

Chapter 5

Applications

5.1 The Ket State in the Schroedinger Picture

Many textbooks [41]-[44] follow the “*derivation*” of the Schroedinger equation in the interaction picture by beginning with the time dependent Schroedinger equation (TDSE). In the following we consider a Hamiltonian \mathbf{H} such that it can be split into two parts.

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{V}(\mathbf{x}, t) \quad (5.1)$$

where \mathbf{H}_0 does not contain time explicitly. The problem, which is $\mathbf{V}(\mathbf{x}, t) = 0$, is assumed to be solved in the sense that the energy eigen kets $|n\rangle$ and the energy eigenvalues ε_n defined by

$$\mathbf{H}_0 |n\rangle = \varepsilon_n |n\rangle \quad (5.2)$$

are completely known. More generally, we are interested in how an arbitrary ket state changes as time goes on, where the total Hamiltonian is the sum of \mathbf{H}_0 and $\mathbf{V}(\mathbf{x}, t)$. Suppose at $t = 0$, the ket state of a physical system is given by

$$|\alpha\rangle = \sum_n c_n(0) |n\rangle. \quad (5.3)$$

We wish to find $c_n(t)$ for $t > 0$ such that

$$|\alpha, t\rangle_S = \sum_n c_n(t) e^{-\frac{i}{\hbar} \varepsilon_n t} |n\rangle \quad (5.4)$$

where the ket on the left side stands for the ket state in the Schroedinger picture of a physical system whose ket state at $t = 0$ was found to be $|\alpha\rangle$.

We now derive the fundamental differential equation that characterizes the time evolution of the ket state $|\alpha, t\rangle$. Let us take the time derivation of $|\alpha, t\rangle$ with



the TDSE for the full \mathbf{H} given by Eq.(5.1):

$$\begin{aligned}
 i\hbar \frac{\partial}{\partial t} |\alpha, t\rangle_S &= (\mathbf{H}_0 + \mathbf{V}(\mathbf{x}, t)) |\alpha, t\rangle_S \\
 i\hbar \frac{\partial}{\partial t} \left(\sum_n c_n(t) e^{-\frac{i}{\hbar} \varepsilon_n t} |n\rangle \right) &= (\mathbf{H}_0 + \mathbf{V}(\mathbf{x}, t)) \left(\sum_n c_n(t) e^{-\frac{i}{\hbar} \varepsilon_n t} |n\rangle \right) \\
 \sum_n \left(i\hbar \frac{\partial}{\partial t} c_n(t) e^{-\frac{i}{\hbar} \varepsilon_n t} |n\rangle \right) &= \sum_n \mathbf{V}(\mathbf{x}, t) c_n(t) e^{-\frac{i}{\hbar} \varepsilon_n t} |n\rangle.
 \end{aligned} \tag{5.5}$$

Multiplying both sides of Eq.(5.5) by $\langle m|$ from the left, we obtain

$$i\hbar \frac{\partial}{\partial t} c_m(t) = \sum_n \langle m| \mathbf{V}(\mathbf{x}, t) |n\rangle c_n(t) e^{-i\omega_{mn}t} \tag{5.6}$$

where $\omega_{mn} = (\varepsilon_m - \varepsilon_n)/\hbar = -\omega_{nm}$. Explicitly,

$$i\hbar \frac{\partial}{\partial t} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} V_{11} & V_{12} e^{-i\omega_{12}t} & \cdots \\ V_{21} e^{-i\omega_{21}t} & V_{22} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \end{bmatrix} \tag{5.7}$$

where $V_{mn} = \langle m| \mathbf{V}(\mathbf{x}, t) |n\rangle$. This is the basic coupled differential equation that must be solved to obtain the probability of finding $|m\rangle$ as a function of time t . Eqs.(5.6) and (5.7) can be rewritten in a convenient form:

$$i\hbar \frac{\partial}{\partial t} |\alpha, t\rangle_I = \mathbf{V}(\mathbf{x}, t) |\alpha, t\rangle_I \tag{5.8}$$

where

$$\mathbf{V}_I(\mathbf{x}, t) = e^{\frac{i}{\hbar} H_0 t} \mathbf{V}(\mathbf{x}, t) e^{-\frac{i}{\hbar} H_0 t} \tag{5.9}$$

