

ตัวแบบเบสสำหรับการสร้างแผนที่ความยากจนในประเทศไทย



วิทยานิพนธ์นี้เป็นส่วนหนึ่งของการศึกษาตามหลักสูตรปริญญาวิทยาศาสตรมหาบัณฑิต

สาขาวิชาคณิตศาสตร์ประยุกต์และวิทยาการคณนา

ภาควิชาคณิตศาสตร์และวิทยาการคอมพิวเตอร์

คณะวิทยาศาสตร์ จุฬาลงกรณ์มหาวิทยาลัย

ปีการศึกษา 2562

ลิขสิทธิ์ของจุฬาลงกรณ์มหาวิทยาลัย

BAYESIAN MODELS FOR POVERTY MAPPING IN THAILAND

Miss Sarasinee Somjettana



จุฬาลงกรณ์มหาวิทยาลัย

CHULALONGKORN UNIVERSITY

A Thesis Submitted in Partial Fulfillment of the Requirements
for the Degree of Master of Science Program in Applied Mathematics and
Computational Science

Department of Mathematics and Computer Science

Faculty of Science

Chulalongkorn University

Academic Year 2019

Copyright of Chulalongkorn University

สรานีนี้ สมเจตนา : ตัวแบบเบส์สำหรับการสร้างแผนที่ความยากจนในประเทศไทย.
(BAYESIAN MODELS FOR POVERTY MAPPING IN THAILAND) อ.ที่
ปริกษาวิทยานิพนธ์หลัก : ผศ.ดร.จิราพรรณ สุนทรโชติ, 139 หน้า.

แผนที่ความยากจนเป็นแหล่งข้อมูลที่สำคัญแหล่งหนึ่งที่ใช้ในการแก้ปัญหาด้านสังคม เศรษฐกิจ และสิ่งแวดล้อม เริ่มจากที่ธนาคารโลกใช้วิธีอีแอลแอลสร้างแผนที่ความยากจนเพื่อนำมาใช้ในการออกแบบ กำหนดเป้าหมายและจัดสรรงบประมาณในประเทศด้อยพัฒนา แม้ว่าวิธีอีแอลแอลนั้นแสดงให้เห็นว่ามีข้อดีมากมายในการทำแผนที่ความยากจน แต่อีแอลแอลนั้นไม่ได้นำข้อมูลจากแบบสำรวจมาใช้ให้เกิดประโยชน์สูงสุด ดังนั้น จึงมีการเสนอวิธีแบบเบส์เชิงประจักษ์ และวิธีแบบเบส์เชิงลำดับชั้น ในอีกประเด็น หลุยส์แสดงให้เห็นว่าวิธีเบส์แบบปกติมีข้อจำกัดบางอย่าง ดังนั้นเขาจึงเสนอตัวประมาณใหม่ที่เรียกว่า วิธีเบส์แบบมีข้อจำกัด ด้วยเหตุนี้เราจึงนำวิธีนี้มาประยุกต์ใช้กับวิธีแบบเบส์เชิงประจักษ์ และวิธีแบบเบส์เชิงลำดับชั้น งานวิจัยนี้แบ่งออกเป็นสองส่วน ในส่วนแรกเป็นการศึกษาประสิทธิภาพของวิธีแบบเบส์เชิงประจักษ์ และวิธีแบบเบส์เชิงลำดับชั้น โดยเปรียบเทียบกับวิธีการดั้งเดิมอีแอลแอล โดยใช้ข้อมูลไทยกับตัวชี้วัดความยากจนเอฟจีที สำหรับส่วนที่สองเป็นการศึกษาประสิทธิภาพของเบส์แบบมีข้อจำกัดซ้ำได้แก่ วิธีแบบเบส์เชิงประจักษ์ แบบมีข้อจำกัด และวิธีแบบเบส์เชิงลำดับชั้นแบบมีข้อจำกัด โดยใช้กับข้อมูลค่าใช้จ่ายของไทย

จุฬาลงกรณ์มหาวิทยาลัย
CHULALONGKORN UNIVERSITY

ภาควิชา	คณิตศาสตร์และ	ลายมือชื่อนิสิต
	วิทยาการคอมพิวเตอร์	ลายมือชื่อ อ.ที่ปรึกษาหลัก
สาขาวิชา	คณิตศาสตร์ประยุกต์	
	และวิทยาการคณนา	
ปีการศึกษา	2562	

6072003423 : MAJOR APPLIED MATHEMATICS AND COMPUTATIONAL SCIENCE

KEYWORDS : POVERTY MAPS / BAYESIAN MODELS / ELL METHOD / EMPIRICAL BAYES METHOD / HIERARCHICAL BAYES METHOD / CONSTRAINED BAYES METHOD

SARASINEE SOMJETTANA : BAYESIAN MODELS FOR POVERTY MAPPING IN THAILAND. ADVISOR : ASST. PROF. JIRAPHAN SUNTORNCHOST, Ph.D., 139 pp.

Poverty maps are important sources of information for solving social, economic, and environmental problems. Initially, the World Bank used the ELL method to produce poverty maps for used in designing, targeting, prioritizing interventions and allocating the budgets for underdeveloped countries. Even though the ELL method has been shown to have many advantages in poverty mapping, it does not use a survey for the most benefit. Therefore, the Empirical Bayes (EB) method and the hierarchical Bayes (HB) method were proposed in literature. In another aspect, Louis shows that the usual Bayes has a limitation. Therefore, he proposed a new method called constrained Bayes (CB) method. For this reason, we apply this to Empirical Bayes and hierarchical Bayes. This research is divided into two parts. First to study the efficiency of EB and HB compare with the original ELL method by applying to Thai data with the FGT poverty indicators. Second to study the efficiency of constrained Bayes with constrained Empirical Bayes (CEB) and constrained hierarchical Bayes (CHB) by applying to Thai expenditure data.

Department : .. Mathematics and Student's Signature

..... Computer Science Advisor's Signature

Field of Study : .. Applied Mathematics and ..

..... Computational Science

Academic Year : .. 2019

ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to my thesis advisor Assistant Professor Jiraphan Suntornchost for the support of my Master's study for motivation patience, motivation, and the guide that helped me in all the time of my research and writing of the thesis.

Besides my advisor, I would like to thank my thesis committee: Associate Professor Phantipa Thipwiwatpotjana, Assistant Professor Kitiporn Plaimas, and Assistant Professor Monthira Duangsaphon, for their insightful comments to made my thesis more complete.

In addition, I would like to thank the Thailand National Statistic Office for recommendations and their help in collecting the Socio-Economic Survey data.

Lastly, I would like to thank my family and my friends for supporting and encouraging me all of everything.



จุฬาลงกรณ์มหาวิทยาลัย
CHULALONGKORN UNIVERSITY

CONTENTS

	Page
ABSTRACT IN THAI	iv
ABSTRACT IN ENGLISH	v
ACKNOWLEDGEMENTS	vi
CONTENTS	vii
LIST OF TABLES	x
LIST OF FIGURES	xi
LIST OF ALGORITHMS	xii
CHAPTER	
1 INTRODUCTION	1
2 PRELIMINARIES	5
2.1 Bayesian Analysis	5
2.1.1 Bayes' Theorem	5
2.1.2 Bayesian Inference	7
2.2 Simple Random Sampling	8
2.3 FGT Poverty indicator	8
2.3.1 Poverty Incidence	9
2.3.2 Poverty Gap	10
2.3.3 Poverty Severity	10
2.4 Normal Distribution	11
2.5 Nested Error Linear Regression Model	11
2.6 Bootstrap Method	13
2.7 Criteria	13
2.7.1 Akaike Information Criteria (AIC)	13
2.7.2 Analysis of Variance (ANOVA)	14
2.7.3 Absolute Bias and Mean Square Error	15
3 BAYESIAN MODELS FOR POVERTY INDICATORS AT UNIT LEVEL	16
3.1 The Household Socio - Economic Survey data	16

CHAPTER	Page
3.1.1	Descriptive Statistics 18
3.1.2	Variable Selection 23
3.2	Methodology 35
3.2.1	ELL Method 43
3.2.1.1	Algorithm for ELL Method 45
3.2.1.2	Result for the ELL method 47
3.2.2	Empirical Bayes Method 62
3.2.2.1	Algorithm for EB method 64
3.2.2.2	Result for the EB method 66
3.2.3	Hierarchical Bayes Method 73
3.2.3.1	Algorithm for HB Method 75
3.2.3.2	Result for the HB method 77
3.3	Result of Bayesian models for poverty indicators 91
4	BAYESIAN MODELS FOR EXPENDITURE VARIABLE AT AREA- LEVEL 105
4.1	The Household Socio-Economic Survey data at area-level 106
4.2	Methodology 113
4.2.1	Constrained Bayes (CB) method 113
4.2.2	Constrained Empirical Bayes (CEB) method 115
4.2.2.1	Algorithm and Process for CEB method 116
4.2.2.2	Result for CEB method 119
4.2.3	Constrained Hierarchical Bayes (CHB) method 121
4.2.3.1	Algorithm and Process for CHB method 122
4.2.3.2	Result for CHB method 124
4.3	Result of Bayesian models for expenditure variable 126
5	DISCUSSIONS 129
5.1	Discussion of the Bayesian Models for Poverty Indicators at Unit Level . . 129
5.2	Discussion of the Bayesian Models for Expenditure Variable at Area-Level 130

CHAPTER	Page
REFERENCES	132
APPENDICES	134
BIOGRAPHY	139



จุฬาลงกรณ์มหาวิทยาลัย
CHULALONGKORN UNIVERSITY

LIST OF TABLES

Table	Page
3.1 The descriptive statistics for the average monthly total expenditure per capita.	18
3.2 The set of qualitative candidate auxiliary variables and its descriptive statistics for selection.	21
3.3 The set of numerical candidate auxiliary variables and its descriptive statistics for selection.	22
3.4 The variable selection by using a forward selection technique with the Socio-Economic Survey data.	25
3.5 ANOVA table for test HH17.	26
3.6 The variable selection by using a backward selection technique with the Socio-Economic Survey data.	29
3.7 ANOVA Table for test covariates.	30
3.8 The list of included auxiliary variables.	31
3.9 The descriptive statistics for all parameter estimates.	54
3.10 The auxiliary variables of Mr.A.	56
3.11 Percentages of absolute biases and mean square errors for the ELL method. . .	61
3.12 The auxiliary variables of Ms.C.	66
3.13 Percentages of absolute biases and mean square errors for the EB method. . .	72
3.14 The descriptive statistics for posterior means and posterior variances over $L = 100$	84
3.15 Percentages of absolute biases and mean square errors for the HB method. . .	90
3.16 Percentages of averages across areas of absolute biases.	98
3.17 Percentages of averages across areas of mean square errors.	99
4.1 The list of area level auxiliary variables.	107
4.2 The variable selection by using a forward selection for area level model. . . .	108
4.3 ANOVA Table for test area auxiliary variables.	109
4.4 The variable selection by using a backward selection for area level model. . . .	110
4.5 ANOVA table for the variables $\mathbf{x}_{(4)}$, $\mathbf{x}_{(12)}$ and $\mathbf{x}_{(13)}$	111
1 Poverty line 2017 for 77 provinces in Thailand	137

LIST OF FIGURES

Figure	Page
3.1 Provincial average monthly total expenditure per capita for Thailand in 2017.	19
3.2 The simple random sampling without replacement for 77 provinces.	36
3.3 Bar chart for in-sample and out-of-sample sizes.	37
3.4 Average poverty incidence in Thailand from SES 2017.	39
3.5 Average poverty gap in Thailand from SES 2017.	40
3.6 Average poverty severity in Thailand from SES 2017.	41
3.7 Diagram for the ELL method.	46
3.8 The histogram of parameter estimation for $L = 100$	47
3.9 Diagram for the EB method.	65
3.10 Diagram for the HB method.	76
3.11 The histogram of posterior mean and posterior variance for $L = 100$	77
3.12 Percentages of absolute biases for poverty incidence of all 77 provinces.	92
3.13 Percentages of absolute biases for poverty gap of all 77 provinces.	93
3.14 Percentages of absolute biases for poverty severity of all 77 provinces.	94
3.15 Percentages of mean square errors for poverty incidence of all 77 provinces.	95
3.16 Percentages of mean square errors for poverty gap of all 77 provinces.	96
3.17 Percentages of mean square errors for poverty severity of all 77 provinces.	97
3.18 Average of poverty incidence in Thailand from the HB method.	101
3.19 Average of poverty gap in Thailand from the HB method.	102
3.20 Average of poverty severity in Thailand from the HB method.	103
4.1 The comparisons of the differences between sample variance and posterior expected sample variance for the EB and CEB methods.	120
4.2 The comparisons of the differences between sample variance and posterior expected sample variance for the HB and CHB methods.	125
4.3 The comparison plot of the Index for the EB and CEB estimator for 100 times.	127
4.4 The comparison plot of the Index for the HB and CHB estimator for 100 times.	127

LIST OF ALGORITHMS

1	Algorithm for bootstrap method.	13
2	Algorithm for forward variable selection.	23
3	Algorithm for backward variable selection.	27
4	Algorithm for the FGT poverty indicator from SES 2017.	38
5	Algorithm for the FGT poverty indicator of ELL method.	45
6	Algorithm for the FGT poverty indicator of EB method.	64
7	Algorithm for the FGT poverty indicator of HB method.	75
8	Algorithm for the constrained Empirical Bayes (CEB) method.	116
9	Algorithm for the constrained Hierarchical Bayes (CHB) method.	122



CHAPTER I

INTRODUCTION

Poverty is a problem that has arisen for a long time and has inevitably occurred in many countries around the world including Thailand. Poverty can be caused by the lack of basic living factors or basic needs, which include food, shelter, medicine and cloth. Also including lack of money, lack of education, natural disasters or no opportunities provided. The World Bank is one organization that works on the poverty of people in each country in order to eliminate these poverties. They have defined the meaning of poverty as Poverty is hunger. Poverty is lack of shelter. Poverty is being sick and not being able to see a doctor. Poverty is not being able to go to school and not knowing how to read. Poverty is not having a job, is fear of the future, living one day at a time. Poverty is losing a child to illness brought about by unclean water. Poverty is powerlessness, lack of representation and freedom. Poverty will lead to the social inequality, which will reduce the quality of the population and lead to many other problems.

Therefore, poverty is the one of the main problems that all sectors, both government and private organizations, for many countries around the world must pay attention to and jointly solve in order to reduce the gap of poverty and inequality. To determine whether a person is poor or not, we consider if his or her expenditure falls below the poverty line. The line is defined as the minimum level necessary of the cost of basic human need. Each country has an organization that defines its own poverty line. For Thailand, the poverty line was defined by the Office of the National Economic and Social Development Council.

Consequently, these poverty data are used to create the maps to identify the poverty areas. The poverty map is therefore one important source of information that can be used to solve social or economic problems. The poverty map can be presented on the Geographic Information System and displayed at the geographic level of regional, provincial, districts, sub-districts, or even to village levels. In other words, the poverty map is used to identify the poor, the level of poverty, and the severity of poverty any specific area. The poverty map is not only used by the policymaker of the governments to determine which areas are worthwhile for allocating government budgets to eradicate the poverty and inequality, but also used by business sector to make investment decisions.

The origin of method to create the poverty map was introduced by the World Bank, presented by Elbers et al. (2013), also called the ELL method. Since then, the ELL method has been widely used to produce poverty maps in many countries all over the world, for example, Neri et al. (2005) and Ballini et al. (2009) used the ELL method to construct the poverty inequalities in Tuscany and the Commonwealth of Dominica, respectively. The method was also applied to Thai poverty, by Healy et al. (2003) and Healy and Jitsuchon (2007), where they applied the model to estimate Thai poverty indicators at amphoe and tambon levels. First of all, the data used in the ELL method comes from 2 sources: survey data and census data. The survey data cover information about economics data such as income, expenditure, and consumption but does not cover all households in the area. The census data cover all households but lack of information about economics data. For this reason, the concept of the ELL method is to model the new population by using both surveys and census data.

Molina and Rao (2010) shown that the ELL method can poorly perform when unexplained between-area variation is significant. For this reason, they proposed

another method also call the Empirical Bayes (EB) method. The concept of the EB method to use survey data to generate only the out of surveys data with the variable from the census and then combine with the survey again. The advantage of this method is the use of real data from the survey.

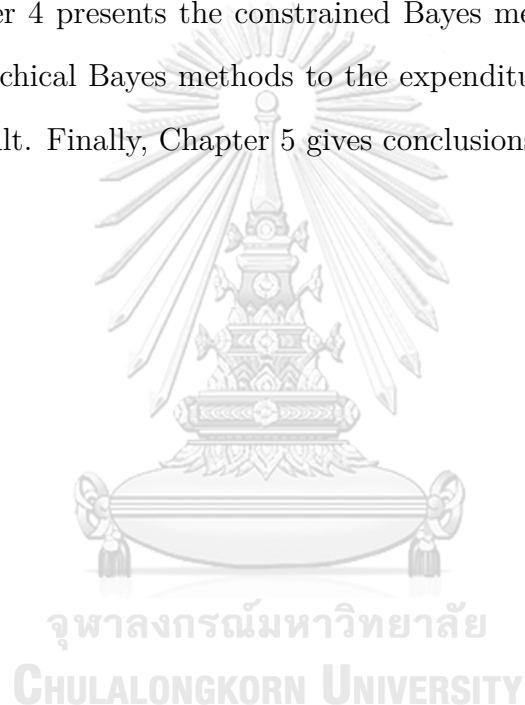
Molina et al. (2014) presented another alternative method, the Hierarchical Bayes method. The method does not require the use of bootstrap for the MSE estimation. The concept of this method is using the stepwise or multi-level model with a noninformative prior.

In another aspect, Louis (1984) shows that the usual Bayes has a limitation that the sampling variability of Bayes estimates is always smaller than the posterior expectation. For this reason, he proposed a new estimator called the constrained Bayes estimator by adding the new constraint, normally it should be equal. Thus, we are interested in applying this constrained Bayes of the Empirical Bayes and Hierarchical Bayes, also call constrained Empirical Bayes (CEB) and constrained Hierarchical Bayes (CHB), respectively, to model average expenditures at provincial levels of Thailand.

Our work is divided into two parts, Bayesian models for poverty indicators at unit level and Bayesian model for expenditure variable at area-level. Firstly, to study the efficiency of three methods for poverty mapping, the original ELL, EB and the HB methods. We apply Thai expenditure data with FGT poverty indicators of Foster et al. (1984). Then compare the methods by the absolute bias (AB) and mean square error (MSE) and then take the average across areas on the absolute bias (\overline{AB}) and on mean square error (\overline{MSE}). The criteria for determining the best method is that the absolute bias and mean square error should be closest to zero, which also for the average across the areas. Secondly, to study the efficiency of constrained Bayes with constrained Empirical Bayes and

constrained Hierarchical Bayes by applying to Thai expenditure data.

This thesis book is organized as follows. Chapter 2 presents the preliminary used as the basis for the thesis. Chapter 3 describes the Thai data and AIC technique for select the auxiliary variables and introduces the three methods for poverty mapping: the ELL method by Elbers et al. (2013), the Empirical Bayes (EB) method of Molina and Rao (2010) and the Hierarchical Bayes (HB) method including the result of comparing the HB and EB methods with the original ELL methods. Chapter 4 presents the constrained Bayes method with the Empirical Bayes and Hierarchical Bayes methods to the expenditure variable including the data and the result. Finally, Chapter 5 gives conclusions.



CHAPTER II

PRELIMINARIES

Before describing the methodology and the data, we present background knowledge used in this thesis as follows.

2.1 Bayesian Analysis

In this section, we give some definitions of Bayes' Theorem and Bayesian Inference.

2.1.1 Bayes' Theorem

Definition 2.1.1. A probability function is any function that satisfies the following conditions:

- (a) for any event A , $0 \leq P(A) \leq 1$,
- (b) for the event E which always occurs, $P(E) = 1$,
- (c) for any possibilities events to be pairwise mutually disjoint A_1, A_2, \dots ,

$$P\left(\bigcup_{i \geq 1} A_i\right) = \sum_{i \geq 1} P(A_i)$$

Definition 2.1.2. The probability that both event A and event B occur is $P(A \cap B)$.

Definition 2.1.3. The conditional probability $P(B|A)$ of event B occurring given

that event A has occurred is defined by

$$P(B|A) = \frac{P(A \cap B)}{P(A)}, \quad \text{where } P(A) > 0.$$

Theorem 2.1.1. (Bayes' Theorem) Bayes' Theorem states the conditional probability $P(B|A)$ of event B occurring given that event A has occurred as

$$P(B|A) = \frac{P(A|B) P(B)}{P(A)},$$

where

- $P(B|A)$ is the probability of event B occurring, given event A has occurred
- $P(A|B)$ is the probability of event A occurring, given event B has occurred
- $P(A)$ is the probability of event A occurring
- $P(B)$ is the probability of event B occurring.

Definition 2.1.4. Let A_1, A_2, \dots, A_n be mutually disjoint of n events conditional on event B . For the event B and the conditional probabilities $P(B|A_i)$, $i = 1, \dots, n$, the total probability $P(B)$ for event B can be written as

$$P(B) = \sum_{i=1}^n P(B \cap A_i) = \sum_{i=1}^n P(B|A_i)P(A_i).$$

Theorem 2.1.2. (Bayes' Theorem with Multiple Events) In general, let B be an event set and A_1, A_2, \dots, A_n be mutually disjoint of n events conditional on event B . For any $i = 1, \dots, n$,

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_{i=1}^n P(B|A_i)}.$$

2.1.2 Bayesian Inference

Bayesian Statistics method *is a method for summarising uncertainty and making estimates and predictions using probability statements conditional on observed data and an assumed model* - Gelman (2008).

Theorem 2.1.3. Bayes' Theorem with Bayesian Inference.

$$P(\theta|A) = \frac{P(A|\theta) P(\theta)}{P(A)},$$

where

$$P(A) = \int P(A|\theta)P(\theta)d\theta,$$

with

- $P(\theta)$ is the prior probability,
- $P(\theta|A)$ is the posterior probability,
- $P(A|\theta)$ is the likelihood probability,
- $P(A)$ is the evidence probability.

Definition 2.1.5. The prior probability $P(\theta)$ represents the probability of θ before the evidence A is observed.

Definition 2.1.6. The posterior probability $P(\theta|A)$ represents the probability of θ when the evidence A is observed.

Definition 2.1.7. The likelihood probability $P(A|\theta)$ represents the probability of A is observing and prior θ is observed.

Definition 2.1.8. The evidence probability $P(A)$ represents the probability of A is observing the data A according to the model.

2.2 Simple Random Sampling

Simple random sampling (SRS) is a process to sample selected n individuals from N individuals in population where each individual has an equal probability for selected.

Definition 2.2.1. The sample mean of sample data $\mathbf{Y} = \{Y_1, \dots, Y_n\}$ is

$$\bar{Y} = \frac{1}{n} \sum_{i=1}^n Y_i.$$

Definition 2.2.2. The sample variance of sample data $\mathbf{Y} = \{Y_1, \dots, Y_n\}$ is

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (Y_i - \bar{Y})^2.$$

2.3 FGT Poverty indicator

This section introduces an overview of the FGT poverty indicators of Foster et al. (1984), also known as the FGT family of poverty indicators. The FGT family includes poverty incidence, poverty gap, and poverty severity.

Definition 2.3.1. The poverty line is the threshold to indicate a person into two groups which are poor and not poor. The poverty line is defined for each area by the Office of the National Economic and Social Development Council.

Definition 2.3.2. (FGT Poverty indicators) Suppose that the population P of size N was partitioned into D areas. The population sizes of all D areas are N_1, \dots, N_D , respectively. The formula of the FGT poverty indicator is defined by

$$F_{\alpha d} = \frac{1}{N_d} \sum_{i=1}^{N_d} \left(\frac{z_d - E_{di}}{z_d} \right)^{\alpha} I(E_{di} < z_d), \quad \alpha = 0, 1, 2, \quad d = 1, \dots, D, \quad (2.1)$$

where z_d is the fixed poverty line for area d , E_{di} is the measure of welfare for individual i in area d , α is the parameter which can be 0, 1 or 2.

Remark. The function $I(\cdot)$ appeared in (2.1) is the indicator function. For this indicator function, $I(E_{di} < z) = 1$ if $E_{di} < z$ that means this person welfare is under poverty and $I(E_{di} < z) = 0$ if $E_{di} \geq z$ that means this person welfare is not under poverty.

2.3.1 Poverty Incidence

Consider $\alpha = 0$, we can see that the term of the relative distance between the poverty line and welfare variable is disappeared.

Definition 2.3.3. (Poverty Incidence) The poverty incidence ($\alpha = 0$) of area d ($d = 1, \dots, D$) is defined as

$$F_{0d} = \frac{1}{N_d} \sum_{i=1}^{N_d} I(E_{di} < z_d). \quad (2.2)$$

Obviously, poverty incidence for area d can be defined as the proportion of the total population in area d living below the poverty line. This poverty incidence is widely used for a poverty measure because it's interpretation is simple. However, the poverty incidence indicates only if the area is poor or not poor. It does not give information on how poor the area is.

2.3.2 Poverty Gap

In the situation of $\alpha = 1$, we consider more complexity of poverty.

Definition 2.3.4. (Poverty Gap) The poverty gap ($\alpha = 1$) of area d ($d = 1, \dots, D$) is defined as

$$F_{1d} = \frac{1}{N_d} \sum_{i=1}^{N_d} \left(\frac{z_d - E_{di}}{z_d} \right) I(E_{di} < z_d). \quad (2.3)$$

This indicator is called the poverty gap. It measures the area mean of the relative distance to the poverty line. Therefore, the poverty gap is an expansion of the poverty incidence by adding information about how far off individuals are from the poverty line.

2.3.3 Poverty Severity

The poverty severity is defined by the square of the poverty gap. The poverty severity shows the indication of inequality among the poor.

Definition 2.3.5. (Poverty Severity) The poverty severity ($\alpha = 2$) of area d ($d = 1, \dots, D$) is defined as

$$F_{2d} = \frac{1}{N_d} \sum_{i=1}^{N_d} \left(\frac{z_d - E_{di}}{z_d} \right)^2 I(E_{di} < z_d). \quad (2.4)$$

This measure considers the distance away from the poverty line adding higher weight on those who are poor.

2.4 Normal Distribution

In this section, we give the definition of the normal distribution and the standard normal distribution.

Definition 2.4.1. Let X be a random variable following the normal or Gaussian distribution with mean μ and variance σ^2 . Then its probability density function (pdf) is defined as

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left\{ -\frac{(x - \mu)^2}{2\sigma^2} \right\}. \quad (2.5)$$

Definition 2.4.2. The standard normal distribution is a special case of the normal distribution with mean zero and variance one. The probability density function (pdf) of the normal distribution is

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp \left\{ -\frac{x^2}{2} \right\}. \quad (2.6)$$

2.5 Nested Error Linear Regression Model

Linear regression is a model used to analyze the linear relationship between the response variable and explanatory variables. The response variable is usually denoted by Y and explanatory variables are denoted by the value x .

Definition 2.5.1. Let $Y_i, x_{i1}, \dots, x_{ip}$ be the data set of individual i ($i = 1, \dots, n$) for total n units. A linear regression has an equation of the form

$$Y_i = \beta_0 + \beta_1 x_{i1} + \dots + \beta_p x_{ip} + \varepsilon_i = \mathbf{x}'_i \boldsymbol{\beta} + \varepsilon_i, \quad i = 1, \dots, n, \quad (2.7)$$

where \mathbf{x} is the p -vector of auxiliary variables, $\boldsymbol{\beta} = [\beta_0 \ \beta_1 \ \dots \ \beta_p]'$ is the vector of regression coefficients and ε_i is random error.

Definition 2.5.2. A linear regression can be written in matrix form as

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \quad (2.8)$$

$$\text{where } \mathbf{Y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}, \mathbf{X} = \begin{pmatrix} 1 & x_{11} & \cdots & x_{1p} \\ 1 & x_{21} & \cdots & x_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & \cdots & x_{np} \end{pmatrix}, \boldsymbol{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_p \end{pmatrix} \text{ and } \boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix}.$$

Theorem 2.5.1. (Nested Error Linear Regression Model) The nested error linear regression model of Battese et al. (1988) of response variable Y_{di} is defined as

$$Y_{di} = \mathbf{x}'_{di}\boldsymbol{\beta} + u_d + e_{di}, \quad i = 1, \dots, N_d, \quad d = 1, \dots, D, \quad (2.9)$$

where \mathbf{x}_{di} is vector of auxiliary variables for individual i in area d , $\boldsymbol{\beta}$ is a vector of regression coefficients, $u_d \stackrel{iid}{\sim} N(0, \sigma_u^2)$ is random effect for area d , $e_{di} \stackrel{ind}{\sim} N(0, \sigma_e^2 k_{di}^2)$ is residual error for individual i in area d , u_d and e_{di} are independent and k_{di} are known constants.

Remark. If the sequence of variables Y_1, Y_2, \dots, Y_n are independent of each other and they have the same probability distribution, then the variables are called independent and identically distributed (i.i.d.) variables.

Definition 2.5.3. If the variables Y_1, Y_2, \dots, Y_n are i.i.d. with the same distribution as Y , then

1. $E(Y_1 + Y_2 + \cdots + Y_n) = nE(Y)$,
2. $V(Y_1 + Y_2 + \cdots + Y_n) = nV(Y)$.

2.6 Bootstrap Method

The bootstrap method is one of the resampling techniques inherited from the Monte Carlo Simulation. The new sample is created by sampling observations from the existing data and returning to the sample after they have been chosen. This process called sampling with replacement. Repeat this process for a sufficient number of iterations. Theoretically, we will be able to see the distribution of the new data set so that we can calculate the statistical characteristics of the population for more reliably. We can describe the process as follows:

Algorithm 1 Algorithm for bootstrap method.

- 1: Specify the number of bootstrap samples (number of iterations).
 - 2: Specify the sample size.
 - 3: For each bootstrap sample (iteration),
 - Draw a sample with replacement with the sample size in Step 2.
 - Calculate the statistic on the sample.
 - 4: Calculate the mean of the calculated sample statistic in Step 3.
-

2.7 Criteria

This section gives an overview of the Akaike Information Criterion (AIC), Analysis of Variance (ANOVA) test for variable selection and the absolute bias (AB) and mean square error (MSE) for comparing the methods.

2.7.1 Akaike Information Criteria (AIC)

Definition 2.7.1. Suppose that the joint probability density of X_1, \dots, X_n is $f(X_1, \dots, X_n|\theta)$. Let $X_1 = x_1, \dots, X_n = x_n$ be the observed sample. Therefore, the likelihood function, $L(\theta|x_1, \dots, x_n)$, as the function of the parameter θ defined as

$$L(\theta) = L(\theta|x_1, \dots, x_n) = f(x_1, \dots, x_n|\theta).$$

Definition 2.7.2. Akaike Information Criteria (AIC) of Akaike (1973) is defined by

$$AIC = -2 \log L(\theta) + 2p, \quad (2.10)$$

where $\log L(\theta)$ is the logarithm of likelihood function and p is the number of parameters in the model.

2.7.2 Analysis of Variance (ANOVA)

Given a data set $\{y_i, x_{i1}, \dots, x_{ip}\}$ for $i = 1, \dots, n$. That is, $\mathbf{y} = (y_1, \dots, y_n)'$ and $\mathbf{x} = (\mathbf{x}_1, \dots, \mathbf{x}_p)'$.

The model under the null hypothesis is

$$\text{Model 1 (m1): } \mathbf{y} = \beta_1 \mathbf{x}_1 + \dots + \beta_{p-1} \mathbf{x}_{p-1}.$$

The model under the alternative hypothesis is

$$\text{Model 2 (m2): } \mathbf{y} = \beta_1 \mathbf{x}_1 + \dots + \beta_{p-1} \mathbf{x}_{p-1} + \beta_p \mathbf{x}_p.$$

The following is table of Analysis of Variance (ANOVA)

Model	Res. Df	RSS	Df	Sum of Square
1	$Df_{m1} = n - p - 2$	$RSS_{m1} = \sum_{i=1}^n (y_i - \hat{y}_{m1})^2$		
2	$Df_{m2} = n - p - 1$	$RSS_{m2} = \sum_{i=1}^n (y_i - \hat{y}_{m2})^2$	$Df_{m1} - Df_{m2}$	$RSS_{m1} - RSS_{m2}$

The F-statistics is

$$F = \frac{(R_{m2}^2 - R_{m1}^2)(n - p - 1)}{(1 - R_{m2}^2)(p - 1)},$$

where $R_{m1}^2 = 1 - \frac{RSS_{m1}}{TSS_{m1}}$ and $R_{m2}^2 = 1 - \frac{RSS_{m2}}{TSS_{m2}}$
 with $TSS_{m1} = \sum_{i=1}^n (y_i - \bar{y}_{m1})^2$ and $TSS_{m2} = \sum_{i=1}^n (y_i - \bar{y}_{m2})^2$.

The P-value is

$$P(F > F_{\alpha, p-1, n-p-1}),$$

where α is the significance level. The null hypothesis is rejected at the significance level α if P-value $\leq \alpha$.

2.7.3 Absolute Bias and Mean Square Error

This section presents the criteria for comparing the model. For $k = 1, \dots, K$, let $\theta^{(k)}$ be the parameter of interest for k^{th} process, $\hat{\theta}^{(k)}$ be the corresponding estimated parameter $\theta^{(k)}$.

