

## CHAPTER V

### Discussion and conclusion

We have investigated the thermodynamic and the electrodynamic properties of high- $T_c$  superconductors in theoretical work. We have shown how to obtain Ginzburg  $\psi$ -theory from the framework of the conventional Ginzburg-Landau theory and the scaling theory. We have obtained the free energy density function for the high- $T_c$  superconductors

$$F_s = F_{no} + A |\psi|^2 + 1/2 B |\psi|^4 + 1/3 C |\psi|^6 + \frac{1}{2m^*} \left( (-i\hbar \nabla - \frac{e}{c} \vec{A}) \psi \right)^2 + \frac{\hbar^2}{8\pi} \quad (4.8)$$

where

$$A = -A_0 \left( \frac{T_c - T}{T_c} \right)^{4/3} \quad (4.7a)$$

$$B = B_0 \left( \frac{T_c - T}{T_c} \right)^{2/3} \quad (4.7b)$$

$$C = C_0 \quad (4.7c)$$

From the Ginzburg-Landau equations which we have obtained in Eqs. (4.10) and (4.11).

$$A + B |\psi|^2 \psi + C |\psi|^4 \psi + \frac{1}{2m^*} \left( (-i\hbar \nabla - \frac{e}{c} \vec{A})^2 \psi \right) = 0 \quad (4.10)$$

$$\vec{J} = \frac{c}{4\pi} \nabla \times \vec{h} = -\frac{i e \hbar}{2m^*} (\Psi^* \nabla \Psi - \Psi \nabla \Psi^*) - \frac{(e^*)^2}{m^* c} |\Psi|^2 \vec{A} \quad (4.11)$$

We have obtained the expressions for the penetration depth  $\lambda$ , the coherence length  $\xi$ , the upper critical field  $H_{c2}$ , the thermodynamic critical field  $H_c$ , the discontinuity specific heat  $\Delta c$ , the lower critical field  $H_{c1}$ , and the surface critical field  $H_{c3}$  as follows

$$\lambda(T) = \lambda(0) \left( \frac{1-T}{T_c} \right)^{-1/2} \quad (4.13)$$

$$\xi(T) = \lambda(0) \left( \frac{1-T}{T_c} \right)^{-2/3} \quad (4.27)$$

$$H_{c2}(T) = \frac{2 m^* c A_0}{c} \left( \frac{1-T}{T_c} \right)^{2/3} \quad (4.28)$$

$$= \frac{\phi_0}{2\pi \xi(T)} \quad (4.26)$$

$$H_c = \left( \frac{4\pi A_0}{B_0} \right)^{1/2} \left( \frac{1-T}{T_c} \right) \quad (4.32)$$

$$\Delta c = \frac{A_0^2}{B_0 T_c} \quad (4.33)$$

$$H_{c1} = \frac{\phi_0}{4\pi \lambda^2(0) T_c} \left( \frac{1-T}{T_c} \right)^{2/3} \ln \left[ K(0) \left( \frac{1-T}{T_c} \right)^{1/3} \right] \quad (4.53)$$

$$H_{c3} = 1.66 H_{c2} \quad (4.67)$$

Where

$$\lambda(\infty) = \left( \frac{m^* c^2 B_0}{4\pi (e^*)^2 A_0} \right)^{1/2} \quad (5.1)$$

$$\xi(0) = \left( \frac{\hbar^2}{2 m^* A_0} \right)^{1/2} \quad (5.2)$$

Substituting  $A_0$  for a pure superconductor Eq. (3.98) into Eq. (5.1) gives,

$$\xi^2(0) = \frac{\hbar^2 \cdot 7 \zeta(3) \epsilon_F^0}{2 m^* 6\pi^2 (k_B T_c)^2} = \frac{\hbar^2 \cdot 7 \zeta(3)}{2 m^* 6\pi^2 (k_B T_c)^2} \cdot \frac{1}{2} m^* v_F^2$$

$$\text{or } \xi(0) \sim \frac{\hbar v_F}{k_B T_c}$$

Thus  $\xi(0)$  becomes to Pippard coherence length in Eq. (2.26).