is observable in the interaction picture and

$$|\alpha, t\rangle_I = e^{\frac{i}{\hbar} H_0 t} |\alpha, t\rangle_S = \sum_n c_n(t) |n\rangle \tag{5.10}$$

is a ket state in the interaction picture. We thus see Eq.(5.8) as a Schroedinger-like equation with the total \mathbf{H} replaced by $\mathbf{V}_I(\mathbf{x}, t)$. Quite generally the time-dependent potential $\mathbf{V}_I(\mathbf{x}, t)$ can cause transitions to states other than the initial state. The probability of finding $|m\rangle$ at time t if $\mathbf{V}_I(\mathbf{x}, t)$ is present is determined by evaluating $|c_m(t)|^2$.

5.2 The Entangled Wave Function in Classical Limit

We begin by considering the composite system comprising a system S and its environment ε

$$\mathbf{H}\Psi(x, \vec{R}) = E\Psi(x, \vec{R}) \quad (5.11)$$

i.e.

$$(\mathbf{H}_\varepsilon + \mathbf{H}_s + \mathbf{V}_{\varepsilon s})\Psi(x, \vec{R}) = E\Psi(x, \vec{R}) \quad (5.12)$$

with the Hamiltonian \mathbf{H}_ε for the environment, \mathbf{H}_s for the quantum system and $\mathbf{V}_{\varepsilon s}$ represents the interaction between S and ε . For convenience we assume a coordinate representation for ε with standard form of $\mathbf{H}_\varepsilon = \mathbf{K} + \mathbf{V}_\varepsilon$ where $\mathbf{V}_\varepsilon = \mathbf{V}_\varepsilon(\vec{R})$ is the potential energy which is a function of coordinates only, and \mathbf{K} is the kinetic energy which is written in mass scale coordinates $\vec{R} = (R_1, R_2, \dots, R_k)$ as assume over all degrees of freedom, $i = 1, \dots, k$:

$$\mathbf{K} = -\frac{\hbar^2}{2M} \sum_i^k \frac{\partial^2}{\partial R_i^2} = -\frac{\hbar^2}{2M} \nabla_R^2. \quad (5.13)$$

We can use the basic state $\{\psi_n(x) = \langle x | n \rangle\}$ which are eigenstates of \mathbf{H}_s and are a function of x only.

$$\mathbf{H}_s \psi_n(x) = \varepsilon_n \psi_n(x) \quad (5.14)$$

for expanding the total wave function as

$$\Psi(x, \vec{R}) = \sum_n \kappa_n(\vec{R}) \psi_n(x). \quad (5.15)$$

In Eq.(5.15) it is in form of “entangled wave function” for the total system described by the Hamiltonian \mathbf{H} . It is different from the entangled wave function in the Born-Oppenheimer approximation (BOA) [45] which was used in Briggs and Rost [46] to derive the time dependent Schrodinger equation (TDSE). We can define the environment wave function $\kappa_n(\vec{R})$ as

$$\kappa_n(\vec{R}) = \int dx \psi_n^*(x) \Psi(x, \vec{R}). \quad (5.16)$$

It means that $|\kappa_n(\vec{R})|^2$ is a probability for finding the quantum system in a state $\psi_n(x)$. We can see that this probability is a function of the environment variable \vec{R} .

Substitution of the expansion Eq.(5.15) into Eq.(5.11) and projection of a state $\psi_m^*(x)$ onto Eq.(5.12) leads to the coupled time dependent Schroedinger equation (TISE) for the environment wave function

$$\sum_n \int dx \psi_m^*(x) [\mathbf{H}_\varepsilon + \mathbf{H}_s + \mathbf{V}_{\varepsilon s}] \kappa_n(\vec{R}) \psi_n(x) = E \kappa_m(\vec{R}) \quad (5.17)$$

For convenience we write the state $\psi_m(x)$ in the ket-bra notation

$$\sum_n \langle m | (\mathbf{H}_\varepsilon + \mathbf{H}_s + \mathbf{V}_{\varepsilon s}) \kappa_n(\vec{R}) | n \rangle = E \kappa_m(\vec{R}) \quad (5.18)$$

or

$$\mathbf{H}_\varepsilon \kappa_m(\vec{R}) + \varepsilon_m \kappa_m(\vec{R}) + \sum_n \langle m | \mathbf{V}_{\varepsilon s} | n \rangle \kappa_n(\vec{R}) = E \kappa_m(\vec{R}). \quad (5.19)$$