Definition 2.7.3. The absolute bias (AB) criteria of θ is defined as

$$AB = \frac{1}{K} \sum_{k=1}^K |(\hat{\theta}^{(k)} - \theta^{(k)})|. \quad (2.11)$$

Definition 2.7.4. The mean square error (MSE) criteria of θ is defined as

$$MSE = \frac{1}{K} \sum_{k=1}^K (\hat{\theta}^{(k)} - \theta^{(k)})^2. \quad (2.12)$$

Remark. The absolute bias and mean square error area always non-negative. A good estimator should provide small absolute bias and small mean square error.

CHAPTER III

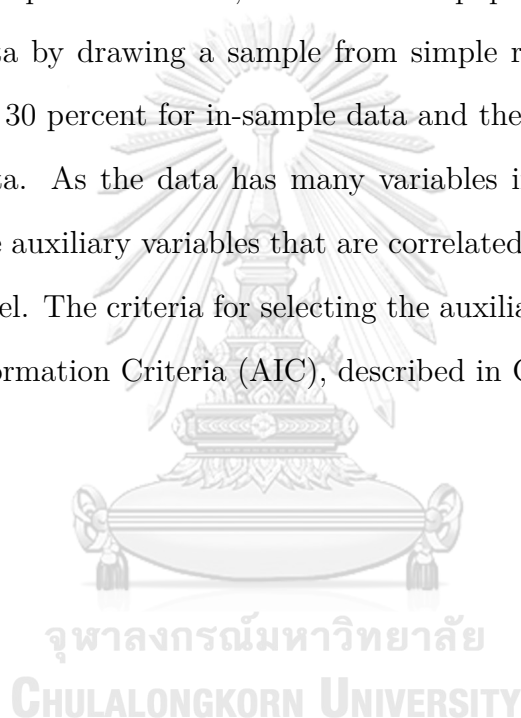
BAYESIAN MODELS FOR POVERTY INDICATORS AT UNIT LEVEL

This chapter presents unit level models that are applied to the three types of FGT poverty indicators. The indicator includes poverty incidence, poverty gap, and poverty severity introduced in Chapter 2. The Bayesian methods are applied to the welfare data of Thailand based on household levels for 77 provinces. We use the data from the Socio-Economic Survey of Thailand. The welfare variable used in the models is the average monthly total expenditure per capita and related auxiliary variables. The organization of this chapter is as follows. Section 3.1 gives descriptive statistics and the variable selection for the data. Section 3.2 presents the three models for poverty mappings which are Elbers, Lanjouw, and Lanjouw (ELL) method, Empirical Bayes (EB) method and Hierarchical Bayes (HB) method. Finally, Section 3.3 shows the result of absolute bias and mean square error according to these average across areas and gives a discussion.

3.1 The Household Socio - Economic Survey data

The data used in this chapter is the Thai Household Socio-Economic Survey (SES) in 2017 produced by Thailand National Statistic Office. The SES covers approximately 43,200 households from 77 provinces of Thailand. This survey is a collection of data on incomes, expenses, conditions, liabilities, household property, as well as housing characteristics. They collect data from sample households in all provinces nationwide both in the municipality and non-municipality. The response variable that we are interested in is the log-transformation of expenditure, which is

the average monthly total expenditure per capita. Therefore, the transformation is $Y_{di} = \log(E_{di})$ where E_{di} is the average monthly total expenditure per capita of individual i in province d . The Thailand poverty line for 77 provinces in 2017 shown in Appendix A. In general, we have information about the auxiliary variables for both in-sample and out-of-sample data from census data. However, the welfare variable is only available in in-sample data but not in the census data. Due to the limitation of the data available to us, our study is performed by treating the SES data as the population. Then, we divide this population into in-sample and out-of-sample data by drawing a sample from simple random sampling without replacement with 30 percent for in-sample data and the remaining 70 percent for out-of-sample data. As the data has many variables in the survey, we have to select appropriate auxiliary variables that are correlated with the welfare variable to use in the model. The criteria for selecting the auxiliary variables in the model is the Akaike Information Criteria (AIC), described in Chapter 2.



3.1.1 Descriptive Statistics

This section shows some descriptive statistics for the welfare variable, the average total expenditure (**E**) in baht per month per capita, and all candidate auxiliary variables used in our study. Table 3.1 shows the descriptive statistics for the average monthly total expenditure per capita from the Thailand Household Socio-Economic Survey.

Statistics	Value (Baht/month/capita)
Minimum	616
1st Quartile	3954
Median	6107
Mean	8236
3rd Quartile	9882
Maximum	435895

Table 3.1: The descriptive statistics for the average monthly total expenditure per capita.

From Table 3.1, we can see that the average monthly total expenditure per capita of people in Thailand range between 616 to 435895. In addition, about 50% of people in Thailand have the average monthly total expenditure per capita between 3954 and 9882. However, consider the difference between the minimum and maximum, we can see that people have quite different expenditure and there are big gaps between those who have low expenditure and high expenditure. This indicates inequality of living standard among people in the country.

Figure 3.1 shows the provincial average monthly total expenditure per capita from Household Socio-Economic Survey of Thailand

From Figure 3.1, the dark provinces have low provincial average monthly total expenditure per capita while the light provinces have high expenditure. It can be seen that the darkest province is Kalasin, which is 3826. Northeastern and northern regions have relatively low provincial expenditure, while the central region has relatively high expenditure. Table 3.2 shows the set of qualitative candidate auxiliary variables and its descriptive statistics for selection.

Qualitative variables	Code	Frequency
Type of dwelling (HH01)	1 - Detached house	33838
	2 - Row house	6285
	3 - Town house / twin house	1654
	4 - Flat or apartment, etc.	1186
	5 - Room or rooms	172
	6 - Improvised quarter	45
	7 - Others	19
Materials of construction (HH02)	1 - Cement or brick	24560
	2 - Wood	7044
	3 - Wood and cement or brick	11315
	4 - Local materials	162
	5 - Re-used materials	50
	6 - Others	68
Tenure (HH03)	1 - Own dwelling and land	31370
	2 - Own dwelling on rented land	1250
	3 - Own dwelling on public area	842
	4 - Hire - purchased	481
	5 - Rent	6018
	6 - Rent paid by others	1407
	7 - Occupied, rented free	1773
	8 - Others	58

Continued on next page...

Table 3.2 – continued from previous page.

Qualitative variables	Code	Frequency
Cooking fuel (HH10)	0 - No cooking	3956
	1 - Charcoal	3515
	2 - Wood	4315
	3 - Kerosene	50
	4 - Gas	29855
	5 - Electricity	1508
Toilet facilities (HH15)	0 - No facility nearby	96
	1 - Flush latrine	16105
	2 - Squat	23512
	3 - Bath flush and squat latrine	3486
Drinking water (HH11)	0 - Bottle-Water/Water from vending machine	28165
	1 - Inside piped water supply	1806
	2 - Inside piped underground water	872
	3 - Outside piped or public tap	40
	4 - Well or underground water	588
	5 - River, stream, etc.	316
	6 - Rain water	4439
	7 - Treated tap water	6912
8 - Others	61	

Table 3.2: The set of qualitative candidate auxiliary variables and its descriptive statistics for selection.

From Table 3.2, we can see the frequency of each type of qualitative candidate auxiliary variables. If we sum up the frequency of each qualitative candidate auxiliary variables, we get 43199 units which is the population size for this data set. From Table 3.2, we notice that three quarters have detached houses and their own houses and lands. However, around 900 people still drink water from the river and underground. As well as there are still 96 households among the population

do not have a toilet facility nearby. These results show that the poverty and inequality are still in Thailand.

Table 3.3 shows the minimum, 1st quartile, median, mean, 3rd quartile and maximum for the quantitative auxiliary variables.

Quantitative Variables	Minimum	1st Quartile	Median	Mean	3rd Quartile	Maximum
Number of owned						
LCD / LED / PLASMA (HH32)	0	0	0	0.5025	1	6
Video/VCD/ DVD Player (HH33)	0	0	0	0.3484	1	6
Mobile phone (HH40)	0	1	2	2.081	3	12
Home computer (HH37)	0	0	0	0.2503	0	9
Refrigerator (HH27)	0	1	1	0.9475	1	6
Microwave oven (HH25)	0	0	0	0.2251	0	3
Washing machine (HH34)	0	0	0	0.0409	0	5
Air conditioner (HH35)	0	0	0	0.4377	1	9
Automobile (HH18)	0	0	0	0.2134	0	8
Motorcycle (HH17)	0	1	1	1.212	2	9

Table 3.3: The set of numerical candidate auxiliary variables and its descriptive statistics for selection.

From Table 3.3, the average number of refrigerators per household is 0.9475, which shows that almost every home has a refrigerator. Furthermore, the 3rd quartile of a home computer, microwave oven, washing machine, and the automobile is zero, shows that up to 75 percent do not own these items.

3.1.2 Variable Selection

For the selection of appropriate variables in the model, we select the auxiliary variables that are related to the welfare variables using forward and backward selections with Akaike Information Criterion (AIC) technique proposed by Akaike (1973).

Let $\mathbf{Y} = \log(\mathbf{E})$ where \mathbf{E} is the average monthly total expenditure per capita or A10 in the SES. Since the SES has many auxiliary variables (\mathbf{X}), we must choose appropriate variables to be included in the model. These variables must be related to the welfare variable. We use the variable selection with both forward and backward to select suitable variables in the model.

Forward Selection

The idea of the forward selection method is to select the auxiliary variables into the model one by one, choosing the one that is the most related to the welfare variable. Continue the process until getting the appropriate model. The model that provides the smallest AIC will be the most appropriate model. The Algorithm of the forward selection is presented as follows.

Algorithm 2 Algorithm for forward variable selection.

- 1: Begin with no candidate variable in the model.
 - 2: Select one significant candidate variable by comparing the AIC values of all models with one variable. Select the candidate variable giving the smallest AIC value. If there is no selection, go to Step 4.
 - 3: Select one more significant candidate variable. Compare the AIC values of all models that include the variable in Step 2 and one additional variable.
 - 4: Select the new variable if the model with the additional variable gives the smallest AIC value when the variables in Step 3 is already in the model. If there is no selection go to Step 5. If there are additional variables for putting in the model go to Step 2.
 - 5: Stop the process for variable selection.
-

Applying the algorithm to the Socio-Economic Survey data, we get the following results.

Step	Add	Model	AIC
1	No	$\log(\mathbf{E}) \sim 1$	-34819
2	HH35	$\log(\mathbf{E}) \sim \text{HH35}$	-44503
3	HH10	$\log(\mathbf{E}) \sim \text{HH35}+\text{HH10}$	-52316
4	HH03	$\log(\mathbf{E}) \sim \text{HH35}+\text{HH10}+\text{HH03}$	-54754
5	HH18	$\log(\mathbf{E}) \sim \text{HH35}+\text{HH10}+\text{HH03}+$ HH18	-56846
6	HH25	$\log(\mathbf{E}) \sim \text{HH35}+\text{HH10}+\text{HH03}+$ HH18+HH25	-58326
7	HH40	$\log(\mathbf{E}) \sim \text{HH35}+\text{HH10}+\text{HH03}+$ HH18+HH25+HH40	-59650
8	HH15	$\log(\mathbf{E}) \sim \text{HH35}+\text{HH10}+\text{HH03}+$ HH18+HH25+HH40+HH15	-60661
9	HH11	$\log(\mathbf{E}) \sim \text{HH35}+\text{HH10}+\text{HH03}+$ HH18+HH25+HH40+HH15+ HH11	-61533
10	HH32	$\log(\mathbf{E}) \sim \text{HH35}+\text{HH10}+\text{HH03}+$ HH18+HH25+HH40+HH15+ HH11+HH32	-62280
11	HH37	$\log(\mathbf{E}) \sim \text{HH35}+\text{HH10}+\text{HH03}+$ HH18+HH25+HH40+HH15+ HH11+HH32+HH37	-62747
12	HH02	$\log(\mathbf{E}) \sim \text{HH35}+\text{HH10}+\text{HH03}+$ HH18+HH25+HH40+HH15+ HH11+HH32+HH37+HH02	-63180

Continued on next page...

Table 3.4 – continued from previous page.

Step	Add	log(E)	Model	AIC
13	HH01	log(E)	~ HH35+HH10+HH03+ HH18+HH25+HH40+HH15+ HH11+HH32+HH37+HH02+ HH01	-63508
14	HH33	log(E)	~ HH35+HH10+HH03+ HH18+HH25+HH40+HH15+ HH11+HH32+HH37+HH02+ HH01+HH33	-63801
15	HH27	log(E)	~ HH35+HH10+HH03+ HH18+HH25+HH40+HH15+ HH11+HH32+HH37+HH02+ HH01+HH33+HH27	-63897
16	HH34	log(E)	~ HH35+HH10+HH03+ HH18+HH25+HH40+HH15+ HH11+HH32+HH37+HH02+ HH01+HH33+HH27+HH34	-63927
17	HH17	log(E)	~ HH35+HH10+HH03+ HH18+HH25+HH40+HH15+ HH11+HH32+HH37+HH02+ HH01+HH33+HH27+HH34+ HH17	-63925
End	The final model is add all candidate auxiliary variable except HH17.			

Table 3.4: The variable selection by using a forward selection technique with the Socio-Economic Survey data.

We see that the AIC in Step 16 and Step 17 are quite close, so we need to make sure whether the variable HH17 should be added into the model by applying the analysis of variance (ANOVA) test.

The model under the null hypothesis is Model 1 (no HH17):

$$\log(\mathbf{E}) \sim \text{HH35} + \text{HH10} + \text{HH03} + \text{HH18} + \text{HH25} + \\ \text{HH40} + \text{HH15} + \text{HH11} + \text{HH32} + \text{HH37} + \\ \text{HH02} + \text{HH01} + \text{HH33} + \text{HH27} + \text{HH34}$$

The model under the alternative hypothesis is Model 2 (with HH17):

$$\log(\mathbf{E}) \sim \text{HH35} + \text{HH10} + \text{HH03} + \text{HH18} + \text{HH25} + \\ \text{HH40} + \text{HH15} + \text{HH11} + \text{HH32} + \text{HH37} + \\ \text{HH02} + \text{HH01} + \text{HH33} + \text{HH27} + \text{HH34} + \\ \text{HH17}$$

Model	Res. Df	RSS	Df	Sum of Square	F	P-value
1	43155	9815.4				
2	43154	9815.4	1	0.03463	0.1523	0.6964

Table 3.5: ANOVA table for test HH17.

Because the P-value is 0.6964, which is greater than the significance level of 0.05. Then, we do not reject the null hypothesis. Then the variable HH17 should not be included in the model. Therefore, the final model from the forward selection is

$$\log(\mathbf{E}) \sim \text{HH01} + \text{HH02} + \text{HH03} + \text{HH10} + \text{HH15} + \text{HH11} + \text{HH32} + \\ \text{HH33} + \text{HH40} + \text{HH37} + \text{HH27} + \text{HH25} + \text{HH34} + \text{HH35} + \text{HH18}.$$

Backward Selection

The backward selection is the method of reversing the forward selection. Begin with all available candidates in the model. Then gradually remove the variable with the least relationship with the welfare variable one by one. Therefore, check the remaining candidate to remove from the model. Continue the process until we cannot remove another variable. The last model is the model with the smallest AIC. Algorithm 3 presents the algorithm for variable selection using the backward selection technique.

Algorithm 3 Algorithm for backward variable selection.

- 1: Begin with all candidate variables in the model.
 - 2: Drop one variable from the model and compute AIC of the new model. Do this for all variables. Compare the AIC values of all model and choose the one with the smallest AIC. If there is no selection, go to Step 4.
 - 3: Delete one more significant candidate variable. Compare the AIC values of all models that delete the variable in Step 2 and delete another one remaining variable.
 - 4: Delete the new variable if the model without this candidate gives the smallest AIC value when the variable in Step 2 is already delete from the model. If there is no selection go to Step 4. If there are more than three variables for deleting in the model go to Step 2.
 - 5: Stop the process for backward variable selection.
-

The result from applying the backward selection to Socio-Economic Survey data is presented in Table 3.6.

Step	Remove		Model	AIC
1	No	$\log(\mathbf{E}) \sim$	HH01 + HH02 + HH03 + HH10 + HH15 + HH11 + HH32 + HH33 + HH40 + HH37 + HH27 + HH25 + HH34 + HH35 + HH18 + HH17	-63925
2	HH17	$\log(\mathbf{E}) \sim$	HH01 + HH02 + HH03 + HH10 + HH15 + HH11 + HH32 + HH33 + HH40 + HH37 + HH27 + HH25 + HH34 + HH35 + HH18	-63927
3	HH01	$\log(\mathbf{E}) \sim$	HH02 + HH03 + HH10 + HH15 + HH11 + HH32 + HH33 + HH40 + HH37 + HH27 + HH25 + HH34 + HH35 + HH18	-63582
	HH02	$\log(\mathbf{E}) \sim$	HH01 + HH03 + HH10 + HH15 + HH11 + HH32 + HH33 + HH40 + HH37 + HH27 + HH25 + HH34 + HH35 + HH18	-63607
	HH03	$\log(\mathbf{E}) \sim$	HH01 + HH02 + HH10 + HH15 + HH11 + HH32 + HH33 + HH40 + HH37 + HH27 + HH25 + HH34 + HH35 + HH18	-62982
	HH10	$\log(\mathbf{E}) \sim$	HH01 + HH02 + HH03 + HH15 + HH11 + HH32 + HH33 + HH40 + HH37 + HH27 + HH25 + HH34 + HH35 + HH18	-61491
	HH15	$\log(\mathbf{E}) \sim$	HH01 + HH02 + HH03 + HH10 + HH11 + HH32 + HH33 + HH40 + HH37 + HH27 + HH25 + HH34 + HH35 + HH18	-63420
	HH11	$\log(\mathbf{E}) \sim$	HH01 + HH02 + HH03 + HH10 + HH15 + HH32 + HH33 + HH40 + HH37 + HH27 + HH25 + HH34 + HH35 + HH18	-63294
	HH32	$\log(\mathbf{E}) \sim$	HH01 + HH02 + HH03 + HH10 + HH15 + HH11 + HH33 + HH40 + HH37 + HH27 + HH25 + HH34 + HH35 + HH18	-63454
	HH33	$\log(\mathbf{E}) \sim$	HH01 + HH02 + HH03 + HH10 + HH15 +	-63646

Continued on next page...

Table 3.6 – continued from previous page.

Step	Remove	Model	AIC
		HH11 + HH32 + HH40 + HH37 + HH27 + HH25 + HH34 + HH35 + HH18	
HH40	$\log(\mathbf{E}) \sim$	HH01 + HH02 + HH03 + HH10 + HH15 + HH11 + HH32 + HH33 + HH37 + HH27 + HH25 + HH34 + HH35 + HH18	-61532
HH37	$\log(\mathbf{E}) \sim$	HH01 + HH02 + HH03 + HH10 + HH15 + HH11 + HH32 + HH33 + HH40 + HH27 + HH25 + HH34 + HH35 + HH18	-63511
HH27	$\log(\mathbf{E}) \sim$	HH01 + HH02 + HH03 + HH10 + HH15 + HH11 + HH32 + HH33 + HH40 + HH37 + HH25 + HH34 + HH35 + HH18	-63831
HH25	$\log(\mathbf{E}) \sim$	HH01 + HH02 + HH03 + HH10 + HH15 + HH11 + HH32 + HH33 + HH40 + HH37 + HH27 + HH34 + HH35 + HH18	-63346
HH34	$\log(\mathbf{E}) \sim$	HH01 + HH02 + HH03 + HH10 + HH15 + HH11 + HH32 + HH33 + HH40 + HH37 + HH27 + HH25 + HH35 + HH18	-63897
HH35	$\log(\mathbf{E}) \sim$	HH01 + HH02 + HH03 + HH10 + HH15 + HH11 + HH32 + HH33 + HH40 + HH37 + HH27 + HH25 + HH34 + HH18	-63246
HH18	$\log(\mathbf{E}) \sim$	HH01 + HH02 + HH03 + HH10 + HH15 + HH11 + HH32 + HH33 + HH40 + HH37 + HH27 + HH25 + HH34 + HH35	-62482
End	The final model is remove only HH17 from all candidate.		

Table 3.6: The variable selection by using a backward selection technique with the Socio-Economic Survey data.

We need to make sure whether the remaining candidate variables, which are HH01, HH02, HH03, HH10, HH15, HH11, HH32, HH33, HH40, HH37, HH27, HH25, HH34, HH35 and HH18, should be removed from the model by analyzing of variance (ANOVA) test.

Initial Model: $\log(\mathbf{E}) \sim \text{HH01} + \text{HH02} + \text{HH03} + \text{HH10} + \text{HH15} + \text{HH11} + \text{HH32} + \text{HH33} + \text{HH40} + \text{HH37} + \text{HH27} + \text{HH25} + \text{HH34} + \text{HH35} + \text{HH18}$

Null hypothesis: Initial model with removing each below variable

Alternative hypothesis: Initial model

Table 3.7 shows the ANOVA Table for test covariates.

Remove	Res.Df	RSS	Df	Sum of Square	F	P-value
No	43155	9815.4				
HH01	43161	9896.8	6	81.357	59.616	<2.2E-16
HH02	43160	9890.6	5	75.159	66.09	<2.2E-16
HH03	43162	10035.8	7	220.34	138.39	<2.2E-16
HH10	43160	10387.2	5	571.8	502.8	<2.2E-16
HH15	43158	9932.6	3	117.15	171.7	<2.2E-16
HH11	43163	9964	8	148.52	81.625	<2.2E-16
HH32	43156	9923.9	1	108.48	476.95	<2.2E-16
HH33	43156	9880	1	64.563	283.86	<2.2E-16
HH40	43156	10375.4	1	559.92	2461.8	<2.2E-16
HH37	43156	9910.9	1	95.424	419.55	<2.2E-16
HH27	43156	9837.7	1	22.272	97.922	<2.2E-16
HH25	43156	9948.9	1	133.44	586.67	<2.2E-16
HH34	43156	9822.6	1	7.184	31.586	1.92E-08
HH35	43156	9971.8	1	156.39	687.58	<2.2E-16
HH18	43156	10149.8	1	334.33	1469.9	<2.2E-16

Table 3.7: ANOVA Table for test covariates.

Table 3.7 shows that the P-values of all cases are smaller than the significance level of 0.05. Then, we reject the null hypothesis. Therefore, these variables should not be removed from the model. Therefore, the final model from the backward selection is

$$\log(\mathbf{E}) \sim \text{HH01} + \text{HH02} + \text{HH03} + \text{HH10} + \text{HH15} + \text{HH11} + \text{HH32} + \\ \text{HH33} + \text{HH40} + \text{HH37} + \text{HH27} + \text{HH25} + \text{HH34} + \text{HH35} + \text{HH18}.$$

From Table 3.4 - Table 3.7, we can see that the forward selection and the backward selection give the same set of auxiliary variables. The final auxiliary variables are presented in Table 3.8

Name of included auxiliary variables	
1	Type of dwelling
2	Materials of construction
3	Tenure
4	Cooking fuel
5	Toilet facilities
6	Drinking water
	Number of items owned by household
7	– LCD/LED/Plasma
8	– Video/VCD/DVD Player
9	– Mobile phone
10	– Home computer
11	– Refrigerator
12	– Microwave oven
13	– Washing machine
14	– Air conditioner
15	– Automobile

Table 3.8: The list of included auxiliary variables.

Since we have both qualitative and quantitative auxiliary variables, we must treat these variables to matrix form before putting in the model. If there are g types of qualitative variables, we can create $g - 1$ different independent variables. If $i = 1, \dots, g$ is the index of variables, the $(i - 1)th$ is 1 and the rest are 0.

For example, the type of dwelling variable is classified into 7 lists which are detached house, row house, townhouse/twin house, flat or apartment, room, improvised quarter and others. Then, we can create 6 new variables that represent the type of dwelling without using 7 variables, which makes the redundancy. Therefore, the first type of qualitative variable is unnecessary. For instance, if Mr.Chan has a dwelling of type 3. Then the 2nd variable is 1 and the rest are 0. Hence the corresponding vector of a dwelling of Mr.Chan is $[0 \ 1 \ 0 \ 0 \ 0 \ 0]'$. We construct the vector of all categorical variables. Then we will get the 44-vector of auxiliary variables with the intercept term.

For more clarification, the matrix of auxiliary variables can be presented as

- $\mathbf{x}_{(1)}$ is the vector of code row house in type of dwelling variable
- $\mathbf{x}_{(2)}$ is the vector of code town house or twin house in type of dwelling variable
- $\mathbf{x}_{(3)}$ is the vector of code flat or apartment and etc. in type of dwelling variable
- $\mathbf{x}_{(4)}$ is the vector of code room or rooms in type of dwelling variable
- $\mathbf{x}_{(5)}$ is the vector of code improvised quarter in type of dwelling variable
- $\mathbf{x}_{(6)}$ is the vector of code others in type of dwelling variable
- $\mathbf{x}_{(7)}$ is the vector of code wood in materials of construction variable

- $\mathbf{x}_{(8)}$ is the vector of code wood and cement or brick in materials of construction variable
- $\mathbf{x}_{(9)}$ is the vector of code local materials in materials of construction variable
- $\mathbf{x}_{(10)}$ is the vector of code re-used materials in materials of construction variable
- $\mathbf{x}_{(11)}$ is the vector of code others in materials of construction variable
- $\mathbf{x}_{(12)}$ is the vector of code own dwelling on rented land in tenure variable
- $\mathbf{x}_{(13)}$ is the vector of code own dwelling on public in tenure variable
- $\mathbf{x}_{(14)}$ is the vector of code hire - purchased in tenure variable
- $\mathbf{x}_{(15)}$ is the vector of code rent in tenure variable
- $\mathbf{x}_{(16)}$ is the vector of code rent paid by others in tenure variable
- $\mathbf{x}_{(17)}$ is the vector of code occupied or rented free in tenure variable
- $\mathbf{x}_{(18)}$ is the vector of code others in tenure variable
- $\mathbf{x}_{(19)}$ is the vector of code charcoal in cooking fuel variable
- $\mathbf{x}_{(20)}$ is the vector of code wood in cooking fuel variable
- $\mathbf{x}_{(21)}$ is the vector of code kerosene in cooking fuel variable
- $\mathbf{x}_{(22)}$ is the vector of code gas in cooking fuel variable
- $\mathbf{x}_{(23)}$ is the vector of code electricity in cooking fuel variable
- $\mathbf{x}_{(24)}$ is the vector of code flush latrine in toilet facilities variable
- $\mathbf{x}_{(25)}$ is the vector of code squat in toilet facilities variable

- $\mathbf{x}_{(26)}$ is the vector of code bath flush and squat latrine in toilet facilities variable
- $\mathbf{x}_{(27)}$ is the vector of code inside piped water supply in drinking water variable
- $\mathbf{x}_{(28)}$ is the vector of code inside piped underground water in drinking water variable
- $\mathbf{x}_{(29)}$ is the vector of code outside piped or public tap in drinking water variable
- $\mathbf{x}_{(30)}$ is the vector of code well or underground water in drinking water variable
- $\mathbf{x}_{(31)}$ is the vector of code river or stream and etc. in drinking water variable
- $\mathbf{x}_{(32)}$ is the vector of code rain water in drinking water variable
- $\mathbf{x}_{(33)}$ is the vector of code treated tap water in drinking water variable
- $\mathbf{x}_{(34)}$ is the vector of code others in drinking water variable
- $\mathbf{x}_{(35)}$ is the vector of number of LCD/LED/PLASMA owned variable
- $\mathbf{x}_{(36)}$ is the vector of number of video/VCD/DVD player owned variable
- $\mathbf{x}_{(37)}$ is the vector of number of mobile phone owned variable
- $\mathbf{x}_{(38)}$ is the vector of number of home computer owned variable
- $\mathbf{x}_{(39)}$ is the vector of number of refrigerator owned variable
- $\mathbf{x}_{(40)}$ is the vector of number of microwave oven owned variable
- $\mathbf{x}_{(41)}$ is the vector of number of washing machine owned variable
- $\mathbf{x}_{(42)}$ is the vector of number of air conditioner owned variable
- $\mathbf{x}_{(43)}$ is the vector of number of automobile owned variable

3.2 Methodology

This section gives a short review of the three methods for poverty mapping discussed in this thesis: the Elbers, Lanjouw, and Lanjouw (ELL) method, the Empirical Bayes (EB) method, and the Hierarchical Bayes (HB) method. Consider the population P of size N partitioned into $D(= 77)$ provinces having population sizes N_1, \dots, N_D . We draw for L sets of samples. The vector \mathbf{Y}_d of the logarithm of average monthly total expenditure per capita for area d is divided into in-sample and out-of-sample data. For each $l = 1, \dots, L(= 100)$,

$$\mathbf{Y}_d^{(l)} = \begin{pmatrix} \mathbf{Y}_{ds}^{(l)} \\ \mathbf{Y}_{dr}^{(l)} \end{pmatrix},$$

where $\mathbf{Y}_d^{(l)}$ is decomposed into 30% for in-sample data $\mathbf{Y}_{ds}^{(l)}$ with sample size n_d and the remaining 70% for out-of-sample data $\mathbf{Y}_{dr}^{(l)}$ with out-of-sample size $N_d - n_d$ for province d .

Figure 3.2 shows the simple random sampling without replacement for 77 provinces and Figure 3.3 shows the in-sample and out-of-sample size for each province in Thailand.

Province	Population	
	In-sample	Out-of-sample
$d = 1$	30% (sizes n_1)	70% (sizes $N_1 - n_1$)
$d = 2$	30% (sizes n_2)	70% (sizes $N_2 - n_2$)
\vdots	\vdots	\vdots
$d = 77$	30% (sizes n_{77})	70% (sizes $N_{77} - n_{77}$)

Figure 3.2: The simple random sampling without replacement for 77 provinces.



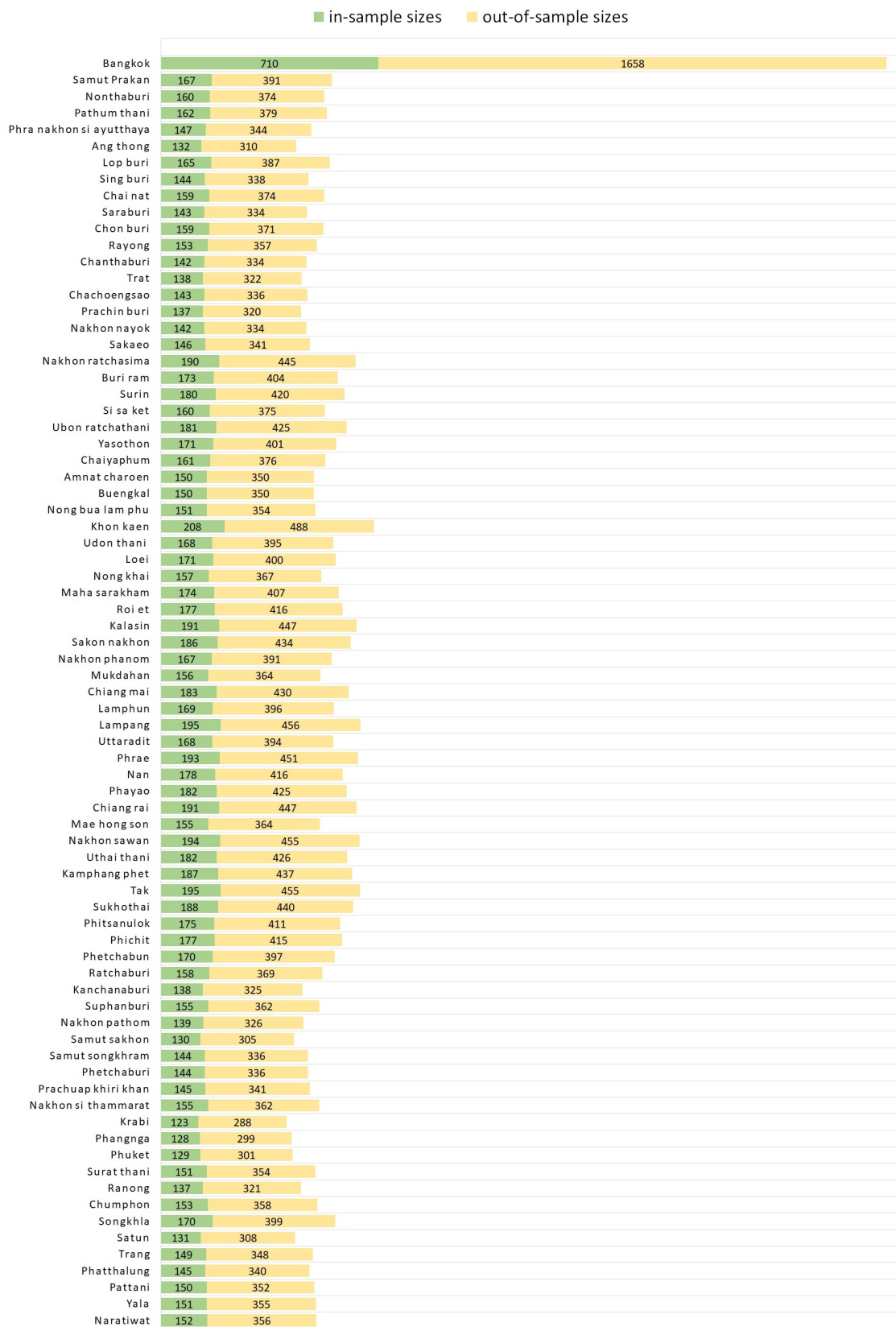


Figure 3.3: Bar chart for in-sample and out-of-sample sizes.

