

Clayton Copula Value-at-Risk in Crisis and the Gold Optimal
Weight: Evidence in Thailand

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Value-at-Risk โดยใช้ Clayton Copula ในช่วงวิกฤตและสัดส่วนทองคำที่

เหมาะสม: ข้อมูลในประเทศไทย



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พิมพ์สุดา รัชฎ์ถัยณากุล : Value-at-Risk โดยใช้ Clayton Copula ในช่วงวิกฤตและสัปดาห์ทอง
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In recent days, investors are facing higher market risk due to the pandemic situation, but this is not the only time, investors also experienced similar risk during the Global Financial Crisis in 2007. We are interested in the tool to accurately estimate the market risk and ways to keep the portfolio maintaining the good performance in the extreme situation. This paper particularly investigates the Value-at-risk (VaR), which is one of the simplest and helpful tools to estimate market risk. By using data of SET50 and Thai Baht Gold, this paper demonstrates the best way to compute VaR among the various models of joint distribution of SET50 return and Thai Baht Gold return. There are two important components of joint distribution. The first one is the marginal distributions of SET50 and Gold. The study compares between the normal distribution and the extreme value distribution. Another component is the dependence structure of SET50 and Gold which will be described as copula. The study compares the gaussian copula with the clayton copula, the dependence structure capable of capturing lower tail dependence during the extreme negative return. Nevertheless, the result shows that value-at-risk using extreme value distribution and gaussian copula is our best model. To maintain the portfolio performance, the study sets the optimization problem and find the optimal weight by maximization the risk-adjusted return and use value-at-risk each model represents the risk instead of the standard deviation. The study shows that the optimal weight improves the portfolio performance during crisis, but the portfolio performance is worse during the non-crisis period.



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KEYWORD: Value-at-risk (VaR), the gaussian copula, clayton copula, the optimal weight improves the portfolio performance during crisis, the various VaR models of joint distribution of SET50 return and Thai Baht Gold return

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1. Introduction

1.1 Background

In the world of uncertainty, stock markets across the world volatily swing every day due to factors predictable and unpredictable. Since 2000, the market boomed and kept increasing dramatically. In 2008, the Global Financial Crisis happened. Investors faced a huge loss due to the critical market downturn. A year ago, people got to know about the COVID-19 pandemic. After that, the pandemic spread widely out rapidly and most of the countries locked down. This affects the stock market around the world. Dow Jones Industrial Average huge drops from approximately 29,000 to 18,000 points in one month and The Stock Exchange of Thailand (“SET”) also drops from approximately 1,500 to 1,000 points in one month too. As a result of these events, investors and financial institutions have daily faced with high market risk and must be monitoring the market risk closely.

Basel Committee on Banking Supervision (1996) at the Bank of International Settlements (BIS) has announced that the Basel II standard will determine the adequacy of capital requirement for financial institutions based on Value-at-Risk (“VaR”) estimation. VaR is a measurement of the market risk level within a financial institution at a specific time frame. The model is applied to estimate the maximum potential loss and then reserve the capital for the capital adequacy to sustain the liquidity of the financial institutions. The past market financial crisis shows that an accurate VaR is needed. Because if VaR is underestimated the financial institution will lack capital adequate. On the other hand, the financial institution reserves the excess cash and will lose the opportunities to invest more in case of the overestimated VaR. Therefore, many papers aim to find an accurate model of VaR even when the market fluctuation even or when the normal market situation. To forecast VaR, there are many steps to do.

The first step to forecast VaR is to forecast the conditional variance. There are many models such as Historical Simulation, Exponential Weighted Moving Average (EWMA), Generalized Autoregressive Conditional Heteroskedasticity (“GARCH”), etc. The drawbacks of the historical simulation and EWMA are the model using the past data. However, GARCH by [Engle \(1982\)](#) and [Bollerslev \(1986\)](#) model will be the

tool using the past return to predict the future volatility. The researchers mostly use the GARCH for forecasting the conditional variance and forecast VaR. Therefore, there are extensions of GARCH such as IGARCH, EGARCH, FIGARCH, MSGARCH, etc. But several pieces of evidence stated the original GARCH is outperforming or indifferent to others developed GARCH. In our paper, I did not deeply focus on the forecast volatility model too much. The focusing point of the study will be other selection steps of VaR such as the distribution and the Copula function.

The second step is the distribution simulation. Practically, the financial institutions use the normal distribution following the BASEL. In the real world, the distribution is not the normal distribution. Because when a crisis happened, the stock return sharply decreases, and the return distribution tends to have a skewness. Moreover, there are other distributions like the t-student's statistics, Generalized Pareto distribution from Extreme Value Theory, etc. The last step is calculating VaR at 95% of the confidence level.

The studied portfolio is SET50 Index and Thai Baht Gold. Because, in Thailand, there are many passive portfolios in SET50. SET50 is the basket of 50 selected stocks that have large market capitalization with criteria listing in SET. For mutual funds in Thailand, there are 27 SET50 Index funds¹. Currently, the trading value of SET50 is around 70% of the overall SET trading value. The investment money in stock concentrates in SET50. The downside movement of the SET50 index which represents the movement of constituents in the SET50 index can be caused a huge loss during the crisis. When a crisis occurred, investors tend to move their money to other assets such as gold, bond, or cash. Gold is a popular asset which people tend to invest in gold during the crisis. And gold has a diversification feature due to the correlation of gold and stock. Gold also has high liquidity in Thailand Most investors tend to have gold in a portfolio to diversify the risk when market uncertainty. Therefore, the study selects SET50 Index and gold to be our sample portfolio.

¹ Data from Association of Investment Management Companies (AIMC)

The return sampling distribution of two assets must be generated before forecast VaR. The study should plugin the dependence structure to make the two sampling returns dependently. The dependence structure is called Copula. Copulas mean link in the Latin language. Copulas have been used widely in many fields like engineering, medicine, climate and weather research, and especially quantitative finance. For the quantitative finance model, most of the applications use to minimize tail risk and portfolio optimization. There are many copula functions, and each function has a different advantage. Copulas divide into two main types. The first one is Elliptical Copula such as the Gaussian Copula and T-Copula whose dependence structure is symmetry distribution. The second is the Archimedean Copula such as Gumbel Copula, Clayton Copula, and Frank Copula whose dependence structure is asymmetry distribution. Gumbel Copula is the famous copula that can capture the strong upper tail dependence and weak lower tail independence. Next, the Clayton Copula can capture the lower tail dependence and weak for the upper tail independence. The last one is the Frank copula which suitable for the modeling data characterized by weak tail dependence. Hence, our study is interesting and selects in the Clayton Copula function due to the feature that can catch the strong correlation in the lower tail of the distribution as the important role for the crisis.

After forecasting the market risk, the investor should find a way to reduce the risk. Now, we have gold in a portfolio to help diversification but how much we should invest in gold. To answer this question, we found the framework that reduces risk by minimizing VaR. But there is a trade-off between return and risk. If the study sets the optimization on risk, the study will lose the opportunity to get a high return. Therefore, there is another interesting framework to study the maximization problem in terms of the return-risk ratio. Then the study can determine the gold weight in our portfolio.

1.2 Objectives and Contributions

Our study focuses on the crisis period which is the Global Financial Crisis and currently COVID-19 situation. High market volatility in crisis provides the hard condition to forecast the accurate VaR. The market risk is harder to predict than the normal market situation period. This reason causes investors to face uncertainty. VaR is a helpful tool to predict that uncertainty. Hence, the study would like to propose the model, which is suitable for especially in the crisis period, and compare the performance with the traditional model. When we know how much the risk is, another objective is to reduce that risk. Therefore, the detail is as below.

