CHAPTER III
MATHEMATICAL MODEL FORMULATION

3.1 Problem Definition

This work addresses the planning of crude oil purchasing to satisfy both specification and demand with the highest profit. Through a model, planning activities involve the maximization of profit by the manipulation of crude oil supply purchase decisions, processing, inventory management, and blending over time periods. The model represents a scheme of a refinery that includes product paths to each production unit. The product paths are recognized by the composition of their key elements, e.g. sulfur and aromatic content. Capacities and yields of several units are also taken into account.

A unit model consists of blending relations and production yields. Blending relations represent intermediate blending possibilities for producing each product. In general, the production yield may be a simple yield relation or a complex system of equations but in this thesis, the unit model is assumed to be a simple yield relation for simplification of solving purpose. Yield expressions are based on averaged values obtained from plant data. Processing of a unit must satisfy bound constraints, which include maximum and minimum unit feed. The mass balance in a unit may not be satisfied due to material losses and to the possible existence of streams that are not modelled by the reason that they are not involved in the model problem.

Physical and chemical properties are calculated using volume and weight average (linear relations) whereas the properties that cannot be blended linearly are calculated by using blending index numbers.

The model programming is based on a discretisation of the time horizon and is linear.

The goal is to maximize the gross refinery margin (GRM) function, which is the difference in dollars between its product revenue (sum of the barrels of each product times the price of each product) minus the cost of raw materials (crude oil, purchased MTBE, and DCC) and the operating costs, including inventory.

The model was first implemented as a deterministic model that is risk-free. Next, uncertainty of demands and prices were incorporated in the model. Finally, financial risk management is discussed.

3.2 General Mathematical Formulation

3.2.1 Assumption

The following assumptions are proposed to the model:

(a) Perfect mixing is assumed in the process.

(b) Material losses are neglected.

(c) The property state of each crude oil or product is decided only by specific key components such as sulfur and aromatic contents.

3.2.2 General Mathematical Model [a]

A set-up of input-output balancing is based on the network structure proposed by Pinto *et al.* (2000). Figure 3.1 shows the general representation of balancing a productive unit.

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[a] Nomenclature definition can be found in Appendix A and Abbreviations.

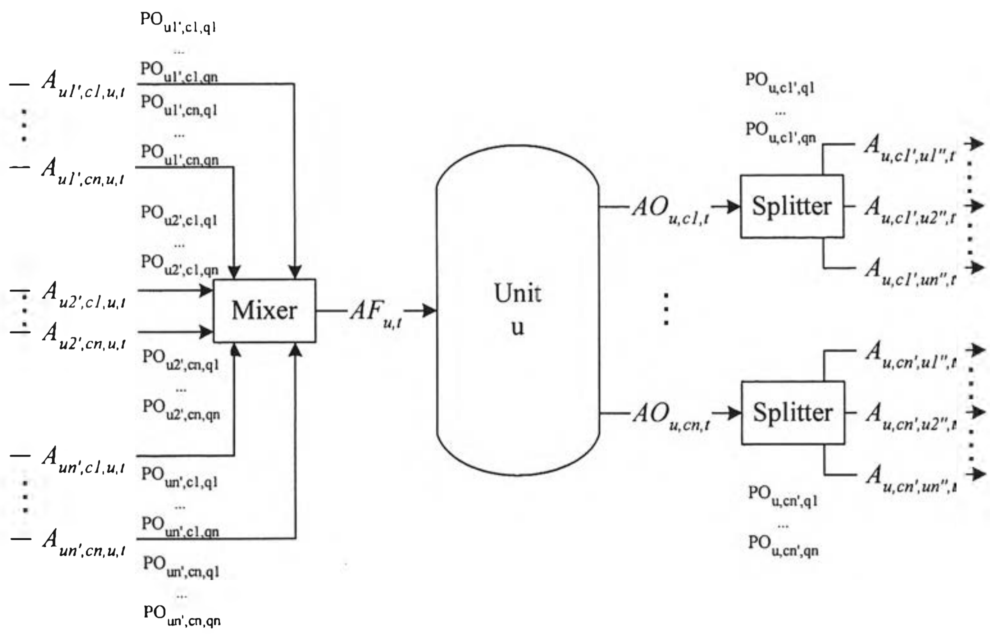

Figure 3.1 Balancing of a typical unit.

From Figure 3.1, commodity c_1 from unit u_1' is sent to unit u at flow rate $A_{u1',c1,u,t}$. The same unit u_1' can send different commodity c $(c_2, c_3, \ldots, c_n)$ to unit u. In addition, a set of units u' $(u_2', u_3', \ldots, u_n')$ can feed to unit u. The summation of feed for unit u is represented by $AF_{u,t}$. Parameters $PO_{u',c,q}$ denote properties of flow from unit u'. Variables $AO_{u,c,t}$ represents the outlet flow rate of commodity c from unit u. A splitter is represented at every outlet stream because a product stream can be sent to more than one unit for further processing or storage.

The model of a typical unit u in Figure 3.1 is represented by two sets of equations. The first is stream balance equations and the other is stream property equations.

