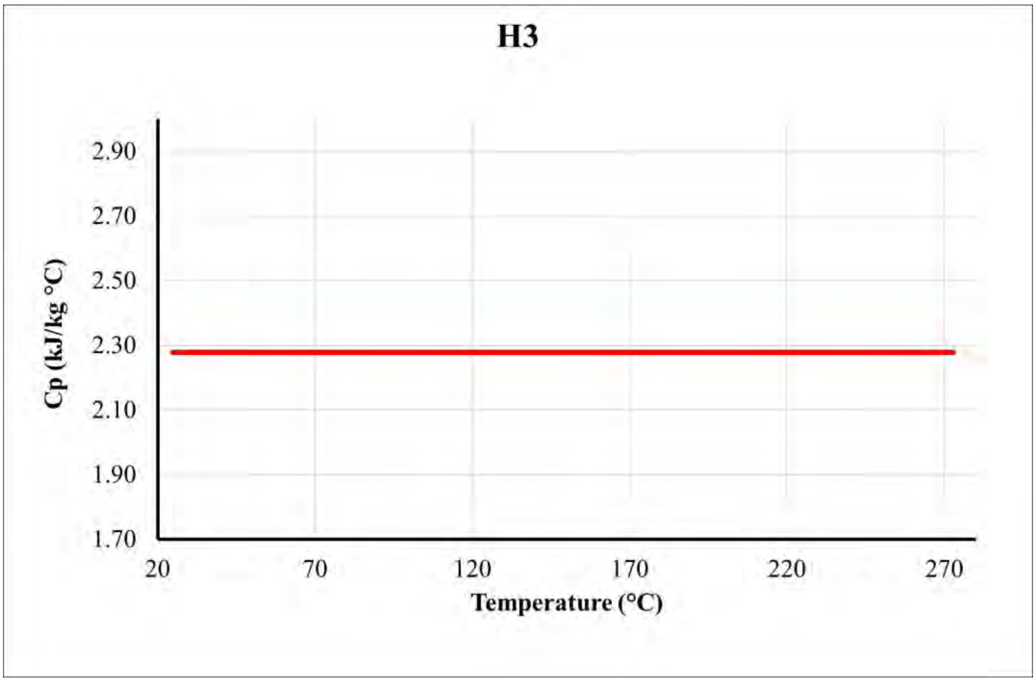
Streams	$Cp_{constant}$ (kJ/kg °C)
H1	2.0115729
H2	2.1982975
H3	2.2798556
H4	2.4213852
H5	2.4131606
C1	2.7665094


H1
2.70
2.50
2.30
2.10
1.90
1.70
1.50
Cp (kJ/kg °C)
24
29
34
39
44
Temperature (°C)

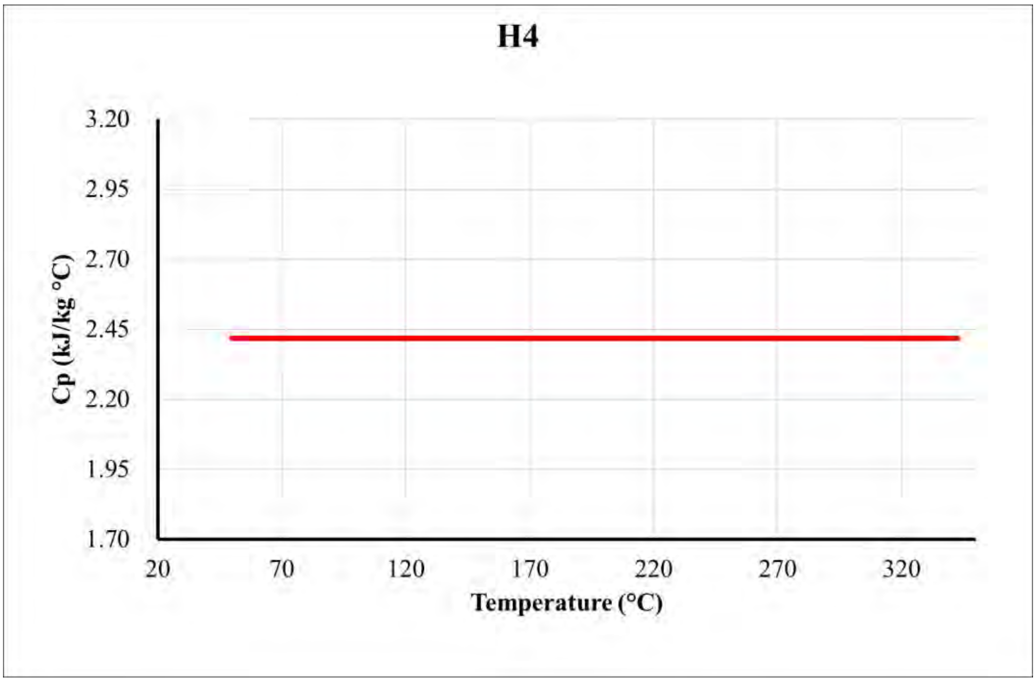
(a)


H2
2.60
2.40
2.20
2.00
1.80
Cp (kJ/kg °C)
20
70
120
170
220
Temperature (°C)

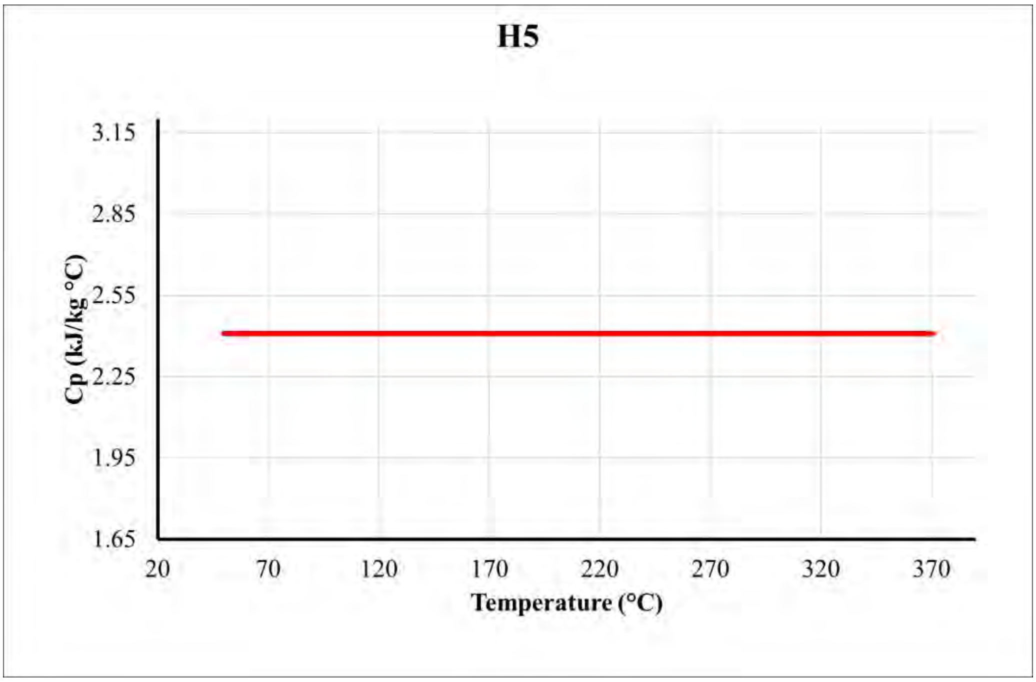
(b)


H3
2.90
2.70
2.50
2.30
2.10
1.90
1.70
Cp (kJ/kg °C)
20
70
120
170
220
270
Temperature (°C)

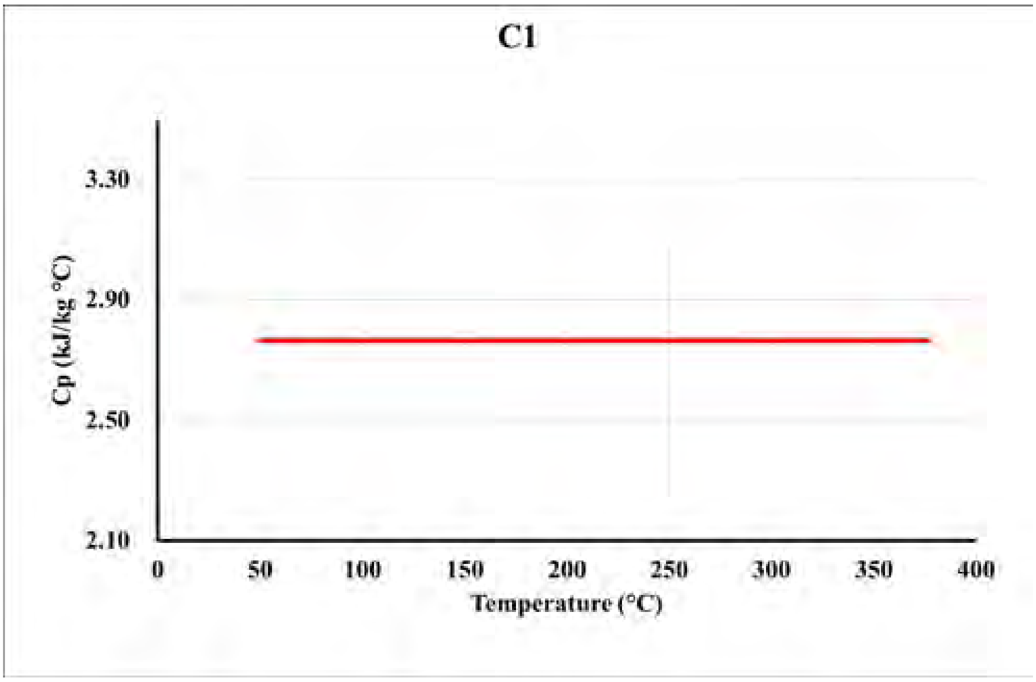
(c)


H4
3.20
2.95
2.70
2.45
2.20
1.95
1.70
Cp (kJ/kg °C)
20
70
120
170
220
270
320
Temperature (°C)

(d)



(e)



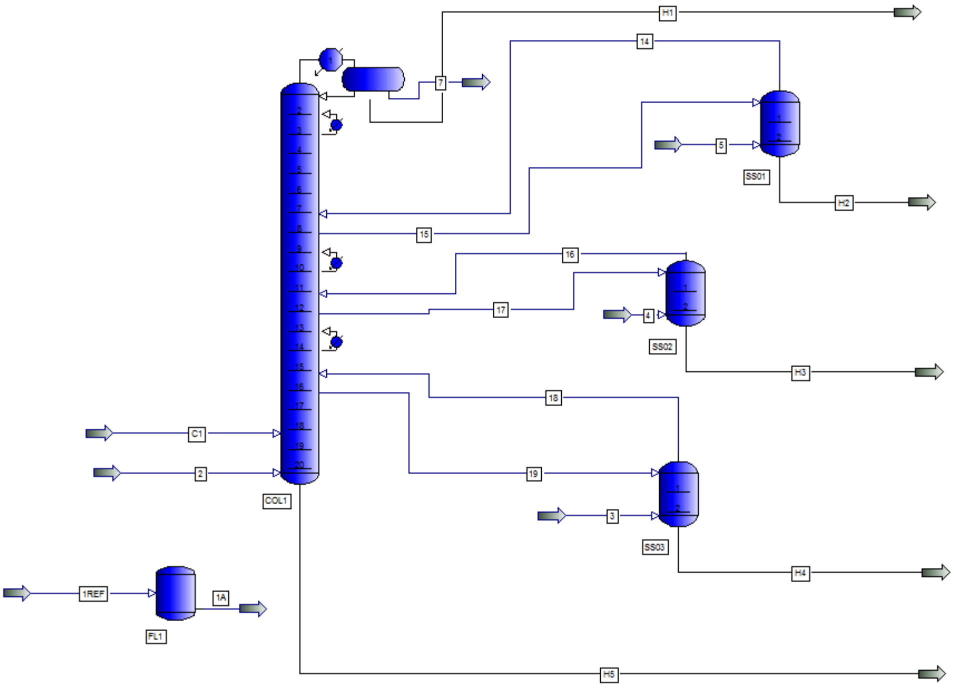
(f)

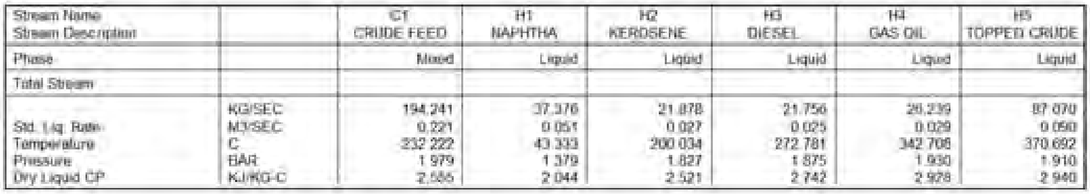
Figure 4.1 Constant Cp graph of case study 1; (a-e) represent for hot stream (H1-H5), respectively. (f) represent for cold stream.

HEN of constant heat capacity assumption is shown in Figure 4.3 and Figure 4.4 for GAMS result and Pro/II results, respectively. The design from Pro/II simulation is shown in Figure 4.5. Grid diagram show the Cp of each stream is set to constant in GAMS results but Pro/II results show that Cp is varied along the stage. Results data of constant heat capacity is shown in Table 4.3 and Table 4.4. It can be observed that percent error of utility consumption in constant Cp case is not high because of heat balance theory (Cp data graph area is equal to Cp average graph area illustrate in Figure 3.2,a) but constant Cp affect temperature outlet of heat exchanger results to increasing percent error of heat exchanger area. For example, observing from E6 in Table 4.4 that hot temperature outlet from heat exchanger is obviously changed when fixed heat duty E6 because Cp that calculate in Pro/II (2.08 kJ/kg °C) is really different from constant Cp (2.28 kJ/kg °C) set by user. Thus, outlet temperature calculation will change from 69.04 °C in constant Cp to 77.52 °C in variable Cp and it can be compared to Pro/II result that increasing percent error of area calculation to 30.79 %. Therefore, this error is unacceptable in reality design that should be specified specific heat capacity as function of temperature.

Table 4.3 Duty data comparison between GAMS and Pro/II of constant Cp.

Heat Exchanger	GAMS Duty (kW)	Pro/II Duty (kW)	Percent Error (%)
E1	8,469.98	8,469.98	-
E2	15,882.16	15,882.16	-
E3	59,939.83	59,939.83	-
E4	6,148.81	6,148.81	-
E5	1,176.17	1,176.17	-
E6	1,626.79	1,626.79	-
CU1	1,378.39	1,378.60	0.02
CU2	2,252.00	2,245.70	0.28
CU3	2,182.15	2,168.60	0.62
CU4	1,522.28	1,495.40	1.80
CU5	7,367.34	7,362.20	0.07
HU1	82,109.79	82,109.90	0.00





Stream Name Stream Description		C1 CRUDE FEED	H1 NAPHTHA	H2 KEROSENE	H3 DIESEL	H4 GAS OIL	H5 TOPPED CRUDE
Phase		Mixed	Liquid	Liquid	Liquid	Liquid	Liquid
Total Stream							
	KG/SEC	194.241	37.376	21.878	21.756	28.239	87.070
Std. Liq. Rate	M3/SEC	0.221	0.051	0.027	0.025	0.029	0.090
Temperature	C	232.222	43.333	200.034	272.781	342.708	370.692
Pressure	BAR	1.979	1.379	1.827	1.875	1.930	1.910
Dry Liquid CP	KJ/KG-C	2.586	2.044	2.521	2.742	2.978	2.940

Figure 4.2 Crude preheat train case from Pro/II library.

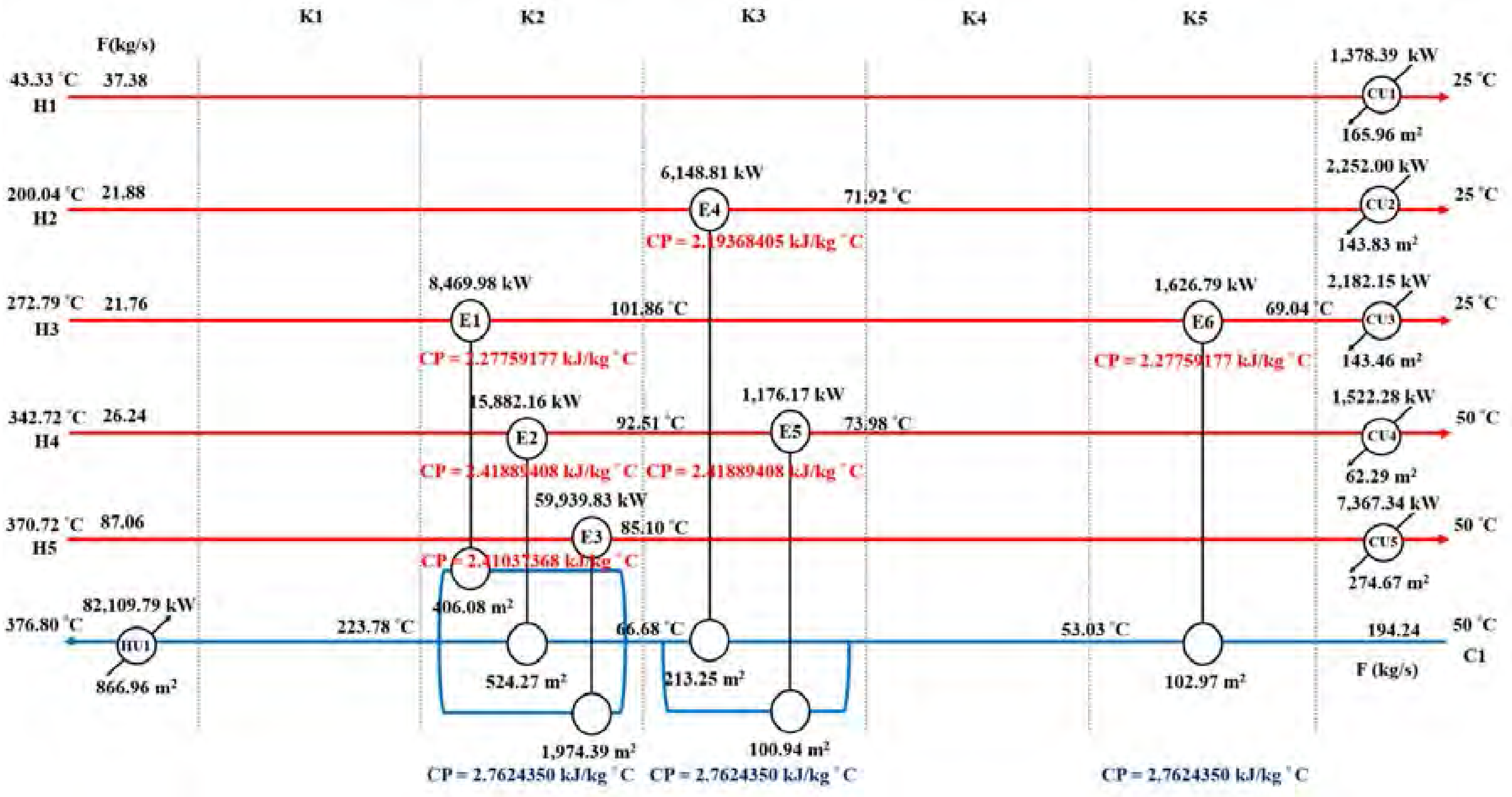


Figure 4.3 HEN from constant Cp GAMS model.

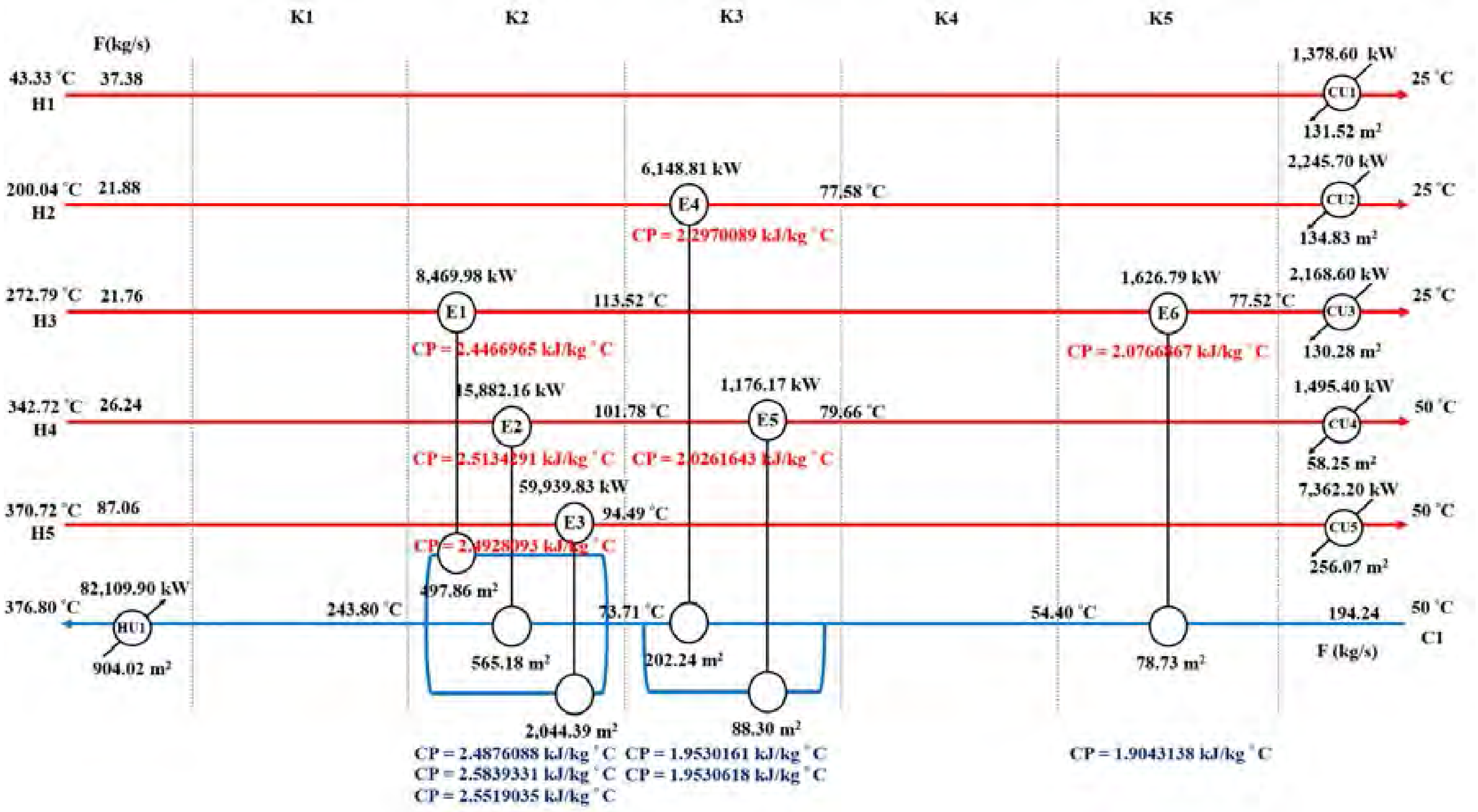
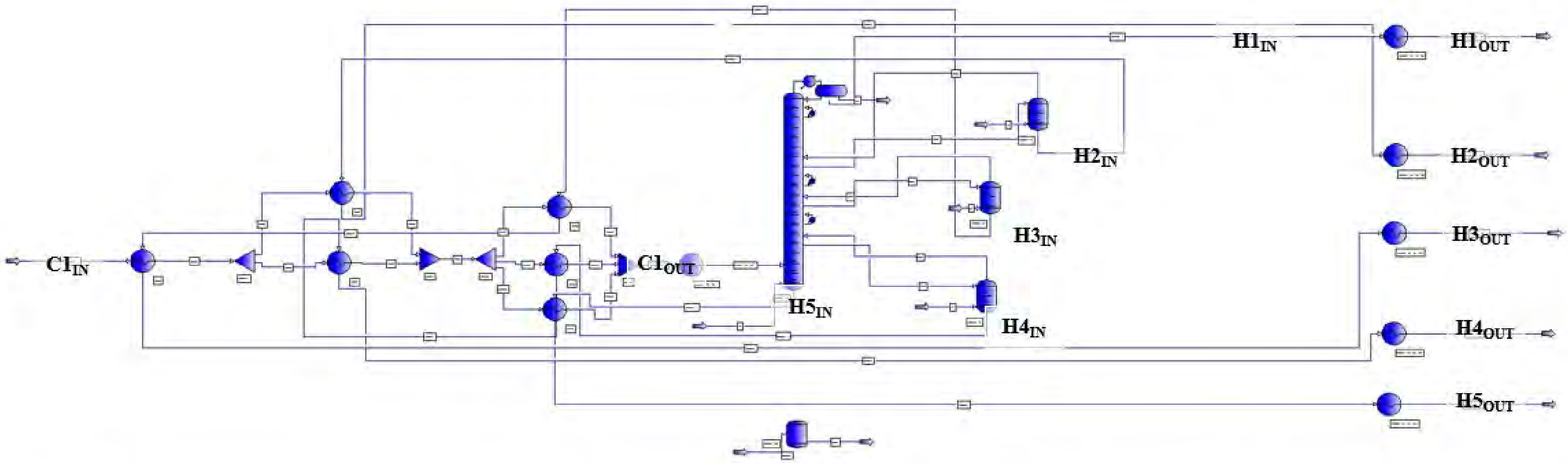


Figure 4.4 Validation of HEN from constant Cp GAMS model by Pro/II simulation.



Stream Name Stream Description		C1IN CRUDE FEED	H1IN NAPHTHA	H2IN KEROSENE	H3IN DIESEL	H4IN GAS OIL	H5IN TOPPED CRUDE
Phase		Liquid	Liquid	Liquid	Liquid	Liquid	Liquid
Total Stream							
	KG/HR	699266.768	134549.849	78758.853	78314.685	94452.107	313470.526
Std. Liq. Rate	M3/HR	794.933	182.800	95.452	90.078	104.468	322.416
Temperature	C	50.000	43.333	200.027	272.769	342.683	370.657
Pressure	BAR	1.979	1.379	1.827	1.875	1.930	1.910
Dry Liquid CP	KJ/KG-C	1.895	2.044	2.521	2.742	2.928	2.940

Stream Name Stream Description		C1OUT	H1OUT	H2OUT	H3OUT	H4OUT	H5OUT
Phase		Mixed	Mixed	Mixed	Mixed	Mixed	Mixed
Total Stream							
	KG/HR	699266.768	134549.849	78758.853	78314.685	94452.107	313470.526
Std. Liq. Rate	M3/HR	794.933	182.800	95.452	90.078	104.468	322.416
Temperature	C	243.803	25.000	25.000	25.000	50.000	50.000
Pressure	BAR	1.979	1.379	1.827	1.875	1.930	1.910
Dry Liquid CP	KJ/KG-C	2.591	1.985	1.842	1.787	1.861	1.808

Figure 4.5 HEN from constant Cp case study by Pro/II simulation.

Table 4.4 Area data comparison between GAMS and Pro/II of constant Cp.

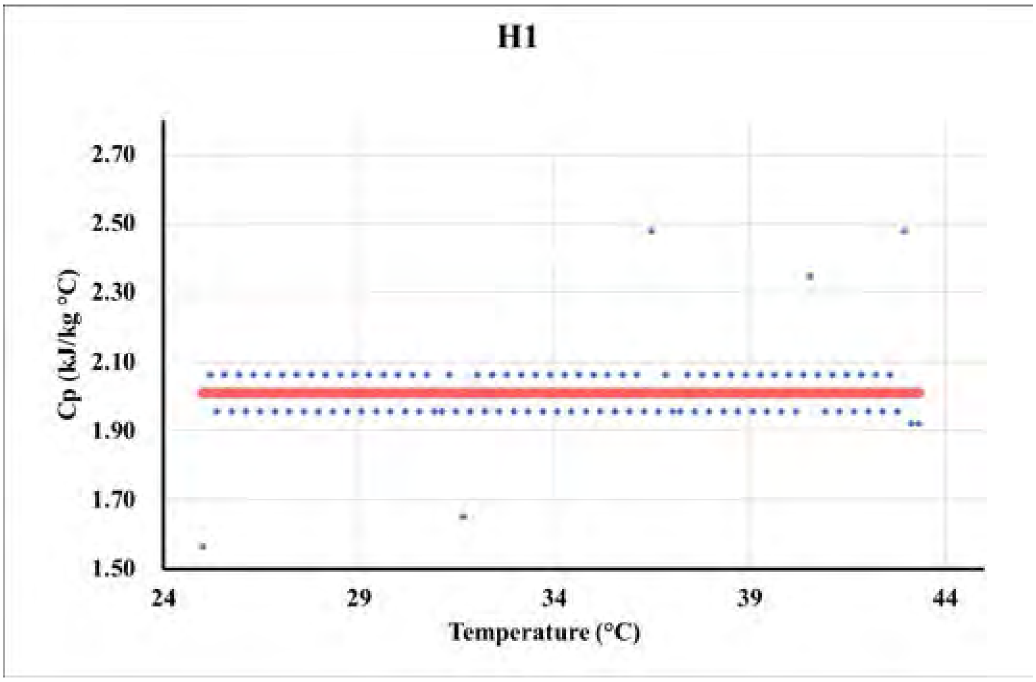
Heat Exchanger	GAMS Area (m^2)	Pro/II Area (m^2)	Percent Error (%)
E1	406.08	497.86	18.44
E2	524.27	565.18	7.24
E3	1,974.39	2,044.39	3.42
E4	213.25	202.24	5.45
E5	100.94	88.30	14.32
E6	102.97	78.73	30.79
CU1	165.96	131.52	26.19
CU2	143.83	134.83	6.67
CU3	143.76	130.28	10.34
CU4	62.29	58.25	6.94
CU5	274.67	256.07	7.26
HU1	866.96	904.02	4.10

4.1.2 1 Partition

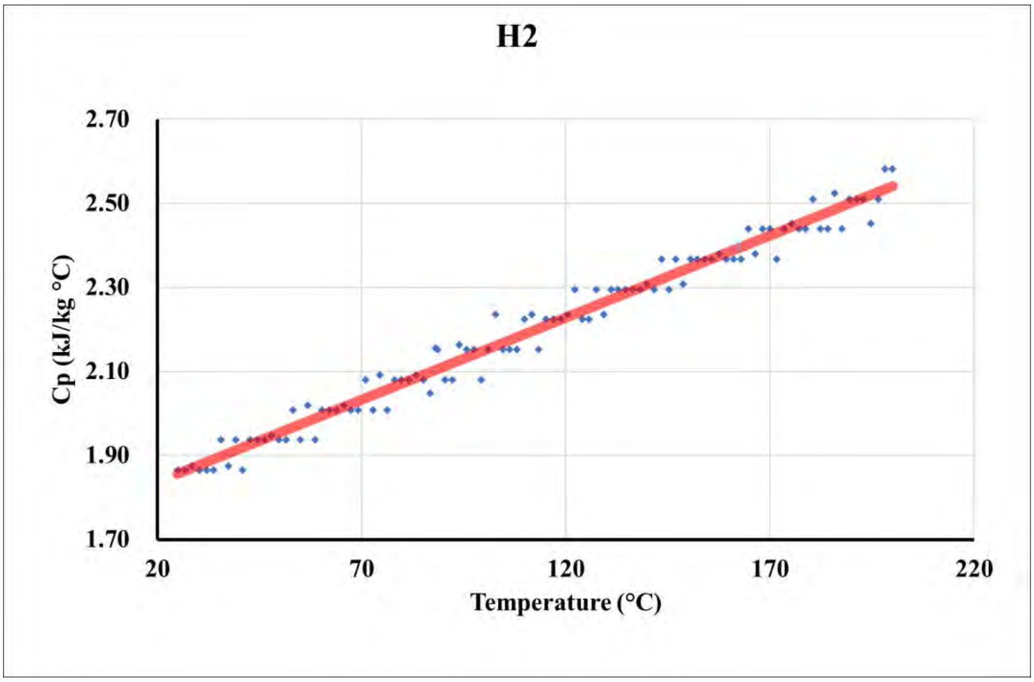
Next case is the new technique of this thesis (partitioning technique) that using 1 partition or one linear equation represent for Cp variable. The Cp linearization of case study 1 parameters is shown in Table 4.5. Cp graph shown in Figure 4.6 and it can be observed that stream H2 to H5 is fit well with Cp data but stream C1 is not enough for only one linear equation (1 partition). However, high range of supply and target temperature lead to high R^2 but it is actually not fit well with the data.

Table 4.5 Specific heat capacity linearization of case study 1 by 1 partition.

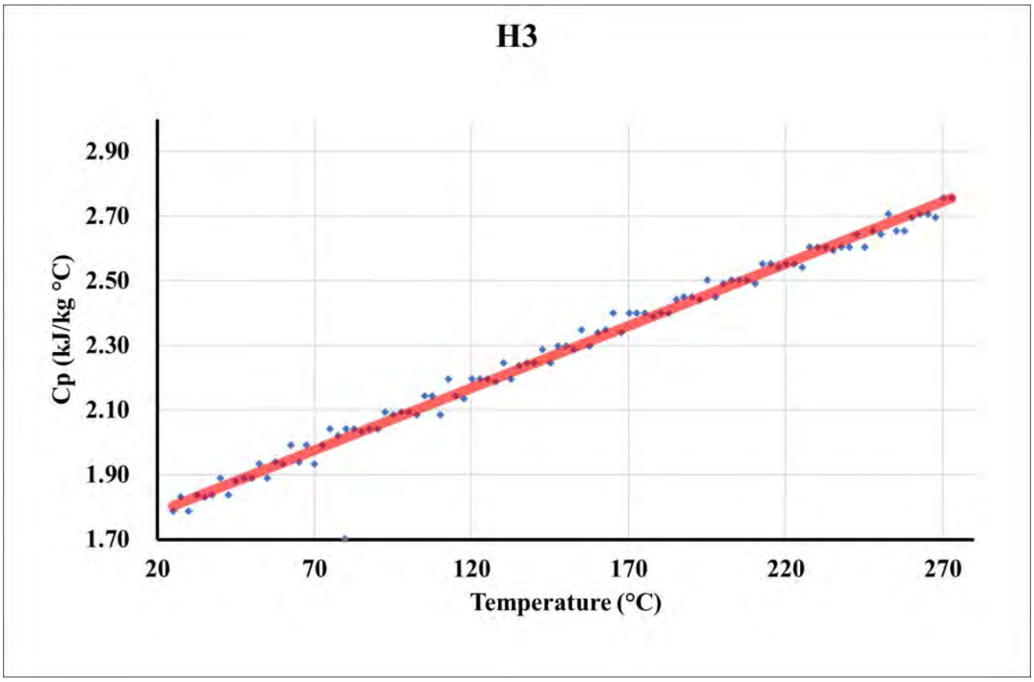
Streams	Partition Number (n)	$Cp_n = A_n \times T_{mean} + B_n$		R^2	$Cp_{average}$ (kJ/kg °C)
		A_n	B_n		
H1	1	0	2.0115729	-	2.0115729
H2	1	0.0039128	1.7580281	0.98	2.1982975
H3	1	0.0038395	1.7081741	0.98	2.2798556
H4	1	0.0036817	1.6984336	0.99	2.4213852
H5	1	0.0035960	1.6567033	0.99	2.4131606
C1	1	0.0047104	1.7613130	0.95	2.7665094


H1
2.70
2.50
2.30
2.10
1.90
1.70
1.50
Cp (kJ/kg °C)
24
29
34
39
44
Temperature (°C)

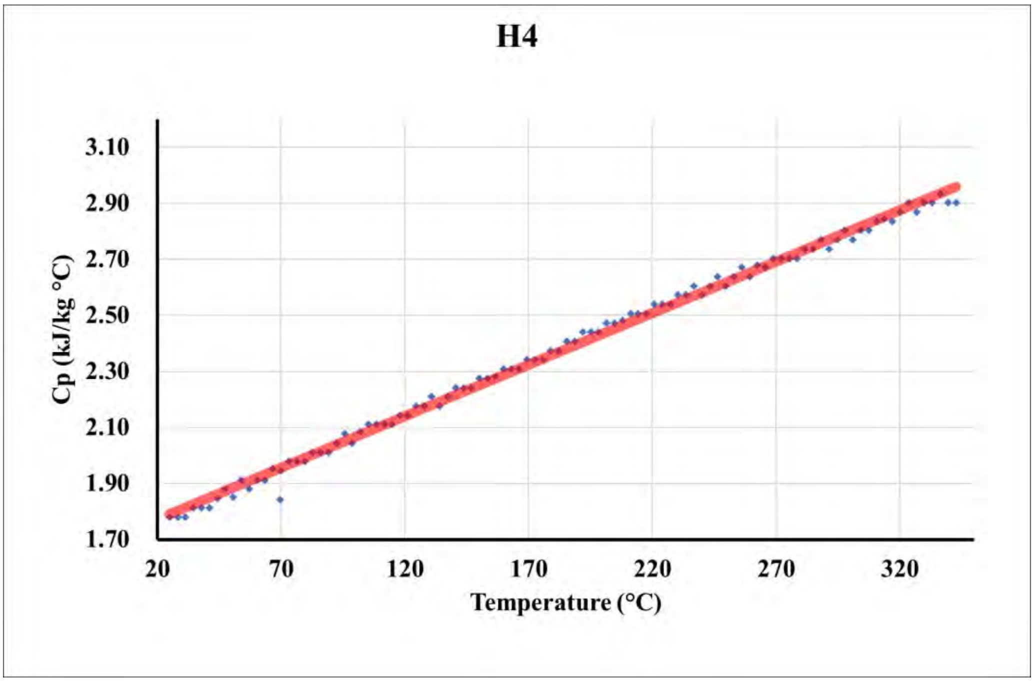
(a)


H2
2.70
2.50
2.30
2.10
1.90
1.70
Cp (kJ/kg °C)
20
70
120
170
220
Temperature (°C)

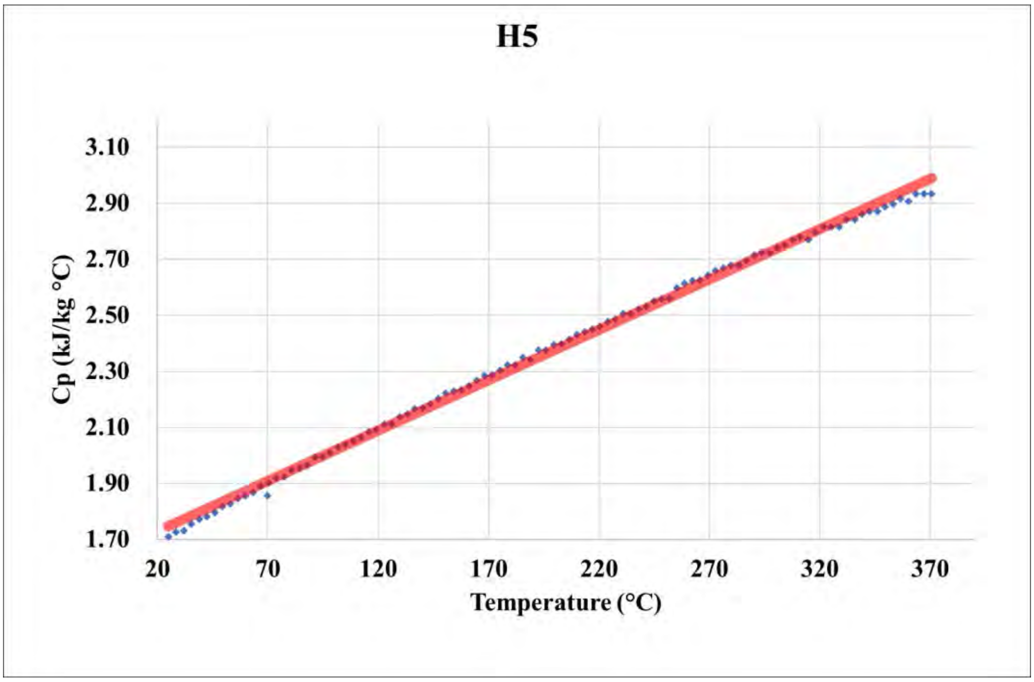
(b)


H3
Cp (kJ/kg °C)
2.90
2.70
2.50
2.30
2.10
1.90
1.70
20
70
120
170
220
270
Temperature (°C)

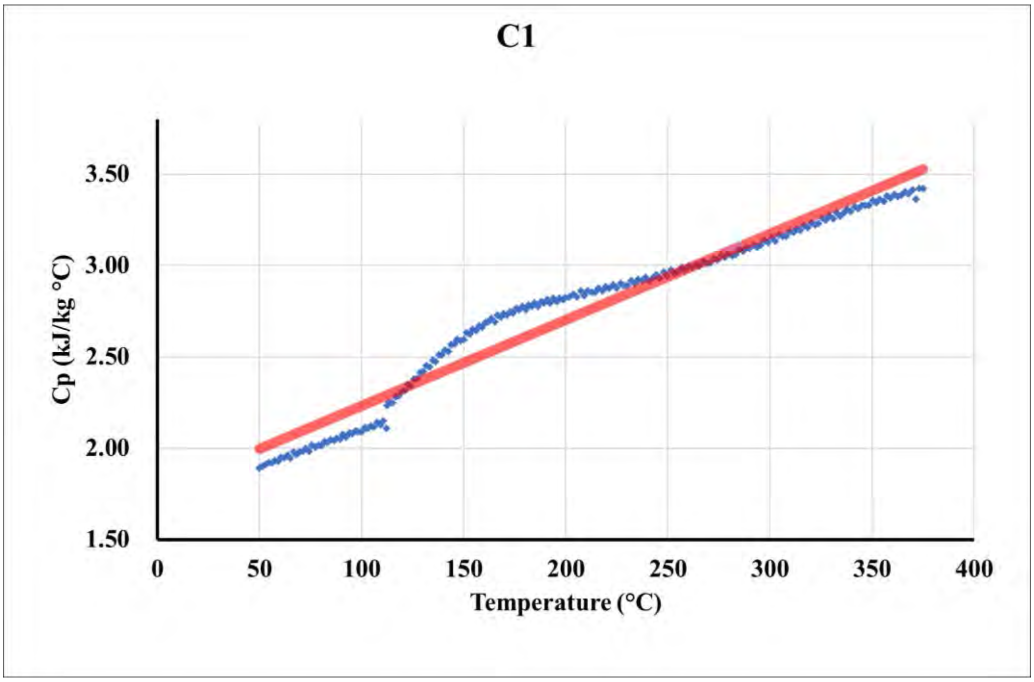
(c)


H4
Cp (kJ/kg °C)
3.10
2.90
2.70
2.50
2.30
2.10
1.90
1.70
20
70
120
170
220
270
320
Temperature (°C)

(d)



(e)



(f)

◆ Cp Data —— Linear Equation

Figure 4.6 Temperature-dependent Cp graph and 1 partition of case study 1; (a-e) represent for hot stream (H1-H5), respectively. (f) represent for cold stream.

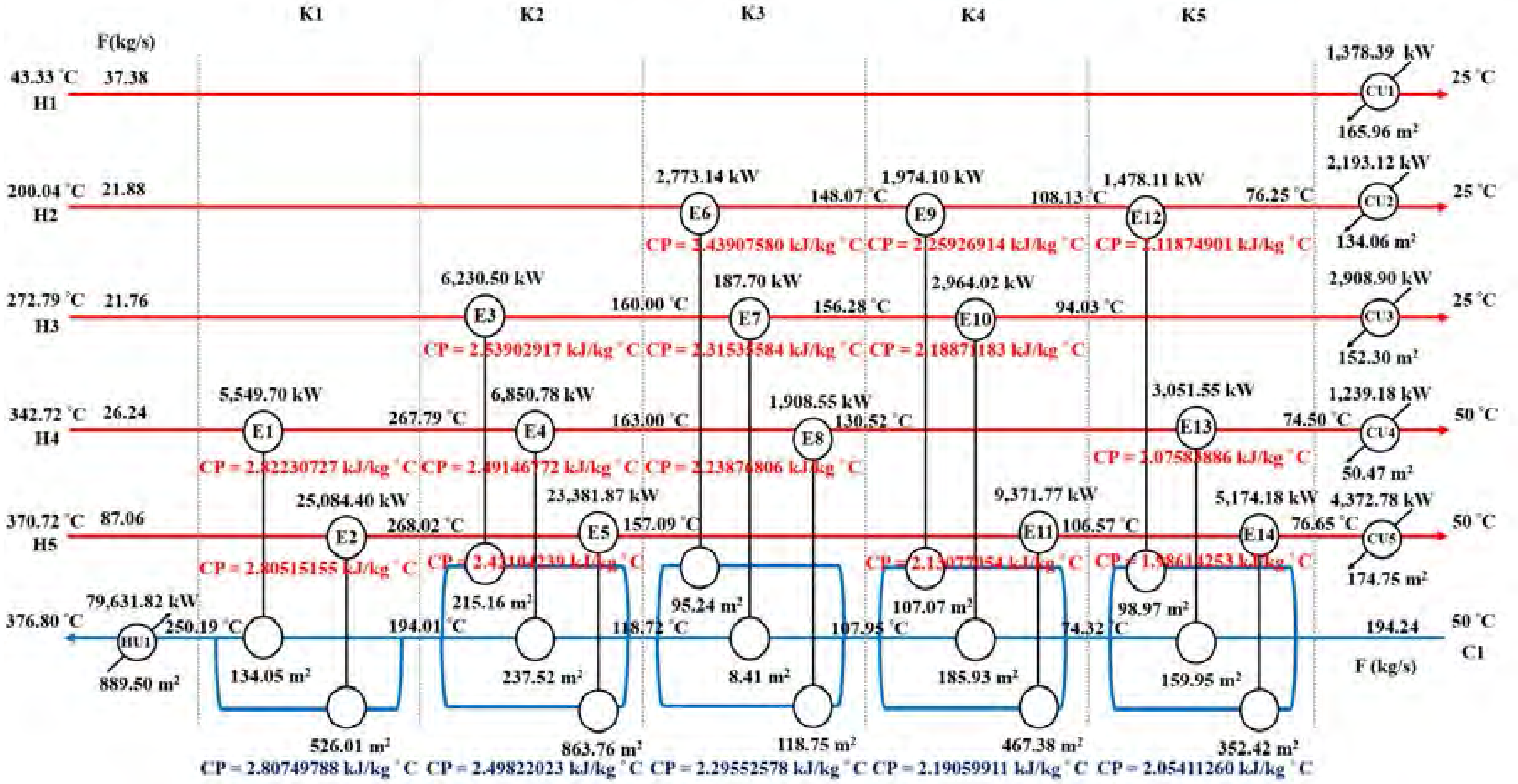


Figure 4.7 HEN from partitioning technique GAMS model of 1 partition.
`

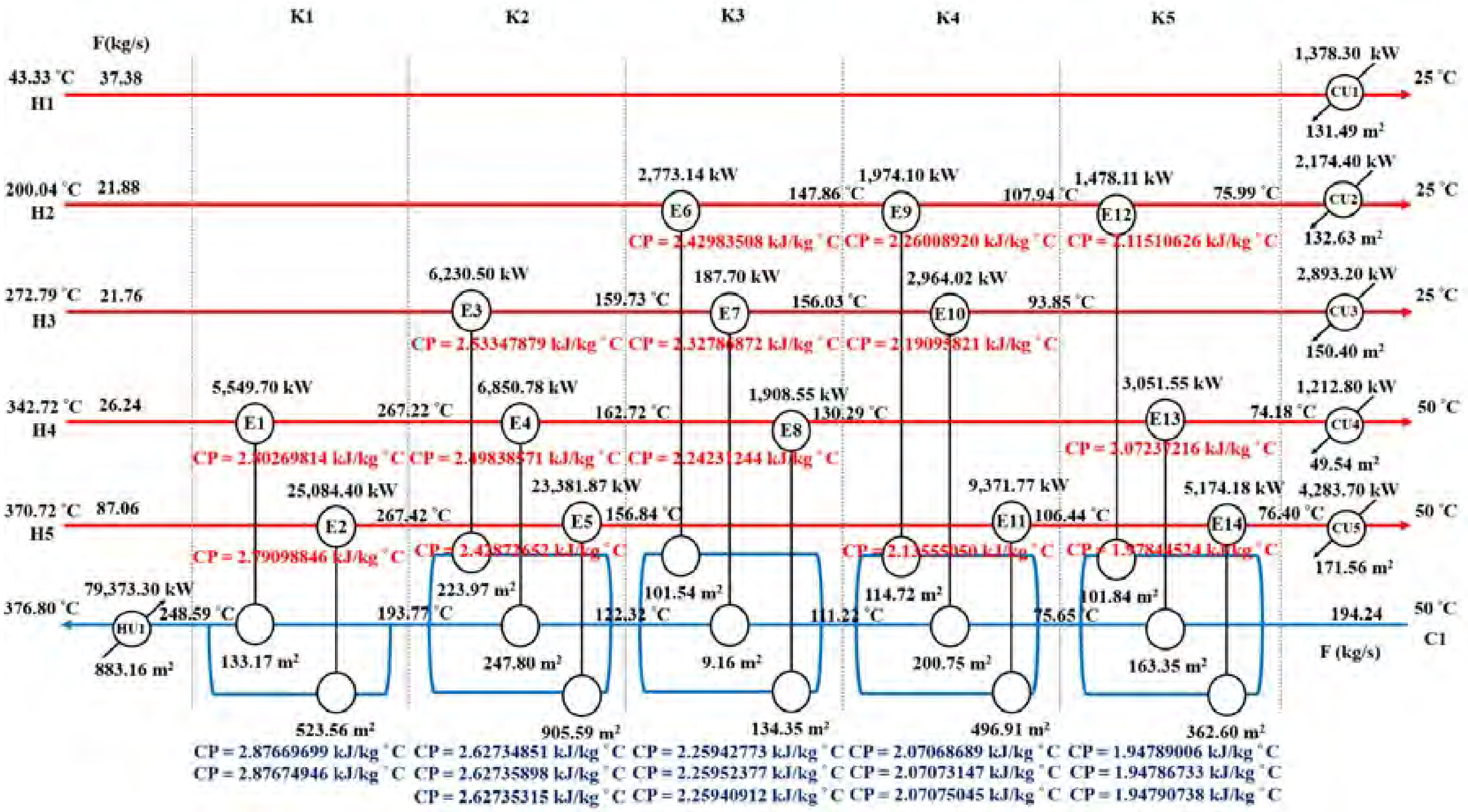
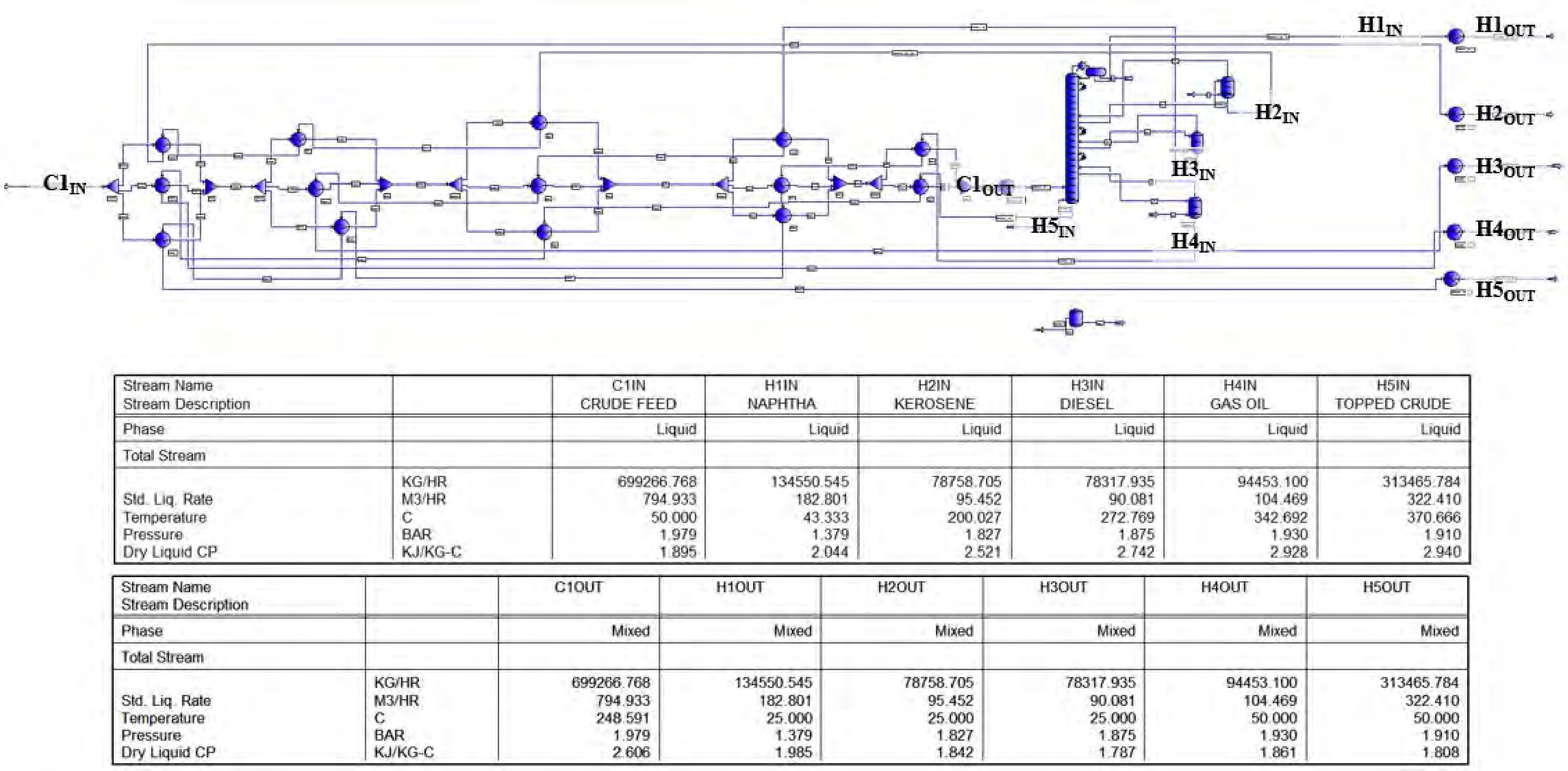


Figure 4.8 Validation of HEN from partitioning technique GAMS model by Pro/II simulation of 1 partition.