First, our study would like to propose the VaR forecasting by using the Extreme Value Theory and using Clayton Copula application to assess how accurate the model comparing with the VaR using traditional Normal Distribution assumption and Gaussian Copula and VaR using Extreme Value Theory and Gaussian Copula. To compare whether our selected distribution assumption and the copula selection is the better tool to forecast VaR during the crisis.

The second objective is to find out the optimal weight of the portfolio how much weight of gold to invest by using the maximization problem of the return to risk from the study proposed by [Campbell et al \(2001\)](#). While VaR focuses only on the market risk, this optimization problem further satisfies the portfolio performance. And the study compares the optimal weight from the VaR optimization model comparing with the hedge ratio from the mean-variance method. And the study will compare VaR and Sharpe ratio before and after changing the weight whether this new optimal weight gives the better Sharpe ratio rather than the pre-optimal weight portfolio.

Currently, most of the VaR forecast tools in Thailand usually use the normal distribution or the historical simulation method and it is hard to predict the accurate VaR. That makes the company face higher market risk. Our study would like to examine VaR in crisis in both Global Financial Crisis and COVID-19 by using the portfolio in Thai assets. The study has not been explored yet. Accordingly, this study tries to help-see the model that will be the better tool for forecast the VaR rather than the traditional forecast model. The study examines the model by back testing the accuracy and find the application of accurate VaR to reduce the risk along with the reliable return. The risk management

manager or the portfolio manager will be a benefit for the better tool forecasting VaR to be more accurate during this situation.

2. Literature Review

The study of VaR is developing since the original VaR was created. VaR is the standard tool for quantifying the market risk and is classified into three categories as Parametric, Non-parametric, and Semi-parametric VaR models.

First, the Parametric model assumes the return distribution to be the normal distribution. The drawback of the parametric model is the normal distribution assumption is violated for forecasting VaR and critically when forecasting in the period of the stock market crisis. But the advantage of the model is the easy way to estimate VaR.

Second, the Non-parametric VaR is the estimation by using the historical data called historical simulation. The advantage is no distribution assumption is required. On the other hand, the drawback is that the model uses historical data, and it will be not defining the future return distribution.

Last, the Semi-Parametric model is the Monte-Carlo simulation and applies the Extreme Value Theory (“EVT”). EVT helps to find the freely tail of distribution behavior. And Monte-Carlo helps to randomly create several of the possible scenarios of the return.

As you have seen, the last method is the most flexible overall approach. [Danielsson and de Vries \(1997\)](#), models based on conditional normality are not fit with the estimation of the large quantiles of the return distribution. On the other hand, the estimation of return distributions of financial time series by using EVT is a popular issue and apply to many research papers because the return distribution has heavy-tailed rather than the symmetry normal distribution ([Embrechts, Resnick, and Samorodnitsky \(1998\)](#), [Longin \(1997\)](#), [McNeil \(1997\)](#)).

EVT has two approaches to estimate the distribution which are the block maxima and the peak-over-threshold (“POT”). [Longin \(1997\)](#) and [McNeil \(1998\)](#) use estimation techniques based on limit theorems for block maxima. The model advantages are that theoretical assumptions are less critical in practice and easy to apply. But the drawback is that the estimation is uncertain if the sample size is small, and it will miss some of the

outlier observation. Nevertheless, the other approach called POT provides a famous method to model the related risk measures of VaR. The POT model is used in several papers (Franke, Härdle, and Hafner (2008), Gilli and Këllezi (2006), McNeil and Frey (2000), and McNeil et al. (2005)). The advantage of POT is correct all data even the maximum information and extreme event and independent. and the drawback is needing the threshold selection. There are several ways to select the threshold and the simple way is using a rule of thumb and commonly used is the 90th percentile². Hence, my study selects POT to be the EVT estimate approach because the observation of the crisis will be very extreme, and it is important to correct the outlier information. And, during the crisis, EVT estimation is the appropriate distribution to use to calculating VaR. T. Berger, M. Missong (2014) studied the best fit model during the Global Financial Crisis in 2008 and the period before the crisis. The study mainly finds the optimal VaR model. They studied several portfolios with different asset classes such as German stocks, national indices, and FX Rates. The result was divided into 2 results. For the normal market situation, the model under the normal distribution assumption is the best estimator. For the crisis period, the result shows EVT is the best distribution applying with Gaussian and T-student's Copula VaR. In Thailand, the paper of Sethapramote, Prukumpai, Kanyamee (2014) used data in the period from 1996 to 2012. But there is some inconsistency. The inconsistency is that the study found the return distribution of the SET50 index as the fat-tailed, but the result shows that the VaR based on the normal distribution assumption is accurate more than the T-student distribution.

Another additional tool based on my study is the Copula. The Copula study is on the early works of Hoeffding (1940, 1941) and Fréchet, Sklar (1959). And the copula is developing over time. Currently, Copula models have more powerful performance than other models and are more suitable for dealing with the nonlinear and tail dependence between random variables (Kayalar, Küçüközmenb, Selcuk-Kestel (2017), Tiwari, Aye, Gupta, Gkillas (2020), Mendes, Souza (2004), Ning (2010), Patton (2012)).

² Max Rydman (2018), Application of the Peaks-Over-Threshold Method on Insurance Data

As mentioned before, Copulas have mainly 2 types. The elliptical copula has Gaussian and T-student copula which have the symmetry distribution. For the crisis period, our study is considering the Archimedean copulas.

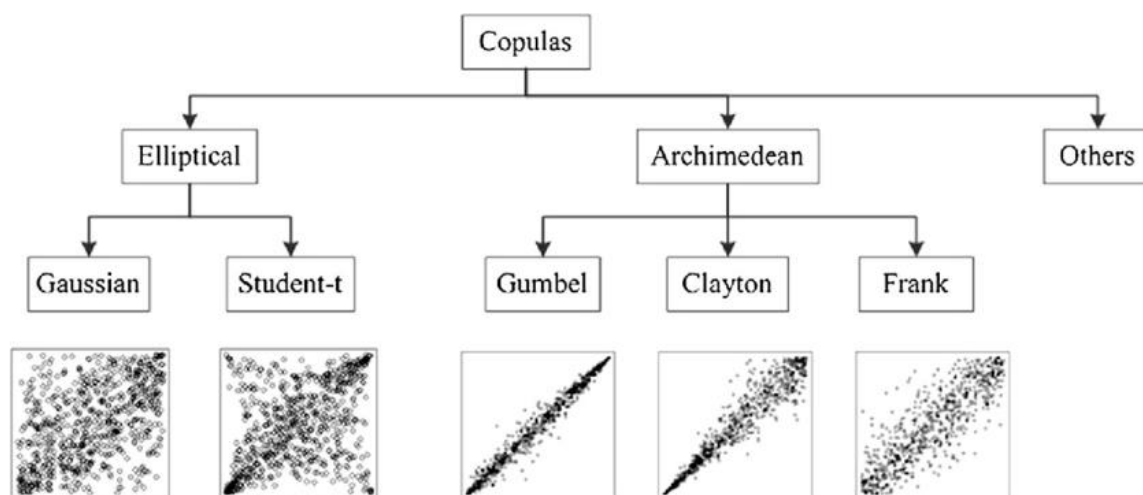


Figure 1: Scatter plots of different copula models³

As you have seen in Figure 1, the Clayton copula has a strong correlation in the lower tail as the scatterplot concentrate in the lower left of the scatter. On the other hand, the Gumbel copula has a strong correlation in the upper tail. And the Gaussian and T-student's statistic Copula is symmetry and dispels all the area. [Demarta, J. McNeil \(2005\)](#) states that the Clayton copula seemed to be the best model for the most extreme observations in the joint lower tail. That is the characteristic of the crisis or when the market extremely down. And Copula concept has been generally applied to calculate the VaR ([Clemente, Romano \(2006\)](#), [Hotta, Lucas, Palaro \(2008\)](#), [Huang, Lee, Liang, Lin \(2009\)](#), [Wang, Chen, Jin, Zhou \(2010\)](#)) Mostly of the paper try to develop the copula families to forecast VaR and back-testing for finding the accurate one.