Stream balance equations include:

1. Balance of feeds to unit u which is represented by

$$AF_{u,t} = \sum_{c} \sum_{u'} A_{u',c,u,t} \tag{3.1}$$

2. Balance of products from splitter which is represented by

$$AO_{u,c,t} = \sum_{u'} A_{u,c,u',t} \tag{3.2}$$

3. Balance of products from unit u which is represented by two methods:

(a) For percent yields that do not depend on the feed properties, the amount of products is equal to the total inlet flow multiply by a constant percent yield of that unit.

$$AO_{u,c,t} = AF_{u,t} \times yield_{u,c} \tag{3.3}$$

(b) For percent yields that depend on the feed properties, e.g. CDU the amount of products is equal to the sum of each inlet flow times percent yield of each inlet flow.

$$AO_{u,c,t} = \sum_{u'} \sum_{c'} (A_{u',c',u,t} \times cyield_{c',c}) \quad \forall\ u' \in \text{ctank}, u \in \text{CDU}, c \in \text{C}_{\text{CDU}}, c' \in \text{C}_{\text{O}} \tag{3.4}$$

Stream property equations:

The calculation of product property can be accomplished by two methods:

1. Product properties leaving unit u can be calculated by the sum of the flow fraction times the properties of each flow as in the following equation. These are called blending equations.

$$PO_{u,c,q,t} = \frac{\sum_{u'} \sum_{c'} (A_{u',c',u,t} \times pro_{u',c,q})}{\sum_{u'} \sum_{c'} A_{u',c',u,t}} \tag{3.5}$$

The blending equation for each property can be found in Appendix B.

2. Product properties from unit u that can be determined over average values obtained from plant data, e.g. isomerate from isomerization unit and reformate from reformer unit:

$$PO_{u,c,q,t} = pro_{u,c,q} \tag{3.6}$$

Capacity constraint

The production amount of each oil product should be greater than or equal to the economic production quantity.

$$AF_{u,t} \geq un_u \tag{3.7}$$

The production quantity must not be over its maximum production capacity.

$$AF_{u,t} \leq ux_u \tag{3.8}$$

The allowable quantity of crude oil purchased in each time period is shown in the following equation:

$$ox_o \geq AC_{o,t} \geq on_o \tag{3.9}$$

The allowable quantity of product stored in each time period is limited by Equation (3.10).

$$stox_p \geq AS_{p,t} \tag{3.10}$$

Note that strategic stock regulated by law is not concerned in this thesis since the objective of this thesis is to find the most economic stock for commercial planning under uncertainty.

Quality constraint

The product quality must be greater or equal to its minimum specifications.

$$PO_{u,c,q,t} \geq pn_{c,q} \qquad \forall\, c \in C_p \qquad (3.11)$$

The product quality must not be over its maximum specifications.

$$PO_{u,c,q,t} \leq px_{c,q} \qquad \forall\, c \in C_p \qquad (3.12)$$

Demand constraint

The amount of each product sold in each period of time must be equal to the amount of product demand in that time period.

$$dem_{p,t} = sales_{p,t} \qquad (3.13)$$

3.3 Case Study

The model was applied to the production planning of Bangchak Refinery. Figure 3.2 shows a simplified scheme illustrating the application of the model. The refinery composes two atmospheric distillation units (CDU2 and CDU3), two naphtha pretreating units (NPU2 and NPU3), one light naphtha isomerization unit (ISOU), two catalytic reforming units (CRU2 and CRU3), one kerosene treating unit (KTU), one gas oil hydrodesulfurization (GO-HDS), and one deep gas oil hydrodesulfurization (DGO-HDS). The commercial products from the refinery are liquefied petroleum gas (LPG), gasoline RON 91 (SUPG), gasoline RON 95 (ISOG), jet fuel (JP-1), high speed diesel (HSD), fuel oil 1 (FO1), fuel oil 2 (FO2), and low sulfur fuel oil (FOVS). Fuel gas (FG) and some amount of FOVS produced from the process are used as an energy source for the plant. The summary of feeds and products that are processed in each unit can be found in the Table 3.1.