Stream Name Stream Description		C1IN CRUDE FEED	H1IN NAPHTHA	H2IN KEROSENE	H3IN DIESEL	H4IN GAS OIL	H5IN TOPPED CRUDE
Phase		Liquid	Liquid	Liquid	Liquid	Liquid	Liquid
Total Stream							
	KG/HR	699266.768	134550.545	78758.705	78317.935	94453.100	313465.784
Std. Liq. Rate	M3/HR	794.933	182.801	95.452	90.081	104.469	322.410
Temperature	C	50.000	43.333	200.027	272.769	342.692	370.666
Pressure	BAR	1.979	1.379	1.827	1.875	1.930	1.910
Dry Liquid CP	KJ/KG-C	1.895	2.044	2.521	2.742	2.928	2.940

Stream Name Stream Description		C1OUT	H1OUT	H2OUT	H3OUT	H4OUT	H5OUT
Phase		Mixed	Mixed	Mixed	Mixed	Mixed	Mixed
Total Stream							
	KG/HR	699266.768	134550.545	78758.705	78317.935	94453.100	313465.784
Std. Liq. Rate	M3/HR	794.933	182.801	95.452	90.081	104.469	322.410
Temperature	C	248.591	25.000	25.000	25.000	50.000	50.000
Pressure	BAR	1.979	1.379	1.827	1.875	1.930	1.910
Dry Liquid CP	KJ/KG-C	2.606	1.985	1.842	1.787	1.861	1.808

Figure 4.9 HEN from 1 partition case study by Pro/II simulation.

New model (partitioning technique) is used to design HEN which shown GAMS and Pro/II results in Figure 4.7 and Figure 4.8, respectively. The design from Pro/II simulation is shown in Figure 4.9. The results show that specific heat capacity change from stage to stage and it increase accuracy of area calculation (Table 4.6). Compared on exchanger E6, 30.79 % error of heat exchanger area reduce to 6.21 % likewise to another exchanger because Cp calculation from our new model vary with temperature results to Cp that shows in Figure 4.7 (partitioning technique) and Figure 4.8 (Cp calculation from Pro/II) close to each other. Average error of area calculation from 1 partition is 5.02 % which reduced from constant Cp of 11.76 %. Thus, it shows that only 1 partition or take Cp as variable is highly impact on reduction of error calculation and it can be less for higher number of partitions. Moreover, effective scenario from Cp variable can be found in this case but it is traded-off by adding more equation which lead to complex model.

Table 4.6 Duty data comparison between GAMS and Pro/II of 1 partition.

Heat Exchanger	GAMS Duty (kW)	Pro/II Duty (kW)	Percent Error (%)
E1	5,549.70	5,549.70	-
E2	25,084.40	25,084.40	-
E3	6,230.50	6,230.50	-
E4	6,850.78	6,850.78	-
E5	23,381.87	23,381.87	-
E6	2,773.14	2,773.14	-
E7	187.70	187.70	-
E8	1,908.55	1,908.55	-
E9	1,974.10	1,974.10	-
E10	2,964.02	2,964.02	-
E11	9,371.77	9,371.77	-
E12	1,478.11	1,478.11	-
E13	3,051.55	3,051.55	-
E14	5,174.18	5,174.18	-
CU1	1,378.39	1,378.30	0.01
CU2	2,193.12	2,174.40	0.86
CU3	2,908.90	2,893.20	0.54
CU4	1,239.18	1,212.80	2.18
CU5	4,372.78	4,283.70	2.08
HU1	79,631.82	79,373.30	0.33

Table 4.7 Area data comparison between GAMS and Pro/II of 1 partition.

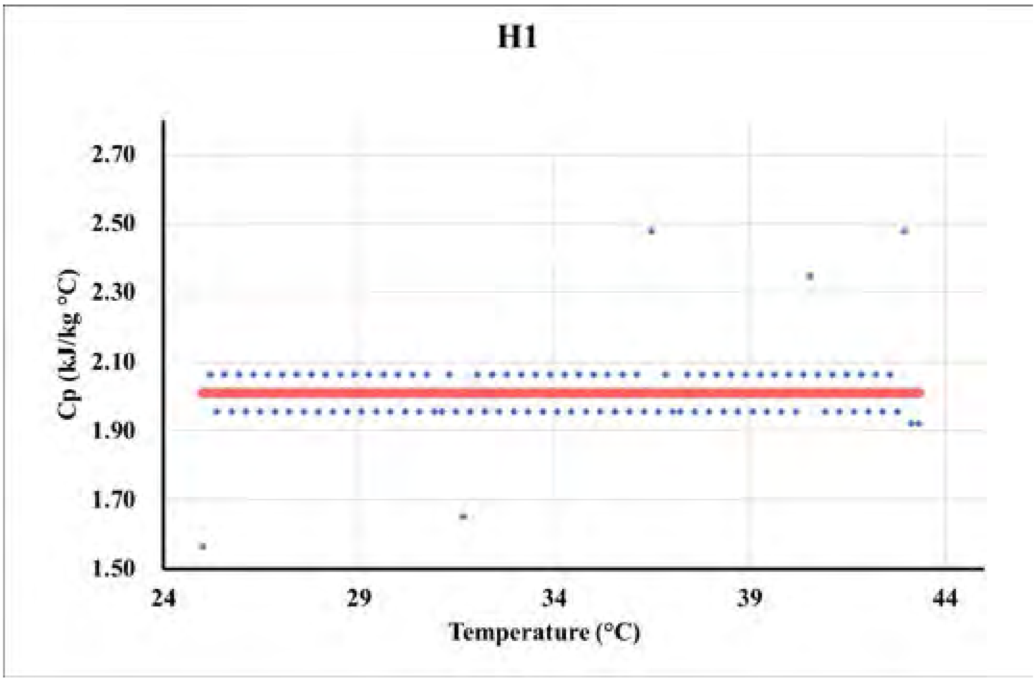
Heat Exchanger	GAMS Area (m^2)	Pro/II Area (m^2)	Percent Error (%)
E1	134.05	133.17	0.65
E2	526.01	523.56	0.47
E3	215.16	223.97	3.93
E4	237.52	247.80	4.15
E5	863.76	905.59	4.62
E6	95.24	101.54	6.21
E7	8.41	9.16	8.15
E8	118.75	134.35	11.61
E9	107.07	114.72	6.67
E10	185.93	200.75	7.38
E11	467.38	496.91	5.94
E12	98.97	101.84	2.82
E13	159.95	163.35	2.08
E14	352.42	362.60	2.81
CU1	165.96	131.49	26.21
CU2	134.06	132.63	1.08
CU3	152.30	150.40	1.27
CU4	50.47	49.54	1.88
CU5	174.75	171.56	1.86
HU1	889.50	883.16	0.72

4.1.3 2 Partitions

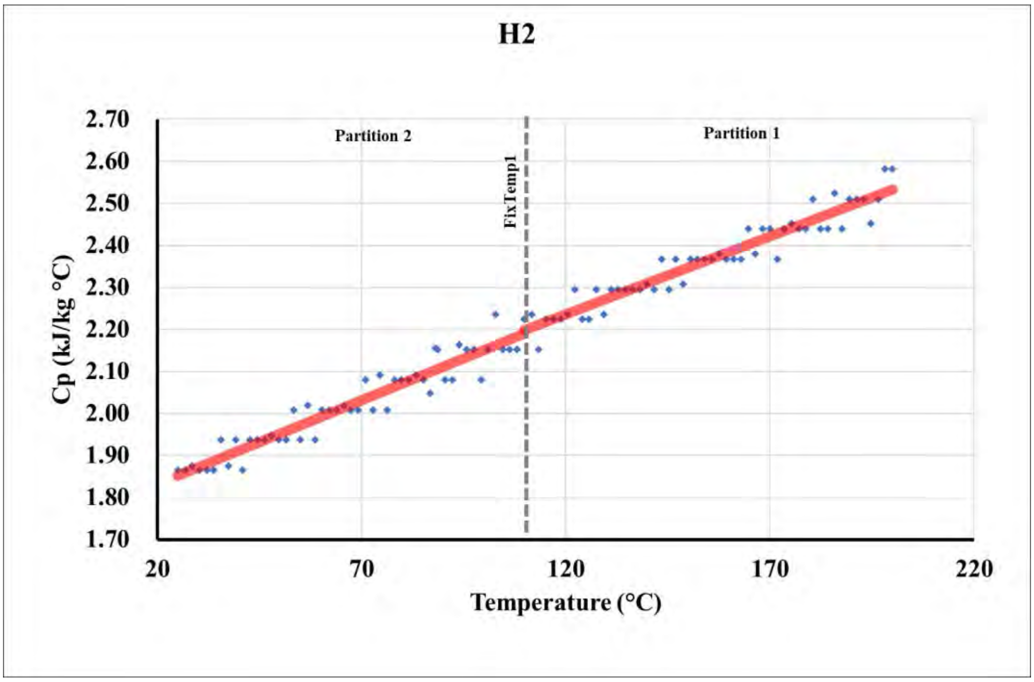
Cp data is divided and increased number of partitions to 2 for more accuracy that shown parameters and Cp graph in Table 4.8 and Figure 4.10, respectively. In general, the partition should divide based on jumping point such as boiling point temperature but this case separate stream H2 to H5 in the middle temperature of supply and target temperature due to linearity Cp data except stream C1. However, even increasing number of partitions to 2, it still be not fit with Cp data of crude oil (stream C1) and it is not appropriate to use only 2 partitions with dome curve Cp data as shows the example of this shape in Case Study 3.

Table 4.8 Specific heat capacity linearization of case study 1 by 2 partitions.

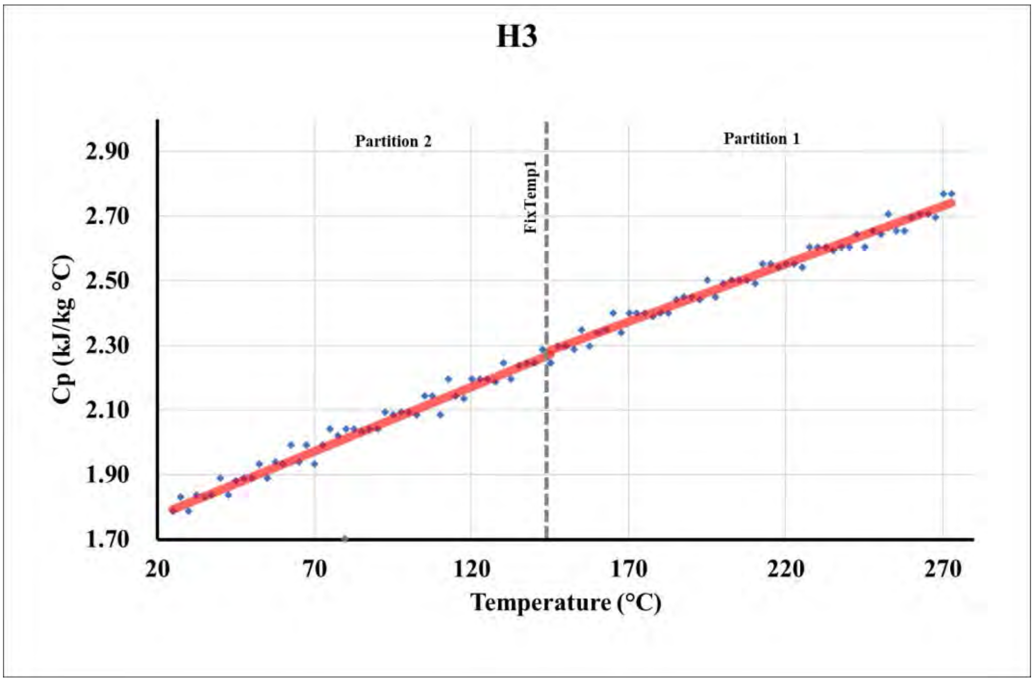
Streams	Partition Number (n)	$Cp_n = A_n \times T_{mean} + B_n$		R^2	Fix Temp 1 (°C)	$Cp_{average}$ (kJ/kg °C)
		A_n	B_n			
H1	1	0	2.0115729	-	-	2.0115729
H2	1	0.0037324	1.7872397	0.92	109.87	2.1986417
	2	0.0039917	1.7520906	0.91		
H3	1	0.0035715	1.7663352	0.98	145.15	2.2799229
	2	0.0039823	1.6938661	0.89		
H4	1	0.0033502	1.7869467	0.99	179.05	2.4233018
	2	0.0039403	1.6706077	0.99		
H5	1	0.0032887	1.7442809	0.99	192.63	2.4155978
	2	0.0039309	1.6202846	0.99		
C1	1	0.0038748	1.9929526	0.96	112.12	2.7662411
	2	0.0041118	1.6917482	0.96		



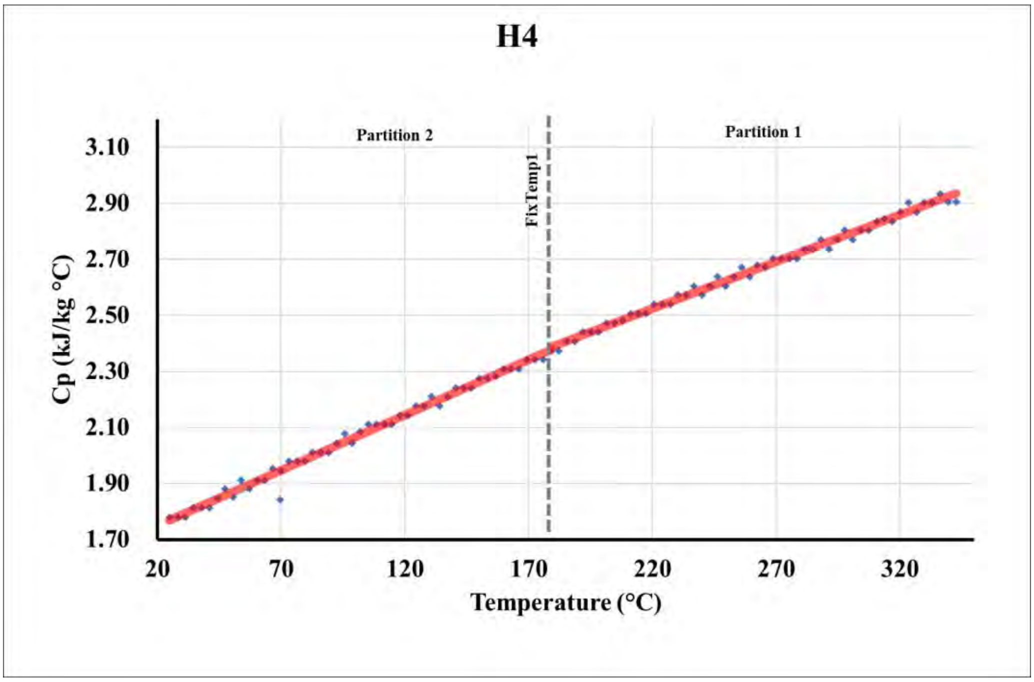
(a)


H2
Partition 2
Partition 1
FixTemp1
Cp (kJ/kg °C)
2.70
2.60
2.50
2.40
2.30
2.20
2.10
2.00
1.90
1.80
1.70
20
70
120
170
220
Temperature (°C)

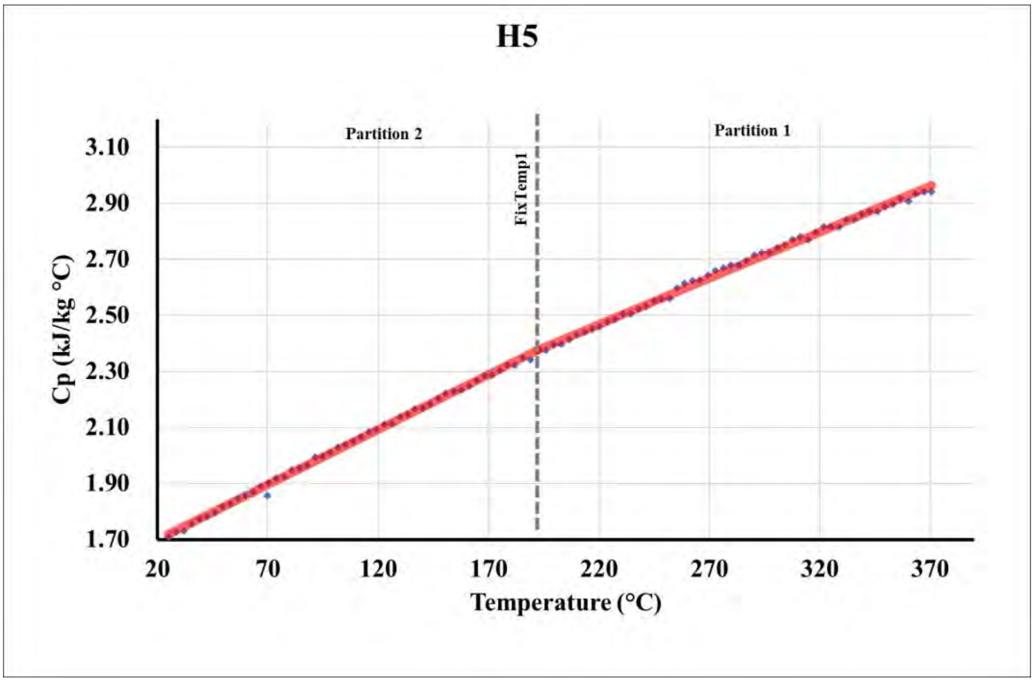
(b)


H3
Partition 2
Partition 1
FixTemp1
Cp (kJ/kg °C)
2.90
2.70
2.50
2.30
2.10
1.90
1.70
20
70
120
170
220
270
Temperature (°C)

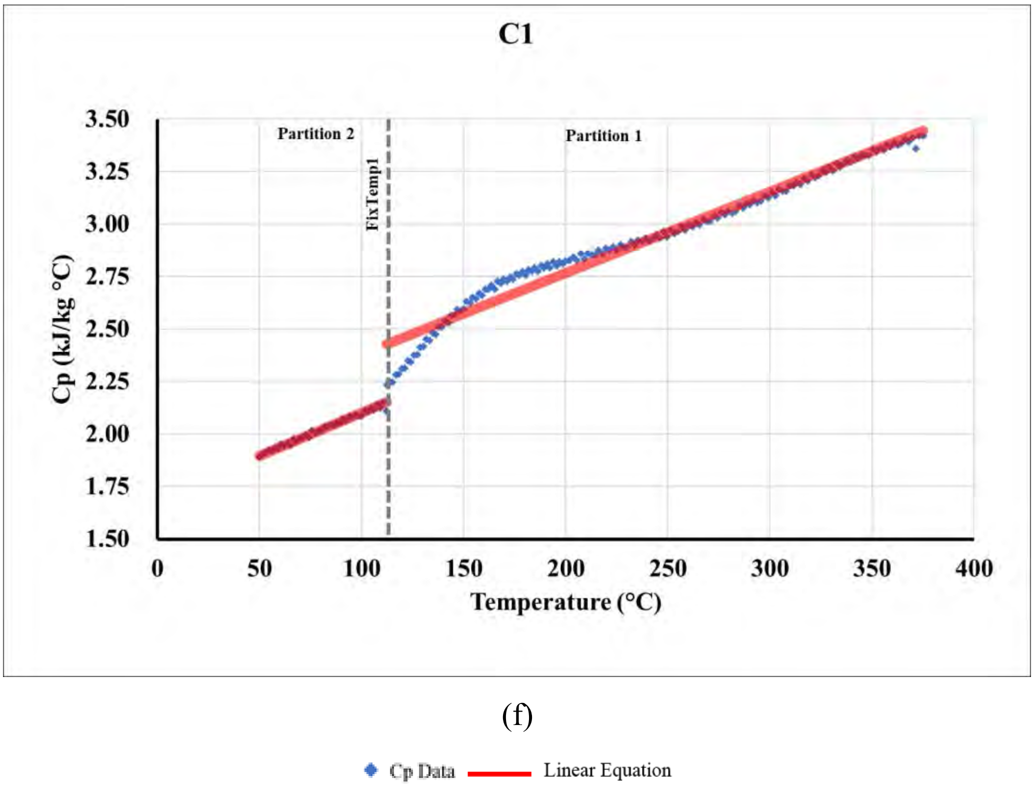
(c)


H4
Partition 2
Partition 1
FixTemp1
Cp (kJ/kg °C)
3.10
2.90
2.70
2.50
2.30
2.10
1.90
1.70
20
70
120
170
220
270
320
Temperature (°C)

(d)


H5
Partition 2
Partition 1
FixTemp1
Cp (kJ/kg °C)
3.10
2.90
2.70
2.50
2.30
2.10
1.90
1.70
20
70
120
170
220
270
320
370
Temperature (°C)

(e)



(f)

Figure 4.10 Temperature-dependent Cp graph and 2 partitions of case study 1; (a-e) represent for hot stream (H1-H5), respectively. (f) represent for cold stream.

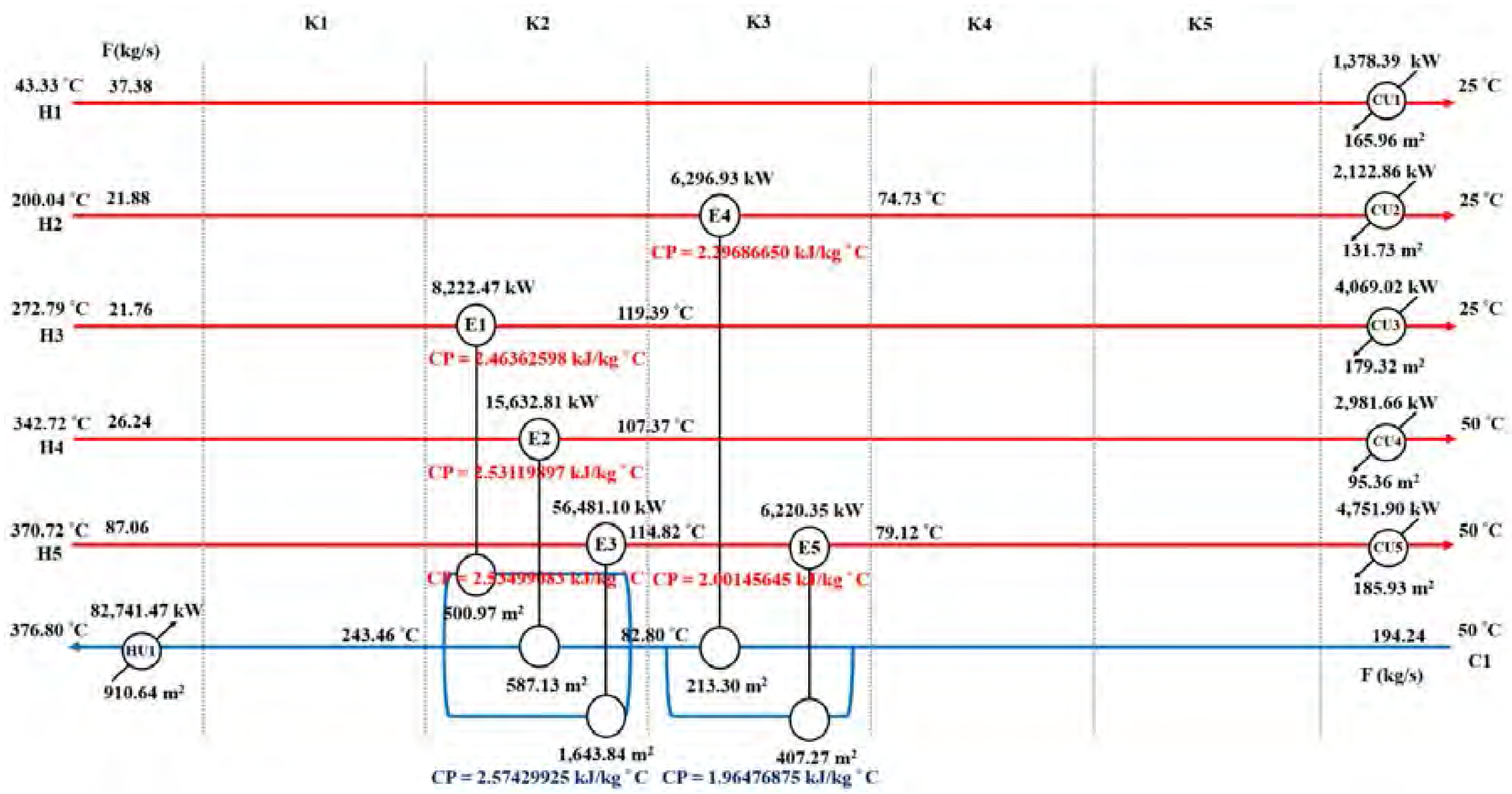


Figure 4.11 HEN from partitioning technique GAMS model of 2 partitions.

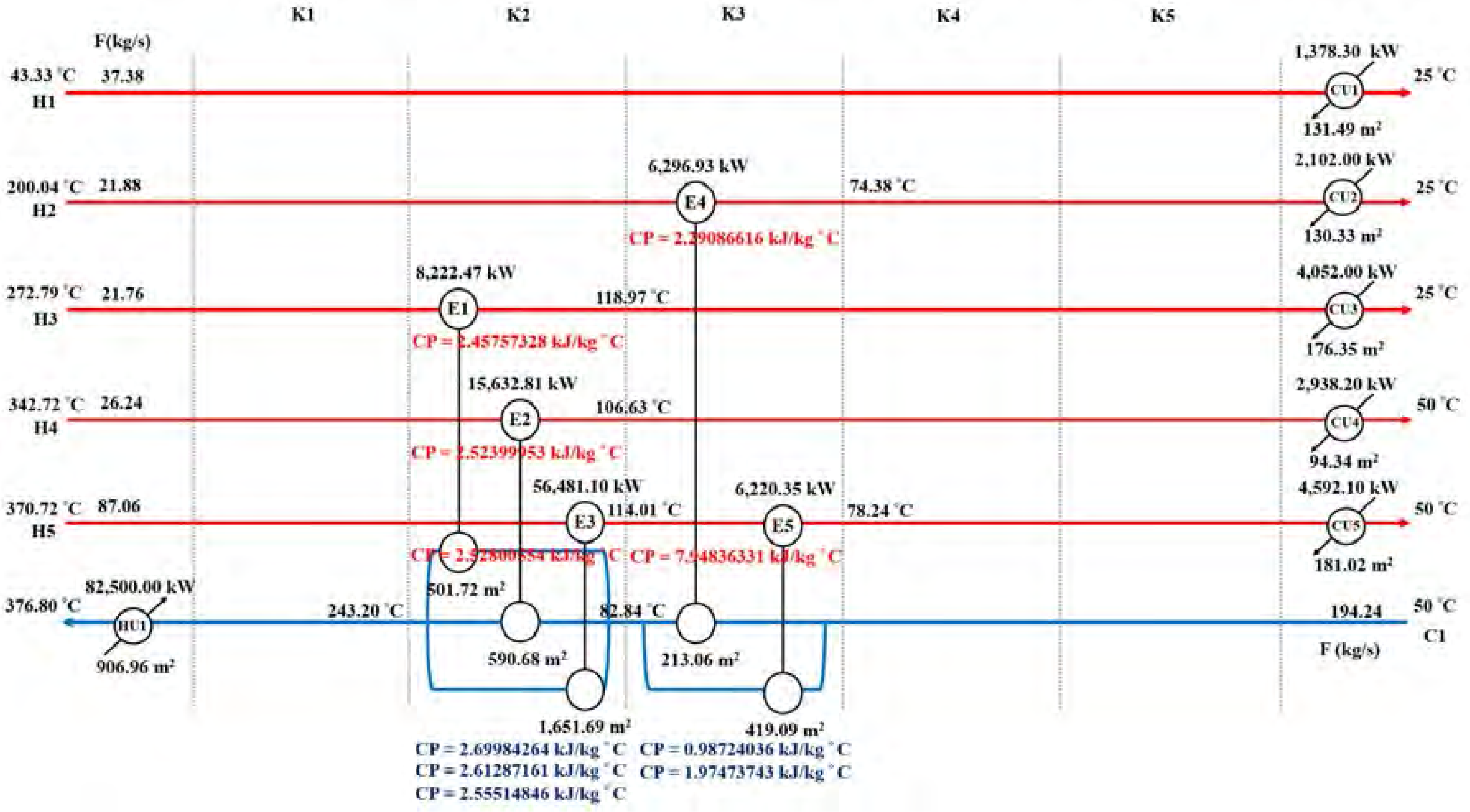
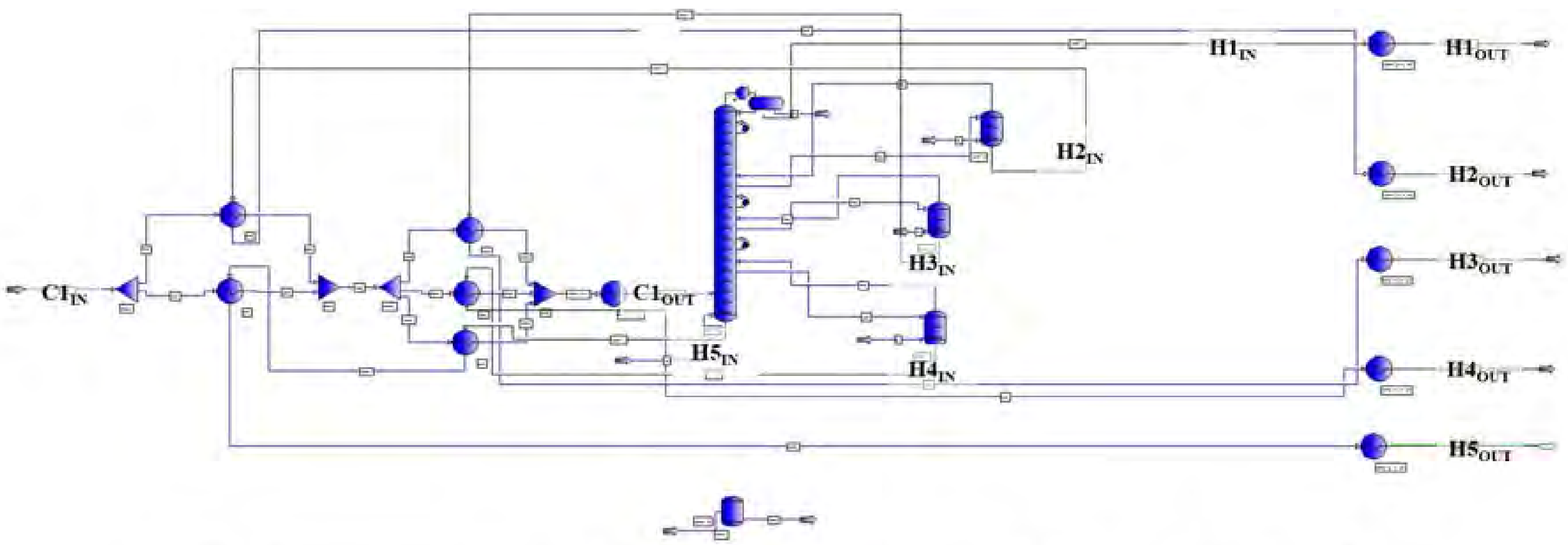


Figure 4.12 Validation of HEN from partitioning technique GAMS model by Pro/II simulation of 2 partitions.



Stream Name Stream Description		C1IN CRUDE FEED	H1IN NAPHTHA	H2IN KEROSENE	H3IN DIESEL	H4IN GAS OIL	H5IN TOPPED CRUDE
Phase		Liquid	Liquid	Liquid	Liquid	Liquid	Liquid
Total Stream							
	KG/HR	699266.768	134552.131	78759.572	78321.187	94460.279	313452.820
Std. Liq. Rate	M3/HR	794.933	182.803	95.453	90.085	104.476	322.398
Temperature	C	50.000	43.333	200.033	272.778	342.705	370.858
Pressure	BAR	1.979	1.379	1.827	1.875	1.930	1.910
Dry Liquid CP	KJ/KG-C	1.895	2.944	2.521	2.742	2.926	2.942

Stream Name Stream Description		C1OUT	H1OUT	H2OUT	H3OUT	H4OUT	H5OUT
Phase		Mixed	Mixed	Mixed	Mixed	Mixed	Mixed
Total Stream							
	KG/HR	699266.768	134552.131	78759.572	78321.187	94460.279	313452.820
Std. Liq. Rate	M3/HR	794.933	182.803	95.453	90.085	104.476	322.398
Temperature	C	243.119	25.000	25.000	25.000	50.000	50.000
Pressure	BAR	1.979	1.379	1.827	1.875	1.930	1.910
Dry Liquid CP	KJ/KG-C	2.589	1.965	1.842	1.787	1.861	1.888

Figure 4.13 HEN from 2 partitions case study by Pro/II simulation.

The results of partitioning technique and validated one by Pro/II are illustrated in Figure 4.11 and Figure 4.12, respectively. Duty data and area data comparison between GAMS and Pro/II are shown in Table 4.9 and Table 4.10, respectively. The design from Pro/II simulation is shown in Figure 4.13. The results follow the theory that error of area calculation reduce from increasing number of partitions because of higher accuracy for Cp calculation results to less error of area calculation. Average error of area calculation from 2 partitions is 3.40 %. Note that with linearization technique percent error of duty will not decrease but this new technique affects directly on area calculation error.

Table 4.9 Duty data comparison between GAMS and Pro/II of 2 partitions

Heat Exchanger	GAMS Duty (kW)	Pro/II Duty (kW)	Percent Error (%)
E1	8,222.47	8,222.47	-
E2	15,632.81	15,632.81	-
E3	56,481.10	56,481.10	-
E4	6,296.93	6,296.93	-
E5	6,220.35	6,220.35	-
CU1	1,378.39	1,378.30	0.01
CU2	2,122.86	2,102.00	0.99
CU3	4,069.02	4,052.00	0.42
CU4	2,981.66	2,938.20	1.48
CU5	4,751.59	4,592.10	3.47
HU1	82,741.47	82,500.00	0.29

Table 4.10 Area data comparison between GAMS and Pro/II of 2 partitions.

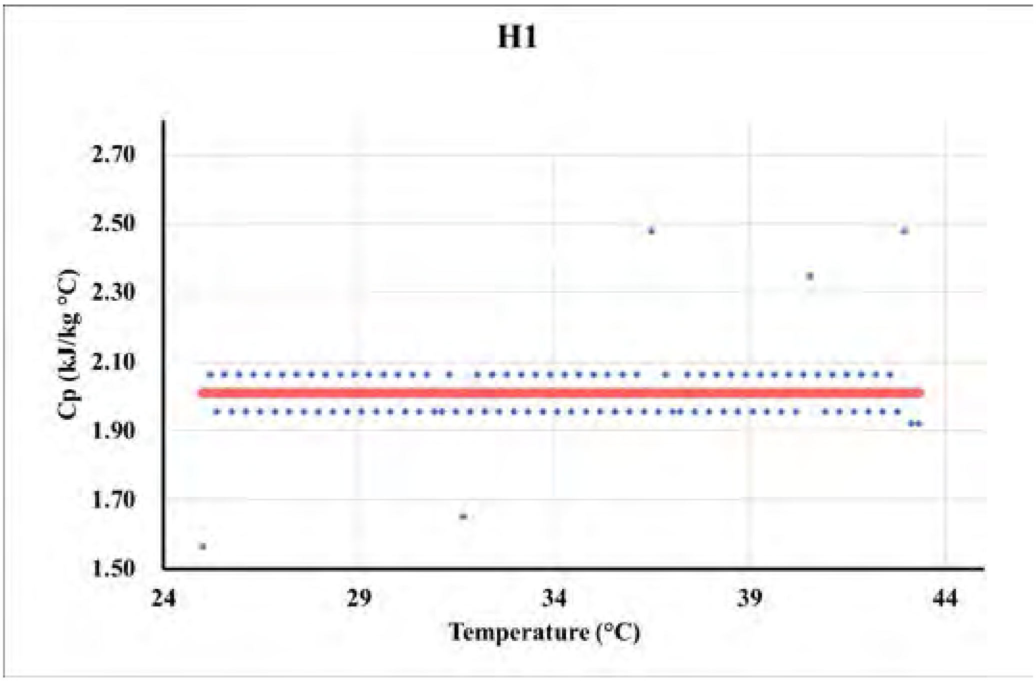
Heat Exchanger	GAMS Area (m^2)	Pro/II Area (m^2)	Percent Error (%)
E1	500.97	501.72	0.15
E2	587.13	590.68	0.60
E3	1,643.84	1,651.69	0.48
E4	213.30	213.06	0.12
E5	407.27	419.09	2.82
CU1	165.96	131.49	26.22
CU2	131.73	130.33	1.08
CU3	179.32	176.35	1.69
CU4	95.36	94.34	1.08
CU5	185.93	181.02	2.71
HU1	910.64	906.96	0.41

4.1.4 3 Partitions

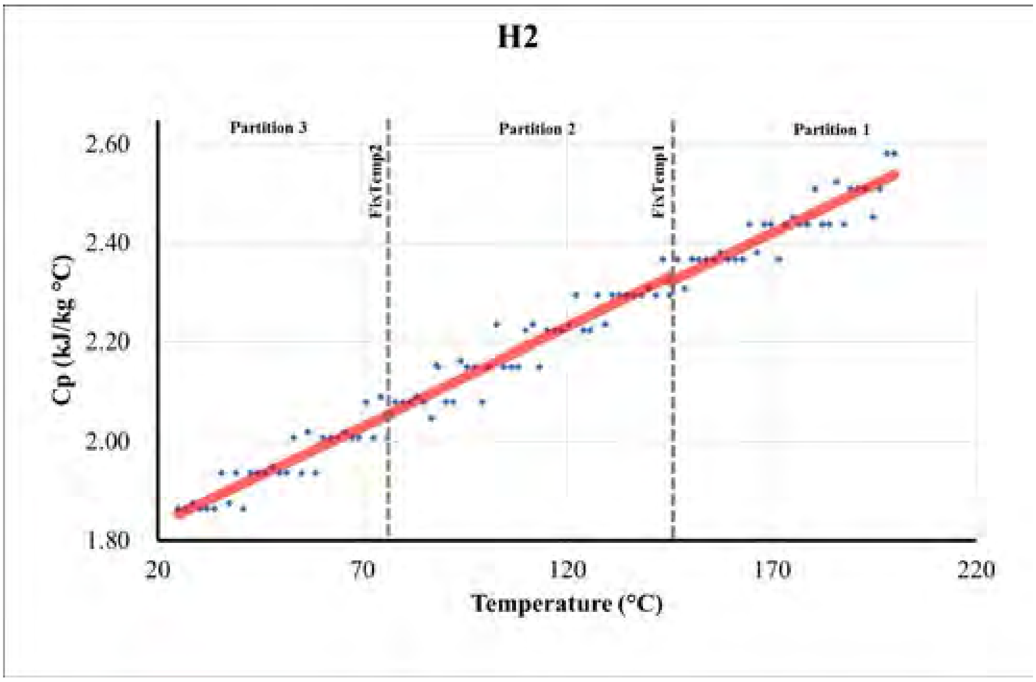
Now, every stream is split into 3 partitions which make model more realistic due to less error in Cp data (Fitting well with stream C2). Cp data parameter are formulated illustrated in Table 4.11 and Figure 4.14.

Table 4.11 Specific heat capacity linearization of case study 1 by 3 partitions.

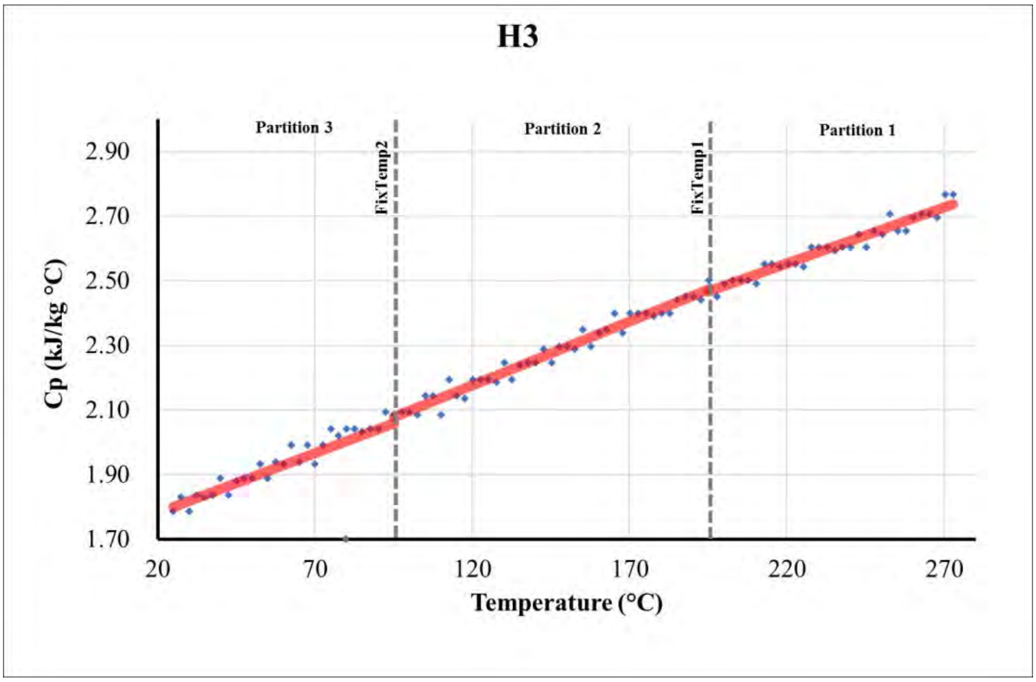
Streams	Partition Number (n)	$Cp_n = A_n \times T_{mean} + B_n$		R^2	Fix Temp 1 (°C)	Fix Temp 2 (°C)	$Cp_{average}$ (kJ/kg °C)
		A_n	B_n				
H1	1	0	2.0115729	-	-	-	2.0115729
H2	1	0.0039408	1.7499202	0.81	145.23	76.27	2.1975324
	2	0.0040235	1.7485183	0.86			
	3	0.0038983	1.7559456	0.82			
H3	1	0.0034835	1.7871469	0.93	195.21	95.08	2.2807703
	2	0.0039627	1.7013860	0.96			
	3	0.0037159	1.7077719	0.62			
H4	1	0.0032687	1.8110935	0.97	243.24	114.86	2.4234212
	2	0.0037746	1.6956386	0.99			
	3	0.0040287	1.6644258	0.95			
H5	1	0.0030150	1.8334726	0.99	262.47	122.78	2.4161113
	2	0.0035929	1.6721186	0.99			
	3	0.0040832	1.6098383	0.99			
C1	1	0.0033967	2.1322310	0.99	168.18	112.12	2.7666217
	2	0.0090825	1.2318505	0.98			
	3	0.0041118	1.6917482	0.96			


H1
Cp (kJ/kg °C)
2.70
2.50
2.30
2.10
1.90
1.70
1.50
24
29
34
39
44
Temperature (°C)

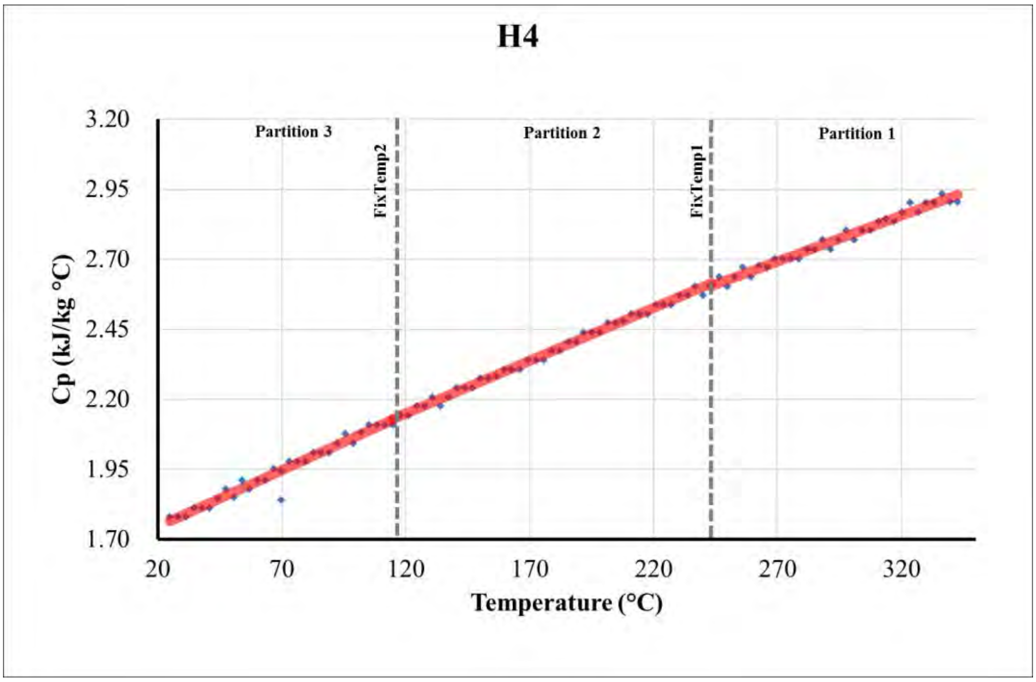
(a)


H2
Partition 3
Partition 2
Partition 1
FixTemp2
FixTemp1
Cp (kJ/kg °C)
2.60
2.40
2.20
2.00
1.80
20
70
120
170
220
Temperature (°C)

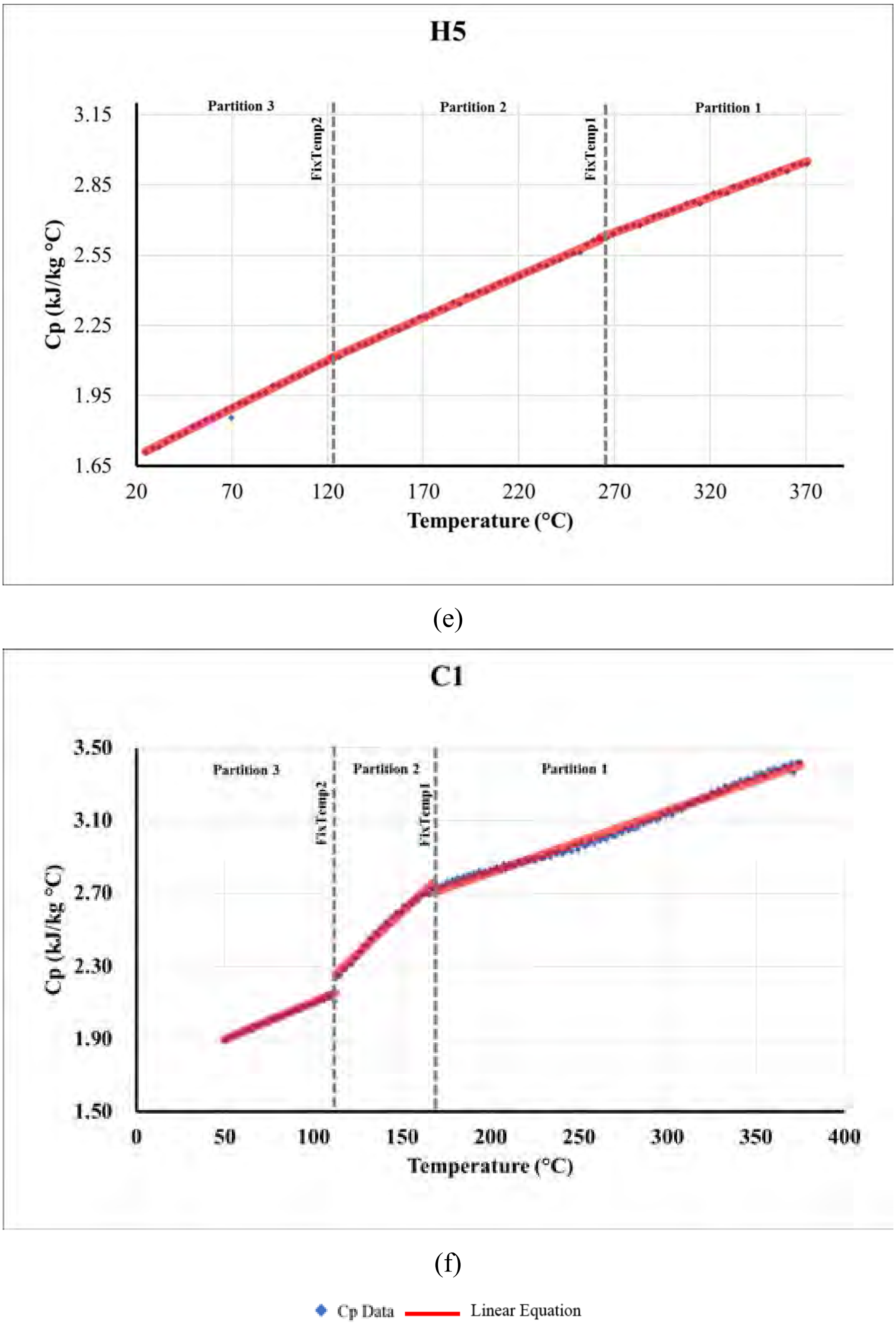
(b)


H3
Partition 3
Partition 2
Partition 1
FixTemp2
FixTemp1
2.90
2.70
2.50
2.30
2.10
1.90
1.70
Cp (kJ/kg °C)
20
70
120
170
220
270
Temperature (°C)

(c)


H4
Partition 3
Partition 2
Partition 1
FixTemp2
FixTemp1
3.20
2.95
2.70
2.45
2.20
1.95
1.70
Cp (kJ/kg °C)
20
70
120
170
220
270
320
Temperature (°C)

(d)



(e)

(f)

Figure 4.14 Temperature-dependent Cp graph and 3 partitioning of case study 1; (a-e) represent for hot stream (H1-H5), respectively. (f) represent for cold stream.