After that, apply the following Algorithm to produce the FGT poverty indicators from SES 2017.

Algorithm 4 Algorithm for the FGT poverty indicator from SES 2017.

1: Calculate the poverty incidence ($\alpha = 0$) for area d , F_{0d} , as

$$F_{0d} = \frac{1}{N_d} \sum_{i=1}^{N_d} I(\exp(Y_{di}) < z_d), \quad d = 1, \dots, D$$

2: Calculate the poverty gap ($\alpha = 1$) for area d , F_{1d} , as

$$F_{1d} = \frac{1}{N_d} \sum_{i=1}^{N_d} \left(\frac{z_d - \exp(Y_{di})}{z_d} \right) I(\exp(Y_{di}) < z_d), \quad d = 1, \dots, D$$

3: Calculate the poverty severity ($\alpha = 2$) for area d , F_{2d} , as

$$F_{2d} = \frac{1}{N_d} \sum_{i=1}^{N_d} \left(\frac{z_d - \exp(Y_{di})}{z_d} \right)^2 I(\exp(Y_{di}) < z_d), \quad d = 1, \dots, D$$

4: **end**

Figure 3.4 - Figure 3.6 show the average poverty incidence, poverty gap and poverty severity from SES 2017, respectively.

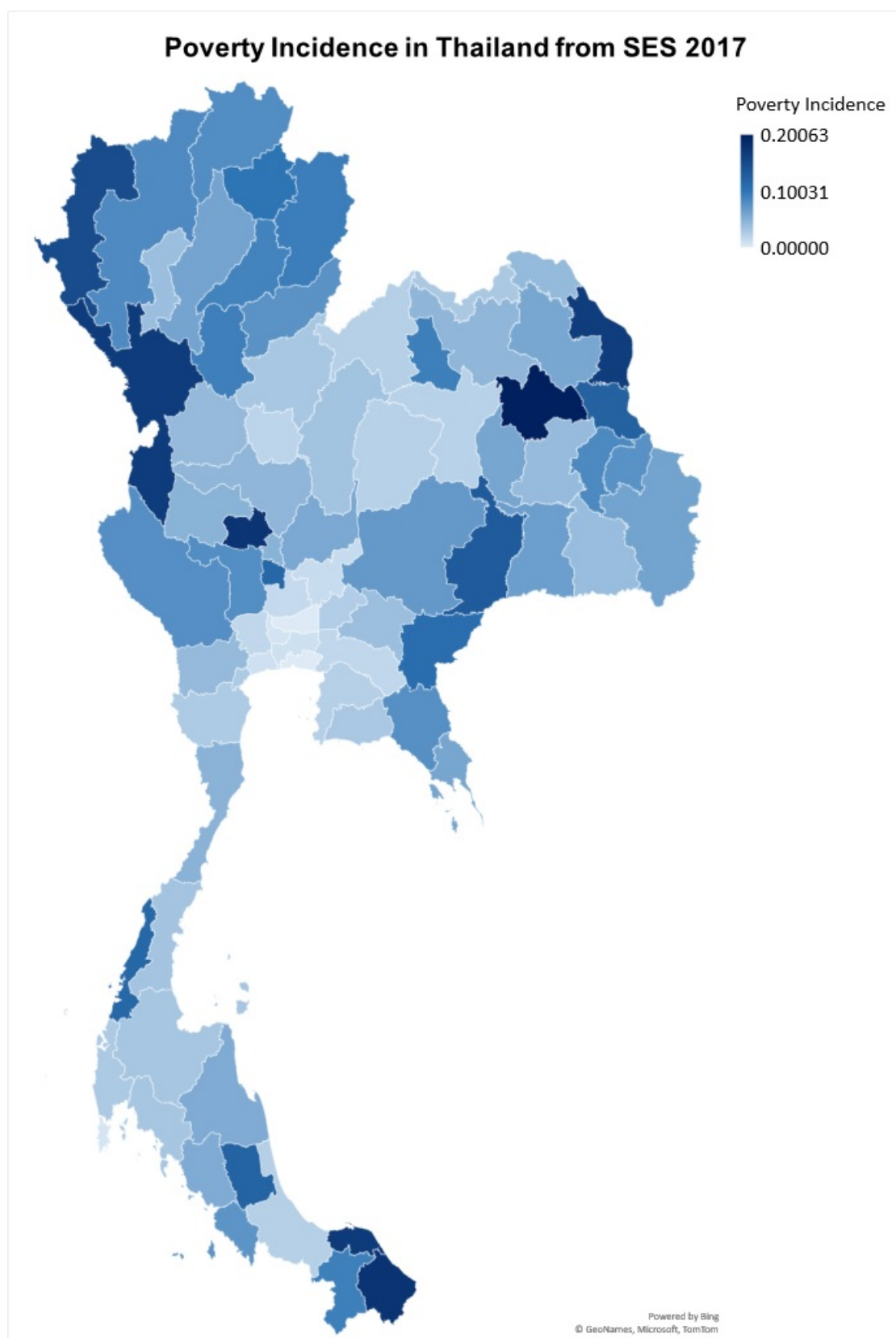


Figure 3.4: Average poverty incidence in Thailand from SES 2017.

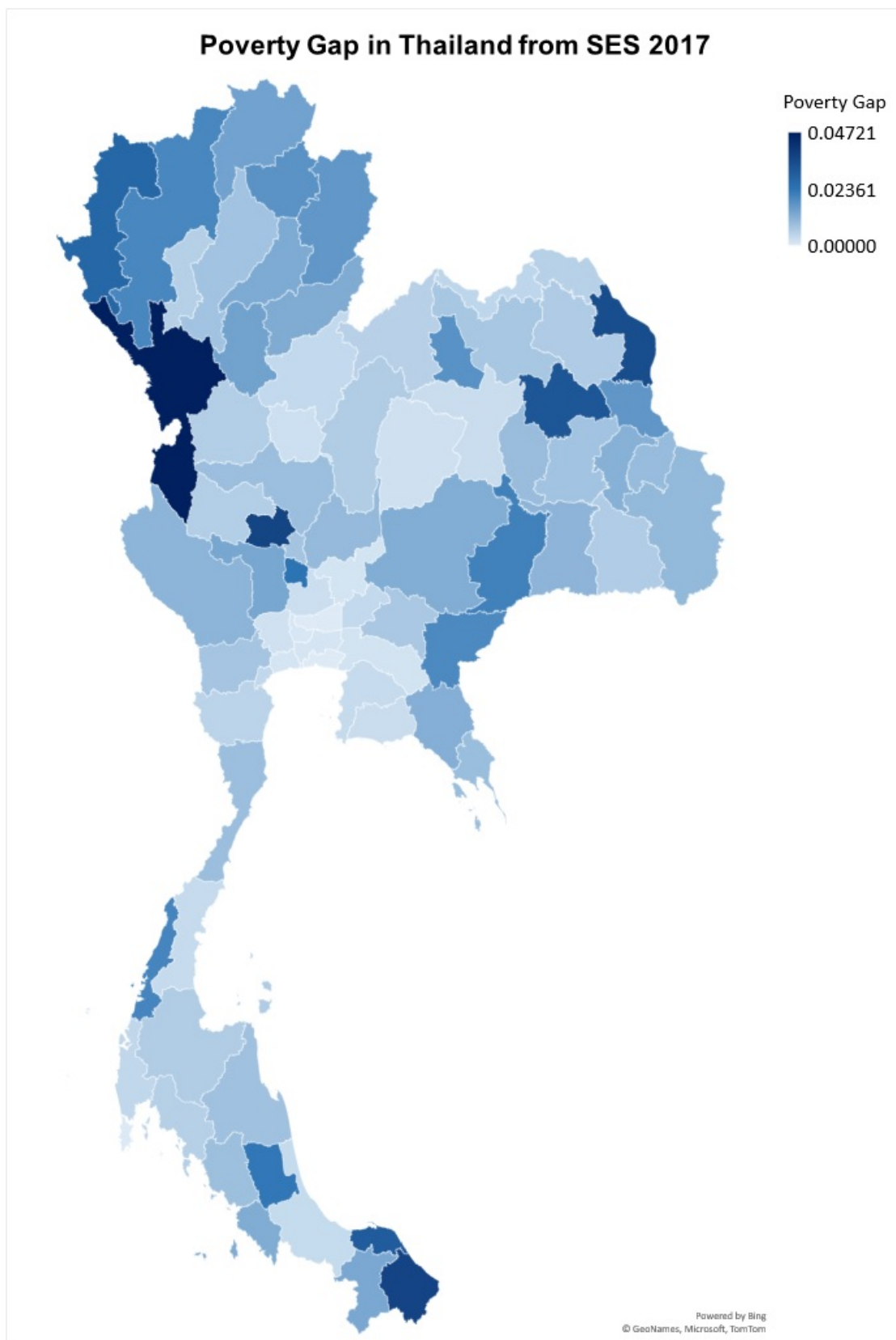


Figure 3.5: Average poverty gap in Thailand from SES 2017.

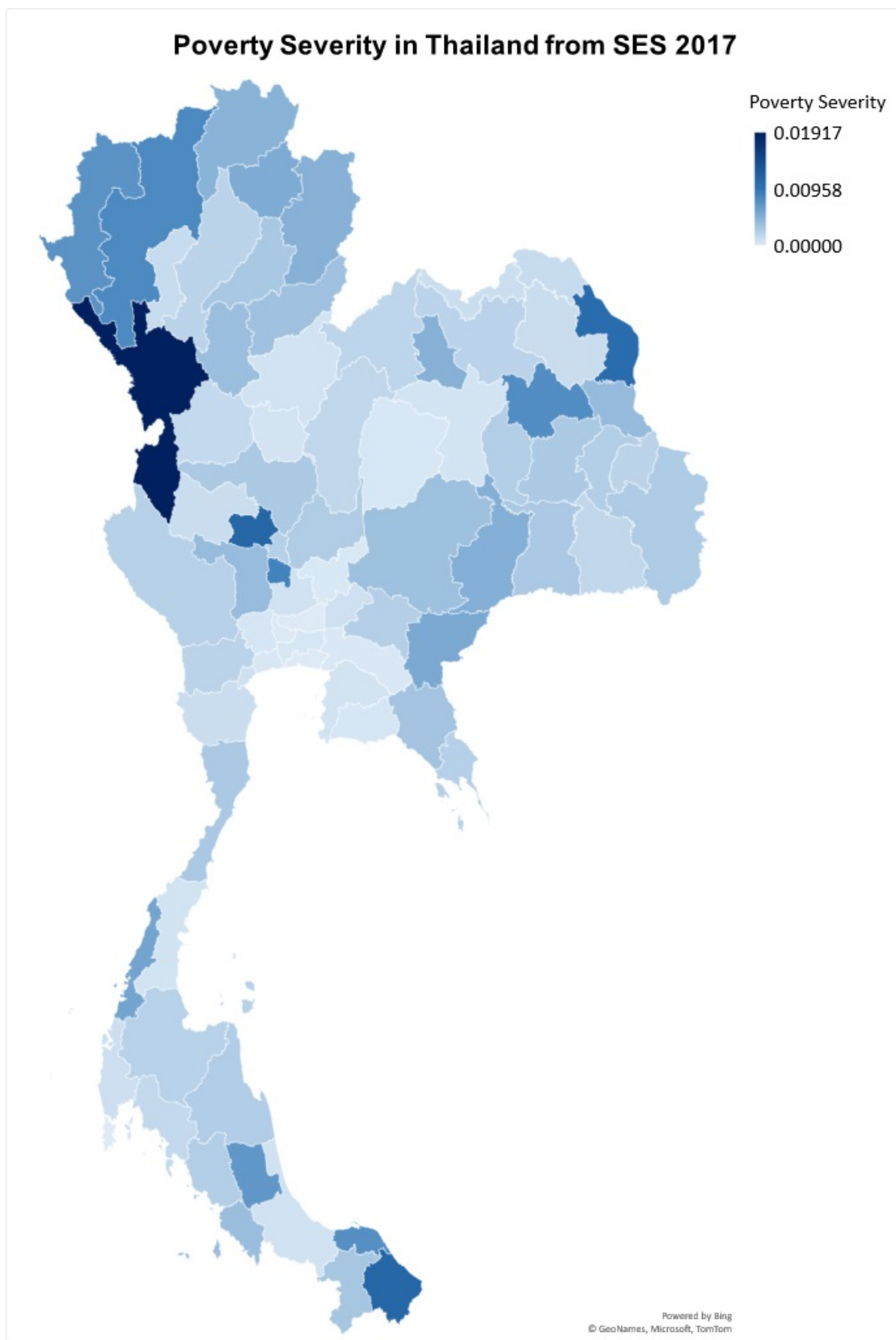


Figure 3.6: Average poverty severity in Thailand from SES 2017.

In Figure 3.4 - Figure 3.6, the dark-colored provinces correspond to the provinces with a high value of the FGT poverty indicators. In another direction, the light-colored provinces correspond to the provinces with a low value of the FGT poverty indicators.

From Figure 3.4, the dark provinces have large proportions of poor people. That is, Northeastern region has many poor while the Central region is relatively small. We can see that Kalasin has the largest proportion of the poor people in Thailand.

From Figure 3.5, the dark provinces have large of the poverty gaps. We can see that Tak has the largest gap of poor people, that is poor people are far from the poverty line, followed by Narathiwat, Chai Nat, Nakhon Phanom and Kalasin.

From Figure 3.6, the dark provinces have large of poverty severities, which adds more weight to poor people. We can see that Tak is still the most violent province of poverty.

Next, we present the three methods, which are the Elbers, Lanjouw, and Lanjouw (ELL) method, the Empirical Bayes (EB) method, and the Hierarchical Bayes (HB) method that will be applied to the FGT poverty indicators as follows.

3.2.1 ELL Method

The ELL method of Elbers et al. (2013) assumes the regression model in the form of the log-transformation of the welfare variable and auxiliary variables of individuals in the population. The model can be seen as the nested error model of Battese et al. (1988), presented in Section 2.5. Let E_{di} be the average total expenditure per capita for individual i in province d . That is, the nested error model of the logarithm of expenditure $Y_{di} = \log(E_{di})$ is defined as

$$Y_{di} = \mathbf{x}'_{di}\boldsymbol{\beta} + u_d + e_{di}, \quad i = 1, \dots, N_d, \quad d = 1, \dots, D \quad (3.1)$$

where \mathbf{x}_{di} is a vector of auxiliary variables for individual i in province d shown in Table 3.8, $\boldsymbol{\beta}$ is a vector of regression coefficients, $u_d \stackrel{iid}{\sim} N(0, \sigma_u^2)$ is random effect for area d , $e_{di} \stackrel{ind}{\sim} N(0, \sigma_e^2 k_{di}^2)$ is a residual error for individual i in area d , u_d and e_{di} are independent and k_{di} is a known constant.

The ELL estimator of $F_{\alpha d}$ is given by the marginal expectation $\hat{F}_{\alpha d}^{ELL} = E[F_{\alpha d}]$ under model (3.1) and is approximated by a bootstrap method. Therefore, use sample data to fit model (2.9) and then generate the new census of \mathbf{Y} .

Accordingly, a bootstrap census of \mathbf{Y} for Q bootstrap sample size is generated as

$$Y_{di}^* = \mathbf{x}'_{di}\hat{\boldsymbol{\beta}} + u_d^* + e_{di}^*, \quad i = 1, \dots, N_d, \quad d = 1, \dots, D, \quad (3.2)$$

where $\hat{\boldsymbol{\beta}}$ is an estimator of $\boldsymbol{\beta}$, $u_d^* \stackrel{iid}{\sim} N(0, \hat{\sigma}_u^2)$ is the new random effect for area d and $e_{di}^* \stackrel{ind}{\sim} N(0, \hat{\sigma}_e^2 k_{di}^2)$ is the new regression error for individual i in area d obtained by fitting model (3.1) to sample data. Doing this process Q times. For each bootstrap

$q = 1, 2, \dots, Q$, the FGT poverty indicators, $F_{\alpha d}^{*(q)}$, can be calculated as

$$F_{\alpha d}^{*(q)} = \frac{1}{N_d} \sum_{i=1}^{N_d} \left(\frac{z_d - \exp(Y_{di}^{*(q)})}{z_d} \right)^{\alpha} I(\exp(Y_{di}^{*(q)}) < z_d). \quad (3.3)$$

Consequently, the ELL estimator of $F_{\alpha d}$ for area d is

$$\hat{F}_{\alpha d}^{ELL} \approx \frac{1}{Q} \sum_{q=1}^Q F_{\alpha d}^{*(q)}. \quad (3.4)$$



3.2.1.1 Algorithm for ELL Method

The algorithm of the ELL method is explained in Algorithm 5 and Figure 3.7 as follows.

Algorithm 5 Algorithm for the FGT poverty indicator of ELL method.

- 1: Decompose population into 30% for in-sample and 70% for out-of-sample

$$\mathbf{Y}_d = \begin{pmatrix} \mathbf{Y}_{ds} \\ \mathbf{Y}_{dr} \end{pmatrix}.$$

- 2: Use in-sample Y_{di} for $i = 1, \dots, n_d$, to fit the model (2.5)

$$Y_{di} = \mathbf{x}'_{di} \boldsymbol{\beta} + u_d + e_{di}.$$

Output: The estimate $\hat{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}$, the estimate $\hat{\sigma}_u^2$ of σ_u^2 , the estimate $\hat{\sigma}_e^2$ of σ_e^2 .

- 3: **for** $q : 1$ to $Q (= 500)$ **do**

- 4: Generate $u_d^{*(q)} \stackrel{iid}{\sim} N(0, \hat{\sigma}_u^2)$ for $q = 1, \dots, Q$.

- 5: Generate $e_{di}^{*(q)} \stackrel{ind}{\sim} N(0, \hat{\sigma}_e^2 k_{di}^2)$ with $k_{di} = 1$ for $q = 1, \dots, Q$.

- 6: Generate Q bootstrap census of \mathbf{Y} -values from the model

$$Y_{di}^{*(q)} = \mathbf{x}'_{di} \hat{\boldsymbol{\beta}} + u_d^{*(q)} + e_{di}^{*(q)}, \quad i = 1, \dots, N_d, \quad d = 1, \dots, D, \quad q = 1, \dots, Q.$$

- 7: Calculate the FGT poverty indicators for area $d = 1, \dots, D$, $F_{\alpha d}^{*(q)}$, as

$$F_{\alpha d}^{*(q)} = \frac{1}{N_d} \sum_{i=1}^{N_d} \left(\frac{z_d - \exp(Y_{di}^{*(q)})}{z_d} \right)^{\alpha} I(\exp(Y_{di}^{*(q)}) < z_d), \quad \alpha = 0, 1, 2.$$

- 8: Calculate a bootstrap of the ELL estimator as

$$\hat{F}_{\alpha d}^{ELL} \approx \frac{1}{Q} \sum_{q=1}^Q F_{\alpha d}^{*(q)}.$$

- 9: **end**
-

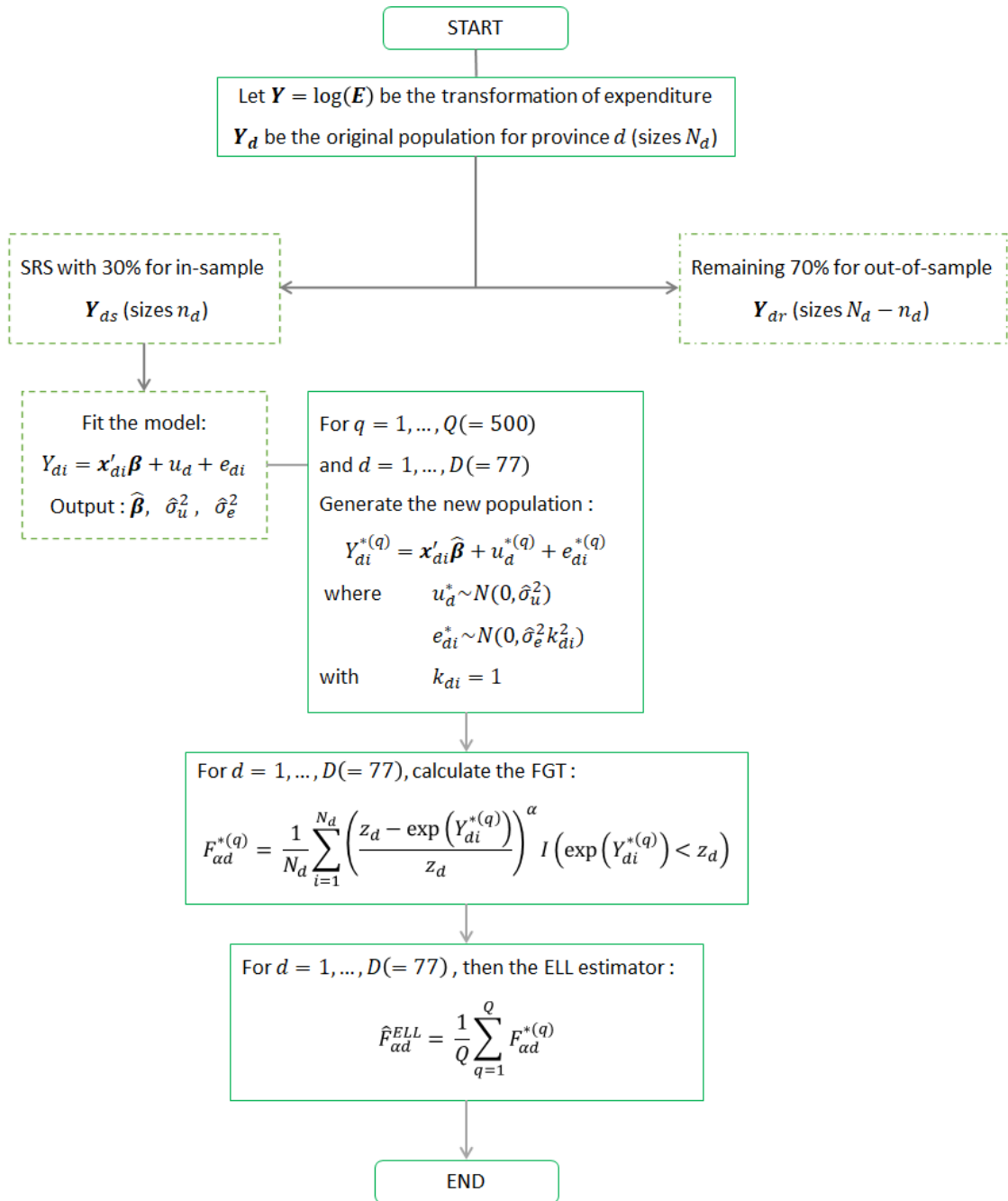
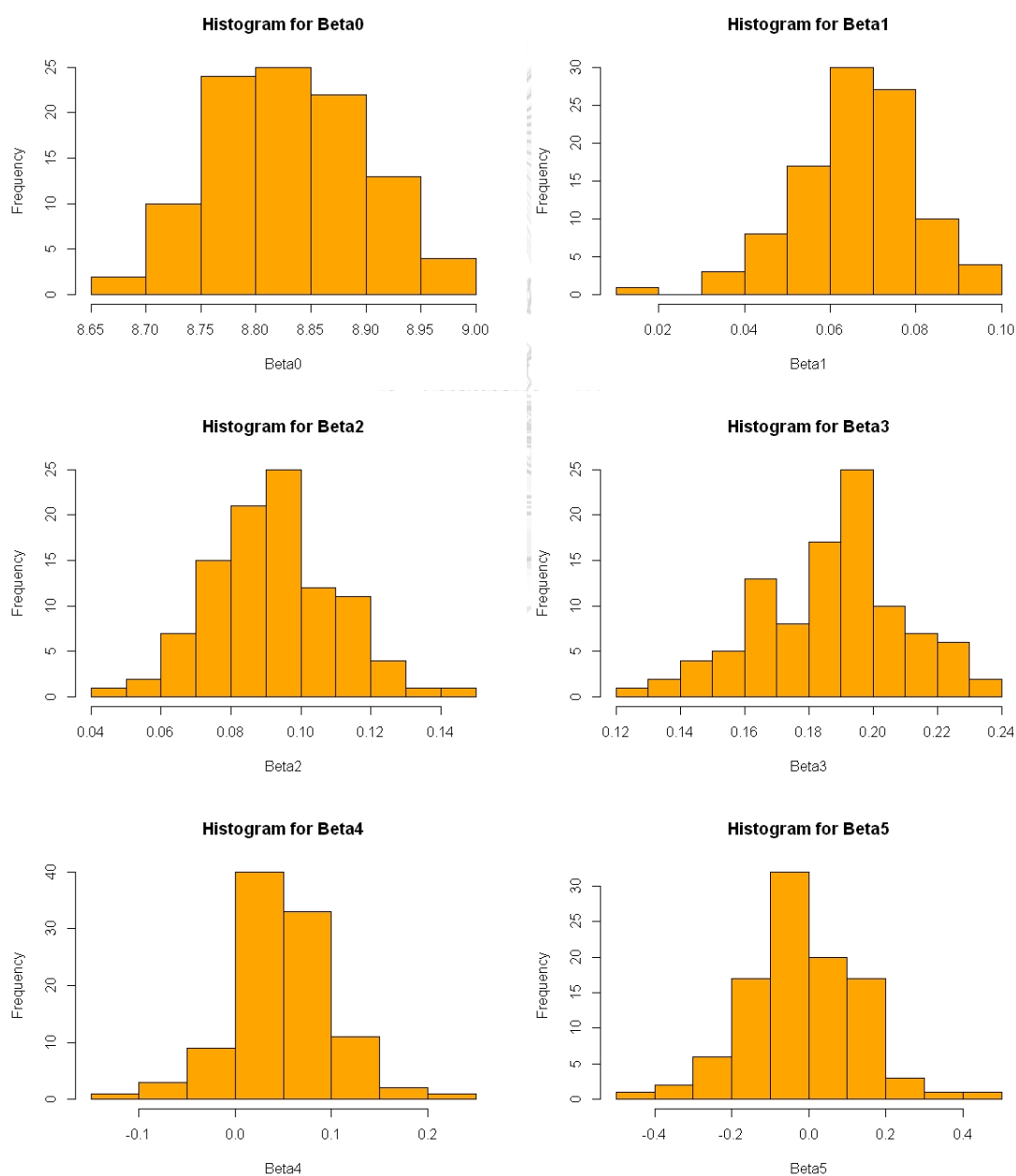


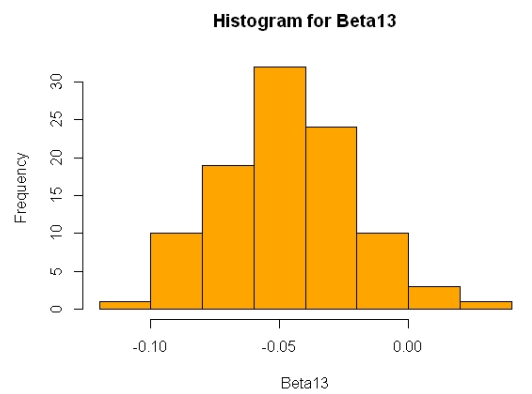
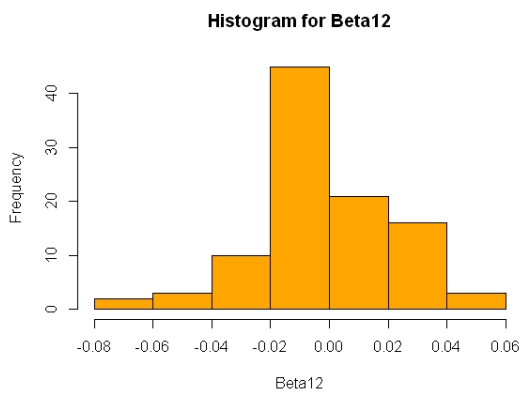
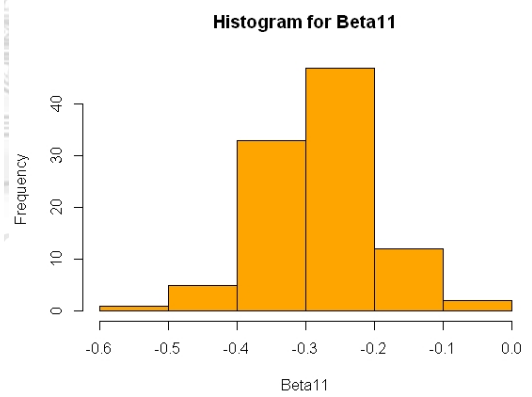
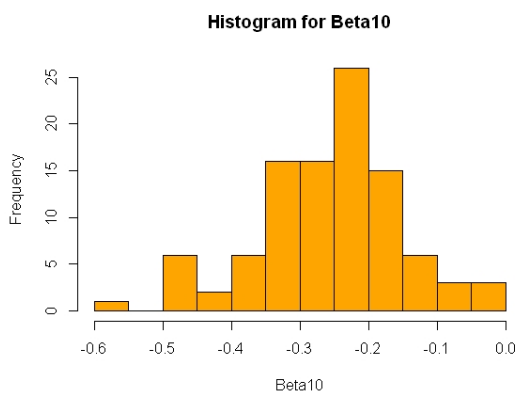
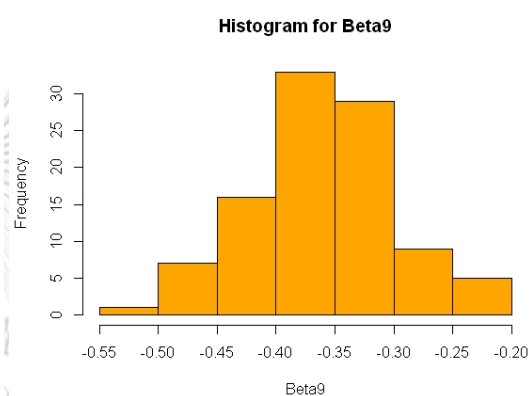
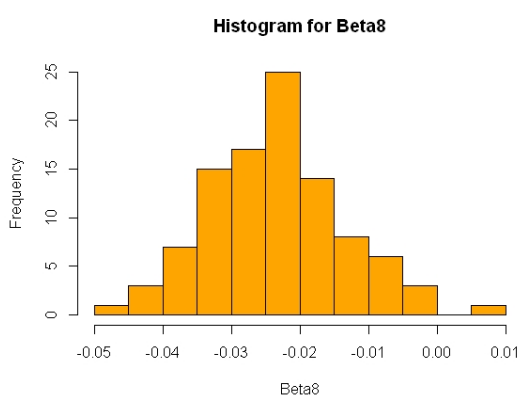
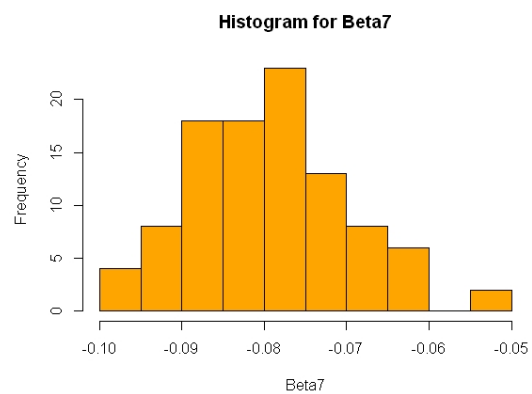
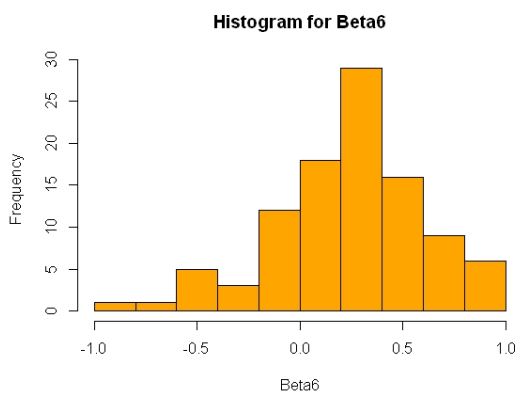
Figure 3.7: Diagram for the ELL method.

3.2.1.2 Result for the ELL method

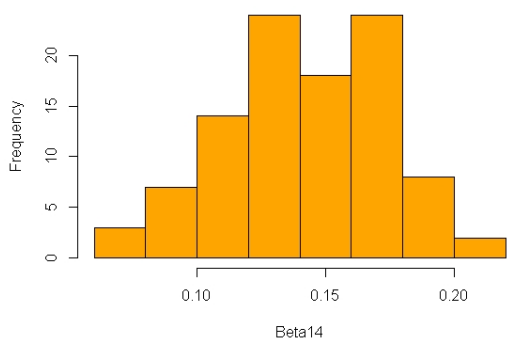
Draw 30% for in-sample data where the remaining 70% for out-of-sample data and doing the process following the diagram and algorithm of the ELL method for $L = 100$ times. Figure 3.8 shows the estimate of β ($\hat{\beta}$), the estimate of σ_u^2 ($\hat{\sigma}_u^2$) and the estimate of σ_e^2 ($\hat{\sigma}_e^2$) for $L = 100$ times in the form of the histogram.

Figure 3.8: The histogram of parameter estimation for $L = 100$.