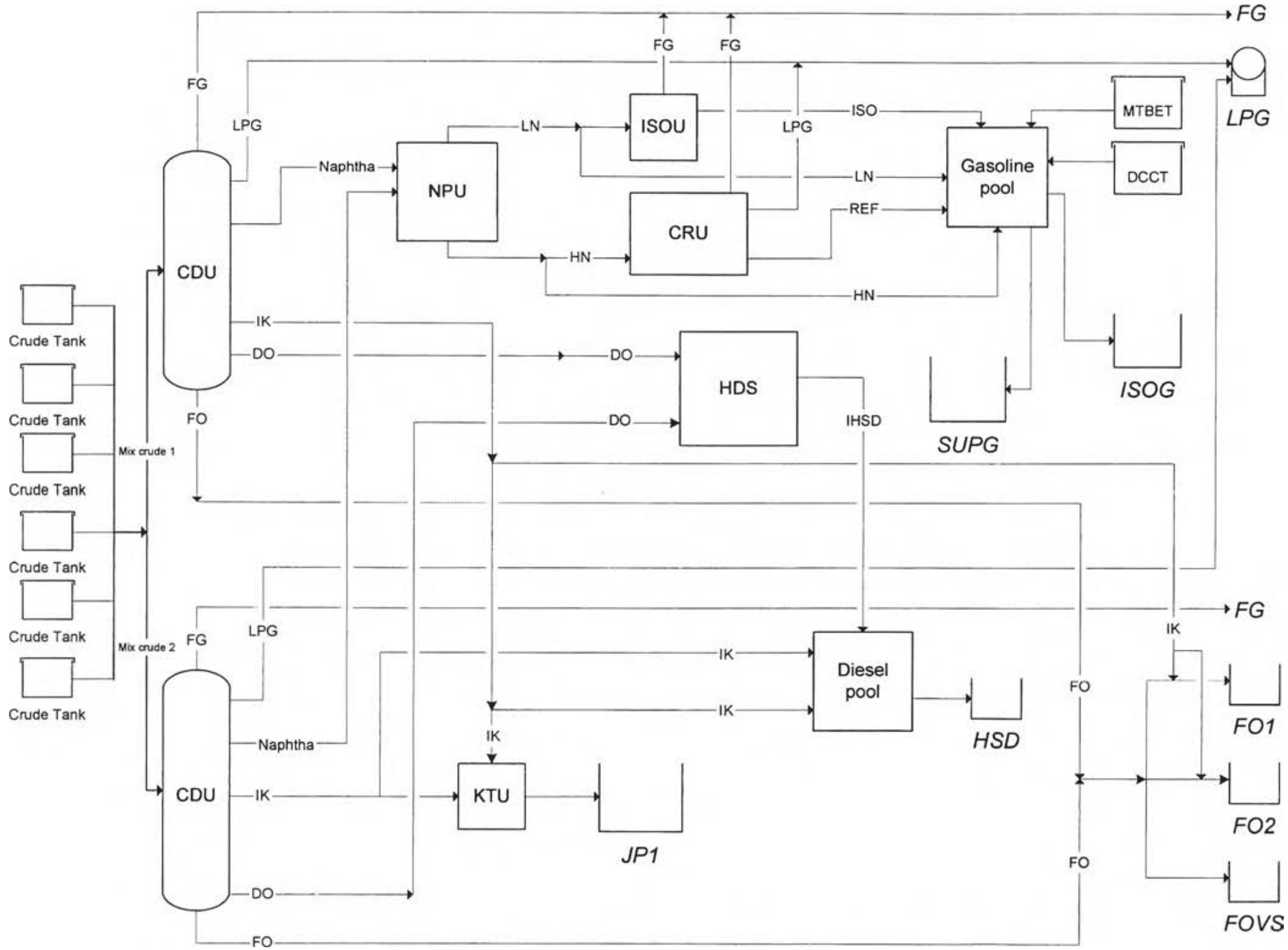

Figure 3.2 Simplified scheme of Bangchak Petroleum Public Company Limited.

Table 3.1 Summary of feeds and products for each unit

Unit	Feed	Product
CDU	• Crude mixture	• FG • LPG • LN • MN • HN • IK • DO • FO
NPU	• LN • MN • HN	• LN • MN • HN
ISOU	• LN	• FG • ISO
CRU	• MN • HN	• FG • LPG • REF
KTU	• IK	• JP-1
HDS	• DO	• IHSD
ISOT	• ISO	• ISO
REFT	• REF	• REF
LNT	• LN	• LN
HNT	• HN	• HN
MTBET	• MTBE (purchased)	• MTBE
DCCT	• DCC (purchased)	• DCC

The intermediate streams from each unit are blended in product pools to satisfy product specification. There are three product pools for blending products: gasoline pool (GSP), diesel pool (DSP), and fuel oil pool (FOP). The streams that are used to blend to attain each product are shown in Table 3.2. The process of both gasoline pools can be found in Figure 3.3.

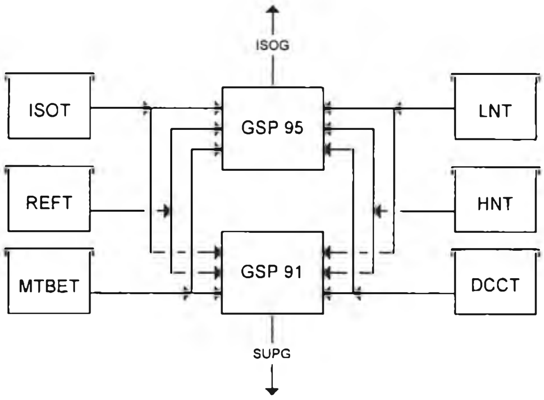

Figure 3.3 Gasoline pool blending.

Table 3.2 Intermediate streams for product blending in each pool

Pool	Intermediate	Product
GSP91	• ISO • REF • LN • HN • MTBE • DCC	• SUPG
GSP95	• ISO • REF • LN • HN • MTBE • DCC	• ISOG
DSP	• IK • DO • IHSD	• HSD
FO1P	• IK • FO	• FO1
FO2P	• IK • FO	• FO2
FOVSP	• FO	• FOVS

There are six crude oil types for feeding the refinery: Oman, Tapis, Labuan, Seria light, Phet, and Murban. Data of all units and commodities (crude oils, intermediates, products) can be found in Appendix C.

The properties concerned in this work are different upon the product. For FG and LPG, the properties of these two products are not taken into account since FG is burned as an energy source in the plant whereas LPG properties are mostly in the range of its specification. Other properties for each product can be described as follows:

Intermediates for blending gasoline and gasoline products

The properties involving with these products are octane number (RON), aromatic content (ARO), and Reid vapor pressure (RVP). These properties can be estimated after leaving from the pretreating unit for LN and HN and processing unit for ISO and REF. MTBE and DCC intermediates are purchased so their properties are constant. These properties are of importance for production of SUPG and ISOG products.

Kerosene (IK)

Since IK can be blended with several products, it involves seven important properties that include freezing point index (FPI), aromatic content (ARO), cetane index (CI), sulfur content (S), viscosities (V50 and V100), and pour point index (PPI) for the IK production. Properties of FPI and ARO, used for jet fuel (JP-1) production, are not very important since most IK product is in the range of the jet fuel specification. CI and S are crucial for the HSD production whereas V50, V100, and PPI are necessary for the fuel oil production.