[Zong-Run Wang, Xiao-Hong Chen, Yan-Bo Jin, and Yan-Ju Zhou \(2010\)](#) study VaR in the foreign exchange portfolio by using the GARCH-EVT-Copula multivariate model with different copula such as Gaussian copula, T-Copula, and Clayton copula to find the optimal weight within a portfolio. The study period is July 2005 to 2008. The result shows

³ Li, Jianping & Zhu, Xiaoqian & Lee, Cheng-Few & Wu, Dengsheng & Feng, Jichuang & Shi, Yong. (2015). On the aggregation of credit, market and operational risks. *Review of Quantitative Finance and Accounting*

that the optimal model is VaR applying with T-copula. For the higher confidence level, the result shows the Clayton copula is better to capture the correlations of the assets like the T-copula. As you have seen, even in the normal market period, the copula that practically uses maybe the Clayton Copula or the T-copula rather than the Gaussian Copula.

Moreover, there is the application of VaR using an optimization problem. [Harris and Shen \(2006\)](#) were applying VaR in the context of hedging, they developed minimum-VaR hedge ratios from the minimum variance framework and develop the framework to apply with the semi-parametric model. In Thailand, [Lertwattanasak, Leemakdej, Mokkhavesa \(2009\)](#) investigated the VaR and hedging by using the minimum-VaR approach. They studied the 20 big-cap stocks in Thailand, using SET50 futures to hedge and finding the hedging ratio during the normal market situation period. For this framework, they only concern about the reduction of market risk. But the reduction of risk sometimes needs to be a trade-off with a return. Another interesting framework came from [Campbell, Huisman, and Koedijk \(2001\)](#). They proposed the method to find the optimal weight from VaR. Many following papers applied this framework. [Shawkat Hammoudeh, Paulo Araújo Santos, Abdullah Al-Hassan \(2013\)](#) study the portfolio of various assets and find the optimal portfolio which applies the method from the framework of [Campbell et al. \(2001\)](#).

Our study finds out more evidence about gold using for hedge and diversify. [Dirk G. Baur, Brian M. Lucey \(2010\)](#) found that gold is a hedge against stocks on average and a safe haven in extreme stock market conditions. [Ghazali, Lean, Bahari \(2013\)](#) investigated the role of gold in Malaysia for the period of July 2001 to February 2013. The result was the domestic gold play an important role as a hedge and critically in the crisis period. Although, some papers have contrary results. Therefore, gold is the one that can use to be a hedge. Gold is one of the best options to satisfy diversification and hedging.

3. Data

The study is interesting in the portfolio composed of the SET50 Index which represents the stock in the SET50 and the Thai Baht gold price. The study uses Thai Baht Gold to be not concern about the foreign exchange. The data is based on a daily basis. The time frame is split into 3 periods. The first period is the crisis period dividing into 2 further periods. The first one is Global Financial Crisis data is collected from mid of 2007 to 2009 and the second is the COVID-19 pandemic which data is collected in 2020. The second is a non-

crisis period which is 2010 to 2019. Daily SET50 Index is gathered from SETSMART and Daily Thai Baht gold price is gathered from Aspen Program. And there is a risk-free rate for the optimization problem, the study uses a 1-year treasury bond which is gathered from Thai Bond Market Association (ThaiBMA).

4. Methodology

This section illustrates the methodology in estimating conditional variance by GARCH and simulating the univariate return distribution by different assumptions which are normal distribution and Generalized Pareto distribution by extreme value theory and then simulating the joint distribution by applying the 2 different dependence structure concepts which are the Gaussian Copula and Clayton Copula. The assumed weight of the SET50 Index and gold in the portfolio is 90% and 10% respectively. 10% comes from the basic rule of thumbs of the portfolio diversification with gold. But, in the last part of the methodology, the study examines the new optimal gold weight of our portfolio by using the optimization problem. Models are as follows.

Model 1: VaR using Normal Distribution and Gaussian Copula concept

Model 2: VaR using Extreme Value Theory and Gaussian Copula concept

Model 3: VaR using Extreme Value Theory and Clayton Copula concept

For models 1 and 2, the study can get the comparable result for the different distribution assumptions, and for models 2 and 3, the study can get the comparable result for copula selection. The last model aims to be the best model among the 3 models.

To answer the first objective, the study will use the [Kupiec's Test \(1995\)](#) to the accuracy. And for the second objective, the study uses the pre-optimal weight portfolio compare with the optimal weight portfolio and compares with the portfolio with the optimal weight which uses the traditional mean-variance approach to compare the portfolio performance and the market risk.

4.1 Return and the Conditional Variance

Begin with the return calculation, this step is preparing the data by calculating the log-return of SET50 Index and gold by using daily price. The formula is below.

$$r_t = \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right)$$

Where r_t is actual return and $P_{i,t}$ is the price of asset i at time t . Next, the study finds the conditional variance by using GARCH, the volatility forecasting model. In this section, we describe various GARCH models that are widely used. The return and the simple standard GARCH (1,1) model are below.

$$r_t = \mu + \varepsilon_t$$

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

Where $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$, $\alpha + \beta < 1$, μ is expected return and σ_t is the volatility of the return on day t .

Then, the study uses the return and conditional variance to simulate the sampling distribution. The next sections are the section of the distribution and the simulation via copula, respectively. To briefly illustrate the model, the study can be summarized in detail below.

Step	Model 1: Normal-Gaussian Copula VaR	Model 2: EVT-Gaussian Copula VaR	Model 3: EVT-Clayton Copula VaR
1	Generate Normal Distribution of SET50 Index and Gold	Generate Normal Distribution of SET50 Index and Gold	Generate Normal Distribution of SET50 Index and Gold
2	Find the correlation matrix to use with the Gaussian Copula	Estimate tail distribution and create the distribution	Estimate tail distribution and create the distribution
3	Apply to Gaussian Copula	Find the correlation matrix to use with the Gaussian Copula	Find the copula parameter to use with Clayton Copula

Step	Model 1: Normal-Gaussian Copula VaR	Model 2: EVT-Gaussian Copula VaR	Model 3: EVT-Clayton Copula VaR
4	Simulate sampling distribution and Find VaR with a 95% confidence level	Apply to Gaussian Copula	Apply to Clayton Copula
5		Simulate sampling distribution and Find VaR with a 95% confidence level	Simulate sampling distribution and Find VaR with a 95% confidence level

4.2 The Distribution

For the distribution, we consider 2 types of distribution that is the standard normal distribution and the other one is Generalized Pareto distribution which has the normal distribution and the tail distribution by estimating the tails via the extreme value theory.

4.2.1 Standard Normal Distribution

The normal distribution is the famous and basic univariate probability distribution. The function is as below.

$$F(z) = \phi(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z-\mu}{\sigma}\right)^2}$$

Where μ is the mean or expectation of the distribution, while the parameter σ is its standard deviation. Although it is the symmetry distribution, the skewness is zero and the kurtosis is three. The study will use the normal distribution assumption to apply to the non-crisis period. In the case of the crisis period, the study focuses on the heavy-tailed return distribution which must use the Extreme Value Theory model to estimate the tailed of the distribution.