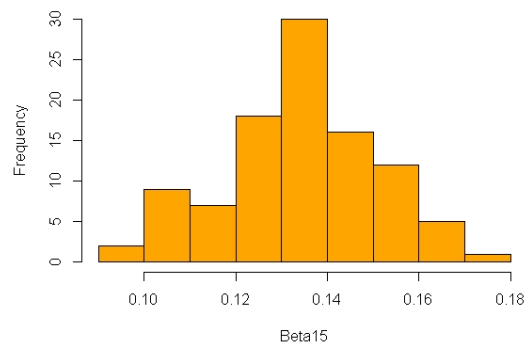




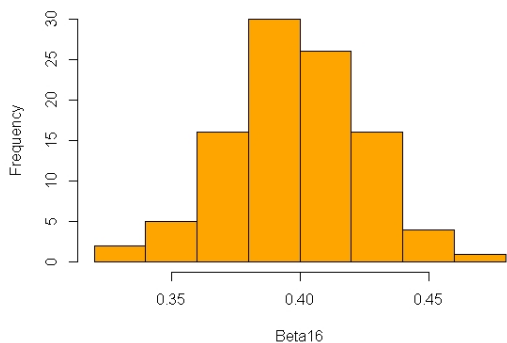
Histogram for Beta14



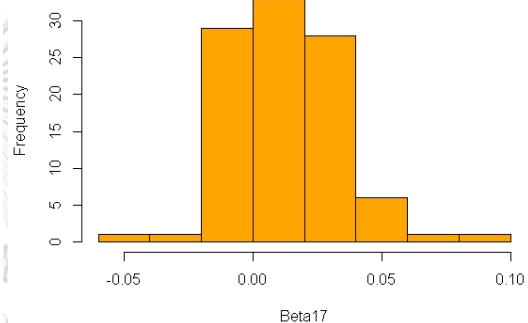
Histogram for Beta15



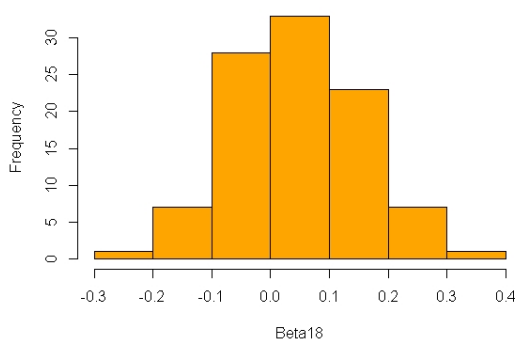
Histogram for Beta16



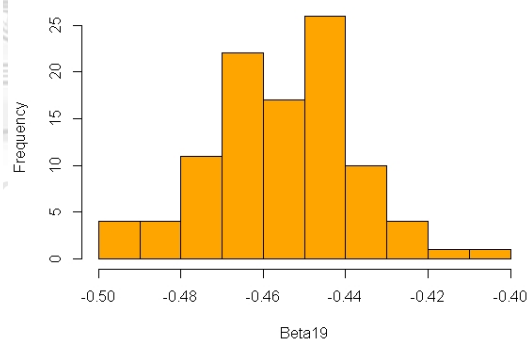
Histogram for Beta17



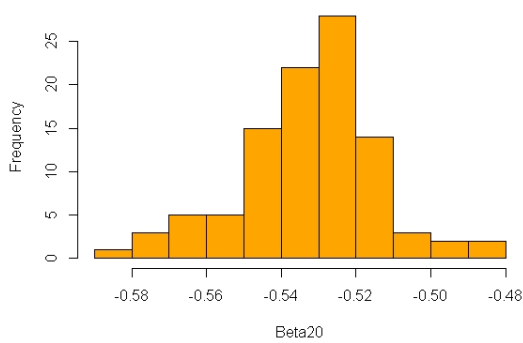
Histogram for Beta18



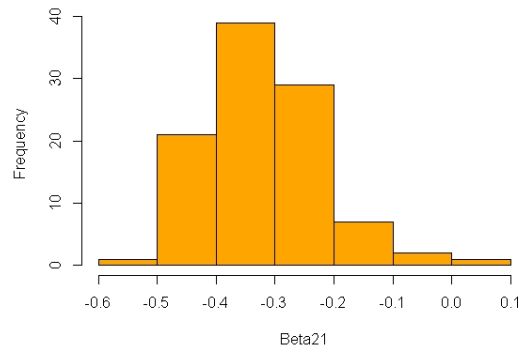
Histogram for Beta19

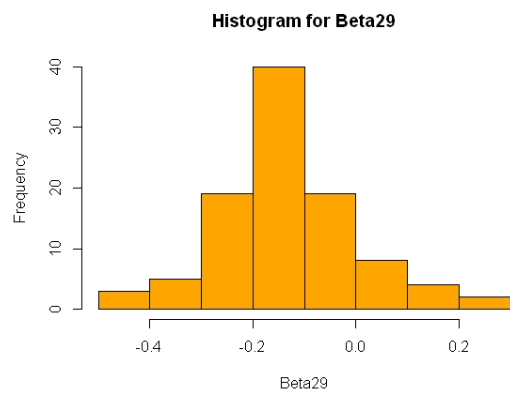
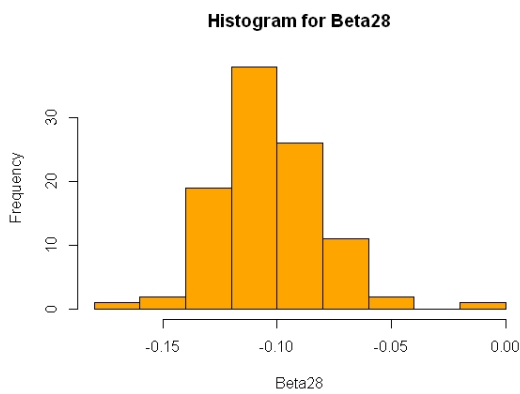
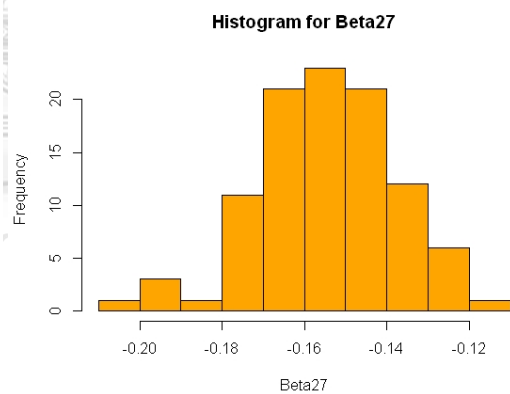
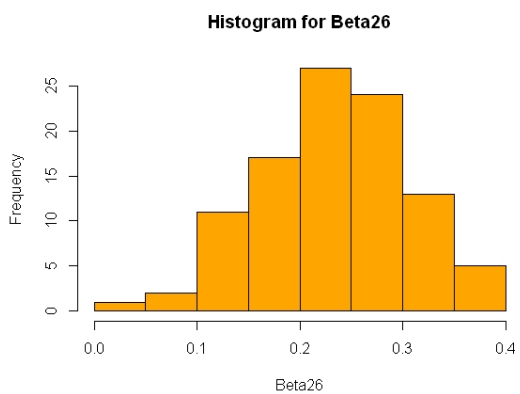
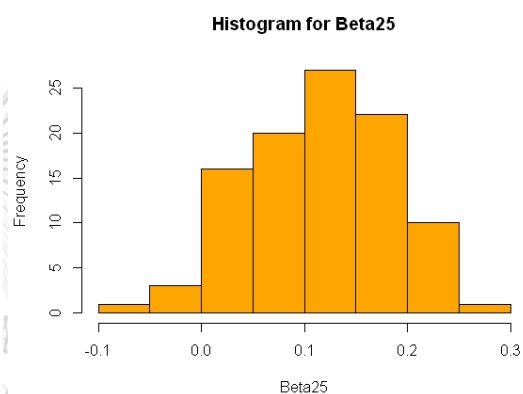
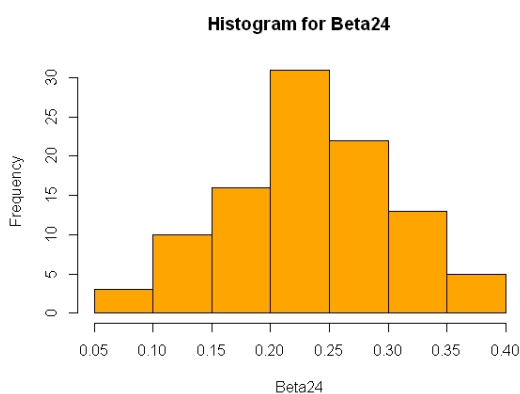
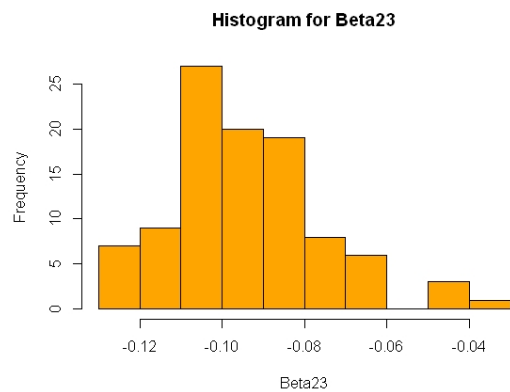
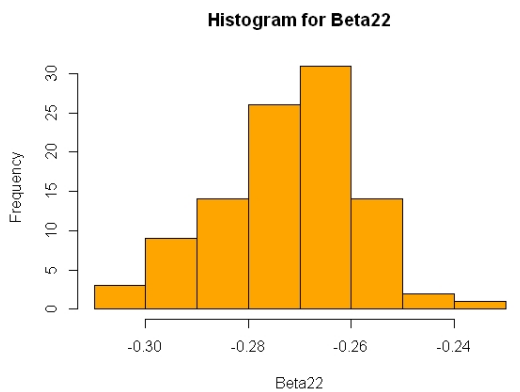


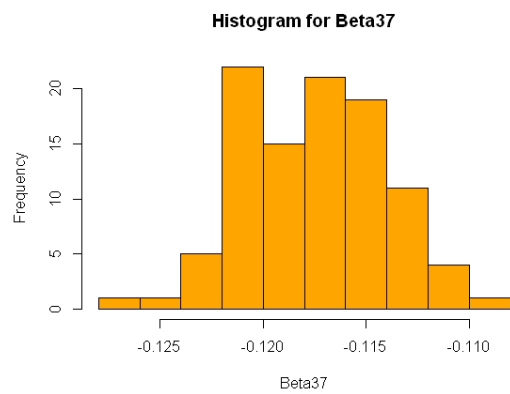
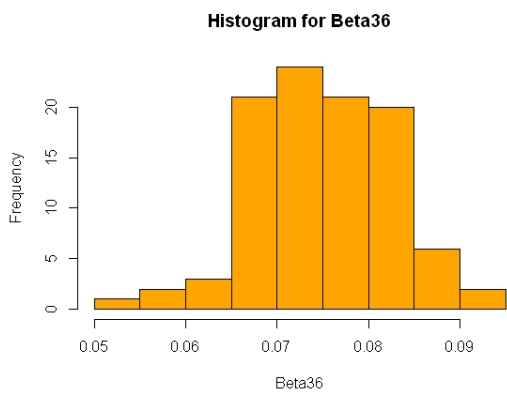
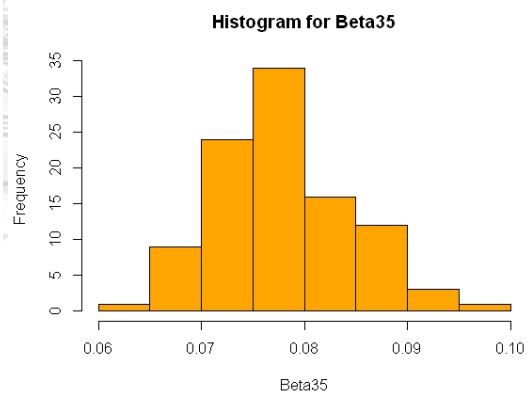
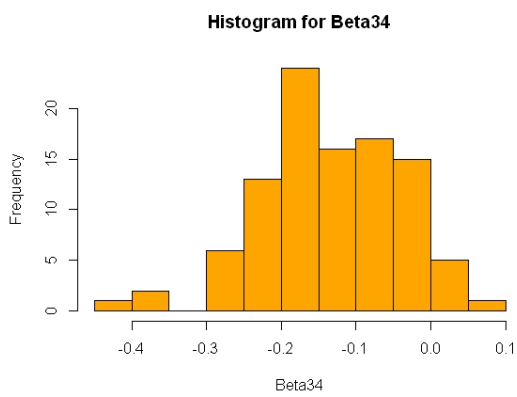
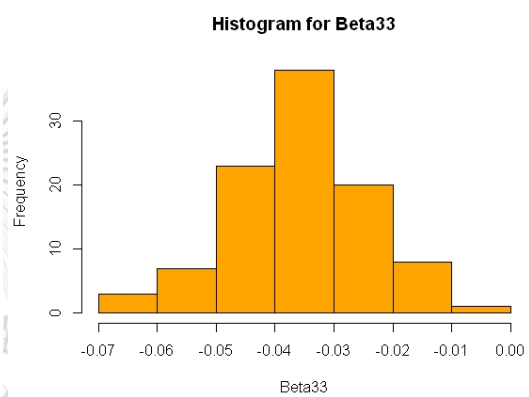
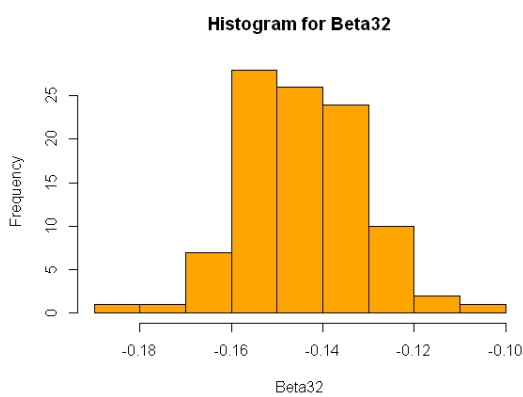
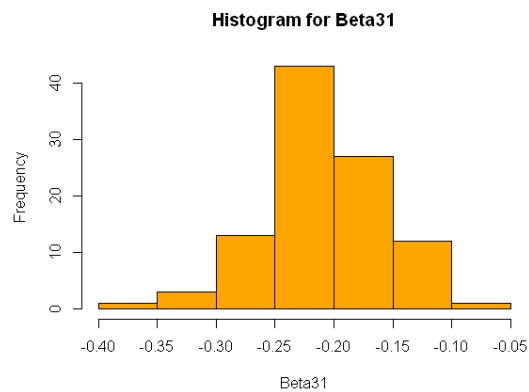
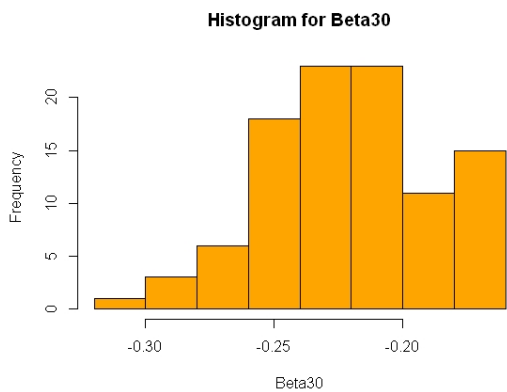
Histogram for Beta20

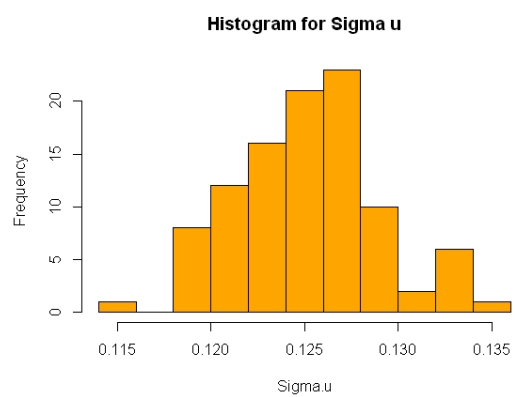
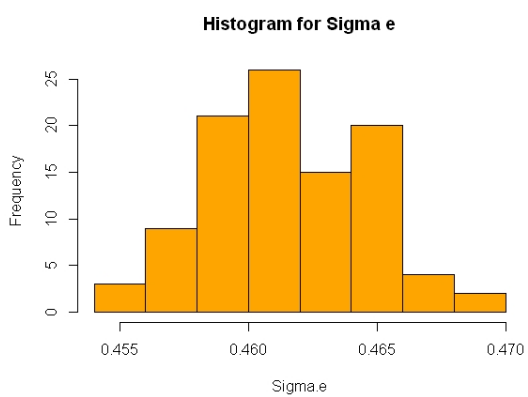
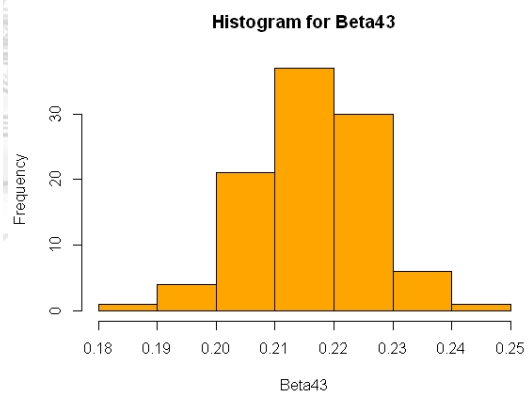
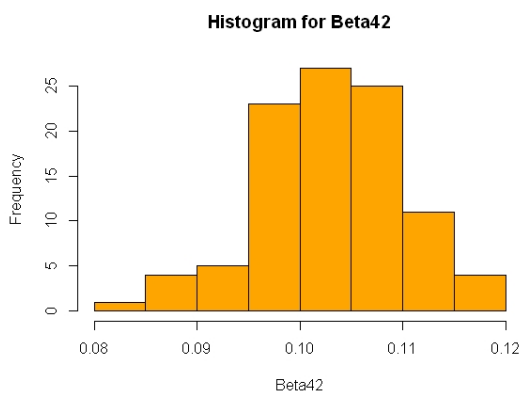
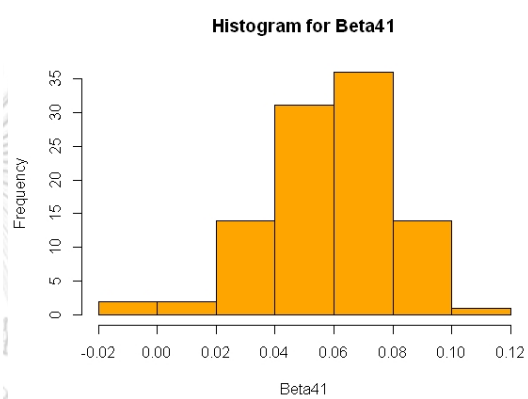
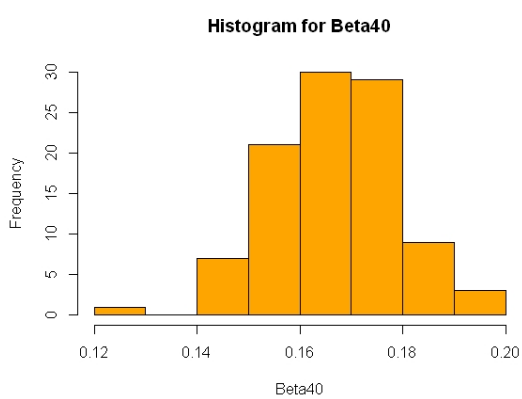
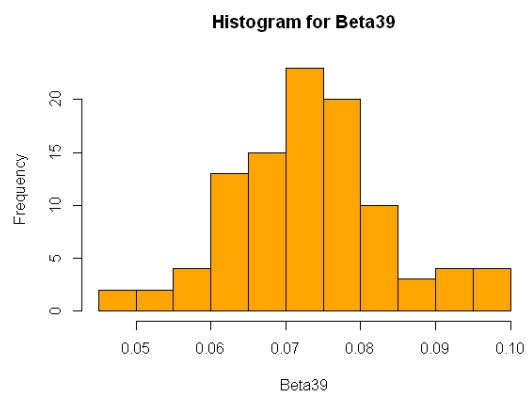
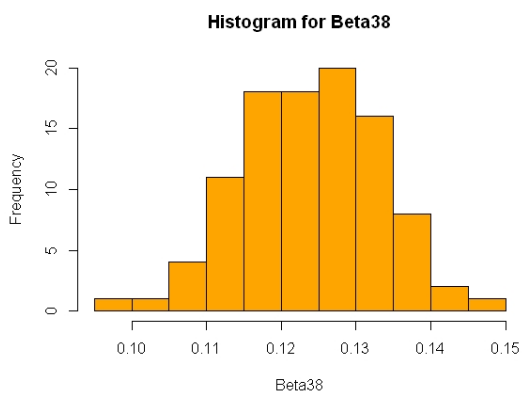


Histogram for Beta21









The corresponding descriptive statistics for all parameters are given in Table 3.9.

Parameter Estimation	Minimum	1st Qu.	Median	Mean	3rd Qu.	Maximum
$\hat{\beta}_0$	8.681	8.768	8.826	8.829	8.886	8.999
$\hat{\beta}_1$	0.018	0.058	0.067	0.066	0.074	0.099
$\hat{\beta}_2$	0.046	0.080	0.092	0.092	0.103	0.140
$\hat{\beta}_3$	0.128	0.173	0.190	0.188	0.200	0.238
$\hat{\beta}_4$	-0.112	0.015	0.046	0.046	0.081	0.200
$\hat{\beta}_5$	-0.409	-0.107	-0.019	-0.014	0.085	0.439
$\hat{\beta}_6$	-0.826	0.066	-0.276	0.250	0.494	0.965
$\hat{\beta}_7$	-0.100	-0.086	-0.079	-0.079	-0.073	-0.050
$\hat{\beta}_8$	-0.047	-0.030	-0.023	-0.023	-0.017	0.006
$\hat{\beta}_9$	-0.508	-0.396	-0.359	-0.362	-0.322	-0.224
$\hat{\beta}_{10}$	-0.583	-0.310	-0.248	-0.256	-0.198	-0.042
$\hat{\beta}_{11}$	-0.503	-0.350	-0.265	-0.277	-0.218	-0.027
$\hat{\beta}_{12}$	-0.066	-0.014	-0.004	-0.002	0.014	0.059
$\hat{\beta}_{13}$	-0.118	-0.064	-0.047	-0.047	-0.032	0.031
$\hat{\beta}_{14}$	0.070	0.125	0.143	0.144	0.168	0.217
$\hat{\beta}_{15}$	0.092	0.124	0.134	0.134	0.145	0.177
$\hat{\beta}_{16}$	0.321	0.380	0.399	0.399	0.416	0.471
$\hat{\beta}_{17}$	-0.050	-0.002	0.011	0.013	0.025	0.087
$\hat{\beta}_{18}$	-0.233	-0.029	0.039	0.045	0.127	0.349
$\hat{\beta}_{19}$	-0.494	-0.467	-0.455	-0.456	-0.445	-0.406
$\hat{\beta}_{20}$	-0.581	-0.543	-0.530	-0.532	-0.521	-0.483
$\hat{\beta}_{21}$	-0.500	-0.386	-0.341	-0.322	-0.267	0.077
$\hat{\beta}_{22}$	-0.309	-0.280	-0.270	-0.272	-0.263	-0.240
$\hat{\beta}_{23}$	-0.129	-0.105	-0.097	-0.094	-0.084	-0.037
$\hat{\beta}_{24}$	0.064	0.191	0.236	0.235	0.286	0.370

Continued on next page...

Table 3.9 – continued from previous page.

Parameter Estimation	Minimum	1st Qu.	Median	Mean	3rd Qu.	Maximum
$\hat{\beta}_{25}$	-0.058	0.070	0.115	0.117	0.171	0.251
$\hat{\beta}_{26}$	0.037	0.178	0.239	0.233	0.282	0.388
$\hat{\beta}_{27}$	-0.207	-0.166	-0.155	-0.154	-0.142	-0.117
$\hat{\beta}_{28}$	-0.161	-0.117	-0.105	-0.104	-0.091	-0.007
$\hat{\beta}_{29}$	-0.483	-0.223	-0.146	-0.134	-0.053	0.256
$\hat{\beta}_{30}$	-0.303	-0.242	-0.221	-0.220	-0.199	-0.162
$\hat{\beta}_{31}$	-0.359	-0.234	-0.211	-0.210	-0.181	-0.091
$\hat{\beta}_{32}$	-0.181	-0.153	-0.145	-0.144	-0.137	-0.105
$\hat{\beta}_{33}$	-0.061	-0.043	-0.036	-0.035	-0.028	-0.008
$\hat{\beta}_{34}$	-0.438	-0.191	-0.140	-0.133	-0.057	0.085
$\hat{\beta}_{35}$	0.064	0.073	0.078	0.078	0.081	0.098
$\hat{\beta}_{36}$	0.052	0.070	0.075	0.075	0.080	0.091
$\hat{\beta}_{37}$	-0.127	-0.120	-0.117	-0.118	-0.115	-0.110
$\hat{\beta}_{38}$	0.096	0.118	0.124	0.124	0.131	0.148
$\hat{\beta}_{39}$	0.047	0.066	0.074	0.073	0.078	0.097
$\hat{\beta}_{40}$	0.129	0.159	0.167	0.167	0.175	0.199
$\hat{\beta}_{41}$	-0.007	0.044	0.061	0.058	0.072	0.110
$\hat{\beta}_{42}$	0.084	0.099	0.103	0.103	0.107	0.117
$\hat{\beta}_{43}$	0.185	0.210	0.218	0.216	0.222	0.243
$\hat{\sigma}_e$	0.456	0.459	0.461	0.461	0.464	0.469
$\hat{\sigma}_u$	0.114	0.123	0.125	0.125	0.125	0.135

Table 3.9: The descriptive statistics for all parameter estimates.

Next, consider the regression coefficient, it can be either positive or negative. The positive regression coefficient indicates that the response variable and auxiliary variables change in the same direction. That is, the value of the response variable increases when the auxiliary variables increases. On the other hand, if they are negative it will change in the opposite direction.

That is, the regression coefficient indicates the relationship between expenditure and the auxiliary variables. The variable does not affect the response variable if the regression coefficient is zero. Each beta represents a different variable that shown each variable which appears in Section 3.1.2. For example, the $\hat{\beta}_1$ represents the $x_{(1)}$ row house in the type of dwelling variable.

Form Table 3.9 and Figure 3.8, the range of $\hat{\beta}_1$ is between 0.018 to 0.099, that is the row house variable affects the expenditure in a positive direction. For toilet facilities, both $\hat{\beta}_{24}$ and $\hat{\beta}_{26}$ values are both positive, while $\hat{\beta}_{25}$ is negative and less than $\hat{\beta}_{24}$ and $\hat{\beta}_{26}$. That is, having either flush ($\mathbf{x}_{(24)}$) or both flush & squat ($\mathbf{x}_{(26)}$) for a toilet indicates higher expenditure. In the opposite direction, for those who have squat toilets ($\mathbf{x}_{(25)}$), the expenditure is less than those with other types of toilets. Moreover, from the value of $\hat{\beta}_{43}$ it can be seen that the number of automobile ($\mathbf{x}_{(43)}$) owned affects the increase of expenditure significantly.

Next, we show an example of how to calculate the FGT poverty indicators by the ELL method. Start with an example for Bangkok ($d = 1$). From Figure 3.3, we can see the population size (N_1) for Bangkok is 2368 units are divided into in-sample of size 710 and out-of-sample of size 1658. For in-sample data, we apply the model following the Algorithm 5. From the process, we then obtain the following set of parameter estimates of $\hat{\beta}$, $\hat{\sigma}_u^2$ and $\hat{\sigma}_e^2$.

$$\hat{\beta} = \begin{pmatrix} 8.837 \\ 0.061 \\ 0.077 \\ 0.170 \\ \vdots \\ 0.220 \end{pmatrix}, \quad \hat{\sigma}_u^2 = 0.212, \quad \hat{\sigma}_e^2 = 0.015. \quad (3.5)$$

Having obtained these estimates, we generate Y_{di}^* of Y_{di} defined in (3.2) as follows

$$Y_{di}^* = \mathbf{x}_{di}' \hat{\beta} + u_1^* + e_{di}^* \quad (3.6)$$

where $\hat{\beta}$ is an estimator of β , $u_d^* \stackrel{iid}{\sim} N(0, \hat{\sigma}_u^2)$ and $e_{di}^* \stackrel{ind}{\sim} N(0, \hat{\sigma}_e^2 k_{di}^2)$.

For illustration, let consider the first unit ($i = 1$) of Bangkok ($d = 1$), called Mr.A having the following auxiliary variables \mathbf{x}_{di} .

Name of auxiliary variable	Input of Mr.A
Type of dwelling (HH01)	1-Detached house
Materials of construction (HH02)	3-Wood and cement or brick
Tenure (HH03)	1-Own dwelling and land
Cooking fuel (HH10)	5-Electricity
Toilet facilities (HH15)	2-Flush latrine
Drinking water (HH11)	8-Others
Number of items owned by household	
– LCD/LED/Plasma (HH32)	1
– Video/VCD/DVD Player (HH33)	1
– Mobile phone (HH40)	2
– Home computer (HH37)	1
– Refrigerator (HH27)	1
– Microwave oven (HH25)	1
– Washing machine (HH34)	0
– Air conditioner (HH35)	1
– Automobile (HH18)	3

Table 3.10: The auxiliary variables of Mr.A.

As described in Section 3.1.2, we treat these variables of Mr.A as a matrix form. That is, the auxiliary variable \mathbf{x}_{11} of Mr.A with the intercept term is $[1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \dots\ 1\ 3]'$. Then we get the vector with length of 44.

From the estimate of parameters shown in (3.5), so we generate u_1^* from $N(0, 0.212)$ and e_{11}^* from $N(0, 0.015)$ where $k = 1$, obtain $u_1^* = 0.285$ and $e_{11}^* = -0.123$. Next step, substitute all of Mr.A's values into the equation. Mr.A obtain

$$\begin{aligned}
 Y_{11}^* &= (1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ \dots\ 1\ 3) \begin{pmatrix} 8.837 \\ 0.061 \\ 0.077 \\ 0.170 \\ \vdots \\ 0.220 \end{pmatrix} + 0.285 - 0.123 \\
 &= 9.769 + 0.285 - 0.123 \\
 &= 9.931.
 \end{aligned}$$

That is the estimated log expenditure of Mr.A is 9.931. The same process is applied for all individuals ($N_1 = 2368$). Having obtained generated log expenditure of people in the population, we then calculate the poverty incidence according to (3.3). Then, the poverty incidence F_{01}^* is 0.0456.

Calculate the poverty incidence for all $Q = 500$. Accordingly, sum the $F_{01}^{*(q)}$ and then divided by $Q = 500$, we obtain the poverty incidence \hat{F}_{01}^{ELL} for Bangkok.

Doing the same process for the poverty gap ($\alpha = 1$) and poverty severity ($\alpha = 2$), we obtain poverty incidence, poverty gap and poverty severity for all 77 provinces in Thailand. Continue these process $L = 100$ times. For $l = 1, \dots, 100$, let $\hat{F}_{\alpha d}^{(l)ELL}$ is the FGT poverty indicator of ELL estimator for l^{th} process and $F_{\alpha d}^{(l)}$ is the true FGT poverty indicator for l^{th} process. We then evaluate the performance of the estimates via absolute bias and mean square error as follows.

The absolute bias (AB) and mean square error (MSE) of \hat{F}_{ad}^{ELL} are defined as

$$AB_{ad}^{ELL} = \frac{1}{L} \sum_{l=1}^L |(\hat{F}_{ad}^{(l)ELL} - F_{ad}^{(l)})|, \quad (3.7)$$

$$MSE_{ad}^{ELL} = \frac{1}{L} \sum_{l=1}^L (\hat{F}_{ad}^{(l)ELL} - F_{ad}^{(l)})^2. \quad (3.8)$$

The results of the ELL method are presented as follows:

Province	Poverty Incidence	Poverty Gap	Poverty Severity
	AB(%) MSE(%)	AB(%) MSE(%)	AB(%) MSE(%)
1	3.206846 0.103060	0.694535 0.004837	0.228416 0.000524
2	3.011061 0.090808	0.537676 0.002898	0.154012 0.000238
3	2.012596 0.040615	0.391552 0.001538	0.124865 0.000156
4	2.928248 0.085800	0.563130 0.003174	0.170149 0.000290
5	3.794513 0.144223	0.783339 0.006152	0.247036 0.000612
6	4.857900 0.236236	1.062200 0.011305	0.417663 0.001749
7	3.462083 0.120368	0.961847 0.009289	0.388366 0.001514
8	0.516909 0.002821	0.267703 0.000727	0.108823 0.000120
9	4.973178 0.248057	0.836355 0.007064	0.141180 0.000212
10	5.339254 0.285242	1.216850 0.014820	0.410835 0.001690
11	3.008762 0.090705	0.733605 0.005394	0.263529 0.000696
12	1.069624 0.011534	0.458020 0.002103	0.169344 0.000287
13	0.899471 0.008467	0.632593 0.004028	0.282429 0.000802
14	0.519900 0.002893	0.368449 0.001370	0.175059 0.000308
15	3.905077 0.152726	0.953322 0.009103	0.327499 0.001075
16	1.635654 0.026852	0.362340 0.001319	0.095935 0.000093
17	2.971605 0.088631	0.712859 0.005100	0.240787 0.000582

Continued on next page...