Figure 4.15 and Figure 4.16 show HEN results of partitioning technique and Pro/II results, respectively. The duty data and area data comparison are illustrated in Table 4.12 and Table 4.13, respectively. The design from Pro/II simulation is shown in Figure 4.17. Average error of area calculation from 3 partitions is 3.09 % which less than number of partition 1 and 2 cases. As mention earlier, the Cp data fit when using 3 partitions impact on less error of area calculation.

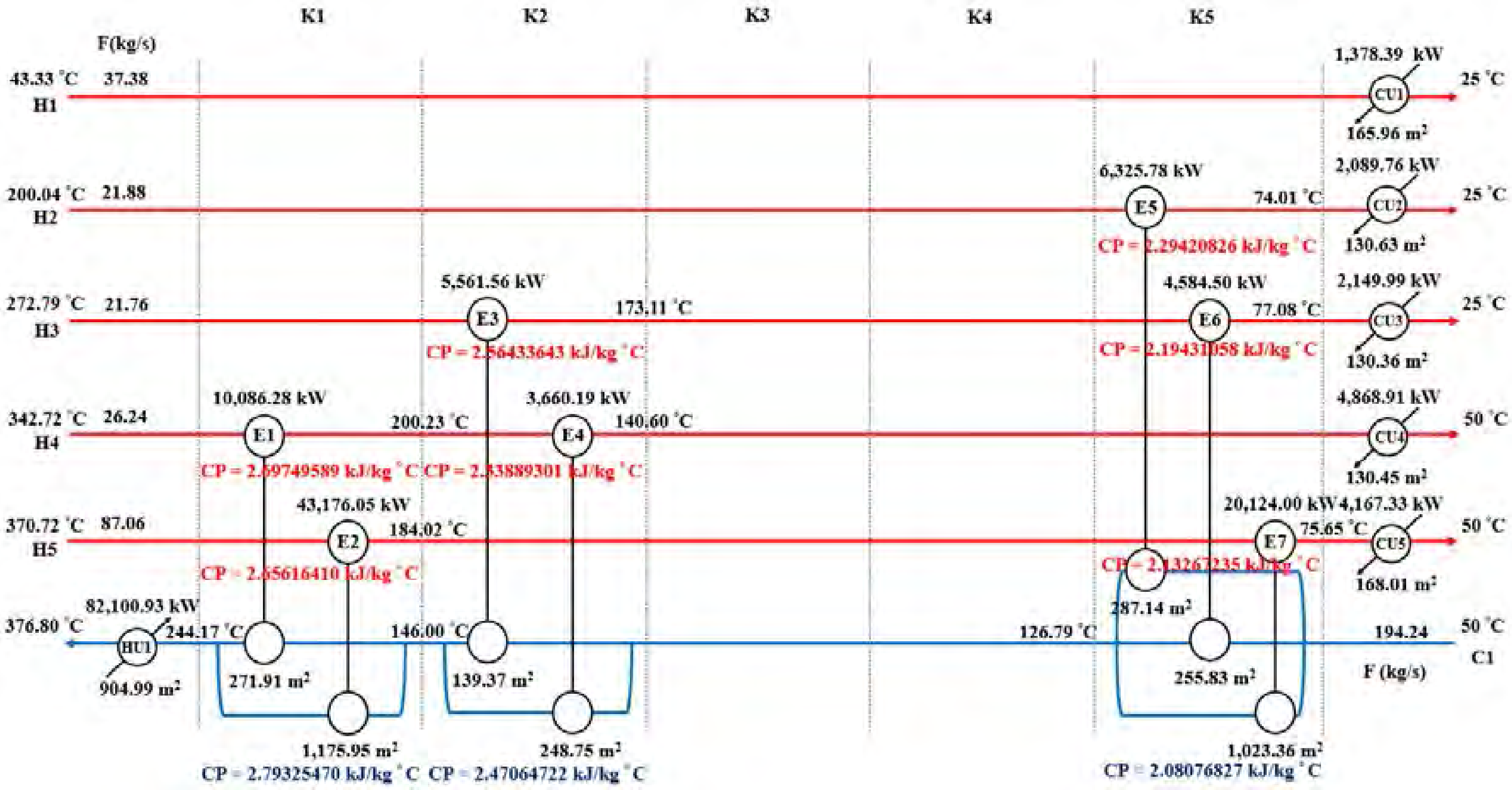


Figure 4.15 HEN from partitioning technique GAMS model of 3 partitions

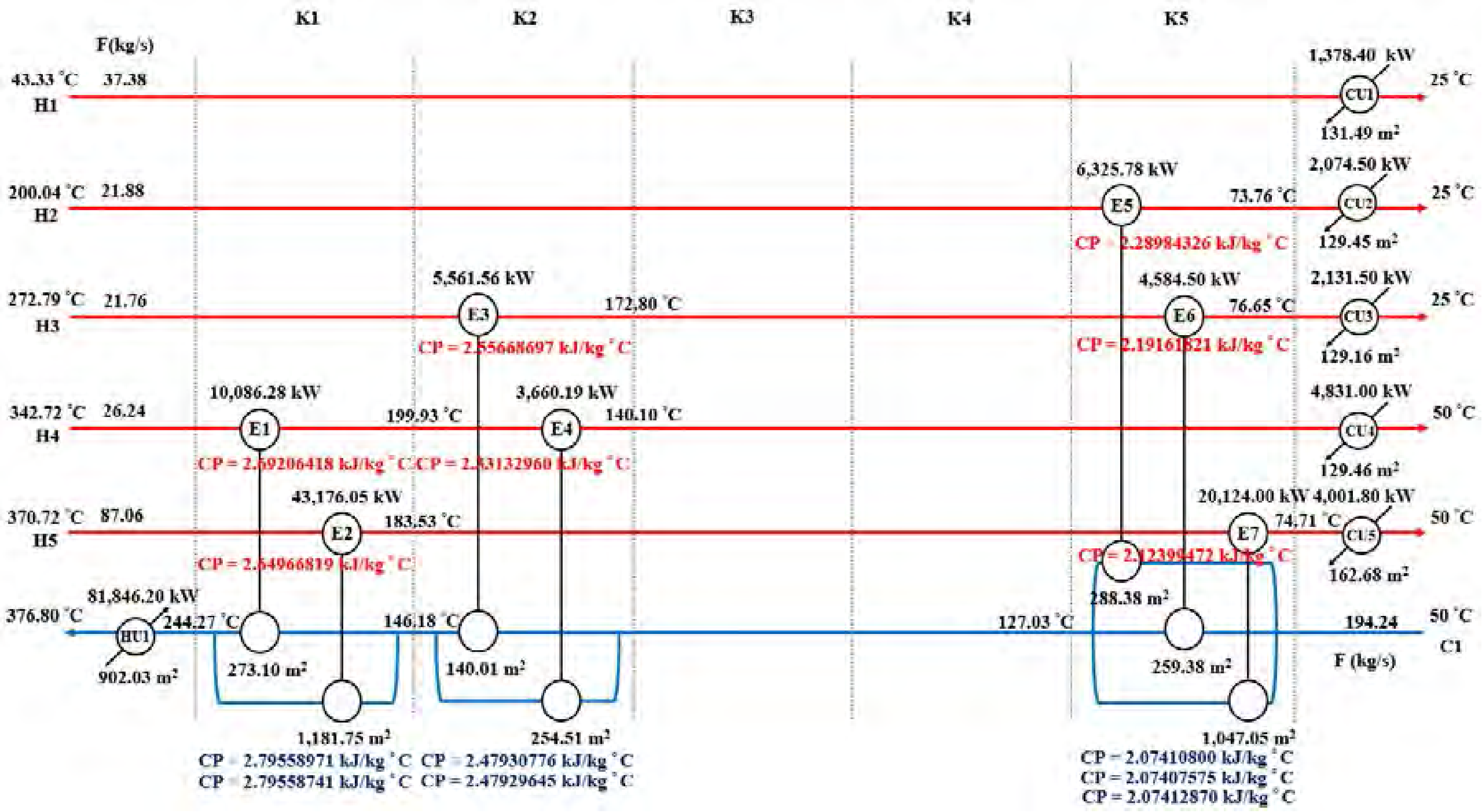
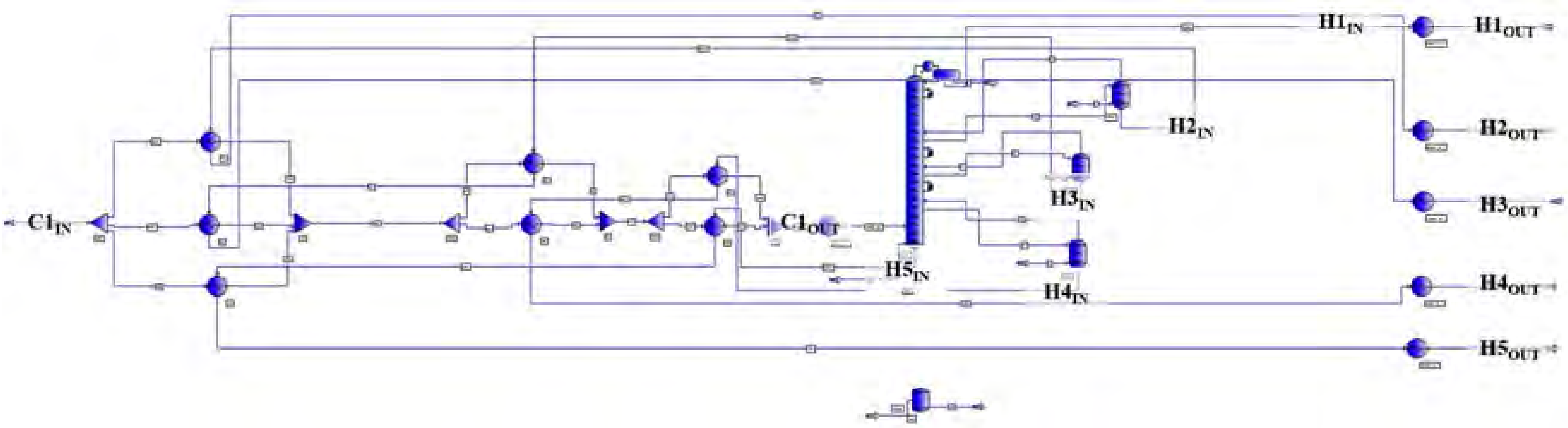
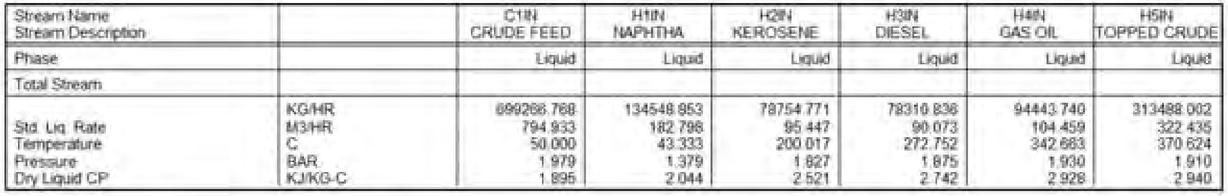
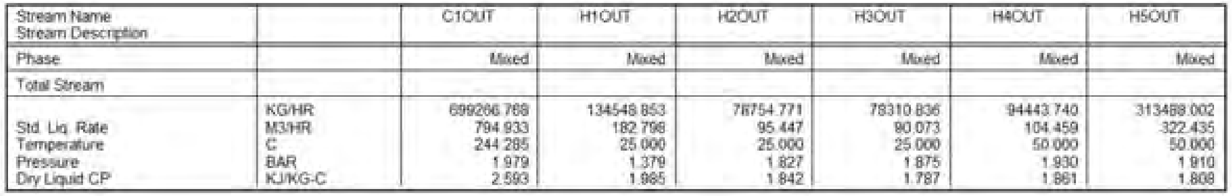


Figure 4.16 Validation of HEN from partitioning technique GAMS model by Pro/II simulation of 3 partitions.





Stream Name Stream Description		C1IN CRUDE FEED	H1IN NAPHTHA	H2IN KEROSENE	H3IN DIESEL	H4IN GAS OIL	H5IN TOPPED CRUDE
Phase		Liquid	Liquid	Liquid	Liquid	Liquid	Liquid
Total Stream							
	KG/HR	699266.768	134548.853	78754.771	78310.836	94443.740	313488.002
Std. Liq. Rate	M3/HR	794.933	182.798	95.447	90.073	104.459	322.435
Temperature	C	50.000	43.333	200.017	272.752	342.683	370.624
Pressure	BAR	1.979	1.379	1.827	1.875	1.930	1.910
Dry Liquid CP	KJ/KG-C	1.895	2.044	2.521	2.742	2.928	2.940



Stream Name Stream Description		C1OUT	H1OUT	H2OUT	H3OUT	H4OUT	H5OUT
Phase		Mixed	Mixed	Mixed	Mixed	Mixed	Mixed
Total Stream							
	KG/HR	699266.768	134548.853	78754.771	78310.836	94443.740	313488.002
Std. Liq. Rate	M3/HR	794.933	182.798	95.447	90.073	104.459	322.435
Temperature	C	244.285	25.000	25.000	25.000	50.000	50.000
Pressure	BAR	1.979	1.379	1.827	1.875	1.930	1.910
Dry Liquid CP	KJ/KG-C	2.593	1.985	1.842	1.787	1.861	1.808

Figure 4.17 HEN from 3 partitions case study by Pro/II simulation.

Table 4.12 Duty data comparison between GAMS and Pro/II of 3 partitions.

Heat Exchanger	GAMS Duty (kW)	Pro/II Duty (kW)	Percent Error (%)
E1	10,086.30	10,086.30	-
E2	43,176.10	43,176.10	-
E3	5,561.60	5,561.60	-
E4	3,660.20	3,660.20	-
E5	6,325.80	6,325.80	-
E6	4,584.50	4,584.50	-
E7	20,124.00	20,124.00	-
CU1	1,378.39	1,378.40	0.00
CU2	2,089.76	2,074.50	0.74
CU3	2,149.99	2,131.50	0.87
CU4	4,868.91	4,831.00	0.78
CU5	4,167.33	4,001.80	4.14
HU1	82,100.93	81,846.20	0.31

Table 4.13 Area data comparison between GAMS and Pro/II of 3 partitions.

Heat Exchanger	GAMS Area (m^2)	Pro/II Area (m^2)	Percent Error (%)
E1	271.91	273.10	0.44
E2	1,175.95	1,181.75	0.49
E3	139.37	140.01	0.46
E4	248.75	254.51	2.26
E5	287.14	288.38	0.43
E6	255.83	259.38	1.37
E7	1,023.36	1,047.05	2.26
CU1	165.96	131.49	26.21
CU2	130.63	129.45	0.91
CU3	130.36	129.16	0.93
CU4	130.45	129.46	0.76
CU5	168.01	162.68	3.28
HU1	904.99	902.03	0.33

4.1.5 5 Partitions

Finally, only cold stream C2 is divided into 5 partitions to fit the Cp data (Table 4.14) because another hot stream (H2 – H5) is already enough fitting data on 3 partitions. Cp graph of stream C2 illustrate in Figure 4.18. The Cp is divided based on boiling point and percent difference of Cp data effect on higher R^2.

Table 4.14 Specific heat capacity linearization of cold stream by 5 partitions

Streams	Partition Number (n)	$Cp_n = A_n \times T_{mean} + B_n$		R^2	Fix Temp$_n$ (°C)	$Cp_{average}$ (kJ/kg °C)
		A_n	B_n			
C1	1	0.0039054	1.9686592	0.99	260.27	2.7656439
	2	0.0027264	2.2753068	0.95	207.30	
	3	0.0039302	2.0497463	0.94	151.52	
	4	0.0107456	1.0128977	0.97	112.52	
	5	0.0041118	1.6917482	0.96		

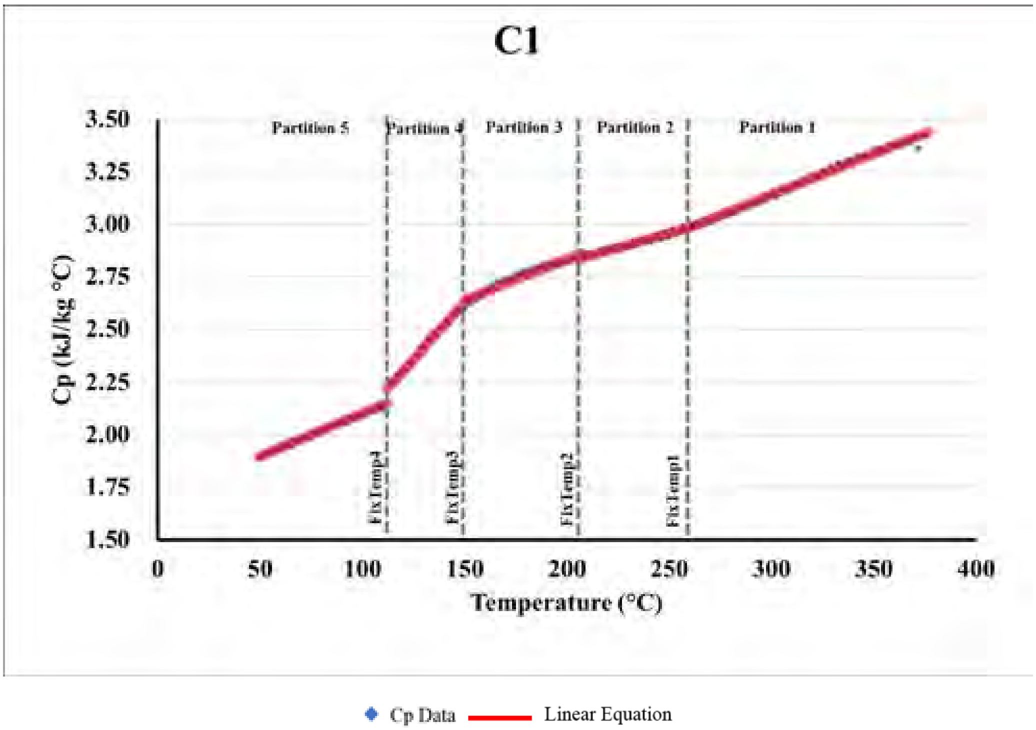


Figure 4.18 Temperature-dependent Cp graph and 5 partition of cold stream.

HEN results from partitioning technique and Pro/II validation are shown in Figure 4.19 and Figure 4.20, respectively. Results data is concluded in Figure 4.20 and Table 4.16. The design from Pro/II simulation is shown in Figure 4.21. Average error of area calculation from 5 partition is 3.12 %.

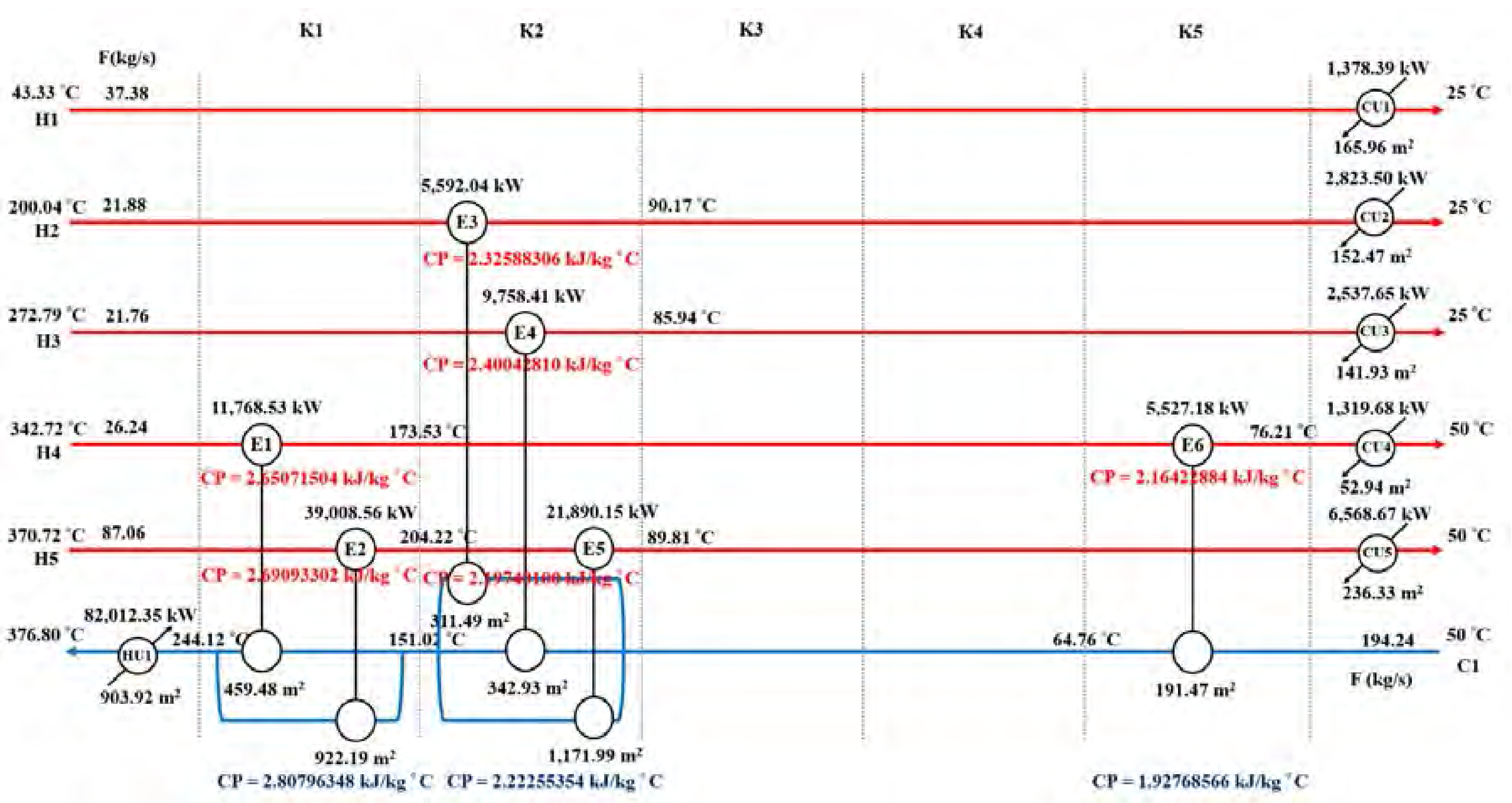


Figure 4.19 HEN from partitioning technique GAMS model of 5 partitions

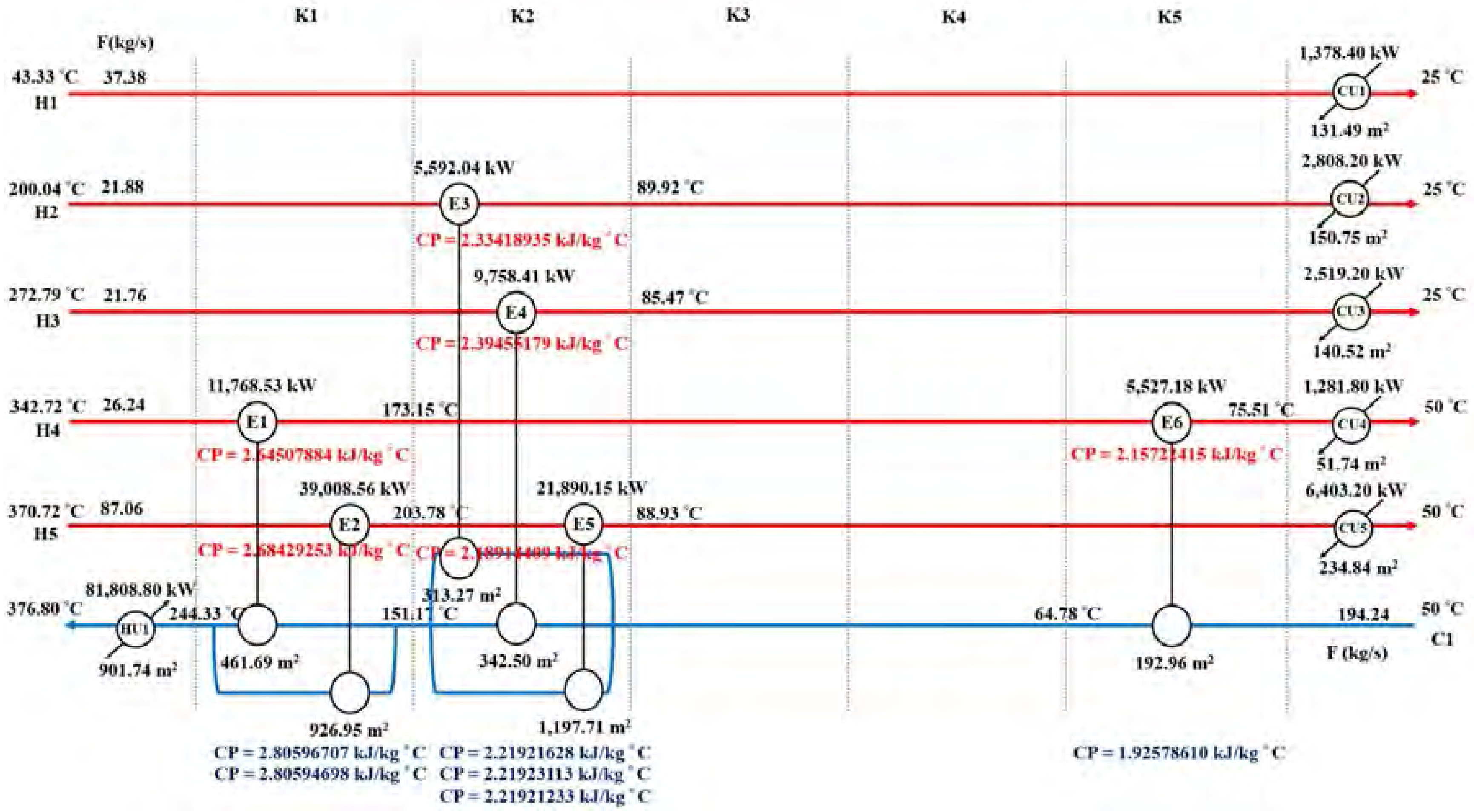
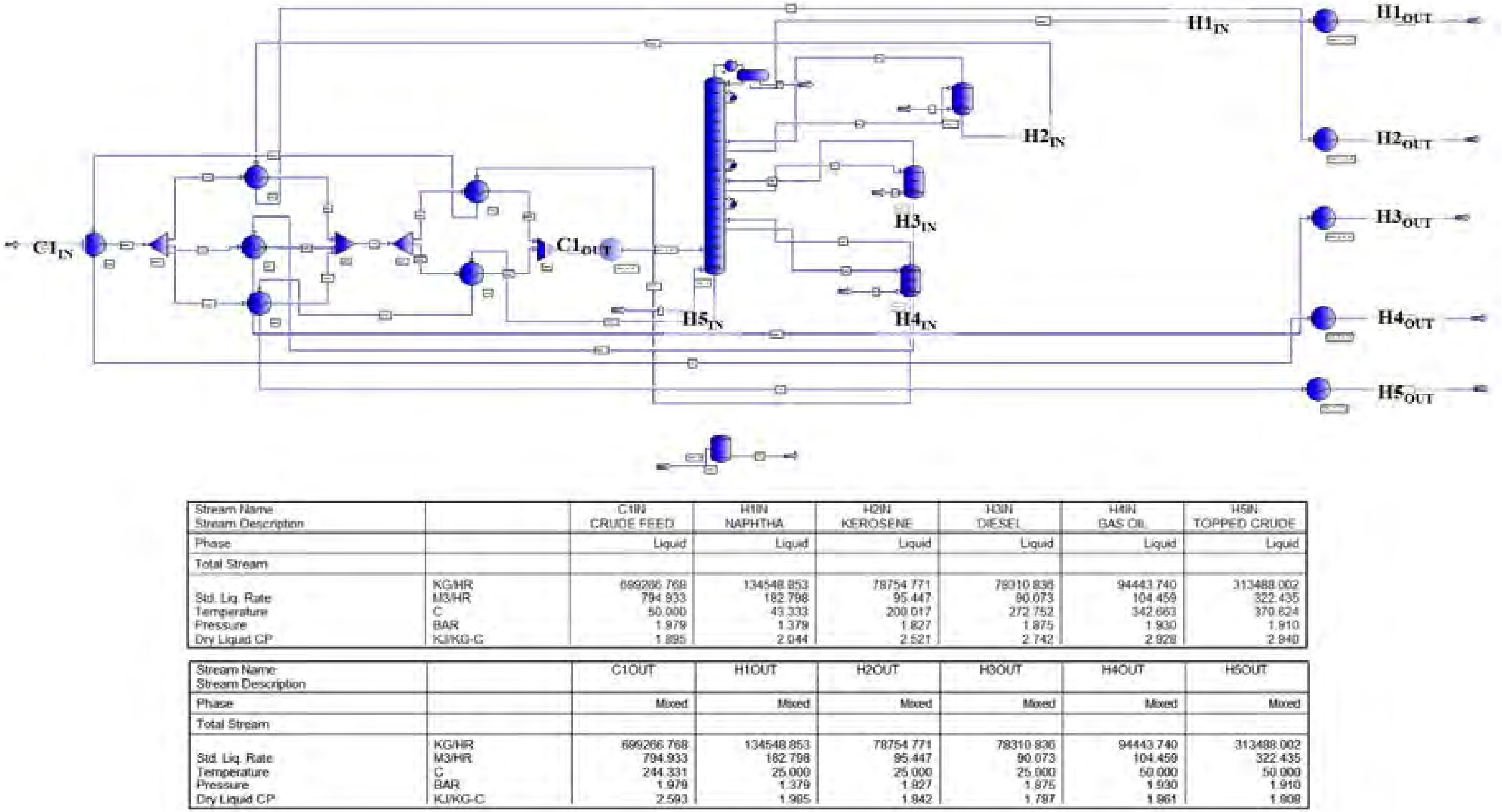


Figure 4.20 Validation of HEN from partitioning technique GAMS model by Pro/II simulation of 5 partitions.



Stream Name Stream Description		C1IN CRUDE FEED	H1IN NAPHTHA	H2IN KEROSENE	H3IN DIESEL	H4IN GAS OIL	H5IN TOPPED CRUDE
Phase		Liquid	Liquid	Liquid	Liquid	Liquid	Liquid
Total Stream							
	KG/HR	699266.768	134548.853	78754.771	78310.836	94443.740	313488.002
Std. Liq. Rate	M3/HR	794.933	182.798	95.447	90.073	104.459	322.435
Temperature	C	50.000	43.333	200.017	272.752	342.663	370.624
Pressure	BAR	1.979	1.379	1.827	1.875	1.930	1.910
Dry Liquid CP	KJ/KG-C	1.895	2.044	2.521	2.742	2.928	2.940

Stream Name Stream Description		C1OUT	H1OUT	H2OUT	H3OUT	H4OUT	H5OUT
Phase		Mixed	Mixed	Mixed	Mixed	Mixed	Mixed
Total Stream							
	KG/HR	699266.768	134548.853	78754.771	78310.836	94443.740	313488.002
Std. Liq. Rate	M3/HR	794.933	182.798	95.447	90.073	104.459	322.435
Temperature	C	244.331	25.000	25.000	25.000	50.000	50.000
Pressure	BAR	1.979	1.379	1.827	1.875	1.930	1.910
Dry Liquid CP	KJ/KG-C	2.593	1.985	1.842	1.787	1.961	1.908

Figure 4.21 HEN from 5 partition case study by Pro/II simulation.

Table 4.15 Duty data comparison between GAMS and Pro/II of 5 partitions

Heat Exchanger	GAMS Duty (kW)	Pro/II Duty (kW)	Percent Error (%)
E1	11,768.53	11,768.53	-
E2	39,008.56	39,008.56	-
E3	5,592.04	5,592.04	-
E4	9,758.41	9,758.41	-
E5	21,890.15	21,890.15	-
E6	5,527.18	5,527.18	-
CU1	1,378.39	1,378.40	0.00
CU2	2,823.50	2,808.20	0.54
CU3	2,537.65	2,519.20	0.73
CU4	1,319.68	1,281.80	2.96
CU5	6,568.67	6,403.20	2.58
HU1	82,012.35	81,808.80	0.25

Table 4.16 Area data comparison between GAMS and Pro/II of 5 partitions

Heat Exchanger	GAMS Area (m^2)	Pro/II Area (m^2)	Percent Error (%)
E1	459.48	461.69	0.48
E2	922.19	926.95	0.51
E3	311.49	313.27	0.57
E4	342.93	342.50	0.13
E5	1,171.99	1,197.71	2.15
E6	191.47	192.96	0.77
CU1	165.96	131.49	26.21
CU2	152.47	150.75	1.14
CU3	141.93	140.52	1.00
CU4	52.94	51.74	2.33
CU5	236.33	231.84	1.94
HU1	903.92	901.74	0.24

The increasing number of partitions case study use to show that even increase number of partitions results to lower average error of area calculation but it affects the optimum solution searching technique. On mathematical programming, increasing number of non-linear equation impact on solver which normally require good initial point to find optimum solution. Thus, partition 5 case study is the lowest average error of area calculation (Figure 4.22) but it actually not change much in partition 2 case study (3.39 % in partition 2 and 3.08 % in partition 3). However, divided Cp data into 2 partitions is not enough in reality work which cannot fit linear equation to nonlinearity Cp data for dome curve as shown in Case Study 3. Thus, three number of partitions divided is the most suitable and it is selected in further discussion and case study.

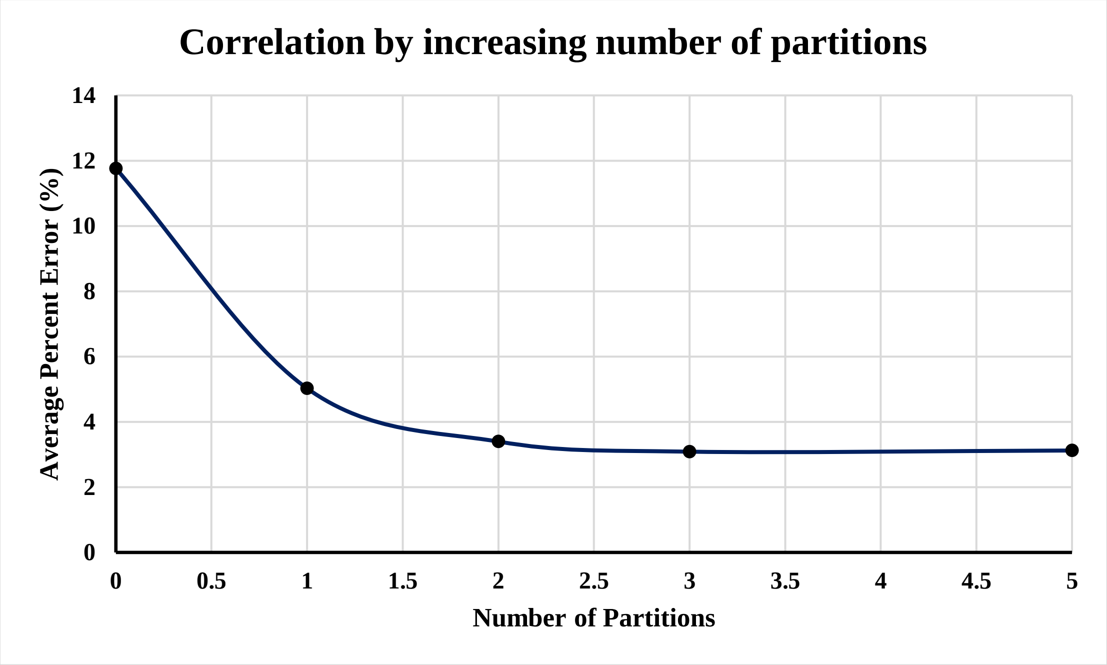


Figure 4.22 Correlation by increasing number of partitions from constant Cp to 5 partitions.

From accurate results by partitioning technique, economic results shown in Table 4.17 that error of TAC reduce from 0.51 % to 0.27 % (3 partitions) and the error effect on increasing TAC in reality design that increasing from 6,535,428.00 $/y to 6,568,864.35 $/y but partitioning technique has different TAC only 0.27 % that 6,554,118.00 $/y in GAMS and 6,536,373.03 $/y in Pro/II simulation. Capital cost error reduce from 2.15 % to 0.08 % but utility cost increase from 0.01 % to 0.33 %. The increasing error of utility occur in average energy calculation from linearization concept that linear line is absolutely not fit with non-linear line. Table 4.18 represent utility consumption error and Table 4.19 represent overall area of heat exchanger. Actually, the results of constant Cp may contain percent error of process area more than 6.16 % but it is reduced from their overestimate and underestimate calculation. For example, Table

4.4 shows the underestimate of exchanger area E1 (401.08 – 497.86 = –96.78 m^2) and overestimate of exchanger area E6 (102.97 – 78.73 = +24.24 m^2) that reduce error of overall area calculation when combining which impact to TAC error calculation (Table 4.19) With this reason, percent error will not obviously reduce when looking at overall area calculation (Table 4.19) and increasing error of utility calculation (Table 4.18) from partitioning technique. However, new model is always getting more accuracy results from area calculation and Cp calculation for each partition. All of this accurate data need in reality HEN design that could be found only in temperature-dependent specific heat capacity model.

Table 4.17 Economic cost data comparing between GAMS and Pro/II results of case study 1.

Cases	GAMS TAC ($/y)	Pro/II TAC ($/y)	TAC Error (%)	Capital Cost (%)	Utility ErrorCost (%)	Error
Constant Cp	6,535,428.00	6,568,864.35	0.51	2.15	0.01	
1 Partition	6,462,740.51	6,448,871.00	0.21	1.84	0.34	
2 Partitions	6,585,501.00	6,562,390.62	0.35	0.48	0.31	
3 Partitions	6,554,118.00	6,536,373.03	0.27	0.08	0.33	
5 Partitions	6,551,335.00	6,534,949.13	0.25	0.19	0.27	

Table 4.18 Utility data comparing between GAMS and Pro/II results of case study 1.

Cases	GAMS CU (kW)	Pro/II CU (kW)	CU Error (%)	GAMS HU (kW)	Pro/II HU (kW)	HU Error (%)
Constant Cp	14,702.15	14,650.50	0.35	82,109.79	82,109.90	0.00
1 Partition	12,092.37	11,942.40	1.26	79,631.82	79,373.30	0.33
2 Partitions	15,303.52	15,062.60	1.60	82,741.47	82,500.00	0.29
3 Partitions	14,654.39	14,417.20	1.65	82,100.93	81,846.20	0.31
5 Partitions	14,627.89	14,390.80	1.62	82,012.35	81,808.80	0.25

Table 4.19 Overall area data of heat exchanger comparing between GAMS and Pro/II results of case study 1.

Cases	GAMS Process Area (m^2)	Pro/II Process Area (m^2)	Process Area Error (%)	GAMS Utility Area (m^2)	Pro/II Utility Area (m^2)	Utility Area Error (%)
Constant Cp	3,321.89	3,476.69	4.45	1,657.47	1,614.97	2.63
1 Partition	3,570.62	3,719.32	4.00	1,567.04	1,518.78	3.18
2 Partitions	3,352.51	3,376.24	0.70	1,668.94	1,620.48	2.99
3 Partitions	3,402.30	3,444.17	1.22	1,630.40	1,584.28	2.91
5 Partitions	3,399.56	3,435.07	1.03	1,653.55	1,608.09	2.83

4.2 Case Study 2

Case study 2 and case study 3 are referenced from Kim and Bagajewicz (2017) to compare between our new technique; partitioning technique and ordinary empirical form fitted data of Cp; cubic equation technique. Case study 2 contains three hot streams and two cold streams which shows specific heat capacity parameter and process stream data in Table 4.20 and Table 4.21, respectively. EMAT is set at 10 °C. The constraints to design HEN for this case are allowing two or less stream splitting per stage on hot and cold streams and three stages are set. Ten years project life time are used for capital cost calculation with no interest rate.

Table 4.20 Cubic equation parameter of variable Cp for case study 2 ($Cp = a + bT + cT^2$).

Streams	a	b	c
H1	0.16135	0.01083	-2.49681×10^{-5}
H2	0.70678	0.00334	-5.05484×10^{-5}
H3	0.77039	0.00198	-2.46313×10^{-5}
C1	0.25693	0.01445	-5.13029×10^{-5}
C2	0.57327	0.00372	-5.25405×10^{-5}

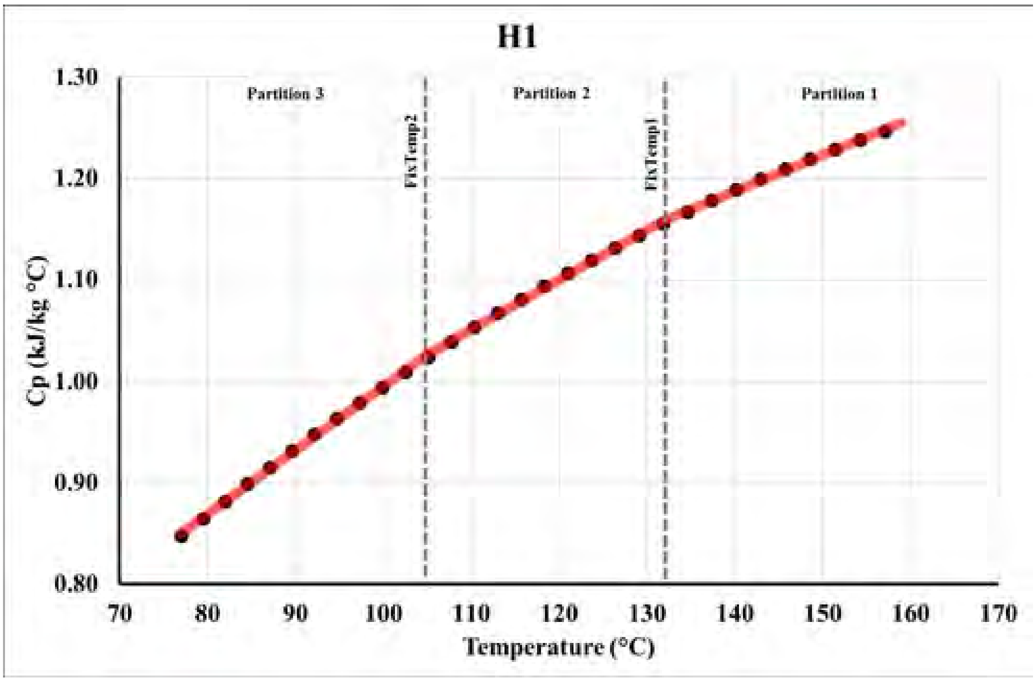
Table 4.21 Process streams data of case study 2.

Streams	T_{in} (°C)	T_{out} (°C)	F (kg/s)	h (kW/m^2 °C)	Cost ($/kW y)
H1	159	77	210	0.40	-
H2	267	88	18	0.30	-
H3	343	90	50	0.25	-
C1	26	127	90	0.15	-
C2	118	265	180	0.50	-
HU	500	499	-	0.53	100
CU	20	40	-	0.53	10

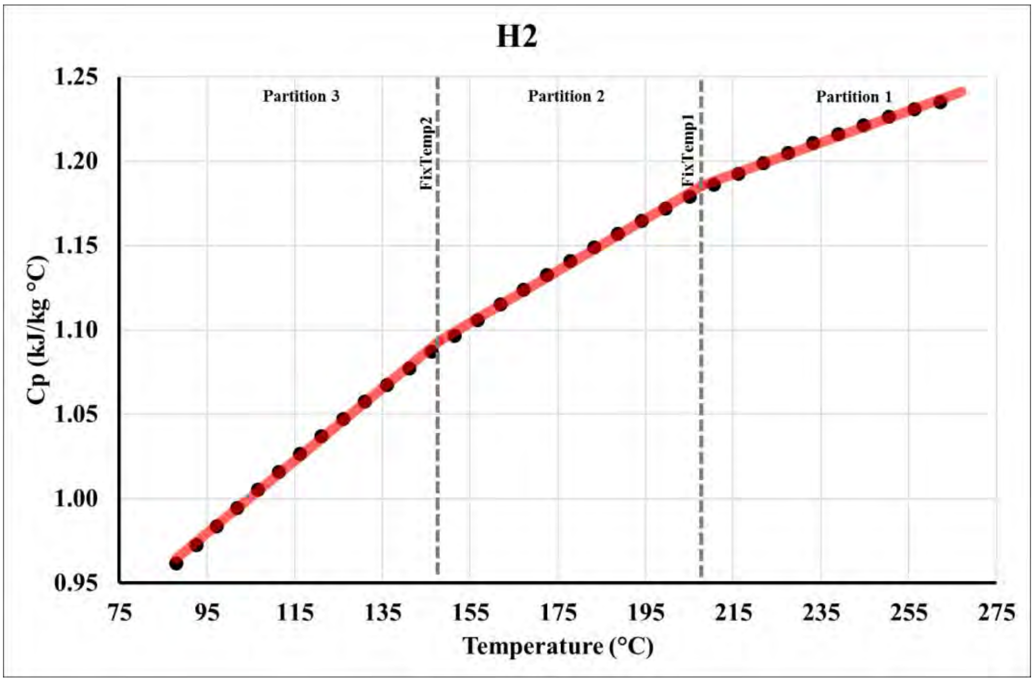
Annual investment cost ($/y) = 250,000 + 550 × (Area; m^2) for all exchangers

Table 4.22 Specific heat capacity linearization of case study 2.

