CHAPTER 1

INTRODUCTION

It frequently occurs that when a character of a physical phenomena is expressed in mathematical form, it is identical to another character of a different physical phenomena. If it is so, both physical systems are said to be analogous.

In the case where two systems are analogous, if the problem of one system is solved, its solution is automatically the solution of the other too.

The equations governing a physical system are often difficult to solve analytically and it is at time equally difficult to solve numerically. Direct physical measurement on the system, or on its scale model, may not be feasible. However it is frequently observed that measurement on the analogous system is practicable. Thus the analogous system may serve as a calculating machine for solving the equation governing the original system.

There are five main steps to be considered in the analogical method:-

1. The equations governing the original physical system are determined.

2. The equations governing the analogous physical system are determined.

3. Consider if any functions of both systems have identical mathematical expressions.

4. The reliability of the analogous system is verified.

5. The functions of the analogous system are measured and transformed to the functions of the original system.

In each of these steps, factors may limit the range of applicability or the accuracy of the results.

A very useful analogue method in solving the boundary value problem of many physical systems is "The Electrical Analogue." The analogy is made between the electrical potential distributed in a thin conducting medium and a character of the analogous system which satisfy either Laplace's or Poisson's equations, such as, the stream function of two-dimensional flow problem of vicous fluid, the conjugate function and the shearing stress function of torsion problem, the sum of direct stresses of problem of plate subjected to uniaxial tension. The conducting medium may be solid, liquid or network analyser, etc. For example the use of conducting paper in conducting paper analogue by R.S.Ross and I.H.Qureshi[1], the use of steel plate in Jacobsen's analogy[2], the use of electrolytic tank in Thum and Bautz's analogy[3], the use of network analyser in network

---

[1] Ross,D.S.and Qureshi,I.H."Boundary Value Problem in Two Dimensions Elasticity by Conducting Paper Analogue," Journal of Scientific Instruments.Vol.40(1963):P.513-517.

[2,3] Hėtenyi,M. Handbook of Experimental Stress Analysis. P.747-751.

analogue by S.C.Redshaw[1] and also J.H.Weiner[2]. The first three cases which are continuous conducting medium, provide a direct analogy to the differential equation, whereas, the network analyser can only provide an analogy to the required equation when expressed in a finite difference form.

In the investigation concerned with the torsion problem of the cylindrical shaft twisted by the twisting moment, the direction of which is along the axis of the shaft, it is not yet possible to measure the shear stress inside the shaft directly. Several functions are introduced to solve the torsion problem analytically but the solutions are very difficult to obtain since they must satisfy either Laplace's equation or Poisson's equation. For example the conjugate function ψ must satisfy Laplace's equation,

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0$$

and the boundary condition

$$\psi = \frac{1}{2}(x^2+y^2) + c$$

[1] Redshaw, S.C. "Use of an Electrical Analogue for the Solution of a Variety of Torsion Problem." British Journal of Applied Physics. Vol.11(1960):P.461-468.

[2] Weiner, J.H. "An Resistor-Network Solution for the Torsion of Hollow Section." Journal of Applied Mechanics. Vol.30(1953):P.562-564.

It is very difficult to determine the value of ψ analytically. The analytical solutions of only a few sections, with simple boundaries are established, e.g. circular, elliptic, equitriangular, etc. Several methods are introduced to express the approximate solutions but they can not be used in general cases. Numerical method is considered as the most commonly employed method of approximation, but it is cumbersome and sometime inaccurate, for example:-

If the finite difference method is used, a set of equations stated the value of functions at the interior points will be obtained. In order to secure sufficient accuracy of the result, the area of cross section has to be divided fine enough which entails necessarily large number of equations to be solved.

If the iteration procedure of finite difference method or the relaxation method is used, the convergence is very slow. Besides the disadventage described above, it is very easy to make mistake in such a long procedure.

If the potential(V) distributed in a conducting medium of constant thickness is considered, the system is governed by the equation:-

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

This equation is similar to the equation of conjugate function, so ψ and V are said to be analogous. When the value of V on the conducting sheet is measured, it is automatically the value of ψ of shaft which cross section is similar to the shape of conducting sheet. So it is clear that unless a high speed computer is available, the use of electrical analogue method is more convenient.

The purpose of this thesis is to investigate the electrical analogue of a torsion problem by using the conducting paper as a conducting medium. This is called -"The conducting paper analogue." Equipotential lines are determined experimentally, then the shear stress lines, maximum shear stress and the point occurred are determined from equipotential lines respectively. From the shear stress lines, the torsional stiffness of the cross-section can be approximated.

The equipments and method of investigation in this experiment are prepared to investigate shafts which cross-sections are simply connected region (no hole inside it) such as rectangular, ellipse, triangle, rolled profile sections, etc. It can not be used for shafts which cross sections are multiply connected region such as hollow shafts or composite shafts, since there are more conditions necessary to be satisfied.

Three shapes of cross - sections are investigated in this thesis, namely, the square, the rectangular and the I cross-sections.

A square conducting sheet which represents a square cross section shaft in pure tension is selected to verify the reliability of the equipment of normal case. The potential distribution, shearing stress function, stress components and torsional stiffness are determined, they are then compared with the analytical value. The reliability is verified again by rectangular cross section.

In case of rolled profile sections, the boundary is also simply connected but it is not normal since the boundary curves inward forming a big groove. I - cross - section is selected to represent the rolled profile section to investigate how the groove effect the result.

The conducting paper analogue comparing with other method is useful to obtain the result rapidly. The method can be extended to determine the characters of other physical systems which are governed by either Laplace's or Poisson's equations.