Table 3.11 – continued from previous page.

Province	Poverty	Incidence	Poverty	Gap	Poverty	Severity
	AB(%)	MSE(%)	AB(%)	MSE(%)	AB(%)	MSE(%)
18	0.423979	0.002433	0.492314	0.002493	0.269138	0.000736
19	0.917112	0.008733	0.043816	0.000030	0.037211	0.000017
20	3.269293	0.107654	0.087844	0.000111	0.161764	0.000269
21	1.676210	0.028488	0.637456	0.004092	0.272739	0.000748
22	2.901604	0.084453	0.717979	0.005170	0.254119	0.000648
23	2.387802	0.057660	0.780380	0.006138	0.324931	0.001063
24	3.888133	0.152242	1.381355	0.019158	0.584438	0.003426
25	6.825829	0.466764	1.690977	0.028651	0.572268	0.003283
26	4.815068	0.233336	1.690140	0.028671	0.661999	0.004397
27	3.676900	0.135530	1.040966	0.010855	0.359967	0.001298
28	0.641691	0.004551	0.271227	0.000769	0.194994	0.000385
29	4.906253	0.240944	1.180465	0.013952	0.407704	0.001665
30	2.671744	0.071602	0.740837	0.005504	0.284949	0.000814
31	5.840168	0.341653	1.170276	0.013737	0.364078	0.001332
32	1.557695	0.024363	0.402278	0.001624	0.132742	0.000177
33	1.600062	0.025945	0.650567	0.004257	0.270582	0.000736
34	7.593491	0.577971	1.657569	0.027570	0.529467	0.002817
35	9.860599	0.973160	1.099446	0.012142	0.109491	0.000127
36	4.747761	0.225949	1.561912	0.024433	0.592050	0.003510
37	7.891122	0.623183	1.610607	0.025974	0.452192	0.002050
38	1.652100	0.028202	1.374317	0.018959	0.627077	0.003943
39	1.323047	0.018358	0.197577	0.000479	0.036655	0.000020
40	5.313257	0.282931	1.396484	0.019554	0.512501	0.002635

Continued on next page...

Table 3.11 – continued from previous page.

Province	Poverty	Incidence	Poverty	Gap	Poverty	Severity
	AB(%)	MSE(%)	AB(%)	MSE(%)	AB(%)	MSE(%)
41	0.759659	0.006160	0.583592	0.003435	0.258880	0.000674
42	1.879203	0.035585	0.153675	0.000252	0.011922	0.000002
43	2.831376	0.080330	0.105114	0.000120	0.074890	0.000057
44	2.692963	0.072800	0.388135	0.001524	0.076153	0.000060
45	0.918962	0.008964	0.133690	0.000211	0.089662	0.000086
46	1.008878	0.010890	0.511722	0.002694	0.205710	0.000438
47	4.260902	0.187797	0.318781	0.001382	0.173655	0.000432
48	0.319803	0.001153	0.109226	0.000126	0.034827	0.000013
49	0.276938	0.001071	0.376637	0.001442	0.181188	0.000331
50	1.675769	0.028550	0.550335	0.003056	0.205794	0.000427
51	8.005268	0.641663	2.724604	0.074352	1.224360	0.015017
52	3.386841	0.115158	0.317976	0.001041	0.016711	0.000004
53	0.730051	0.005478	0.362252	0.001320	0.152175	0.000233
54	3.318389	0.110450	0.789350	0.006253	0.255417	0.000655
55	1.464995	0.021697	0.348480	0.001230	0.136013	0.000187
56	1.791954	0.032279	0.465646	0.002181	0.210843	0.000447
57	1.018302	0.010668	0.832872	0.006972	0.430380	0.001860
58	0.892692	0.008348	0.453374	0.002082	0.198039	0.000396
59	2.872108	0.082622	0.672081	0.004524	0.220583	0.000487
60	4.499186	0.202670	0.917001	0.008421	0.282009	0.000797
61	4.202221	0.177182	1.016625	0.010372	0.352100	0.001245
62	5.334113	0.284868	1.222883	0.014984	0.442347	0.001962
63	1.401486	0.019838	0.343371	0.001192	0.099434	0.000101

Continued on next page...

Table 3.11 – continued from previous page.

Province	Poverty	Incidence	Poverty	Gap	Poverty	Severity
	AB(%)	MSE(%)	AB(%)	MSE(%)	AB(%)	MSE(%)
64	1.160309	0.013673	0.484149	0.002359	0.172295	0.000299
65	1.850735	0.034443	0.434643	0.001901	0.147723	0.000220
66	2.575035	0.066512	0.618073	0.003831	0.213865	0.000459
67	3.712419	0.137940	0.757133	0.005738	0.233320	0.000545
68	4.663366	0.217767	0.990633	0.009834	0.298393	0.000893
69	4.517764	0.204352	0.611421	0.003755	0.128108	0.000166
70	4.110481	0.169170	1.189566	0.014165	0.406471	0.001654
71	4.294612	0.184638	0.960665	0.009241	0.328077	0.001078
72	0.228032	0.000699	0.190576	0.000382	0.087704	0.000080
73	0.903819	0.008374	0.375534	0.001423	0.142803	0.000206
74	3.164668	0.100554	0.497138	0.002497	0.108899	0.000122
75	9.260124	0.858155	1.433818	0.020606	0.242794	0.000597
76	1.355289	0.018900	0.206326	0.000461	0.180294	0.000330
77	9.159732	0.840741	1.924605	0.037167	0.544524	0.002983

Table 3.11: Percentages of absolute biases and mean square errors for the ELL method.

From Table 3.11, if the AB is very far from zero shows that the estimated is not as good as it should be. But if close to zero, the estimate is very close to the actual value. For the province that has large AB, the MSE is also large as well. For example, it can be seen that the AB value of poverty incidence of the Kalasin (35th) is up to 9.86%, followed by the Pattani (75th) and Narathiwat (77th) that gives the value of 9.26% and 9.15%. That means the ELL estimates perform poorly in the case of the poverty incidence for those provinces. We may conclude that the provinces with large FGT poverty indicators from SES may provide the ELL estimate perform poorly.

3.2.2 Empirical Bayes Method

In this section, we discuss another method for the FGT poverty indicators which is the Empirical Bayes (EB) method of Molina and Rao (2010). In their method, they assume the nested error model for in-sample data with random effect (\mathbf{v}) and the unit error ($\boldsymbol{\epsilon}$) are normally distributed. For the vector of transformed average total monthly expenditure per capita for province d denoted by (\mathbf{Y}_d), the matrix of the auxiliary variables (\mathbf{x}_d) shown in Table 3.8 and covariance matrix (\mathbf{V}_d) for province d , they are decomposed into in-sample and out-of-sample elements:

$$\mathbf{Y}_d = \begin{pmatrix} \mathbf{Y}_{ds} \\ \mathbf{Y}_{dr} \end{pmatrix}, \quad \mathbf{x}_d = \begin{pmatrix} \mathbf{x}_{ds} \\ \mathbf{x}_{dr} \end{pmatrix}, \quad \mathbf{V}_d = \begin{pmatrix} \mathbf{V}_{ds} & \mathbf{V}_{dsr} \\ \mathbf{V}_{drs} & \mathbf{V}_{dr} \end{pmatrix}.$$

The best predictor of FGT poverty indicator for out-of-sample data is obtained by its expectation with respect to the conditional distribution of the population data given sample data which can be approximated by a Monte Carlo approximation. The conditional model of out-of-sample data given in-sample data defined as

$$\mathbf{Y}_{dr} | \mathbf{Y}_{ds} = \boldsymbol{\mu}_{dr|s} + v_d \mathbf{1}_{N_d - n_d} + \boldsymbol{\epsilon}_{dr}, \quad (3.9)$$

where

$$\boldsymbol{\mu}_{dr|s} = \mathbf{x}_{dr} \boldsymbol{\beta} + \sigma_u^2 \mathbf{1}_{N_d - n_d} \mathbf{1}'_{n_d} \mathbf{V}_{ds}^{-1} (\mathbf{Y}_{ds} - \mathbf{x}_{ds} \boldsymbol{\beta}), \quad (3.10)$$

for

$$\mathbf{V}_{ds} = \sigma_u^2 \mathbf{1}_{n_d} \mathbf{1}'_{n_d} + \sigma_e^2 \mathbf{I}_{n_d}, \quad (3.11)$$

and

$$\gamma_d = \sigma_u^2 (\sigma_u^2 + \sigma_e^2 / n_d)^{-1}, \quad (3.12)$$

with the random effect v_d and the model error $\boldsymbol{\epsilon}_{dr}$ are independent and satisfy

$$v_d \sim N(0, \sigma_u^2 (1 - \gamma_d)) \quad \text{and} \quad \boldsymbol{\epsilon}_{dr} \sim N(\mathbf{0}_{N_d - n_d}, \sigma_e^2 \mathbf{I}_{N_d - n_d}). \quad (3.13)$$

Then they compose the average total monthly expenditure per capita of in-sample and out-of-sample by $\mathbf{Y}_d = (\mathbf{Y}'_{ds}, \mathbf{Y}'_{dr})'$. The generation is repeated for $q = 1, \dots, Q$, to obtain Q censuses. Then, for each q , the FGT poverty indicator for province d ($d = 1, \dots, D$) is calculated as

$$F_{\alpha d}^{(q)} = \frac{1}{N_d} \sum_{i=1}^{N_d} \left(\frac{z_d - \exp(Y_{di}^{(q)})}{z_d} \right)^{\alpha} I(\exp(Y_{di}^{(q)}) < z_d), \quad \alpha = 0, 1, 2. \quad (3.14)$$

The EB estimator of $F_{\alpha d}$ for area d was calculated as

$$\hat{F}_{\alpha d}^{EB} \approx \frac{1}{Q} \sum_{q=1}^Q F_{\alpha d}^{(q)}. \quad (3.15)$$

3.2.2.1 Algorithm for EB method

The algorithm of the EB method is explained in Algorithm 6 and Figure 3.9 as follows.

Algorithm 6 Algorithm for the FGT poverty indicator of EB method.

- 1: Decompose population into two parts: 30% for in-sample and 70% for out-of-sample

$$\mathbf{Y}_d = \begin{pmatrix} \mathbf{Y}_{ds} \\ \mathbf{Y}_{dr} \end{pmatrix}.$$

- 2: Use in-sample Y_{di} for $i = 1, \dots, n_d$, to fit the model (2.9)

$$Y_{di} = \mathbf{x}'_{di}\boldsymbol{\beta} + u_d + e_{di}.$$

Output: The estimate $\hat{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}$, the estimate $\hat{\sigma}_u^2$ of σ_u^2 , the estimate $\hat{\sigma}_e^2$ of σ_e^2 .

- 3: **for** $q : 1$ to $Q (= 500)$ **do**

- 4: Generate out-of-sample (3.9) from $\mathbf{Y}_{dr} | \mathbf{Y}_{ds} = \boldsymbol{\mu}_{dr|s} + v_d \mathbf{1}_{N_d - n_d} + \boldsymbol{\epsilon}_{dr}$.

- 5: Calculate $\gamma_d = \sigma_u^2 (\sigma_u^2 + \sigma_e^2 / n_d)^{-1}$.

- 6: Calculate $\mathbf{V}_{ds} = \sigma_u^2 \mathbf{1}_{n_d} \mathbf{1}'_{n_d} + \sigma_e^2 \mathbf{I}_{n_d}$.

- 7: Calculate $\boldsymbol{\mu}_{dr|s} = \mathbf{x}_{dr} \boldsymbol{\beta} + \sigma_u^2 \mathbf{1}_{N_d - n_d} \mathbf{1}'_{n_d} \mathbf{V}_{ds}^{-1} (\mathbf{Y}_{ds} - \mathbf{x}_{ds} \boldsymbol{\beta})$.

- 8: Generate $v_d \sim N(0, \sigma_u^2 (1 - \gamma_d))$.

- 9: Generate $\boldsymbol{\epsilon}_{dr} \sim N(\mathbf{0}_{N_d - n_d}, \sigma_e^2 \mathbf{I}_{N_d - n_d})$.

- 10: Construct the new population $\mathbf{Y}_d = (\mathbf{Y}'_{ds}, \mathbf{Y}'_{dr})'$.

- 11: Calculate the FGT poverty indicators $F_{\alpha d}^{(q)}$ for province $d = 1, \dots, D$ as

$$F_{\alpha d}^{(q)} = \frac{1}{N_d} \sum_{i=1}^{N_d} \left(\frac{z_d - \exp(Y_{di}^{(q)})}{z_d} \right)^{\alpha} I(\exp(Y_{di}^{(q)}) < z_d), \quad \alpha = 0, 1, 2.$$

- 12: Calculate a bootstrap of the EB estimator as

$$\hat{F}_{\alpha d}^{EB} \approx \frac{1}{Q} \sum_{q=1}^Q F_{\alpha d}^{(q)}.$$

- 13: **end**
-

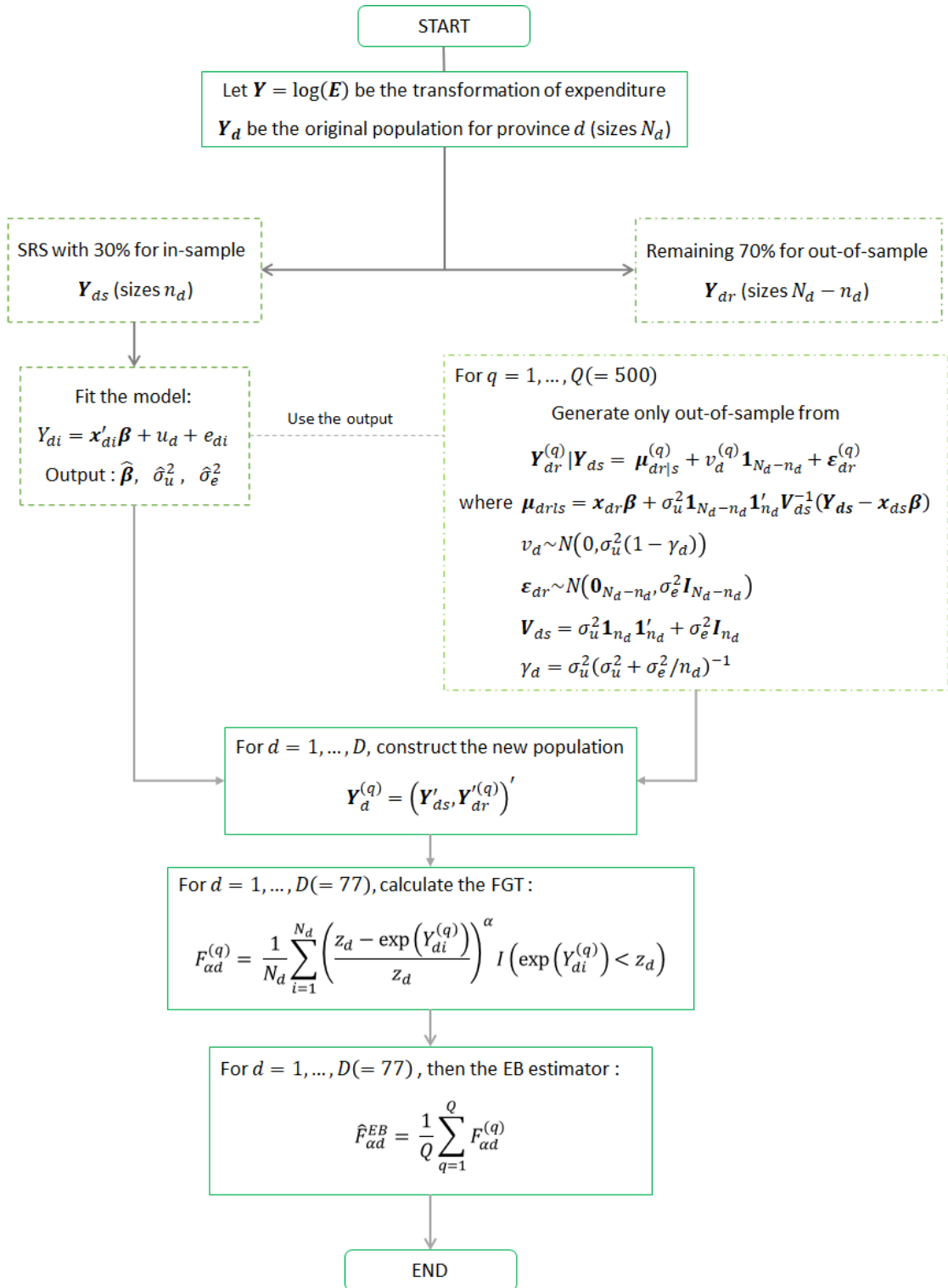


Figure 3.9: Diagram for the EB method.

3.2.2.2 Result for the EB method

Follow the EB method, we apply in-sample data to the model following Algorithm 8. Since the EB method assumes the same model as in the ELL method, then the output for $\hat{\beta}$, $\hat{\sigma}_u^2$ and $\hat{\sigma}_e^2$ are the same. That is, the set of parameter estimates of $\hat{\beta}$, $\hat{\sigma}_u^2$ and $\hat{\sigma}_e^2$ are

$$\hat{\beta} = \begin{pmatrix} 8.837 \\ 0.061 \\ 0.077 \\ 0.170 \\ \vdots \\ 0.220 \end{pmatrix}, \quad \hat{\sigma}_u^2 = 0.212, \quad \hat{\sigma}_e^2 = 0.015.$$

For illustration, let consider the first unit ($i = 1$) of out-of-sample units of Bangkok ($d = 1$), called Ms.C. The auxiliary variables $\mathbf{x}_{(1)}$ of Ms.C are as follows.

Name of auxiliary variable	Input of Ms.C
Type of dwelling (HH01)	2-Row house
Materials of construction (HH02)	1-Cement or brick
Tenure (HH03)	1-Own dwelling and land
Cooking fuel (HH10)	5-Electricity
Toilet facilities (HH15)	2-Squat
Drinking water (HH11)	8-Other
Number of items owned by household	
– LCD/LED/Plasma (HH32)	2
– Video/VCD/DVD Player (HH33)	1
– Mobile phone (HH40)	4
– Home computer (HH37)	1
– Refrigerator (HH27)	1
– Microwave oven (HH25)	1
– Washing machine (HH34)	0
– Air conditioner (HH35)	3
– Automobile (HH18)	1

Table 3.12: The auxiliary variables of Ms.C.

As described in Section 3.1.2, we treat these variables of Ms.C as a matrix form. Thus, the auxiliary variables $\mathbf{x}_{(11)}$ of Ms.C with the intercept term is $[1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ \dots \ 3 \ 1]'$ with length of 44. Doing for all out-of-sample 1658 units of Bangkok and then construct the auxiliary variable for all out-of-sample units of Bangkok denoted by \mathbf{x}_{1r} .

For Bangkok ($d = 1$), calculate \mathbf{V}_{1s} from (3.11) and γ_1 from (3.12) as

$$\begin{aligned}
 \mathbf{V}_{1s} &= 0.212 \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}_{710 \times 1} \begin{pmatrix} 1 & 1 & \dots & 1 \end{pmatrix}_{1 \times 710} + 0.015 \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix}_{710 \times 710} \\
 &= \begin{pmatrix} 0.212 & 0.212 & \dots & 0.212 \\ 0.212 & 0.212 & \dots & 0.212 \\ \vdots & \vdots & \ddots & \vdots \\ 0.212 & 0.212 & \dots & 0.212 \end{pmatrix}_{710 \times 710} + \begin{pmatrix} 0.015 & 0 & \dots & 0 \\ 0 & 0.015 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0.015 \end{pmatrix}_{710 \times 710} \\
 &= \begin{pmatrix} 0.227 & 0.212 & \dots & 0.212 \\ 0.212 & 0.227 & \dots & 0.212 \\ \vdots & \vdots & \ddots & \vdots \\ 0.212 & 0.212 & \dots & 0.227 \end{pmatrix}_{710 \times 710} \\
 \gamma_1 &= \frac{0.212}{0.212 + 0.015/710} = 0.999.
 \end{aligned}$$

For the first out-of-sample unit ($i = 1$), calculate μ_{11} from (3.10) obtain 8.2437. Next, generate v_1 and ϵ_{11} from (3.13). That is, $v_1 \sim N(0, 0.212(1-0.999))$ and $\epsilon_{11} \sim N(0, 0.015)$, obtain $v_1 = 0.0085$ and $\epsilon_{11} = 0.133$.

That is the estimated out-of-sample (3.9) of log expenditure for Ms.C is 8.385. Putting the in-sample and out-of-sample together. The same process is applied to the remaining out-of-sample units in Bangkok. Therefore, we obtain the new population for Bangkok $\mathbf{Y}_1 = (\mathbf{Y}_{1s}, \mathbf{Y}_{1r})'$.

For the same, Bangkok must generate another $q(= 2, \dots, 500)$ denote the new population for Bangkok by $\mathbf{Y}_1^{(q)}$. Calculate the poverty incidence ($\alpha = 0$) for all $Q = 500$. Accordingly, sum the $F_{01}^{(q)}$ and then divided by $Q = 500$, we obtain the poverty incidence \hat{F}_{01}^{EB} for Bangkok. Doing the same process for the poverty gap ($\alpha = 1$) and poverty severity ($\alpha = 2$) and also for other provinces, we obtain the poverty incidence, poverty gap and poverty severity for all 77 provinces in Thailand. Continue these processes $L = 100$ times. For $l = 1, \dots, L(= 100)$, let $\hat{F}_{\alpha d}^{(l)EB}$ be the FGT poverty indicator of EB estimator for the l^{th} process and $F_{\alpha d}^{(l)}$ be the true FGT poverty indicator for the l^{th} process. We then evaluate the performance of the estimates via absolute bias and mean square error as follows.

The absolute bias (AB) and mean square error (MSE) of $\hat{F}_{\alpha d}^{EB}$ are

$$AB_{\alpha d}^{EB} = \frac{1}{L} \sum_{l=1}^L |\hat{F}_{\alpha d}^{(l)EB} - F_{\alpha d}^{(l)}|, \quad (3.16)$$

$$MSE_{\alpha d}^{EB} = \frac{1}{L} \sum_{l=1}^L (\hat{F}_{\alpha d}^{(l)EB} - F_{\alpha d}^{(l)})^2. \quad (3.17)$$

The results of the EB method show as follows:

Province	Poverty	Incidence	Poverty	Gap	Poverty	Severity
	AB(%)	MSE(%)	AB(%)	MSE(%)	AB(%)	MSE(%)
1	0.412307	0.001733	0.048892	0.000025	0.011463	0.000001
2	0.352495	0.001268	0.051153	0.000027	0.012834	0.000002
3	0.177753	0.000516	0.025408	0.000009	0.013318	0.000002
4	0.538713	0.002952	0.089165	0.000081	0.022917	0.000005
5	0.739772	0.006862	0.100380	0.000141	0.029415	0.000013
6	2.635032	0.081272	0.567988	0.003929	0.230107	0.000636
7	0.580286	0.005599	0.193869	0.000544	0.100425	0.000123
8	0.488212	0.004291	0.154112	0.000358	0.057865	0.000048
9	5.799242	0.348398	1.161454	0.014184	0.314045	0.001073
10	0.848277	0.008361	0.193907	0.000410	0.070549	0.000053
11	1.367562	0.019073	0.182722	0.000345	0.029104	0.000009
12	1.989667	0.040092	0.153168	0.000241	0.015855	0.000003
13	3.808710	0.149562	0.507305	0.002795	0.117357	0.000169
14	2.075439	0.046508	0.277012	0.000879	0.050398	0.000033
15	0.326033	0.001474	0.039281	0.000024	0.033537	0.000012
16	0.701138	0.006844	0.148658	0.000308	0.061292	0.000051
17	0.733202	0.007964	0.199255	0.000500	0.072578	0.000063
18	2.953786	0.097718	0.427387	0.002320	0.102136	0.000142
19	2.061395	0.051118	0.551837	0.003565	0.223994	0.000576
20	3.509591	0.138981	1.368917	0.019646	0.635518	0.004156
21	3.610470	0.141132	0.961838	0.009871	0.370832	0.001453
22	4.989783	0.255977	1.131890	0.013205	0.382250	0.001509
23	3.749941	0.150893	1.046101	0.011558	0.387123	0.001576
24	3.698311	0.145970	1.139737	0.013461	0.453519	0.002106
25	5.587374	0.318281	1.310014	0.017439	0.428291	0.001867

Continued on next page...

Table 3.13 – continued from previous page.

Province	Poverty	Incidence	Poverty	Gap	Poverty	Severity
	AB(%)	MSE(%)	AB(%)	MSE(%)	AB(%)	MSE(%)
26	2.867288	0.094444	1.009671	0.010692	0.388768	0.001566
27	3.097372	0.104595	0.830842	0.007270	0.277175	0.000807
28	1.421711	0.030563	0.381138	0.002105	0.185552	0.000429
29	1.069750	0.013218	0.278665	0.000844	0.097852	0.000102
30	3.370799	0.121151	0.815470	0.007012	0.294513	0.000909
31	2.783093	0.080543	0.483232	0.002547	0.128961	0.000202
32	2.643508	0.074129	0.574845	0.003474	0.180764	0.000346
33	1.274262	0.023360	0.479039	0.002791	0.184455	0.000402
34	1.527460	0.026719	0.262685	0.000900	0.065768	0.000069
35	2.122483	0.064038	1.600014	0.027159	0.801923	0.006657
36	4.244713	0.188250	1.285895	0.016944	0.482221	0.002371
37	1.569355	0.033175	0.599109	0.004715	0.303834	0.001139
38	1.490223	0.033417	0.977493	0.010350	0.436558	0.002001
39	1.898062	0.043097	0.403307	0.002137	0.109704	0.000189
40	2.365515	0.059987	0.642966	0.004327	0.226812	0.000536
41	3.594820	0.136144	1.119511	0.012999	0.423215	0.001855
42	3.068292	0.102544	0.938874	0.009323	0.360462	0.001367
43	2.924112	0.096378	1.077516	0.012246	0.459229	0.002192
44	2.403202	0.070009	0.788466	0.007006	0.313960	0.001089
45	3.268043	0.117861	1.079592	0.012317	0.428602	0.001935
46	2.000313	0.046986	0.692173	0.005300	0.263307	0.000775
47	1.683249	0.040302	1.150493	0.014887	0.635379	0.004335
48	3.134456	0.103547	0.631917	0.004255	0.207288	0.000464
49	2.976424	0.095921	0.906530	0.008548	0.339783	0.001191
50	4.570715	0.216592	1.052106	0.011445	0.351995	0.001284

Continued on next page...

Table 3.13 – continued from previous page.

Province	Poverty	Incidence	Poverty	Gap	Poverty	Severity
	AB(%)	MSE(%)	AB(%)	MSE(%)	AB(%)	MSE(%)
51	1.666763	0.043649	0.389644	0.002311	0.186395	0.000533
52	4.092541	0.181205	1.203049	0.015364	0.502376	0.002641
53	1.529614	0.026510	0.473819	0.002353	0.162076	0.000273
54	5.043723	0.258677	1.106381	0.012445	0.361421	0.001332
55	3.288039	0.112970	0.787090	0.006513	0.288927	0.000878
56	0.593412	0.005144	0.127348	0.000228	0.031516	0.000017
57	2.471983	0.065896	0.187011	0.000473	0.050572	0.000036
58	1.287478	0.020997	0.139627	0.000303	0.041332	0.000026
59	0.207204	0.000671	0.082370	0.000088	0.038545	0.000016
60	0.175784	0.000429	0.032500	0.000018	0.007629	0.000001
61	0.496454	0.003516	0.158081	0.000334	0.057871	0.000041
62	0.654308	0.006320	0.124328	0.000236	0.058594	0.000042
63	1.169642	0.016641	0.212966	0.000581	0.085333	0.000094
64	1.535099	0.027596	0.149330	0.000341	0.046649	0.000033
65	1.163625	0.015939	0.195511	0.000476	0.057329	0.000041
66	0.380464	0.002300	0.081389	0.000110	0.038619	0.000021
67	0.173451	0.000329	0.011795	0.000002	0.009446	0.000001
68	0.950677	0.011046	0.238331	0.000667	0.099322	0.000126
69	5.346197	0.294940	0.885844	0.008281	0.238638	0.000620
70	1.139159	0.014327	0.073292	0.000083	0.035329	0.000016
71	0.557220	0.003943	0.079063	0.000080	0.011363	0.000002
72	1.229248	0.020137	0.165104	0.000421	0.054901	0.000049
73	1.616720	0.029258	0.197173	0.000512	0.048527	0.000034
74	5.128214	0.271520	1.040829	0.011209	0.318987	0.001068
75	5.242566	0.292549	0.742400	0.006291	0.108971	0.000163

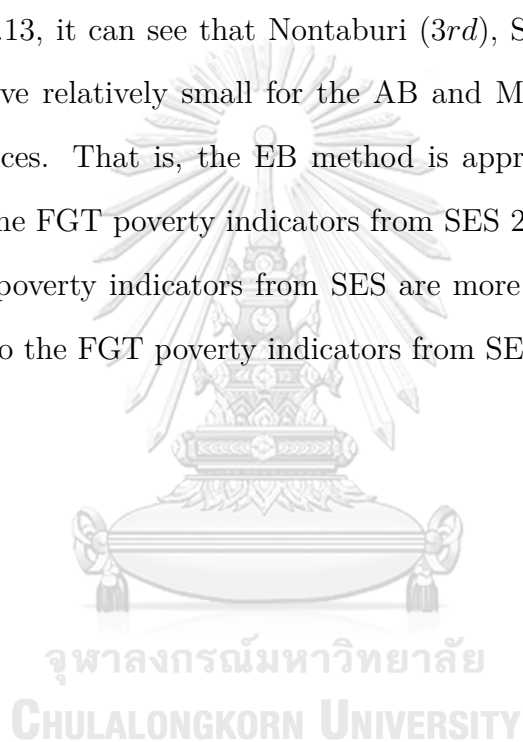
Continued on next page...

Table 3.13 – continued from previous page.

Province	Poverty	Incidence	Poverty	Gap	Poverty	Severity
	AB(%)	MSE(%)	AB(%)	MSE(%)	AB(%)	MSE(%)
76	2.936269	0.092123	0.248247	0.000804	0.037408	0.000022
77	3.379740	0.133073	0.580460	0.004453	0.143668	0.000289

Table 3.13: Percentages of absolute biases and mean square errors for the EB method.

For Table 3.13, it can see that Nontaburi (3rd), Samut Sakhon (60th) and Phuket (67th) have relatively small for the AB and MSE, which are also small of poverty provinces. That is, the EB method is appropriate and the EB estimate is close to the FGT poverty indicators from SES 2017. Moreover, provinces with small FGT poverty indicators from SES are more likely to provide the EB estimator closer to the FGT poverty indicators from SES.



3.2.3 Hierarchical Bayes Method

In this section, we discuss another method for the FGT poverty indicators which is the Hierarchical Bayes (HB) method. The HB method is based on reparameterizing the nested error model (2.9) and considering priors for the model parameters $(\boldsymbol{\beta}, \rho, \sigma_e^2)$.

The Hierarchical Bayes model with informative prior can be written as

- (i) $Y_{di}|u_d, \rho, \boldsymbol{\beta}, \sigma_e^2 \stackrel{\text{ind}}{\sim} N(\mathbf{x}'_{di}\boldsymbol{\beta} + u_d, \sigma_e^2), \quad i = 1, \dots, N_d, \quad d = 1, \dots, D,$
- (ii) $u_d|\rho, \boldsymbol{\beta}, \sigma_e^2 \stackrel{\text{iid}}{\sim} N\left(0, \frac{\rho}{1-\rho}\sigma_e^2\right)$ for $\rho = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2},$
- (iii) $\pi(\boldsymbol{\beta}, \rho, \sigma_e^2) \propto \frac{1}{\sigma_e^2}, \quad \epsilon \leq \rho \leq 1 - \epsilon, \quad \sigma_e^2 > 0, \quad \boldsymbol{\beta} \in \mathbb{R}^p.$

For Thai data, Y_{di} is the average total monthly expenditure per capita and \mathbf{x}'_{di} is the final auxiliary variables for individual i in province d was shown in Table 3.8.

For the HB method for the FGT poverty indicators, the vector of transformed average total monthly expenditure per capita for province d denoted by (\mathbf{Y}_d) , the matrix of the auxiliary variables (\mathbf{x}_d) shown in Table 3.8 and covariance matrix (\mathbf{V}_d) of area d , were decomposed into in-sample and out-of-sample elements same as the EB method:

$$\mathbf{Y}_d = \begin{pmatrix} \mathbf{Y}_{ds} \\ \mathbf{Y}_{dr} \end{pmatrix}, \quad \mathbf{x}_d = \begin{pmatrix} \mathbf{x}_{ds} \\ \mathbf{x}_{dr} \end{pmatrix}, \quad \mathbf{V}_d = \begin{pmatrix} \mathbf{V}_{ds} & \mathbf{V}_{dsr} \\ \mathbf{V}_{drs} & \mathbf{V}_{dr} \end{pmatrix}.$$

Use the in-sample data to fit the HB model (i)-(iii). While the distribution of out-of-sample of average total monthly expenditure per capita $\{Y_{di}, i \in r_d\}$ given parameters $\mathbf{u}, \boldsymbol{\beta}, \sigma_e^2$ and ρ is

$$Y_{di} | \mathbf{u}, \boldsymbol{\beta}, \sigma_e^2, \rho \stackrel{\text{ind}}{\sim} N(\mathbf{x}'_{di} \boldsymbol{\beta} + u_d, \sigma_e^2), \quad i \in r_d, \quad d = 1, \dots, D, \quad (3.18)$$

where $u_d | \rho, \boldsymbol{\beta}, \sigma_e^2 \stackrel{\text{iid}}{\sim} N\left(0, \frac{\rho}{1-\rho} \sigma_e^2\right)$ for $\rho = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2}$.

