

ยุงชาวท่าควยขุนทรายเสริมไมไผ่



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A STUDY OF BAMBOO AS REINFORCEMENT FOR RICE BINS

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Thesis Title A Study of Bamboo as Reinforcement for Rice Bins

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Thesis Title A Study of Bamboo as Reinforcement for Rice  
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#### ABSTRACT

The purpose of this investigation is to study the suitability of bamboo as reinforcement for conical rice bins, and to compare the theoretical analysed structure of conical rice bins as composite and isotropic materials to the field test measurement of deflections and strains of a conical rice bin. The mechanical properties of bamboo and mortar were determined by tests, the result of which will be used in the design.

Bamboo, one of the most common materials available in many parts of Thailand, may be proved to be a good substitute for steel because of its high tensile strength and is also the low-cost construction material. In the investigation a variety of bamboo scientifically called *Thrsostachys Oliveri* Gamble known locally as Pai Ruak in Thailand was used. The test results also showed that the average ultimated tensile strength, the average modulus of elasticity and the average bond stress between bamboo and

mortar are 1937 kg/sq.cm,  $2.64 \times 10^5$  kg/sq.cm and 8.35 kg/sq.cm respectively.

The mortar used for all the test samples had cement-sand ratio of 1 : 2 by weight with a water cement ratio of 0.45. The average ultimate compressive strength and the average modulus of elasticity of mortar were determined by cylindrical control specimens (15 cm. dia. x 30 cm.) as 408 kg/sq.cm and  $3.0 \times 10^5$  kg/sq.cm respectively and from cube control specimens (5.0 cm x 5.0 cm x 5.0 cm) as 325 kg/sq.cm and  $3.25 \times 10^5$  kg/sq.cm respectively.

A prototype rice bin was constructed at the site at Chulalongkorn University. Measurements were carried out, using dial gages for deflections and electrical resistance strain gauges for strains at various positions shown in Figs. (22), (23). Test results showed that the bin occurred higher deflections than the deflections calculated from theoretical analysis about 50 percent. The bin did not crack but sparse permeation in the bin occurred. This is because there is a loss of moisture content of mortar during construction, and because the mortar had not been mixed with sealing compound and also because the water pressure was 2.25 m. height which was too high. The compressive strength of mortar occurred lower than the compressive strength calculated from theoretical analysis about 19 percent but the tensile strength of bamboocement was equal to those calculated.

หัวข้อวิทยานิพนธ์      ยุงชวาทำคยปูนทรายเสริมไม้ไผ่  
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บทคัดย่อ



การศึกษานี้มีจุดประสงค์ เพื่อศึกษาดังความเหมาะสมของการเอาไม้ไผ่ มาทำเป็นโครงรูปกรวย แล้วฉาบคยปูนทราย โดยการวิเคราะห์โครงสร้างของ ยุงชวารูปกรวยเป็นวัสดุผสม (composite materials) และวัสดุเหมือนทุก ทิศทาง (isotropic materials) แล้วเปรียบเทียบระยะโก่งตามแนวราบ และความเค้นกับการทดลองในสนาม พร้อมทั้งหาคุณสมบัติทางกลของไม้ไผ่ และ ปูนทราย เพื่อใช้ในการออกแบบ

ไม้ไผ่เป็นวัสดุที่มีมากมายในประเทศ ราคาถูก และหาได้ง่ายในทุกภาค ของประเทศไทย อาจจะมีผู้นึกได้ว่าใช้แทนเหล็กก็ได้ เพราะมีแรงดึงสูง เนื่อง จากไม้ไผ่มีอยู่มากมายหลายชนิด มีชนิดหนึ่ง ที่ทางวิทยาศาสตร์เรียกว่า ไทรโซส แทคซ์ ออลลิเวอร์ แกมเบิล (Thyrsostachys Oliveri Gamble) ในประเทศ ไทยที่เราเรียกกันตามท้องถิ่นว่าไม้ไผ่ลวก ซึ่งเป็นชนิดที่ใช้ในการวิจัยครั้งนี้ ผลการ ทดลองแสดงควยว่า ค่าเฉลี่ยของแรงดึงประลัย โมดูลัสแห่งความยืดหยุ่น และค่าเฉลี่ย ของแรงยึดหน่วงระหว่างไม้ไผ่กับปูนทราย เป็น 1937 ก.ก. ต่อตาราง ซม.,  $2.64 \times 10^5$  ก.ก. ต่อตาราง ซม. และ 3.35 ก.ก. ต่อ ตาราง ซม. ตาม ลำดับ

