SOLVING ELLIPTIC AND PARABOLIC PARTIAL DIFFERENTIAL EQUATIONS USING EXCEL

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ABSTRACT

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Linear partial differential equations of the second order are frequently classified as of the elliptic, parabolic or hyperbolic type. Elliptic and parabolic equations are used for modeling engineering problems such as heat transfer, boundary-layer flow, diffusion, etc. The purposes of this project are to transform partial differential equations into finite difference form and to solve the finite difference equations by numerical methods using computer programs written in Microsoft Visual Basic. Then the results are divided into two parts. The first part is the numerical results displayed in Microsoft Excel tables and the second is the graphical results displayed by the MATLAB software.

บทคัดย่อ

มาลินี สินทวีเลิศ : การแก้ปัญหาสมการเชิงอนุพันธ์ย่อยอิลิปติกและพาราโบลิกโดยใช้ไมโครซอฟเอ็กเซล (Solving Elliptic and Parabolic Partial Differential Equations Problems Using Excel) อ. ที่ปรึกษา : ศ. ไบรส์ คานาฮาน ดร. กิตติพัฒน์ สีมานนท์ 123 หน้า ISBN 974-13-0709-8

สมการเชิงอนุพันธ์ย่อยเชิงเส้นแบ่งออกได้เป็น 3 ประเภท คือ สมการอิลิปติก สมการพาราโบลิก และสมการไฮเพอร์โบลิก สมการอิลิปติกและสมการพาราโบลิกถูกใช้ในการแก้ปัญหาทางด้านวิศวกรรมเคมี ตัวอย่างเช่น การถ่ายเทความร้อน, การไหลของชั้นของไหล, การแพร่ ฯลฯ ขั้นตอนของโปรแกรมเริ่มจาก เปลี่ยนรูปสมการเชิงอนุพันธ์ย่อยให้อยู่ในรูปของวิธีผลต่างสืบเนื่องแล้วใช้ระเบียบวิธีเชิงตัวเลขเพื่อแก้ปัญหาสมการเชิงอนุพันธ์เชิงย่อยนั้น หลังจากนั้นนำระเบียบวิธีเชิงตัวเลขนั้นมาเขียนเป็นโปรแกรมคอมพิวเตอร์โดยใช้ภาษาวิชวลเบสิก ผลที่ได้จากการคำนวณจะถูกนำมาแสดงแบบตารางโดยใช้ไมโครซอฟเอกเซลและแบบกราฟโดยใช้เมทแล็ป

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LIST OF SYMBOLS

Symbol	Definition
ρ	density
k	thermal conductivity
C_p	specific heat of the plate
ζ	vorticity
ν	kinematic viscosity
ε	relative tolerance