In order to compute the coherence length  $\xi$  and the penetration depth  $\lambda$  from Eqs. (4.27) and (4.13), the coefficients  $A_0$  and  $B_0$  must be obtained first. If we assume that high- $T_c$  superconductor is a superconductor in the dirty limit; we may then obtain  $A_0$  and  $B_0$  from the microscopic theory of alloy (63)

$$A_0 = 0.7 \frac{k_B T_c \hbar v_F}{l \epsilon_F} \quad (5.3)$$

$$B_0 = 0.7 \frac{k_B T_c \hbar v_F}{l \epsilon_F n} \quad (5.4)$$

Where  $v_F$  is the Fermi velocity,  $\epsilon_F^0$  is the fermi energy,  $\lambda$  is the electron mean free path and  $n$  is the electron density. Therefore, substituting Eqs. (5.3) and (5.4) into Eqs. (5.1) and (5.2), yields

$$\lambda(0) = \left( \frac{\hbar^2 c^2}{4\pi (e^*)^2 n} \right)^{1/2} \quad (5.5)$$

$$\lambda(0) = 0.6 \left( \frac{\hbar v_F \lambda}{k_B T_c} \right)^{1/2} \quad (5.6)$$

For high  $T_c$  superconductor, the oxygen-deficient mixed crystals ceramics oxide carries the electron density the experimental (18), (64) works  $n = (2-4) \times 10^{21} \text{ cm}^{-3}$ . smaller than conventional alloy superconductor (65) ( $n = (6-8) \times 10^{22} \text{ cm}^{-3}$ ). Substituting  $n = (2-4) \times 10^{21} \text{ cm}^{-3}$ ,  $m^* = 2m_0$ ,  $e^* = 2e$  into Eq. (5.5) gives  $\lambda(0) \approx (600-1200) \text{ \AA}$ . Since the mean free path (66)  $\lambda = (5-10) \text{ \AA}$  which is very short with respect to the alloy superconductor (67)  $\lambda = (300-1200) \text{ \AA}$  and the Fermi velocity (68)  $v_F = (1.5-4) \times 10^7 \text{ cm. sec}^{-1}$  which is less than the one for standard metal (65) ( $v_F = 1.4 \times 10^8 \text{ cm. sec}^{-1}$ ) we thus substitutes  $\lambda = (5-10) \text{ \AA}$ ,  $v_F = (1.5-4) \times 10^7 \text{ cm. sec}^{-1}$  into Eq.(5.6), to yield,  $\xi(0) \approx (16-30) \text{ \AA}$ . These show that, for the high- $T_c$  superconductor the penetration depth at the absolute zero temperature very short with respect to alloy superconductor (69). Consequently, the Ginzburg-Landau parameter  $K = \lambda / \xi$  is very large, the high- $T_c$  superconductor is thus the extreme type II superconductor, Eqs. (4.26) and (4.52) show high value of the upper critical field  $H_{c2}$  and the low value of lower critical field  $H_{c1}$ , respectively. These results are in agreement with many experimental works (70-74).



We have also found the temperature dependence at near  $T_c$  of the penetration depth  $\lambda$ , the coherence length  $\xi$ , the upper critical field  $H_{c2}$ , the lower critical field  $H_{c1}$ , and the thermodynamic critical field  $H_c$  as follows

$$\begin{aligned} \lambda(T) &\sim (1 - T/T_c)^{-1/2} \\ \xi(T) &\sim (1 - T/T_c)^{-2/3} \\ H_{c2}(T) &\sim (1 - T/T_c)^{4/3} \\ H_{c2}(T) &\sim (1 - T/T_c)^{2/3} \ln [K_0 (1 - T/T_c)^{1/3}] \\ H_c(T) &\sim (1 - T/T_c) \end{aligned}$$

which are the same as the results from the work of Lobb (75), but we obtain the factor in more details, as well as the work of Kulic and Stenschke (76). Whereas, the temperature dependence of the upper critical field  $H_{c2}$  is the same as the work of Shapiro (77-78).

In addition to the work of Ref. (75-78), we have extended the calculation in obtaining the critical current density  $J_c$  and the parallel critical field  $H_{c1||}$  for thin film of the high- $T_c$  superconductor. We have found that the temperature dependence of  $J_c$  and  $H_{c1||}$ , at near  $T_c$  from Eqs. (4.74), (4.78), (4.79),

$$\begin{aligned} J_c(T) &\sim (1 - T/T_c)^{4/3} \\ H_{c1||}(T) &\sim (1 - T/T_c)^{2/3} \end{aligned}$$

Finally, the author is interested in a phenomenological model for the high- $T_c$  superconductivity from Ginzburg-Landau free energy of  $n$ th layer for calculating the critical temperature  $T_c$  (79-81). The author would like to propose that it may be viewed as a modification of Ref. (79-81), when the temperature dependence of the coefficients  $A = -A_0 (1 - T/T_c)^{4/3}$  and  $B = B_0 (1 - T/T_c)^{2/3}$

and then follows the author's work, which may be leading essentially to the perfect agreement with the experimental result for the values of  $T_c$ .