ปูนทรายซึ่งใช้ในการทดลองบ่อตราส่วนของ ปูนซีเมนต์ก่อทราย เท่ากับ 1 : 2 โดยมีน้ำหนัก กับ อัตราส่วนของน้ำกับปูนซีเมนต์เท่ากับ 0.45 ผลการทดลอง

โศคาเฉลี่ยแรงดึงประลัยและโมเมนต์แห่งความยืดหยุ่นของปูนทราย ตัวอย่างรูปทรง  
 กระบอกมาตรฐานที่มีเส้นผ่าศูนย์กลาง 15 ซม. สูง 30 ซม. มีค่าเป็น 408  
 ก.ก. ต่อ(ซ.ม)<sup>2</sup> และ  $3.0 \times 10^5$  ก.ก. ต่อ(ซ.ม)<sup>2</sup> ตามลำดับ และหาจาก  
 ตัวอย่างรูปทรงลูกบาศก์ มาตรฐาน ขนาด 5 ซม. x 5 ซม. x 5 ซม.  
 มีค่าเป็น 325 ก.ก. ต่อ (ซ.ม)<sup>2</sup> และ  $3.25 \times 10^5$  ก.ก. ต่อ (ซ.ม)<sup>2</sup>  
 ตามลำดับ

ยุงขาวขนาดเตาของจริง ซึ่งใช้ในการทดลอง ได้สร้างในบริเวณจุฬาลงกรณ์มหาวิทยาลัย โดยใช้ Vial Gages สำหรับวัดระยะโคงตามแนวราบ และ Electrical Resistance Strain Gauges สำหรับวัดความเค้นที่ระยะต่าง ๆ ตามที่แสดงในรูป (22) และ (23) ผลการทดลองแสดงว่ายุงขาวมีระยะโคงตามแนวราบที่วัดได้สูงกว่าค่าประมาณการจากการวิเคราะห์ 50 เปอร์เซ็นต์ แต่ไม่มรอยแตก มีแครอยซึมประปรายตามผนังของยุงขาว เนื่องจากปูนทรายไม่เค้นสมน้ำยากันซึมออกทั้งแรงคั้นของน้ำที่สูง 2.25 น. มีค่าสูงมาก แรงอัดของปูนทรายที่หาได้จากการทดลองมีค่าต่ำกว่าค่าที่ได้จากการวิเคราะห์ 19 เปอร์เซ็นต์ ส่วนแรงดึงของใบไม้ผสมปูนทราย ที่หาได้จากการทดลองมีค่าใกล้เคียงกับค่าที่ได้จากการวิเคราะห์

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## LIST OF DEFINITIONS AND SYMBOLS

- $A$  = gross sectional area of ring girder;
- $A_b$  = bamboo area in ring girder;
- $A_r$  = effective sectional area of ring girder;
- $A_{sk}$  = skeletal bamboo area per unit area of shell section;
- $a_1$  = radius of ring support of circular bin lid;
- $a_2$  = outer radius of circular bin lid;
- $E[x, f_1(z), f_2(z)]$  = combination of Bessel functions of 2<sup>nd</sup> order  
 =  $ber_2 \cdot x \cdot f_1(z) + bei_2 \cdot x \cdot f_2(z)$ ;
- $b$  = subscript denoting functions belonging to bottom cone;
- $c$  = subscript denoting part of solution according to bending analysis;
- $D$  = flexural rigidity of shell  
 =  $Eh^3/12(1 - \nu^2)$  ;
- $d$  = diameter of skeletal bamboo;
- $E_b$  = modulus of elasticity of bamboo;