4.2.2 Extreme Value Theory and Estimating Parameters

Extreme Value Theory (“EVT”) is the theory used for estimating the tail of return distribution. Finding the Generalized Pareto Distribution (GPD) of the log-return distribution. The study has to find the appropriate tail parameter (ξ) and the scaling parameter (β). The maximum likelihood estimation (MLE) can be applied with the following log-likelihood function (Max Rydman (2018)).

$$L(\xi, \beta | x) = \begin{cases} -N_u \ln \beta - \left(\frac{1}{\xi} + 1\right) \sum_{i=1}^{N_u} \left(1 + \frac{\xi}{\beta} (x_i - u)\right), & \text{if } \xi \neq 0 \\ -N_u \ln \beta - \left(\frac{1}{\beta}\right) \sum_{i=1}^{N_u} (x_i - u), & \text{if } \xi = 0 \end{cases}$$

where N_u is the number of exceeding log-returns over threshold u , and x_i is the log-returns that exceed the threshold u . The exceedances and the parameters, ξ , and β , can be estimated by using MATLAB software. The following equation is the marginal distribution used to simulate the risk factor standardized log returns.

$$F(z) = \begin{cases} N_{u^L} \left\{ 1 + \xi^L \frac{u^L - z}{\beta^L} \right\} & ; z < u^L \\ \phi(z) & ; u^L < z < u^R \\ 1 - \frac{N_{u^R}}{N} \left\{ 1 + \xi^R \frac{u^R - z}{\beta^R} \right\}^{-\frac{1}{\xi}} & ; z > u^R \end{cases}$$

Where N_{u^L} is the number of the negative log-returns exceeding the threshold u^L which is the lower tail threshold and N_{u^R} is the number of log-returns exceeding the threshold u^R is the upper tail threshold. The rule of thumbs threshold is the 90th percentile (Max Rydman (2018)).

4.3 Simulation via Copula

The part of the simulation is the use of the 2 of the univariate distributions and the copula function to simulate the joint distribution and use it to find VaR in the upcoming step. The study selected 2 interesting copulas Gaussian and Clayton Copula.

4.3.1 Simulation via Gaussian Copula

The Gaussian Copula is belonging to the family of elliptical copula and is the joint distribution constructed by the univariate normal distribution and the dependence structure. We applied the simulation method from Wang, Chen, Jin, Zhou (2010). The detail is below.

Let ϕ_{Σ} is the joint standardized bivariate normal distribution with the correlation matrix.

$$C^{Gu}(u_1, u_2) = \phi_{\Sigma}(\phi^{-1}(u_1), \phi^{-1}(u_2))$$

Where ϕ_{Σ} is the multivariate standard normal cumulative distribution function with the linear correlation matrix Σ and ϕ^{-1} is the inverse univariate standardized normal cumulative distribution function. From $F(z)$ in section 4.2.1. for Model 1 and 4.2.2. for Model 2, Based on the historical data $\{z_{1t}, z_{2t}\}, t = 1, 2, \dots, T$ we set:

$$u_t = (u_{1t}, u_{2t}) = (F_1(z_{1t}), F_2(z_{2t}))$$

$$\xi_t = (\phi^{-1}(u_{1t}), \phi^{-1}(u_{2t}))$$

Therefore, we can get the expression: $C^{Gu}(u_t) = \phi_{\Sigma}(\xi_t)$. And using the maximum likelihood method. Σ is estimated from $\hat{\Sigma} = \frac{1}{T} \sum_{t=1}^T \xi_t \xi_t'$ and simulate by the following steps as below.

1. For $\hat{\Sigma}$ derived, we use Cholesky decomposition and then have $\Sigma = AA'$
2. Generate independent 2-dimensional vectors which are from the 2 normal distribution variables $x = (x_1, x_2)'$, $x_i \sim N(0,1)$ Let $y = A'x$, then $z = (F_1^{-1}(\phi(y_1)), F_2^{-1}(\phi(y_2)))$ where $F_i^{-1}, i = 1, 2$ is the inverse distribution of F_i from $F(z)$;
3. Repeating the above steps M times, get the vector $(z_{1m}, z_{2m})', m = 1, 2, \dots, M$. Then restoring it into the formula of GARCH and return in section 4.1 and then get M returns at time $t + 1$. Finally, the returns residuals' joint distribution is this Gaussian

Copula. The returns can be defined by $r_{T+1} = (r_{1m}, r_{2m})' = (\mu_1 + z_{1m}\sigma_{1,T+1}, \mu_2 + z_{2m}\sigma_{2,T+1})'$; where $\sigma_{i,T+1}, \mu_i, i = 1, 2$ are calculated by the GARCH model.

4.3.2 Simulation via Clayton Copula

The Clayton Copula (1978) is belonging to the Archimedean copula family and its feature can catch a strong correlation in the lower tails. The Clayton copula function for the bivariate model is as below.

$$C^{Clayton}(u_1, u_2) = (u_1^{-\theta} + u_2^{-\theta} - 1)^{-\frac{1}{\theta}}, \quad \theta \geq 0$$

About the simulation, the study applied the simulation of the multivariate method from Wang, Chen, Jin, and Zhou (2010) to our bivariate model. The detail is below.

Based on the historical data $\{z_{1t}, z_{2t}\}, t = 1, 2, \dots, T$ and from $F(z)$ in the section 4.2.2, we have $u_i = F_i(z_{it})$. The study can estimate the copula parameter (θ) of 2 variables Clayton Copula using the Maximum Likelihood Method.

And considering the inverse function of $Gamma\left(\frac{1}{\theta}, 1\right)$'s LT transformation $\widehat{G}_\alpha^{-1} = t^{-\theta} - 1$, it is closely related to the generator of Clayton Copula: $\phi(t) = \frac{1}{\theta}(t^{-\theta} - 1)$ The two expressions are only different by a factor of $\frac{1}{\theta}$, which would not affect generating Clayton Copula. As a result, Clayton Copula is an LT-Archimedean Copula and the algorithms of simulating returns via Clayton Copula are as follows:

1. Generate M random variables $Y_m, m = 1, 2, \dots, M$ which satisfy the function $Gamma\left(\frac{1}{\theta}, 1\right)$ where the parameter θ is estimated using MATLAB program by generating from inverse cumulative distribution function.

2. Simulate independent uniform random variables y_1, y_2 ;

3. Let $u_m = G_\alpha\left(\frac{-\ln y_i}{Y_M}\right), i = 1, 2, m = 1, \dots, M$ then the joint distribution of $u = (u_1, u_2)$ is the 2-dimensional Clayton Copula with the parameter θ , then $z_m = F_i^{-1}(u_m), i = 1, 2, m = 1, \dots, M$;

4. Like the last step of simulating Gaussian Copula as illustrated above, the result is to get M returns at time $t + 1$. And the returns can be defined by $r_{T+1} = (r_{1m}, r_{2m})' = (\mu_1 + z_{1m}\sigma_{1,T+1}, \mu_2 + z_{2m}\sigma_{2,T+1})'$.

4.4 Value-at-Risk Calculation

The estimate VaR at time $t+1$ is simply the quantile of the vector of simulated portfolio return based on the data at time t . As we assume the weight is composed of 90% of the SET50 Index and 10% of gold. Therefore, the weight is $w = (0.9, 0.1)'$ and r_{T+1} from section 4.3. The study obtains the return that is $r_{T+1} * w = (r_{1m}, r_{2m}) * w$. Therefore, we get the distribution and can find the 95% VaR from the simulated distribution and will compare the accuracy of the model in the next step.