Diesel oil (DO)

Only two properties are used in the DO production that is CI and S. Since these properties are the specification for HSD products.

Fuel oil (FO)

There are four properties (S, V50, V100, PPI) used in the fuel oil production. S, V100, and PPI are used for the low sulfur fuel oil (FOVS) production while S, V50, and PPI are required for the low pour point fuel oil (FO1 and FO2) production. These fuel oils, FO1 and FO2, are different in viscosity after being blended with IK.

The refinery planning model can be described as follows:

Charging tank model

All crude oils are unloaded to the charging tanks and then delivered to the CDU. Since in this refinery there is no storage tank that keeps each type of crude oil separately, the crude oils are mixed together and stored in charging tanks. The process is assumed to have two charging tanks for each CDU in each period and no capacity limit in order to find the exact amount of each crude oil refined to satisfy demand in each month. In this process, each charging tank also works as a mixer. Equation (3.14) is used to model the outlet stream from the crude storage tanks and Figure 3.4 can be used to illustrate the charging tank model.

$$AO_{ctank,o,t} = \sum_{CDU} A_{ctank,o,CDU,t} \tag{3.14}$$

In addition, the PHET crude has to be fed to CDU2 only due to the limitation of unit. This operation rule is represented in Equation (3.15).

$$A_{PHETT,PHET,CDU3,t} = 0 \tag{3.15}$$

Crude distillation unit (CDU) model

There are two CDUs that are modeled in the same manner. Equations (3.1), (3.4), (3.5), (3.7), and (3.8) are used to model both CDUs. Total feed flow to the both CDUs are represented by the following equation:

$$AF_{CDU,t} = \sum_{o} \sum_{ctank} A_{ctank,o,CDU,t} \tag{3.16}$$

Furthermore, the feed flow must satisfy both CDUs operating capacity:

For CDU2: $$190784 \geq AF_{CDU2,t} \geq 95392 \tag{3.17}$$

For CDU3: $$381569 \geq AF_{CDU3,t} \geq 190784 \tag{3.18}$$

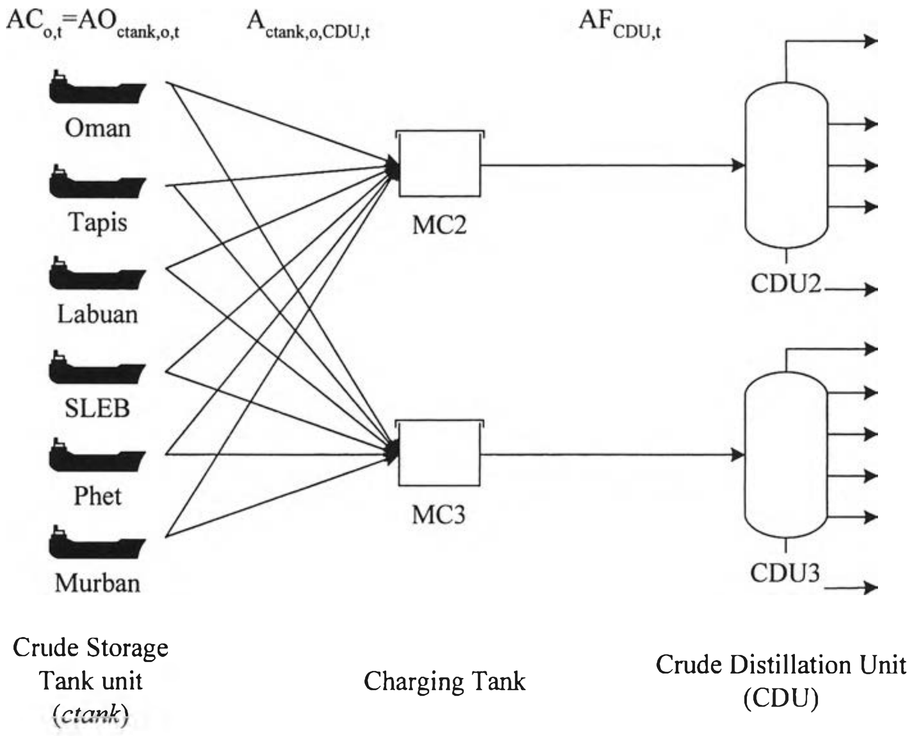

Figure 3.4 Overview of crude storage tanks and charging tanks.

The amount of product yield depends on feed flow and feed properties:

$$AO_{CDU,c,t} = \sum_{ctank} \sum_{o} (A_{ctank,o,CDU,t} \times cyield_{o,c}) \; \forall \; c \in \mathrm{C}_{\mathrm{CDU}} \tag{3.19}$$

The properties of product streams are determined from properties of each fraction from each crude oil. The properties based on volume basis, which are RON, RVPI, ARO, CI, PPI, and SG, can be calculated from:

$$PO_{CDU,c,q,t} = \frac{\sum_{o}\sum_{ctank}(A_{ctank,o,CDU,t} \times cyield_{o,c} \times pro_{ctank,c,q})}{\sum_{o}\sum_{ctank}(A_{ctank,o,CDU,t} \times cyield_{o,c})} \quad (3.20)$$

$$\forall\ c \in C_{CDU}, q \in AV_q$$

The properties based on weight basis, which are FPI, S, V50, and V100, can be calculated from:

$$PO_{CDU,c,q,t} = \frac{\sum_{o}\sum_{ctank}(A_{ctank,o,CDU,t} \times cyield_{o,c} \times pro_{ctank,c,q} \times pro_{ctank,c,SG})}{\sum_{o}\sum_{ctank}(A_{ctank,o,CDU,t} \times cyield_{o,c} \times pro_{ctank,c,SG})} \quad (3.21)$$

$$\forall\ c \in C_{CDU}, q \in AW_q$$

Properties of products leaving from both CDUs are bounded by Equations (3.11) and (3.12). These constraints are shown in Table3.3.