We assume that $\kappa_n(\vec{R})$ can be written as

$$\kappa_m(\vec{R}) = e^{\frac{i}{\hbar} W(\vec{R})} c'_m(\vec{R}). \quad (5.20)$$

Substituting Eq.(5.20) into Eq.(5.19) and multiplying both sides by $e^{-\frac{i}{\hbar} W(\vec{R})}$, we obtain

$$\begin{aligned} & \left[-\frac{i\hbar}{M} \vec{\nabla}_R W(\vec{R}) \cdot \vec{\nabla}_R c'_m(\vec{R}) + \sum_n \langle m | \mathbf{V}_{\varepsilon s} | n \rangle c'_n(\vec{R}) + \varepsilon_m c'_m(\vec{R}) \right] \\ & + \left[\frac{1}{2M} \left(\vec{\nabla}_R W(\vec{R}) \right)^2 + V_\varepsilon(\vec{R}) - E \right] c'_m(\vec{R}) + \left[-\frac{\hbar^2}{2M} \nabla_R^2 c'_m(\vec{R}) - \frac{i\hbar}{2M} c'_m(\vec{R}) \nabla_R^2 W(\vec{R}) \right] = 0. \end{aligned} \quad (5.21)$$

This is exact and the is full quantum equation for the environment wave function which is mixed by the “back-coupling” from the system.

Let us consider an approximation. This approximation was used by Aharonov and Bohm [26] for considering the nature of time in quantum mechanics by using

the variable determining the time of measurement. It was placed on the quantum-mechanical side of the “cut” defined in Chapter 2. The approximation is to assume the great mass M of the environment and the environment energy to be large compared with the quantum system energy (so that the environment wave function is narrow enough in \vec{R} - space and it will move in an essentially classical way). So we can suppose that we can determine an orbit $\vec{R} = \vec{R}(t)$ of the environment motion by assuming $W(\vec{R})$ to satisfy

$$\frac{1}{2M} \left(\vec{\nabla}_R W(\vec{R}) \right)^2 + V_\varepsilon(\vec{R}) - E = 0. \quad (5.22)$$

We recognize Eq.(5.22) to be the *Hamilton – Jacobi equation* in classical mechanics [47]. Eq.(5.22) implies that $W(\vec{R})$ is *Hamilton’s characteristic function* for environment system. The momentum in classical Hamilton-Jacobi theory is given by

$$\vec{\nabla}_R W(\vec{R}) = \vec{P} = M \frac{\partial \vec{R}}{\partial t}. \quad (5.23)$$

In this approximation, if M is large then the last term on the left-hand side of Eq.(5.21) may be neglected, i.e.

$$\left[-\frac{\hbar^2}{2M} \nabla_R^2 c'_m(\vec{R}) - \frac{i\hbar}{2M} c'_m(\vec{R}) \nabla_R^2 W(\vec{R}) \right] = 0. \quad (5.24)$$

So Eq.(5.21) becomes

$$-\frac{i\hbar}{M} \vec{\nabla}_R W(\vec{R}) \cdot \vec{\nabla}_R c'_m(\vec{R}) + \sum_n \langle m | \mathbf{V}_{\varepsilon_s} | n \rangle c'_n(\vec{R}) + \varepsilon_m c'_m(\vec{R}) = 0 \quad (5.25)$$

We may simplify Eq.(5.24) by replacing \vec{R} by a new parameter t , defined through a trajectory $\vec{R}(t)$

$$t = \int \frac{M dR_i}{P_i}, i = 1, \dots, k \quad (5.26)$$

(P_i = is the momentum in the i direction) that the new parameter t means to be “time” and the partial differential with respect to t is determined by

$$\frac{\partial}{\partial t} = \vec{P} \cdot \vec{\nabla}_R = \frac{\partial \vec{R}}{\partial t} \cdot \vec{\nabla}_R. \quad (5.27)$$

Since the \vec{R} are reduced to the classical variables $\vec{R}(t)$, Eq.(5.25), $c'_m(\vec{R}(t)) = c'_m(t)$, can be rewritten as

$$i\hbar \frac{\partial}{\partial t} c'_m(t) = \sum_n \langle m | \mathbf{V}_{\varepsilon_s} | n \rangle c'_n(t) + \varepsilon_m c'_m(t). \quad (5.28)$$