4.5 VaR Back-testing

After the study obtains VaR from the different copula. Next step, the study examines the efficiency of each model which one is the most accurate model. [Kupiec's Test \(1995\)](#) provides the standard test for the validity of VaR estimation. The test statistics are computed as. where N equals the number of days that loss exceeds the estimated VaR and T is the total number of observations. This approach presents the method to do the hypothesis testing. The null hypothesis:

$$H_0: p = 0.05$$

And the log-likelihood ratio test statistic is given by:

$$LR = -2 \ln((1 - p)^{T-N} p^N) + 2 \ln \left\{ \left(1 - \frac{N}{T}\right)^{T-N} \left(\frac{N}{T}\right)^N \right\}$$

which is asymptotically distributed chi-square with one degree of freedom under the null hypothesis that p is true probability. Therefore, the null hypothesis can be rejected at the 95% confidence level if $LR > 3.84$. And the VaR model would be accepted, if $LR < 3.84$.

4.6 Optimal Weight

To find the optimal weight, the study would like to find the weight which maximizes the risk-adjusted return of each model. The study applies the portfolio optimization model proposed by Campbell et al. (2001). Where $W(0)$ denotes to the invested amount, r_f is the risk-free asset, α is the confidence level of VaR and h is the weight of the gold in the portfolio.

$$\varphi(\alpha, h) = W(0)r_f - VaR(\alpha, h)$$

The optimization problem is to find the optimal weight of gold by the maximization of the return-risk ratio $S(h)$. The optimization problem is as below.

$$\max_h S(h) = \frac{r(h) - r_f}{\varphi(\alpha, h)}$$

According to the study, assume our portfolio has $W(0)$ equals to 1 unit. Therefore, the equation that applies to our portfolio is below.

$$\max_h S(h) = \frac{r(h) - r_f}{r_f - VaR(\alpha, h)}$$

Lastly, the study compares the optimal weight from each model by comparing VaR to compare the market risk and comparing the Sharpe ratio to compare the portfolio performance per unit of risk.

5. Empirical Result

Before the study calculates the Value-at-Risk, the study analyses our SET50 and gold data first. There are two important compositions. The first composition is the return distribution, and the second is the copula.

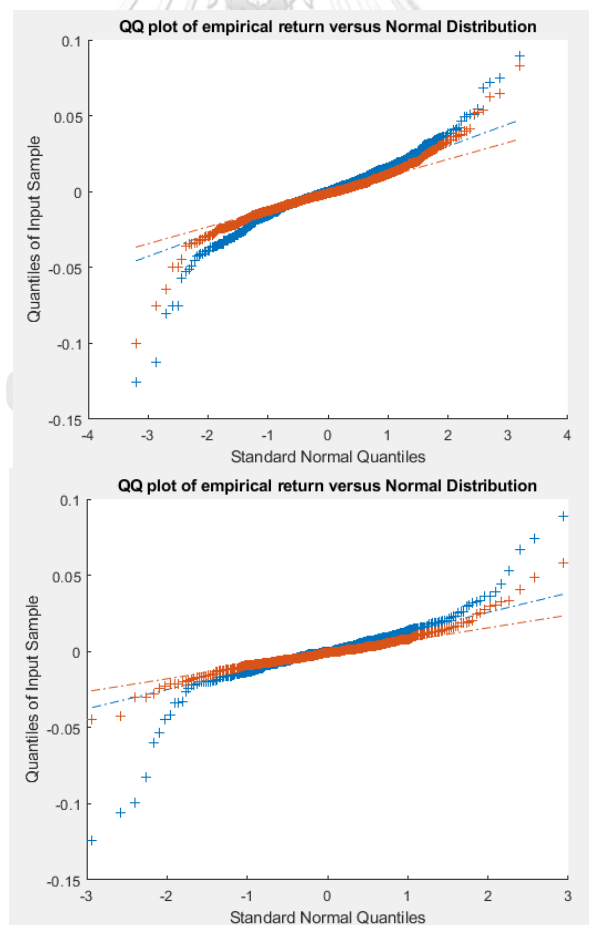
5.1 Return Distribution

According to the distribution, we try to fit our data by using a quantile-quantile plot (“Q-Q Plot”). Q-Q Plot is a probability plot by using a graphic for comparing two probability distributions by plotting their quantiles against each other. In the study, the

main consideration is the Q-Q Plot of empirical data comparing with the standard normal distribution. The Q-Q Plot for the extreme value theory is complex to compare. Because the extreme distribution has many Q-Q Plots, the distribution and Q-Q plots vary based on the shape and scale parameters of each distribution.

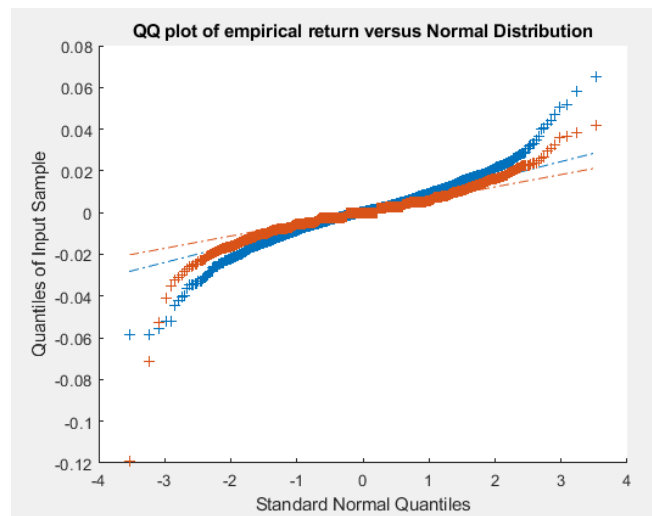
About the Q-Q Plot of normal distribution, the straight line (line-dash) represents the normal distribution, and the scatter represents the empirical return data. The blue scatter represents the return of SET50 and the orange scatter represents the return of Thai Baht gold. If the scatter plot is aligning with the straight line, it means the return distribution is implied to be the same as the normal distribution. But if the scatter deviate from the straight line, it means the return distribution might be another distribution shape. The result shows as below.

Crisis period



Global Financial Crisis Period (2007-2009) COVID-19 Period (2020 – 24 March 2021)

Non-Crisis Period



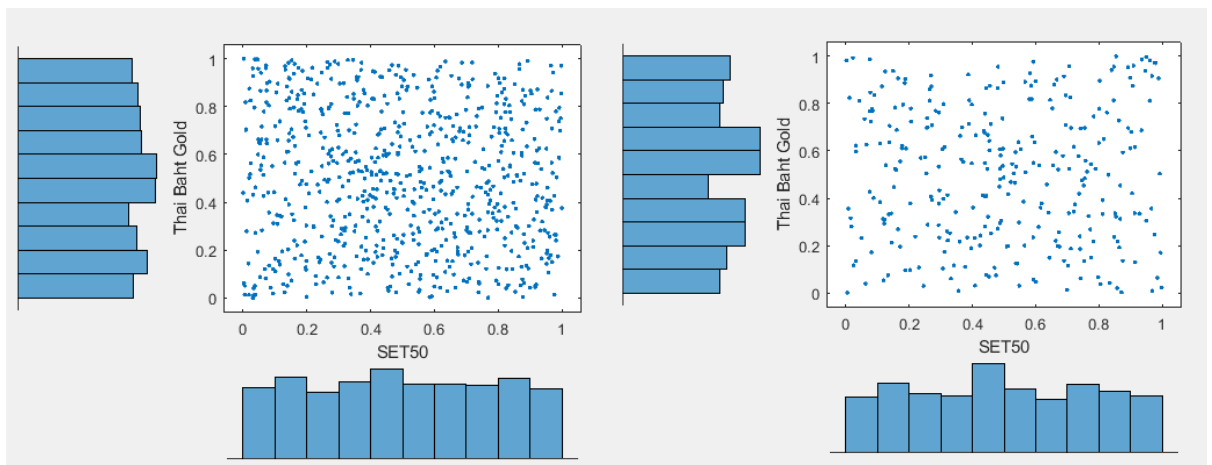
Non-crisis Period (2010 – 2019)

According to the scatter plot, the tails of all scatters deviate from the straight line every period. It means that the return distribution tends to be different from the standard normal distribution. Even in the non-crisis period, the scatter of SET50 and Thai Baht Gold also deviate from normal distribution either.