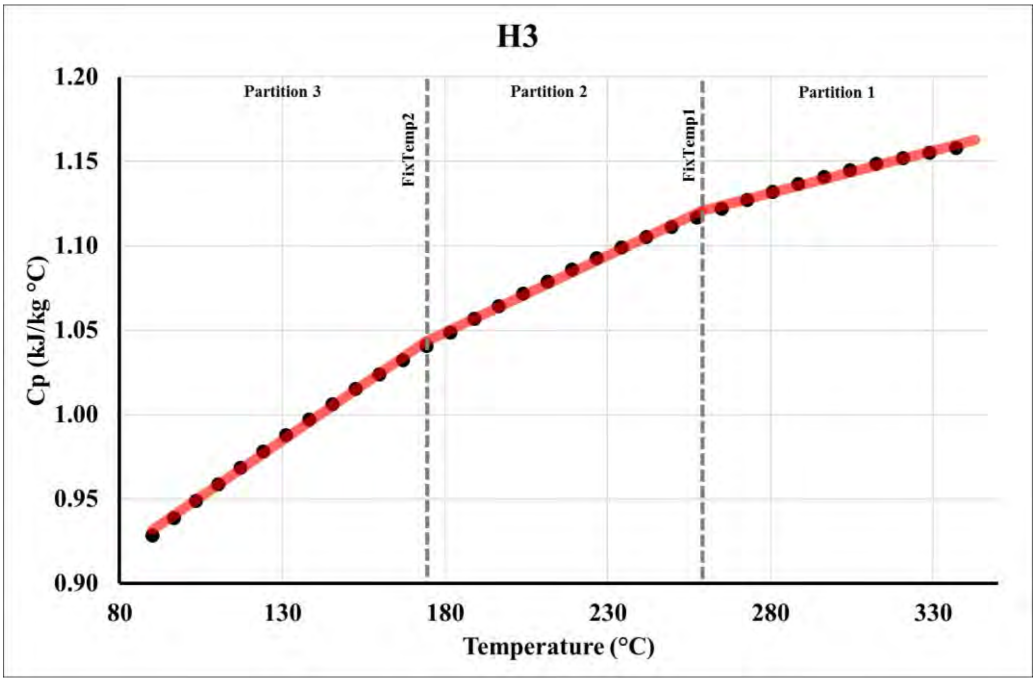
Streams	Partition Number (n)	$Cp_n = A_n \times T_{mean} + B_n$		R^2	Fix Temp 1 (°C)	Fix Temp 2 (°C)	$Cp_{average}$ (kJ/kg °C)
		A_n	B_n				
H1	1	0.0035628	0.6885283	0.99	132.00	105.00	1.0775422
	2	0.0049127	0.5103291	0.99			
	3	0.0062879	0.3661944	0.99			
H2	1	0.0009389	0.9903885	0.99	208.00	148.00	1.1268238
	2	0.0015405	0.8653706	0.99			
	3	0.0021471	0.7755966	0.99			
H3	1	0.0004972	0.9920692	0.99	259.00	174.00	1.0704344
	2	0.0009135	0.8843245	0.99			
	3	0.0013297	0.8118248	0.99			
C1	1	0.0031121	0.8784133	0.98	94.00	60.00	1.0182179
	2	0.0065494	0.5558720	0.99			
	3	0.0100380	0.3465562	0.99			
C2	1	0.0011928	0.8760714	0.99	216.00	167.00	1.0834680
	2	0.0017077	0.7648537	0.99			
	3	0.0022226	0.6788659	0.99			


H1
Cp (kJ/kg °C)
1.30
1.20
1.10
1.00
0.90
0.80
Partition 3
Partition 2
Partition 1
FixTemp2
FixTemp1
70
80
90
100
110
120
130
140
150
160
170
Temperature (°C)

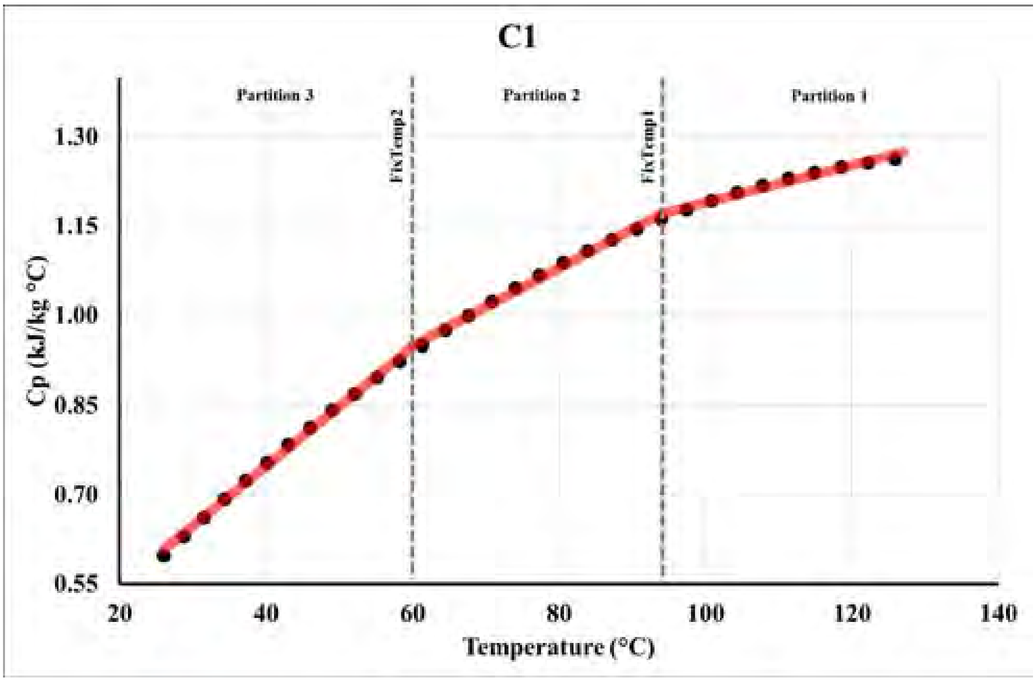
(a)


H2
Cp (kJ/kg °C)
1.25
1.20
1.15
1.10
1.05
1.00
0.95
Partition 3
Partition 2
Partition 1
FixTemp2
FixTemp1
75
95
115
135
155
175
195
215
235
255
275
Temperature (°C)

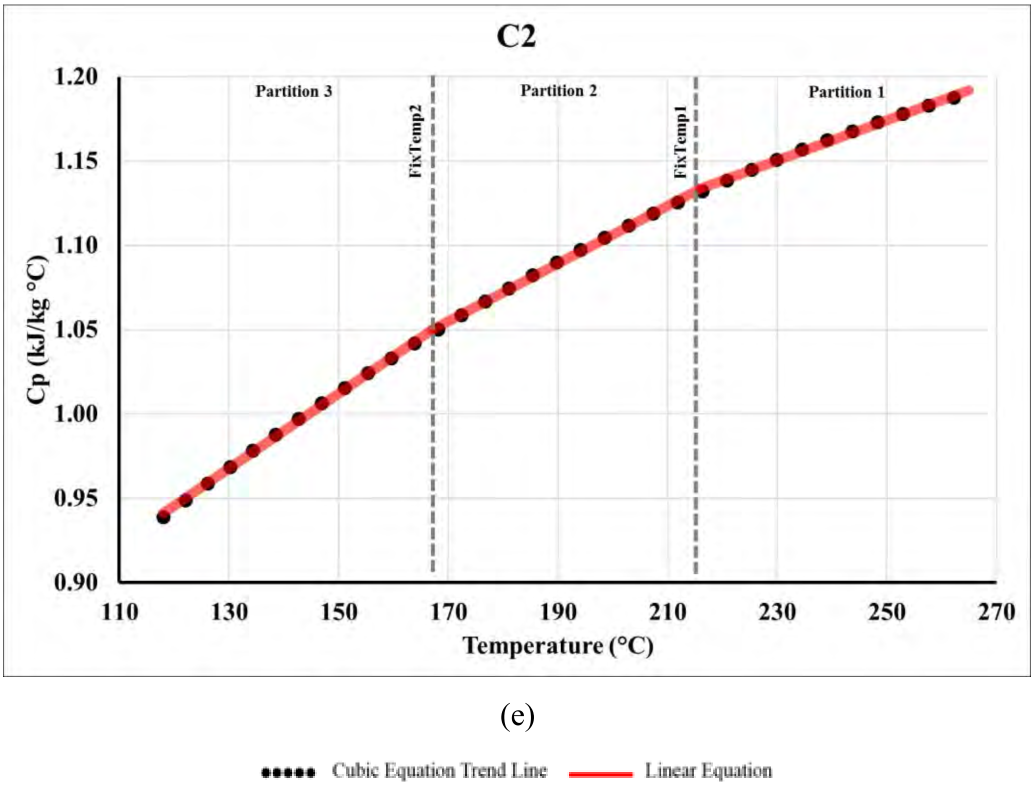
(b)


H3
Partition 3
Partition 2
Partition 1
FixTemp2
FixTemp1
1.20
1.15
1.10
1.05
1.00
0.95
0.90
Cp (kJ/kg °C)
80
130
180
230
280
330
Temperature (°C)

(c)


C1
Partition 3
Partition 2
Partition 1
FixTemp2
FixTemp1
1.30
1.15
1.00
0.85
0.70
0.55
Cp (kJ/kg °C)
20
40
60
80
100
120
140
Temperature (°C)

(d)



(e)

Figure 4.23 Temperature-dependent Cp graph and partitioning of case study 2; (a-c) represent for hot stream (H1-H3), respectively. (d-e) represent for cold stream (C1-C2), respectively.

Linear parameters of specific heat capacity are shown in Table 4.22 and Figure 4.23 shows Cp graph of the streams. HEN result is shown in Figure 4.25 which can be observed that Cp change from stage to stage such as cold stream C2 that vary from 0.97 kJ/kg °C in stage K2 to 1.07 kJ/kg °C in stage K1. The results data are concluded in Table 4.23 and Table 4.24. Capital cost of new model is higher than previous but it affects utility consumption that greatly decrease. TAC of new model is 1,737,401.57 \$/y (from capital cost 551,379.60 \$/y and utility cost 1,186,021.97 \$/y) which is slightly low than previous solution Figure 4.25about 2.59 % (1,783,724.90 \$/y).

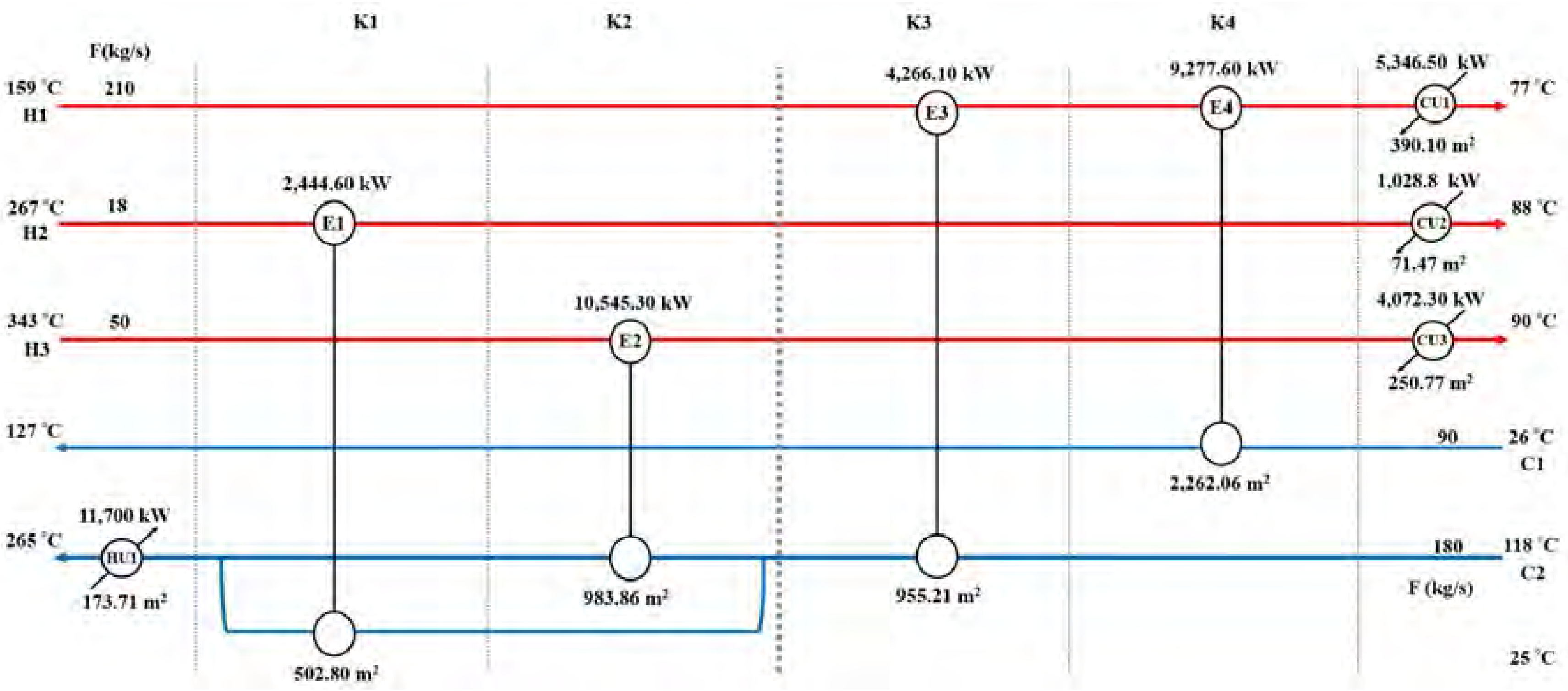


Figure 4.24 Previous solution of case study 2 from Kim and Bagajewicz (2017).

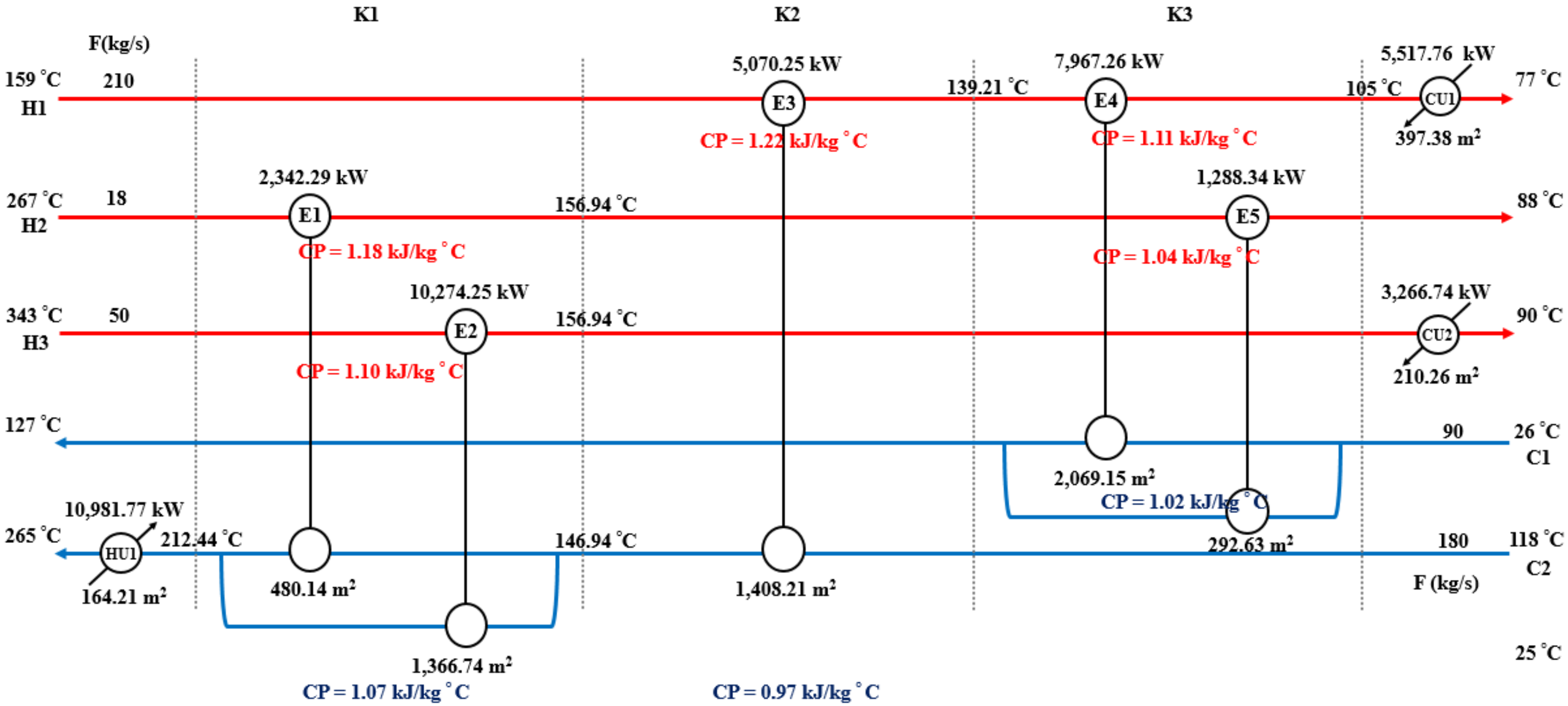


Figure 4.25 GAMS results of temperature-dependent Cp for cast study 2.

Table 4.23 HEN results of case study 2.

Heat Exchanger	Duty (kW)	Area (m^2)
E1	2,342.29	480.14
E2	10,274.25	1,366.74
E3	5,070.25	1,408.21
E4	7,967.26	2,069.15
E5	1,288.34	292.63
CU1	5,517.76	397.38
CU2	3,266.74	210.26
HU1	10,981.77	164.21

Table 4.24 Economic cost data of case study 2.

Cases	TAC (\$/y)	Capital Cost (\$/y)	Utility Cost (\$/y)
Previous Solution	1,783,724.90	507,448.90	1,276,276.00
New Solution	1,737,401.57	551,379.60	1,186,021.97

4.3 Case Study 3

Last case study is study crude preheat train including 11 hot streams and 2 cold streams which shows process stream data in Table 4.26. Cubic equation in Table 4.25 represent Cp of process streams. Table 4.27 shows Cp linearization of this case study. Non-linear specific heat capacity graphs show in Figure 4.26 and it can be observed that every graph is dome curve especially decrease slope of C2 stream. It shows that our new model capability to design large problems and dome curve Cp graph. EMAT is set at 10 °C. No splitting allow and 4 splitting allow for hot streams and cold streams, respectively. To reduce to much complex of model, stage of calculation is set at 5 stages. Finally, project life time is specified at 10 years for capital cost economic calculation with no interest rate.

Table 4.25 Cubic equation parameter of variable Cp for case study 3 ($Cp = a + 2bT + 3cT^2$).

Streams	a	b	c
H1	1.27	0.011	-3.54×10^{-5}
H2	1.70	0.004	-6.29×10^{-6}
H3	1.28	0.008	-1.93×10^{-5}
H4	1.87	0.003	-4.54×10^{-6}
H5	1.94	0.003	-6.02×10^{-6}
H6	-0.20	0.013	-2.05×10^{-5}
H7	-1.41	0.015	-1.74×10^{-5}
H8	-0.31	0.006	-5.95×10^{-6}
H9	0.89	0.010	-2.01×10^{-5}
H10	1.89	0.004	-4.06×10^{-6}
H11	1.01	0.003	-4.03×10^{-6}
C1	0.89	0.014	-4.75×10^{-5}
C2	2.48	0.001	-2.38×10^{-6}

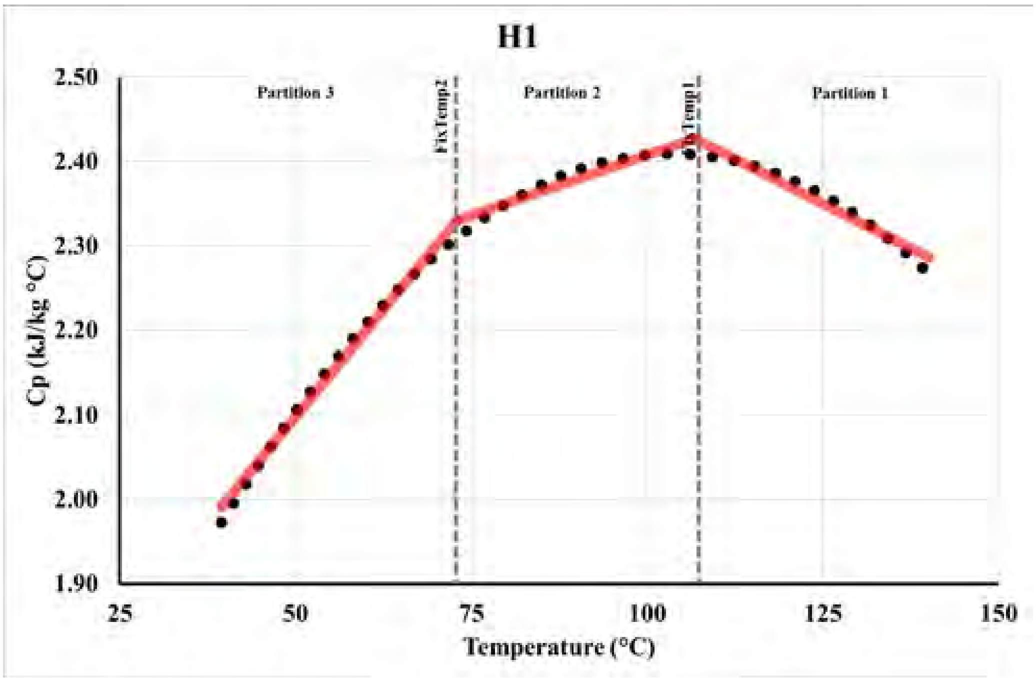
Table 4.26 Process streams data of case study 3.

Streams	T_{in} (°C)	T_{out} (°C)	F (kg/s)	h (kW/m^2 °C)	Cost ($/kW y)
H1	140.2	39.5	46.30	0.26	-
H2	248.8	110	12.70	0.72	-
H3	170.1	60	14.75	0.45	-
H4	277	121.9	9.83	0.57	-
H5	250.6	90	55.08	0.26	-
H6	210	163	46.03	0.33	-
H7	303.6	270.2	82.03	0.41	-
H8	360	290	23.42	0.47	-
H9	178.6	108.9	19.14	0.60	-
H10	359.6	280	7.66	0.47	-
H11	290	115	23.42	0.47	-
C1	30	130	96.41	0.26	-
C2	130	350	96.64	0.72	-
HU	500	499	-	0.53	100
CU	20	40	-	0.53	10

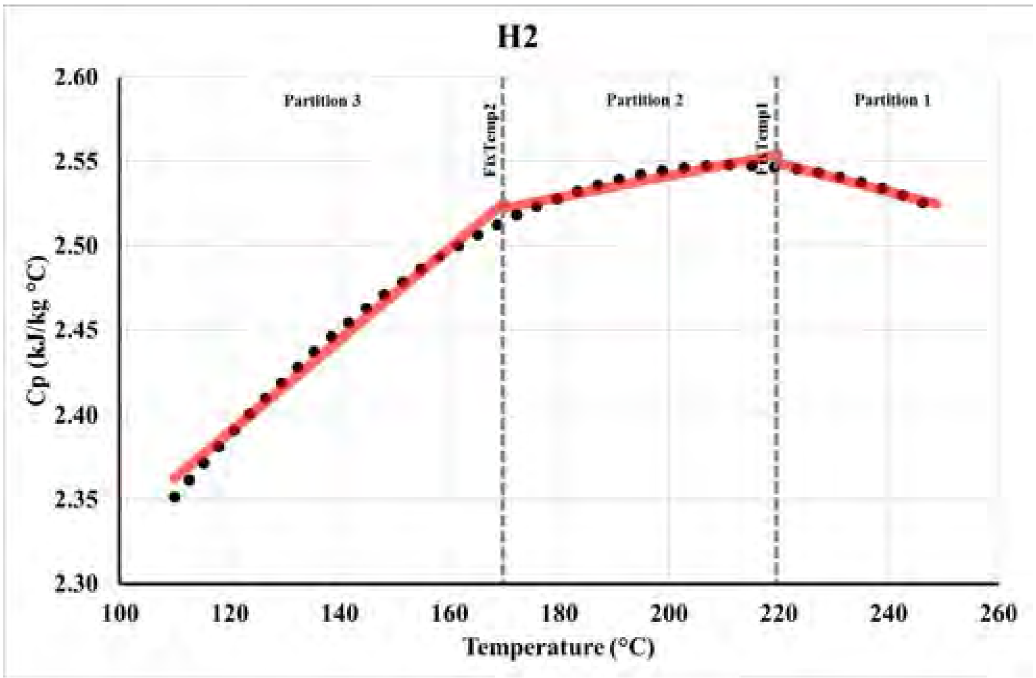
Annual investment cost ($/y) = 250,000 + 550 × (Area) for all exchangers.

Table 4.27 Specific heat capacity linearization of case study 3.

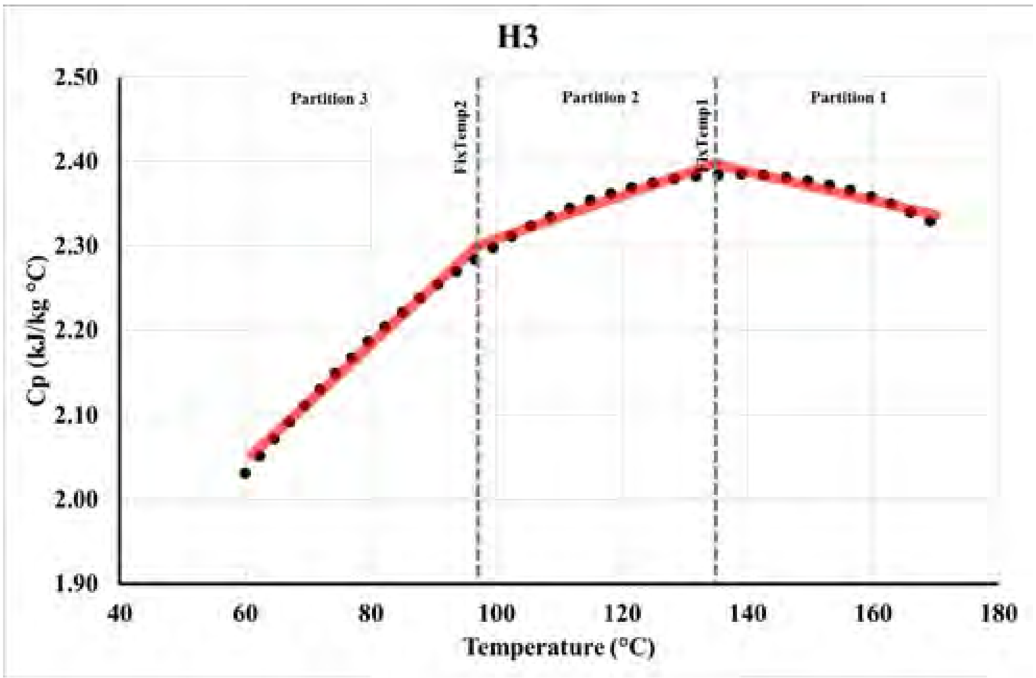
Streams	Partition Number (n)	$Cp_n = A_n \times T_{mean} + B_n$		R^2	Fix Temp 1 (°C)	Fix Temp 2 (°C)	$Cp_{average}$ (kJ/kg °C)
		A_n	B_n				
H1	1	-0.0042361	2.8801306	0.95	107.00	73.00	2.2989566
	2	0.0028840	2.1193876	0.90			
	3	0.0100945	1.5928813	0.99			
H2	1	-0.0008491	2.7360399	0.97	220.00	170.00	2.4974261
	2	0.0006407	2.4134432	0.87			
	3	0.0027164	2.0640023	0.99			
H3	1	-0.0017073	2.6272855	0.91	135.00	97.00	2.2956067
	2	0.0025672	2.0517684	0.95			
	3	0.0068518	1.6347533	0.99			
H4	1	-0.0008509	2.7284277	0.96	226.00	173.00	2.4974405
	2	0.0005656	2.4087709	0.90			
	3	0.0019940	2.1613910	0.99			
H5	1	-0.0021079	2.8456216	0.99	198.00	144.00	2.3990249
	2	-0.0001765	2.4635413	0.32			
	3	0.0017740	2.1826722	0.98			
H6	1	0.0010925	2.3205775	0.95	195.00	179.00	2.4984098
	2	0.0029990	1.9491175	0.99			
	3	0.0049670	1.5968455	0.99			
H7	1	-0.0011524	3.2372663	0.98	293.00	281.00	2.8953734
	2	0.0000372	2.8889310	0.04			
	3	0.0012322	2.5529276	0.98			
H8	1	-0.0004414	1.8570674	0.94	337.00	314.00	1.6972356
	2	0.0003797	1.5803566	0.92			
	3	0.0012186	1.3170632	0.99			
H9	1	-0.0001934	2.5777379	0.22	156.00	132.00	2.4942332
	2	0.0026336	2.1372452	0.98			
	3	0.0055132	1.7570131	0.99			
H10	1	-0.0004516	3.3553994	0.97	334.00	307.00	3.1962360
	2	0.0001926	3.1403379	0.83			
	3	0.0008503	2.9384179	0.99			
H11	1	-0.0003323	1.8355475	0.76	233.00	173.00	1.6981135
	2	0.0010915	1.5043480	0.97			
	3	0.0025181	1.2570753	0.99			
C1	1	-0.0043475	2.7120050	0.92	97.00	64.00	2.0984582
	2	0.0050575	1.7997200	0.94			
	3	0.0146050	1.1902475	0.99			
C2	1	-0.0024268	3.1622508	0.99	270.00	200.00	2.5198504
	2	-0.0013558	2.8713077	0.99			
	3	-0.0003562	2.6713877	0.88			


H1
Partition 3
Partition 2
Partition 1
FixTemp2
FixTemp1
Cp (kJ/kg °C)
2.50
2.40
2.30
2.20
2.10
2.00
1.90
25
50
75
100
125
150
Temperature (°C)

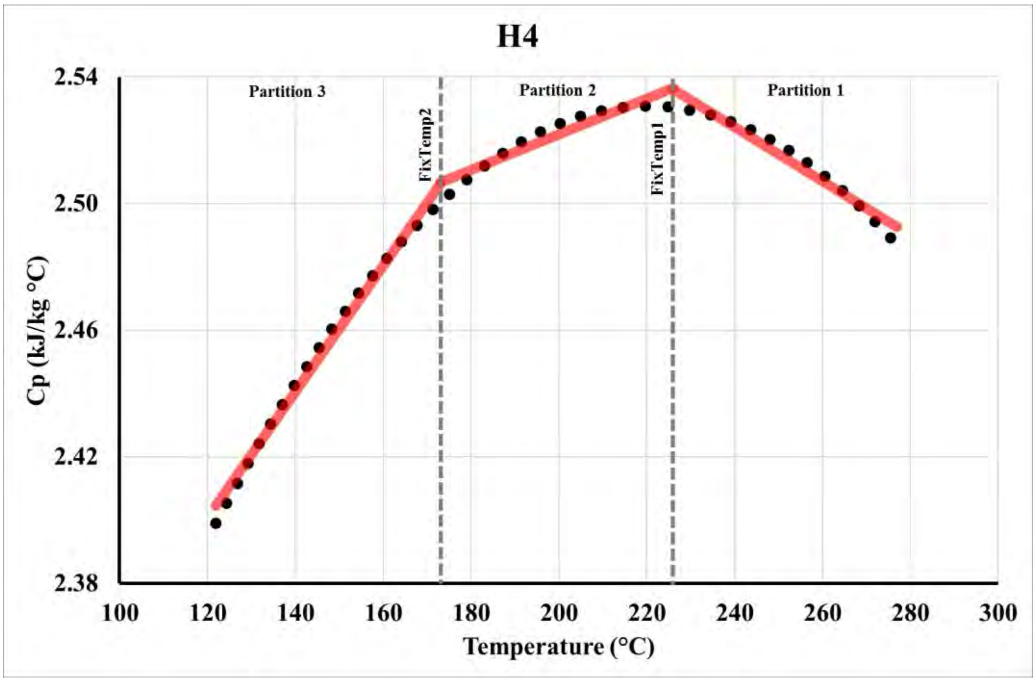
(a)


H2
Partition 3
Partition 2
Partition 1
FixTemp2
FixTemp1
Cp (kJ/kg °C)
2.60
2.55
2.50
2.45
2.40
2.35
2.30
100
120
140
160
180
200
220
240
260
Temperature (°C)

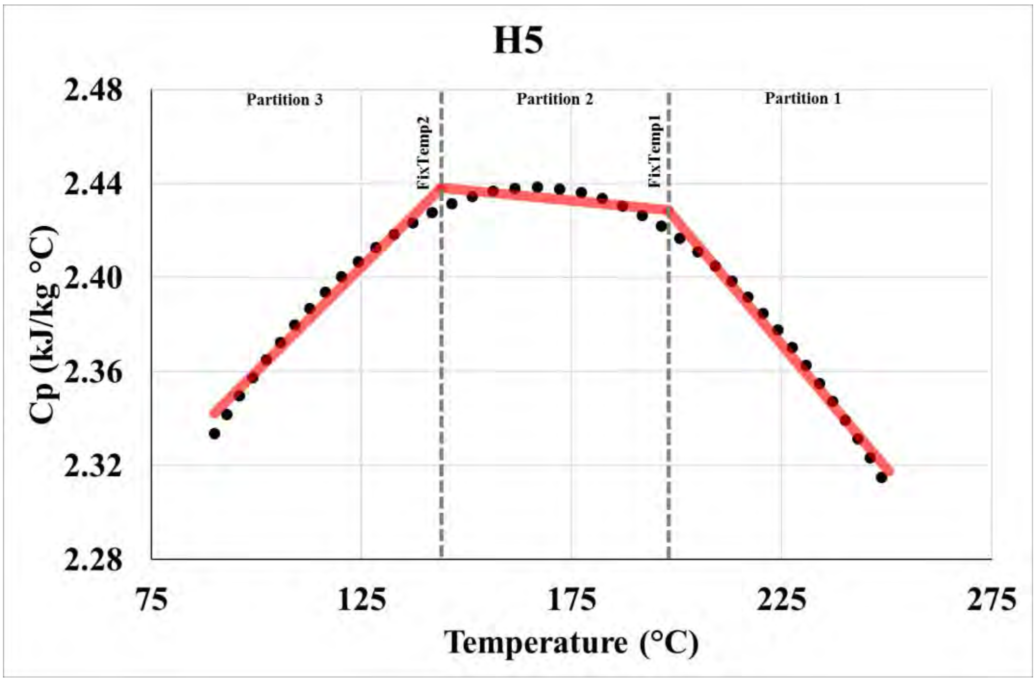
(b)


H3
Partition 3
FixTemp2
Partition 2
FixTemp1
Partition 1
Cp (kJ/kg °C)
2.50
2.40
2.30
2.20
2.10
2.00
1.90
40
60
80
100
120
140
160
180
Temperature (°C)

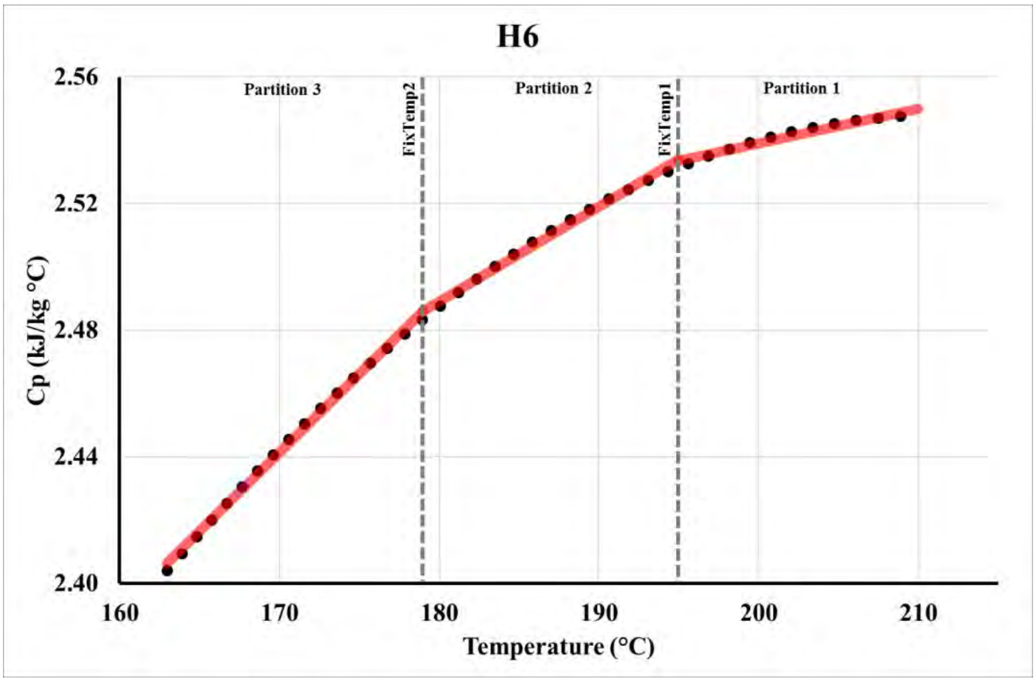
(c)


H4
Partition 3
FixTemp2
Partition 2
FixTemp1
Partition 1
Cp (kJ/kg °C)
2.54
2.50
2.46
2.42
2.38
100
120
140
160
180
200
220
240
260
280
300
Temperature (°C)

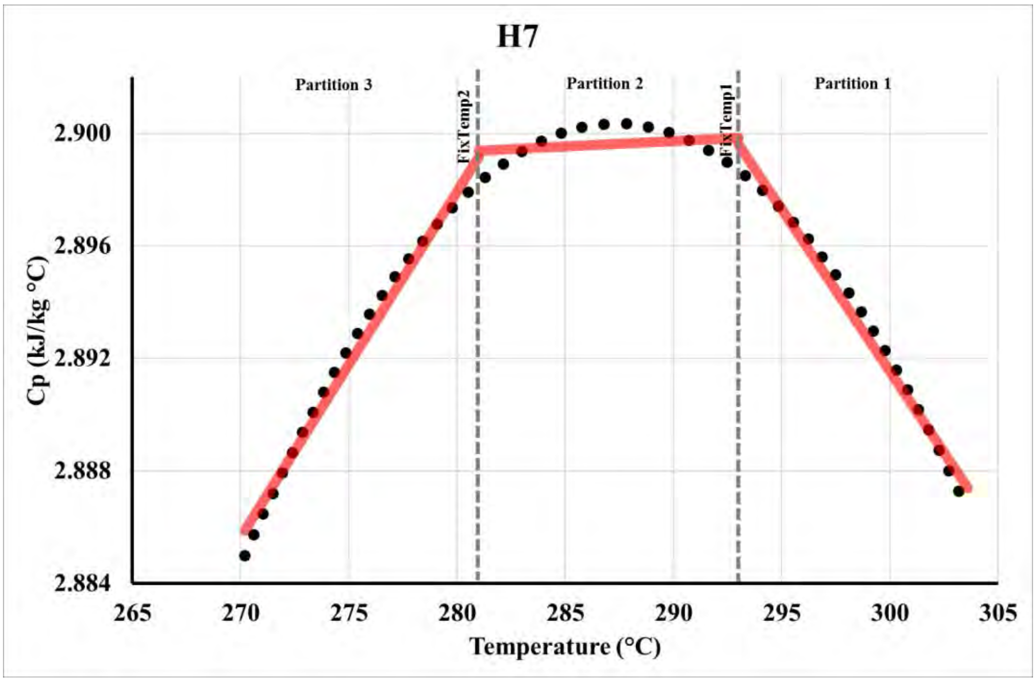
(d)


H5
Partition 3
Partition 2
Partition 1
FixTemp2
FixTemp1
2.48
2.44
2.40
2.36
2.32
2.28
Cp (kJ/kg °C)
75
125
175
225
275
Temperature (°C)

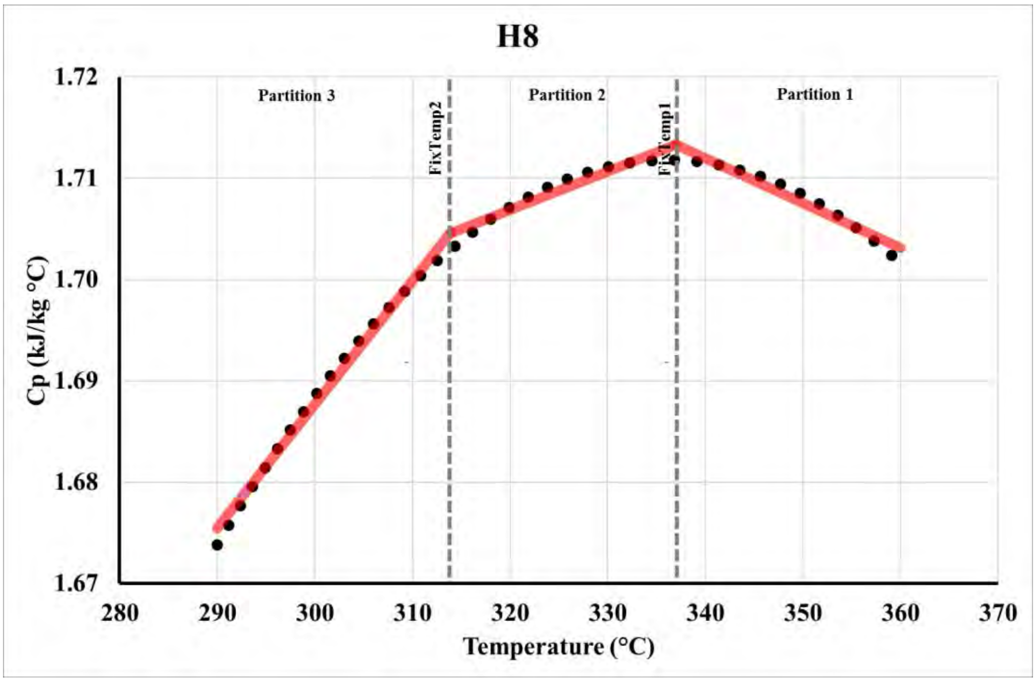
(e)


H6
Partition 3
Partition 2
Partition 1
FixTemp2
FixTemp1
2.56
2.52
2.48
2.44
2.40
Cp (kJ/kg °C)
160
170
180
190
200
210
Temperature (°C)

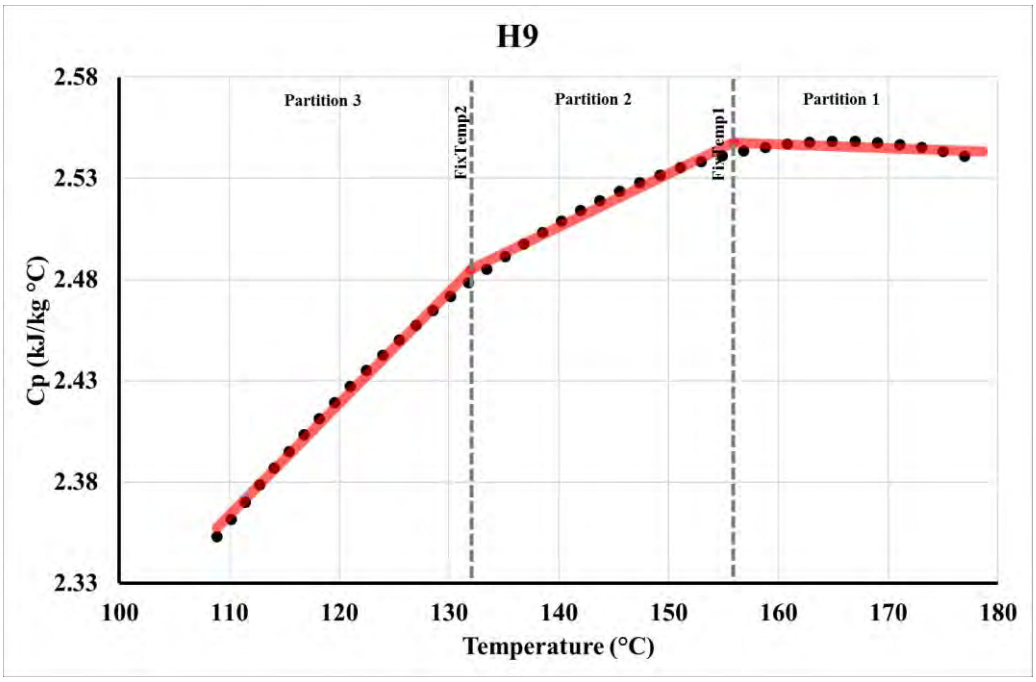
(f)


H7
Partition 3
Partition 2
Partition 1
FixTemp2
FixTemp1
Cp (kJ/kg °C)
2.900
2.896
2.892
2.888
2.884
265
270
275
280
285
290
295
300
305
Temperature (°C)

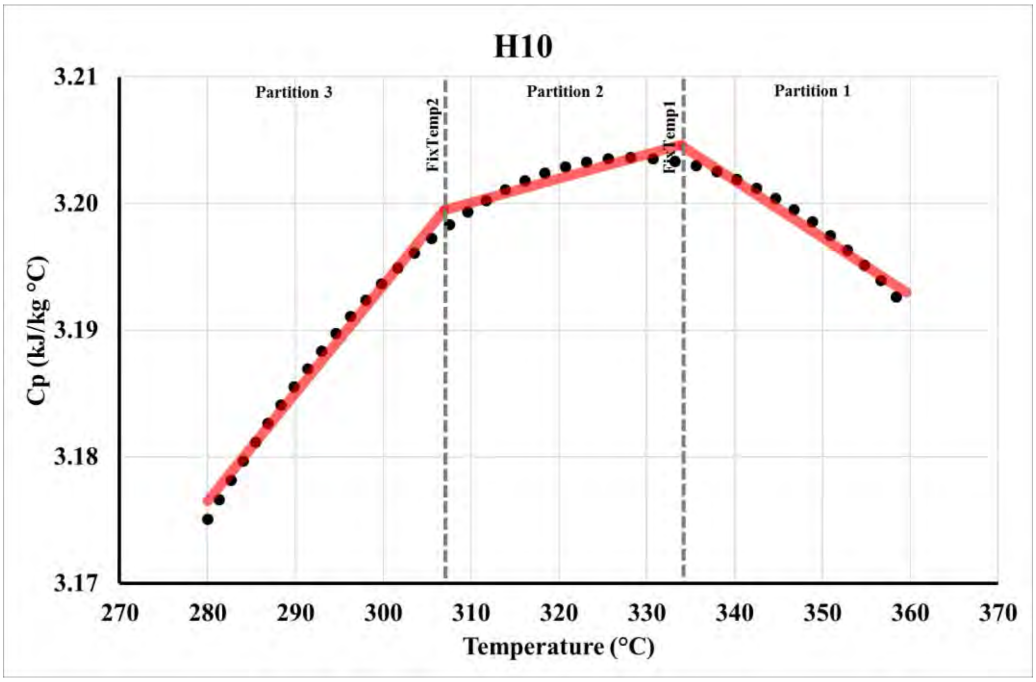
(g)


H8
Partition 3
Partition 2
Partition 1
FixTemp2
FixTemp1
Cp (kJ/kg °C)
1.72
1.71
1.70
1.69
1.68
1.67
280
290
300
310
320
330
340
350
360
370
Temperature (°C)

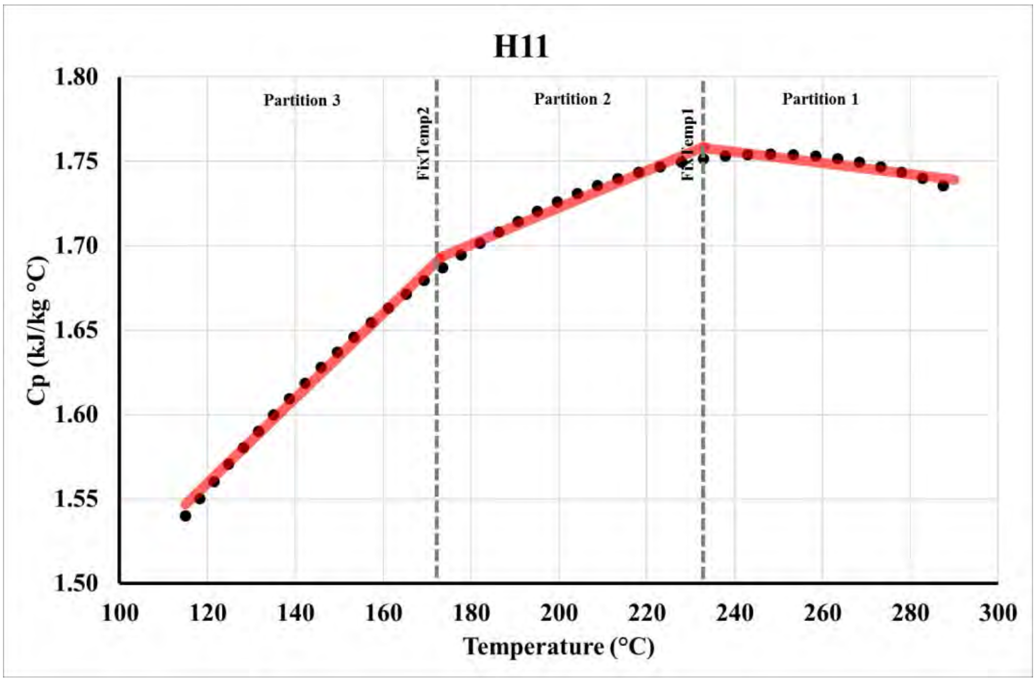
(h)


H9
Partition 3
Partition 2
Partition 1
FixTemp2
FixTemp1
2.58
2.53
2.48
2.43
2.38
2.33
Cp (kJ/kg °C)
100
110
120
130
140
150
160
170
180
Temperature (°C)

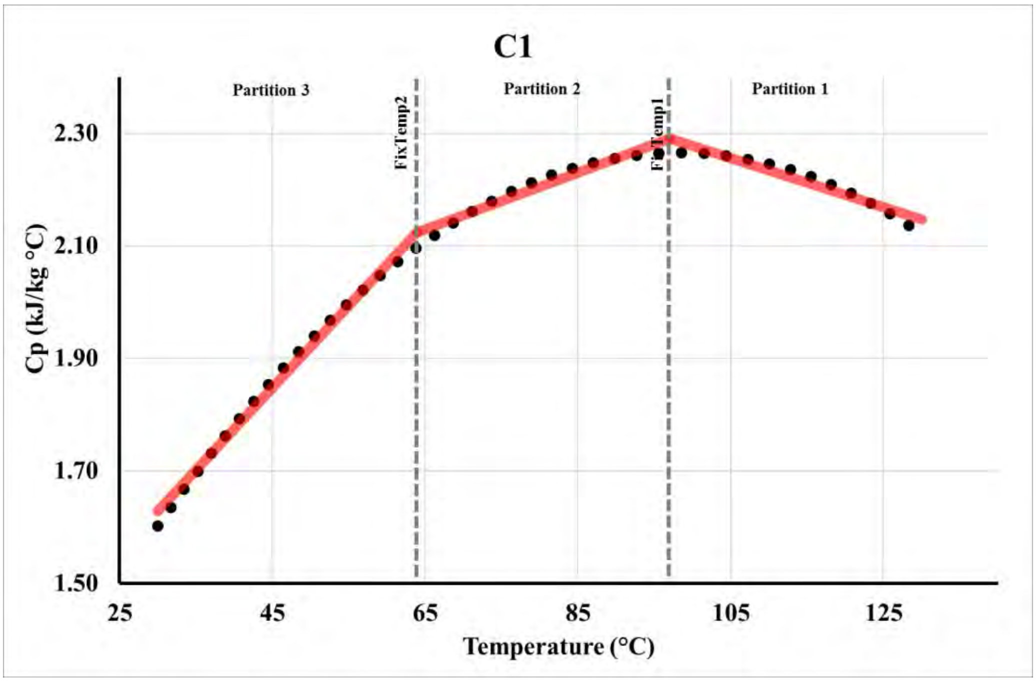
(i)


H10
Partition 3
Partition 2
Partition 1
FixTemp2
FixTemp1
3.21
3.20
3.19
3.18
3.17
Cp (kJ/kg °C)
270
280
290
300
310
320
330
340
350
360
370
Temperature (°C)

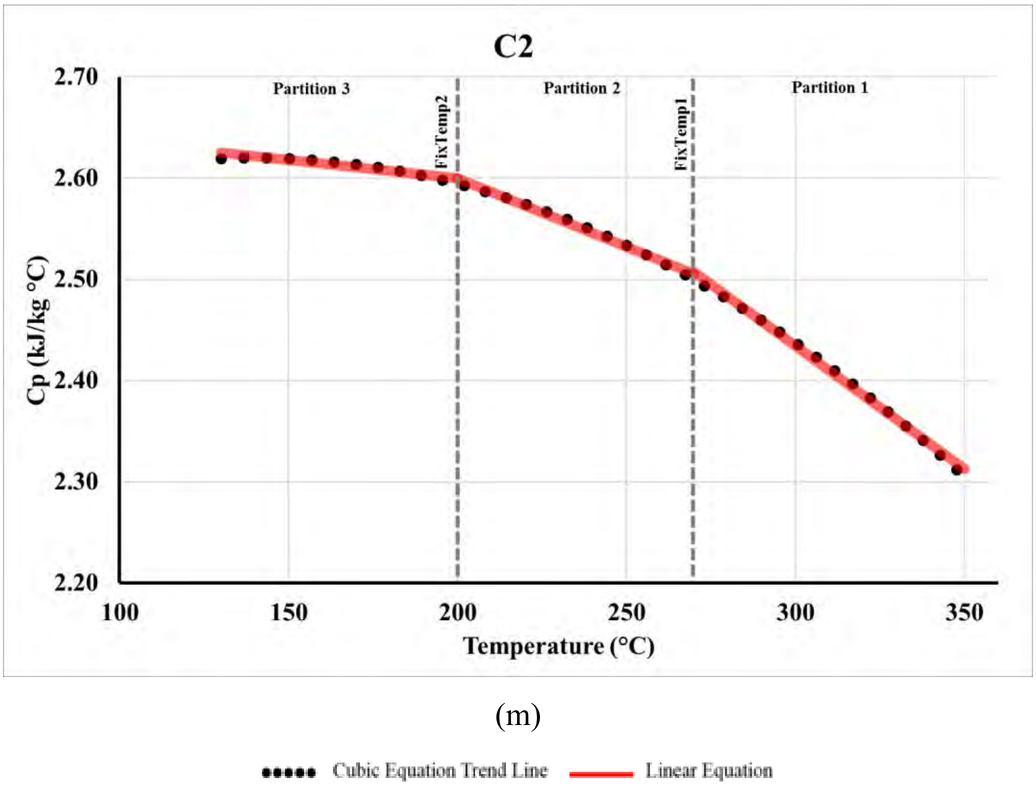
(j)


H11
Partition 3
Partition 2
Partition 1
FixTemp2
FixTemp1
Cp (kJ/kg °C)
1.80
1.75
1.70
1.65
1.60
1.55
1.50
100
120
140
160
180
200
220
240
260
280
300
Temperature (°C)

(k)


C1
Partition 3
Partition 2
Partition 1
FixTemp2
FixTemp1
Cp (kJ/kg °C)
2.30
2.10
1.90
1.70
1.50
25
45
65
85
105
125
Temperature (°C)

(l)



(m)

Figure 4.26 Temperature-dependent Cp graph and partitioning of case study 3; (a-k) represent for hot stream (H1-H11), respectively. (m-l) represent for cold stream (C1-C2), respectively.