If we assume $c'_m(t) = c_m(t)e^{-\frac{i}{\hbar}\varepsilon_m t}$ then Eq.(5.28) becomes

$$i\hbar \frac{\partial}{\partial t} c_m(t) = \sum_n \langle m | \mathbf{V}_{\varepsilon_s} | n \rangle c_n(t) e^{-i\omega_{mn}t} \quad (5.29)$$

where $\omega_{mn} = (\varepsilon_m - \varepsilon_n)/\hbar = -\omega_{nm}$. We thus see Eq.(5.29) the same as Eq.(5.6) in Sec.5.1 which is the differential equation for the ket state in the interaction picture.

By this way, the total wave function, Eq.(5.15) becomes

$$\Psi(x, t) = \Psi(x, \vec{R}(t)) = e^{\frac{i}{\hbar}W(t)} \sum_n c_n(t) e^{-\frac{i}{\hbar}\varepsilon_n t} \psi_n(x). \quad (5.30)$$

$e^{\frac{i}{\hbar}W(t)}$ means a phase factor, depending on t *only*. It does not affect on the expectation value of the system. We can neglect this phase factor. So we can write the state wave function for the quantum system in the Schroedinger picture as

$$\Psi(x, t) = \sum_n c_n(t) e^{-\frac{i}{\hbar}\varepsilon_n t} \psi_n(x). \quad (5.31)$$

Now we obtain the state wave function in the Schroedinger picture Eq.(5.31) and the fundamental differential equation for characterizing the time evolution of the quantum system Eq.(5.29) without the time-dependent potential $\mathbf{V}(\mathbf{x}, t)$ from the TISE by investigating environment system in the classical limit of a definite orbit, $\vec{R}(t)$.

5.3 The Jaynes-Cummings Model [50]

In this section we consider the specific example of the Jaynes-Cummings model [48] in more detail. The Jaynes-Cummings model is a two-level atom which

is coupled to a laser field consisting of only one quantized mode. The total Hamiltonian is written as

$$\mathbf{H}_{JC} = \mathbf{H}_s + \mathbf{H}_F + \mathbf{V}_{FS}$$

with

$$\begin{aligned} \mathbf{H}_F &= \frac{1}{2} (\mathbf{P}^2 + \omega^2 \mathbf{Q}^2), \mathbf{P} = -i\hbar \frac{\partial}{\partial \mathbf{Q}} \\ \mathbf{H}_s &= \frac{1}{2} \hbar \omega_0 \sigma_z, \mathbf{V}_{FS} = \sqrt{2\hbar \omega g} \mathbf{Q} \sigma_x. \end{aligned} \quad (5.32)$$

Here \mathbf{Q} is the field amplitude operator with the eigenvalues Q , g is the electric-dipole matrix element, the energy of a single photon mode is $\hbar\omega$. $\hbar\omega_0$ is the energy of the atomic transition, and σ_x, σ_z are (2×2) Pauli matrices. In case of the two-level atom one requires two states $|+\rangle$ and $|-\rangle$ corresponding to the atom in the upper or lower state of doublet (see Fig 5.1).

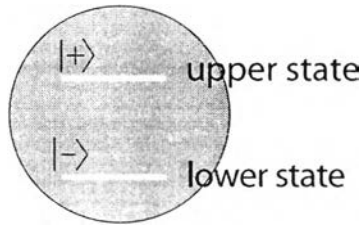


Figure 5.1: The two-level atom has two states $|+\rangle$ (the upper state) and $|-\rangle$ (lower state).

The total state of the system can be written in the form analogous to Eq.(5.15).

$$\Psi = \kappa_+(Q) |+\rangle + \kappa_-(Q) |-\rangle$$

or $(\kappa_+(Q) = \kappa(Q)c'_+(Q)$ and $\kappa_-(Q) = \kappa(Q)c'_-(Q)$)

$$\Psi = \kappa(Q) [c'_+(Q) |+\rangle + c'_-(Q) |-\rangle]. \quad (5.33)$$

Eq.(5.33) is in the form of the entangled state which has been studied frequently in laser-atom interactions. We substitute Eq.(5.33) into the TISE $(\mathbf{H} - E)\Psi = 0$