5.2 Copula

The copula scatter is also the graphical method that can help us to see the relationship between the return distribution of two assets. The copula scatter rescales the return data for each asset to be in the same scale $[0,1]$ interval, called unit square to see their relationship. The copula scatters are shown as below.

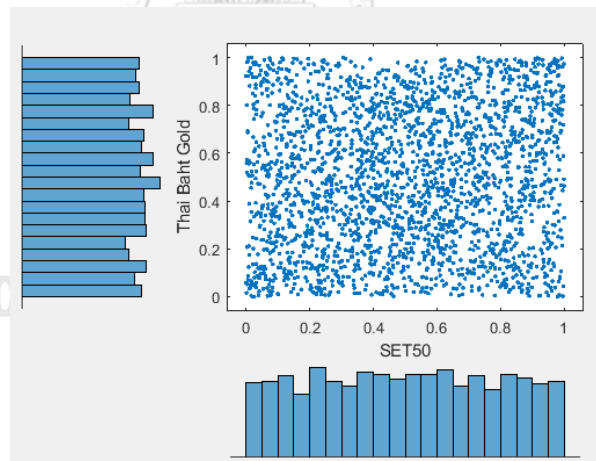
Crisis Period



Global Financial Crisis Period (2007-2009)

COVID-19 Period (2020 – 24 March 2021)

Non-Crisis Period



Non-crisis Period (2010 – 2019)

Even though the copula scatters are a beneficial tool, sometimes the graphical method is hard to identify which copula it is. The graphical analysis can be easier to identify if there is enough of the number of observations. If there are many observations, the study can see the relationship more clearly like the non-crisis period. On the contrary, if there is a little observation, the scatter will hard identify like the crisis period.

According to the copula scatter, the scatter spreads all over the area. During the crisis, the scatter plot is not having the exact shape. But the scatter tends to look like gaussian copula rather than clayton copula. In the non-crisis period, the scatter tends to look more clearly. It aligns with the gaussian copula with low correlation. To illustrate more, we show the Gaussian Copula with low correlation to graphical comparison in the figure below.

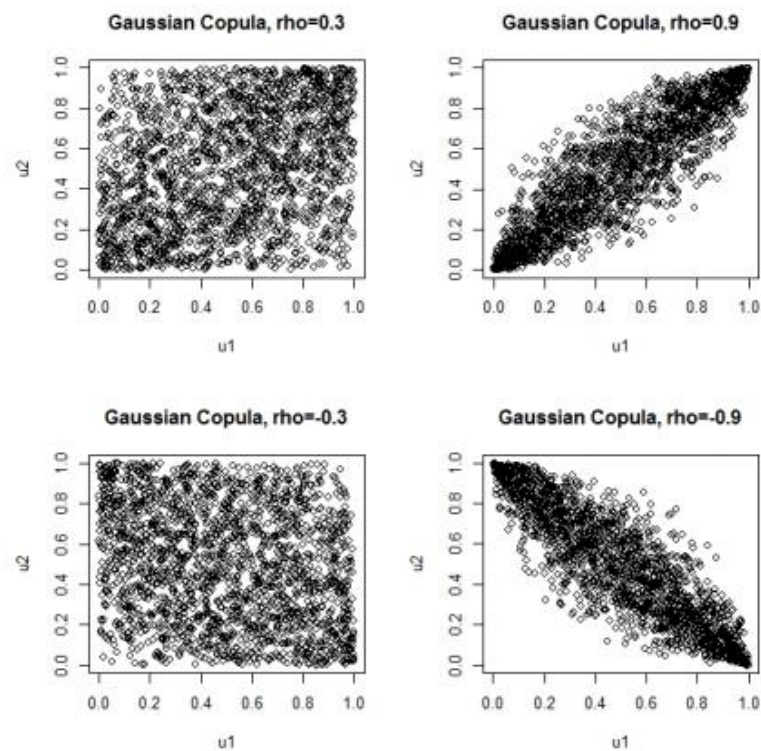


Figure 2: Bivariate Gaussian Copula with low and high correlation⁴

5.3 Value-at-Risk Calculation

The calculation is separated into 3 periods: Global financial crisis, COVID-19 crisis, and non-crisis. We use the rolling historical return data to create the model and calculate 1-day Value-at-Risk based on the input from the 240-day rolling estimate window. Since the 240-day estimation window comes from the approximately one-year trading day, the 240-day historical return is used as well to forecast volatility and other

⁴ See Johannes Krouthén, “Extreme joint dependencies with copulas A new approach for the structure of C-Vines” (June 2015)

parameters such as shape parameter, scale parameter, correlation matrix, and copula parameter.

Rolling Window Period



T stands for the day reporting Value-at-Risk.

5.4 Back Testing Result

The calculation is divided into two crisis periods and one non-crisis period. In the Global Financial crisis period, the data is from 2007 to 2009 containing 735 observations. COVID-19 period is from 2020 to 24 March 2021 and containing 299 observations. In the non-crisis period, the data is from 2010 to 2019, and there are 2,441 observations. We get the rolling 1-day Value-at-risk and use Kupiec's test (1995) to do the backtesting. The result shows in the table below.

	Kupiec's Test (POF)	Likelihood Ratio (LR)	P-value	Observations	Failures	%Failures
Crisis Period						
Global Financial Crisis						
Model 1	accept	3.32	0.07	735	48	6.53%
Model 2	accept	1.83	0.18	735	45	6.12%
Model 3	reject	5.22	0.02	735	51	6.94%
COVID-19						
Model 1	accept	0.06	0.80	299	14	4.68%
Model 2	accept	0.28	0.59	299	17	5.69%
Model 3	accept	0.62	0.43	299	18	6.02%
Non-Crisis Period						
Non-Crisis Period						
Model 1	reject	9.17	0.00	2441	156	6.39%
Model 2	accept	0.00	1.00	2441	122	5.00%
Model 3	reject	9.17	0.00	2441	156	6.39%

- Model 1 represents VaR using Normal Distribution and Gaussian Copula, Model 2 represents VaR using Extreme Value Theory Distribution and Gaussian Copula, and Model 3 represents VaR using Extreme Value Theory Distribution and Clayton Copula.
- Kupiec's Test result is from MATLAB 2020b software.

Table 1: Back Testing Result at 95% Confidence Level

Table 1 shows the backtesting result. In Global Financial crisis, Model 1 and Model 2 are accepted by Kupiec's Test at 95% confidence level due to $LR < 3.84$. it implied that no. of failure days is not significant. Therefore, these models are suitable estimators to calculate value-at-risk in this period. Especially for Model 2, the failure day is less

than the Model 1. In COVID-19 period, all models are accepted by Kupiec's test because all models have $LR < 3.84$. It implied that all models are suitable models to estimate Value-at-Risk. Moreover, the failure days are quite indifferent to all the models.