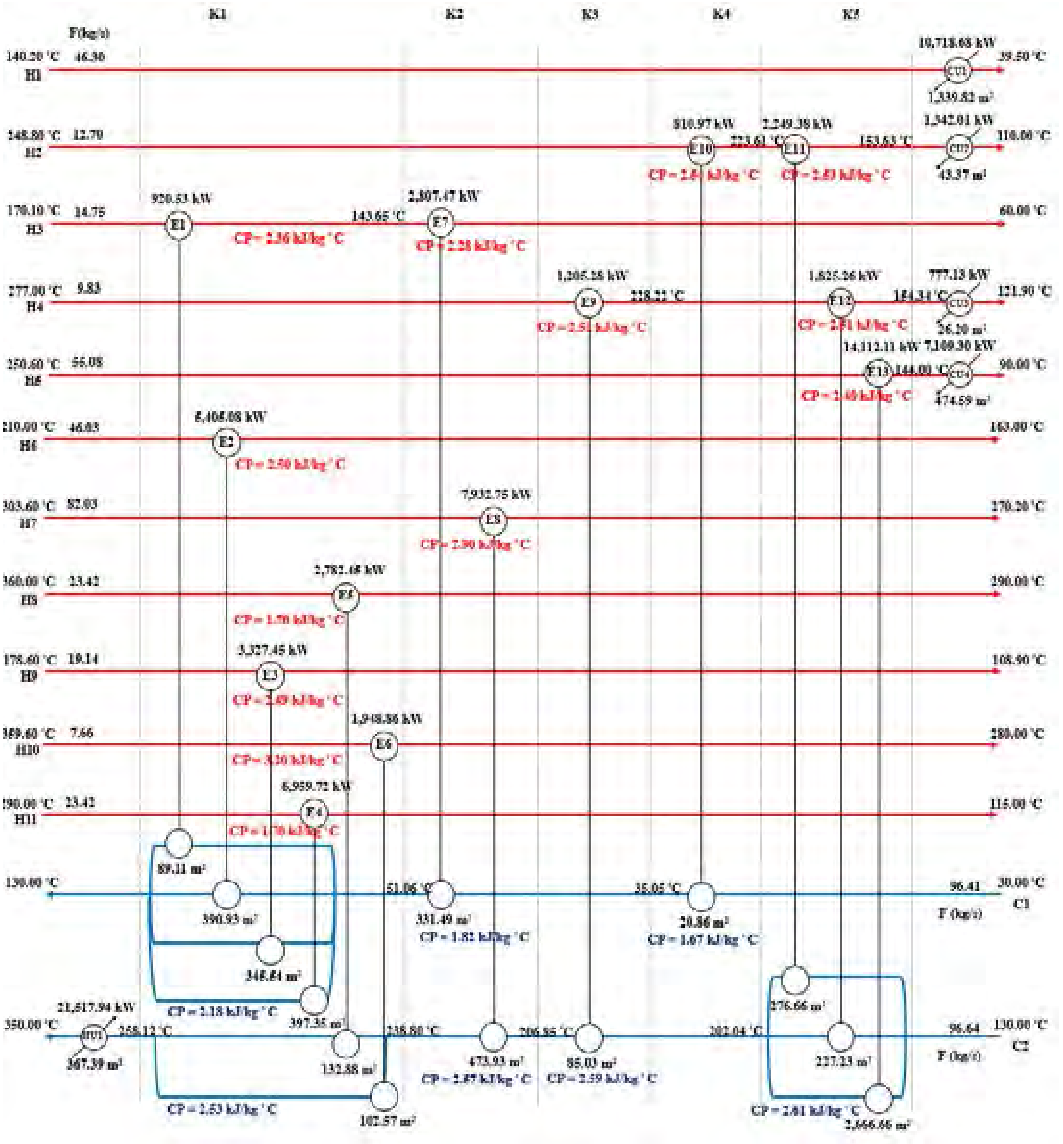


Figure 4.27 GAMS results of temperature-dependent Cp for case study 3.

13 heat exchangers, 1 hot utility and 4 cold utilities exchanger are installed in this case study which show HEN and data in Figure 4.27 and Table 4.26, respectively. TAC of new solution is 3,229,803.75 $/y from capital cost 878,538.55 $/y and utility cost 2,351,265.20 $/y. Compared to previous solution (Figure 4.28), install 2 more exchanger which increasing capital cost from 874,980.00 to 878,538.55 $/y but the results show that utility cost decrease from 2,576,605.00 to 2,351,265.20 $/y. It reduces TAC 6.42 % (3,451,585.00 to 3,229,803.75 $/y) which shows comparison data

between new solution and previous solution in Table 4.27. The reason, that our model has lower TAC in case study 2 and case study 3, is no lower limits specified by user in calculation step which different from previous solution that has constraint of lower limits of total area and total utility heat consumption. That mean our technique do not require any special constraint or limits. From this main reason, our model with no constraint has better solution from installed more heat exchanger to reduce utility consumption. Another advantage from our technique is no special require for solving strategy at this step. The three partitioning technique is solved under commercial solver become friendly interface model. In contrast to many author, such as Kim *et al.* (2017) used RYSIA for solve their problem and Li *et al.* (2012) used genetic algorithm, it make model more complex and their model of Cp variable cannot solve the problem by normal solver in GAMS which compare all reasons in Table 4.30.

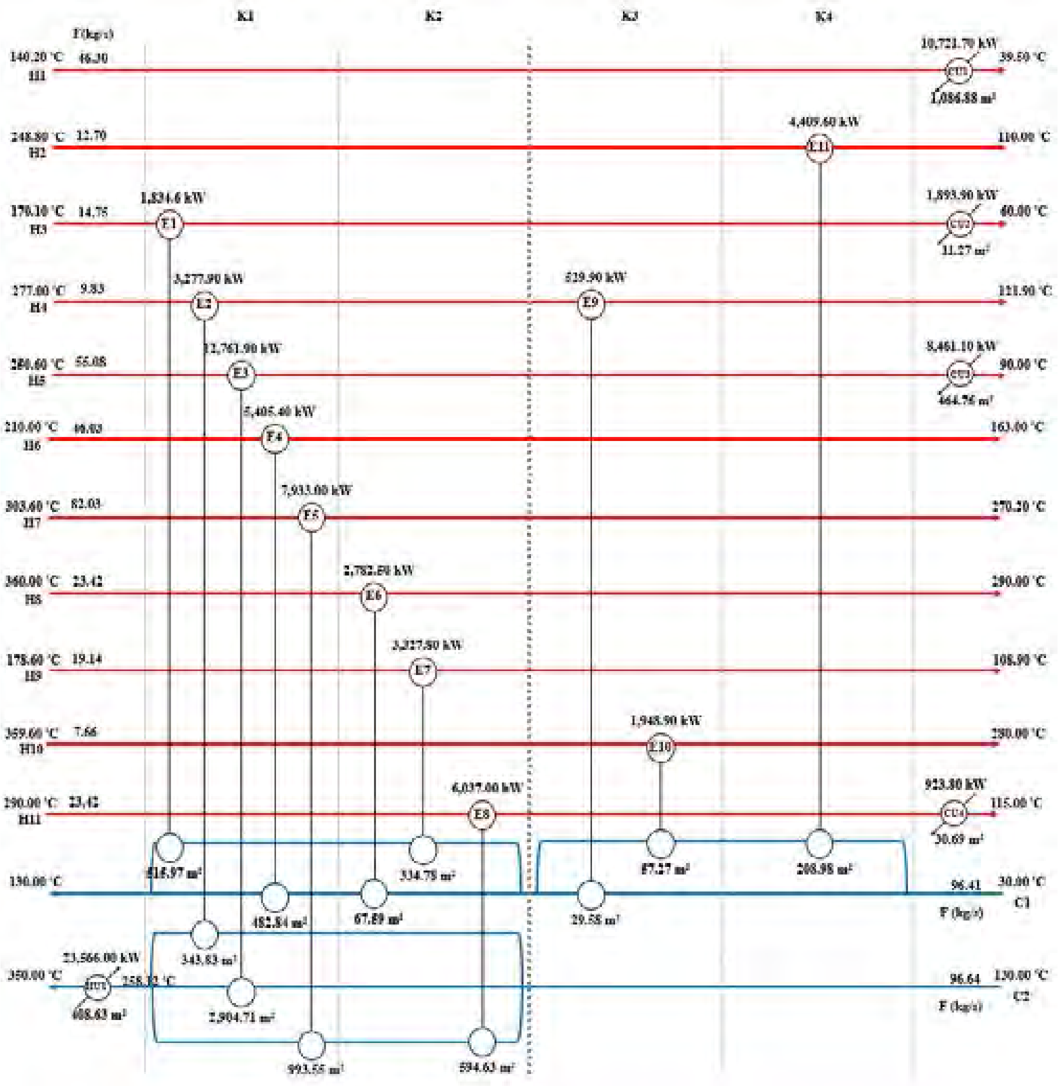


Figure 4.28 Previous solution of case study 3 from Kim and Bagajewicz (2017).

Table 4.28 HEN results of case study 3.

Heat Exchanger	Duty (kW)	Area (m^2)
E1	920.53	89.11
E2	5,405.08	390.93
E3	3,327.45	345.54
E4	6,959.72	397.35
E5	2,782.45	132.88
E6	1,948.86	102.57
E7	2,807.47	331.49

E8	7,932.75	473.93
E9	1,205.28	85.03
E10	810.97	20.86
E11	2,249.38	276.66
E12	1,825.26	227.23
E13	14,112.11	2,666.66
CU1	10,718.68	1,339.82
CU2	1,342.01	43.37
CU3	777.13	26.20
CU4	7,109.30	474.59
HU1	21,517.94	367.39

Table 4.29 Economic cost data of case study 3.

Cases	TAC ($/y)	Capital Cost ($/y)	Utility Cost ($/y)
Previous Solution	3,451,585.00	874,980.00	2,576,605.00
New Solution	3,229,803.75	878,538.55	2,351,265.20

Table 4.30 Comparison between partitioning technique and Kim and Bagajewicz (2017) technique of case study 2 and 3.

Features	Partitioning Technique	Kim and Bagajewicz (2017)
Model	Stage-wise superstructure	Stages/Substages Superstructure (Based on stage-wise superstructure)
EMAT (°C)	10	10
Solver Technique	Dicopt solver	RYSIA solution strategy (Global optimize solver)
Upper bound	-	Fixed (MINLP model)
Lower bound	-	Fixed (Pinch analysis)
Number of splitting	Based on journal	Based on journal
Economic cost	Based on journal	Based on journal

Moreover, our model can solve alternative design by fix computational time. From author experience, good solution is obtained by less computational time than 1,800 s because good solution is generally obtained by good initial point. Just a few cases that good solution is solved in other range of computational time. However, all of these solutions are guaranteed that they are not global solution but it is nearly perfect one which user can interact and use it as good starting point. The alternative solutions of case study 3 are shown only 3 alternatives by fix computational time divided to; no fix, 600 s and 3,600 s. They show that even using computational time about 1 hour but the greatest solution still be on 2,400 s but the advantage of alternative solution is to give the option for real design.

4.3.1 Alternative 1 (First Solution)

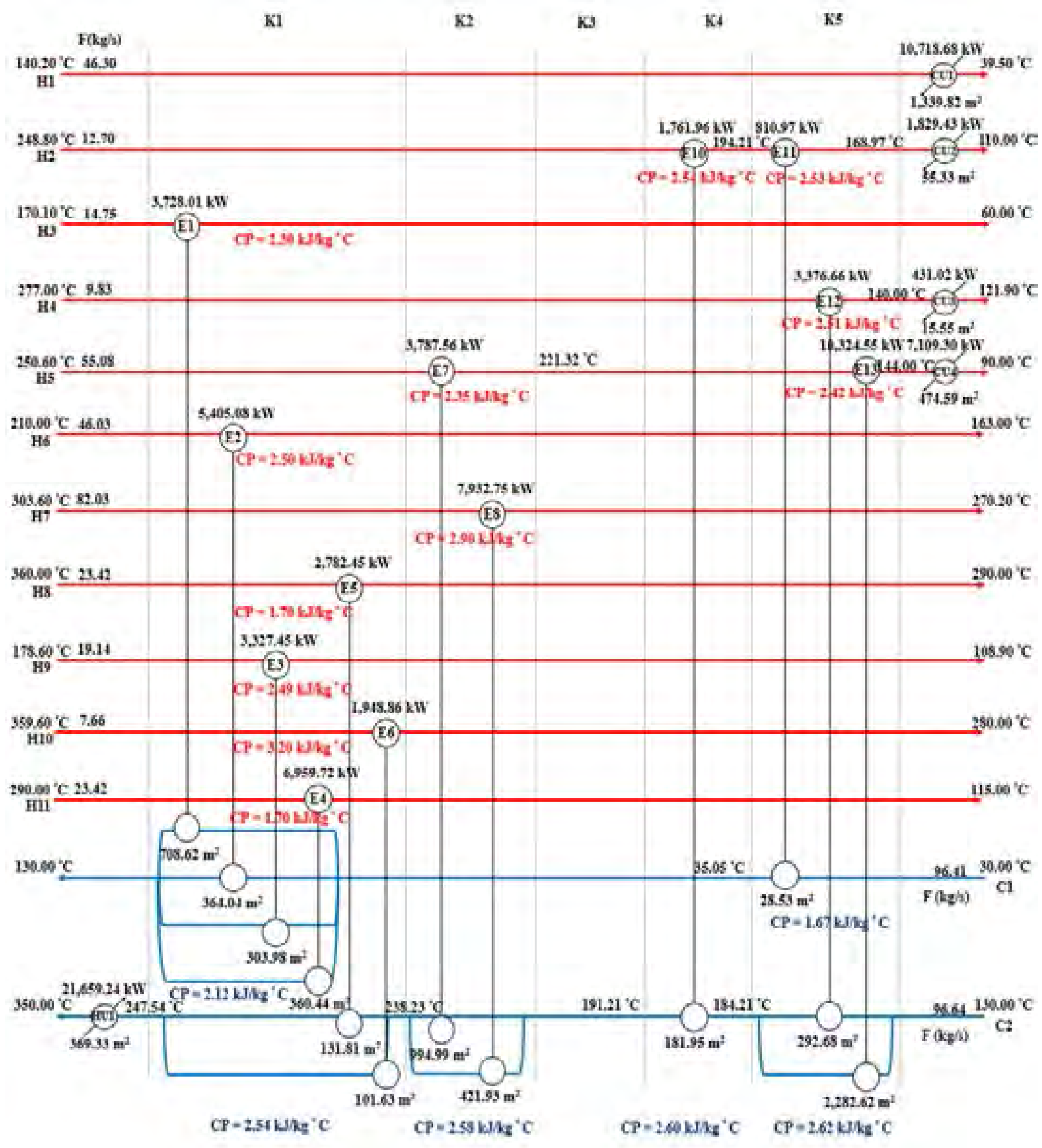


Figure 4.29 GAMS results alternative 1 of temperature-dependent Cp.

The alternative 1 is set by default of Dicopt solver which solution (Figure 4.29) obtains in 1,111 s computational time. TAC is 3,344,959.49 \$/Y combining from utility cost = 2,366,808.49 \$/Y and capital cost = 978,151.00 \$/Y.

4.3.2 Alternative 2 (600 s.)

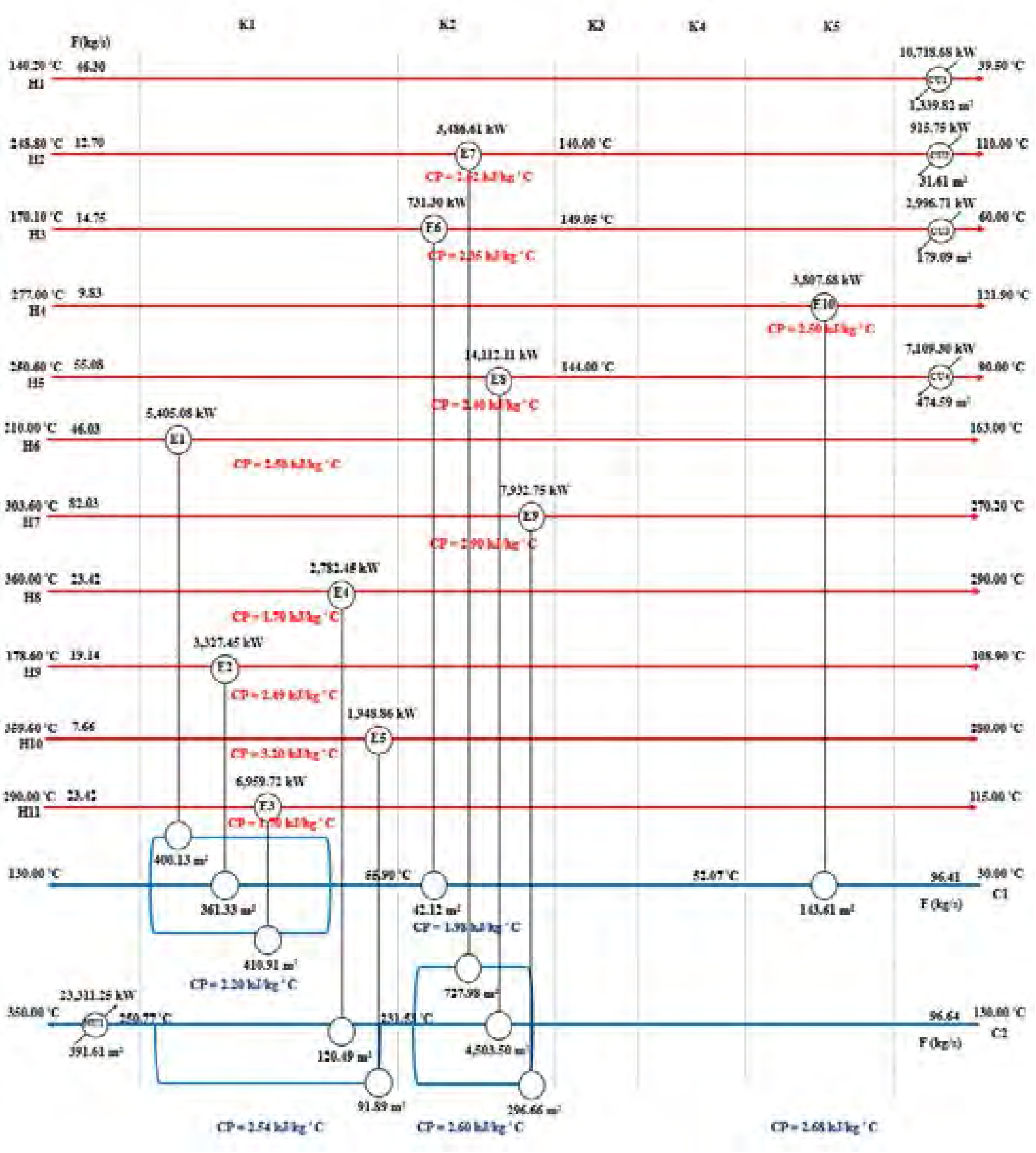


Figure 4.30 GAMS results alternative 2 of temperature-dependent Cp.

Alternative 2 is set computational time as 600 s. HEN result illustrate in Figure 4.30 which install less exchanger than atoner solution (it requires only 10 exchangers).

Thus, TAC is 3,601,112.74 \$/Y. Utility cost and capital cost are 2,548,529.74 \$/Y and 1,052,583 \$/Y, respectively.

4.3.3 Alternative 3 (3600 s.)

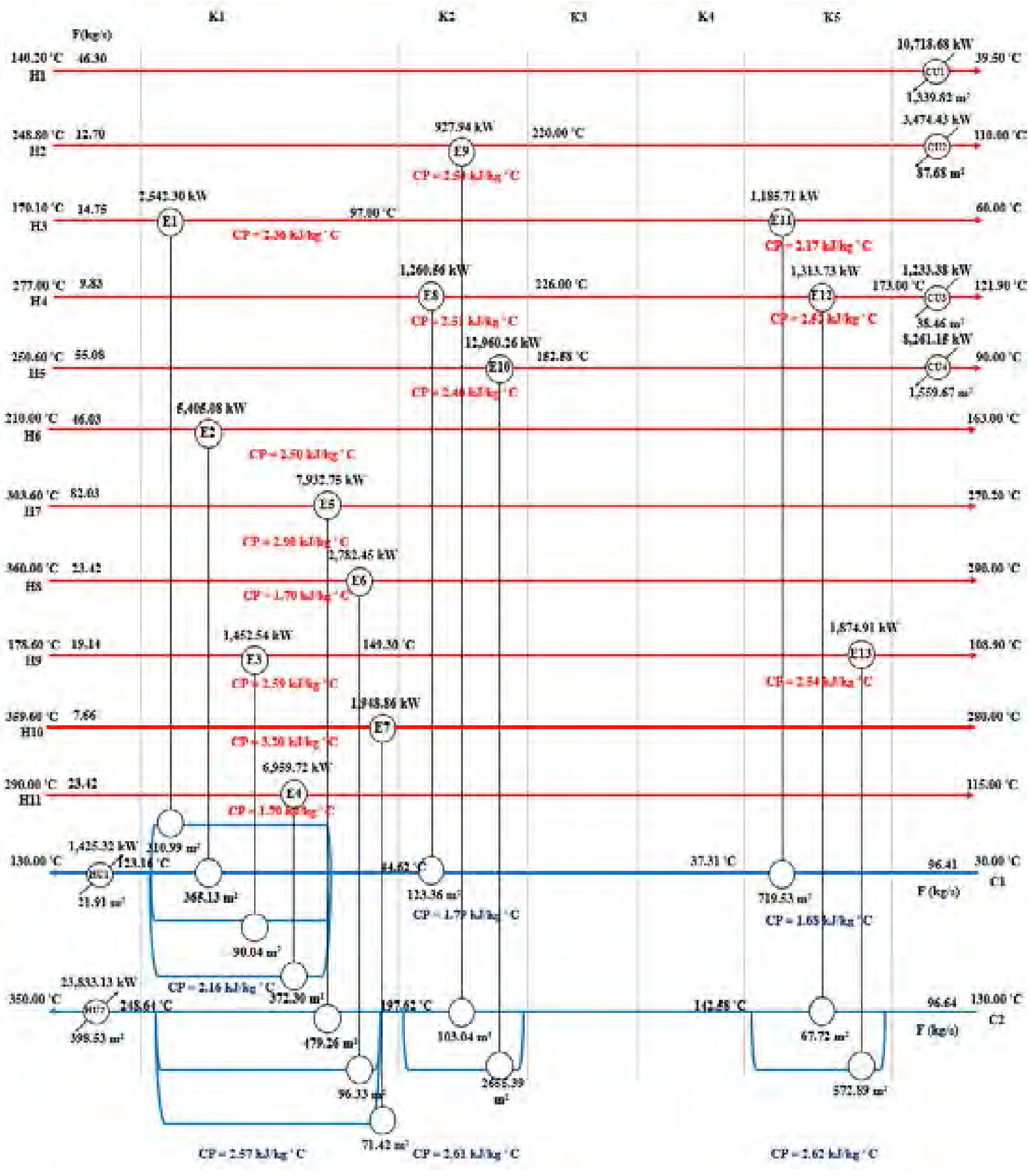


Figure 4.31 GAMS results alternative 3 of temperature-dependent Cp.

Last alternative is set computational time as 3,600 s which require capital cost = 1,060,661.19 \$/Y and utility cost = 2,762,722.06 \$/Y. TAC is 3,823,383.25 \$/Y.

From these results (Best solution to alternative 1,2 and 3) of case study 3, TAC is one of the best criteria to adjudge HEN results but some alternative is more proper in some reality work based on user experience. Therefore, final decision should come from user and alternative design is one of the greatest choices solved by mathematical programming technique.

4.4 Case Study 4 (Retrofit case)

This example shows the potential of new model for HEN retrofit case. Base case of this is created from case study 1 (following HEN by Figure 4.15 and Figure 4.17 which conclude Table 4.12 in and Table 4.13) and it has been assumed that utility cost is increasing over the time. Hot and cold utility cost increase from 60 to 80 and 5 to 10, respectively (Table 4.31). EMAT is reduced to 1 °C. Splitting constrain is no allowing for hot stream and 3 splitting for cold stream. Capital cost parameters are not changed and objective function is to maximize NPV over 3 years. From retrofit equation, process heat exchanger area, utility consumption and area of utility heat exchanger are identified as base case (Table 4.32).

Table 4.31 Process streams data of case study 4.

Streams	T_{in} (°C)	T_{out} (°C)	F (kg/s)	h (kW/m^2 °C)	Cost ($/kW y)
H1	43.33	25	37.38	1	-
H2	200.04	25	21.88	1	-
H3	272.79	25	21.76	1	-
H4	342.72	50	26.24	1	-
H5	370.72	50	87.06	1	-
C1	50	376.80	194.24	1	-
HU	500 (steam)	500 (condensate)	-	1	80
CU	10	15	-	1	10

Annual investment cost ($/y) = 3,460 + 300 × (Area; m2) for all exchangers (Pan *et al.* (2013)).

Table 4.32 Energy loading and exchanger area parameter of base case.

Heat Exchanger	Energy Loading (kW)	Area of Heat Exchanger (m^2)
E1	-	271.9076
E2	-	1,175.9500
E3	-	139.3715
E4	-	248.7469
E5	-	287.1363
E6	-	255.8308
E7	-	1,023.3600
CU1	1,378.3890	165.9617
CU2	2,089.7620	130.6266
CU3	2,149.9900	130.3613
CU4	4,868.9130	130.4485
CU5	4,167.3340	168.0144
HU1	82,100.9300	904.9860

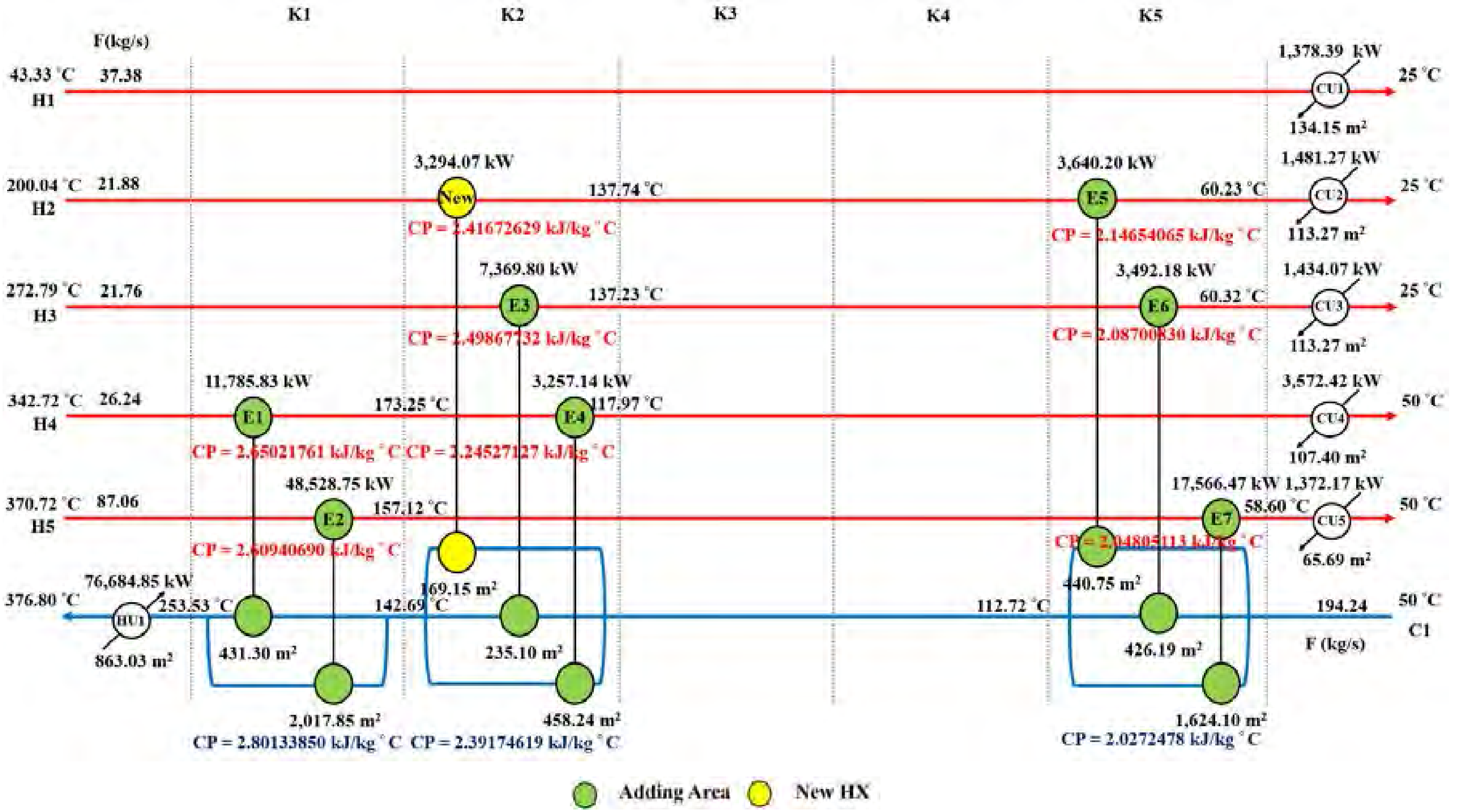


Figure 4.32 HEN retrofit result by GAMS.

After optimize by new model, new HEN is shown in Figure 4.32 and conclude in Table 4.33. The retrofit case by Pro/II simulation is shown in Figure 4.34. It shows that area of existing process heat exchanger is increased and one new heat exchanger is added. From this result, over all area of heat exchanger is increasing from 5,032.71 to 5,767.68 but hot utility and cold utility consumption is reduced from 82,100.93 to 76,684.85 and from 14,654.38 to 9,238.32, respectively (Table 4.34). NPV of this solution is 731,849.17 in 3 years.

Table 4.33 HEN retrofit result data.

Heat Exchanger	Base Case Heat Duty (kW)	New Heat Duty (kW)	Base Case Area (m^2)	New Area (m^2)
E1	10,086.28	11,785.83	271.91	431.30
E2	43,176.05	48,528.75	1,175.95	2,017.85
E3	5,561.56	7,369.80	139.37	235.10
E4	3,660.19	3,257.14	248.75	458.24
E5	6,325.78	3,640.20	287.14	440.75
E6	4,584.50	3,492.18	255.83	426.19
E7	20,124.00	17,566.47	1,023.36	1,624.10
New	-	3,294.07	-	169.15
CU1	1,378.39	1,378.39	165.96	134.15
CU2	2,089.76	1,481.27	130.63	113.27
CU3	2,149.99	1,434.07	130.36	113.27
CU4	4,868.91	3,572.42	130.45	107.40
CU5	4,167.33	1,372.17	168.01	65.69
HU1	82,100.93	76,684.85	904.99	863.03

Table 4.34 Retrofit results comparison between base case and retrofit case.

Cases	Area of Heat Exchanger (m^2)	Hot Utility Consumption (kW)	Cold Utility Consumption (kW)
Base Case	5,032.71	82,100.93	14,654.38
Retrofit Case	5,767.68	76,684.85	9,238.32
Difference	734.97	-5,416.08	-5,416.06

These results are validated by fixing heat load of each exchanger and HEN in Pro/II simulation which illustrated in Figure 4.33 . The results show that outlet temperature different of each exchanger is not more than 1 °C and Cp calculation of each exchanger is close to each other. However, small approach temperature drive in large area calculation. For example, outlet/inlet temperature of hot and cold streams for exchanger E4 are not majorly change (about 1 °C) but area calculation for GAMS and Pro/II are 458.24 m^2 and 481.99 m^2, respectively. The error of area calculation is 4.93 %. Thus, the error shows how important of Cp calculation (comparing between new model that still have major error in some point and constant Cp which, of course, have much error than new model). Duty data and area data comparison are shown in Table 4.35 and, respectively.

Table 4.35 Duty data comparison between GAMS and Pro/II of case study 4.

Heat Exchanger	Duty GAMS (kW)	Duty Pro/II (kW)	Percent Error (%)
E1	11,785.83	11,785.83	-
E2	48,528.75	48,528.75	-
E3	7,369.80	7,369.80	-
E4	3,257.14	3,257.14	-
E5	3,640.20	3,640.20	-
E6	3,492.18	3,492.18	-
E7	17,566.47	17,566.47	-
NEW	3,294.08	3,294.08	-
CU1	1,378.39	1,378.40	0.00
CU2	1,481.27	1,466.30	1.02
CU3	1,434.07	1,415.20	1.33
CU4	3,572.42	3,536.90	1.00
CU5	1,372.17	1,206.90	13.69
HU1	76,684.85	76,419.20	0.35

Table 4.36 Area data comparison between GAMS and Pro/II of case study 4.

Heat Exchanger	Area GAMS (m^2)	Area Pro/II (m^2)	Percent Error (%)
E1	431.30	434.43	0.72
E2	2,017.85	2,016.74	0.05
E3	235.10	235.19	0.04
E4	458.24	481.99	4.93
E5	440.75	448.01	1.62
E6	426.19	438.51	2.81
E7	1,624.10	1,702.44	4.60
NEW	169.15	170.21	0.62
CU1	134.15	131.49	2.02

CU2	113.27	107.43	5.43
CU3	110.26	103.75	6.28
CU4	107.40	106.55	0.80
CU5	65.69	58.47	12.36
HU1	863.03	860.16	0.33

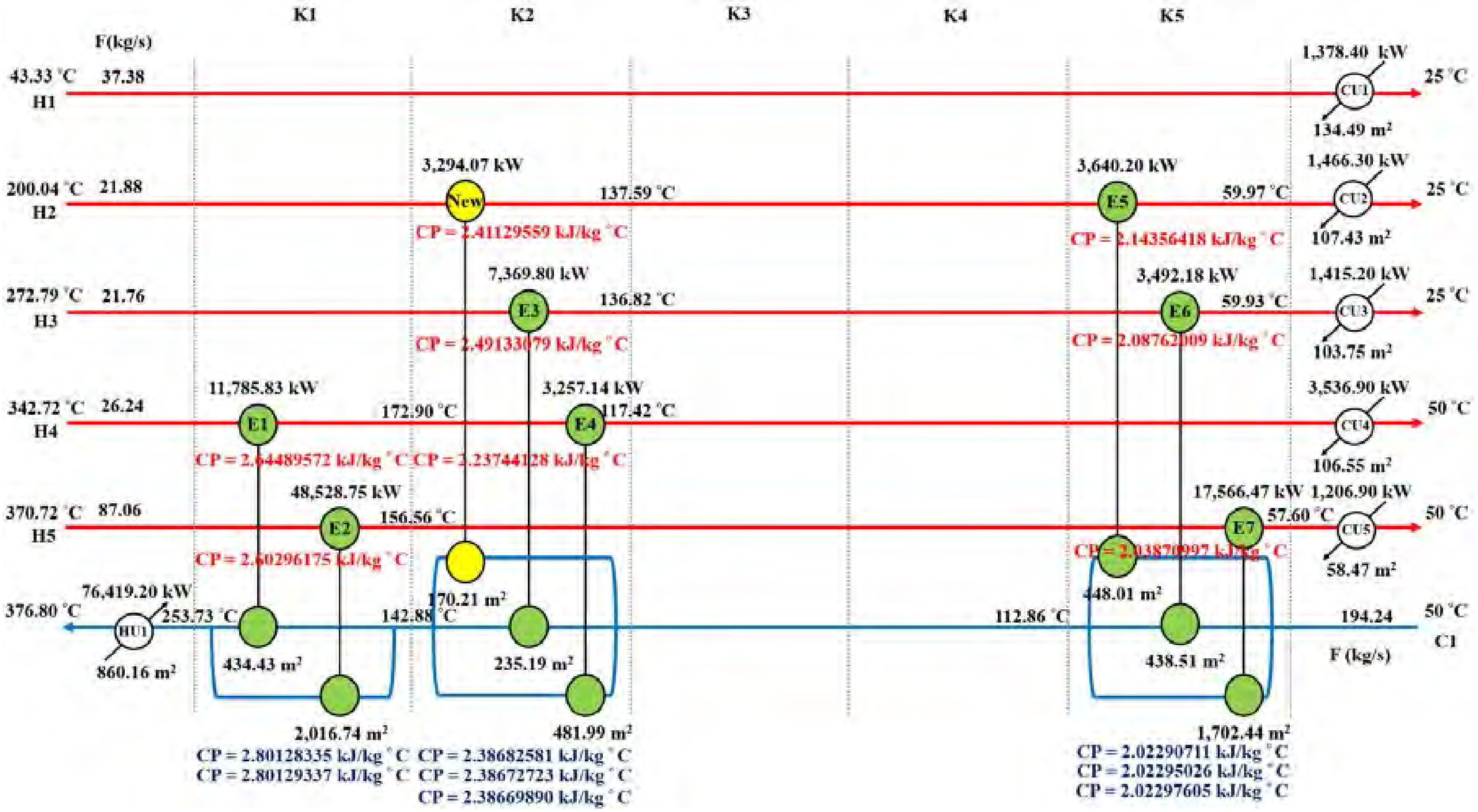
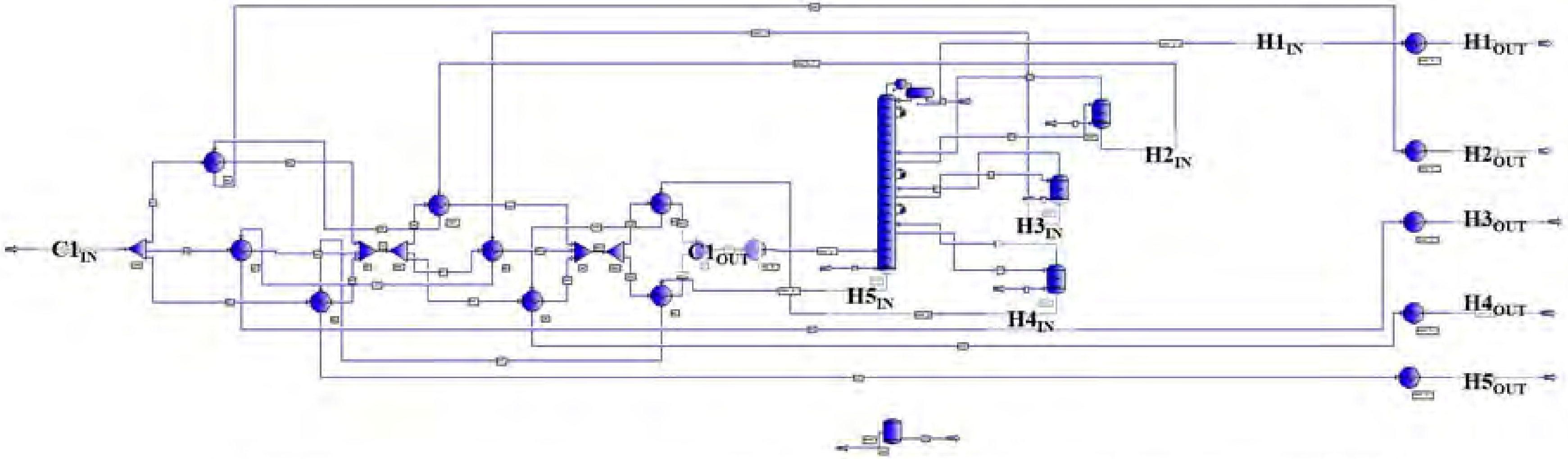
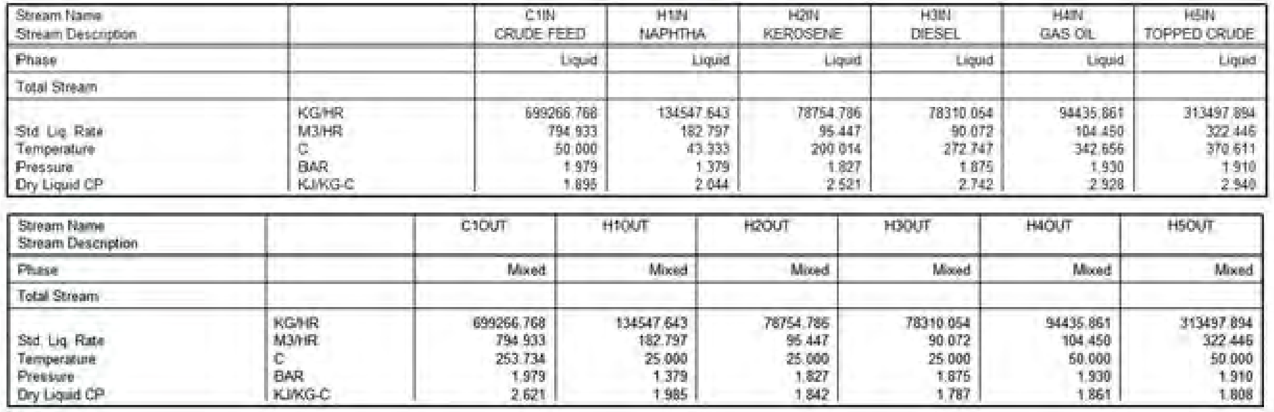


Figure 4.33 HEN retrofit result by Pro/II.





Stream Name Stream Description		C1IN CRUDE FEED	H1IN NAPHTHA	H2IN KEROSENE	H3IN DIESEL	H4IN GAS OIL	H5IN TOPPED CRUDE
Phase		Liquid	Liquid	Liquid	Liquid	Liquid	Liquid
Total Stream							
	KG/HR	699266.768	134547.643	78754.786	78310.054	94435.861	313497.894
Std. Liq. Rate	M3/HR	794.933	182.797	95.447	90.072	104.450	322.446
Temperature	C	50.000	43.333	200.014	272.747	342.656	370.611
Pressure	BAR	1.979	1.379	1.827	1.875	1.930	1.910
Dry Liquid CP	KJ/KG-C	1.895	2.044	2.521	2.742	2.928	2.940

Stream Name Stream Description		C1OUT	H1OUT	H2OUT	H3OUT	H4OUT	H5OUT
Phase		Mixed	Mixed	Mixed	Mixed	Mixed	Mixed
Total Stream							
	KG/HR	699266.768	134547.643	78754.786	78310.054	94435.861	313497.894
Std. Liq. Rate	M3/HR	794.933	182.797	95.447	90.072	104.450	322.446
Temperature	C	253.734	25.000	25.000	25.000	50.000	50.000
Pressure	BAR	1.979	1.379	1.827	1.875	1.930	1.910
Dry Liquid CP	KJ/KG-C	2.621	1.985	1.842	1.787	1.861	1.808

Figure 4.34 HEN retrofit case study by Pro/II simulation.

Table 4.37 to Table 4.39 show percent error of NPV, capital cost, utility cost, cold utility consumption, hot utility consumption process area and utility area. As mention early, main error of each part is occurred from any small different in temperature point of exchanger but it really effects on area calculation and utility consumption.

Table 4.37 Economic cost data comparing between GAMS and Pro/II results of case study 4.

Cases	GAMS NPV ($)	Pro/II NPV ($)	NPV Error (%)	Capital Cost Error (%)	Utility Cost Saving Error (%)
Partitioning Technique	738,769.00	701,318.45	5.34	4.92	0.00

Table 4.38 Utility data comparing between GAMS and Pro/II results of case study 4.

Cases	GAMS CU (kW)	Pro/II CU (kW)	CU Error (%)	GAMS HU (kW)	Pro/II HU (kW)	HU Error (%)
Partitioning Technique	9,238.31	9,003.70	2.61	76,684.85	76,419.20	0.35

Table 4.39 Overall area data of heat exchanger comparing between GAMS and Pro/II results of case study 4.

Cases	GAMS Process Area (m^2)	Pro/II Process Area (m^2)	Process Area Error (%)	GAMS Utility Area (m^2)	Pro/II Utility Area (m^2)	Utility Area Error (%)
Partitioning Technique	5,802.68	5,927.51	2.11	1,393.79	1,367.84	1.90

CHAPTER V
CONCLUSIONS AND RECOMMENDATIONS

Stage-wise superstructure with temperature-dependent Cp model had been developed in this journal by partitioning technique aiming to obtain high quality results instead of using ordinary polynomial cubic equation. Three partitions are selected to fit curvature of non-linear Cp data. Constant Cp affect directly to outlet temperature of exchanger results to area calculation error. Validation case study from Pro/II library shows that partitioning technique can reduce error of area calculation from about 30 % error to about 1 % impact on overall error (TAC) reduce from 0.51 to 0.27 %. To compared between empirical fitting of polynomial Cp equation and partitioning technique, case studies are used. Case study 2 shows that new solution has lower TAC than the previous solution about 2.59 %. From case study 3 that contains 11 hot streams and 2 cold streams, it has many HEN design scenarios and no lower limit constraints setting. This case study shows that our new model can solve another HEN design which reduce TAC from 3,451,585.00 (Previous solution) to 3,229,803.75 (New solution). It reduces about 6.42 % of TAC. Thus, this new model by partitioning technique has less complexity of third order equation, high accuracy and better solution. Moreover, retrofit case study can be applied for exiting HEN (case study 4).

