CHAPTER I

INTRODUCTION

Motivation

At present there are several theoretical approaches to the quantum transport in condensed matter physics, such as Wigner function[1], the density matrix[2], Green's function[3] and Feynman's path integrals[4]. All these approaches are all equivalent in the presentation of the quantum nature into the transport process. Unfortunately, there is no single theory that can unify and describe the transport phenomena correctly. All these theories have their applications and computational strength and weaknesses.

The simplest Boltzmann approach to the transport theory is a semiclassical scheme.It deals with a well-defined probability distribution that changes in space and time locally and are governed by an integro-differential equation involving complicated scattering terms. In the quantum transport theory the problems are completely different, in the con text that the distribution function are nonlocal in space and time and carriers scatter so rapidly that the scattering processes are no longer represented in terms of scattering rate alone. Instead, more details of scattering amplitude must be considered and included into the description of the transport equation.

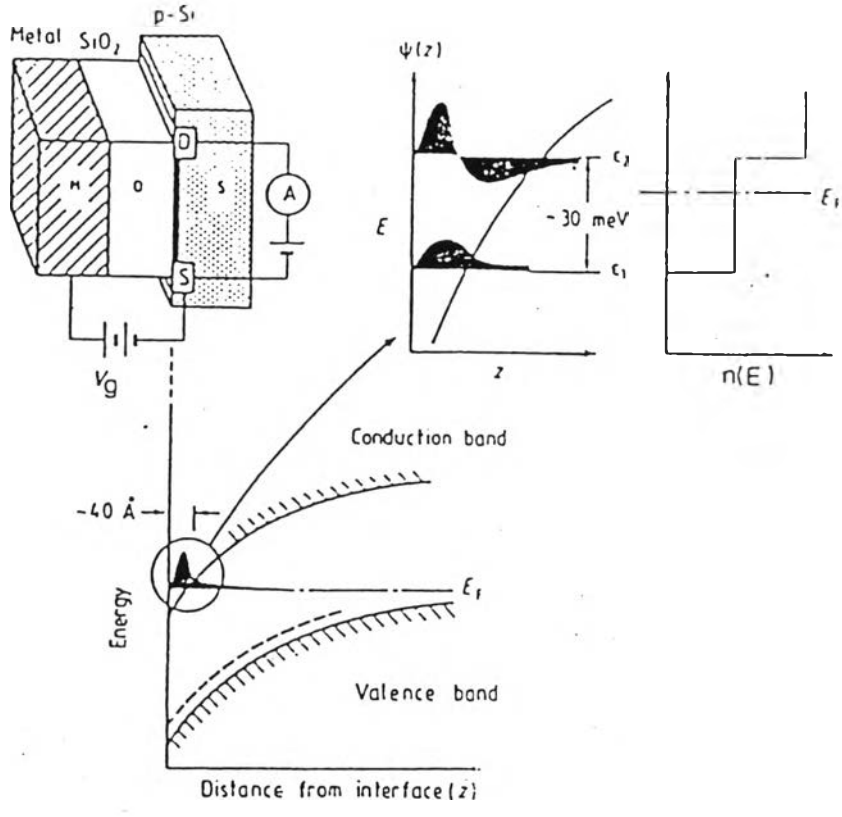
The quantum transport problem can be handled by using the Wigner function, the density matrix, or the Green's function approaches. In the Wigner function scheme, one attempts to retain as much as the classical formalism in order to be able to express the result in terms of momentum and velocity which is of greatest experimental interest. The Wigner function scheme has a maximum flexibility. By contrast, the density matrix and the Green's function schemes adhere closely to the actual quantum states. These approaches can be obtained at the greatest sensitivity but are relatively inflexible in studying the non-linear properties in the presence of full scattering processes.

The Feynman path integrals scheme relies on the influence functional technique[5] in which the sources of dissipations such as phonons, plasmons, imperfections, have been integrated out and result to the influence functionals in the action function of carriers[6]. This scheme has greatest flexibility in studying the non-linear transport and channeled transport as in the case of Landauer's equation[7]. The approximations can be made in different levels, i.e., using modeled influence functional, consider the influence functional in some limit or make the cumulant expansion[8] of the influence functional into series. In this thesis, we would like to apply the Feynman path integrals method to the problem of electron transport in two-dimensional systems which has the greatest physical phenomenon, the quantum Hall effect. The system we used here is modeled to be the one that is believed to be for quantum Hall system. The details of our model will be discussed later.