Table 3.3 Property constraints of products leaving from both CDU (x = maximum, n = minimum)

Product	Property	CDU 2	CDU 3
IK	ARO lv%	25x	25x
	FPI index	11.8x	11.8x
DO	CI index	47n	47n
FO	S wt%	0.5x	2.0x
	Vis50 cSt	-	300x
	Vis100 cSt	3-30	-
	PP °C	57x	24x

Naphtha pretreating unit (NPU) model

Equations (3.1), (3.3), (3.7), and (3.8) model both NPUs. Feed flow is determined by Equations (3.22) and (3.23):

For NPU2: $$AF_{NPU2,t} = \sum_{nap} \sum_{CDU} A_{CDU,nap,NPU2,t} \tag{3.22}$$

For NPU3: $$AF_{NPU3,t} = \sum_{nap} \sum_{CDU} A_{CDU,nap,NPU3,t} \tag{3.23}$$

NPUs operate within the following range:

For NPU2: $$65820 \geq AF_{NPU2,t} \geq 32910 \tag{3.24}$$

For NPU3: $$35772 \geq AF_{NPU3,t} \geq 19078 \tag{3.25}$$

The amount of product from both NPUs can be calculated from Equations (3.26) and (3.27):

For NPU2: $$AO_{NPU2,nap,t} = (\sum_{CDU} A_{CDU,nap,NPU2,t}) \times yield_{NPU2,nap} \tag{3.26}$$

For NPU3: $$AO_{NPU3,nap,t} = (\sum_{CDU} A_{CDU,nap,NPU3,t}) \times yield_{NPU3,nap} \tag{3.27}$$

The NPU duty is to reduce sulfur content of all naphthas before feeding to both CRUs. The sulfur content calculation is not necessary in this process since sulfur in gasoline is lower than gasoline specification.

Catalytic reformer unit (CRU) model

Equations (3.1), (3.3), (3.6), (3.7), and (3.8) model both CDUs. Feed flow is considered by Equations (3.28) and (3.29):

For CRU2: $$AF_{CRU2,t} = \sum_{c} \sum_{NPU} A_{NPU,c,CRU2,t} \tag{3.28}$$

For CRU3: $$AF_{CRU3,t} = \sum_{c} \sum_{NPU} A_{NPU,c,CRU3,t} \tag{3.29}$$

CRUs operate within the following range:

For CRU2: $$33387 \geq AF_{CRU2,t} \geq 19078 \quad (3.30)$$

For CRU3: $$35772 \geq AF_{CRU3,t} \geq 19078 \quad (3.31)$$

The amount of product yield depends on feed flow and feed properties:

$$AO_{CRU,c,t} = AF_{CRU,t} \times yield_{CRU,c} \qquad \forall\, c \in \mathrm{C_{CRU}} \quad (3.32)$$

Percent yield of LPG and FG from CRUs are calculated using the following equations:

$$yield_{CRU,LPG} = (100 - yield_{CRU,REF}) \times 0.75 \quad (3.33)$$

$$yield_{CRU,FG} = (100 - yield_{CRU,REF}) \times 0.25 \quad (3.34)$$

The properties of reformate from both CRUs are constants:

$$PO_{CRU,REF,q,t} = pro_{CRU,REF,q} \quad (3.35)$$

These properties are octane number, aromatic content, and RVP.

Isomerization unit (ISOU) model

Equations (3.1), (3.3), (3.6), and (3.8) model ISOU. Since ISOU is fed only with LN from NPU, inlet variables are equal to outlet variables of the LN stream:

$$AF_{ISOU,t} = \sum_{NPU} A_{NPU,LN,ISOU,t} \quad (3.36)$$

ISOU has only maximum capacity boundary because it can recycle feed flow back to the process:

$$35772 \geq AF_{ISOU,t} \tag{3.37}$$

There are two products, FG and ISO, from ISOU. Its production yield can be estimated from the following equations:

$$AO_{ISOU,FG,t} = AF_{ISOU,t} \times yield_{ISOU,FG} \tag{3.38}$$

$$AO_{ISOU,ISO,t} = AF_{ISOU,t} \times yield_{ISOU,ISO} \tag{3.39}$$

Properties of the products are constant and can be estimated from:

$$PO_{ISOU,ISO,q,t} = pro_{ISOU,ISO,q} \tag{3.40}$$

These properties are octane number, aromatic content, and RVP.

Kerosene treating unit (KTU) model

Equations (3.1), (3.2), (3.7), and (3.8) model KTU. Total feed flow and operating range are shown in the Equations (3.41) and (3.42):

$$AF_{KTU,t} = \sum_{CDU} A_{CDU,IK,KTU,t} \tag{3.41}$$

$$65820 \geq AF_{KTU,t} \geq 32910 \tag{3.42}$$

The product from KTU is estimated to equal to the feed.

$$AO_{KTU,JP\text{-}1,t} = AF_{KTU,t} \tag{3.43}$$

KTU converts mercaptan sulfur to sulfur since mercaptan is limited in jet fuel (JP-1). However, the level of mercaptan is very low so it is not taken into account here.