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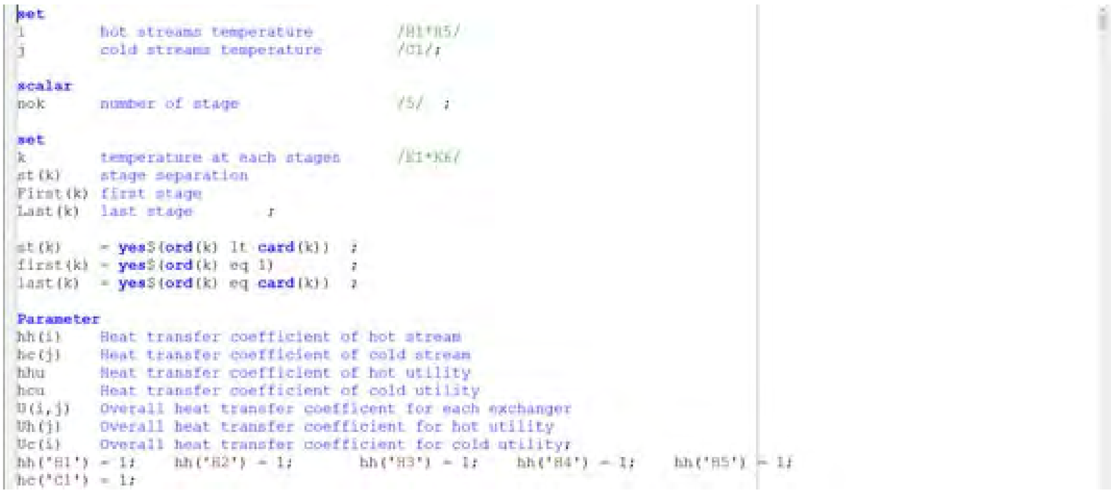
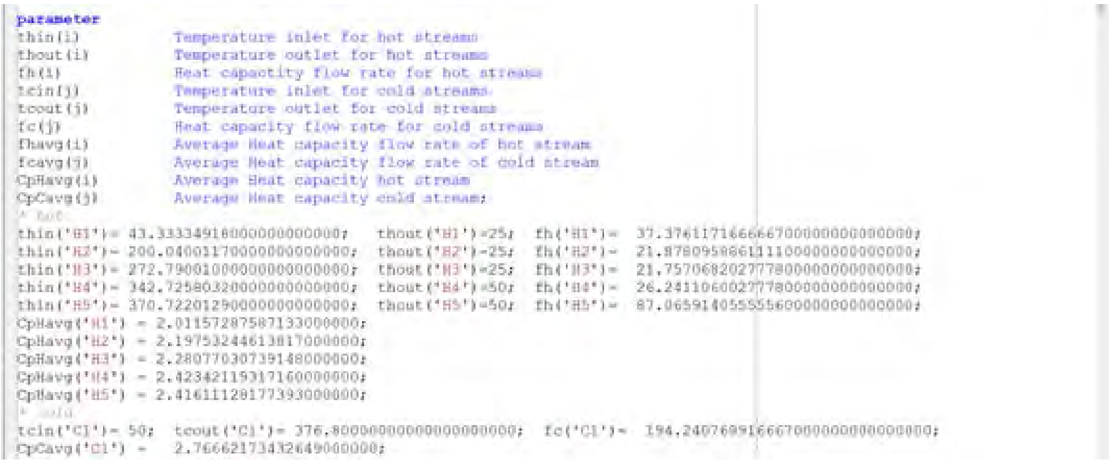
APPENDICES

Appendix A GAMS Code of Case Study 1 (HEN Synthesis)



```
set
i          hot streams temperature          /H1*H5/
j          cold streams temperature         /C1/;

scalar
nok        number of stage                  /5/  ;

set
k          temperature at each stages       /K1*K6/
st(k)      stage separation
First(k)   first stage
Last(k)    last stage          ;

st(k)      = yes$(ord(k) lt card(k))  ;
first(k)   = yes$(ord(k) eq 1)        ;
last(k)    = yes$(ord(k) eq card(k))  ;

Parameter
hh(i)      Heat transfer coefficient of hot stream
hc(j)      Heat transfer coefficient of cold stream
hhu        Heat transfer coefficient of hot utility
hcu        Heat transfer coefficient of cold utility
U(i,j)     Overall heat transfer coefficent for each exchanger
Uh(j)      Overall heat transfer coefficient for hot utility
Uc(i)      Overall heat transfer coefficient for cold utility;
hh('H1') = 1;    hh('H2') = 1;       hh('H3') = 1;    hh('H4') = 1;    hh('H5') = 1;
hc('C1') = 1;
```

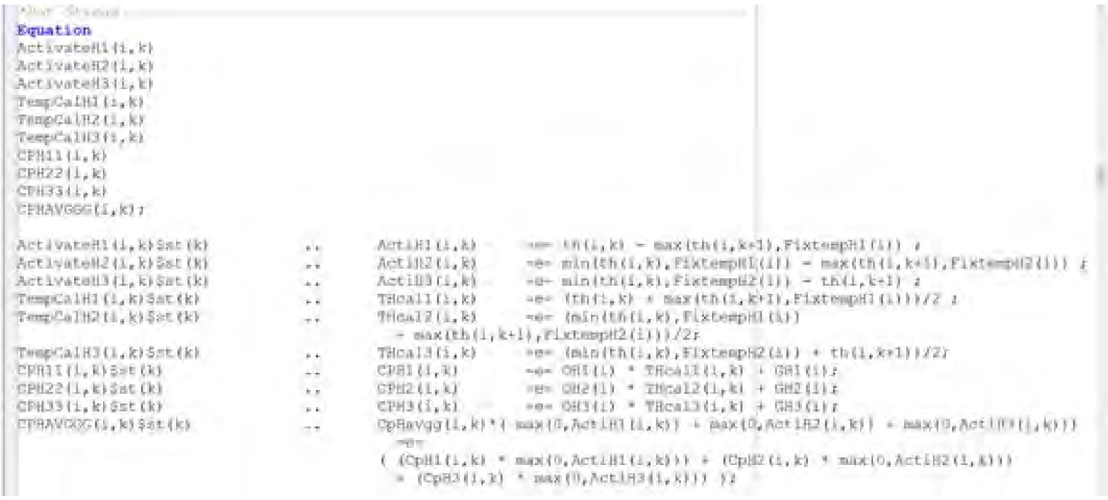
```
hhu        = 1;
hcu        = 1;
U(i,j) = ( (hh(i) * hc(j))/(hh(i) + hc(j)) );
Uh(j)  = ( (hhu * hc(j))/(hhu + hc(j)) );
Uc(i)  = ( (hh(i) * hcu)/(hh(i) + hcu) );
display U,uh,uc;
*-----------------------------------------------------------------*
*                          Data UT                                *
*-----------------------------------------------------------------*
parameter
TINhu     Temperature inlet hot utility
TOUThu    Temperature outlet hot utility
TINcu     Temperature inlet cold utility
TOUTcu    Temperature outlet cold utility;
TINhu     =        500;
TOUThu    =        500;
TINcu     =        10;
TOUTcu    =        15;
*-----------------------------------------------------------------*
*                          Data HEN                               *
*-----------------------------------------------------------------*
```



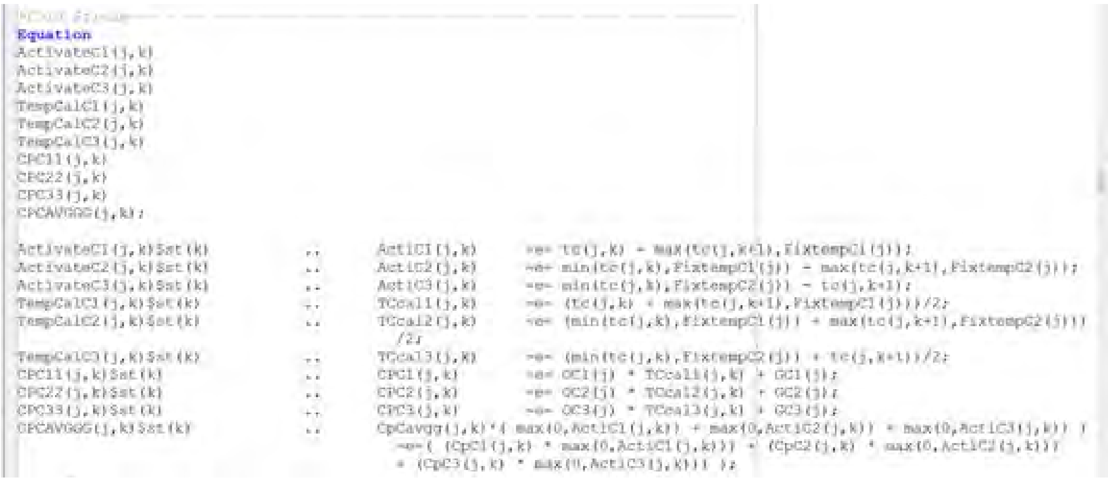
```
parameter
thin(i)              Temperature inlet for hot streams
thout(i)             Temperature outlet for hot streams
fh(i)                Heat capactity flow rate for hot streams
tcin(j)              Temperature inlet for cold streams
tcout(j)             Temperature outlet for cold streams
fc(j)                Heat capacity flow rate for cold streams
fhavg(i)             Average Heat capacity flow rate of hot stream
fcavg(j)             Average Heat capacity flow rate of cold stream
CpHavg(i)            Average Heat capacity hot stream
CpCavg(j)            Average Heat capacity cold stream;
* hot
thin('H1')= 43.3333491600000000000;   thout('H1')=25;  fh('H1')=  37.3761171666667000000000000000;
thin('H2')= 200.040011700000000000;   thout('H2')=25;  fh('H2')=  21.8780958861111000000000000000;
thin('H3')= 272.790010000000000000;   thout('H3')=25;  fh('H3')=  21.7570682027778000000000000000;
thin('H4')= 342.725803200000000000;   thout('H4')=50;  fh('H4')=  26.2411060027778000000000000000;
thin('H5')= 370.722012900000000000;   thout('H5')=50;  fh('H5')=  87.0659140555556000000000000000;
CpHavg('H1') = 2.01157287587133000000;
CpHavg('H2') = 2.19753244613817000000;
CpHavg('H3') = 2.28077030739148000000;
CpHavg('H4') = 2.42342119317160000000;
CpHavg('H5') = 2.41611120177393000000;
* cold
tcin('C1')= 50;  tcout('C1')= 376.800000000000000000000;  fc('C1')=  194.240769916667000000000000000;
CpCavg('C1') =    2.76662173432649000000;
```

```
*Avg
fhavg(i)= fh(i) * CpHavg(i);
fcavg(j)= fc(j) * CpCavg(j);
Display fhavg,fcavg;
Scalar
EMAT     Exchanger minimum approximation temperature
HRAT     Heat recovery approximation temperature
Small    Small Value;
EMAT     = 10;
HRAT     = 10;
Small    = 1e-7;
*-------------------------------------------------------------------------*
*                              COST DATA                                  *
*-------------------------------------------------------------------------*
Parameter
CHU             Hot utility cost
CCU             Cold utility cost
CF              Fixed charge for exchanger
CFH             Fixed charge for Hot UT
CFC             Fixed charge for Cold UT
C               Area cost coefficient
CH              Area cost coefficient for hot UT
CC              Area cost coefficient for Cold UT
B               Exoponent for HX area cost
BH              Exponent for hot UT area cost
BC              Exponent for cold UT area cost;
CHU             =       60;
CCU             =       5;
CF              =       3460;
CFH             =       3460;
CFC             =       3460;
C               =       300;
CH              =       300;
CC              =       300;
B               =       1;
BH              =       1;
BC              =       1;
positive Variable
dT(i,j,k)       temperature approach for match (ij) at temperature location k
dTHU(j)         Temperature approach for the match of hot stream i and cold utility
dTHU2(j)        Temperature approach for the match of hot stream i and cold utility
dTCU(i)         Temperature approach for the match of cold stream j and hot utility
dTCU2(i)         Temperature approach for the match of cold stream j and hot utility
q(i,j,k)        Heat exchanged between hot process stream i and cold process streams j in stage k
qcu(i)          heat exchanged between hot stream i and cold utility
qhu(j)          Heat exchanged between hot utility and cold stream j
AreaHX(i,j,k)   Area of each exchanger
AreaH(j)        Area of hot UT
AreaC(i)        Area of cold UT;
free variable
th(i,k)         temperature of hot stream i at hot end of stage k
tc(j,k)         temperature of cold stream j at hot end of stage k
s               objective function;
Binary variable
z(i,j,k)        binary variable to denote existence of match ij in stage k
zcu(i)          binary variable to denote that cold utility exchanges heat with hot stream i
zhu(j)          binary variable to denote that hot utility exchanges heat with cold stream j;
*--------------------Inconstant Heat Capacity---------------------------------
parameter
OH1(i) Cp Coefficient of Hot stream
OH2(i) Cp Coefficient of Hot stream
OH3(i) Cp Coefficient of Hot stream
GH1(i) Cp Coefficient of Hot stream
GH2(i) Cp Coefficient of Hot stream
GH3(i) Cp Coefficient of Hot stream
OC1(j) Cp Coefficient of cold stream
OC2(j) Cp Coefficient of cold stream
OC3(j) Cp Coefficient of cold stream
GC1(j) Cp Coefficient of cold stream
GC2(j) Cp Coefficient of cold stream
GC3(j) Cp Coefficient of cold stream
FixtempH1(i)
FixtempH2(i)
FixtempC1(j)
FixtempC2(j)
TestH(i)
TestC(j);

FixtempH1('H1') = 30;
FixtempH2('H1') = 30;
```

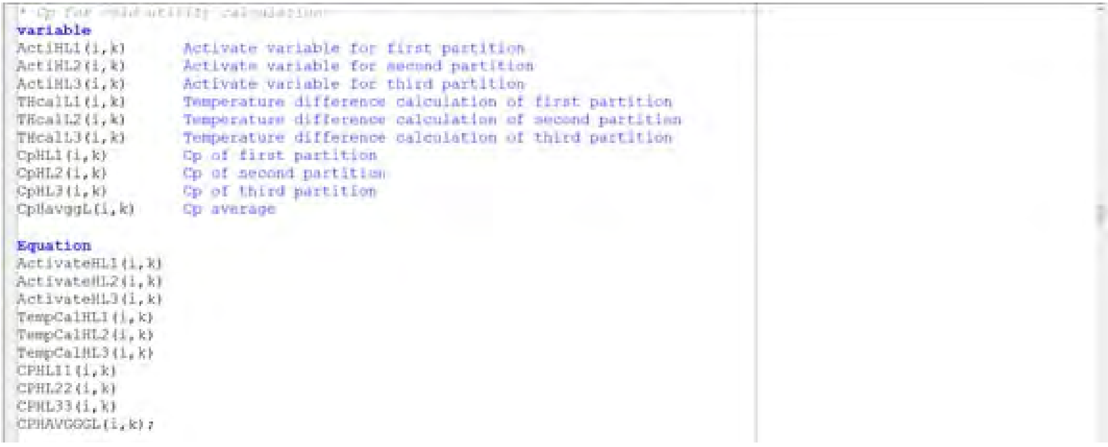
```
FixtempH1('H2') = 145.23000000000000000000;
FixtempH2('H2') = 76.27000000000000000000;

FixtempH1('H3') = 195.21000000000000000000;
FixtempH2('H3') = 95.08000000000000000000;

FixtempH1('H4') = 243.24000000000000000000;
FixtempH2('H4') = 114.86000000000000000000;

FixtempH1('H5') = 262.47000000000000000000;
FixtempH2('H5') = 122.78000000000000000000;

FixtempC1(j) = 160.18000000000000000000;
FixtempC2(j) = 112.12000000000000000000;

*Hot Stream
OH1('H1') = 0.00000000000000000000; GH1('H1') = 2.01157287587133000000;
OH2('H1') = 0.00000000000000000000; GH2('H1') = 2.01157287587133000000;
OH3('H1') = 0.00000000000000000000; GH3('H1') = 2.01157287587133000000;

OH1('H2') = 0.00394085499726679000; GH1('H2') = 1.74992048732344000000;
OH2('H2') = 0.00402348592188883000; GH2('H2') = 1.74851827105308000000;
OH3('H2') = 0.00389832605203208000; GH3('H2') = 1.75594557210110000000;

OH1('H3') = 0.00348347848525961000; GH1('H3') = 1.78714691436846000000;
OH2('H3') = 0.00396272018163033000; GH2('H3') = 1.70138615192470000000;
OH3('H3') = 0.00371589145434134000; GH3('H3') = 1.70777187532426000000;
OH2('H4') = 0.00377460391613860000; GH2('H4') = 1.69563864167670000000;
OH3('H4') = 0.00402866554468413000; GH3('H4') = 1.66442582879783000000;

OH1('H5') = 0.00301500014925505000; GH1('H5') = 1.83347265339945000000;
OH2('H5') = 0.00359288674396509000; GH2('H5') = 1.67211857583920000000;
OH3('H5') = 0.00408324170762641000; GH3('H5') = 1.60983832605874000000;

*Cold Stream
OC1('C1') = 0.00339671360203902000; GC1('C1') = 2.13223102521803000000;
OC2('C1') = 0.00909253970312521000; GC2('C1') = 1.23185053302393000000;
OC3('C1') = 0.00411179201266024000; GC3('C1') = 1.69174820755582000000;

TestH(i) = ( (OH1(i) * (thin(i)+FixtempH1(i))/2         + GH1(i)) * (thin(i) - FixtempH1(i)) +
             (OH2(i) * (FixtempH1(i) + FixtempH2(i))/2  + GH2(i)) * (FixtempH1(i) - FixtempH2(i)) +
             (OH3(i) * (FixtempH2(i) + thout(i))/2      + GH3(i)) * (FixtempH2(i) - thout(i))
           )
           / ( (thin(i) - FixtempH1(i)) + (FixtempH1(i) - FixtempH2(i)) + (FixtempH2(i) - thout(i))
             )
           - CpHavg(i) ;

TestC(j) = ( (OC1(j) * (tcout(j)+FixtempC1(j))/2         + GC1(j)) * (tcout(j) - FixtempC1(j)) +
             (OC2(j) * (FixtempC1(j)+ FixtempC2(j))/2   + GC2(j)) * (FixtempC1(j) - FixtempC2(j))  +
             (OC3(j) * (FixtempC2(j)+ tcin(j))/2        + GC3(j)) * (FixtempC2(j) - tcin(j))
            )
           / ( (tcout(j) - FixtempC1(j)) + (FixtempC1(j) - FixtempC2(j)) + (FixtempC2(j) - tcin(j))
              )
           - CpCavg(j) ;
Display TestH,TestC;

Variable
CpHL(i,k)          Heat capacity for cold utility
CpCF(j,k)          Heat capacity for hot utility
ActiH1(i,k)        Activate variable for first partition
ActiH2(i,k)        Activate variable for second partition
ActiH3(i,k)        Activate variable for third partition
THcal1(i,k)        Temperature difference calculation of first partition
THcal2(i,k)        Temperature difference calculation of second partition
THcal3(i,k)        Temperature difference calculation of third partition
CpH1(i,k)          Cp of first partition
CpH2(i,k)          Cp of second partition
CpH3(i,k)          Cp of third partition
CpHavgg(i,k)       Cp average
ActiC1(j,k)        Activate variable for first partition
ActiC2(j,k)        Activate variable for second partition
ActiC3(j,k)        Activate variable for third partition
TCcal1(j,k)        Temperature difference calculation of first partition
TCcal2(j,k)        Temperature difference calculation of second partition
TCcal3(j,k)        Temperature difference calculation of third partition
CpC1(j,k)          Cp of first partition
CpC2(j,k)          Cp of second partition
CpC3(j,k)          Cp of third partition
CpCavgg(j,k)       Cp average     ;
```



```
*Hot Stream
Equation
ActivateH1(i,k)
ActivateH2(i,k)
ActivateH3(i,k)
TempCalH1(i,k)
TempCalH2(i,k)
TempCalH3(i,k)
CPH11(i,k)
CPH22(i,k)
CPH33(i,k)
CPHAVGGG(i,k);

ActivateH1(i,k)$st(k)          ..     ActiH1(i,k)      =e= th(i,k) - max(th(i,k+1),FixtempH1(i)) ;
ActivateH2(i,k)$st(k)          ..     ActiH2(i,k)      =e= min(th(i,k),FixtempH1(i)) - max(th(i,k+1),FixtempH2(i)) ;
ActivateH3(i,k)$st(k)          ..     ActiH3(i,k)      =e= min(th(i,k),FixtempH2(i)) - th(i,k+1) ;
TempCalH1(i,k)$st(k)           ..     THcal1(i,k)      =e= (th(i,k) + max(th(i,k+1),FixtempH1(i)))/2 ;
TempCalH2(i,k)$st(k)           ..     THcal2(i,k)      =e= (min(th(i,k),FixtempH1(i))
                                        + max(th(i,k+1),FixtempH2(i)))/2;
TempCalH3(i,k)$st(k)           ..     THcal3(i,k)      =e= (min(th(i,k),FixtempH2(i)) + th(i,k+1))/2;
CPH11(i,k)$st(k)               ..     CPH1(i,k)        =e= OH1(i) * THcal1(i,k) + GH1(i);
CPH22(i,k)$st(k)               ..     CPH2(i,k)        =e= OH2(i) * THcal2(i,k) + GH2(i);
CPH33(i,k)$st(k)               ..     CPH3(i,k)        =e= OH3(i) * THcal3(i,k) + GH3(i);
CPHAVGGG(i,k)$st(k)            ..     CpHavgg(i,k)*( max(0,ActiH1(i,k)) + max(0,ActiH2(i,k)) + max(0,ActiH3(i,k)))
                                        =e=
                                      ( (CpH1(i,k) * max(0,ActiH1(i,k))) + (CpH2(i,k) * max(0,ActiH2(i,k)))
                                        + (CpH3(i,k) * max(0,ActiH3(i,k))) );
```



```
*Cold Stream
Equation
ActivateC1(j,k)
ActivateC2(j,k)
ActivateC3(j,k)
TempCalC1(j,k)
TempCalC2(j,k)
TempCalC3(j,k)
CPC11(j,k)
CPC22(j,k)
CPC33(j,k)
CPCAVGGG(j,k);

ActivateC1(j,k)$st(k)          ..     ActiC1(j,k)      =e= tc(j,k) - max(tc(j,k+1),FixtempC1(j));
ActivateC2(j,k)$st(k)          ..     ActiC2(j,k)      =e= min(tc(j,k),FixtempC1(j)) - max(tc(j,k+1),FixtempC2(j));
ActivateC3(j,k)$st(k)          ..     ActiC3(j,k)      =e= min(tc(j,k),FixtempC2(j)) - tc(j,k+1);
TempCalC1(j,k)$st(k)           ..     TCcal1(j,k)      =e= (tc(j,k) + max(tc(j,k+1),FixtempC1(j)))/2;
TempCalC2(j,k)$st(k)           ..     TCcal2(j,k)      =e= (min(tc(j,k),FixtempC1(j)) + max(tc(j,k+1),FixtempC2(j)))
                                        /2;
TempCalC3(j,k)$st(k)           ..     TCcal3(j,k)      =e= (min(tc(j,k),FixtempC2(j)) + tc(j,k+1))/2;
CPC11(j,k)$st(k)               ..     CPC1(j,k)        =e= OC1(j) * TCcal1(j,k) + GC1(j);
CPC22(j,k)$st(k)               ..     CPC2(j,k)        =e= OC2(j) * TCcal2(j,k) + GC2(j);
CPC33(j,k)$st(k)               ..     CPC3(j,k)        =e= OC3(j) * TCcal3(j,k) + GC3(j);
CPCAVGGG(j,k)$st(k)            ..     CpCavgg(j,k)*( max(0,ActiC1(j,k)) + max(0,ActiC2(j,k)) + max(0,ActiC3(j,k)) )
                                        =e=( (CpC1(j,k) * max(0,ActiC1(j,k))) + (CpC2(j,k) * max(0,ActiC2(j,k)))
                                        + (CpC3(j,k) * max(0,ActiC3(j,k))) );
```



```
* Cp for cold utility calculation
variable
ActiHL1(i,k)        Activate variable for first partition
ActiHL2(i,k)        Activate variable for second partition
ActiHL3(i,k)        Activate variable for third partition
THcalL1(i,k)        Temperature difference calculation of first partition
THcalL2(i,k)        Temperature difference calculation of second partition
THcalL3(i,k)        Temperature difference calculation of third partition
CpHL1(i,k)          Cp of first partition
CpHL2(i,k)          Cp of second partition
CpHL3(i,k)          Cp of third partition
CpHavggL(i,k)       Cp average

Equation
ActivateHL1(i,k)
ActivateHL2(i,k)
ActivateHL3(i,k)
TempCalHL1(i,k)
TempCalHL2(i,k)
TempCalHL3(i,k)
CPHL11(i,k)
CPHL22(i,k)
CPHL33(i,k)
CPHAVGGGL(i,k);
```

```
ActivateHL1(i,k)$last(k)          ..     ActiHL1(i,k)     =e= th(i,k) - FixtempH1(i);
ActivateHL2(i,k)$last(k)          ..     ActiHL2(i,k)     =e= min(th(i,k),FixtempH1(i)) - FixtempH2(i);
ActivateHL3(i,k)$last(k)          ..     ActiHL3(i,k)     =e= min(th(i,k),FixtempH2(i)) - thout(i);
TempCalHL1(i,k)$last(k)           ..     THcalL1(i,k)     =e= (th(i,k) + FixtempH1(i))/2 ;
TempCalHL2(i,k)$last(k)           ..     THcalL2(i,k)     =e= (min(th(i,k),FixtempH1(i)) + FixtempH2(i))/2;
TempCalHL3(i,k)$last(k)           ..     THcalL3(i,k)     =e= (min(th(i,k),FixtempH2(i)) + thout(i))/2;
CPHL11(i,k)$last(k)               ..     CPHL1(i,k)       =e= OH1(i) * THcalL1(i,k) + GH1(i);
CPHL22(i,k)$last(k)               ..     CPHL2(i,k)       =e= OH2(i) * THcalL2(i,k) + GH2(i);
CPHL33(i,k)$last(k)               ..     CPHL3(i,k)       =e= OH3(i) * THcalL3(i,k) + GH3(i);
CPHAVGGGL(i,k)$last(k)            ..     CpHavggL(i,k)*(max(0,ActiHL1(i,k)) + max(0,ActiHL2(i,k))
                                         + max(0,ActiHL3(i,k)) ) =e=( (CpHL1(i,k) * max(0,ActiHL1(i,k)))
                                        + (CpHL2(i,k) * max(0,ActiHL2(i,k))) + (CpHL3(i,k) * max(0,ActiHL3(i,k))) );


* Cp for hot utility calculation
variable
ActiCF1(j,k)        Activate variable for first partition
ActiCF2(j,k)        Activate variable for second partition
ActiCF3(j,k)        Activate variable for second partition
TCcalF1(j,k)        Temperature difference calculation of first partition
TCcalF2(j,k)        Temperature difference calculation of second partition
TCcalF3(j,k)        Temperature difference calculation of second partition
CpCF1(j,k)          Cp of first partition
CpCF2(j,k)          Cp of second partition
CpCF3(j,k)          Cp of second partition
CpCavggF(j,k)       Cp average

Equation
ActivateCF1(j,k)
ActivateCF2(j,k)
ActivateCF3(j,k)
TempCalCF1(j,k)
TempCalCF2(j,k)
TempCalCF3(j,k)
CPCF11(j,k)
CPCF22(j,k)
CPCF33(j,k)
CPCAVGGGF(j,k);

ActivateCF1(j,k)$first(k)          ..     ActiCF1(j,k)     =e= tcout(j) - max(FixtempC1(j),tc(j,k));
ActivateCF2(j,k)$first(k)          ..     ActiCF2(j,k)     =e= FixtempC1(j) - max(FixtempC2(j),tc(j,k)) ;
ActivateCF3(j,k)$first(k)          ..     ActiCF3(j,k)     =e= FixtempC2(j) - tc(j,k) ;
TempCalCF1(j,k)$first(k)           ..     TCcalF1(j,k)     =e= (tcout(j) + max(FixtempC1(j),tc(j,k)))/2 ;
TempCalCF2(j,k)$first(k)           ..     TCcalF2(j,k)     =e= (FixtempC1(j) + max(FixtempC2(j),tc(j,k)))/2;
TempCalCF3(j,k)$first(k)           ..     TCcalF3(j,k)     =e= (FixtempC2(j) + tc(j,k))/2;
CPCF11(j,k)$first(k)               ..     CPCF1(j,k)       =e= OC1(j) * TCcalF1(j,k) + GC1(j);
CPCF22(j,k)$first(k)               ..     CPCF2(j,k)       =e= OC2(j) * TCcalF2(j,k) + GC2(j);
CPCF33(j,k)$first(k)               ..     CPCF3(j,k)       =e= OC3(j) * TCcalF3(j,k) + GC3(j);
CPCAVGGGF(j,k)$first(k)            ..     CpCavggF(j,k)*(max(0,ActiCF1(j,k)) + max(0,ActiCF2(j,k))
                                        + max(0,ActiCF3(j,k)) ) =e=( (CpCF1(j,k) * max(0,ActiCF1(j,k)))
                                        + (CpCF2(j,k) * max(0,ActiCF2(j,k))) + (CpCF3(j,k) * max(0,ActiCF3(j,k))) );
*--------------------[illegible] Heat Capacity'--------------------

Parameter
ech(i)            Overall heat transfer at hot streams
ecc(j)            Overall heat transfer at cold streams
Omega(i,j)        Upper bound set to the smallest heat content of the two streams involved in the match
Theta(i,j)        maximum temperature difference for each exchanger;
ech(i)            =  fhavg(i) * (thin(i) - thout(i));
ecc(j)            =  fcavg(j) * (tcout(j) - tcin(j));
Omega(i,j)        =  min(ecc(j),ech(i));
Theta(i,j)        =  ( thin(i)-thout(i) ) - ( tcin(j)-tcout(j) );
Display Omega,Theta;

Equation
OverallBalanceHOT(i)   Overall heat balance for hot stream
OverallBalanceCOLD(j)  Overall heat balance for cold stream;
OverallBalanceHOT(i)  .. (thin(i)-thout(i)) * fhavg(i)  =e= sum((j,st),q(i,j,st)) + qcu(i) ;
OverallBalanceCOLD(j) .. (tcout(j)-tcin(j)) * fcavg(j)  =e= sum((i,st),q(i,j,st)) + qhu(j) ;

Equation
StageBalanceHOT(i,k)         Heat balance at each stage for hot stream
StageBalanceCOLD(j,k)        Heat balance at each stage for cold stream ;
StageBalanceHOT(i,k)$st(k)  .. (th(i,k)-th(i,k+1)) * fh(i) * CpHavgg(i,k) =e= sum(j,q(i,j,k)) ;
StageBalanceCOLD(j,k)$st(k) .. (tc(j,k)-tc(j,k+1)) * fc(j) * CpCavgg(j,k) =e= sum(i,q(i,j,k)) ;

Equation
ColdUTload(i,k)    Cold UT loading at hot streams
HotUTload(j,k)     Hot UT loading at cold streams;
ColdUTload(i,k)$last(k) .. (th(i,k)-thout(i)) * fh(i)  * CpHavggL(i,k)   =e= qcu(i);
HotUTload(j,k)$first(k) .. (tcout(j)-tc(j,k)) * fc(j)  * CpCavggF(j,k)   =e= qhu(j);

Equation
AssignmentHot(i,k)          Assignment of superstructure inlet temperatures for hot streams
AssignmentCold(j,k)         Assignment of superstructure inlet temperature for cols streams;
AssignmentHot(i,k)$first(k) .. thin(i) =e= th(i,k);
AssignmentCold(j,k)$last(k) .. tcin(j) =e= tc(j,k);

Equation
Feasibility1(i,k)      Feasibility temperature at each stage for hot stream
Feasibility2(j,k)      Feasibility temperature at each stage for cold stream
Feasibility3(i,k)      Feasibility temperature at final stage for hot stream
Feasibility4(j,k)      Feasibility temperature at first stage for cold stream;
Feasibility1(i,k)$st(k)    .. th(i,k)  =g= th(i,k+1)    ;
Feasibility2(j,k)$st(k)    .. tc(j,k)  =g= tc(j,k+1)    ;
Feasibility3(i,k)$last(k) .. th(i,k)  =g= thout(i)     ;
Feasibility4(j,k)$first(k).. tcout(j) =g= tc(j,k)      ;
```

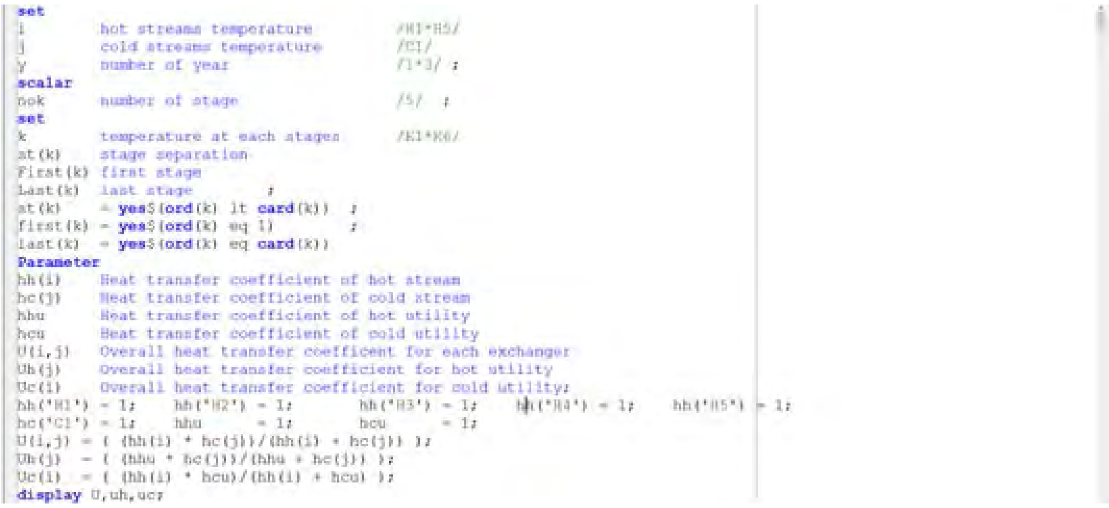
```
Equation
Logic1(i,j,k)    Matching heat exchanger at each stage
Logic2(i)        Hot UT matching
Logic3(j)        Cold UT matching;
Logic1(i,j,k)$st(k)        .. q(i,j,k) - Omega(i,j)*z(i,j,k) =l= 0       ;
Logic2(i)                  .. qcu(i)   - ech(i)*zcu(i)       =l= 0       ;
Logic3(j)                  .. qhu(j)   - ecc(j)*zhu(j)       =l= 0       ;

Equation
Approach1(i,j,k)          Approach temperature for hot inlet and cold outlet
Approach2(i,j,k)          Approach temperature for hot outlet and cold inlet
Approach3(j,k)            Approach temperature for hot UT
Approach4(j)              Approach temperature for hot UT
Approach5(i,k)            Approach temperature for cold UT
Approach6(i)              Approach temperature for cold UT;
Approach1(i,j,k)$st(k)    .. dT(i,j,k)     =l= th(i,k)     - tc(j,k)       + Theta(i,j)*(1-z(i,j,k));
Approach2(i,j,k)$st(k)    .. dT(i,j,k+1)   =l= th(i,k+1)   - tc(j,k+1)     + Theta(i,j)*(1-z(i,j,k)) ;
Approach3(j,k)$first(k)   .. dTHU(j)       =l= TOUThu      - tc(j,k)       + sum(i,Theta(i,j))*(1-zhu(j));
Approach4(j)              .. dTHU2(j)      =l= TINhu       - tcout(j)      + sum(i,Theta(i,j))*(1-zhu(j));
Approach5(i,k)$last(k)    .. dTCU(i)       =l= th(i,k)     - TOUTcu        + sum(j,Theta(i,j))*(1-zcu(i));
Approach6(i)              .. dTCU2(i)      =l= thout(i)    - TINcu         + sum(j,Theta(i,j))*(1-zcu(i));

Equation
EMATconstraint(i,j,k)              minimum temperature approach for each exchanger
EMATconstraintHU(j)                minimum temperature approach for each exchanger
EMATconstraintHU2(j)               minimum temperature approach for each exchanger
EMATconstraintCU(i)                minimum temperature approach for each exchanger
EMATconstraintCU2(i)               minimum temperature approach for each exchanger ;
EMATconstraint(i,j,k)              ..  dT(i,j,k) =g= EMAT;
EMATconstraintHU(j)                ..  dTHU(j)   =g= EMAT;
EMATconstraintHU2(j)               ..  dTHU2(j)  =g= EMAT;
EMATconstraintCU(i)                ..  dTCU(i)   =g= EMAT;
EMATconstraintCU2(i)               ..  dTCU2(i)  =g= EMAT;

Equation
AreasHX(i,j,k)    Area calculation for each exchanger
AreasH(j,k)       Area calculation for hot UT
AreasC(i,k)       Area calculation for cold UT;
AreasHX(i,j,k)$st(k)    .. AreaHX(i,j,k)    =e= (q(i,j,k)/(U(i,j)*((dt(i,j,k)*dt(i,j,k+1)*(dt(i,j,k)+dt(i,j,k+1))/2)
                                                 **(1/3)))) ;
AreasH(j,k)$first(k)    .. AreaH(j)         =e= (qhu(j)/(Uh(j)*((dthu(j)*dTHU2(j)*(dthu(j)+dTHU2(j))/2)**(1/3))));
AreasC(i,k)$last(k)     .. AreaC(i)         =e= (qcu(i)/(Uc(i)*((dtcu(i)*dTCU2(i)*(dtcu(i)+dTCU2(i))/2)**(1/3))));
*SynHEAT Objective Function---------------------------------------------------------
Equation
OBJ    Objective function;
OBJ .. s =e= sum(i,(ccu*qcu(i))) + sum(j,(chu*qhu(j)))
                 +sum((i,j,st),CF*z(i,j,st)) +sum(i,(CFH*(zcu(i))))
                 +sum(j,CFC*zhu(j))
                 +sum((i,j,k),C*(AreaHX(i,j,k))**B )
                 +sum((j),CH*(AreaH(j))**BH  )
                 +sum((i),CC*(AreaC(i))**BC  );
*SynHEAT Objective Function---------------------------------------------------------

Equation
SplitlineH(i,k)  Splitting line constrain for hot streams
SplitlineC(j,k)  Splitting line constrain for cold streams ;
SplitlineH(i,k)$st(k) .. sum(j,z(i,j,k)) =l= 1 ;
SplitlineC(j,k)$st(k) .. sum(i,z(i,j,k)) =l= 3 ;

*Boundary---------------------------------------------------------------------------
*Temperature
th.up(i,k) = thin(i);
th.lo(i,k) = thout(i);
tc.up(j,k) = tcout(j);
tc.lo(j,k) = tcin(j);
*Util Temp
*-----------------------------------------------------------------------------------
loop((i,j),
         if(thin(i) - tcin(j) < 0,
             loop(k,
                 dt.up(i,j,k) = TINhu-EMAT;
                 display dt.up;
             );
                 else
             loop(k,
                 dt.up(i,j,k) = max(0,thin(i)-tcin(j));
                 display dt.up;
             );
         );
);
*-----------------------------------------------------------------------------------
dTHU.up(j)        = max(0,TINhu-tcin(j)-EMAT);
dTHU2.up(j)       = max(0,TINhu-tcin(j)-EMAT);
dTCU.up(i)        = max(0,thin(i)-toutcu-EMAT);
dTCU2.up(i)       = max(0,thin(i)-toutcu-EMAT);
dt.lo(i,j,k)      = EMAT;
dt.lo(i,j,k+1)    = EMAT;
dTHU.lo(j)        = EMAT;
dTHU2.lo(j)       = EMAT;
dTCU.lo(i)        = EMAT;
dTCU2.lo(i)       = EMAT;
```

```
*Heat load
q.up(i,j,k)$ST(k)= min(ech(i),ecc(j));
qcu.up(i)         = ech(i);
qhu.up(j)         = ecc(j);
q.lo(i,j,k)$ST(k)= 0;
qcu.lo(i)         = 0;
qhu.lo(j)         = 0;

*Area
AreaHX.up(i,j,k)$ST(k)    = q.up(i,j,k)/(U(i,j)*EMAT)   ;
AreaH.up(j)               = qhu.up(j)/((Uh(j))*EMAT);
AreaC.up(i)               = qcu.up(i)/((Uc(i))*EMAT);
AreaHX.lo(i,j,k)$ST(k)    = 0;
AreaH.lo(j)               = 0;
AreaC.lo(i)               = 0;

*Initial Value
th.l(i,k)         = thin(i);
tc.l(j,k)         = tcin(j);
dt.l(i,j,k)       = max(EMAT,th.l(i,k)-tc.l(j,k));
dTHU.l(j)         = EMAT;
dTHU2.l(j)        = EMAT;
dTCU.l(i)         = EMAT;
dTCU2.l(i)        = EMAT;
qcu.l(i)          = ech(i)/2;
*Fix value
z.fx('H1','C1',k) = 0;
*Boundary for inconstant Heat Capacity

*CP
ActiH1.l(i,k)            =       thin(i) - FixtempH1(i);
ActiH2.l(i,k)            =       FixtempH1(i) - FixtempH2(i);
ActiH3.l(i,k)            =       FixtempH1(i) - thout(i);

THcal1.l(i,k)            =       (thin(i) + FixtempH1(i))/2;
THcal2.l(i,k)            =       (FixtempH1(i) + FixtempH2(i))/2;
THcal3.l(i,k)            =       (FixtempH2(i) + thout(i))/2;

CpH1.l(i,k)              =       OH1(i) * THcal1.l(i,k) + GH1(i);
CpH2.l(i,k)              =       OH2(i) * THcal2.l(i,k) + GH2(i);
CpH3.l(i,k)              =       OH3(i) * THcal3.l(i,k) + GH3(i);

CpHavgg.l(i,k)           =       CpHavg(i);

ActiC1.l(j,k)            =       tcout(j) - FixtempC1(j);
CpCavgg.l(j,k)           =       CpCavg(j);
*initial guess of cold utility
ActiHL1.l(i,k)$last(k)          =       thin(i) - FixtempH1(i);
ActiHL2.l(i,k)$last(k)          =       FixtempH1(i) - FixtempH2(i);
ActiHL3.l(i,k)$last(k)          =       FixtempH2(i) - thout(i);

THcalL1.l(i,k)$last(k)          =       (thin(i) + FixtempH1(i))/2;
THcalL2.l(i,k)$last(k)          =       (FixtempH1(i) + FixtempH2(i))/2;
THcalL3.l(i,k)$last(k)          =       (FixtempH2(i) + thout(i))/2;

CpHL1.l(i,k)$last(k)            =       OH1(i) * THcalL1.l(i,k) + GH1(i);
CpHL2.l(i,k)$last(k)            =       OH2(i) * THcalL2.l(i,k) + GH2(i);
CpHL3.l(i,k)$last(k)            =       OH3(i) * THcalL3.l(i,k) + GH3(i);

CpHavggL.l(i,k)$last(k)         =       (CpHL1.l(i,k) * max(0,ActiHL1.l(i,k)) + CpHL2.l(i,k)*max(0,ActiHL2.l(i,k))
                                        /( max(0,ActiHL1.l(i,k)) + max(0,ActiHL2.l(i,k)) + max(0,ActiHL3.l(i,k))  +

*initial guess of hot utility

ActiCF1.l(j,k)$first(k)          =       tcout(j) - FixtempC1(j);
ActiCF2.l(j,k)$first(k)          =       FixtempC1(j) - FixtempC2(j);
ActiCF3.l(j,k)$first(k)          =       FixtempC2(j) - tcin(j);

TCcalF1.l(j,k)$first(k)          =       (tcout(j) + FixtempC1(j))/2;
TCcalF2.l(j,k)$first(k)          =       (FixtempC1(j) + FixtempC2(j))/2;
TCcalF3.l(j,k)$first(k)          =       (FixtempC2(j) + tcin(j))/2;
```

```
ActiH1.up(i,k)          =       thin(i) - FixtempH1(i);
ActiH1.lo(i,k)          =       thout(i) - thin(i);
ActiH2.up(i,k)          =       FixtempH1(i) - FixtempH2(i);
ActiH2.lo(i,k)          =       thout(i) - thin(i);
ActiH3.up(i,k)          =       FixtempH2(i) - thout(i);
ActiH3.lo(i,k)          =       thout(i) - thin(i);

THcal1.up(i,k)          =       thin(i);
THcal1.lo(i,k)          =       (thout(i)+FixTempH1(i))/2;
THcal2.up(i,k)          =       (FixTempH1(i)+thin(i))/2;
THcal2.lo(i,k)          =       (thout(i)+FixTempH2(i))/2;
THcal3.up(i,k)          =       (FixTempH2(i)+thin(i))/2;
THcal3.lo(i,k)          =       thout(i);

CpH1.up(i,k)            =       OH1(i) * THcal1.up(i,k) + GH1(i);
CpH1.lo(i,k)            =       OH1(i) * THcal1.lo(i,k) + GH1(i);
CpH2.up(i,k)            =       OH2(i) * THcal2.up(i,k) + GH2(i);
CpH2.lo(i,k)            =       OH2(i) * THcal2.lo(i,k) + GH2(i);
CpH3.up(i,k)            =       OH3(i) * THcal3.up(i,k) + GH3(i);
CpH3.lo(i,k)            =       OH3(i) * THcal3.lo(i,k) + GH3(i);

CpHavgg.up(i,k)         =       (CpH1.up(i,k) * ActiH1.up(i,k) + CpH2.up(i,k) * ActiH2.up(i,k) + CpH3.up(i,k) * Act
                                (ActiH1.up(i,k) + ActiH2.up(i,k) + ActiH3.up(i,k));
CpHavgg.lo(i,k)         =       0;

ActiC1.up(j,k)          =       tcout(j) - FixtempC1(j);
ActiC1.lo(j,k)          =       tcin(j) - tcout(j);
ActiC2.up(j,k)          =       FixtempC1(j) - FixtempC2(j);
ActiC2.lo(j,k)          =       tcin(j) - tcout(j);
ActiC3.up(j,k)          =       FixtempC2(j) - tcin(j);
ActiC3.lo(j,k)          =       tcin(j) - tcout(j);

TCcal1.up(j,k)          =       tcout(j);
TCcal1.lo(j,k)          =       (tcin(j)+FixTempC1(j))/2;
TCcal2.up(j,k)          =       (tcout(j)+FixTempC1(j))/2;
TCcal2.lo(j,k)          =       (tcin(j)+FixTempC2(j))/2;
TCcal3.up(j,k)          =       (tcout(j)+FixTempC2(j))/2;
TCcal3.lo(j,k)          =       tcin(j);

CpC1.up(j,k)            =       OC1(j) * TCcal1.up(j,k) + GC1(j);
CpC1.lo(j,k)            =       OC1(j) * TCcal1.lo(j,k) + GC1(j);
CpC2.up(j,k)            =       OC2(j) * TCcal2.up(j,k) + GC2(j);
CpC2.lo(j,k)            =       OC2(j) * TCcal2.lo(j,k) + GC2(j);
CpC3.up(j,k)            =       OC3(j) * TCcal3.up(j,k) + GC3(j);
CpC3.lo(j,k)            =       OC3(j) * TCcal3.lo(j,k) + GC3(j);

CpCavgg.up(j,k)         =       (CpC1.up(j,k) * ActiC1.up(j,k)  + CpC2.up(j,k) * ActiC2.up(j,k) + CpC3.up(j,k)
                                * ActiC3.up(j,k) )/(ActiC2.up(j,k) + ActiC1.up(j,k) + ActiC3.up(j,k) + sma( ));
CpCavgg.lo(j,k)         =       0;

ActiHL1.up(i,k)          =       thin(i) - FixTempH1(i);
ActiHL1.lo(i,k)          =       thout(i) - FixTempH1(i);
ActiHL2.up(i,k)          =       FixTempH1(i) - FixTempH2(i);
ActiHL2.lo(i,k)          =       thout(i) - FixTempH2(i);
ActiHL3.up(i,k)          =       FixTempH2(i) - thout(i);
ActiHL3.lo(i,k)          =       0;

THcalL1.up(i,k)          =       (thin(i)+FixTempH1(i))/2;
THcalL1.lo(i,k)          =       (thout(i)+FixTempH1(i))/2;
THcalL2.up(i,k)          =       (FixTempH1(i)+FixTempH2(i))/2;
THcalL2.lo(i,k)          =       (thout(i)+FixTempH2(i))/2;
THcalL3.up(i,k)          =       (thout(i)+FixTempH2(i))/2;
THcalL3.lo(i,k)          =       thout(i);

CpHL1.up(i,k)            =       OH1(i) * THcalL1.up(i,k) + GH1(i);
CpHL1.lo(i,k)            =       OH1(i) * THcalL1.lo(i,k) + GH1(i);
CpHL2.up(i,k)            =       OH2(i) * THcalL2.up(i,k) + GH2(i);
CpHL2.lo(i,k)            =       OH2(i) * THcalL2.lo(i,k) + GH2(i);
CpHL3.up(i,k)            =       OH3(i) * THcalL3.up(i,k) + GH3(i);
CpHL3.lo(i,k)            =       OH3(i) * THcalL3.lo(i,k) + GH3(i);

CpHavggL.up(i,k)$last(k) =       (CpHL1.up(i,k)*ActiHL1.up(i,k)+CpHL2.up(i,k)*ActiHL2.up(i,k)+CpHL3.up(i,k)*ActiHL3
                                 (ActiHL1.up(i,k)+ActiHL2.up(i,k)+ActiHL3.up(i,k));
CpHavggL.lo(i,k)$last(k) =       0;
option sysout = on;
option decimals = 8;
option domlim = 5;
option MINLP = DICOPT;
option Threads = 0;
option reslim = 600;
model Synheat /all/;

Synheat.optfile=1;

$onecho > dicopt.opt
$offecho


solve Synheat min z using MINLP ;
Display CpHavgg.l,CpCavgg.l,CpHavggL.l,CpCavggF.l,z.l,zcu.l,zhu.l,q.l,qcu.l,qhu.l,th.l,tc.l,
          s.l,dthu.l,dthu2.l,dtcu.l,dtcu2.l;
```

```
Parameter
AreacalHX(i,j,k)          Area calculation for each heat exchanger
AreacalHU(j)              Area calculation for hot utility
AreacalCU(i)              Area calculation for cold utility
TotalArea                 Total Area calculation
ColdUTcost                Cold utility cost
HotUTcost                 Hot utility cost
TotalUT                   Total utility cost
FixedcostHX               Fixed charge for exchanger
FixedcostHotUT            Fixed charge for hot utility
FixedcostColdUT           Fixed charge for cold utility
AreaHXcost                Area cost for each exchanger
AreaHUcost                Area cost for hot utility
AreaCUcost                Area cost for cold utility
Capitalcost               Capital cost
TAC                       Total Annual Cost;
AreacalHX(i,j,k)$st(k)    = q.l(i,j,k)/(U(i,j)*(dt.l(i,j,k)*dt.l(i,j,k+1)*(dt.l(i,j,k)+dt.l(i,j,k+1))/2)**(1/3)) ;
AreacalHU(j)              = qhu.l(j)/((Uh(j))*((dthu.l(j))*(TINhu-tcout(j))*((dthu.l(j))+(TINhu-tcout(j)))/2)**(1/3))
AreacalCU(i)              = qcu.l(i)/((Uc(i))*((dtcu.l(i))*(thout(i)-TINcu)*((dtcu.l(i))+(thout(i)-TINcu))/2)**(1/3))
TotalArea                 = sum((i,j,k),AreacalHX(i,j,k))+ sum(j,AreacalHU(j)) + sum(i,AreacalCU(i));
ColdUTcost                = sum(i,(ccu*qcu.l(i)));
HotUTcost                 = sum(j,(chu*qhu.l(j)));
TotalUT                   = ColdUTcost + HotUTcost;
FixedcostHX               = sum((i,j,st),CF*z.l(i,j,st));
FixedcostHotUT            = sum(i,(CFH*(zcu.l(i))));
FixedcostColdUT           = sum(j,CFC*zhu.l(j));
AreaHXcost                = C*sum((i,j,k),AreacalHX(i,j,k)**B);
AreaHUcost                = CH*sum(j,AreacalHU(j)**BH);
AreaCUcost                = CC*sum(i,AreacalCU(i)**BC);
Capitalcost               = FixedcostHX + FixedcostHotUT + FixedcostColdUT + AreaHXcost + AreaHUcost + AreaCUcost ;
TAC                       = Capitalcost + TotalUT ;

display AreacalHX,AreacalHU,AreacalCU,TotalArea
,ColdUTcost,HotUTcost,TotalUT,FixedcostHX,FixedcostHotUT,FixedcostColdUT
,AreaHXcost,AreaHUcost,AreaCUcost,Capitalcost,TAC ;
```