During the crisis, the preferred model is Model 1 and Model 2 because it is acceptable for both crisis periods. However, Model 2 which using Extreme Value Theory Distribution and Gaussian Copula provides less failure estimator during Global Financial Crisis.

For a non-crisis period, Model 2 is the only model accepted by Kupiec's Test. Because LR is less than 3.84. It implied that the no. of failure days is not significant.

All the period, Model 2 is our best suitable Value-at-Risk estimators as comparing with the other two models. It aligns with our data fit sections. The Extreme Value Theory is a better assumption than the normal distribution as we see in Q-Q Plot. Gaussian Copula is also a better assumption than Clayton Copula.

As a result, our proposed model, Model 3, is not a suitable assumption for accurate VaR calculation. And Model 2 is a good estimator. There are possible reasons to explain this. First, the crisis affects SET50 Index more critical than affect Thai Baht Gold in term of magnitude. Hence, the correlation of the two assets during a crisis is lower than we expected before. Second, the direction between two assets is sometimes not obvious. The dependence structure of clayton copula is a strong correlation in the lower tail. But these assets' lower-tailed correlation is not matching with the dependence structure of the clayton copula. Third, even though, the dependence structure of gaussian copula cannot capture the different correlation of assets while different market situation. But the rolling estimate window that the study uses in our model affects the change in correlation along the time. This makes the model can be captured the market environment while the market situation changes. Hence, the gaussian copula, in this case, can be used to be the VaR estimator in different market situations.

Even though Model 3 is not satisfied as to the suitable model. But Model 2 is a good predictor which can be applied along our sample period. It provides a more

accurate Value-at-Risk calculation. The study tries to apply Model 2 further to test whether it good for out of our sample data or not.

Out of Sample

For out-of-sample data, the study gathers data in two further periods. The first period is from 2000 to 2006, before Global Financial Crisis. And the second is from 24 March 2021 to 1 June 2021. The methodology aligns with the Model 2 methodology and uses Kupiec's Test to test how accurate the model is. The result is in the table below.

	Kupiec's Test (POF)	Likelihood Ratio	P-value	Observations	Failures	%Failures
Out of Sample 1 : 2000 - 2006	accept	0.99	0.32	1717	77	4.48%
Out of Sample 2 : 24 March - 1 June 2021	accept	0.01	0.94	42	2	4.76%

Table 2: Back Testing Result at 95% Confidence Level of Out of Sample data

According to the out-of-sample data, the result shows that p-value > 0.05 or likelihood ratio (LR) < 3.84 in both periods. Therefore, Kupiec's Test accepts Model 2. The no. of failure day is not significant in these models. It implies that the model can estimate the accurate Value at Risk at a 95% confidence level during those periods. Therefore, VaR calculation using extreme value theory and gaussian copula can be an accurate VaR estimator from 2000 to 1 June 2021.

Next, the study investigates the optimal weight calculating from all models comparing with our current weight 90% for SET50 Index and 10% for Thai Baht Gold.

5.5 Optimal Weight

After that, we use models to find the optimal weight based on Value-at-Risk using different models. We assume no short selling. And we find the optimal weight based on the optimization problem here below.

$$\max_h S(h) = \frac{r(h) - r_f}{r_f - VaR(\alpha, h)}$$

To solve the optimization problem, we use 240 trading days before the calculating day to be the estimate window period to input into optimization problem finding maximize risk-adjusted return, $S(h)$. As a result, we get the optimal weight for each

period on each model. Then, we calculate the return, volatility, and Sharpe ratio by using the old weight, which is 90% weight in SET50 and 10% weight in Thai Baht Gold, and optimal weights.

The study is divided into four portfolios. Portfolio 1 represents the portfolio using the old weight. Portfolio 2 is using optimal weight using Value-at-risk from the model using normal distribution and Gaussian Copula. Portfolio 3 is using Value-at-risk from the model using Extreme value theory distribution and Gaussian Copula. Lastly, Portfolio 4 is using Value-at-risk from the model using Extreme value theory distribution and Clayton Copula. The result shows in Table 2 below.

	Portfolio 1: Old weight	Portfolio 2: Optimal Weight from Normal Distribution and Gaussian Copula	Portfolio 3: Optimal Weight from Extreme Value Theory Distribution and Gaussian Copula	Portfolio 4: Optimal Weight Using Extreme Value Theory Distribution and Clayton Copula
Total Return				
Crisis Period				
Global Financial Crisis	13.18%	49.15%	51.88%	49.88%
COVID-19	-7.79%	13.30%	13.20%	13.03%
Non-Crisis Period				
Non-Crisis Period	66.89%	38.07%	40.95%	39.07%
Average Daily Return				
Crisis Period				
Global Financial Crisis	0.02%	0.07%	0.07%	0.07%
COVID-19	-0.03%	0.04%	0.04%	0.04%
Non-Crisis Period				
Non-Crisis Period	0.03%	0.02%	0.02%	0.02%
Standard Deviation				
Crisis Period				
Global Financial Crisis	1.77%	1.51%	1.51%	1.50%
COVID-19	1.82%	1.15%	1.15%	1.15%
Non-Crisis Period				
Non-Crisis Period	0.95%	0.88%	0.88%	0.87%
Sharpe Ratio				
Crisis Period				
Global Financial Crisis	0.01	0.04	0.04	0.04
COVID-19	-0.02	0.04	0.04	0.04
Non-Crisis Period				
Non-Crisis Period	0.02	0.01	0.01	0.01
Annualized Sharpe Ratio				
Crisis Period				
Global Financial Crisis	0.09	0.61	0.65	0.63
COVID-19	-0.24	0.58	0.58	0.57
Non-Crisis Period				
Non-Crisis Period	0.33	0.14	0.16	0.15

For Sharpe Ratio, the risk-free rate is a one-year Thai government bond yield divided by 365 days. Assume 245 trading days to calculate Annualized Sharpe Ratio.

Table 3: Return, Standard Deviation, and Sharpe Ratio in Different Weight

During the crisis period, the return of optimal weight, which is Portfolio 2, 3, and 4, have a much higher total return than Portfolio 1. The highest total portfolio return in the global financial crisis is from Portfolio 3. During COVID-19, the highest total portfolio return is from Portfolio 4. During the non-crisis period, Portfolio 1 has the highest total portfolio return. Therefore, this optimization problem only works well during the crisis period.