Gas oil hydrodesulfurization and deep gas oil hydrodesulfurization unit (GO-HDS and DGO-HDS) model

Equations (3.1), (3.3), (3.7), and (3.8) model GO-HDS and DGO-HDS. Amount of feed is given by Equations (3.44) and (3.46) whereas the operating range is limited by Equations (3.45) and (3.47).

For GO-HDS:

$$AF_{GO-HDS,t} = \sum_{CDU} A_{CDU,IK,GO-HDS,t} + \sum_{CDU} A_{CDU,DO,GO-HDS,t} \tag{3.44}$$

$$85853 \geq AF_{GO-HDS,t} \geq 42926 \tag{3.45}$$

For DGO-HDS:

$$AF_{DGO-HDS,t} = \sum_{CDU} A_{CDU,IK,DGO-HDS,t} + \sum_{CDU} A_{CDU,DO,DGO-HDS,t} \tag{3.46}$$

$$119240 \geq AF_{DGO-HDS,t} \geq 59620 \tag{3.47}$$

Note that the volumes of DO feed to HDS are 50 and 100% of DO leaving from CDU2 and CDU3, respectively.

Production yield from both HDS units are estimated from Equations (3.48) and (3.49):

For GO-HDS: $$AO_{GO-HDS,IHSD,t} = AF_{GO-HDS,t} \times yield_{GO-HDS,IHDS} \tag{3.48}$$

For DGO-HDS: $$AO_{DGO-HDS,IHSD,t} = AF_{DGO-HDS,t} \times yield_{DGO-HDS,IHDS} \tag{3.49}$$

Product pool model

Fuel gas (FG)

The total production of FG can be found in the following equation:

$$AF_{FGT,t} = \sum_{u'} A_{u',FG,FGT,t} \tag{3.50}$$

The amount of product flow out from FGT is represented by:

$$AO_{FGT,FG,t} = AF_{FGT,t} \tag{3.51}$$

The property of FG is not an issue here because it is burned as an energy source for the plant.

Liquefied petroleum gas (LPG)

The total production of LPG can be found in the following equation:

$$AF_{LPGT,t} = \sum_{u'} A_{u',LPG,LPGT,t} \tag{3.52}$$

The amount of product flow out from LPGT is represented by:

$$AO_{LPGT,LPG,t} = AF_{LPGT,t} \tag{3.53}$$

Likewise, the property of LPG is not taken into account because sulfur and RVP of LPG product from most crude oils are in the range of specification.

Gasoline RON 91 (SUPG) and RON 95 (ISOG)

Gasoline is produced by blending six intermediate streams which include ISOT, REFT, LNT, HNT, MTBET, and DCCT. Equation (3.54) represents the feed flow to both GSPs:

$$\begin{aligned} AF_{GSP,t} &= A_{ISOT,ISO,GSP,t} + A_{REFT,REF,GSP,t} + A_{LNT,LN,GSP,t} \\ &\quad A_{HNT,HN,GSP,t} + A_{MTBET,MTBE,GSP,t} + A_{DCCT,DCC,GSP,t} \end{aligned} \tag{3.54}$$

In addition, there is an operating rule in blending gasoline product with MTBE. The amount of MTBE in gasoline must not be over 10%. This rule is shown in the following equation:

$$A_{MTBET,MTBE,GSP,t} \leq AF_{GSP,t} \times 0.1 \tag{3.55}$$

The amount of product flow out from both GSPs are determined by Equations (3.56) and (3.57):

For GSP91: $$AO_{GSP91,SUPG,t} = AF_{GSP91,t} \tag{3.56}$$

For GSP95: $$AO_{GSP95,ISOG,t} = AF_{GSP95,t} \tag{3.57}$$

The product RON, ARO, and RVPI for both GSPs can be calculated from the following equation:

$$PO_{GSP,c,aref,t} = \frac{\sum_{in} \sum_{int} (A_{int,in,GSP,t} \times pro_{int,in,aref})}{\sum_{in} \sum_{int} A_{int,in,GSP,t}} \tag{3.58}$$

Moreover, the product properties must satisfy the product specifications:

For GSP91: $$px_{SUPG,aref} \geq PO_{GSP91,SUPG,aref,t} \geq pn_{SUPG,aref} \tag{3.59}$$

For GSP95: $$px_{ISOG,aref} \geq PO_{GSP95,ISOG,aref,t} \geq pn_{ISOG,aref} \tag{3.60}$$

In blending gasoline, four intermediate streams including LN, HN, ISO, and REF are produced from the refinery while MTBE and DCC are purchased from the outside.

Jet fuel (JP-1)

JP-1 is a product that is produced by KTU. The total production of JP-1 can be found by the following equation:

$$AF_{JPT,t} = A_{KTU,JP\text{-}1,JPT,t} \tag{3.61}$$

The amount of product flow out from JPT is represented by:

$$AO_{JPT,JP\text{-}1,t} = AF_{JPT,t} \tag{3.62}$$

High speed diesel (HSD)

HSD is produced from six intermediate streams and represented by the following equation:

$$AF_{DSP,t} = \sum_{CDU} A_{CDU,IK,DSP,t} + \sum_{CDU} A_{CDU,DO,DSP,t} + \sum_{HDS} A_{HDS,IHSD,DSP,t} \tag{3.63}$$

The amount of product flow out from DSP is determined by Equation (3.65):

$$AO_{DSP,HSD,t} = AF_{DSP,t} \tag{3.64}$$

Fuel oil #1 (FO1), Fuel oil #2 (FO2), and Low sulfur fuel oil (FOVS)

There are three types of fuel oil with different viscosity, pour point and sulfur content. All fuel oils are blended in FO1P, FO2P, and FOVSP. The following equation represents the feed flow to each fuel oil pool.