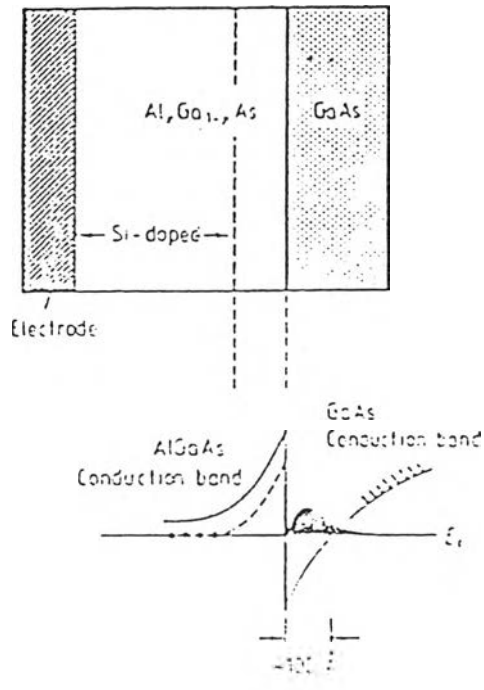
In the folowing section we will discuss the real two-dimensional systems which are used in experiments. We will then discuss that how the transverse component of conductivity of a two-dimensional electron becomes quantized in such a system, and called the quantum Hall phenomenon. The last section will be devoted to discussion of our modeled system.

2-Dimensional System

Two-dimensional systems can be realized in several classes of systems. One example is the electrons trapped on the surface of liquid helium at temperature below 2.1 K. Since we cannot make the concentration of electrons to be high in this system, chiefly because the liquid surface connot sustain too many electrons pressed to the surface by an electric field. There are two classes of systems in that we can make degenerate two-dimensional electrons which electrons occupy up to the Fermi energy, E_F; Metal-Oxide-Semiconductor(MOS) space charge layers, fig.(1.1a), and semiconductor heterojunctions, fig.(1.1b).



Figure(1.1) (a) Two-dimensional electron system in the metal-oxide semiconductor(MOSFET) inversion layer. The S and D represent source and drain (usually n-type doped region) respectively. V_g is the gate voltage and E_F is the Fermi energy. Bending in the valence and conduction bands is depicted together with wave function $\Psi(z)$ and density of state $n(E)$.



Figure(1.1) (b) Two-dimensional electron system in the semiconductor heterostructure. The figure depictes the case of selective doping in GaAs/AlGaAs system.

Quantum Hall Systems

The range of phenomena observed in two-dimensional electronic systems in a magnetic field are large and spectacular. Perhaps most spectacular of all are the quantum Hall effects. The integer quantum Hall effect, discovered by Klaus von Klitzing [9], occurs in two dimensional systems placed in a strong perpendicular magnetic field (typically above 1 Tesla) at low temperatures (typically below 1 K). It is found that under certain conditions of such a system, the conductivity tensor, σ, takes the form

$$\sigma = \begin{bmatrix} 0 & -ie^2/h \\ ie^2/h & 0 \end{bmatrix}. \tag{1}$$

Here h is Plank's constant, e is the charge of the electron and i is a small integer. In other words, the current density $\vec{j}$ is directed precisely perpendicular to the electric field $\vec{E}$ according to

$$j_i = \sum_j \sigma_{ij} E_j \tag{2}$$

and it has the quantized magnitude

$$\frac{j}{E} = \sigma_{xy} \equiv \sigma_H = \frac{ie^2}{h}. \tag{3}$$

The off-diagonal conductivity is thus given by a combination of fundamental constants. Most transport properties of these systems can be simply described by treating them as macroscopically homogeneous and isotropic conducting plates (of size L_x, L_y). Imposing a sufficiently weak electric current (I_x, I_y) gives rise to

voltage drop across the plates,

$$\begin{aligned} U_x &= RI_x + R_H I_y \\ U_y &= -R_H I_x + \tilde{R} I_y \end{aligned} \tag{4}$$

In these linear relations, R and $\tilde{R}$ are the longitudinal resistances and R_H is the Hall resistance. In usual measurements, $I_y = 0$ and eq.(4) reduces to Ohm's law.