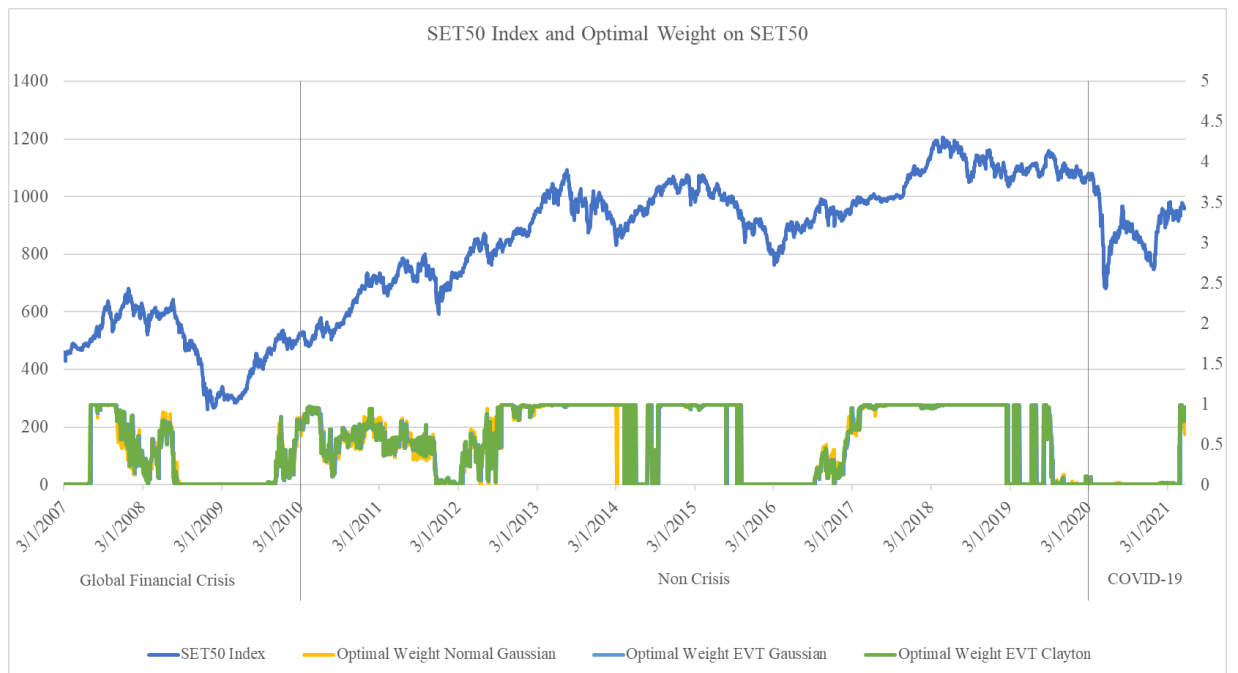


Figure 3: SET50 Index and Optimal Weight of SET50

Figure 3 shows that when the SET50 Index goes down, the optimal weight of SET50 will adjust to being almost 0%. As in the graph, the optimal weight of SET50 tends to decrease after June 2006. The optimal weight made the portfolio preventing the huge loss in Global Financial Crisis. SET50 Index tends to increase after mid-2009 and optimization problems suggest adding more investment in SET50, even there is a lagged time of market recovery recognition. As a result, total portfolio returns during the Global Financial crisis are higher than the old weight in Portfolio 1. Nonetheless, Portfolio 2, 3, and 4 show quite similar results. In COVID-19, the optimization problem is applicable. When the market downturn the optimal weight of SET50 is suggested to be almost 0%.

In the non-crisis period, there is the market downside in mid of 2013 in figure 3. But the optimization does not suggest decreasing weight in SET50. It makes the non-crisis period optimal weight does not beneficial at all. But, during mid of 2016, the market has collapsed, the weight had been adjusted to such an event.

There is some noticeable point that in Global Financial Crisis, COVID-19, and also in mid of 2016. The market looks like sideways down a bit before the market collapse. Therefore, the selection of a 240-day estimate window helps the model suggest dropping SET50 weight. On the contrary, there is an uptrend market before the market goes down in mid of 2013. Then, market recovery like a V-shaped graph. The model has the lagged market recovery signal. The suggested weight is dropped, this makes a return in Portfolio 2, 3, and 4 drops.

For the standard deviation, the standard deviation of Portfolio 2, 3, and 4 have lower portfolio than Portfolio 1 which using the old weight all along period. All optimal weight help reduce the risk even during a crisis or not in the crisis.

For the Sharpe ratio, Portfolio 2, 3, and 4 have a higher Sharpe ratio than Portfolio 1 during the crisis period because of much higher portfolio return. On the other hand, Portfolio 1 has a better Sharpe ratio comparing with the optimal Portfolio 2, 3, and 4 during the non-crisis period.

If you see in figure 3, the optimal weight is quite volatile. And the portfolio requires adjusting weight every day. In the real world, the adjustment caused a higher cost to manage the portfolio. Therefore, 1-day weight might not be practically applicable. The study tries 1-month-adjusted weight. The result is as below.

	Portfolio 1: Old weight	Portfolio 2: Optimal Weight from Normal Distribution and Gaussian Copula	Portfolio 3: Optimal Weight from Extreme Value Theory Distribution and Gaussian Copula	Portfolio 4: Optimal Weight Using Extreme Value Theory Distribution and Clayton Copula
Annualized Sharpe Ratio				
Crisis Period				
Global Financial Crisis	0.09	0.52	0.53	0.50
COVID-19	-0.24	0.34	0.35	0.32
Non-Crisis Period				
Non-Crisis Period	0.33	0.38	0.41	0.39

Table 4: Sharpe Ratio of 1-month-adjusted Weight

The result suggests that Portfolio 2, 3, and 4 are better than Portfolio 1 every period. For crisis, the Sharpe ratio in Portfolio 2,3, and 4 are lower than the 1-day-adjusted weight. On the contrary, Portfolio 2, 3, and 4 make a better performance than Portfolio 2, 3, and 4 using 1-day-adjusted weight. Based on our selected non-crisis period data, we see that there is a V-shaped SET50 Index movement. 1-month-adjusted weight has no critical change in weight. This situation makes the portfolio performance is better than the 1-day-adjusted weight which misses the chance to get a good return during market sudden recovery. However, the indices, in this case, is SET50 Index, mostly move unpredictably. It cannot be summarized that 1-month-adjusted weight or 1-day-adjusted weight is the better way. Both are better than Portfolio 1. But 1-day-adjusted weight surely causes the expense to adjust the portfolio. Therefore, 1-month-adjusted weight might be the better way to use it.

6. Conclusion

During the crisis, investors even individuals or institutional have to be confronted with higher market risk due to higher volatility. Thailand also impacts the crisis both the Global Financial crisis or called the subprime crisis and directly impacts COVID-19 pandemics. A helpful tool to forecast the market risk is Value-at-Risk. Value-at-Risk can forecast the maximum possible loss but there are many methods and assumptions to predict. And many portfolios are interesting. The study selects the portfolio which has SET50. Because most portfolios in Thailand invest in SET50 stocks. Most of it is the passive funds on SET50 Index. Moreover, there is a literature study that gold could be the hedge or the good diversification of portfolio. It is exciting that the study can find a good estimator to predict the accurate market risk and use gold to help reduce this risk, especially during a crisis.

When a crisis happens, the return with deviation from normal distribution due to the high negative return. The study selects the Extreme Value Theory distribution to be the proposed distribution of our study. Moreover, according to the target portfolio, SET50 and Thai Baht Gold tend to have a relationship during the crisis. Therefore, the study selects the clayton copula to be a helpful tool to identify the relationship between these two assets.

In conclusion, the empirical study suggests that the best model among our study models is the extreme value theory distribution. But, for the dependence structure, clayton copula is not suitable for identifying the relation between SET50 and Thai Baht gold. Because when market collapse, SET50 also has a more severe impact than Thai Baht Gold in term of magnitude, and the correlation is not fitted with clayton copula dependence structure. On the contrary, the study found that the gaussian copula is a better tool. And it can be used in both our sample and out-of-sample periods which are from 2000 to 1 June 2021.

After that, the study finds the optimal weight by the risk-adjusted return optimization problem. During the crisis, the result shows that portfolios with optimal weight provide better performance than the old weight. On the contrary, during the non-crisis, portfolios with optimal weight are worse performance than a portfolio using old weight. During the Global financial crisis and COVID-19, the stock market trend is sideways-down market characteristic follow by the market collapse. This market characteristic makes our model catching up downside signal and decrease weight on time. It means the optimal weight is better with the market characteristic is sideways down before the market crash.

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