with the Hamiltonian \mathbf{H} given in Eq.(5.32) and project $|+\rangle$ and $|-\rangle$ onto this equation. After projecting $|+\rangle$ and $|-\rangle$ onto the TISE $(\mathbf{H} - E)\Psi = 0$ we get

$$\begin{aligned} c_+(Q)\left(-\frac{\hbar^2}{2}\frac{\partial^2}{\partial Q^2} + \omega^2 Q^2 - E\right)\kappa(Q) - \frac{\hbar^2}{2}\kappa(Q)\frac{\partial^2}{\partial Q^2}c_+(Q) \\ + \left(\frac{1}{2}\hbar\omega_0 + \sqrt{2\hbar\omega g} Q\right)\kappa(Q)c_-(Q) = \hbar^2\frac{\partial}{\partial Q}\kappa(Q)\frac{\partial}{\partial Q}c_+(Q) \end{aligned} \quad (5.34)$$

and

$$\begin{aligned} c_-(Q)\left(-\frac{\hbar^2}{2}\frac{\partial^2}{\partial Q^2} + \omega^2 Q^2 - E\right)\kappa(Q) - \frac{\hbar^2}{2}\kappa(Q)\frac{\partial^2}{\partial Q^2}c_-(Q) \\ + \left(-\frac{1}{2}\hbar\omega_0 + \sqrt{2\hbar\omega g} Q\right)\kappa(Q)c_+(Q) = \hbar^2\frac{\partial}{\partial Q}\kappa(Q)\frac{\partial}{\partial Q}c_-(Q). \end{aligned} \quad (5.35)$$

Next, we assume that the wave function $\kappa(Q)$ describing the field with an energy close to the total energy E can be determined in the form of the eigenvalue equation as

$$\left(-\frac{\hbar^2}{2}\frac{\partial^2}{\partial Q^2} + \omega^2 Q^2 - E\right)\kappa(Q) = 0. \quad (5.36)$$

For these large (classical) energies, we may replace the true wave function $\kappa(Q)$ by its WKB expression as

$$\kappa(Q) \simeq \exp\left\{\frac{i}{\hbar}\int P(Q)dQ\right\} \quad (5.37)$$

and find in the leading classical order the usual wave vector

$$\frac{1}{\kappa(Q)}\frac{\partial\kappa(Q)}{\partial Q} = \frac{i}{\hbar}P(Q) \quad (5.38)$$

with the classical momentum at position Q determined from energy conservation.

$$P(Q) = \sqrt{2(E - \omega^2 Q^2/2)}. \quad (5.39)$$

So we can replace Q by a new parameter t , defined through a trajectory $Q(t)$

$$t = \int \frac{dQ}{P(Q)}$$

and determined by

$$P(Q)\frac{\partial}{\partial Q} = \frac{\partial}{\partial t}. \quad (5.40)$$

Clearly, $P(Q)$ from Eq.(5.39) is the velocity \dot{Q} , as determined from the classical equation of motion:

$$Q = P \quad \text{and} \quad P = -\omega^2 Q, \quad (5.41)$$

with solution $Q(t) = Q_0 \cos(\omega t)$, so that the parameter t is just classical time.

Thus, using Eqs.(5.34)-(5.40), we get

$$\left(-\frac{\hbar^2}{2} \frac{\partial^2}{\partial Q^2} + \frac{1}{2} \hbar \omega_0 \sigma_z + \sqrt{2\hbar\omega g} Q_0 \sigma_x \right) [C] = i\hbar \frac{\partial}{\partial t} [C] \quad (5.42)$$

where $[C] = \begin{bmatrix} c_+(Q) \\ c_-(Q) \end{bmatrix}$. The parameter t is introduced in $\Psi(x, Q(t)) = \Psi(x, t)$ and Eq.(5.42) is found in the form

$$i\hbar \frac{\partial}{\partial t} [C] = \frac{\hbar^2}{2} \left(\frac{\ddot{Q}}{Q^3} \frac{\partial}{\partial t} - \frac{1}{Q^2} \frac{\partial^2}{\partial t^2} \right) \sigma_z [C] + \frac{1}{2} \hbar \omega_0 \sigma_z + \sqrt{2\hbar\omega g} Q \sigma_x [C]. \quad (5.43)$$