$$U_x = RI_x \tag{5}$$

and the Hall effect is

$$U_y = -R_H I_x \tag{6}$$

in their simplest form. From eqs.(5) and (6) we see that measuring the relative decreases in voltage yields the ratio

$$R_H/R = -U_y/U_x. \tag{7}$$

Let the electric field $\vec{E} = (E_x, E_y)$ be homogeneous, i.e. $E_x = U_x/L_x$, $E_y = U_y/L_y$, then, with the corresponding electric current density $\vec{j} = (j_x, j_y)$, $j_x = I_x/L_x$, $j_y = I_y/L_y$, eq.(2) can be expressed as

$$E_i = \sum_j \rho_{ij} j_j, (i, j = x, y) \tag{8}$$

where

$$\rho_{xx} = \rho_{yy} = R\frac{L_x}{L_y} = \tilde{R}\frac{L_y}{L_x}, \tag{9}$$

$$\rho_{xy} = -\rho_{yx} = R_H. \tag{10}$$

The quantities ρ_{xx} and ρ_{xy}, called the longitudinal and the Hall resistivity, respectively, are macroscopic characteristics of the material. Notice here that (owing to the two-dimensionality of the system) the measured Hall resistance $R_H = -U_y/I_x$ coincides with the Hall resistivity ρ_{xy} whereas in the relation $R = \rho_{xx}L_y/L_x$ the geometrical factor L_y/L_x occurs which experimentally is not known with high precision.

Expressing the current density in terms of the electric field, eq.(2),

$$j_i = \sum_j \sigma_{ij} E_j,$$

the conductivity tensor σ_{ij} defined in this way is the inverse of the resistivity tensor ρ_{ij},

$$\sigma_{ij} = (\rho^{-1})_{ij}, \tag{11}$$

$$\sigma_{xx} = \sigma_{yy} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2}, \tag{12}$$

$$\sigma_{yx} = -\sigma_{xy} = \frac{\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2} \tag{13}$$

or, conversely,

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} \quad , \quad \rho_{xy} = \frac{\sigma_{xy}}{\sigma_{xx}^2 + \sigma - xy^2}. \tag{14}$$

The two related entries, σ_{xx} and σ_{yx} are called the longitudinal and the Hall conductivity, respectively. Notice that (if $\rho_{xy} \neq 0$) the conductivity σ_{xx} and the resistivity ρ_{xx} vanish simultaneously. This is why quantum Hall systems are described as being ideal conductors and ideal insulators at the same time. Thus at first sight a somewhat paradoxial result is due to the fact that, as expressed

by eqs.(8) - (14), in a perpendicularmagnetic field the vectors $\vec{E}$ and $\vec{j}$ are not parallel.

The physical dimension of the conductivity in two dimensions is $(charge)^2/action$. Thus its atomic unit is e^2/h, $-e$ being the electron charge and $h = 2\pi\hbar$ Planck's constant. The task of experimental and theoretical investigations is to determine the conductivities σ_{xx} and σ_{yx} as functions of electron concentration $n = N/A$, $A = L_x L_y$, the magnetic field B and the temperature T.

For constant fields the simple classical friction model characterized by the equation of motion

$$m\dot{\vec{v}} = -\frac{e}{c}\vec{v} \times \vec{B} - e\vec{E} - \frac{m}{\tau}\vec{v} \tag{15}$$

with constant relaxation time τ, and by the average current density formula

$$\vec{j} = -en\vec{v}, \tag{16}$$

permits a steady state ($\dot{\vec{v}} = 0$) such that

$$\begin{aligned} \sigma_{xx} &= \frac{\sigma_0}{1+(\Omega\tau)^2}, \\ \sigma_{yx} &= \frac{\Omega\tau\sigma_0}{1+(\Omega\tau)^2} \doteq \Omega\tau\sigma_{xx} \end{aligned} \tag{17}$$

where $\Omega = eB/mc$ is the cyclotron frequancy and

$$\sigma_0 = \frac{e^2}{m}n\tau \tag{18}$$

the zero-field conductivity. Eq.(17) together with eq.(18) is known as the *Drude-Zener formula.* For $B = 0$ ($\Omega\tau \ll 0$)

$$\sigma_{xx} \simeq \sigma_0 \to \infty \text{ as } \tau \to \infty. \tag{19}$$

Thus friction is needed to get a non-diverging conductivity. On the other hand, in strong magnetic field, i.e. $\Omega\tau \gg 1$,

$$\sigma_{xx} \simeq \frac{\sigma_0}{\Omega^2\tau^2} \sim \frac{1}{\tau} \to 0 \quad \text{for} \quad \tau \to \infty \tag{20}$$