- $E_c$  = modulus of elasticity of uncracked bamboo cement;
- $E_f$  = modulus of elasticity of fibre bamboo;
- $E_m$  = modulus of elasticity of mortar;
- $E_r$  = modulus of elasticity of ring girder;
- $E_t$  = modulus of elasticity of cracked bamboo cement;
- $F$  = horizontal radial force per unit circumferential distance acting on ring girder;
- $F_a$  = axial forces in ring girder;
- $F_b, F_t$  = subscripts denoting functions belonging to bottom and top ring girders respectively;
- $F_y$  = yield strength of ring girder;
- $H$  = horizontal radial stress resultant;
- $H_{bo}$  = horizontal radial force applied uniformly along outer edge of bottom cone in the bending analysis;
- $H_{ti}, H_{to}$  = horizontal radial forces applied uniformly along inner and outer edges of top cone respectively, in the bending analysis;

$h_b, h_t$  = thickness of bottom and top cones  
respectively;

$i$  = subscript denoting functions at inner  
edge of conical shell;

$K[x.f_1(z), f_2(z)]$  = combination of Kelvin functions of 2<sup>nd</sup>  
order

$$= \text{Ker}_2 \cdot x.f_1(z) + \text{Kei}_2 \cdot x.f_2(z) ;$$

$K$  = Rankine's coefficient for cohesiveless  
granular materials on inclined wall;

$$k = \sqrt{12(1-\nu^2)}$$

$L$  = layer numbers of fibre bamboo in bamboo  
cement section ;

$M_o$  = interacting edge couple of the cone union  
per unit circumferential distance;

$M_y, M_\theta$  = bending stress resultants at sections  
across  $y$  and  $\theta$  directions respectively;

$m$  = ratio of thickness to inclined distance  
of outer edge of the cone;

$N_y, N_\theta$  = normal stress resultants in  $y$  and  $\theta$   
directions respectively;

- $n$  = modular ratio of bamboo to uncracked bamboo cement  
 $= E_b/E_c$  ;
- $O$  = subscript denoting functions at outer edge of conical shell;
- $P$  = vertical live load acting uniformly along top edge of bin ;
- $P_b, P_t$  = dead loads per unit distance on bottom and top edge of bin ;
- $P_r$  = intensity of contact pressure of foundation exerted on bin structure;
- $Q_r$  = plate radial shear stress resultant;
- $Q_y$  = shear stress resultant of section across y direction;
- $q$  = intensity of uniformly distributed load on the bin lid ;
- $q_b, q_t$  = dead loads per unit surface area of bottom and top cones respectively;
- $R_1, R_2$  = radii of top and bottom ring girders respectively;

$r_1, r_2$  = functions defined by Eqs. (34) and (35) respectively;

$S$  = Spacing of skeletal bamboo ;

$T_s$  = resisting force in skeletal bamboo ;

$t$  = subscript denoting elements belonging to top cone ;

$V_m, V_f$  = volume fractions of mortar and fibre bamboo in bamboo-cement respectively;

$w(r)$  = plate deflection function ;

$Y, Z$  = intensity of load components in  $y$  and  $z$  directions respectively ;

$y, \bar{y}$  = co-ordinate distance from apexes of top and bottom cones respectively ;

$y_0$  = distance of outer edge of bottom cone from its apex ;

$y_1, y_2$  = distance of inner and outer edges of top cone from its apex ;

$Z, \bar{Z}$  = arguments of kelvin functions corresponding to  $y$  and  $\bar{y}$  co-ordinates

$$= 2 \sqrt{\frac{k \cdot y \cdot \cot \alpha_t}{h_t}} \quad \text{and} \quad 2 \sqrt{\frac{k \cdot \bar{y} \cdot \cot \alpha_b}{h_b}} \quad \text{respectively}$$

- $z_0, z_1, z_2$  = arguments of kelvin functions as defined above corresponding to  $y_0, y_1$  and  $y_2$  respectively ;
- $\alpha$  = cone semi - vertex angle ;
- $\alpha_c$  = ratio of neutral axis distance from compressive extreme fiber to thickness of shell section,  
 =  $h_c/h$  ;
- $\alpha_d$  = ratio of skeletal bamboo diameter to thickness of shell section,  
 =  $d/h$  ;
- $\gamma_r$  = bulk density of rice;
- $\gamma_w$  = density of water;
- $\Delta_b(z), \Delta_k(z)$  = combinations of Bessel and Kelvin functions of 2<sup>nd</sup> order  
 =  $\text{ber}_2 z \phi_{br}(z) + \text{bei}_2 z \phi_{bi}(z)$  and  
 =  $\text{ker}_2 z \phi_{kr}(z) + \text{kei}_2 z \phi_{ki}(z)$  respectively ;
- $\delta$  = horizontal radial deflection ;
- $\epsilon_b$  = axial strain in skeletal bamboo