Repeat the procedure of generating out-of-sample data M times, denoted by $\mathbf{Y}_{dr}^{(m)} = \{Y_{di}^{(m)}, i \in r_d, m = 1, \dots, M\}$. Therefore, we construct the new population by putting them together as $\mathbf{Y}_d^{(m)} = (\mathbf{Y}'_{ds}, (\mathbf{Y}_{dr}^{(m)})')'$.

Now using the new population $\mathbf{Y}_d^{(m)}$ for $m = 1, \dots, M$, compute the FGT poverty indicators for area d ($d = 1, \dots, D$) as

$$F_{\alpha d}^{(m)} = \frac{1}{N_d} \sum_{i=1}^{N_d} \left(\frac{z_d - \exp(Y_{di}^{(m)})}{z_d} \right)^{\alpha} I(\exp(Y_{di}^{(m)}) < z_d), \quad \alpha = 0, 1, 2. \quad (3.19)$$

Then, the HB estimator is approximated as

$$\hat{F}_{\alpha d}^{HB} \approx \frac{1}{M} \sum_{m=1}^M F_{\alpha d}^{(m)}. \quad (3.20)$$

3.2.3.1 Algorithm for HB Method

The algorithm of the HB method is explained in Algorithm 7 and Figure 3.10 as follows.

Algorithm 7 Algorithm for the FGT poverty indicator of HB method.

- 1: Decompose population into two parts: 30% for in-sample and 70% for out-of-sample

$$\mathbf{Y}_d = \begin{pmatrix} \mathbf{Y}_{ds} \\ \mathbf{Y}_{dr} \end{pmatrix}.$$

- 2: Run the HB model (i)-(iii) in WinBUGS14 with $n.chains = 2$, $n.iter = 5000$, $n.burnin = 2500$ and $n.thin = 2$.

Remark: The uniform informative prior on $(0,1)$ was used.

Output: The estimate $\hat{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}$, the estimate $\hat{\sigma}_u^2$ of σ_u^2 , the estimate $\hat{\sigma}_e^2$ of σ_e^2 .

- 3: **for** $m : 1$ to $M (= 2500)$ **do**

- 4: Generate $u_d \stackrel{\text{iid}}{\sim} N\left(0, \frac{\rho}{1-\rho} \hat{\sigma}_e^2\right)$ where $\rho = \frac{\hat{\sigma}_u^2}{\hat{\sigma}_u^2 + \hat{\sigma}_e^2}$.

- 5: Generate out-of-sample $Y_{di}^{(m)}$ (3.18) from $N(\mathbf{x}'_{di} \hat{\boldsymbol{\beta}}^{(m)} + u_d^{(m)}, \hat{\sigma}_e^{2(m)})$ for $i \in r_d$ and $d = 1, \dots, D$.

- 6: Construct the new population by combine the in-sample and out-of-sample data together, denoted by $\mathbf{Y}_d^{(m)} = (\mathbf{Y}'_{ds}, (\mathbf{Y}'_{dr})')'$.

- 7: Calculate the FGT poverty indicators for province d ($d = 1, \dots, D$), $F_{\alpha d}^{(m)}$, as

$$F_{\alpha d}^{(m)} = \frac{1}{N_d} \sum_{i=1}^{N_d} \left(\frac{z_d - \exp(Y_{di}^{(m)})}{z_d} \right)^{\alpha} I(\exp(Y_{di}^{(m)}) < z_d), \quad \alpha = 0, 1, 2.$$

- 8: Calculate a bootstrap of the HB estimator as

$$\hat{F}_{\alpha d}^{HB} \approx \frac{1}{M} \sum_{m=1}^M F_{\alpha d}^{(m)}.$$

- 9: **end**
-

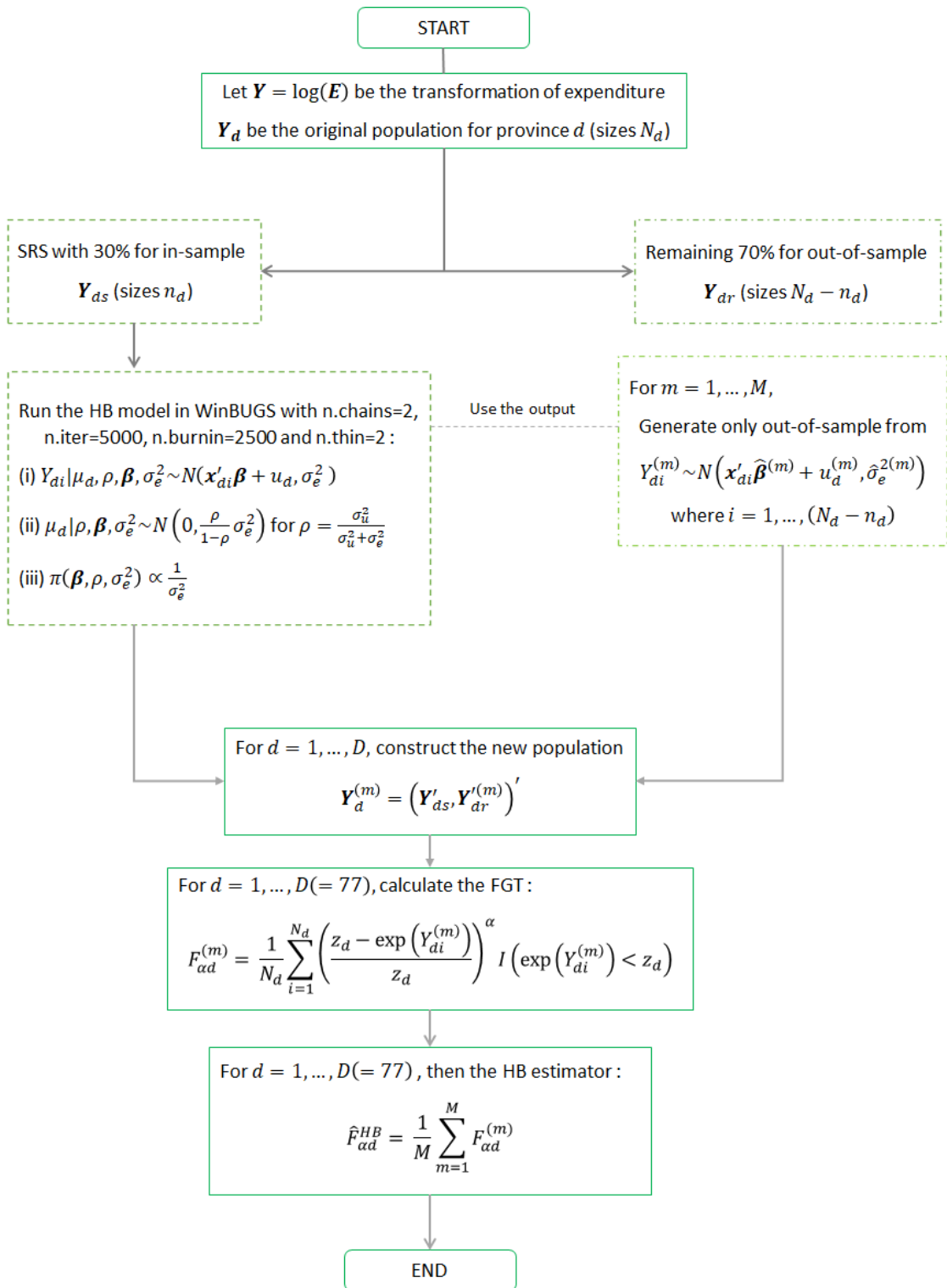
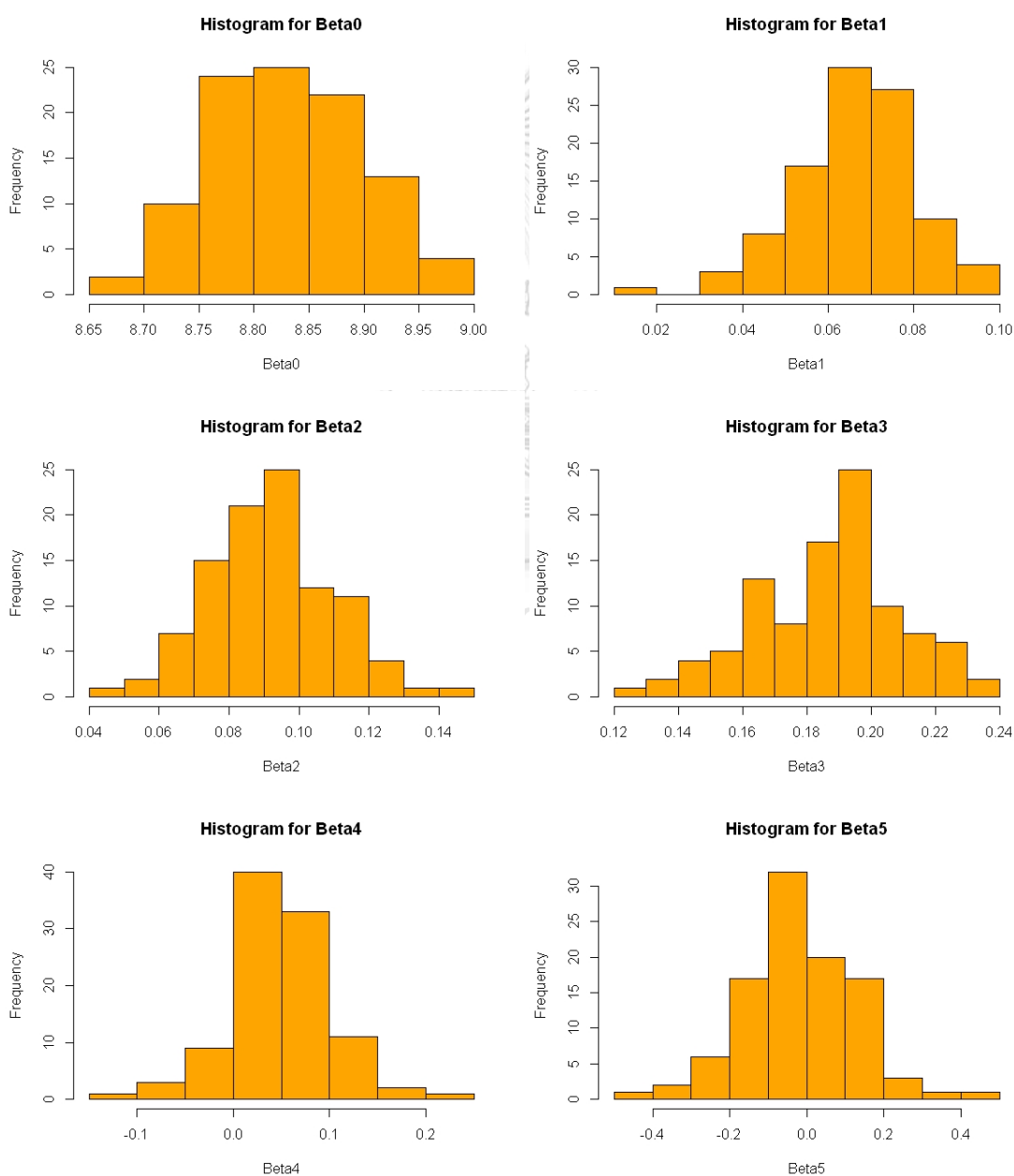


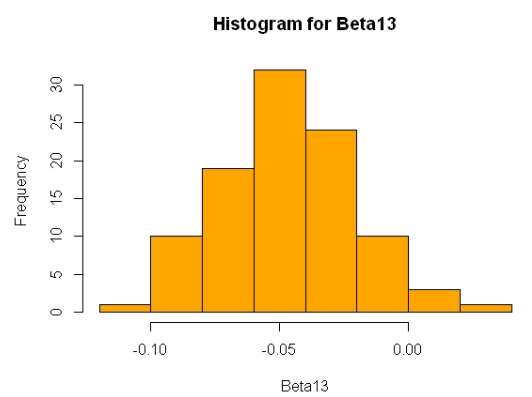
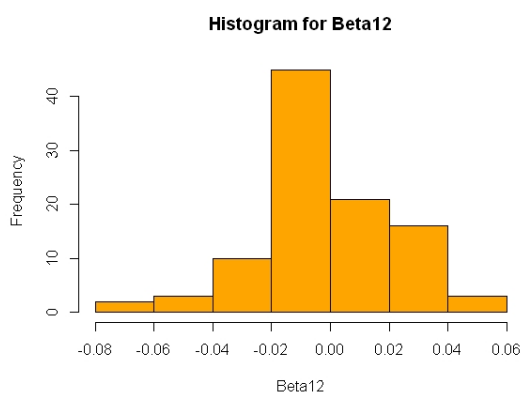
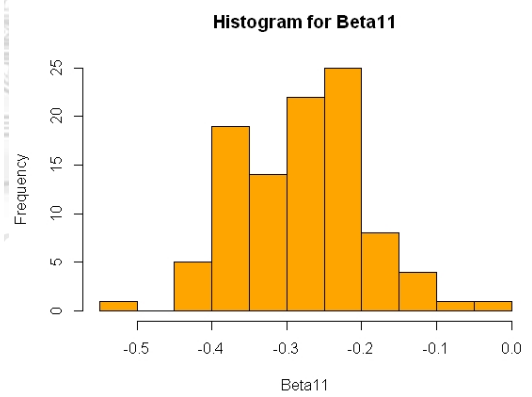
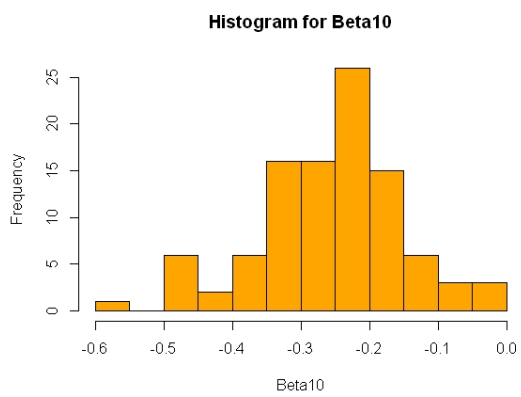
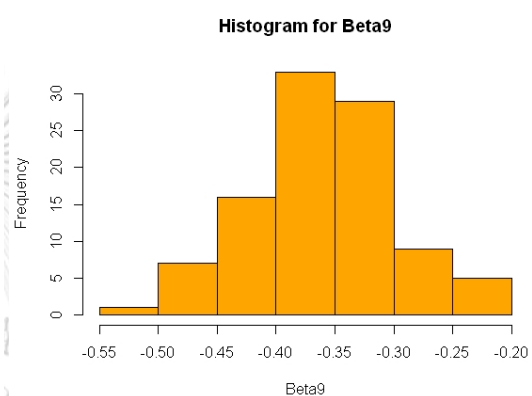
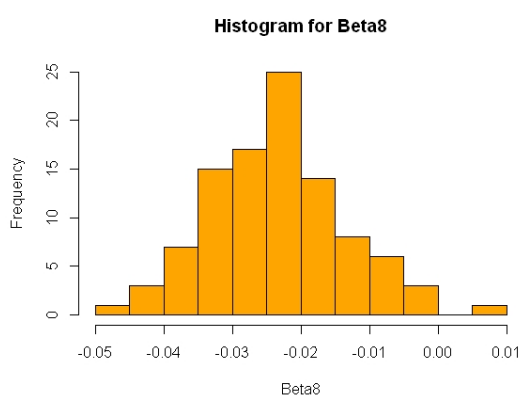
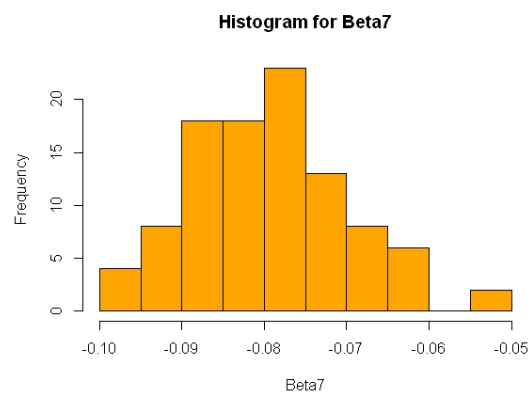
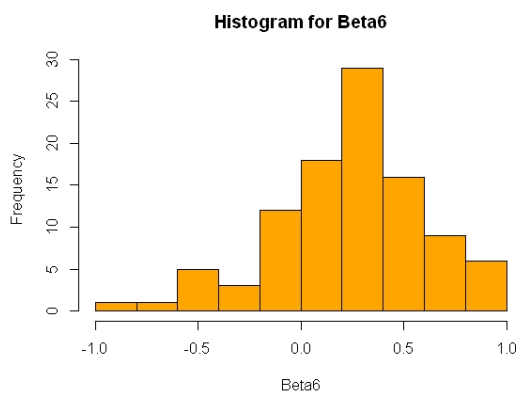
Figure 3.10: Diagram for the HB method.

3.2.3.2 Result for the HB method

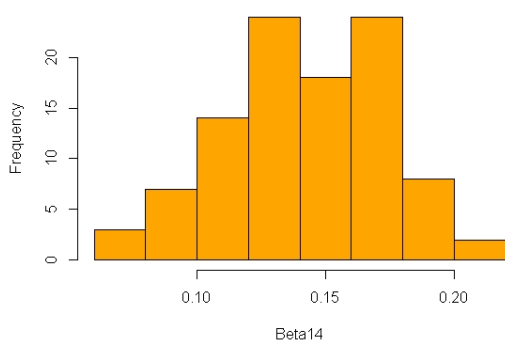
Draw 30% for in-sample data where the remaining 70% for out-of-sample data and doing the process following the diagram and algorithm of HB method for $L = 100$ times. Figure 3.11 shows the histogram of the posterior mean and posterior variance for $L = 100$ times.

Figure 3.11: The histogram of posterior mean and posterior variance for $L = 100$.

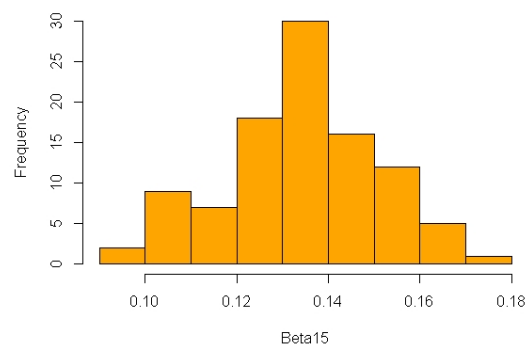




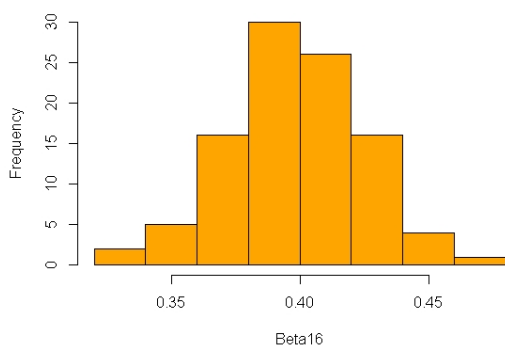
Histogram for Beta14



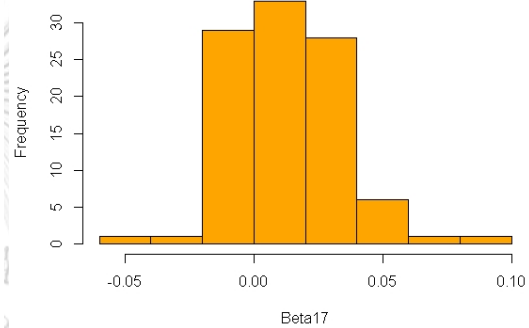
Histogram for Beta15



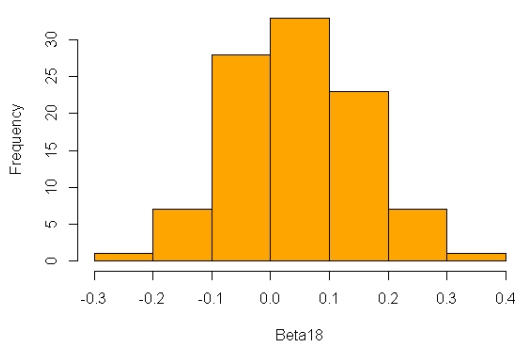
Histogram for Beta16



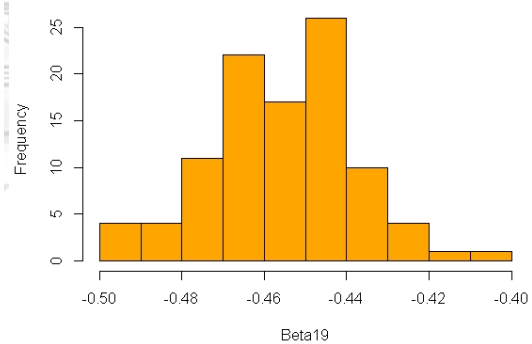
Histogram for Beta17



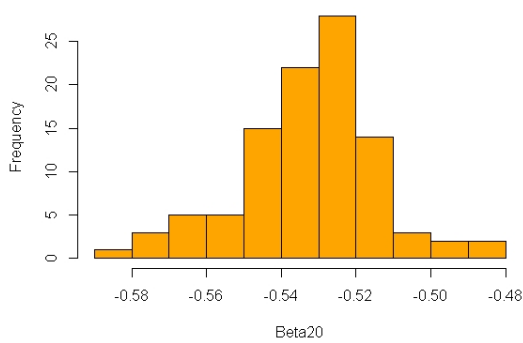
Histogram for Beta18



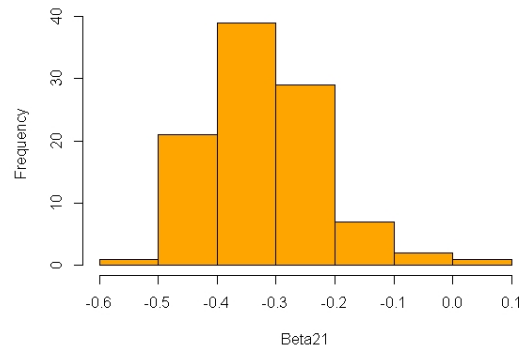
Histogram for Beta19



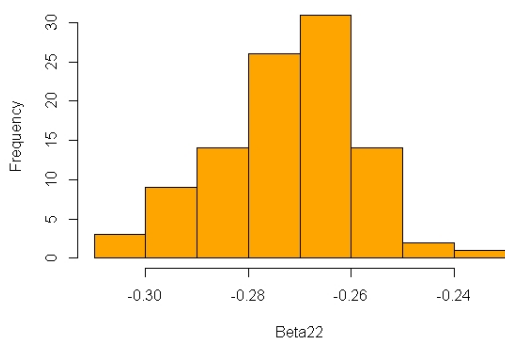
Histogram for Beta20



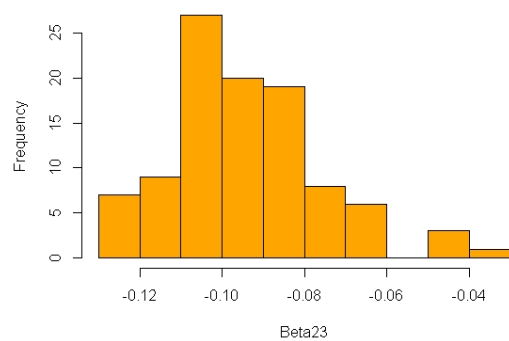
Histogram for Beta21



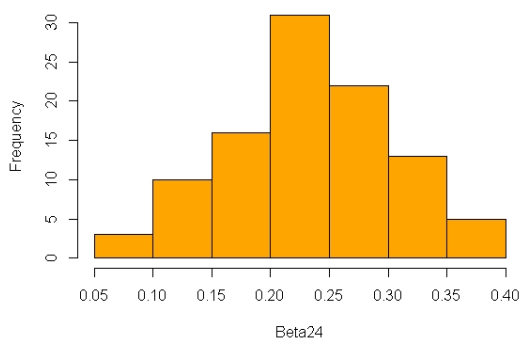
Histogram for Beta22



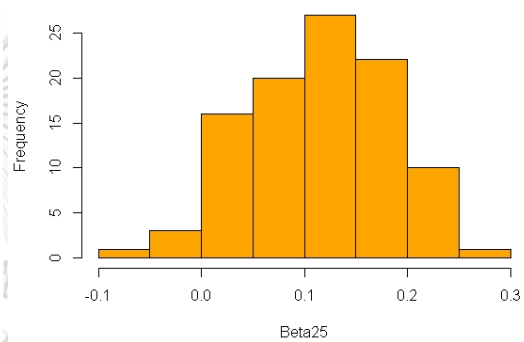
Histogram for Beta23



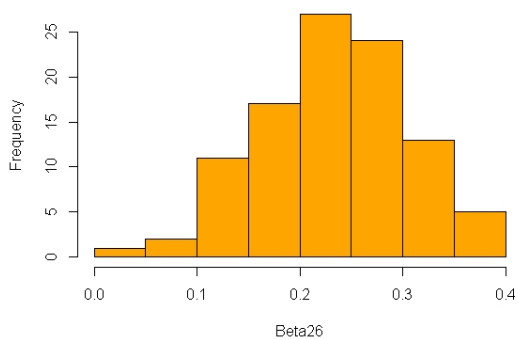
Histogram for Beta24



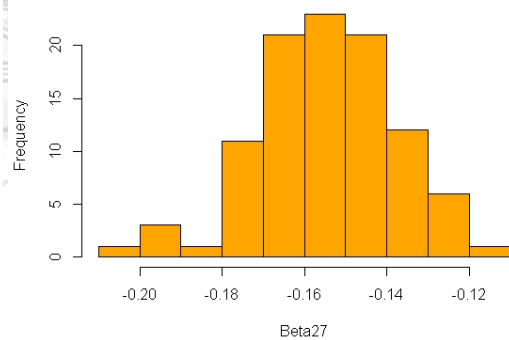
Histogram for Beta25



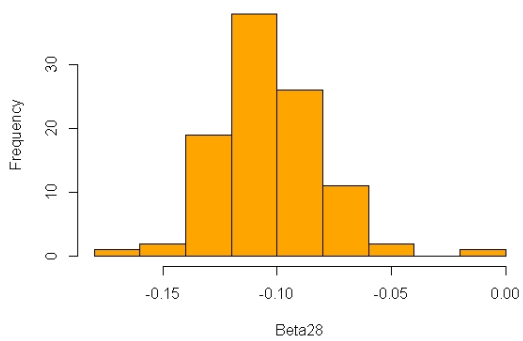
Histogram for Beta26



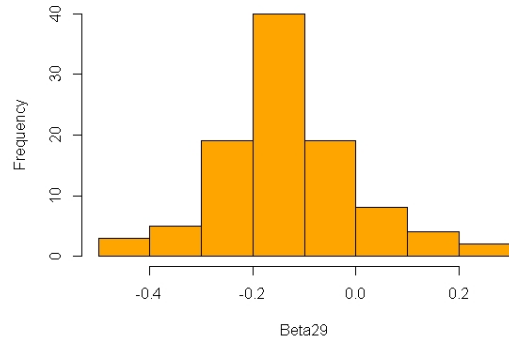
Histogram for Beta27

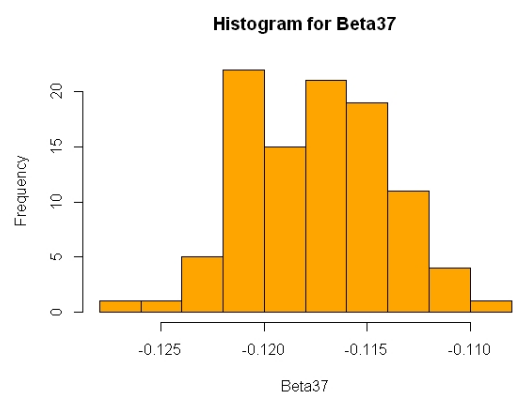
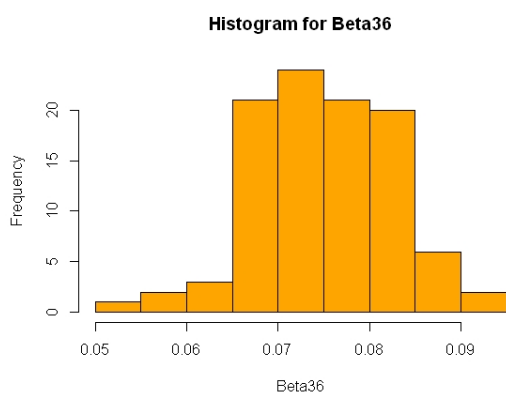
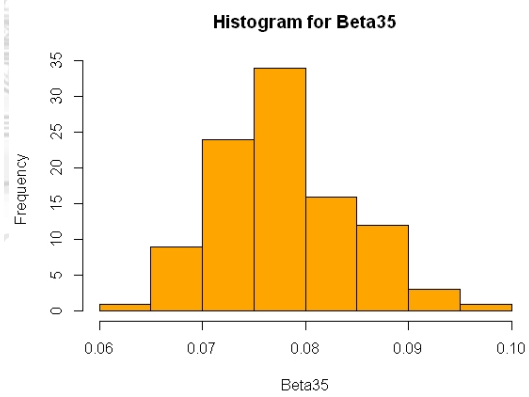
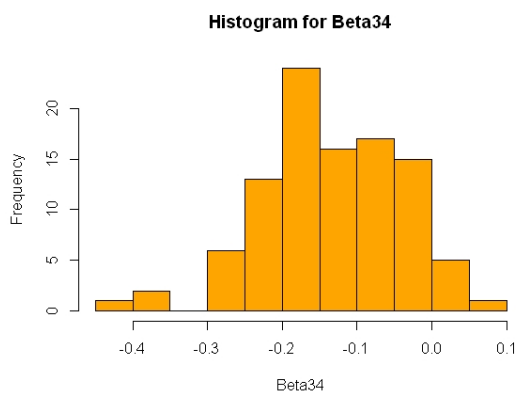
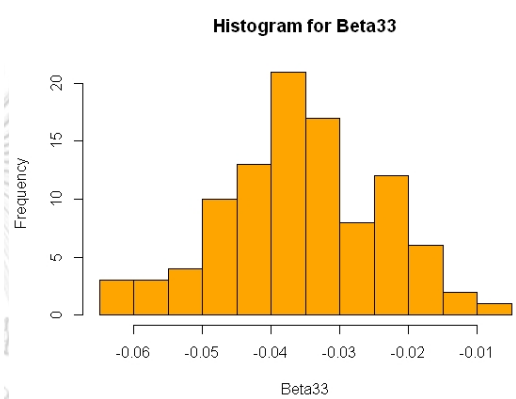
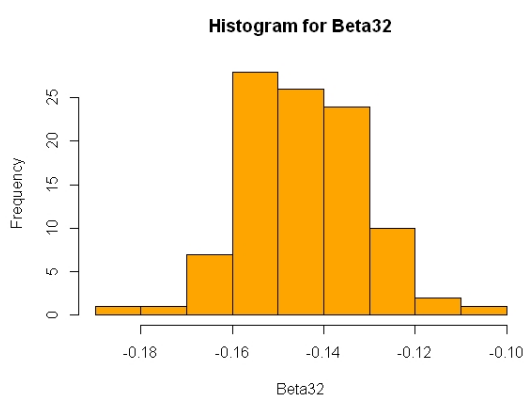
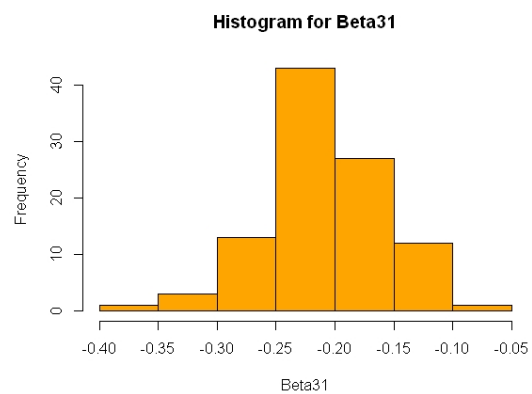
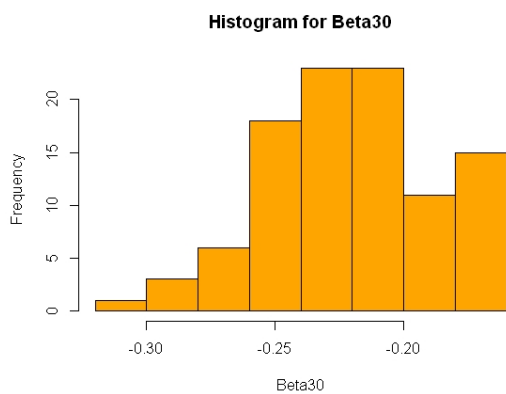