We estimate the order of magnitude. First, $\hbar\partial/\partial t$ is of the order of the energy of the quantum system $E_s = \langle \mathbf{H}_s + \mathbf{V} \rangle$, i.e., small compared to E . Second, $(\ddot{Q}/Q^3)\partial/\partial t$ and $(1/Q^2)\partial^2/\partial t^2$ can be estimated by using $\dot{Q} = P \approx \sqrt{E}$ and $Q \approx \omega Q$ for the harmonic oscillator as

$$\left\langle \frac{\hbar^2 \ddot{Q}(t)}{2 Q^3} \frac{\partial}{\partial t} \right\rangle \approx E_s \frac{\hbar\omega}{E}, \quad \text{and} \quad \left\langle \frac{\hbar^2}{2} \frac{1}{Q^2} \frac{\partial^2}{\partial t^2} \right\rangle \approx E_s^2/E. \quad (5.44)$$

So the additional derivative term $(\ddot{Q}/Q^3)\partial/\partial t$ and $(1/Q^2)\partial^2/\partial t^2$ of Eq.(5.43) are of the order $E_s(E_s + \hbar\omega)/E$. In the limit of large ("classical" amount of) energy E , the additional derivative are smaller by a factor of the order $\hbar\omega/E \approx 1/n$, where n is the number of photons in the field mode. The latter is a tiny number for a classical field. Therefore Eq.(5.43) can be reduced to

$$i\hbar \frac{\partial}{\partial t} [C] = \left(\frac{1}{2} \hbar \omega_0 \sigma_z + \sqrt{2\hbar\omega g} Q(t) \sigma_x \right) [C]. \quad (5.45)$$

If we change $[C] = \begin{bmatrix} c'_+(t) \\ c'_-(t) \end{bmatrix} = \begin{bmatrix} c_+(t)e^{-\frac{i}{\hbar}\frac{1}{2}\hbar\omega_0 t} \\ c_-(t)e^{\frac{i}{\hbar}\frac{1}{2}\hbar\omega_0 t} \end{bmatrix}$ Eq.(5.45) becomes

$$i\hbar\frac{\partial}{\partial t} \begin{bmatrix} c_+(t) \\ c_-(t) \end{bmatrix} = \sqrt{2\hbar\omega g} \begin{bmatrix} 0 & Qe^{\frac{i}{\hbar}\hbar\omega_0 t} \\ Qe^{-\frac{i}{\hbar}\hbar\omega_0 t} & 0 \end{bmatrix} \begin{bmatrix} c_+(t) \\ c_-(t) \end{bmatrix}.$$

This is exactly the TDSE obtained by assuming at the outset a classical field with amplitude $Q(t)$ driving the two-level atom whose state vector is expanded in the interaction picture and the ket state is expanded in the unperturbed basis

$$|\Psi(t)\rangle = c_+(t)e^{-\frac{i}{\hbar}\frac{1}{2}\hbar\omega_0 t} |+\rangle + c_-(t)e^{\frac{i}{\hbar}\frac{1}{2}\hbar\omega_0 t} |-\rangle. \quad (5.46)$$

We have shown that the ket state in the Schrodinger picture at time t can be derived by starting from the entangled wave function for the composite system which satisfies the full quantum mechanical TISE. Eq.(5.11). By beginning with the entangled wave function in Eq.(5.14), which is different from the entangled wave function in the Born-Oppenheimer approximation, we consider the environment in the classical limit. This determines the trajectory $\vec{R}(t)$ for classical motion from the *Hamilton - Jacobi equation*. The full quantum equation for the environment wave function becomes Eq.(5.29) by replacing the quantum variable \vec{R} by the classical variable $\vec{R}(t)$. Eq.(5.29) is the differential equation for $c_m(t)$. We obtain the wave function of the quantum system in the interaction picture. In this way, the entangled wave function reduces to the wave function of the quantum system in the Schrodinger picture at time t .

$$\Psi(x, \vec{R}) = \sum_n \kappa_n(R)\psi_n(x) \rightarrow \sum_n c_n(t)e^{-\frac{i}{\hbar}\varepsilon_n t} \psi_n(x).$$

The environment wave functions have characterized the time evolution of the system $\kappa_n(\vec{R}) \rightarrow c_n(t)e^{-\frac{i}{\hbar}\varepsilon_n t}$ which satisfy Eq.(5.29). The time independent interaction between the system S and its environment ε changes to play the role of the time dependent perturbation for the quantum system.