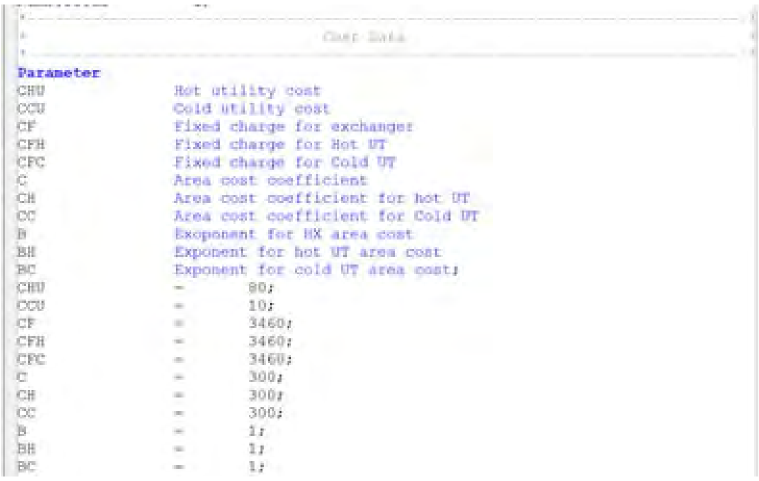
Appendix B GAMS Code of Case Study 4 (Retrofit)



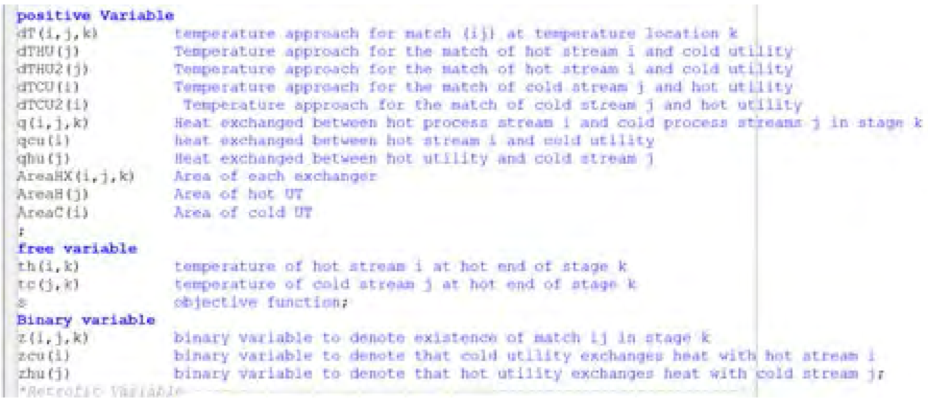
```
set
i         hot streams temperature          /H1*H5/
j         cold streams temperature         /C1/
y         number of year                   /1*3/ ;
scalar
nok       number of stage                  /5/  ;
set
k         temperature at each stages       /K1*K6/
st(k)     stage separation
First(k)  first stage
Last(k)   last stage          ;
st(k)     = yes$(ord(k) lt card(k))   ;
first(k)  = yes$(ord(k) eq 1)         ;
last(k)   = yes$(ord(k) eq card(k))
Parameter
hh(i)     Heat transfer coefficient of hot stream
hc(j)     Heat transfer coefficient of cold stream
hhu       Heat transfer coefficient of hot utility
hcu       Heat transfer coefficient of cold utility
U(i,j)    Overall heat transfer coefficent for each exchanger
Uh(j)     Overall heat transfer coefficient for hot utility
Uc(i)     Overall heat transfer coefficient for cold utility;
hh('H1') = 1;    hh('H2') = 1;       hh('H3') = 1;    hh('H4') = 1;    hh('H5') = 1;
hc('C1') = 1;    hhu      = 1;       hcu      = 1;
U(i,j) = ( (hh(i) * hc(j))/(hh(i) + hc(j)) );
Uh(j)  = ( (hhu * hc(j))/(hhu + hc(j)) );
Uc(i)  = ( (hh(i) * hcu)/(hh(i) + hcu) );
display U,uh,uc;
```

```
*-----------------------------------------------------------------------*
*                               Data HT                                  *
*-----------------------------------------------------------------------*
parameter
TINhu     Temperature inlet hot utility
TOUThu    Temperature outlet hot utility
TINcu     Temperature inlet cold utility
TOUTcu    Temperature outlet cold utility;
TINhu     =       500;
TOUThu    =       500;
TINcu     =       10;
TOUTcu    =       15;
*-----------------------------------------------------------------------*
*                               Data HEN                                 *
*-----------------------------------------------------------------------*
parameter
thin(i)          Temperature inlet for hot streams
thout(i)         Temperature outlet for hot streams
fh(i)            Heat capactity flow rate for hot streams
tcin(j)          Temperature inlet for cold streams
tcout(j)         Temperature outlet for cold streams
fc(j)            Heat capacity flow rate for cold streams
fhavg(i)         Average Heat capacity flow rate of hot stream
fcavg(j)         Average Heat capacity flow rate of cold stream
CpHavg(i)        Average Heat capacity hot stream
CpCavg(j)        Average Heat capacity cold stream;
```

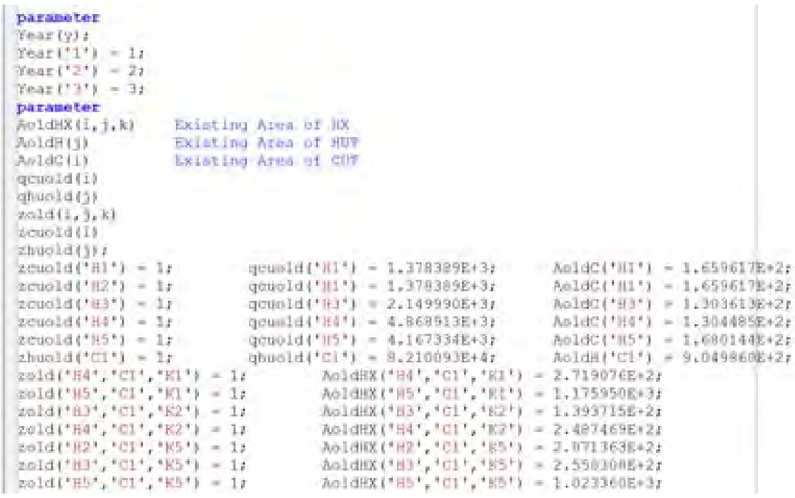
```
* hot
thin('H1')= 43.3333491800000000000000;   thout('H1')=25;  fh('H1')=  37.3761171666667000000000000000;
thin('H2')= 200.040011700000000000000;  thout('H2')=25;  fh('H2')=  21.8780958861111000000000000000;
thin('H3')= 272.790010000000000000000;  thout('H3')=25;  fh('H3')=  21.7570682027778000000000000000;
thin('H4')= 342.725803200000000000000;  thout('H4')=50;  fh('H4')=  26.2411060027778000000000000000;
thin('H5')= 370.722012900000000000000;  thout('H5')=50;  fh('H5')=  87.0659140555556000000000000000;
CpHavg('H1') = 2.01157287587133000000;
CpHavg('H2') = 2.19753244613817000000;
CpHavg('H3') = 2.28077030739148000000;
CpHavg('H4') = 2.42342119317160000000;
CpHavg('H5') = 2.41611128177393000000;
* cold
tcin('C1')= 50;  tcout('C1')= 376.800000000000000000000;  fc('C1')=  194.240768916667000000000000000;
CpCavg('C1') =    2.76662173432649000000;
*Avg
fhavg(i)= fh(i) * CpHavg(i);
fcavg(j)= fc(j) * CpCavg(j);
Display fhavg,fcavg;

Scalar
EMAT      Exchanger minimum approximation temperature
HRAT      Heat recovery approximation temperature
Small     Small Value
NumberYear        Number of Year for Installation Cost;
EMAT      = 1;
HRAT      = 1;
Small     = 1e-7;
NumberYear        = 1;
```



```
*------------------------------------------------------------------------------
*                                  Cost Data
*------------------------------------------------------------------------------
Parameter
CHU                 Hot utility cost
CCU                 Cold utility cost
CF                  Fixed charge for exchanger
CFH                 Fixed charge for Hot UT
CFC                 Fixed charge for Cold UT
C                   Area cost coefficient
CH                  Area cost coefficient for hot UT
CC                  Area cost coefficient for Cold UT
B                   Exoponent for HX area cost
BH                  Exponent for hot UT area cost
BC                  Exponent for cold UT area cost;
CHU                 =         80;
CCU                 =         10;
CF                  =         3460;
CFH                 =         3460;
CFC                 =         3460;
C                   =         300;
CH                  =         300;
CC                  =         300;
B                   =         1;
BH                  =         1;
BC                  =         1;
```

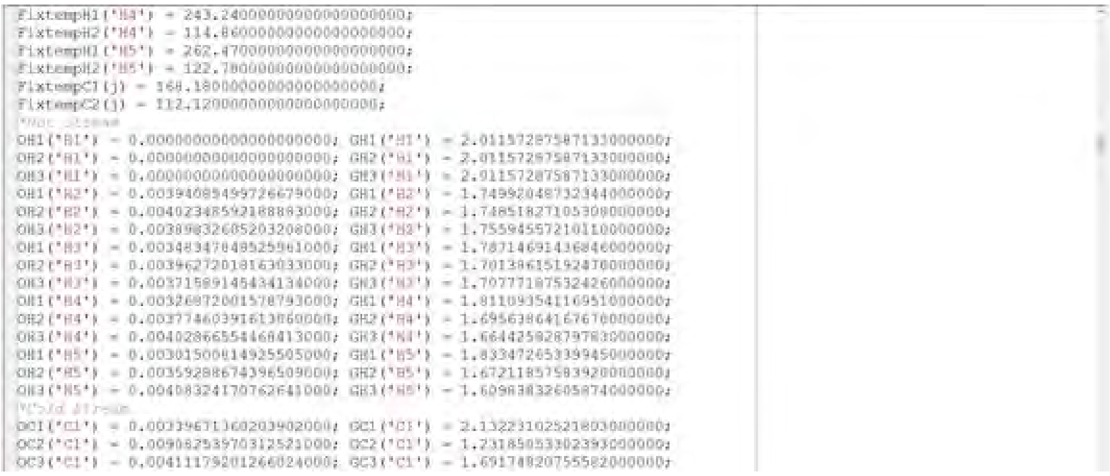


```
positive Variable
dT(i,j,k)           temperature approach for match (ij) at temperature location k
dTHU(j)             Temperature approach for the match of hot stream i and cold utility
dTHU2(j)            Temperature approach for the match of hot stream i and cold utility
dTCU(i)             Temperature approach for the match of cold stream j and hot utility
dTCU2(i)             Temperature approach for the match of cold stream j and hot utility
q(i,j,k)            Heat exchanged between hot process stream i and cold process streams j in stage k
qcu(i)              heat exchanged between hot stream i and cold utility
qhu(j)              Heat exchanged between hot utility and cold stream j
AreaHX(i,j,k)       Area of each exchanger
AreaH(j)            Area of hot UT
AreaC(i)            Area of cold UT
;
free variable
th(i,k)             temperature of hot stream i at hot end of stage k
tc(j,k)             temperature of cold stream j at hot end of stage k
z                   objective function;
Binary variable
z(i,j,k)            binary variable to denote existence of match ij in stage k
zcu(i)              binary variable to denote that cold utility exchanges heat with hot stream i
zhu(j)              binary variable to denote that hot utility exchanges heat with cold stream j;
*Retrofit Variable
```

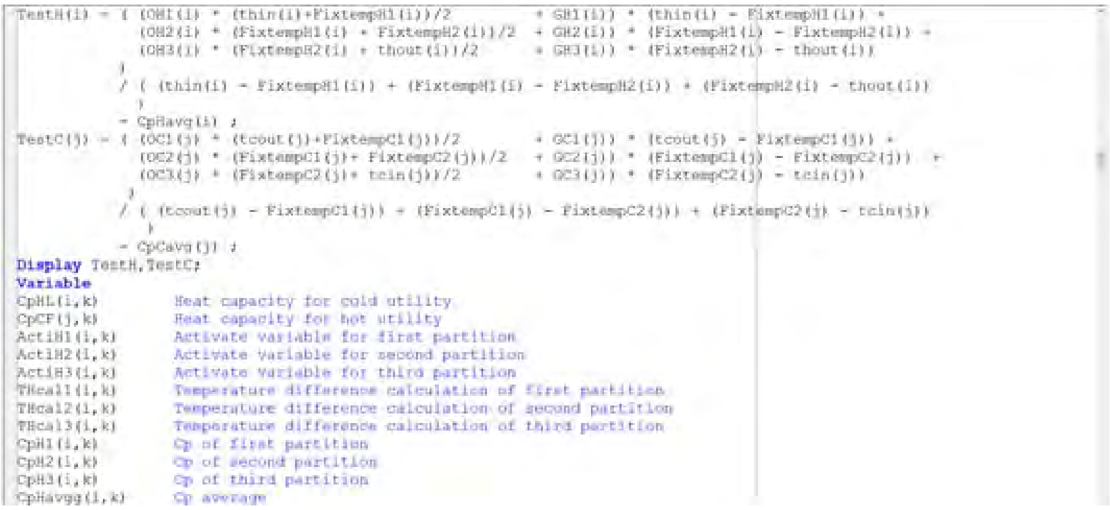


```
parameter
Year(y);
Year('1') = 1;
Year('2') = 2;
Year('3') = 3;
parameter
AoldHX(i,j,k)       Existing Area of HX
AoldH(j)            Existing Area of HUT
AoldC(i)            Existing Area of CUT
qcuold(i)
qhuold(j)
zold(i,j,k)
zcuold(I)
zhuold(j);
zcuold('H1') = 1;        qcuold('H1') = 1.378389E+3;       AoldC('H1') = 1.659617E+2;
zcuold('H2') = 1;        qcuold('H1') = 1.378389E+3;       AoldC('H1') = 1.659617E+2;
zcuold('H3') = 1;        qcuold('H3') = 2.149990E+3;       AoldC('H3') = 1.303613E+2;
zcuold('H4') = 1;        qcuold('H4') = 4.868913E+3;       AoldC('H4') = 1.304485E+2;
zcuold('H5') = 1;        qcuold('H5') = 4.167334E+3;       AoldC('H5') = 1.680144E+2;
zhuold('C1') = 1;        qhuold('C1') = 8.210093E+4;       AoldH('C1') = 9.049860E+2;
zold('H4','C1','K1') = 1;        AoldHX('H4','C1','K1') = 2.719076E+2;
zold('H5','C1','K1') = 1;        AoldHX('H5','C1','K1') = 1.175950E+3;
zold('H3','C1','K2') = 1;        AoldHX('H3','C1','K2') = 1.393715E+2;
zold('H4','C1','K2') = 1;        AoldHX('H4','C1','K2') = 2.407469E+2;
zold('H2','C1','K5') = 1;        AoldHX('H2','C1','K5') = 2.071363E+2;
zold('H3','C1','K5') = 1;        AoldHX('H3','C1','K5') = 2.558308E+2;
zold('H5','C1','K5') = 1;        AoldHX('H5','C1','K5') = 1.023360E+3;
```

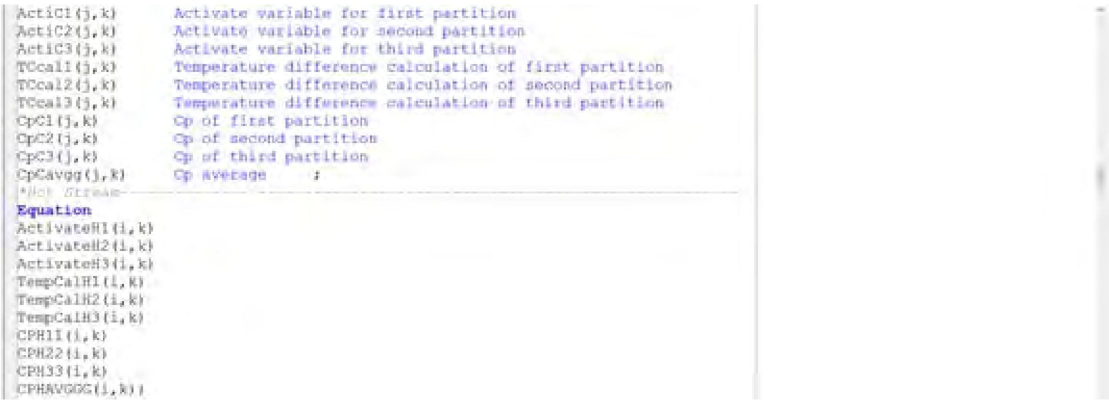
```
*----------------------Inconstant Heat Capacity----------------------*
parameter
OH1(i) Cp Coefficient of Hot stream
OH2(i) Cp Coefficient of Hot stream
OH3(i) Cp Coefficient of Hot stream
GH1(i) Cp Coefficient of Hot stream
GH2(i) Cp Coefficient of Hot stream
GH3(i) Cp Coefficient of Hot stream
OC1(j) Cp Coefficient of cold stream
OC2(j) Cp Coefficient of cold stream
OC3(j) Cp Coefficient of cold stream
GC1(j) Cp Coefficient of cold stream
GC2(j) Cp Coefficient of cold stream
GC3(j) Cp Coefficient of cold stream
FixtempH1(i)
FixtempH2(i)
FixtempC1(j)
FixtempC2(j)
TestH(i)
TestC(j);
FixtempH1('H1') = 30;
FixtempH2('H1') = 30;
FixtempH1('H2') = 145.23000000000000000000;
FixtempH2('H2') = 76.27000000000000000000;
FixtempH1('H3') = 195.21000000000000000000;
FixtempH2('H3') = 95.08000000000000000000;
```



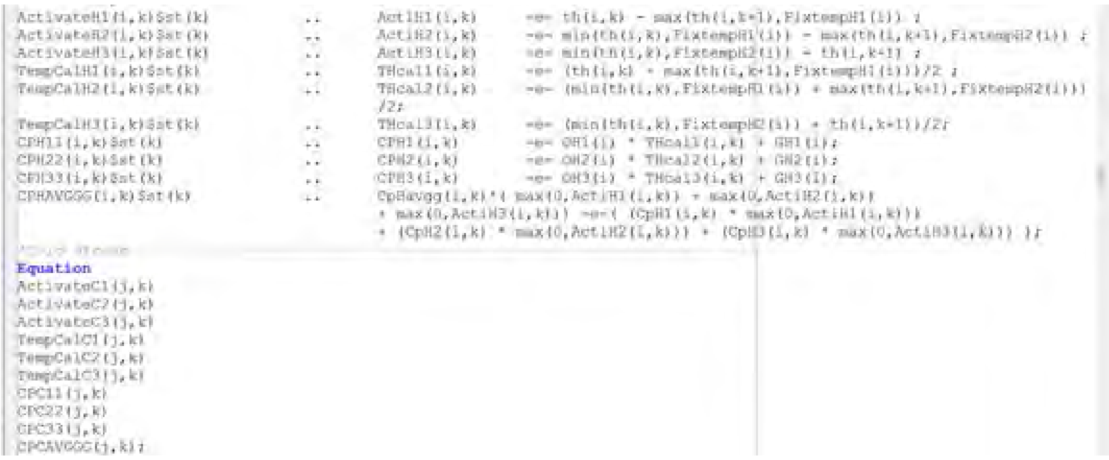
```
FixtempH1('H4') = 243.24000000000000000000;
FixtempH2('H4') = 114.86000000000000000000;
FixtempH1('H5') = 262.47000000000000000000;
FixtempH2('H5') = 122.78000000000000000000;
FixtempC1(j) = 168.18000000000000000000;
FixtempC2(j) = 112.12000000000000000000;
*Hot stream
OH1('H1') = 0.00000000000000000000; GH1('H1') = 2.01157287587133000000;
OH2('H1') = 0.00000000000000000000; GH2('H1') = 2.01157287587133000000;
OH3('H1') = 0.00000000000000000000; GH3('H1') = 2.01157287587133000000;
OH1('H2') = 0.00394085499726679000; GH1('H2') = 1.74992048732344000000;
OH2('H2') = 0.00402348592188883000; GH2('H2') = 1.74851827105308000000;
OH3('H2') = 0.00389832605203208000; GH3('H2') = 1.75594557210110000000;
OH1('H3') = 0.00348347848525961000; GH1('H3') = 1.78714691436846000000;
OH2('H3') = 0.00396272018163033000; GH2('H3') = 1.70138615192476000000;
OH3('H3') = 0.00371589145434134000; GH3('H3') = 1.70777187532426000000;
OH1('H4') = 0.00326872001578793000; GH1('H4') = 1.81109354116951000000;
OH2('H4') = 0.00377460391613060000; GH2('H4') = 1.69563864167676000000;
OH3('H4') = 0.00402866554468413000; GH3('H4') = 1.66442582879783000000;
OH1('H5') = 0.00301500814925505000; GH1('H5') = 1.83347265339945000000;
OH2('H5') = 0.00359288674396509000; GH2('H5') = 1.67211857583920000000;
OH3('H5') = 0.00408324170762641000; GH3('H5') = 1.60983832605874000000;
*Cold stream
OC1('C1') = 0.00339671360203902000; GC1('C1') = 2.13223102521803000000;
OC2('C1') = 0.00906253970312521000; GC2('C1') = 1.23185053302393000000;
OC3('C1') = 0.00411179201266024000; GC3('C1') = 1.69174820755582000000;
```



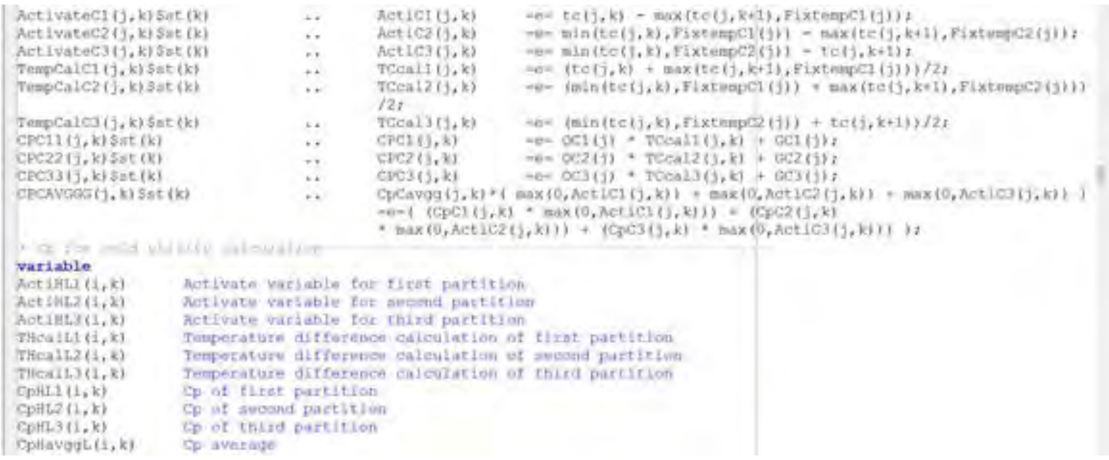
```
TestH(i) = ( (OH1(i) * (thin(i)+FixtempH1(i))/2          + GH1(i)) * (thin(i) - FixtempH1(i)) +
             (OH2(i) * (FixtempH1(i) + FixtempH2(i))/2  + GH2(i)) * (FixtempH1(i) - FixtempH2(i)) +
             (OH3(i) * (FixtempH2(i) + thout(i))/2      + GH3(i)) * (FixtempH2(i) - thout(i))
           )
           / ( (thin(i) - FixtempH1(i)) + (FixtempH1(i) - FixtempH2(i)) + (FixtempH2(i) - thout(i))
             )
           - CpHavg(i) ;
TestC(j) = ( (OC1(j) * (tcout(j)+FixtempC1(j))/2          + GC1(j)) * (tcout(j) - FixtempC1(j)) +
             (OC2(j) * (FixtempC1(j)+ FixtempC2(j))/2   + GC2(j)) * (FixtempC1(j) - FixtempC2(j))  +
             (OC3(j) * (FixtempC2(j)+ tcin(j))/2        + GC3(j)) * (FixtempC2(j) - tcin(j))
            )
           / ( (tcout(j) - FixtempC1(j)) + (FixtempC1(j) - FixtempC2(j)) + (FixtempC2(j) - tcin(j))
              )
           - CpCavg(j) ;
Display TestH,TestC;
Variable
CpHL(i,k)           Heat capacity for cold utility
CpCF(j,k)           Heat capacity for hot utility
ActiH1(i,k)         Activate variable for first partition
ActiH2(i,k)         Activate variable for second partition
ActiH3(i,k)         Activate variable for third partition
THcal1(i,k)         Temperature difference calculation of first partition
THcal2(i,k)         Temperature difference calculation of second partition
THcal3(i,k)         Temperature difference calculation of third partition
CpH1(i,k)           Cp of first partition
CpH2(i,k)           Cp of second partition
CpH3(i,k)           Cp of third partition
CpHavgg(i,k)        Cp average
```



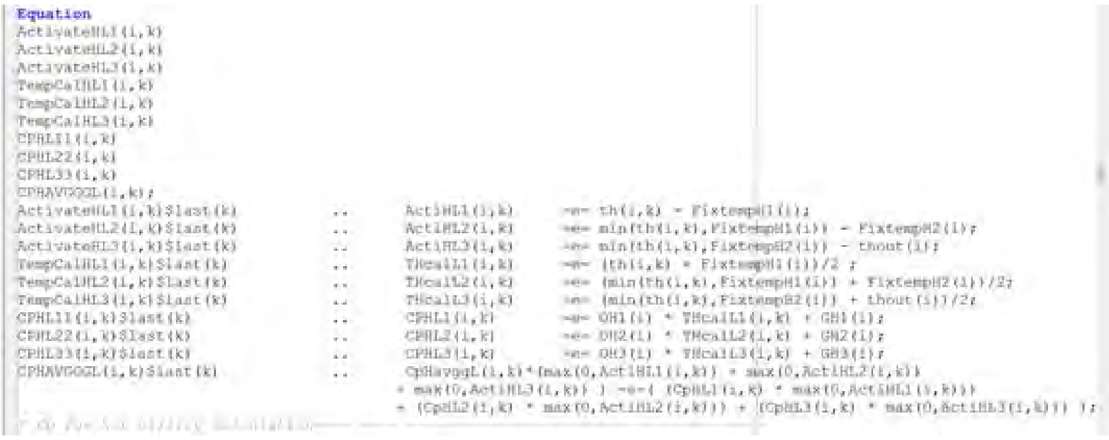
```
ActiC1(j,k)        Activate variable for first partition
ActiC2(j,k)        Activate variable for second partition
ActiC3(j,k)        Activate variable for third partition
TCcal1(j,k)        Temperature difference calculation of first partition
TCcal2(j,k)        Temperature difference calculation of second partition
TCcal3(j,k)        Temperature difference calculation of third partition
CpC1(j,k)          Cp of first partition
CpC2(j,k)          Cp of second partition
CpC3(j,k)          Cp of third partition
CpCavgg(j,k)       Cp average     ;
*Hot Stream-----------------------------------------------------------------------
Equation
ActivateH1(i,k)
ActivateH2(i,k)
ActivateH3(i,k)
TempCalH1(i,k)
TempCalH2(i,k)
TempCalH3(i,k)
CPH11(i,k)
CPH22(i,k)
CPH33(i,k)
CPHAVGGG(i,k);
```



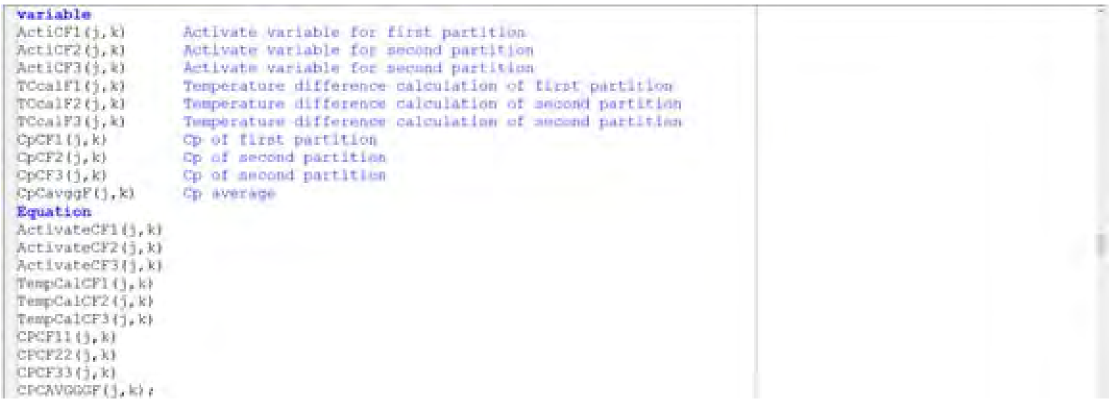
```
ActivateH1(i,k)$st(k)          ..     ActiH1(i,k)      =e= th(i,k) - max(th(i,k+1),FixtempH1(i)) ;
ActivateH2(i,k)$st(k)          ..     ActiH2(i,k)      =e= min(th(i,k),FixtempH1(i)) - max(th(i,k+1),FixtempH2(i)) ;
ActivateH3(i,k)$st(k)          ..     ActiH3(i,k)      =e= min(th(i,k),FixtempH2(i)) - th(i,k+1) ;
TempCalH1(i,k)$st(k)           ..     THcal1(i,k)      =e= (th(i,k) + max(th(i,k+1),FixtempH1(i)))/2 ;
TempCalH2(i,k)$st(k)           ..     THcal2(i,k)      =e= (min(th(i,k),FixtempH1(i)) + max(th(i,k+1),FixtempH2(i)))
                                      /2;
TempCalH3(i,k)$st(k)           ..     THcal3(i,k)      =e= (min(th(i,k),FixtempH2(i)) + th(i,k+1))/2;
CPH11(i,k)$st(k)               ..     CPH1(i,k)        =e= OH1(i) * THcal1(i,k) + GH1(i);
CPH22(i,k)$st(k)               ..     CPH2(i,k)        =e= OH2(i) * THcal2(i,k) + GH2(i);
CPH33(i,k)$st(k)               ..     CPH3(i,k)        =e= OH3(i) * THcal3(i,k) + GH3(I);
CPHAVGGG(i,k)$st(k)            ..     CpHavgg(i,k)*( max(0,ActiH1(i,k)) + max(0,ActiH2(i,k))
                                      + max(0,ActiH3(i,k))) =e=( (CpH1(i,k) * max(0,ActiH1(i,k)))
                                      + (CpH2(i,k) * max(0,ActiH2(I,k))) + (CpH3(i,k) * max(0,ActiH3(i,k))) );
*Cold Stream-----------------------------------------------------------------------
Equation
ActivateC1(j,k)
ActivateC2(j,k)
ActivateC3(j,k)
TempCalC1(j,k)
TempCalC2(j,k)
TempCalC3(j,k)
CPC11(j,k)
CPC22(j,k)
CPC33(j,k)
CPCAVGGG(j,k);
```



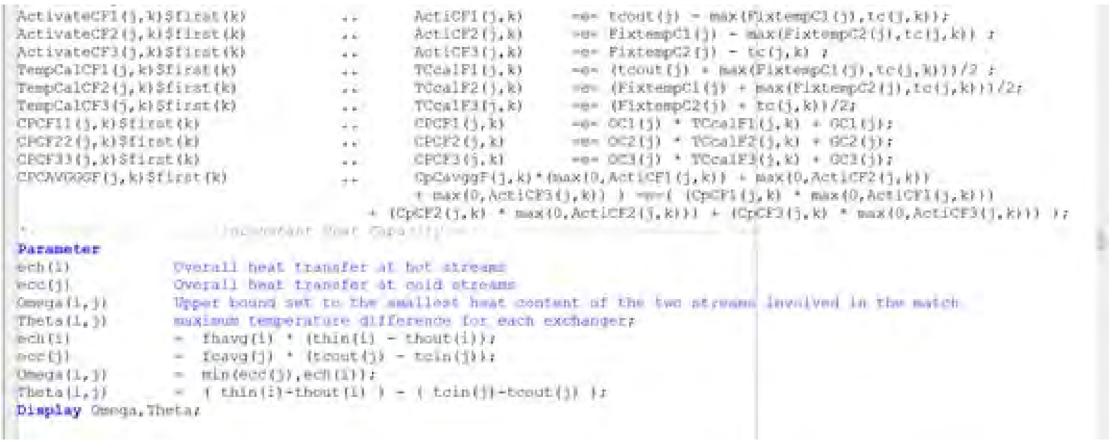
```
ActivateC1(j,k)$st(k)          ..     ActiC1(j,k)      =e= tc(j,k) - max(tc(j,k+1),FixtempC1(j));
ActivateC2(j,k)$st(k)          ..     ActiC2(j,k)      =e= min(tc(j,k),FixtempC1(j)) - max(tc(j,k+1),FixtempC2(j));
ActivateC3(j,k)$st(K)          ..     ActiC3(j,k)      =e= min(tc(j,k),FixtempC2(j)) - tc(j,k+1);
TempCalC1(j,k)$st(k)           ..     TCcal1(j,k)      =e= (tc(j,k) + max(tc(j,k+1),FixtempC1(j)))/2;
TempCalC2(j,k)$st(k)           ..     TCcal2(j,k)      =e= (min(tc(j,k),FixtempC1(j)) + max(tc(j,k+1),FixtempC2(j)))
                                      /2;
TempCalC3(j,k)$st(k)           ..     TCcal3(j,k)      =e= (min(tc(j,k),FixtempC2(j)) + tc(j,k+1))/2;
CPC11(j,k)$st(k)               ..     CPC1(j,k)        =e= OC1(j) * TCcal1(j,k) + GC1(j);
CPC22(j,k)$st(K)               ..     CPC2(j,k)        =e= OC2(j) * TCcal2(j,k) + GC2(j);
CPC33(j,k)$st(k)               ..     CPC3(j,k)        =e= OC3(j) * TCcal3(j,k) + GC3(j);
CPCAVGGG(j,k)$st(k)            ..     CpCavgg(j,k)*( max(0,ActiC1(j,k)) + max(0,ActiC2(j,k)) + max(0,ActiC3(j,k)) )
                                      =e=( (CpC1(j,k) * max(0,ActiC1(j,k))) + (CpC2(j,k)
                                      * max(0,ActiC2(j,k))) + (CpC3(j,k) * max(0,ActiC3(j,k))) );
* Cp for cold utility calculation-------------------------------------------------
variable
ActiHL1(i,k)        Activate variable for first partition
ActiHL2(i,k)        Activate variable for second partition
ActiHL3(i,k)        Activate variable for third partition
THcalL1(i,k)        Temperature difference calculation of first partition
THcalL2(i,k)        Temperature difference calculation of second partition
THcalL3(i,k)        Temperature difference calculation of third partition
CpHL1(i,k)          Cp of first partition
CpHL2(i,k)          Cp of second partition
CpHL3(i,k)          Cp of third partition
CpHavggL(i,k)       Cp average
```



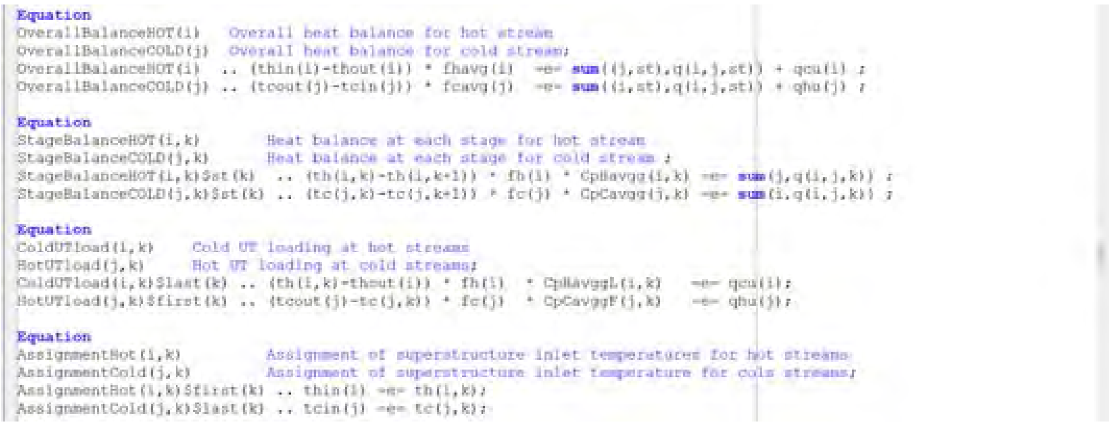
```
Equation
ActivateHL1(i,k)
ActivateHL2(i,k)
ActivateHL3(i,k)
TempCalHL1(i,k)
TempCalHL2(i,k)
TempCalHL3(i,k)
CPHL11(i,k)
CPHL22(i,k)
CPHL33(i,k)
CPHAVGGGL(i,k);
ActivateHL1(i,k)$last(k)        ..      ActiHL1(i,k)      =e= th(i,k) - FixtempH1(i);
ActivateHL2(i,k)$last(k)        ..      ActiHL2(i,k)      =e= min(th(i,k),FixtempH1(i)) - FixtempH2(i);
ActivateHL3(i,k)$last(k)        ..      ActiHL3(i,k)      =e= min(th(i,k),FixtempH2(i)) - thout(i);
TempCalHL1(i,k)$last(k)         ..      THcalL1(i,k)      =e= (th(i,k) + FixtempH1(i))/2 ;
TempCalHL2(i,k)$last(k)         ..      THcalL2(i,k)      =e= (min(th(i,k),FixtempH1(i)) + FixtempH2(i))/2;
TempCalHL3(i,k)$last(k)         ..      THcalL3(i,k)      =e= (min(th(i,k),FixtempH2(i)) + thout(i))/2;
CPHL11(i,k)$last(k)             ..      CPHL1(i,k)        =e= OH1(i) * THcalL1(i,k) + GH1(i);
CPHL22(i,k)$last(k)             ..      CPHL2(i,k)        =e= OH2(i) * THcalL2(i,k) + GH2(i);
CPHL33(i,k)$last(k)             ..      CPHL3(i,k)        =e= OH3(i) * THcalL3(i,k) + GH3(i);
CPHAVGGGL(i,k)$last(k)          ..      CpHavggL(i,k)*(max(0,ActiHL1(i,k)) + max(0,ActiHL2(i,k))
                                       + max(0,ActiHL3(i,k)) ) =e=( (CpHL1(i,k) * max(0,ActiHL1(i,k)))
                                       + (CpHL2(i,k) * max(0,ActiHL2(i,k))) + (CpHL3(i,k) * max(0,ActiHL3(i,k))) );
* Cp for hot utility calculation-----
```



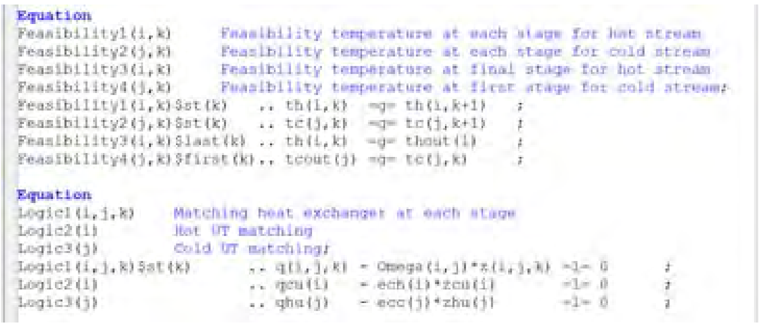
```
variable
ActiCF1(j,k)      Activate variable for first partition
ActiCF2(j,k)      Activate variable for second partition
ActiCF3(j,k)      Activate variable for second partition
TCcalF1(j,k)      Temperature difference calculation of first partition
TCcalF2(j,k)      Temperature difference calculation of second partition
TCcalF3(j,k)      Temperature difference calculation of second partition
CpCF1(j,k)        Cp of first partition
CpCF2(j,k)        Cp of second partition
CpCF3(j,k)        Cp of second partition
CpCavggF(j,k)     Cp average
Equation
ActivateCF1(j,k)
ActivateCF2(j,k)
ActivateCF3(j,k)
TempCalCF1(j,k)
TempCalCF2(j,k)
TempCalCF3(j,k)
CPCF11(j,k)
CPCF22(j,k)
CPCF33(j,k)
CPCAVGGGF(j,k);
```



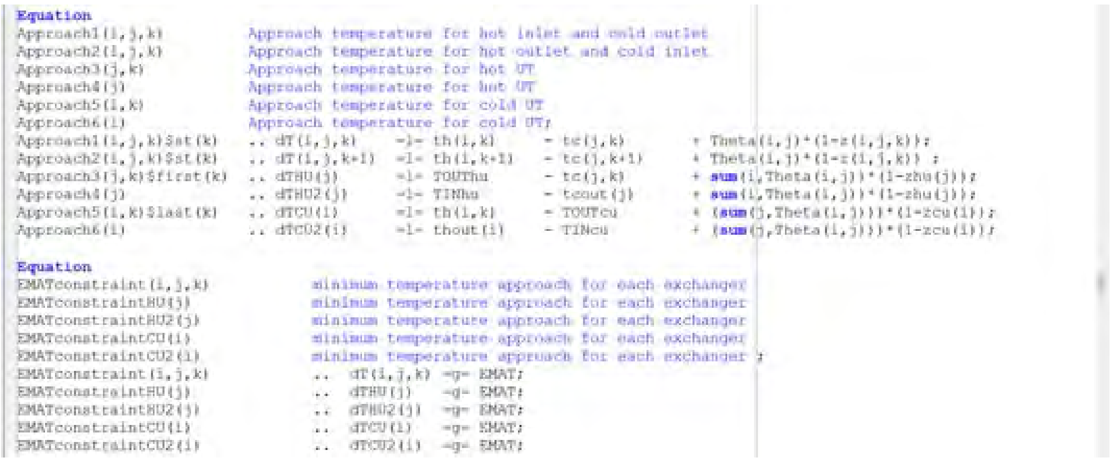
```
ActivateCF1(j,k)$first(k)        ..      ActiCF1(j,k)      =e= tcout(j) - max(FixtempC1(j),tc(j,k));
ActivateCF2(j,k)$first(k)        ..      ActiCF2(j,k)      =e= FixtempC1(j) - max(FixtempC2(j),tc(j,k)) ;
ActivateCF3(j,k)$first(k)        ..      ActiCF3(j,k)      =e= FixtempC2(j) - tc(j,k) ;
TempCalCF1(j,k)$first(k)         ..      TCcalF1(j,k)      =e= (tcout(j) + max(FixtempC1(j),tc(j,k)))/2 ;
TempCalCF2(j,k)$first(k)         ..      TCcalF2(j,k)      =e= (FixtempC1(j) + max(FixtempC2(j),tc(j,k)))/2;
TempCalCF3(j,k)$first(k)         ..      TCcalF3(j,k)      =e= (FixtempC2(j) + tc(j,k))/2;
CPCF11(j,k)$first(k)             ..      CPCF1(j,k)        =e= OC1(j) * TCcalF1(j,k) + GC1(j);
CPCF22(j,k)$first(k)             ..      CPCF2(j,k)        =e= OC2(j) * TCcalF2(j,k) + GC2(j);
CPCF33(j,k)$first(k)             ..      CPCF3(j,k)        =e= OC3(j) * TCcalF3(j,k) + GC3(j);
CPCAVGGGF(j,k)$first(k)          ..      CpCavggF(j,k)*(max(0,ActiCF1(j,k)) + max(0,ActiCF2(j,k))
                                         + max(0,ActiCF3(j,k)) ) =e=( (CpCF1(j,k) * max(0,ActiCF1(j,k)))
                                   + (CpCF2(j,k) * max(0,ActiCF2(j,k))) + (CpCF3(j,k) * max(0,ActiCF3(j,k))) );
*                     Independent Heat Capacity
Parameter
ech(i)            Overall heat transfer at hot streams
ecc(j)            Overall heat transfer at cold streams
Omega(i,j)        Upper bound set to the smallest heat content of the two streams involved in the match
Theta(i,j)        maximum temperature difference for each exchanger;
ech(i)            =  fhavg(i) * (thin(i) - thout(i));
ecc(j)            =  fcavg(j) * (tcout(j) - tcin(j));
Omega(i,j)        =  min(ecc(j),ech(i));
Theta(i,j)        =  ( thin(i)-thout(i) ) - ( tcin(j)-tcout(j) );
Display Omega,Theta;
```



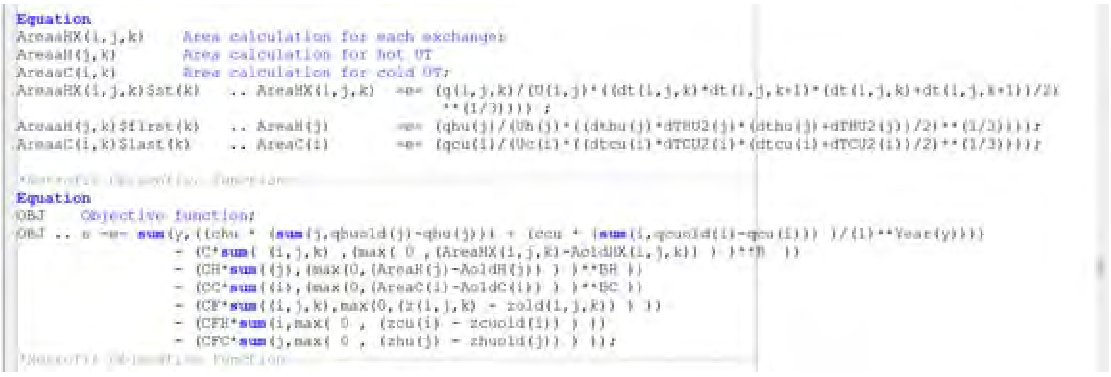
```
Equation
OverallBalanceHOT(i)   Overall heat balance for hot stream
OverallBalanceCOLD(j)  Overall heat balance for cold stream;
OverallBalanceHOT(i)  .. (thin(i)-thout(i)) * fhavg(i)  =e= sum((j,st),q(i,j,st)) + qcu(i) ;
OverallBalanceCOLD(j) .. (tcout(j)-tcin(j)) * fcavg(j)  =e= sum((i,st),q(i,j,st)) + qhu(j) ;

Equation
StageBalanceHOT(i,k)         Heat balance at each stage for hot stream
StageBalanceCOLD(j,k)        Heat balance at each stage for cold stream ;
StageBalanceHOT(i,k)$st(k)  .. (th(i,k)-th(i,k+1)) * fh(i) * CpHavgg(i,k) =e= sum(j,q(i,j,k)) ;
StageBalanceCOLD(j,k)$st(k) .. (tc(j,k)-tc(j,k+1)) * fc(j) * CpCavgg(j,k) =e= sum(i,q(i,j,k)) ;

Equation
ColdUTload(i,k)    Cold UT loading at hot streams
HotUTload(j,k)     Hot UT loading at cold streams;
ColdUTload(i,k)$last(k) .. (th(i,k)-thout(i)) * fh(i)  * CpHavggL(i,k)   =e= qcu(i);
HotUTload(j,k)$first(k) .. (tcout(j)-tc(j,k)) * fc(j)  * CpCavggF(j,k)   =e= qhu(j);

Equation
AssignmentHot(i,k)         Assignment of superstructure inlet temperatures for hot streams
AssignmentCold(j,k)        Assignment of superstructure inlet temperature for cols streams;
AssignmentHot(i,k)$first(k) .. thin(i) =e= th(i,k);
AssignmentCold(j,k)$last(k) .. tcin(j) =e= tc(j,k);
```