Hence in this limit friction is needed to get a nonvanishing conductivity. In the friction model the longitudinal resistivity is independent of the magnetic field,

$$\rho_{xx} = \rho_0 = \frac{1}{\sigma_0} \tag{21}$$

and the Hall resistivity is independent of the friction,

$$\rho_{xy} = \frac{1}{\sigma_0} \ , \ \sigma_0 = \frac{enc}{B}. \tag{22}$$

Both, the conductivity σ_0 and the classical Hall conductivity of free and independent electrons σ_H^0 are strictly proportional to the electron concentration. In terms of atomic units,

$$\sigma_H^0 = \nu\frac{e^2}{\hbar} \tag{23}$$

Owing to eq.(22),

$$n = \nu\frac{eB}{hc} \tag{24}$$

indicating that for any fixed value of the magnetic field, eB/hc is the appropriate unit for the electron concentration. In deed, $eB/hc = 1/2\pi l_B^2$, where $l_B^2 = \sqrt{\hbar/m\Omega}$ is the radius of the classical cyclotron orbit corresponding to the energy $E_0 = \hbar\Omega_c/2$ (oscillator ground state), the so-called *magnetic length.* For reasons to become obvious later, the dimensionless measure of the electron concentration

$$\nu = 2\pi l_B^2 n \tag{25}$$

is called the *filling factor* and can be expressed in terms of the magnetic flux through the plate, $\phi = \int \vec{B} \cdot d\vec{r}$, and the elementary quantum of flux, $\phi_0 = hc/e$,

$$\nu = \frac{N}{\phi/\phi_0} = \frac{\text{No. of electrons}}{\text{No. of flux quanta}}. \tag{26}$$

This equation is a clear indication of the microscopic significant of ν.

As revealed by the experiments of von Klitzing[9] the behavior of a real two dimensional system is dramatically different from that of ideal two dimensional electron gas. Indeed, the Hall resistivity of silicon MOSFET as a function of the gate voltage (which is proportional to the electron concentration) was found to be constant,

$$\rho_{xy} = \frac{1}{i}\frac{h}{e^2}, i = 1, 2, ... \tag{27}$$

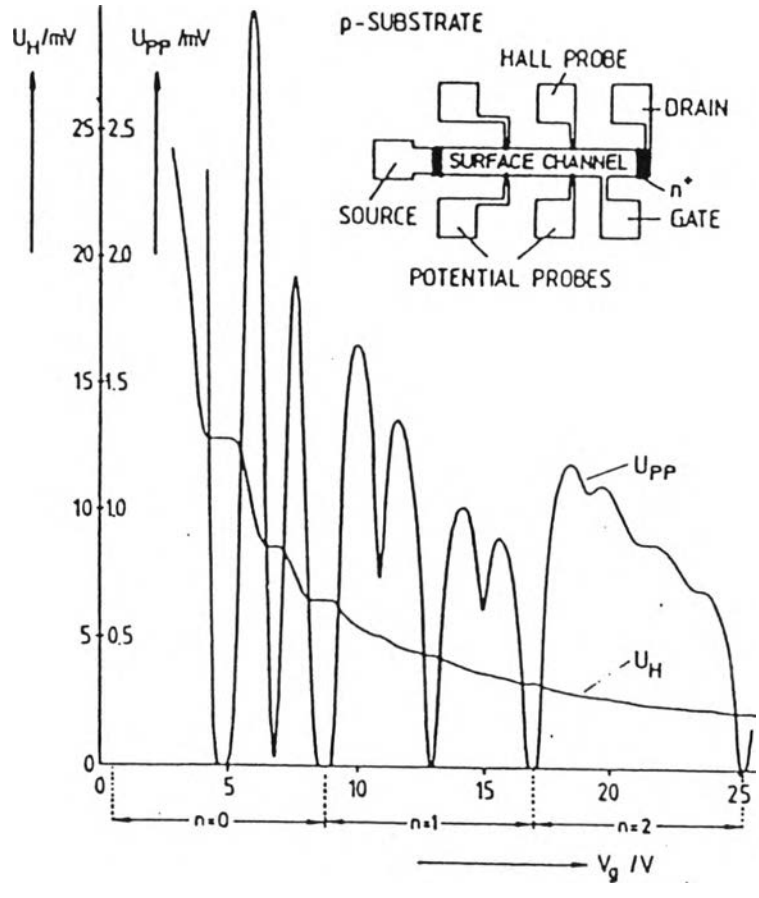
within a certain range around each integer value of the filling factor ν. Simultaneously, the longitudinal resistivity was found to be vanishing ($V_x \simeq 10^{-14}$V in these ranges, so that by eq.(14),