- $\mu$  = tangent of angle of friction between grain and bin wall ,  
 =  $\tan \phi$
- $\theta, \bar{\theta}$  = circumferential co-ordinate variable of top and bottom cones respectively ;
- $\theta_{bi}(Z), \theta_{br}(Z)$  = combinations of Bessel functions of 2<sup>nd</sup> order  
 =  $Z\text{bei}_2' Z - 2\sqrt{Z}\text{bei}_2 Z, Z\text{ber}_2' Z - 2\sqrt{Z}\text{ber}_2 Z$  respectively;
- $\theta_{ki}(Z), \theta_{kr}(Z)$  = combinations of Kelvin functions of 2<sup>nd</sup> order,  
 =  $Z\text{kei}_2' Z - \sqrt{Z}\text{kei}_2 Z, Z\text{ker}_2' Z - 2\sqrt{Z}\text{ker}_2 Z$  respectively;
- $\lambda$  = ratio of modulus of elasticity of uncracked to that of cracked bamboo-cement,  
 =  $E_c/E_t$  ;
- $\lambda_b$  = ratio of axial stiffness of bottom ring girder to bending stiffness of top conical shell,  
 =  $A_{rb} E_r/E_c h_t^2$
- $\lambda_t$  = ratio of axial stiffness of top ring girder to bending stiffness of top conical shell,  
 =  $A_{rt} E_r/E_c h_t^2$

- $\nu$  = Poisson's ratio ;
- $\xi_b, \xi_t$  = arguments =  $\bar{y}/y_0, y/y_2$  respectively ;
- $f_{bi}(Z), f_{br}(Z)$  = Combinations of Bessel functions of 2<sup>nd</sup> order,  
 =  $\sqrt{Z} \text{bei}_2 Z + \text{bei}_2 Z, \sqrt{Z} \text{ber}_2 Z + 2 \text{ber}_2 Z$  respectively ;
- $f_{ki}(Z), f_{kr}(Z)$  = Combinations of Kelvin functions of 2<sup>nd</sup> order,  
 =  $\sqrt{Z} \text{kei}_2 Z + 2 \text{kei}_2 Z + \frac{1}{2} \text{zker}_2 Z + 2 \text{ker}_2 Z$  respectively ;
- $T_t$  = parameter =  $y_1/y_2$
- $\Phi_b[x, f_1(Z), f_2(Z)]$  = Combinations of Bessel functions of 2<sup>nd</sup> order,  
 =  $\psi_{br}(x) \cdot f_1(Z) + \psi_{bi}(x) \cdot f_2(Z)$  ;
- $\Phi_k[x, f_1(Z), f_2(Z)]$  = Combinations of Kelvin functions of 2<sup>nd</sup> order,  
 =  $\psi_{kr}(x) \cdot f_1(Z) + \psi_{ki}(x) \cdot f_2(Z)$  ;
- $\psi_{bi}(Z), \psi_{br}(Z)$  = Combinations of Bessel functions of 2<sup>nd</sup> order ,  
 =  $z \text{bei}_2 Z + 2 \sqrt{Z} \text{bsi}_2 Z, z \text{ber}_2 Z + 2 \sqrt{Z} \text{ber}_2 Z$  respectively ;



- $\phi_{ki}(Z), \phi_{kr}(Z)$  = Combinations of Kelvin functions of  
 $2^{\text{nd}}$  order,  
 =  $Zkei_2 Z + 2i'kei_2 Z, Zker_2 Z + 2i'ker_2 Z$  respectively;
- $\phi$  = angle of repose of grain ;
- $\phi'$  = angle of friction between grain and bin  
 wall ;
- $\psi$  = rotation of tangent to the meridian in  
 generator ; and
- $\omega$  = vertical ring load on ring girder.