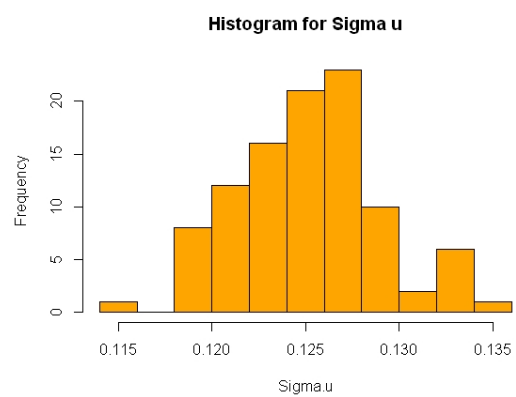
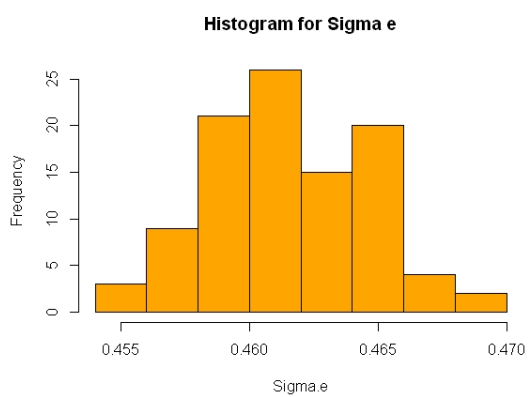
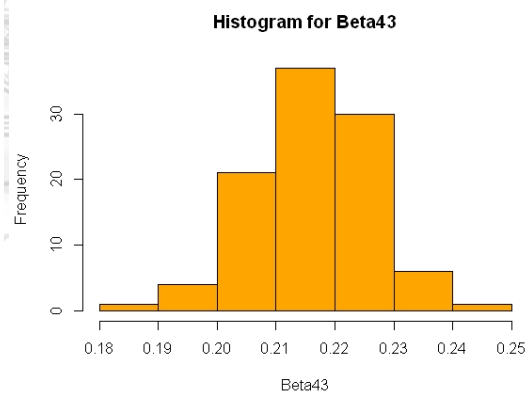
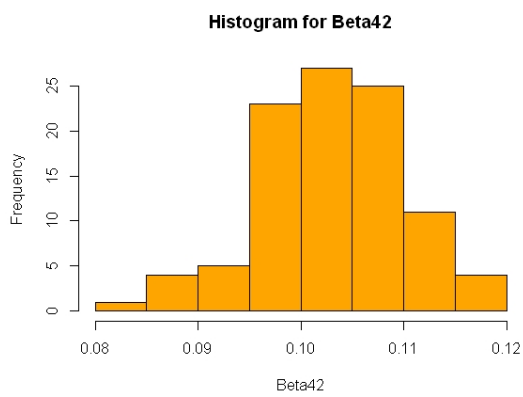
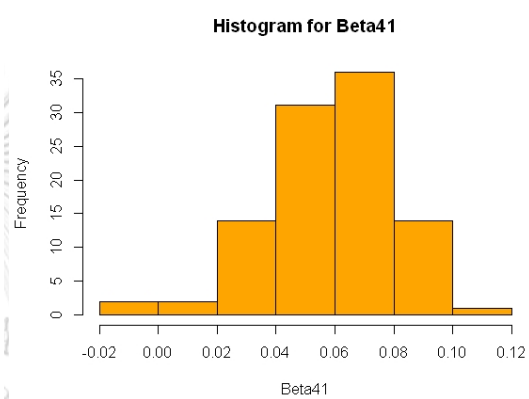
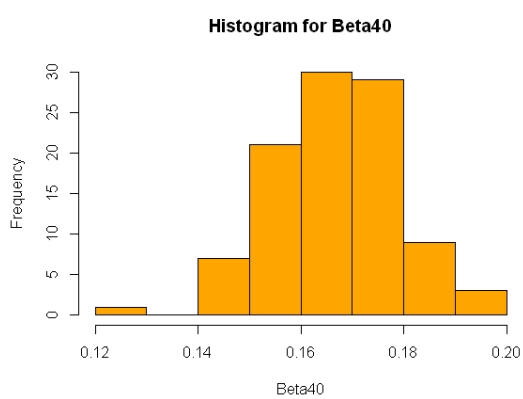
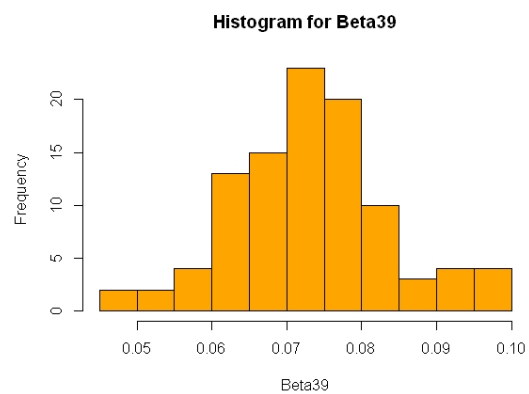
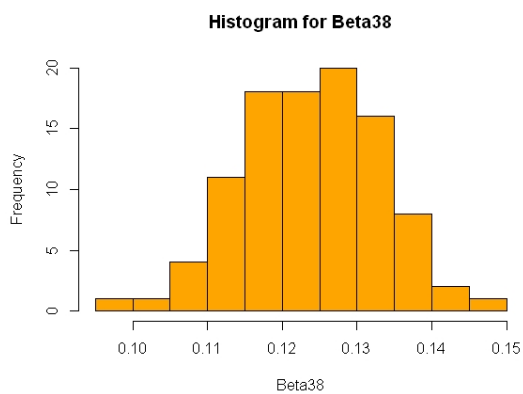
Histogram for Beta28



Histogram for Beta29







The corresponding descriptive statistics for the posterior means and posterior variances are given in Table 3.14.

Posterior	Minimum	1st Qu.	Median	Mean	3rd Qu.	Maximum
$\hat{\beta}_0$	8.674	8.764	8.820	8.823	8.879	8.991
$\hat{\beta}_1$	0.018	0.058	0.067	0.066	0.074	0.099
$\hat{\beta}_2$	0.046	0.079	0.091	0.092	0.104	0.141
$\hat{\beta}_3$	0.128	0.173	0.190	0.188	0.200	0.239
$\hat{\beta}_4$	-0.113	0.016	0.046	0.047	0.081	0.202
$\hat{\beta}_5$	-0.410	-0.105	-0.019	-0.014	0.083	0.433
$\hat{\beta}_6$	-0.827	0.064	0.280	0.249	0.498	0.961
$\hat{\beta}_7$	-0.100	-0.086	-0.079	-0.079	-0.073	-0.051
$\hat{\beta}_8$	-0.047	-0.030	-0.023	-0.023	-0.017	0.006
$\hat{\beta}_9$	-0.507	-0.396	-0.358	-0.361	-0.323	-0.221
$\hat{\beta}_{10}$	-0.583	-0.313	-0.248	-0.255	-0.195	-0.044
$\hat{\beta}_{11}$	-0.502	-0.349	-0.264	-0.276	-0.218	-0.030
$\hat{\beta}_{12}$	-0.066	-0.014	-0.005	-0.002	0.014	0.060
$\hat{\beta}_{13}$	-0.119	-0.064	-0.047	-0.047	-0.032	0.032
$\hat{\beta}_{14}$	0.071	0.124	0.142	0.143	0.168	0.217
$\hat{\beta}_{15}$	0.091	0.124	0.134	0.134	0.145	0.177
$\hat{\beta}_{16}$	0.321	0.380	0.399	0.399	0.416	0.472
$\hat{\beta}_{17}$	-0.049	-0.002	0.012	0.013	0.025	0.088
$\hat{\beta}_{18}$	-0.236	-0.030	0.040	0.045	0.125	0.345
$\hat{\beta}_{19}$	-0.493	-0.467	-0.454	-0.455	-0.445	-0.406
$\hat{\beta}_{20}$	-0.581	-0.543	-0.530	-0.532	-0.521	-0.483
$\hat{\beta}_{21}$	-0.497	-0.387	-0.341	-0.321	-0.266	0.077
$\hat{\beta}_{22}$	-0.308	-0.280	-0.271	-0.272	-0.263	-0.239
$\hat{\beta}_{23}$	-0.129	-0.106	-0.096	-0.094	-0.084	-0.037
$\hat{\beta}_{24}$	0.071	0.197	0.243	0.241	0.291	0.377

Continued on next page...

Table 3.14 – continued from previous page.

Posterior	Minimum	1st Qu.	Median	Mean	3rd Qu.	Maximum
$\hat{\beta}_{25}$	-0.051	0.076	0.121	0.123	0.176	0.256
$\hat{\beta}_{26}$	0.043	0.184	0.245	0.239	0.288	0.395
$\hat{\beta}_{27}$	-0.208	-0.165	-0.155	-0.154	-0.142	-0.117
$\hat{\beta}_{28}$	-0.161	-0.118	-0.104	-0.104	-0.091	-0.006
$\hat{\beta}_{29}$	-0.483	-0.221	-0.143	-0.134	-0.050	0.254
$\hat{\beta}_{30}$	-0.303	-0.242	-0.221	-0.220	-0.199	-0.163
$\hat{\beta}_{31}$	-0.358	-0.234	-0.210	-0.210	-0.180	-0.093
$\hat{\beta}_{32}$	-0.181	-0.153	-0.145	-0.144	-0.137	-0.105
$\hat{\beta}_{33}$	-0.061	-0.043	-0.036	-0.035	-0.028	-0.008
$\hat{\beta}_{34}$	-0.439	-0.194	-0.140	-0.133	-0.058	0.085
$\hat{\beta}_{35}$	0.064	0.073	0.078	0.078	0.082	0.098
$\hat{\beta}_{36}$	0.053	0.070	0.075	0.075	0.080	0.091
$\hat{\beta}_{37}$	-0.127	-0.120	-0.117	-0.118	-0.115	-0.110
$\hat{\beta}_{38}$	0.096	0.118	0.124	0.124	0.131	0.148
$\hat{\beta}_{39}$	0.047	0.066	0.074	0.073	0.078	0.097
$\hat{\beta}_{40}$	0.129	0.159	0.167	0.167	0.175	0.198
$\hat{\beta}_{41}$	-0.007	0.045	0.061	0.058	0.072	0.110
$\hat{\beta}_{42}$	0.085	0.099	0.103	0.103	0.107	0.117
$\hat{\beta}_{43}$	0.185	0.210	0.218	0.216	0.223	0.243
$\hat{\sigma}_e$	0.456	0.459	0.461	0.462	0.464	0.469
$\hat{\sigma}_u$	0.116	0.125	0.128	0.127	0.130	0.137

Table 3.14: The descriptive statistics for posterior means and posterior variances over $L = 100$.

From Table 3.14 and Figure 3.11, we can see that both $\hat{\beta}_{24}$ and $\hat{\beta}_{26}$ values are both positive, while $\hat{\beta}_{25}$ is negative and is less than $\hat{\beta}_{24}$ and $\hat{\beta}_{26}$. That is, also having either flush ($\mathbf{x}_{(24)}$) or both flush & squat ($\mathbf{x}_{(26)}$) for a toilet indicates higher expenditure. In the opposite direction, for those who have squat toilets ($\mathbf{x}_{(25)}$), the expenditure is also less than those with other types of toilets. Similar to $\hat{\beta}_{43}$ for the number of automobiles ($\mathbf{x}_{(43)}$) owned also affects the increase in expenditure significantly.

Next, we show the example of how to calculate the FGT poverty indicators by the HB method. Start with an example for Bangkok ($d = 1$). For in-sample data, we apply the model following Algorithm 7 in WinBUGS program to sample the posterior distributions of parameters. We generated chains of length 5000, discarding the first 2500 as a burn-in with 2 Markov chains and 2 thinning rate. For the first iteration ($m = 1$), we obtain the following

$$\hat{\beta} = \begin{pmatrix} 8.778 \\ 0.058 \\ 0.068 \\ 0.152 \\ \vdots \\ 0.241 \end{pmatrix}, \quad \hat{\sigma}_u^2 = 0.209, \quad \hat{\sigma}_e^2 = 0.018. \quad (3.21)$$

Having obtain these estimates, we generate out-of-sample data from (3.18)

as

$$Y_{di} \stackrel{\text{iid}}{\sim} N(\mathbf{x}'_{di}\hat{\beta} + u_d, \hat{\sigma}_e^2), \quad i \in r_d, \quad d = 1, \dots, D,$$

where $u_d \stackrel{\text{iid}}{\sim} N\left(0, \frac{\rho}{1-\rho}\hat{\sigma}_e^2\right)$ for $\rho = \frac{\hat{\sigma}_u^2}{\hat{\sigma}_u^2 + \hat{\sigma}_e^2}$.

Similar to the EB method in Section 3.2.2, we consider the first unit ($i = 1$) of out-of-sample units of Bangkok ($d = 1$), called Ms.C. So, we can calculate $\rho = \frac{0.209}{0.209+0.018} = 0.9207$ and generate u_1 from $N(0, \frac{0.9207}{1-0.9207} \cdot 0.018)$ obtaine $u_1 = 0.112$.

As describe in Section 3.1.2, we treat these variables of Ms.C as a matrix form. That is, the auxiliary variables \mathbf{x}_{11} of Ms.C is $[1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \dots \ 3 \ 1]'$ with length of 44. After that, we calculate the term of the mean of out-of-sample μ_{11} as follows. That is,

$$\begin{aligned} \mu_{11} &= [1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \dots \ 3 \ 1] \begin{pmatrix} 8.778 \\ 0.058 \\ 0.068 \\ 0.152 \\ \vdots \\ 0.241 \end{pmatrix} + 0.112 \\ &= 9.558. \end{aligned}$$

Next step, we generate Y_{11} of Ms.C from model (3.18). Finally, the estimated log expenditure of Ms.C is 9.152. The same process is applied to all out-of-sample units in Bangkok. Therefore, we obtain the new population for Bangkok $\mathbf{Y}_1 = (\mathbf{Y}_{1s}, \mathbf{Y}_{1r})'$.

For the same processes, we must generate another m ($= 2, \dots, 2500$) for Bangkok, denoted the new population by $\mathbf{Y}_1^{(m)}$. Calculate the poverty incidence ($\alpha = 0$) for all $M = 2500$. Accordingly, sum the $F_{01}^{(m)}$ and then divided by $M = 2500$, we obtain the poverty incidence \hat{F}_{01}^{HB} for Bangkok.

Doing the same process for the poverty gap ($\alpha = 1$) and poverty severity ($\alpha = 2$) and also for other provinces, we obtain the poverty incidence, poverty gap

and poverty severity for all 77 provinces in Thailand. Continue these processes $L = 100$ times. For $l = 1, \dots, L (= 100)$, let $\hat{F}_{\alpha d}^{(l)HB}$ be the FGT poverty indicator of HB estimator for the l^{th} process and $F_{\alpha d}^{(l)}$ be the true FGT poverty indicator for the l^{th} process. We then evaluate the performance of the estimates via absolute bias and mean square error as follows.

Therefore, the absolute bias (AB) and mean square error (MSE) of $\hat{F}_{\alpha d}^{HB}$ are

$$AB_{\alpha d}^{HB} = \frac{1}{L} \sum_{l=1}^L |\hat{F}_{\alpha d}^{(l)HB} - F_{\alpha d}^{(l)}|, \quad (3.22)$$

$$MSE_{\alpha d}^{HB} = \frac{1}{L} \sum_{l=1}^L (\hat{F}_{\alpha d}^{(l)HB} - F_{\alpha d}^{(l)})^2. \quad (3.23)$$

The results of the HB method are presented as follows:

Province	Poverty	Incidence	Poverty	Gap	Poverty	Severity
	AB(%)	MSE(%)	AB(%)	MSE(%)	AB(%)	MSE(%)
1	2.284807	0.052397	0.493629	0.002446	0.161953	0.000264
2	2.230816	0.049896	0.401634	0.001619	0.115852	0.000135
3	1.549825	0.024330	0.307135	0.000953	0.098875	0.000098
4	2.109312	0.044643	0.407921	0.001672	0.124192	0.000155
5	2.727426	0.075199	0.574989	0.003342	0.183300	0.000341
6	3.276466	0.111264	0.732589	0.005682	0.286788	0.000899
7	2.450947	0.062171	0.685375	0.004813	0.278708	0.000790
8	0.435748	0.002740	0.183328	0.000412	0.074008	0.000066
9	3.463393	0.125669	0.566697	0.003583	0.098564	0.000137
10	3.799837	0.145187	0.874752	0.007687	0.296694	0.000885
11	2.035304	0.042555	0.494020	0.002482	0.177031	0.000317
12	0.780362	0.007241	0.329137	0.001101	0.121650	0.000149

Continued on next page...

Table 3.15 – continued from previous page.

Province	Poverty	Incidence	Poverty	Gap	Poverty	Severity
	AB(%)	MSE(%)	AB(%)	MSE(%)	AB(%)	MSE(%)
13	0.709039	0.007622	0.426062	0.001999	0.190082	0.000396
14	0.657448	0.006306	0.313725	0.001106	0.142370	0.000217
15	2.628337	0.069979	0.652472	0.004284	0.226092	0.000514
16	1.134451	0.014630	0.248708	0.000740	0.067562	0.000068
17	2.276464	0.053181	0.536923	0.002934	0.179028	0.000326
18	0.541252	0.004633	0.298090	0.001110	0.172573	0.000329
19	0.613501	0.005396	0.091800	0.000126	0.045760	0.000032
20	2.285222	0.057527	0.116602	0.000224	0.115179	0.000150
21	1.081410	0.014104	0.417565	0.001867	0.180844	0.000340
22	2.250252	0.052170	0.557381	0.003175	0.197975	0.000399
23	1.843402	0.036906	0.592885	0.003650	0.247117	0.000627
24	2.492173	0.064861	0.924574	0.008667	0.396141	0.001581
25	4.845690	0.236207	1.197808	0.014409	0.407546	0.001669
26	3.218370	0.107589	1.173968	0.013932	0.464747	0.002176
27	2.650109	0.072676	0.750095	0.005699	0.261054	0.000690
28	0.515882	0.004430	0.187248	0.000478	0.137921	0.000210
29	3.551960	0.127292	0.848449	0.007239	0.291788	0.000856
30	1.911455	0.038112	0.526230	0.002849	0.203323	0.000422
31	4.117606	0.170969	0.823654	0.006906	0.254676	0.000673
32	1.142688	0.013996	0.292285	0.000891	0.097139	0.000099
33	1.253681	0.018107	0.486277	0.002500	0.198518	0.000411
34	5.334365	0.286552	1.179908	0.014076	0.382829	0.001491
35	6.747835	0.460127	0.729539	0.005539	0.065937	0.000060
36	3.334185	0.113544	1.110429	0.012419	0.423984	0.001806
37	5.548715	0.312350	1.121678	0.012942	0.313788	0.001042

Continued on next page...

Table 3.15 – continued from previous page.

Province	Poverty	Incidence	Poverty	Gap	Poverty	Severity
	AB(%)	MSE(%)	AB(%)	MSE(%)	AB(%)	MSE(%)
38	1.190257	0.018090	0.973836	0.009668	0.445436	0.002009
39	1.023997	0.012945	0.187623	0.000508	0.068507	0.000066
40	3.618214	0.132594	0.964750	0.009396	0.357448	0.001290
41	0.522327	0.003929	0.402579	0.001720	0.182394	0.000346
42	1.189685	0.015900	0.106181	0.000164	0.032378	0.000017
43	1.954472	0.040518	0.097193	0.000138	0.050765	0.000033
44	1.866041	0.037823	0.251397	0.000774	0.048929	0.000037
45	0.662757	0.006390	0.161221	0.000371	0.080226	0.000089
46	0.579311	0.004912	0.296115	0.001075	0.124352	0.000194
47	2.873831	0.091549	0.249200	0.000943	0.154664	0.000354
48	0.300178	0.001382	0.078207	0.000098	0.028545	0.000013
49	0.411248	0.002539	0.292093	0.000903	0.137217	0.000193
50	1.440039	0.022246	0.457210	0.002163	0.170992	0.000301
51	5.586478	0.317674	1.889772	0.036387	0.845839	0.007337
52	2.212115	0.052478	0.190084	0.000475	0.031586	0.000015
53	0.673817	0.005420	0.290144	0.000866	0.117879	0.000141
54	2.301965	0.053693	0.552175	0.003079	0.180282	0.000328
55	1.088738	0.013196	0.269347	0.000803	0.106179	0.000123
56	1.326990	0.019298	0.351978	0.001336	0.158490	0.000261
57	0.885423	0.010490	0.624230	0.004008	0.319900	0.001039
58	0.810280	0.008564	0.354445	0.001384	0.154470	0.000256
59	2.063460	0.043569	0.491916	0.002454	0.163748	0.000271
60	3.203143	0.103606	0.656340	0.004351	0.203429	0.000419
61	3.222306	0.105262	0.780286	0.006149	0.271143	0.000742
62	3.718815	0.139698	0.851920	0.007325	0.309753	0.000968

Continued on next page...

Table 3.15 – continued from previous page.

Province	Poverty	Incidence	Poverty	Gap	Poverty	Severity
	AB(%)	MSE(%)	AB(%)	MSE(%)	AB(%)	MSE(%)
63	1.031062	0.012741	0.253816	0.000776	0.076649	0.000085
64	0.856126	0.009319	0.358019	0.001385	0.130637	0.000187
65	1.371581	0.020709	0.316556	0.001102	0.107047	0.000126
66	1.775647	0.032867	0.437935	0.001966	0.152242	0.000237
67	2.640263	0.070112	0.542227	0.002952	0.168126	0.000284
68	3.424413	0.119029	0.731466	0.005490	0.220483	0.000523
69	3.247844	0.111021	0.445357	0.002324	0.101931	0.000143
70	2.899584	0.085566	0.832434	0.006978	0.285000	0.000818
71	3.192464	0.103301	0.717956	0.005204	0.245230	0.000605
72	0.536083	0.004200	0.134127	0.000304	0.062751	0.000062
73	0.735396	0.007326	0.286853	0.000925	0.109029	0.000132
74	2.454315	0.065175	0.420058	0.002051	0.111597	0.000161
75	6.450198	0.421831	1.002784	0.010304	0.170431	0.000319
76	0.991272	0.012940	0.162669	0.000364	0.130667	0.000183
77	6.240682	0.397216	1.285163	0.017026	0.351907	0.001318

Table 3.15: Percentages of absolute biases and mean square errors for the HB method.

From Table 3.15, the AB of poverty incidence of Kalasin (35th) is up to 6.75%, followed by Pattani (75th) and Narathiwat (77th) which give the values of 6.45% and 6.24%. That means the HB estimates poorly in the case of poverty incidence for those provinces. Moreover, those provinces have large FGT poverty indicators from SES. We may conclude that the provinces with large FGT poverty indicators from SES may provide the HB estimate perform poorly.

3.3 Result of Bayesian models for poverty indicators

In this section, we provide the result by comparing these three methods: ELL, EB and HB described in Section 3.2. The criteria for comparing these three methods are the absolute bias (AB) and the mean square error (MSE) which are described in Section 2.7.3. Next, we consider the AB and MSE on the FGT poverty indicators. For $l = 1, \dots, L (= 100)$, let $F_{\alpha d}^{(l)}$ be the true FGT for area d and $\hat{F}_{\alpha d}^{(l)}$ be the estimator of the FGT shown in Section 3.2 for each method. Calculate the following summary measures to examine the performance of the estimators. The absolute biases (AB) and mean square errors (MSE) on the FGT poverty indicators are calculated as

$$AB_{\alpha d} = \frac{1}{L} \sum_{l=1}^L |(\hat{F}_{\alpha d}^{(l)} - F_{\alpha d}^{(l)})|, \quad (3.24)$$

$$MSE_{\alpha d} = \frac{1}{L} \sum_{l=1}^L (\hat{F}_{\alpha d}^{(l)} - F_{\alpha d}^{(l)})^2. \quad (3.25)$$

Figure 3.12 - Figure 3.14 show the percentages of absolute biases of the ELL, EB and HB methods for the poverty incidence, poverty gap and poverty severity for all 77 provinces, respectively. Moreover, Figure 3.15 - Figure 3.17 show the percentages of mean square errors of the ELL, EB and HB methods for the poverty incidence, poverty gap and poverty severity for all 77 provinces, respectively.

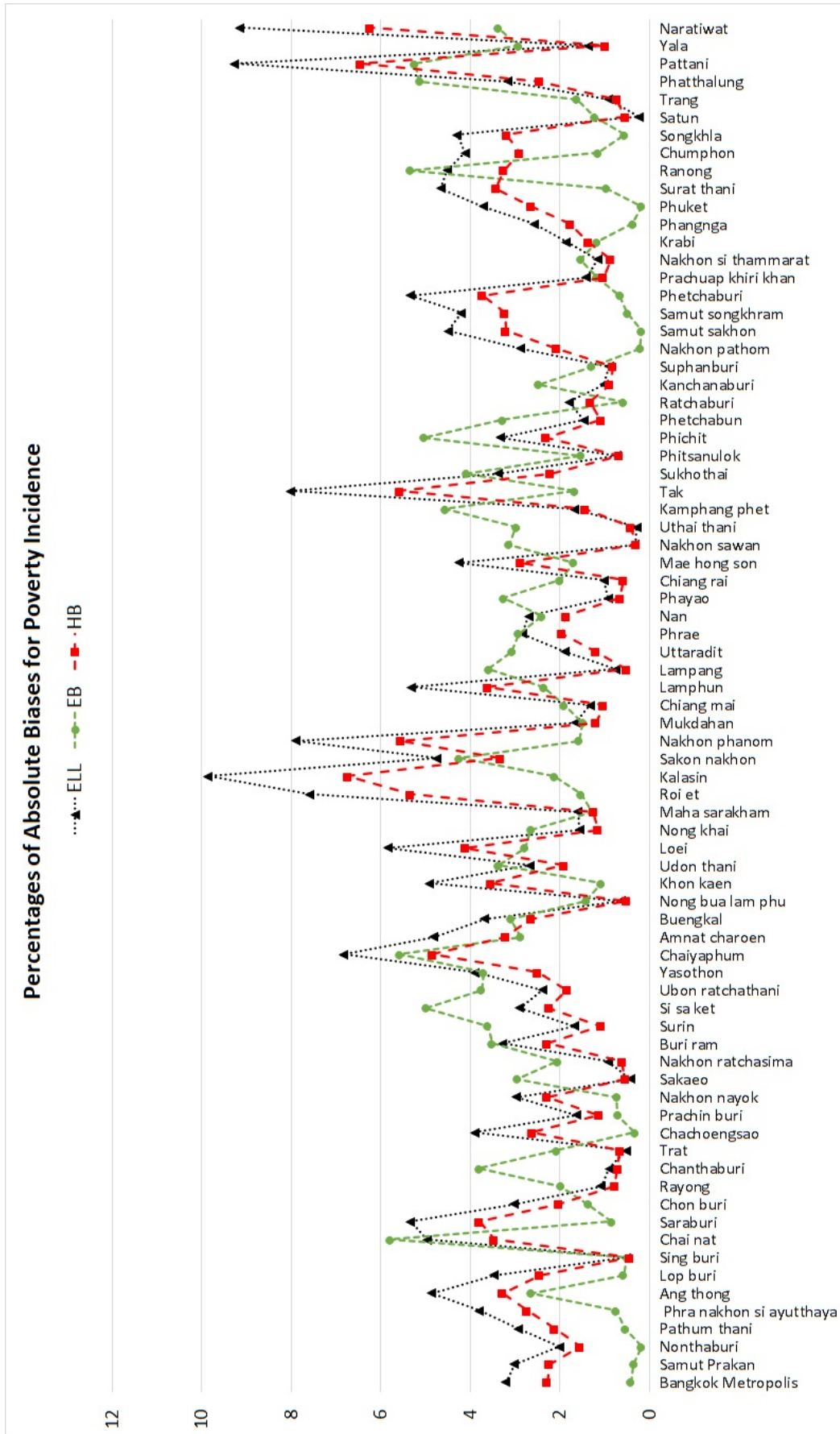


Figure 3.12: Percentages of absolute biases for poverty incidence of all 77 provinces.

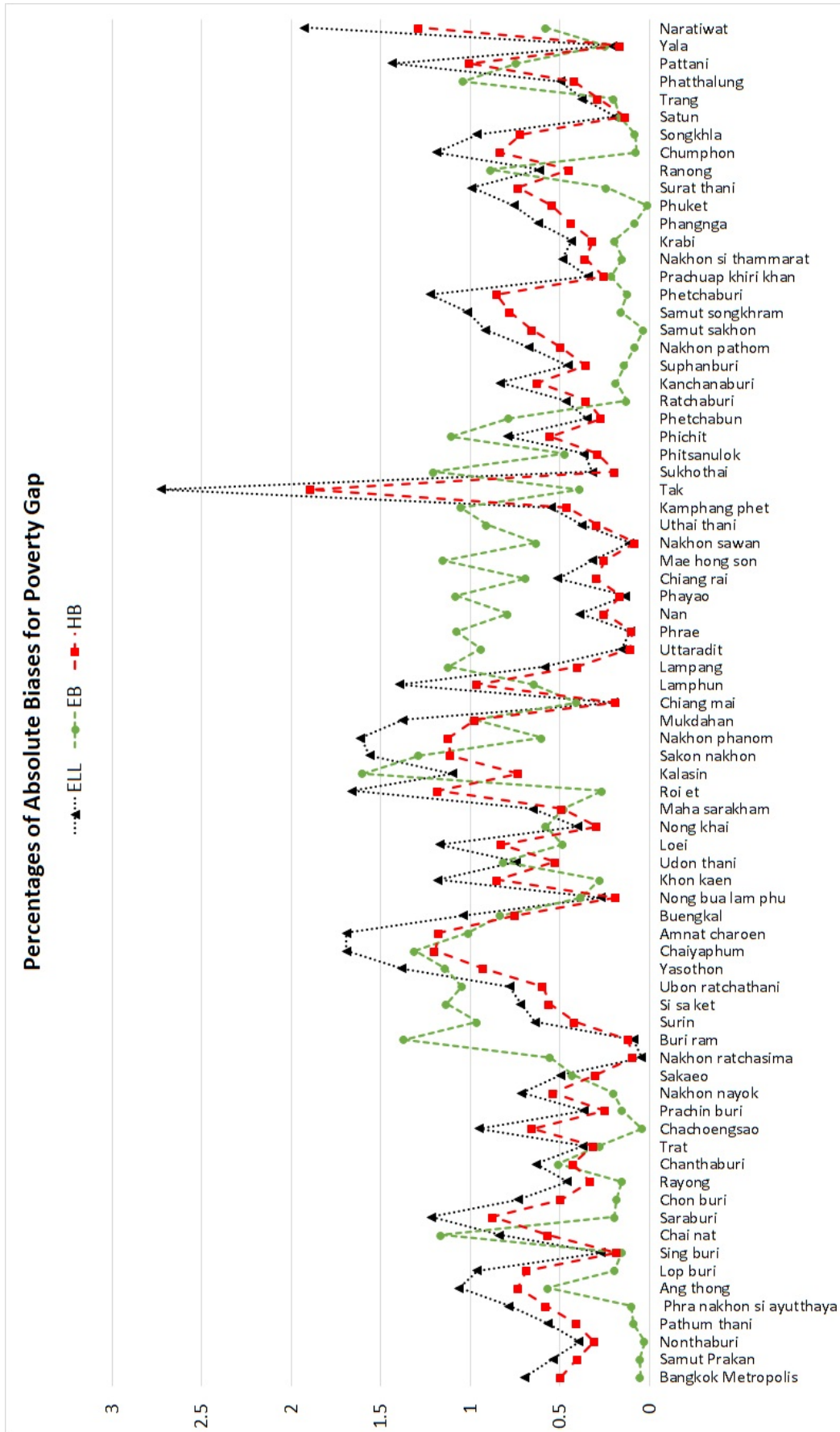


Figure 3.13: Percentages of absolute biases for poverty gap of all 77 provinces.