$$\sigma_{yx} = i\frac{e^2}{h}, i = 1, 2, ... \tag{28}$$

$$\sigma_{xx} = 0. \tag{29}$$

Later this surprising quantum Hall effects has also been observed in GaAs heterostructure when changing the magnetic field and keeping the electron concentration fixed[10]. Owing to eq.(25), these two procedures are equivalent. Thus the quantum Hall effect is characterized by a plateau behavior in σ_{yx} and vanishing dissipation in some interval of ν around $\nu = i = 1, 2, ...$ Figure(1.2) shows

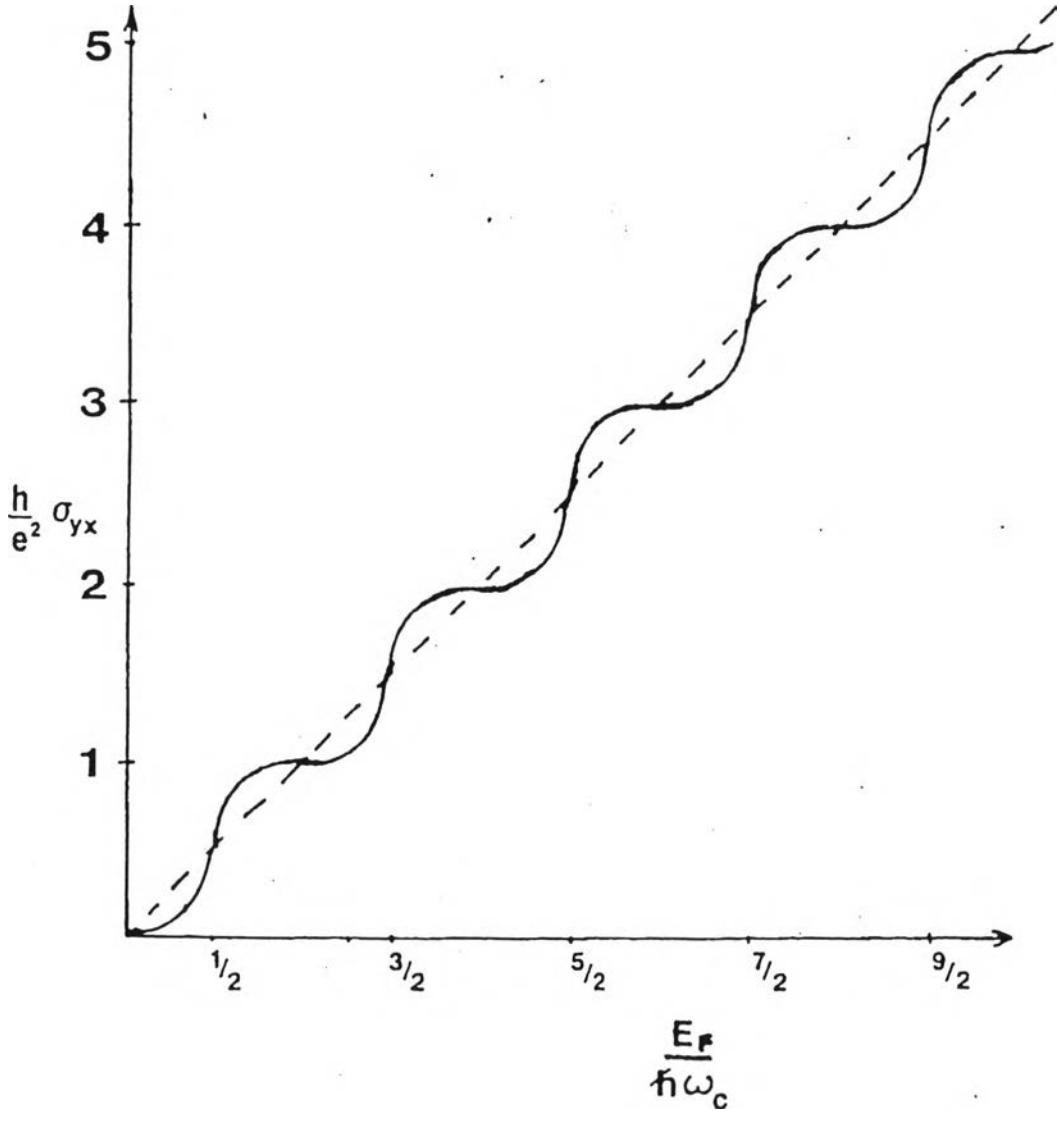
experimental result of von Klitzing[9]. Nowaday, the experimental accuracy of the plateau values is about 10^{-8}. With increasing temperatures the width of the plateaus shrinks gradually to zero. With regard to the quantized plateau values the integer quantum Hall effect is entirely universal.