For FO1P: $$AF_{FO1P,t} = \sum_{CDU} A_{CDU,IK,FO1P,t} + \sum_{CDU} A_{CDU,FO,FO1P,t} \quad (3.65)$$

For FO2P: $$AF_{FO2P,t} = \sum_{CDU} A_{CDU,IK,FO2P,t} + \sum_{CDU} A_{CDU,FO,FO2P,t} \quad (3.66)$$

For FOVSP: $$AF_{FOVSP,t} = \sum_{CDU} A_{CDU,FO,FOVSP,t} \quad (3.67)$$

In addition, the recipe used in blending FO1 and FO2 with IK is 7 and 2.5% of the FO1 and FO2 volume, respectively. This is shown in the following equations:

$$\sum_{CDU} A_{CDU,IK,FO1P,t} = AF_{FO1P,t} \times 0.07 \quad (3.68)$$

$$\sum_{CDU} A_{CDU,IK,FO2P,t} = AF_{FO2P,t} \times 0.025 \quad (3.69)$$

The amount of product flow out from all fuel oil pools are determined by Equations (3.72), (3.73), and (3.74):

For FO1P: $$AO_{FO1P,FO1,t} = AF_{FO1P,t} \quad (3.70)$$

For FO2P: $$AO_{FO2P,FO2,t} = AF_{FO2P,t} \quad (3.71)$$

For FOVSP: $$AO_{FOVSP,FOVS,t} = AF_{FOVSP,t} \quad (3.72)$$

Refinery Fuel Balance

There are two energy sources burnt in this refinery which are FG and FOVS. FG consists of methane and ethane that has been produced in different units. These gases are burnt in the refinery to provide the energy required for operation of the different units and to provide utilities (steam, electricity, etc.). There are no purchasing or selling of these gases and there is no fixed demand. Therefore, production of these gases from the process is equal to the burnt amount:

$$AO_{FGT,FG,t} = Burnt_{FG,t} \tag{3.73}$$

On the other hand, FOVS can be sold as a product and burnt as an energy source for the plant. The amount of FOVS produced can be calculated from the following equation:

$$AO_{FOVSP,FOVS,t} - Burnt_{FOVS,t} = MANU_{FOVS,t} \tag{3.74}$$

where $AO_{FOVSP,FOVS,t}$ is equal to the amount of FOVS leaving from the process.

The refinery fuel balance is calculated by expressing in fuel oil equivalence and based on weight basis. Calorific equivalents of 1 ton FG is estimated to 1.3 ton FO. The refinery fuel balance equation is shown by the following equation:

$$Used_t = (Burnt_{FG,t} \times 0.3 \times 1.3) + (Burnt_{FOVS,t} \times 0.93) \tag{3.75}$$

where the value of 0.3 and 0.93 are specific gravity of FG and FOVS, respectively. $Used_t$ is energy consumption for operating the process expressed in ton of fuel oil equivalence (tFOE) and can be calculated from the following equation:

$$Used_t = \sum_{u}^{UN} (AF_{u,t} \times density_u \times fuel_u) \tag{3.76}$$

where $AF_{u,t}$ is volume of feed and $density_u$ is density of feed to each unit. This density is an average value for each unit except CDUs which are different between crude oil types. Energy consumption for each unit is calculated by using $fuel_u$ which is percent of energy consumption for each unit. The value of $fuel_u$ can be found in Appendix C.

3.4 Objective Function

The objective function in this model is profit that is shown in the following equation:

$$\begin{aligned}\text{Max Profit } &= \sum_t \sum_p TP_{p,t} - \sum_t \sum_o TO_{o,t} - \sum_t \sum_i TI_{i,t} \\ &- \sum_t \sum_p TS_{p,t} - \sum_t \sum_p TL_{p,t} - \sum_t \sum_p TD_{p,t}\end{aligned} \tag{3.77}$$

where:

$TP_{p,t}$ comes from

$$TP_{p,t} = MANU_{p,t} \times cp_{p,t} \qquad \forall\, p \in \mathrm{C_p} \tag{3.78}$$

where $MANU_{p,t}$ is equal to the amount of product produced in that time period.

$$MANU_{p,t} = \sum_u AO_{u,p,t} \qquad \forall\, u \in \mathrm{UP_u}, p \in \mathrm{C_p} \tag{3.79}$$

Note that $AO_{u,p,t}$ is the amount of product flow out from production unit in each time period.

$TO_{o,t}$ comes from

$$TO_{o,t} = AC_{o,t} \times co_{o,t} \qquad \forall\, o \in \mathrm{C_o} \tag{3.80}$$

where $AC_{o,t}$ is equal to the amount of crude oil refined in that time period.

$$AC_{o,t} = \sum_{u} AO_{u,o,t} \qquad \forall\ u \in \mathrm{UC_u}, o \in \mathrm{C_o} \tag{3.81}$$

note that $AO_{u,o,t}$ is amount of crude oil flow out from crude oil storage tank in each time period.