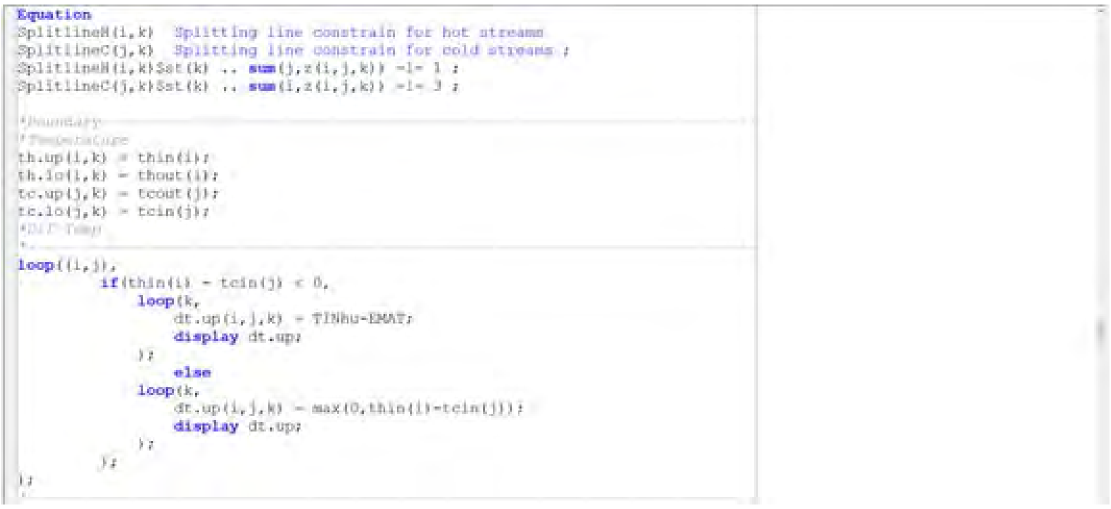
```
Equation
Feasibility1(i,k)      Feasibility temperature at each stage for hot stream
Feasibility2(j,k)      Feasibility temperature at each stage for cold stream
Feasibility3(i,k)      Feasibility temperature at final stage for hot stream
Feasibility4(j,k)      Feasibility temperature at first stage for cold stream;
Feasibility1(i,k)$st(k)    .. th(i,k)   =g= th(i,k+1)    ;
Feasibility2(j,k)$st(k)    .. tc(j,k)   =g= tc(j,k+1)    ;
Feasibility3(i,k)$last(k)  .. th(i,k)   =g= thout(i)     ;
Feasibility4(j,k)$first(k).. tcout(j)  =g= tc(j,k)      ;

Equation
Logic1(i,j,k)    Matching heat exchanger at each stage
Logic2(i)        Hot UT matching
Logic3(j)        Cold UT matching;
Logic1(i,j,k)$st(k)        .. q(i,j,k)  - Omega(i,j)*z(i,j,k) =l= 0       ;
Logic2(i)                  .. qcu(i)    - ech(i)*zcu(i)       =l= 0       ;
Logic3(j)                  .. qhu(j)    - ecc(j)*zhu(j)       =l= 0       ;
```



```
Equation
Approach1(i,j,k)           Approach temperature for hot inlet and cold outlet
Approach2(i,j,k)           Approach temperature for hot outlet and cold inlet
Approach3(j,k)             Approach temperature for hot UT
Approach4(j)               Approach temperature for hot UT
Approach5(i,k)             Approach temperature for cold UT
Approach6(i)               Approach temperature for cold UT;
Approach1(i,j,k)$st(k)    .. dT(i,j,k)     =l= th(i,k)      - tc(j,k)       + Theta(i,j)*(1-z(i,j,k));
Approach2(i,j,k)$st(k)    .. dT(i,j,k+1)   =l= th(i,k+1)    - tc(j,k+1)     + Theta(i,j)*(1-z(i,j,k)) ;
Approach3(j,k)$first(k)   .. dTHU(j)       =l= TOUThu       - tc(j,k)       + sum(i,Theta(i,j))*(1-zhu(j));
Approach4(j)              .. dTHU2(j)      =l= TINhu        - tcout(j)      + sum(i,Theta(i,j))*(1-zhu(j));
Approach5(i,k)$last(k)    .. dTCU(i)       =l= th(i,k)      - TOUTcu        + (sum(j,Theta(i,j)))*(1-zcu(i));
Approach6(i)              .. dTCU2(i)      =l= thout(i)     - TINcu         + (sum(j,Theta(i,j)))*(1-zcu(i));

Equation
EMATconstraint(i,j,k)               minimum temperature approach for each exchanger
EMATconstraintHU(j)                 minimum temperature approach for each exchanger
EMATconstraintHU2(j)                minimum temperature approach for each exchanger
EMATconstraintCU(i)                 minimum temperature approach for each exchanger
EMATconstraintCU2(i)                minimum temperature approach for each exchanger ;
EMATconstraint(i,j,k)               ..  dT(i,j,k) =g= EMAT;
EMATconstraintHU(j)                 ..  dTHU(j)   =g= EMAT;
EMATconstraintHU2(j)                ..  dTHU2(j)  =g= EMAT;
EMATconstraintCU(i)                 ..  dTCU(i)   =g= EMAT;
EMATconstraintCU2(i)                ..  dTCU2(i)  =g= EMAT;
```



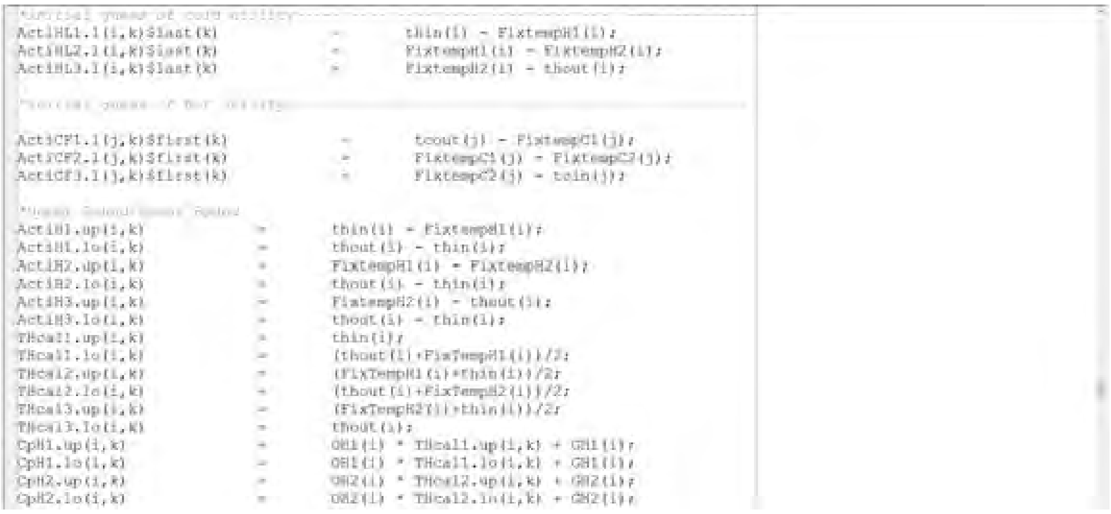
```
Equation
AreaaHX(i,j,k)    Area calculation for each exchanger
AreaaH(j,k)       Area calculation for hot UT
AreaaC(i,k)       Area calculation for cold UT;
AreaaHX(i,j,k)$st(k)    .. AreaHX(i,j,k)   =e= (q(i,j,k)/(U(i,j)*((dt(i,j,k)*dt(i,j,k+1)*(dt(i,j,k)+dt(i,j,k+1))/2)
                                                **(1/3)))) ;
AreaaH(j,k)$first(k)    .. AreaH(j)        =e= (qhu(j)/(Uh(j)*((dthu(j)*dTHU2(j)*(dthu(j)+dTHU2(j))/2)**(1/3)))));
AreaaC(i,k)$last(k)     .. AreaC(i)        =e= (qcu(i)/(Uc(i)*((dtcu(i)*dTCU2(i)*(dtcu(i)+dTCU2(i))/2)**(1/3)))));

*Retrofit Objective function
Equation
OBJ    Objective function;
OBJ .. s =e= sum(y,((chu * (sum(j,qhuold(j)-qhu(j))) + (ccu * (sum(i,qcuold(i)-qcu(i))) )/(1)**Year(y)))
                  - (C*sum( (i,j,k) ,(max( 0 ,(AreaHX(i,j,k)-AoldHX(i,j,k)) ) )**B  ))
                  - (CH*sum((j),(max(0,(AreaH(j)-AoldH(j)) ) )**BH ))
                  - (CC*sum((i),(max(0,(AreaC(i)-AoldC(i)) ) )**BC ))
                  - (CF*sum((i,j,k),max(0,(z(i,j,k) - zold(i,j,k)) ) ))
                  - (CFH*sum(i,max( 0 , (zcu(i) - zcuold(i)) ) ))
                  - (CFC*sum(j,max( 0 , (zhu(j) - zhuold(j)) ) ));
*Retrofit Objective function
```



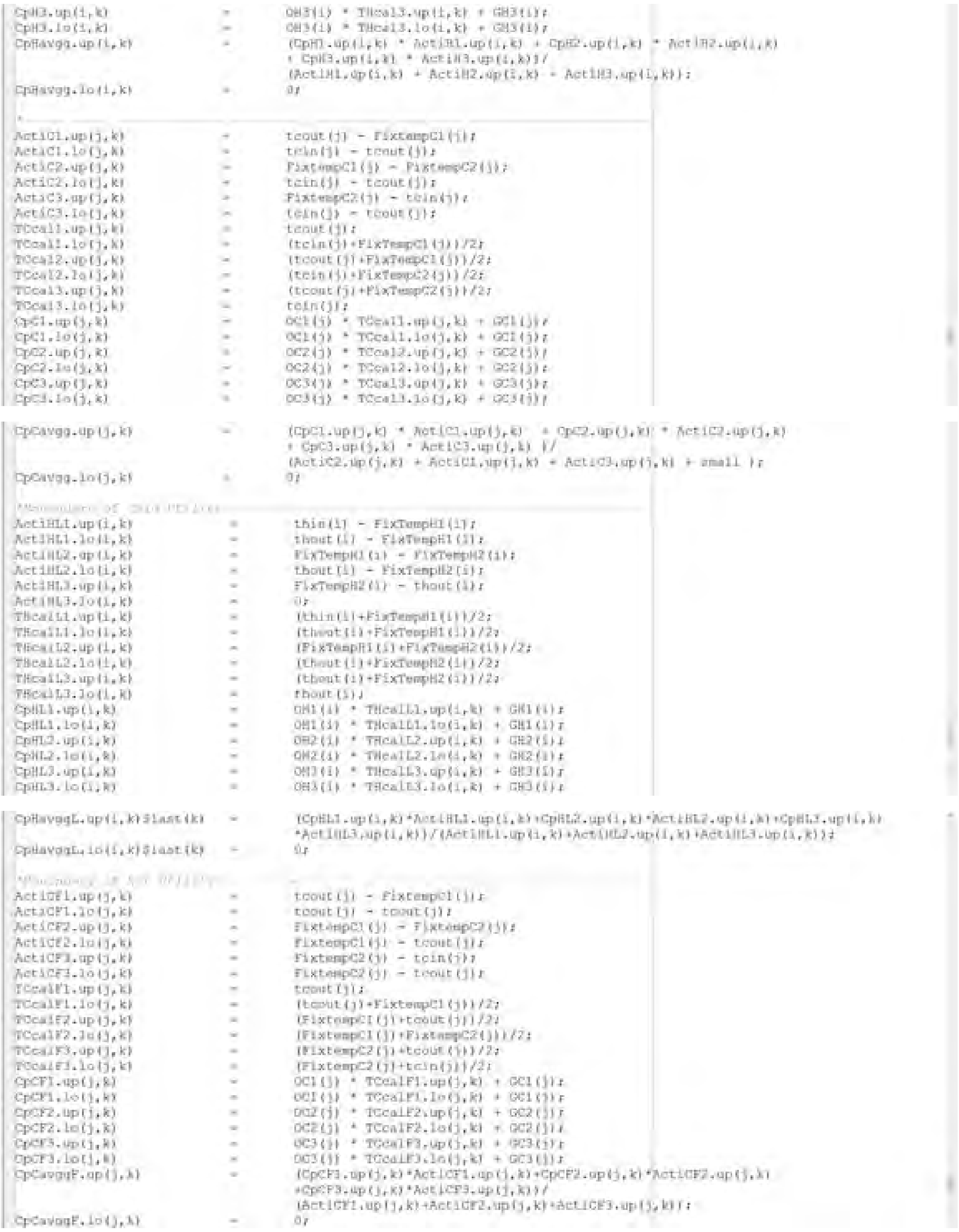
```
Equation
SplitlineH(i,k)  Splitting line constrain for hot streams
SplitlineC(j,k)  Splitting line constrain for cold streams ;
SplitlineH(i,k)$st(k) .. sum(j,z(i,j,k)) =l= 1 ;
SplitlineC(j,k)$st(k) .. sum(i,z(i,j,k)) =l= 3 ;

*Boundary
*Temperature
th.up(i,k) = thin(i);
th.lo(i,k) = thout(i);
tc.up(j,k) = tcout(j);
tc.lo(j,k) = tcin(j);
*DIF Temp

loop((i,j),
          if(thin(i) - tcin(j) < 0,
             loop(k,
                 dt.up(i,j,k) = TINhu-EMAT;
                 display dt.up;
             );
                 else
             loop(k,
                 dt.up(i,j,k) = max(0,thin(i)-tcin(j));
                 display dt.up;
             );
          );
);
```

```
dTHU.up(j)          = max(0,TINhu-tcin(j)-EMAT);
dTHU2.up(j)         = max(0,TINhu-tcin(j)-EMAT);
dTCU.up(i)          = max(0,thin(i)-toutcu-EMAT);
dTCU2.up(i)         = max(0,thin(i)-toutcu-EMAT);
dt.lo(i,j,k)        = EMAT;
dt.lo(i,j,k+1)      = EMAT;
dTHU.lo(j)          = EMAT;
dTHU2.lo(j)         = EMAT;
dTCU.lo(i)          = EMAT;
dTCU2.lo(i)         = EMAT;

*Heat load
q.up(i,j,k)$ST(k)= min(ech(i),ecc(j));
qcu.up(i)           = ech(i);
qhu.up(j)           = ecc(j);
q.lo(i,j,k)$ST(k)= 0;
qcu.lo(i)           = 0;
qhu.lo(j)           = 0;

*Area
AreaHX.up(i,j,k)$ST(k)    = q.up(i,j,k)/(U(i,j)*EMAT)   ;
AreaH.up(j)               = qhu.up(j)/((Uh(j))*EMAT);
AreaC.up(i)               = qcu.up(i)/((Uc(i))*EMAT);
AreaHX.lo(i,j,k)$ST(k)    = 0;
AreaH.lo(j)               = 0;
AreaC.lo(i)               = 0;
```

```
*Initial Value------------------------------------------------------------
th.l(i,k)           = thin(i);
tc.l(j,k)           = tcin(j);
dt.l(i,j,k)         = max(EMAT,th.l(i,k)-tc.l(j,k));
dTHU.l(j)           = EMAT;
dTHU2.l(j)          = EMAT;
dTCU.l(i)           = EMAT;
dTCU2.l(i)          = EMAT;
*Fix values---------------------------------------------------------------
z.fx('H1','C1',k) = 0;
*Boundary for Inconstant Heat Capacity ------------------------------------
*CP
ActiH1.l(i,k)              =       thin(i) - FixtempH1(i);
ActiH2.l(i,k)              =       FixtempH1(i) - FixtempH2(i);
ActiH3.l(i,k)              =       FixtempH1(i) - thout(i);
THcal1.l(i,k)              =       (thin(i) + FixtempH1(i))/2;
THcal2.l(i,k)              =       (FixtempH1(i) + FixtempH2(i))/2;
THcal3.l(i,k)              =       (FixtempH2(i) + thout(i))/2;
CpHavgg.l(i,k)             =       CpHavg(i);

ActiC1.l(j,k)              =       tcout(j) - FixtempC1(j);
ActiC2.l(j,k)              =       FixtempC1(j) - FixtempC2(j);
ActiC3.l(j,k)              =       FixtempC2(j) - tcin(j);
TCcal1.l(j,k)              =       (tcout(j) + FixtempC1(j))/2;
TCcal2.l(j,k)              =       (FixtempC1(j) + FixtempC2(j))/2;
TCcal3.l(j,k)              =       (FixtempC1(j) + tcin(j))/2;
CpCavgg.l(j,k)             =       CpCavg(j);
```



```
*initial guess of cold utility--------------------------------------------
ActiHL1.l(i,k)$last(k)              =       thin(i) - FixtempH1(i);
ActiHL2.l(i,k)$last(k)              =       FixtempH1(i) - FixtempH2(i);
ActiHL3.l(i,k)$last(k)              =       FixtempH2(i) - thout(i);

*initial guess of hot utility---------------------------------------------

ActiCF1.l(j,k)$first(k)              =       tcout(j) - FixtempC1(j);
ActiCF2.l(j,k)$first(k)              =       FixtempC1(j) - FixtempC2(j);
ActiCF3.l(j,k)$first(k)              =       FixtempC2(j) - tcin(j);

*Upper Bound/Lower Bound
ActiH1.up(i,k)              =       thin(i) - FixtempH1(i);
ActiH1.lo(i,k)              =       thout(i) - thin(i);
ActiH2.up(i,k)              =       FixtempH1(i) - FixtempH2(i);
ActiH2.lo(i,k)              =       thout(i) - thin(i);
ActiH3.up(i,k)              =       FixtempH2(i) - thout(i);
ActiH3.lo(i,k)              =       thout(i) - thin(i);
THcal1.up(i,k)              =       thin(i);
THcal1.lo(i,k)              =       (thout(i)+FixTempH1(i))/2;
THcal2.up(i,k)              =       (FixTempH1(i)+thin(i))/2;
THcal2.lo(i,k)              =       (thout(i)+FixTempH2(i))/2;
THcal3.up(i,k)              =       (FixTempH2(i)+thin(i))/2;
THcal3.lo(i,k)              =       thout(i);
CpH1.up(i,k)                =       GH1(i) * THcal1.up(i,k) + GH1(i);
CpH1.lo(i,k)                =       GH1(i) * THcal1.lo(i,k) + GH1(i);
CpH2.up(i,k)                =       GH2(i) * THcal2.up(i,k) + GH2(i);
CpH2.lo(i,k)                =       GH2(i) * THcal2.lo(i,k) + GH2(i);
```



```
CpH3.up(i,k)              =       OH3(i) * THcal3.up(i,k) + GH3(i);
CpH3.lo(i,k)              =       OH3(i) * THcal3.lo(i,k) + GH3(i);
CpHavgg.up(i,k)           =       (CpH1.up(i,k) * ActiH1.up(i,k) + CpH2.up(i,k) * ActiH2.up(i,k)
                                  + CpH3.up(i,k) * ActiH3.up(i,k))/
                                  (ActiH1.up(i,k) + ActiH2.up(i,k) + ActiH3.up(i,k));
CpHavgg.lo(i,k)           =       0;

*------------------------------------------------------------------------
ActiC1.up(j,k)            =       tcout(j) - FixtempC1(j);
ActiC1.lo(j,k)            =       tcin(j) - tcout(j);
ActiC2.up(j,k)            =       FixtempC1(j) - FixtempC2(j);
ActiC2.lo(j,k)            =       tcin(j) - tcout(j);
ActiC3.up(j,k)            =       FixtempC2(j) - tcin(j);
ActiC3.lo(j,k)            =       tcin(j) - tcout(j);
TCcal1.up(j,k)            =       tcout(j);
TCcal1.lo(j,k)            =       (tcin(j)+FixTempC1(j))/2;
TCcal2.up(j,k)            =       (tcout(j)+FixTempC1(j))/2;
TCcal2.lo(j,k)            =       (tcin(j)+FixTempC2(j))/2;
TCcal3.up(j,k)            =       (tcout(j)+FixTempC2(j))/2;
TCcal3.lo(j,k)            =       tcin(j);
CpC1.up(j,k)              =       OC1(j) * TCcal1.up(j,k) + GC1(j);
CpC1.lo(j,k)              =       OC1(j) * TCcal1.lo(j,k) + GC1(j);
CpC2.up(j,k)              =       OC2(j) * TCcal2.up(j,k) + GC2(j);
CpC2.lo(j,k)              =       OC2(j) * TCcal2.lo(j,k) + GC2(j);
CpC3.up(j,k)              =       OC3(j) * TCcal3.up(j,k) + GC3(j);
CpC3.lo(j,k)              =       OC3(j) * TCcal3.lo(j,k) + GC3(j);

CpCavgg.up(j,k)           =       (CpC1.up(j,k) * ActiC1.up(j,k)  + CpC2.up(j,k) * ActiC2.up(j,k)
                                  + CpC3.up(j,k) * ActiC3.up(j,k) )/
                                  (ActiC2.up(j,k) + ActiC1.up(j,k) + ActiC3.up(j,k) + small );
CpCavgg.lo(j,k)           =       0;

*[illegible]--------------------------------------------------------------
ActiHL1.up(i,k)            =       thin(i) - FixTempH1(i);
ActiHL1.lo(i,k)            =       thout(i) - FixTempH1(i);
ActiHL2.up(i,k)            =       FixTempH1(i) - FixTempH2(i);
ActiHL2.lo(i,k)            =       thout(i) - FixTempH2(i);
ActiHL3.up(i,k)            =       FixTempH2(i) - thout(i);
ActiHL3.lo(i,k)            =       0;
THcalL1.up(i,k)            =       (thin(i)+FixTempH1(i))/2;
THcalL1.lo(i,k)            =       (thout(i)+FixTempH1(i))/2;
THcalL2.up(i,k)            =       (FixTempH1(i)+FixTempH2(i))/2;
THcalL2.lo(i,k)            =       (thout(i)+FixTempH2(i))/2;
THcalL3.up(i,k)            =       (thout(i)+FixTempH2(i))/2;
THcalL3.lo(i,k)            =       thout(i);
CpHL1.up(i,k)              =       OH1(i) * THcalL1.up(i,k) + GH1(i);
CpHL1.lo(i,k)              =       OH1(i) * THcalL1.lo(i,k) + GH1(i);
CpHL2.up(i,k)              =       OH2(i) * THcalL2.up(i,k) + GH2(i);
CpHL2.lo(i,k)              =       OH2(i) * THcalL2.lo(i,k) + GH2(i);
CpHL3.up(i,k)              =       OH3(i) * THcalL3.up(i,k) + GH3(i);
CpHL3.lo(i,k)              =       OH3(i) * THcalL3.lo(i,k) + GH3(i);

CpHavggL.up(i,k)$last(k)   =       (CpHL1.up(i,k)*ActiHL1.up(i,k)+CpHL2.up(i,k)*ActiHL2.up(i,k)+CpHL3.up(i,k)
                                   *ActiHL3.up(i,k))/(ActiHL1.up(i,k)+ActiHL2.up(i,k)+ActiHL3.up(i,k));
CpHavggL.lo(i,k)$last(k)   =       0;

*[illegible]--------------------------------------------------------------
ActiCF1.up(j,k)            =       tcout(j) - FixtempC1(j);
ActiCF1.lo(j,k)            =       tcout(j) - tcout(j);
ActiCF2.up(j,k)            =       FixtempC1(j) - FixtempC2(j);
ActiCF2.lo(j,k)            =       FixtempC1(j) - tcout(j);
ActiCF3.up(j,k)            =       FixtempC2(j) - tcin(j);
ActiCF3.lo(j,k)            =       FixtempC2(j) - tcout(j);
TCcalF1.up(j,k)            =       tcout(j);
TCcalF1.lo(j,k)            =       (tcout(j)+FixtempC1(j))/2;
TCcalF2.up(j,k)            =       (FixtempC1(j)+tcout(j))/2;
TCcalF2.lo(j,k)            =       (FixtempC1(j)+FixtempC2(j))/2;
TCcalF3.up(j,k)            =       (FixtempC2(j)+tcout(j))/2;
TCcalF3.lo(j,k)            =       (FixtempC2(j)+tcin(j))/2;
CpCF1.up(j,k)              =       OC1(j) * TCcalF1.up(j,k) + GC1(j);
CpCF1.lo(j,k)              =       OC1(j) * TCcalF1.lo(j,k) + GC1(j);
CpCF2.up(j,k)              =       OC2(j) * TCcalF2.up(j,k) + GC2(j);
CpCF2.lo(j,k)              =       OC2(j) * TCcalF2.lo(j,k) + GC2(j);
CpCF3.up(j,k)              =       OC3(j) * TCcalF3.up(j,k) + GC3(j);
CpCF3.lo(j,k)              =       OC3(j) * TCcalF3.lo(j,k) + GC3(j);
CpCavggF.up(j,k)           =       (CpCF1.up(j,k)*ActiCF1.up(j,k)+CpCF2.up(j,k)*ActiCF2.up(j,k)
                                   +CpCF3.up(j,k)*ActiCF3.up(j,k))/
                                   (ActiCF1.up(j,k)+ActiCF2.up(j,k)+ActiCF3.up(j,k));
CpCavggF.lo(j,k)           =       0;
```

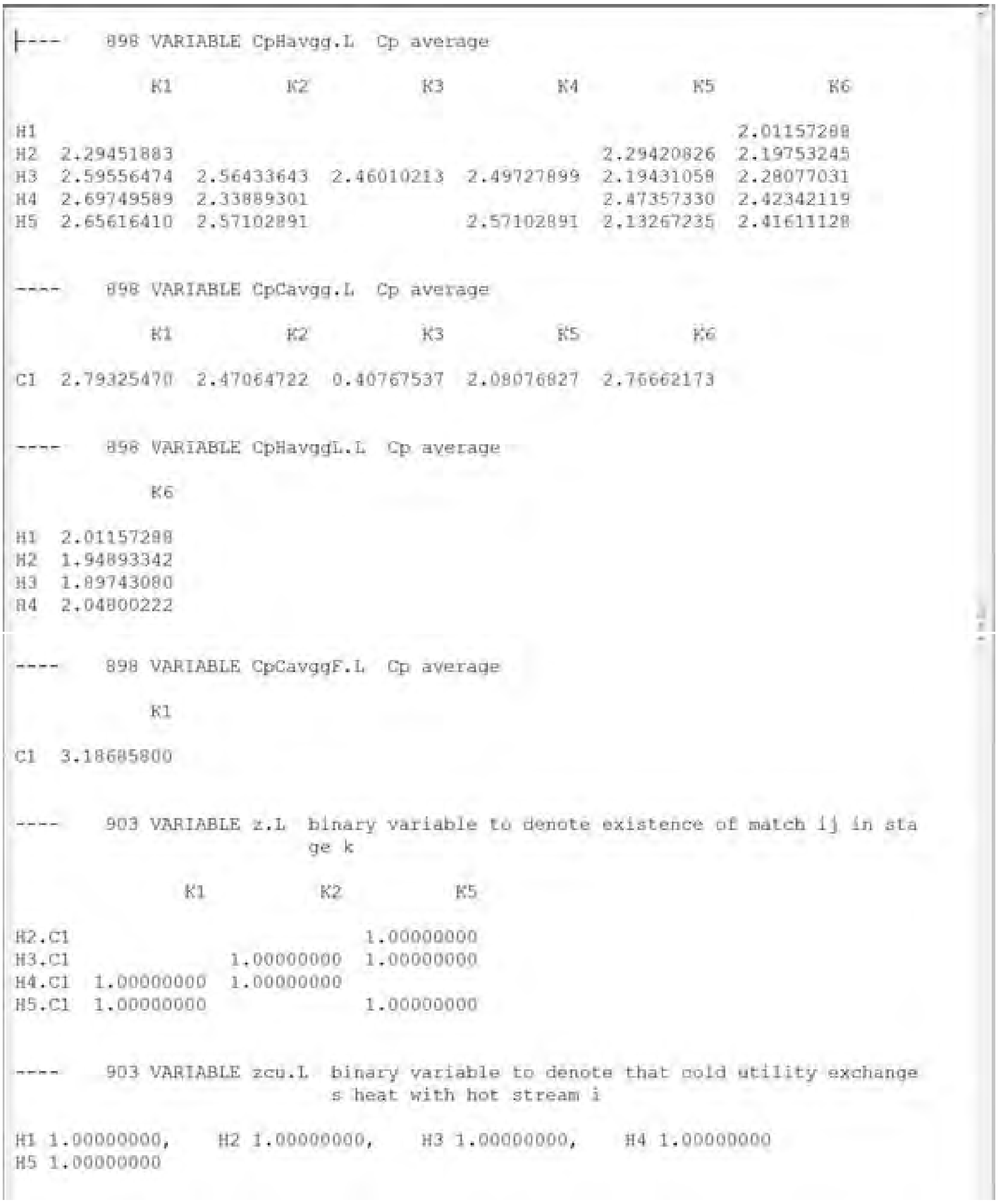
```
q.l('H3','C1','K2') = 5.561564E+3;
q.l('H4','C1','K2') = 3.660193E+3;
q.l('H2','C1','K5') = 6.325781E+3;
q.l('H3','C1','K5') = 4.584490E+3;
q.l('H5','C1','K5') = 2.012400E+4;
z.l('H4','C1','K1') = 1;
z.l('H5','C1','K1') = 1;
z.l('H3','C1','K2') = 1;
z.l('H4','C1','K2') = 1;
z.l('H2','C1','K5') = 1;
z.l('H3','C1','K5') = 1;
z.l('H5','C1','K5') = 1;
qcu.l('H1') = 1.378389E+3;
qcu.l('H2') = 2.089762E+3;
qcu.l('H3') = 2.149990E+3;
qcu.l('H4') = 4.868913E+3;
qcu.l('H5') = 4.167334E+3;
qcu.up('H1') = 1.378389E+3;
qcu.up('H2') = 2.089762E+3;
qcu.up('H3') = 2.149990E+3;
qcu.up('H4') = 4.068913E+3;
qcu.up('H5') = 4.167334E+3;
qhu.l('C1')  = 8.210093E+4;
qhu.up('C1') = 8.210093E+4;
```

```
option sysout = on;
option decimals = 8;
option domlim = 100;
option MINLP = DICOPT;
option Threads = 0;
option reslim = 600;
model Synheat /all/;

Synheat.optfile=1;

$onecho > dicopt.opt
stop 0
maxcycles 50000
$offecho

solve Synheat max s using MINLP ;
Display CpHavgg.l,CpCavgg.l,CpHavggL.l,CpCavggF.l,z.l,zcu.l,zhu.l,q.l,qcu.l,qhu.l,th.l,tc.l,s.l,dtcu.l,dtcu2.l;
```

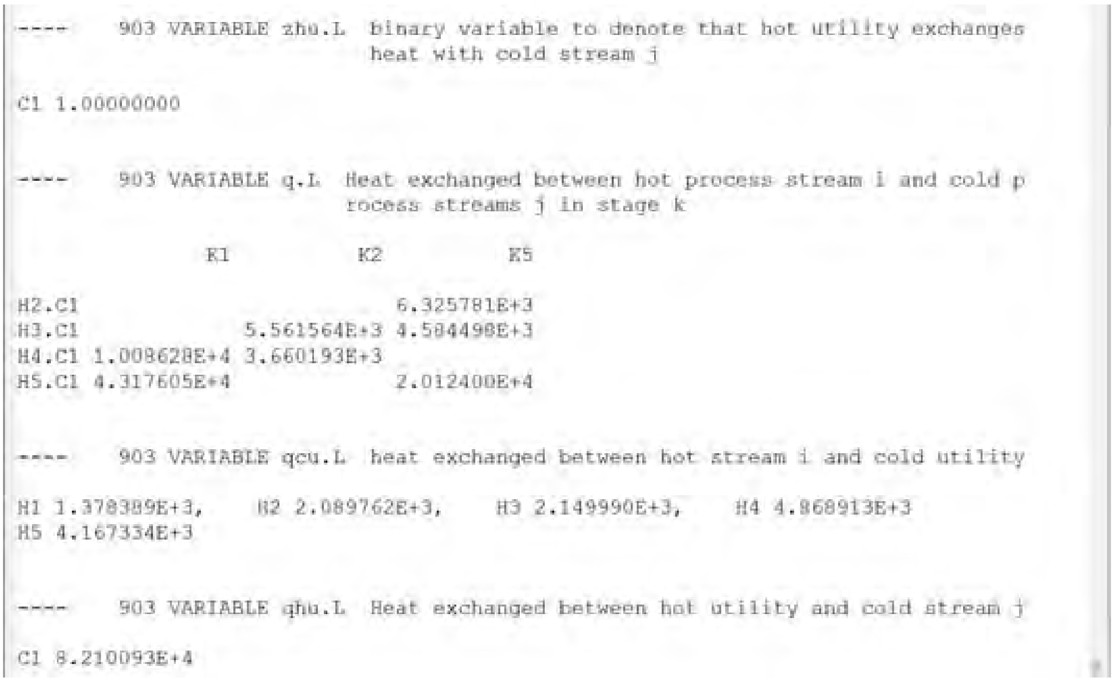
```
Parameter
AreacalHX(i,j,k)         Area calculation for each heat exchanger
AreacalHU(j)             Area calculation for hot utility
AreacalCU(i)             Area calculation for cold utility
TotalArea                Total Area calculation
ColdUTcost               Cold utility cost
HotUTcost                Hot utility cost
ColdUTSave
HotUTSave
TotalSave
FixHXNew
AreaHXNew
ModifyCost
Check
ColdOld
HotOld
OldUT
Save
pay
UTcost;
```

```
AreacalHX(i,j,k)$at(k)   = q.l(i,j,k)/(U(i,j)*(dt.l(i,j,k)*dt.l(i,j,k+1)*(dt.l(i,j,k)+dt.l(i,j,k+1))/2)**(1/3)) ;
AreacalHU(j)             = qhu.l(j)/((Uh(j))*((dthu.l(j))*(TINhu-tcout(j))*((dthu.l(j))+(TINhu-tcout(j)))/2)**(1/3))
AreacalCU(i)             = qcu.l(i)/((Uc(i))*((dtcu.l(i))*(thout(i)-TINcu)*((dtcu.l(i))+(thout(i)-TINcu))/2)**(1/3));
TotalArea                = sum((i,j,k),AreacalHX(i,j,k))+ sum(j,AreacalHU(j)) + sum(i,AreacalCU(i));
ColdUTcost               = sum(i,(ccu*qcu.l(i)));
hotUTcost                = sum(j,(chu*qhu.l(j)));
UTcost                   = ColdUTcost + hotUTcost;
ColdOld                  = ccu*sum(i,qcuold(i))    ;
HotOld                   = chu*sum(j,qhuold(j))     ;
OldUT                    = ColdOld + HotOld;
Save                     = OldUT - ColdUTcost - HotUTcost ;
ColdUTSave               = ccu * (sum(i,qcuold(i)-qcu.l(i)));
HotUTSave                = chu * (sum(j,qhuold(j)-qhu.l(j)));
TotalSave                = ColdUTSave + HotUTSave ;
FixHXNew                 = (CF*sum((i,j,k),max(0,(z.l(i,j,k)-zold(i,j,k)) ) ))
                            + (CFH*sum(i,max( 0 , (zcu.l(i) - zcuold(i)) ) ))
                            + (CFC*sum(j,max( 0 , (zhu.l(j) - zhuold(j)) ) ));
AreaHXNew                = (C*sum( (i,j,k) ,(max( 0 ,(AreaHX.l(i,j,k)-AoldHX(i,j,k)) ) )**B  ))
                            + (CH*sum((j),(max(0,(AreaH.l(j)-AoldH(j)) ) )**BH ))
                            + (CC*sum((i),(max(0,(AreaC.l(i)-AoldC(i)) ) )**BC )) ;
ModifyCost               = FixHXNew + AreaHXNew;
PAy                      = ModifyCost/save  ;
Check                    = TotalSave - FixHXNew - AreaHXNew;

display AreacalHX,AreacalHU,AreacalCU,TotalArea,ColdUTcost,HotUTcost,UTCOST,ColdOld ,
HotOld,OldUT,Save,ColdUTSave,HotUTSave,TotalSave,FixHXNew,AreaHXNew,ModifyCost,Check,PAy ;
```

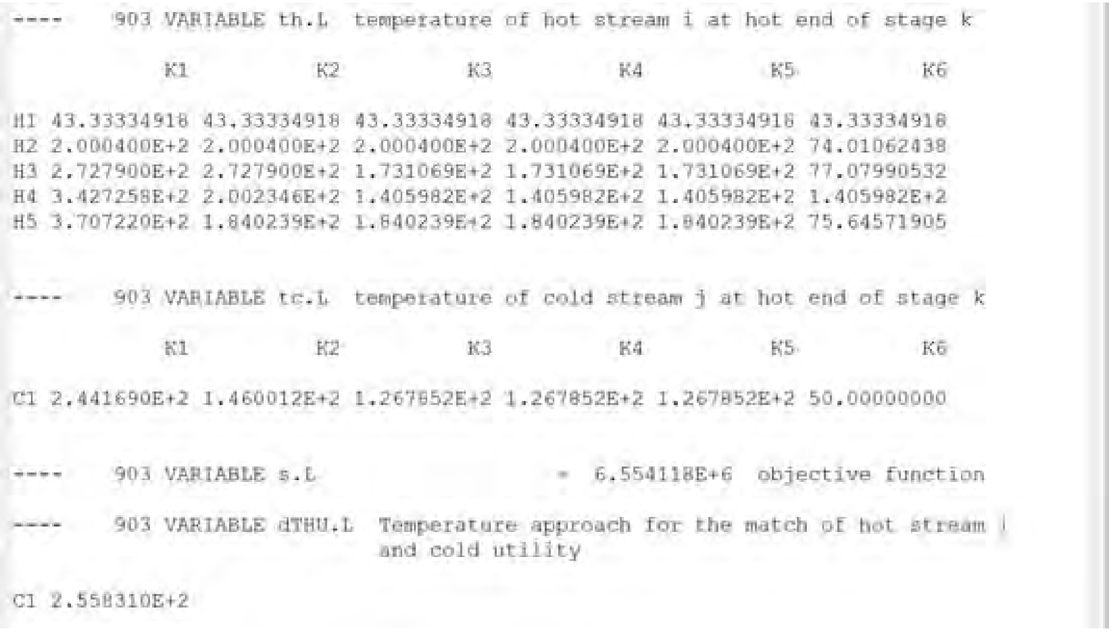
Appendix C GAMS Results of Study 1 (HEN Synthesis)



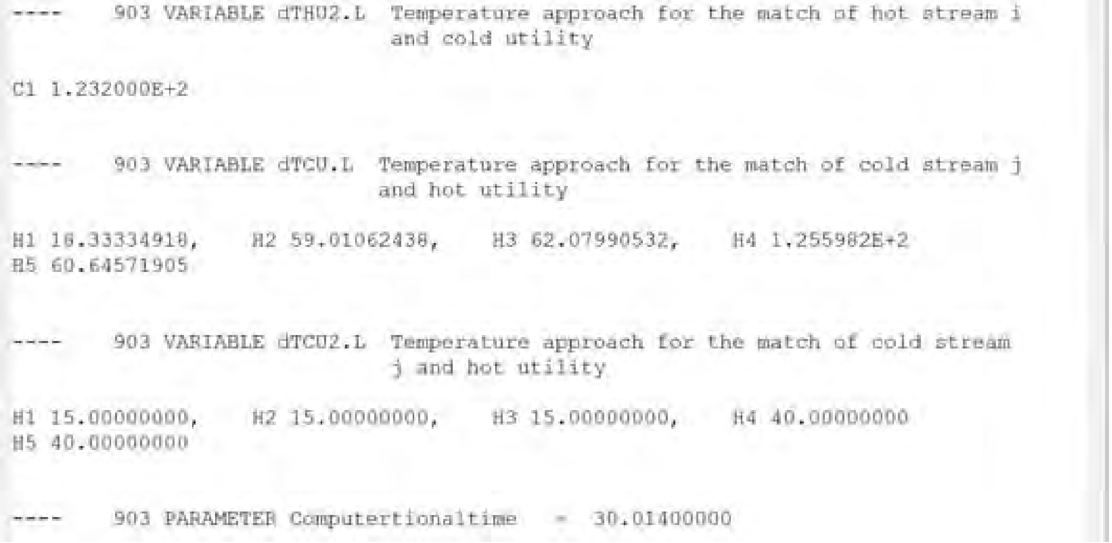
```
----    898 VARIABLE CpHavgg.L  Cp average

            K1          K2          K3          K4          K5          K6

H1                                                              2.01157288
H2  2.29451883                                      2.29420826  2.19753245
H3  2.59556474  2.56433643  2.46010213  2.49727899  2.19431058  2.28077031
H4  2.69749589  2.33889301                          2.47357330  2.42342119
H5  2.65616410  2.57102891              2.57102891  2.13267235  2.41611128


----    898 VARIABLE CpCavgg.L  Cp average

            K1          K2          K3          K5          K6

C1  2.79325470  2.47064722  0.40767537  2.08076827  2.76662173


----    898 VARIABLE CpHavggL.L  Cp average

            K6

H1  2.01157288
H2  1.94893342
H3  1.89743080
H4  2.04800222

----    898 VARIABLE CpCavggF.L  Cp average

            K1

C1  3.18685800


----    903 VARIABLE z.L  binary variable to denote existence of match ij in sta
                          ge k

               K1          K2          K5

H2.C1                          1.00000000
H3.C1              1.00000000  1.00000000
H4.C1  1.00000000  1.00000000
H5.C1  1.00000000              1.00000000


----    903 VARIABLE zcu.L  binary variable to denote that cold utility exchange
                            s heat with hot stream i

H1 1.00000000,    H2 1.00000000,    H3 1.00000000,    H4 1.00000000
H5 1.00000000
```



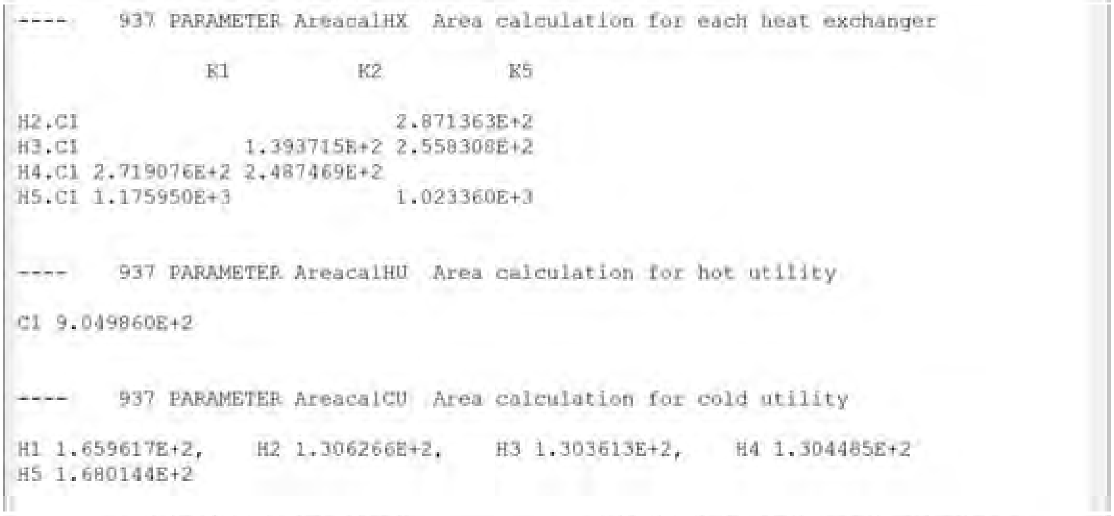
```
----    903 VARIABLE zhu.L  binary variable to denote that hot utility exchanges
                            heat with cold stream j

C1 1.00000000


----    903 VARIABLE q.L  Heat exchanged between hot process stream i and cold p
                          rocess streams j in stage k

               K1          K2          K5

H2.C1                         6.325781E+3
H3.C1             5.561564E+3 4.584498E+3
H4.C1 1.008628E+4 3.660193E+3
H5.C1 4.317605E+4             2.012400E+4


----    903 VARIABLE qcu.L  heat exchanged between hot stream i and cold utility

H1 1.378389E+3,    H2 2.089762E+3,    H3 2.149990E+3,    H4 4.868913E+3
H5 4.167334E+3


----    903 VARIABLE qhu.L  Heat exchanged between hot utility and cold stream j

C1 8.210093E+4
```



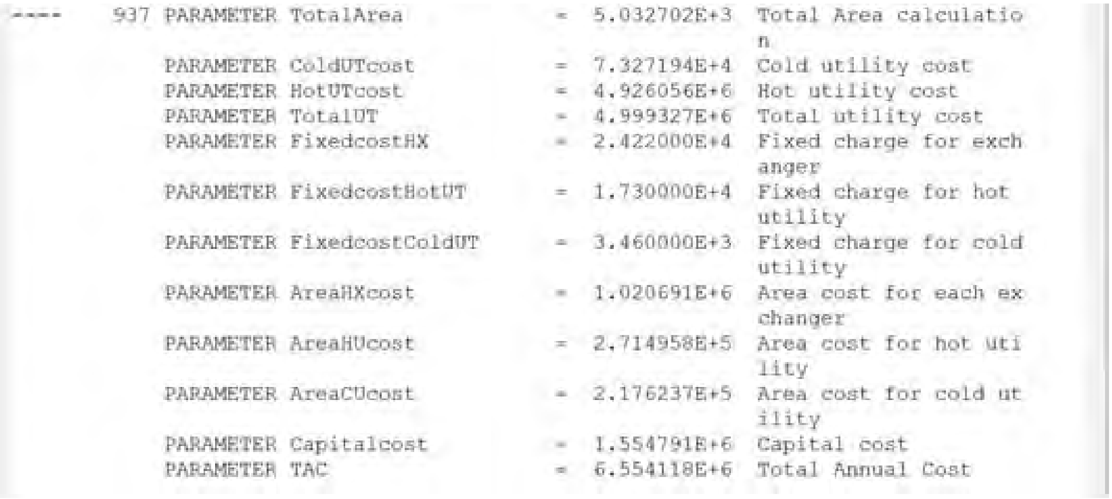
```
----    903 VARIABLE th.L  temperature of hot stream i at hot end of stage k

            K1          K2          K3          K4          K5          K6

H1 43.33334918 43.33334918 43.33334918 43.33334918 43.33334918 43.33334918
H2 2.000400E+2 2.000400E+2 2.000400E+2 2.000400E+2 2.000400E+2 74.01062438
H3 2.727900E+2 2.727900E+2 1.731069E+2 1.731069E+2 1.731069E+2 77.07990532
H4 3.427258E+2 2.002346E+2 1.405982E+2 1.405982E+2 1.405982E+2 1.405982E+2
H5 3.707220E+2 1.840239E+2 1.840239E+2 1.840239E+2 1.840239E+2 75.64571905


----    903 VARIABLE tc.L  temperature of cold stream j at hot end of stage k

            K1          K2          K3          K4          K5          K6

C1 2.441690E+2 1.460012E+2 1.267852E+2 1.267852E+2 1.267852E+2 50.00000000


----    903 VARIABLE s.L                   =  6.554118E+6  objective function

----    903 VARIABLE dTHU.L  Temperature approach for the match of hot stream i
                             and cold utility

C1 2.558310E+2
```



```
----      903 VARIABLE dTHU2.L  Temperature approach for the match of hot stream i
                                and cold utility

C1 1.232000E+2


----      903 VARIABLE dTCU.L  Temperature approach for the match of cold stream j
                               and hot utility

H1 18.33334918,    H2 59.01062438,    H3 62.07990532,    H4 1.255982E+2
H5 60.64571905


----      903 VARIABLE dTCU2.L  Temperature approach for the match of cold stream
                                j and hot utility

H1 15.00000000,    H2 15.00000000,    H3 15.00000000,    H4 40.00000000
H5 40.00000000


----      903 PARAMETER Computertionaltime   =  30.01400000
```



```
----      937 PARAMETER AreacalHX  Area calculation for each heat exchanger

               K1          K2          K5

H2.C1                         2.871363E+2
H3.C1             1.393715E+2 2.558308E+2
H4.C1 2.719076E+2 2.487469E+2
H5.C1 1.175950E+3             1.023360E+3


----      937 PARAMETER AreacalHU  Area calculation for hot utility

C1 9.049860E+2


----      937 PARAMETER AreacalCU  Area calculation for cold utility

H1 1.659617E+2,    H2 1.306266E+2,    H3 1.303613E+2,    H4 1.304485E+2
H5 1.680144E+2
```



```
----      937 PARAMETER TotalArea            =  5.032702E+3  Total Area calculatio
                                                             n
              PARAMETER ColdUTcost           =  7.327194E+4  Cold utility cost
              PARAMETER HotUTcost            =  4.926056E+6  Hot utility cost
              PARAMETER TotalUT              =  4.999327E+6  Total utility cost
              PARAMETER FixedcostHX          =  2.422000E+4  Fixed charge for exch
                                                             anger
              PARAMETER FixedcostHotUT       =  1.730000E+4  Fixed charge for hot
                                                             utility
              PARAMETER FixedcostColdUT      =  3.460000E+3  Fixed charge for cold
                                                             utility
              PARAMETER AreaHXcost           =  1.020691E+6  Area cost for each ex
                                                             changer
              PARAMETER AreaHUcost           =  2.714958E+5  Area cost for hot uti
                                                             lity
              PARAMETER AreaCUcost           =  2.176237E+5  Area cost for cold ut
                                                             ility
              PARAMETER Capitalcost          =  1.554791E+6  Capital cost
              PARAMETER TAC                  =  6.554118E+6  Total Annual Cost
```

CURRICULUM VITAE

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	Company name:	Siam Mitsui PTA Co., Ltd.
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	Company name:	Thaioil Public Co., Ltd.