Figure(1.2) Observed quantum Hall effect by K. von Klitzing and and co-worker[9] in MOS system.

For high mobility samples, quantized plateaus also occur at some fractional effect first observed by Tsui, et.al[10], or more precisely the proliferation of integer plateaus, is generally believed to be due to the coulomb interaction between the electrons. Since this phenomenon is more complex and less understood in details than the integer effect.

Figure(1.3) shows a strongly simplified, schematic view of the integer quantum Hall effect. It is generally believed that Fig.(1.3) displays the correct extrapolation of the experimental data to zero temperature: σ_{yx} plotted against ν is an exact step function with jumps at $\nu = (i + 1/2)$ and step values $\sigma_{yx} = ie^2/h$, and σ_{xx} vanishes everywhere except for $\nu = (i + 1/2)$.



Figure(1.3) Schematic graph of Hall conductance versus Fermi energy. Plateaus have regions of easily measurable width which are completely flat to within 1 ppm.

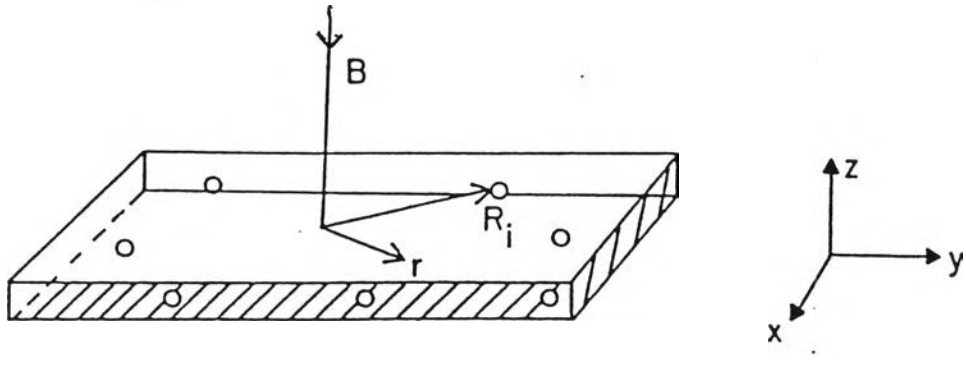
The Modeled System

The two-dimensional system we used here is modeled to be on the x-y plane and described by the coordinates $\vec{r} = (x, y)$ The magnetic field is applied in $-\hat{z}$ direction in order that the minus sign of $\hat{z}$-direction will be canceled with the minus electron charge, see fig.(1.4). Also we apply the DC-electric field $\mathcal{E}$ in the direction parallel to the plane. Furthermore, we add imperfections into the system which will create the electron scattering potential energy $V(\vec{r})$. The imperfections model we used will be discussed later. The Hamiltonian of this system can be written in the form

$$\mathcal{H} = \frac{1}{2m}\left(\vec{p} + \frac{e}{c}\vec{A}\right) + V(\vec{r}) \tag{30}$$

where m is an effective mass of an electron and $\vec{A}$ is the vector potential of the magnetic field, $\vec{B} = \nabla \times \vec{A}$. This kind of system is believed to be the one for the quantum Hall system. The imperfections will cause the appearance of the longitudinal component of electron conductivity at the plateau-plateau transitions of the transverse component as appear in experimental results.

In next chapter, we will discuss the formulation of the Feynman path integrals. The formulation will be for the transition amplitude, the propagator, and then for the density matrix. We will consider these two quantities for the case in our modeled system.



Figure(1.4) Schematic diagram of a two-dimensional system for our model. The dots denote impurities.