CHAPTER 1

Introduction and Scope of the Thesis

Recently there has been substantial interest in the study of the behavior of the extended states of noninteracting electrons in a two-dimensional system under a strong magnetic field with saddle point potential. This problem is important for the understanding of the integer Quantum Hall effect [1], in particular, the understanding of the Landau Level-mixing and levitation of extended states or the floating of extended states. There are several approaches to this problem [1]-[6]. The quadratic saddle point potential $V_{sp}(x, y) = U_y y^2 - U_x x^2$, where U_y and U_x are frequencies of the saddle point potential, has been used by Ferting and Halperin [2] for calculating the transmission coefficient through the saddle point potential in a two-dimensional system with a strong magnetic field. By using the Bogoliubov transformation, they were able to decouple the Hamiltonian into a sum of two commuting Hamiltonians, one being equivalent to that of a one-dimensional particle in an inverted harmonic potential and the other representing a one-dimensional particle confined by a harmonic potential. This potential was also used by Haldane and Yang [3] for studying the effect of mixing of different Landau Levels (LL) for a two-dimensional system with a strong magnetic field and random potential. Using the perturbative approach together with a systematic expansion in both the strength and smoothness of the random potential, they found that the mixing gives rise to the level repulsion effect which tends to lower the energy level but shift the extended states upward with an amount proportional to $(n + \frac{1}{2})/B^3$ for strong magnetic field B where n is the LL index.

In this thesis, we show that by using the Feynman Path Integral method developed by [7], [8] for handling the Quantum Hall problem, one can obtain the free energy of the system. We calculate the classical action associated with this V_{sp} and obtain the exact propagator. Then the free energy and the partition function of system for any value of B of the system can be obtained.

The starting point of our discussion is the following Lagrangian,

$$L = \frac{m}{2}(\dot{x}^2 + \dot{y}^2) + \frac{m}{2}\omega(x\dot{y} - y\dot{x}) - \frac{m}{2}(\Omega_x^2 x^2 - \Omega_y^2 y^2). \qquad (1.1)$$

Note that we have defined $U_x = -(m\ \Omega_x^2)/2$ and $U_y = -(m\ \Omega_y^2)/2$. Here $\omega = eB/mc$ is the cyclotron frequency, Ω_x and Ω_y are parameters representing the saddle point potential. Since the system is quadratic, the path integral can be carried out exactly. The propagator consists of the classical action S_{cl} and the prefactor $F(\tau)$ which can be calculated using the Van Vleck-Pauli formula. To find the classical action, we need to find the classical path which can be achieved by making a variation of Eq. (1.1). Then the classical solution consists of two coupled equations. To solve these two simultaneous equations, we use the method developed by Sa-yakanit, Choosiri and Robkob [9]. We obtain the classical solutions. Starting from the exact propagator, it is possible to obtain the density matrices by taking the trace of the propagator. The density matrices can be obtained and we can find free energy of this system.

The content of this thesis can be abbreviated as follows: In Chapter 2, we review the statistical mechanics. Next, the Feynman path integral is explained in Chapter 3. The propagator of the saddle point potential is introduced and is solved exactly in Chapter 4. In Chapter 5, we introduce the magnetic field to the saddle point potential. The magnetic field leads to a complicated simultaneous differential equation involving a matrix manipulation. In Chapters 4 and 5, we also give the numerical results of the free energy for different cases. The last Chapter, Chapter 6, is devoted to conclusion.