$TI_{i,t}$ comes from

$$TI_{i,t} = AI_{i,t} \times ci_{i,t} \qquad \forall\ i \in \mathrm{C_{ia}} \tag{3.82}$$

where $AI_{i,t}$ is equal to the amount of purchased intermediate added in that time period.

$$AI_{i,t} = \sum_{u} AO_{u,i,t} \qquad \forall\ u \in \mathrm{UI_u}, i \in \mathrm{C_{ia}} \tag{3.83}$$

note that $AO_{u,ia,t}$ is amount of MTBE and DCC flow out from their storage tank in each time period.

$TS_{p,t}$ comes from

$$TS_{p,t} = \left(\frac{AS_{p,t} + AS_{p,t-1}}{2} \right) \times cp_{p,t} \times int \qquad \forall\ p \in \mathrm{C_p} \tag{3.84}$$

where cost of storage is the cost of financing the investment in the working capital that it represents. The financial cost incurred relates to the average stock level over the period. Unless the stock levels are known, they are assumed that the average stock level is equal to the arithmetic mean of the opening and closing stock (Favennec, 2001). $AS_{p,t}$ represents closing stock and $AS_{p,t-1}$ represents opening stock. *Int* represents the average rate of interest payable in that period.

The balance of product storage can be found in the following equation:

$$AS_{p,t} = AS_{p,t-1} + MANU_{p,t} - sales_{p,t} - AD_{p,t} \tag{3.85}$$

where $sales_{p,t}$ is equal to the amount of product demand in that time period (capacity constraint in Equation (3.13)).

$TL_{p,t}$ comes from

$$TL_{p,t} = AL_{p,t} \times cl_{p,t} \qquad \forall\, p \in \mathrm{C_p} \tag{3.86}$$

where $AL_{p,t}$ is the product volume that cannot satisfy its demand. The demand constraint in Equation (3.13) is modified to the following equation:

$$dem_{p,t} = sales_{p,t} + AL_{p,t} \tag{3.87}$$

and $cl_{p,t}$ is assigned to equal $cp_{p,t}$ as follows:

$$cl_{p,t} = cp_{p,t} \qquad \forall\, p \in \mathrm{C_p} \tag{3.88}$$

In Equation (3.87), the demands of each product must be equal to the volume of that product sale plus the volume of lost demand of that product. The volume of the lost demand is taken into account as the opportunity cost if that production cannot satisfy the demand.

$TD_{p,t}$ comes from

$$TD_{p,t} = AD_{p,t} \times cp_{p,t} \times disc \qquad \forall\, p \in \mathrm{C_p} \tag{3.89}$$

where $AD_{p,t}$ is the product volume that over demand and sold as discount. The balance of discount volume can be found in Equation (3.85).

3.5 Heuristic

The procedure used in deciding crude oil purchased for a traditional refinery is as follows:

Step 1: Estimate the demand of each product.

Step 2: Choose the crude oils by considering from their GRM. The mixture of these crude oils must satisfy both demand and specification of products.

Step 3: If the highest GRM crude oil is not available, next highest GRM crude oil is chosen.

Step 4: Purchase crude oil about 1-2 months in advance before production.

3.6 Stochastic Formulation

The stochastic formulation technique used in this thesis is the two-stage stochastic linear program with fixed recourse. The uncertainty is introduced through the demand and product price parameters. First-stage decisions are the amount of crude oil purchased, $AO_{p,t}$, for every planning period. Second-stage decisions are the amount of product production, $MANU^{s}_{p,t}$, amount of product stock, $AS^{s}_{p,t}$, amount of intermediate purchased, $AI^{s}_{p,t}$, amount of product that cannot satisfy demand, $AL^{s}_{p,t}$, and amount of discount sales, $AD^{s}_{p,t}$. These second-stage scenarios are denoted by the index s and assumed to occur with individual probabilities ρ_s. It is assumed that the random events which occur at the second-stage are finite and independent from the first-stage decisions.

The stochastic results are obtained by using sampling algorithm method which was discussed by Aseeri and Bagajewicz (2003). In this method, a full deterministic model is run for each scenario and then, after that scenario is solved, the first stage variables (commitment to buy a certain sets of crudes) are fixed and rerun the same model for all the rest of the scenarios. After that, the highest expected GRM risk curve and non-dominated curve with this one are selected and discussed.

3.7 Model Testing

Data from Bangchak Refinery was used in testing the model. All properties that are not additive were changed to the index form in order to be able to calculate linearly.

The refinery produces eight commercial products (LPG, SUPG, ISOG, JP-1, HSD, FO1, FO2, and FOVS) using two crude distillation units (CDU2 and CDU3) and six productive units (NPU2, NPU3, CRU2, CRU3, ISOU, GO-HDS, DGO-HDS, KTU). The maximum plant production capacity is 120 kbd. The production yields and unit capacity can be found in Appendix C.

Demand in each period was considered to be satisfied by the production. Uncertainty was introduced in market demand and product price.

The model was tested in three-time-period planning. First, a deterministic linear programming model was solved to obtain the results including amount and type of crude oil used, amount and property of product refined, amount of product stored, and profit. The stochastic programming model was then formulated with uncertainty in the product demands and prices by using a deterministic linear programming model as a basis. The stochastic solution was found by using sampling algorithm method. The results are compared with the deterministic model. Moreover, risk curves from both solutions are analyzed.

The proposed model was implemented in the modeling system GAMS. The linear model was solved by CPLEX 9.0 solver. GAMS was run on a Pentium IV / 2.4 GHz PC platform.