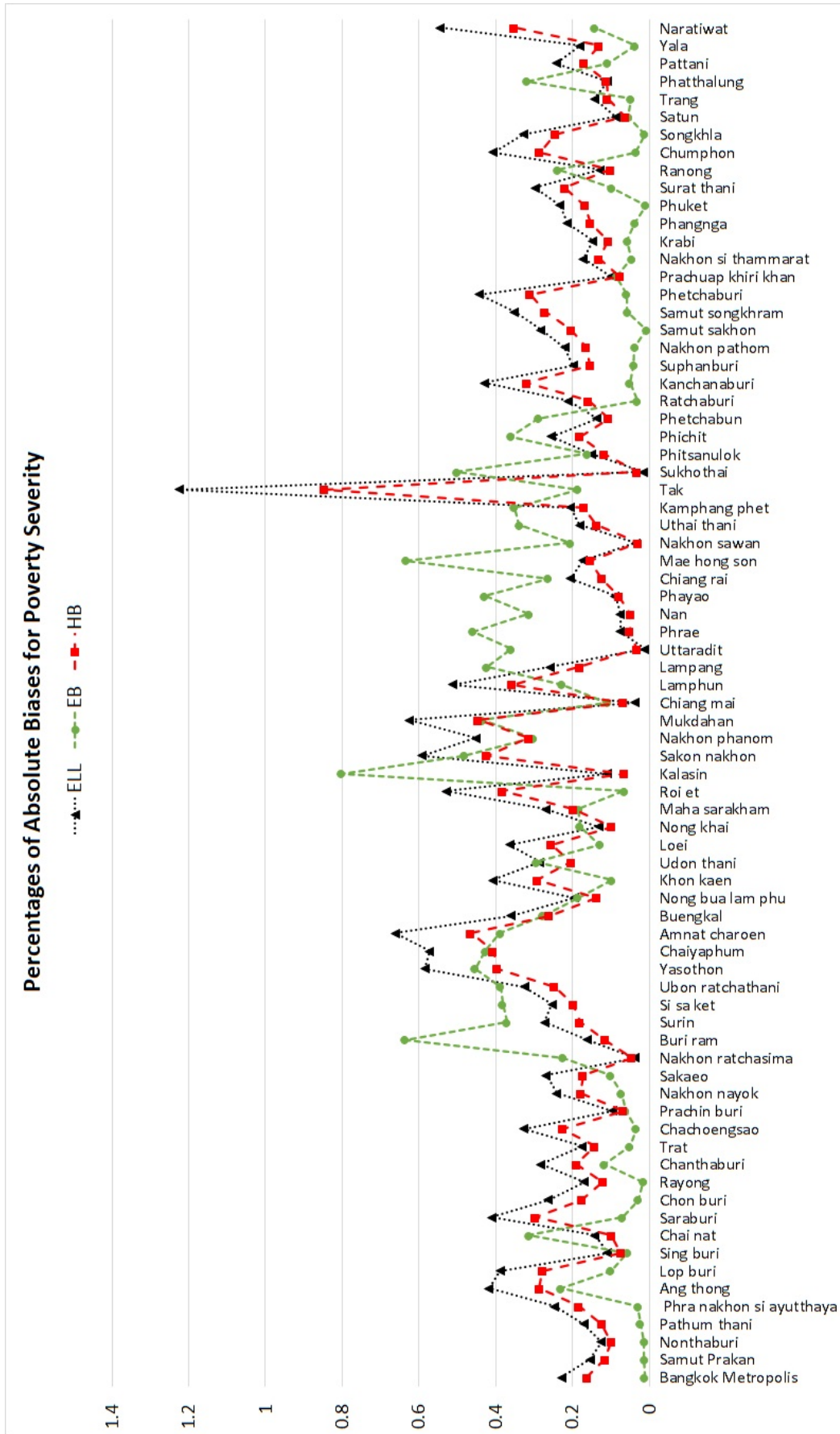


Figure 3.14: Percentages of absolute biases for poverty severity of all 77 provinces.

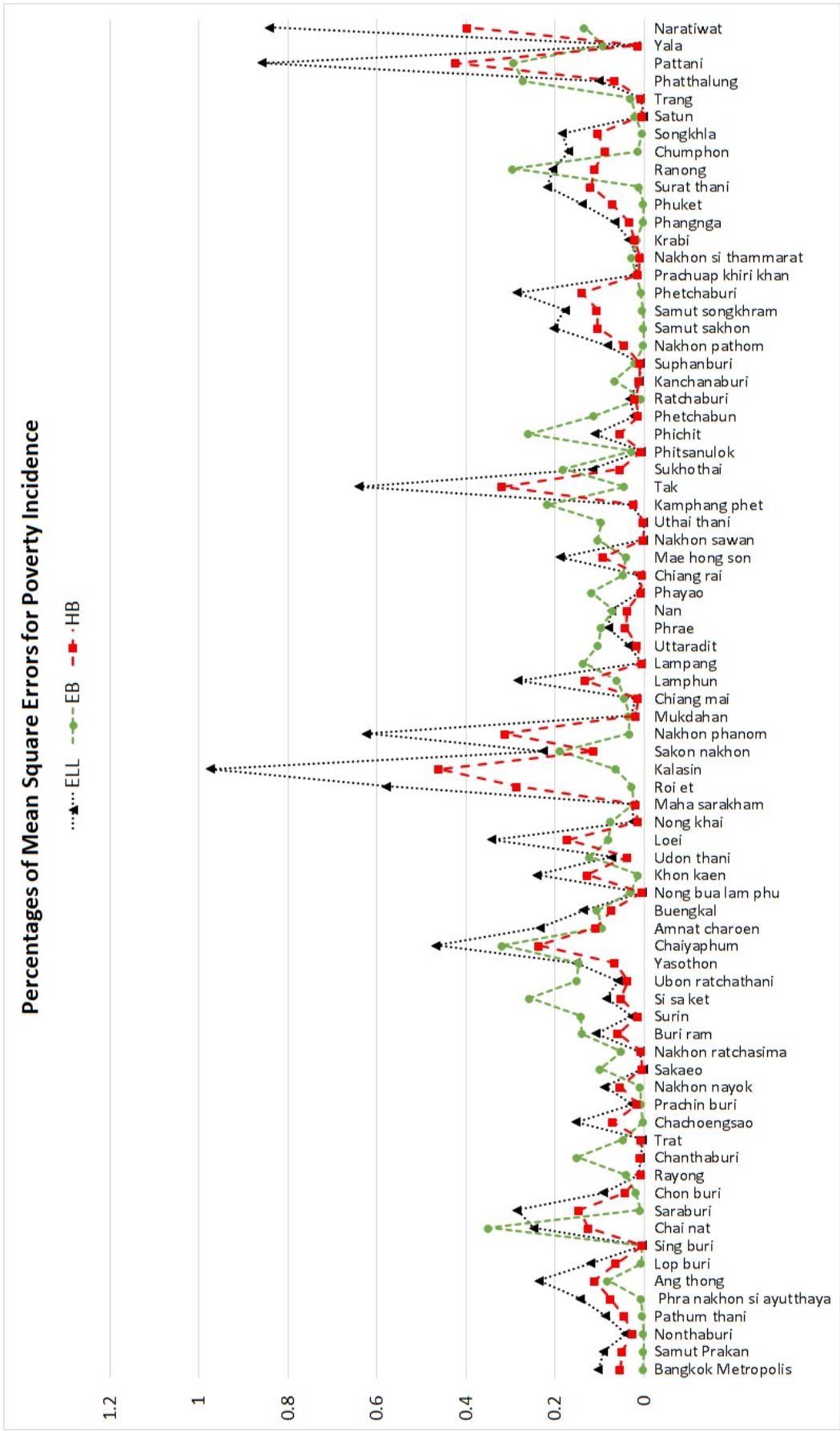


Figure 3.15: Percentages of mean square errors for poverty incidence of all 77 provinces.

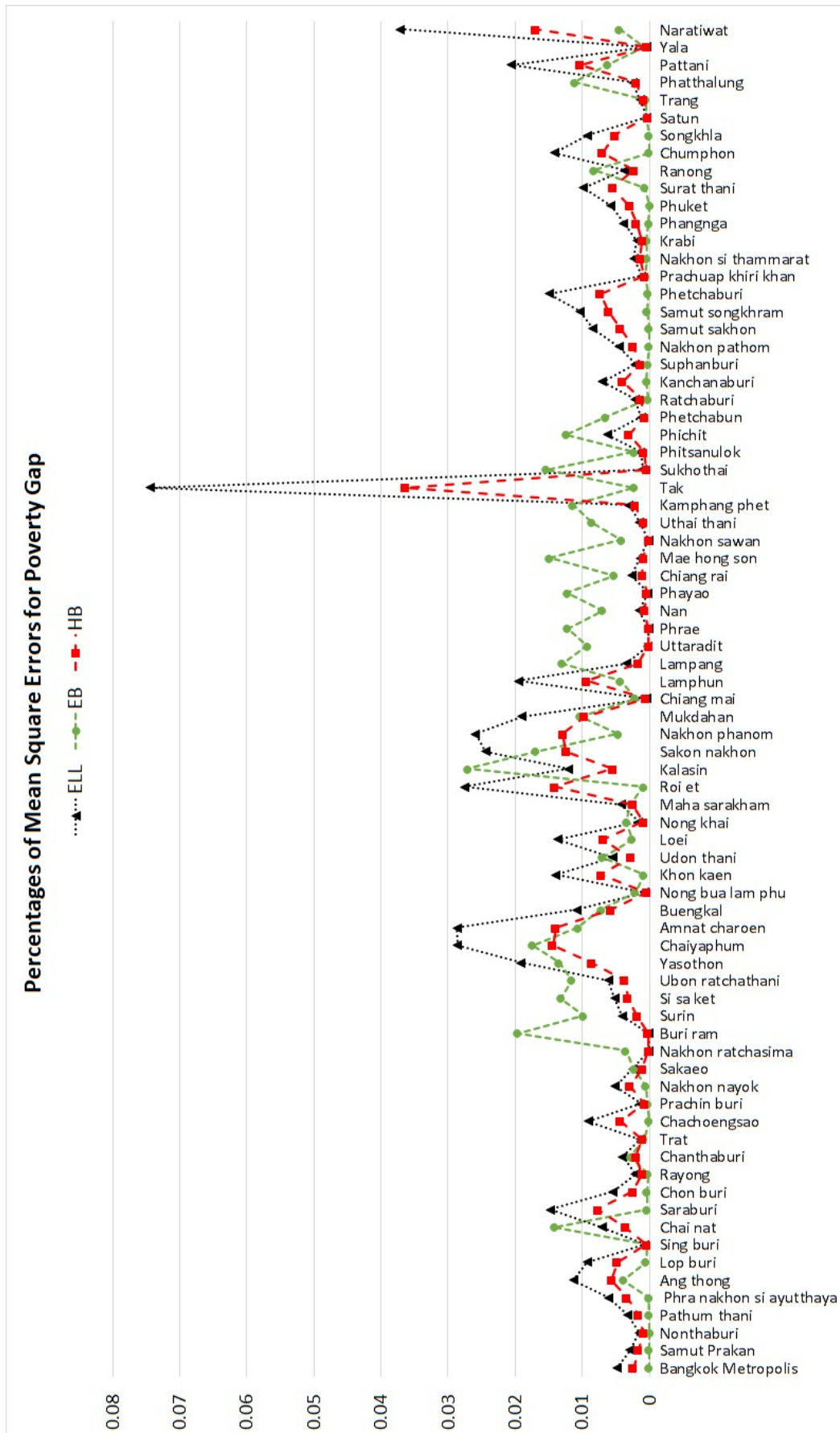


Figure 3.16: Percentages of mean square errors for poverty gap of all 77 provinces.

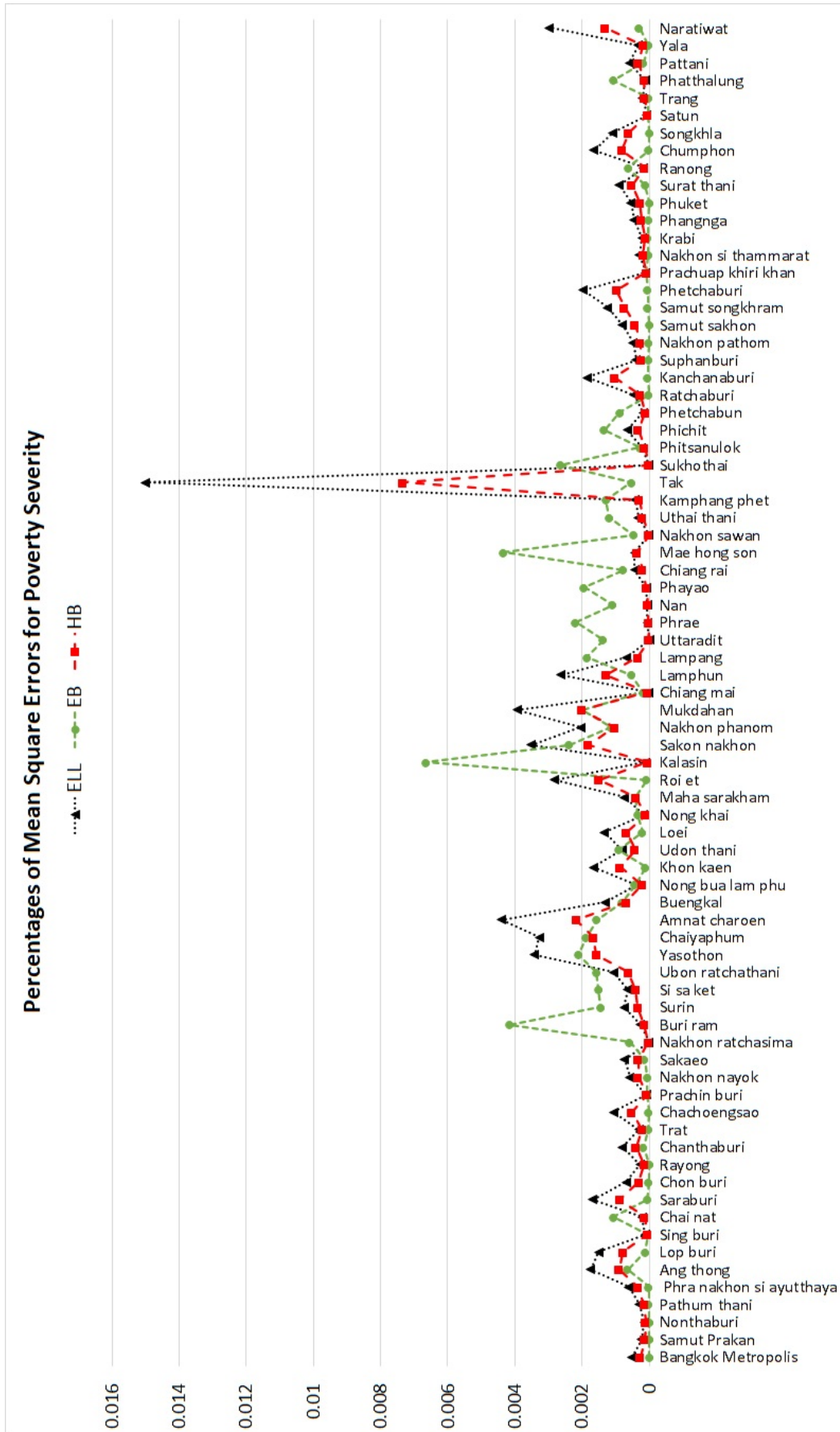


Figure 3.17: Percentages of mean square errors for poverty severity of all 77 provinces.

Figure 3.12 - Figure 3.14 show that in most areas, the absolute biases (ABs) of the ELL method are higher than the EB and HB methods. If we compare only the HB and EB methods, we can see that in some provinces the ABs of the HB are smaller than the EB but some provinces are higher. However, the ABs of the ELL in most provinces are higher than ABs of the HB and EB methods. The same notice can be seen in Figure 3.15 - Figure 3.17 for the mean square errors (MSEs). In overall, the HB method provides smaller ABs and MSEs than the original ELL and EB methods for the FGT poverty indicators of the Household Socio-Economic Survey (SES) in 2017.

Consequently, average across areas of absolute biases (\overline{AB}) and mean square errors (\overline{MSE}) are calculated as

$$\overline{AB}_\alpha = \frac{1}{D} \sum_{d=1}^D AB_{ad}, \quad (3.26)$$

$$\overline{MSE}_\alpha = \frac{1}{D} \sum_{d=1}^D MSE_{ad}. \quad (3.27)$$

Table 3.16 and Table 3.17 present the percentages of averages across areas of absolute biases and mean square errors for the ELL, EB and HB methods, respectively.

Method	Poverty incidence	Poverty gap	Poverty severity
ELL	3.12977	0.75045	0.26659
EB	2.21506	0.54293	0.19866
HB	2.23899	0.53770	0.19286

Table 3.16: Percentages of averages across areas of absolute biases for the ELL, EB and HB methods.

Method	Poverty incidence	Poverty gap	Poverty severity
ELL	0.14880	0.00814	0.00107
EB	0.07794	0.00510	0.00076
HB	0.07571	0.00417	0.00056

Table 3.17: Percentages of averages across areas of mean square errors for the ELL, EB and HB methods.

Table 3.16 reports the percentages of average across areas of absolute biases (\overline{AB}) over 77 provinces for the poverty incidence, poverty gap and poverty severity. It is commonly known that a good estimator should provide small AB and MSE, also for the average across areas of AB and MSE. Consider the \overline{AB} of all FGT poverty indicators, the original ELL method provides the highest value of \overline{AB} . Regarding comparisons between the EB and HB methods, for the poverty gap, the \overline{AB}_0 of the HB which is 0.53770 is smaller than \overline{AB}_0 of the EB method which is 0.54293. Similarly, for the poverty severity, the \overline{AB}_0 of the HB which is 0.19286 is smaller than \overline{AB}_0 of the EB method which is 0.19866.

Table 3.17 reports the percentages of average across areas of mean square errors (\overline{MSE}) over 77 areas for the poverty incidence, poverty gap and poverty severity. For poverty incidence, the \overline{MSE}_0 of the HB method which is 0.07571 is the smallest while the \overline{MSE}_0 of the EB and ELL methods are 0.07794 and 0.14880, respectively. For the poverty gap, the \overline{MSE}_1 of the HB method which is 0.00417 is also the smallest while the \overline{MSE}_1 of the EB and ELL methods are 0.00510 and 0.00814, respectively. Similar to the poverty severity, the \overline{MSE}_2 of the HB method which is 0.00056 is also the smallest while \overline{MSE}_2 for the EB and ELL methods are 0.00076 and 0.00107, respectively.

We can conclude that the HB method provides the smallest \overline{AB} and \overline{MSE} for all FGT poverty indicators except the \overline{AB}_0 of the poverty incidence. However, the original ELL method provides the highest for all FGT poverty indicators. In summary, the original ELL method performs poorly while the HB method is the most appropriate method for FGT poverty indicators of the Household Socio-Economic Survey (SES) in 2017.

In addition, we present the average of the FGT poverty indicators, which are the poverty incidence, poverty gap and poverty severity for the HB method in the form of poverty mapping as shown in Figure 3.18 - Figure 3.20.



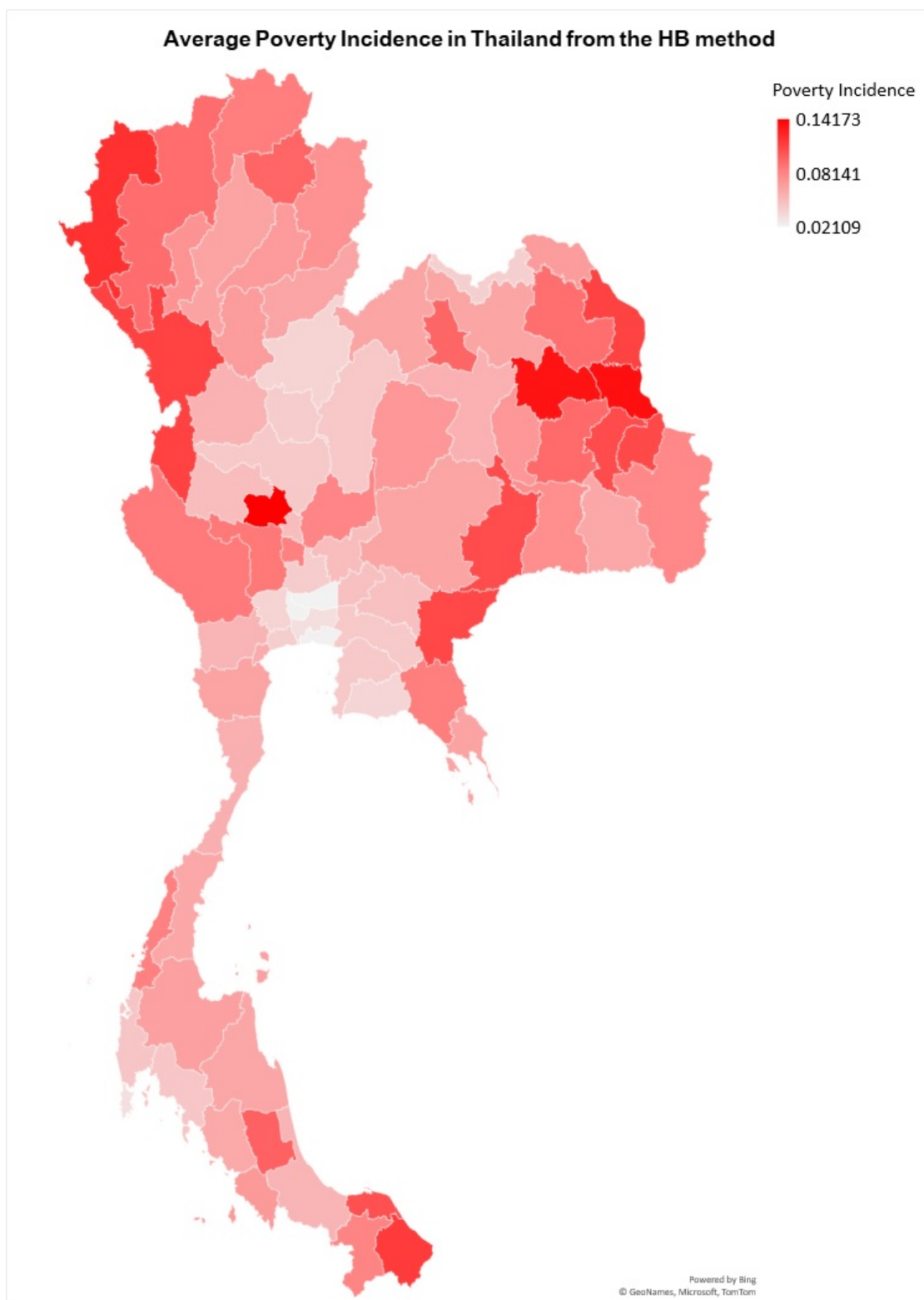


Figure 3.18: Average of poverty incidence in Thailand from the HB method.

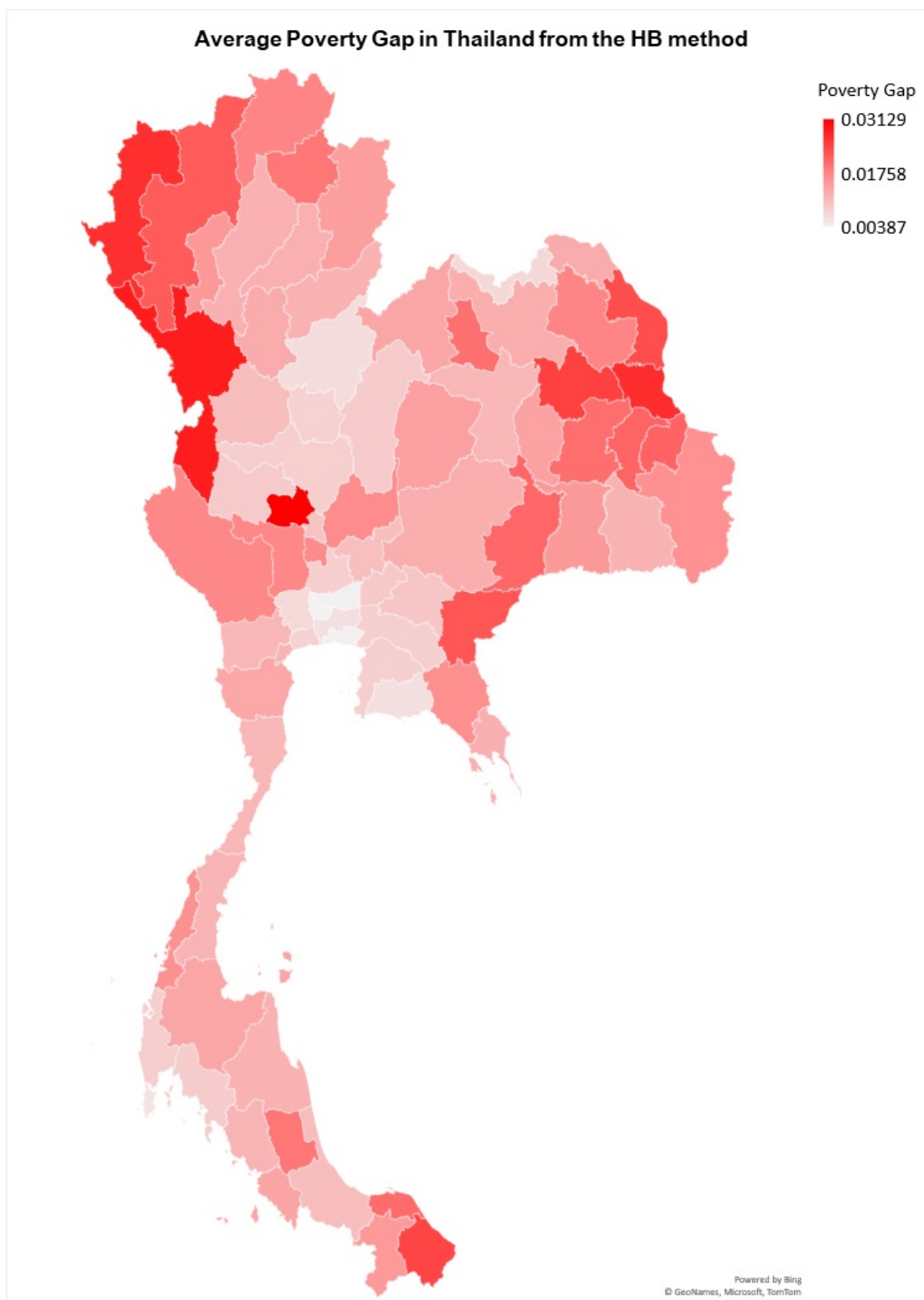


Figure 3.19: Average of poverty gap in Thailand from the HB method.

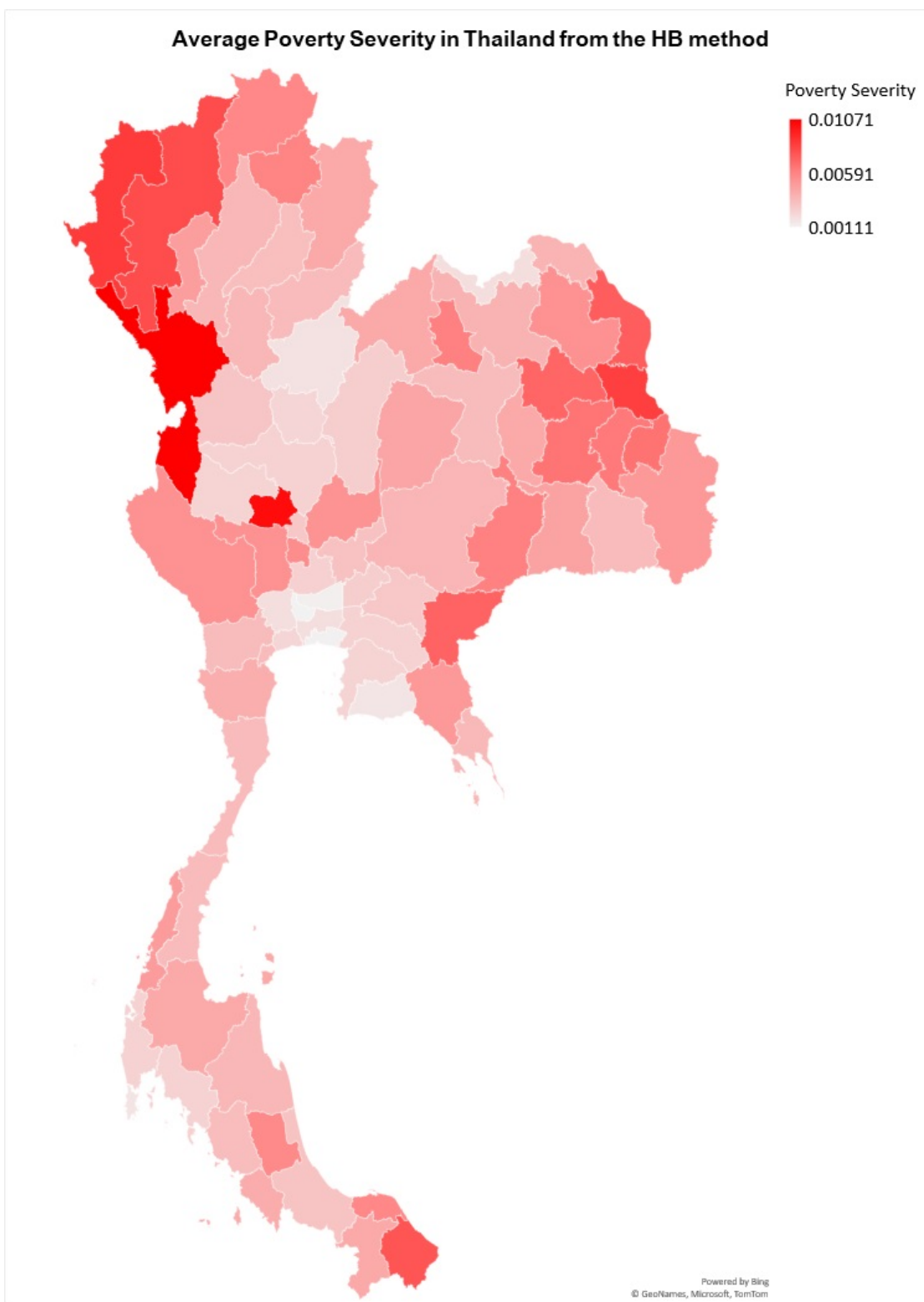


Figure 3.20: Average of poverty severity in Thailand from the HB method.

In Figure 3.18 - Figure 3.20, the dark-colored provinces correspond to provinces with high values of FGT poverty indicators. In another direction, the light-colored provinces correspond to provinces with low values of FGT poverty indicators.

From Figure 3.18, the dark provinces have large proportions of poor people. That is, the Northeastern and Northwestern regions have many poor people while the Central and South regions have relatively small number of poor people. Although the overall Central region has quite a few poor people, Chainat is a province that has a lot of poor people.

From Figure 3.19, the dark provinces have large poverty gaps. We can see that the Northwestern and North regions have large gaps of the poor people, that is poor people are far from the poverty line. While the Northeastern has a smaller poverty gap than the Northwestern and North regions.

From Figure 3.20, the dark provinces have large poverty severities, which adds more weight to poor people. We can see that the Northwestern and North regions also have the most violent of poverty, including Chainat in the Central region.

CHAPTER IV

BAYESIAN MODELS FOR EXPENDITURE VARIABLE AT AREA-LEVEL

In Chapter 3, we have discussed three models for poverty mapping required data at household level, called unit level models. Unit level models are usually precise and give detailed information for the population. However, due to its requirements of detailed household level information on the welfare variable and auxiliary variables, these unit level models can only be done when the census is available. That is every 10 years. In this chapter, we study alternative methods for the situation when such data is not available but aggregated data at the provincial level is available. These models are called the area-level model. The family of area-level models considered in this chapter is the constrained Bayes introduced by Louis (1984). He proposed this method to solve the limitation of the usual Bayes estimator which shows that the sample variance is less than the posterior expected sample variance of the parameter. In particular, we apply the concepts of constrained Bayes to construct the Empirical Bayes and Hierarchical Bayes to study expenditure data of Thailand at the provincial level. The organization of this chapter is as follows. Section 4.1 describes the Socio-Economic Survey (SES) data for area-level models. Furthermore, the constrained Bayes is applied to Empirical Bayes and Hierarchical Bayes, also known as constrained Empirical Bayes (CEB) and constrained Hierarchical Bayes (CHB), are discussed in Section 4.2. Finally, Section 4.3 gives the conclusion of this chapter.

4.1 The Household Socio-Economic Survey data at area-level

To be consistent with Chapter 3, we still treat the SES as the population and use the same set of sampling used in Chapter 3 even though area-level models do not require population data. Consider the population P of size N partitioned into $D(= 77)$ provinces having population size N_1, \dots, N_{77} . We draw for $L(= 100)$ sets of samples. The vector of the logarithm of average monthly expenditure per capita for area d , \mathbf{Y}_d , is divided into in-sample and out-of-sample data. For each $l = 1, \dots, L(= 100)$,

$$\mathbf{Y}_d^{(l)} = \begin{pmatrix} \mathbf{Y}_{ds}^{(l)} \\ \mathbf{Y}_{dr}^{(l)} \end{pmatrix},$$

where $\mathbf{Y}_d^{(l)}$ is decomposed into 30% for in-sample data $\mathbf{Y}_{ds}^{(l)}$ with sample size n_d and the remaining 70% for out-of-sample data $\mathbf{Y}_{dr}^{(l)}$ with out-of-sample size $N_d - n_d$ for province d .

Consider the welfare variable, let E_{di} be the average monthly expenditure per capita for individual i in province d . Therefore, the population mean of the logarithm of average monthly expenditure per capita across province \bar{Y}_{dU} for province d is defined by

$$\bar{Y}_{dU} = \frac{1}{N_d} \sum_{i=1}^{N_d} \log(E_{di}), \quad d = 1, \dots, 77.$$

Consequently, the sample mean of the logarithm of average monthly expenditure per capita across province \bar{Y}_d for province d is defined by

$$\bar{Y}_d = \frac{1}{n_d} \sum_{i=1}^{n_d} \log(E_{di}), \quad d = 1, \dots, 77.$$

The Table 4.1 presents the list of area level auxiliary variables.

\mathbf{x}	Area level auxiliary variables
$\mathbf{x}_{(1)}$	Proportion of detached house, row house or town house
$\mathbf{x}_{(2)}$	Proportion of cement & brick or wood & brick house
$\mathbf{x}_{(3)}$	Proportion of own dwelling and land or rented land
$\mathbf{x}_{(4)}$	Proportion of gas, electric cooking fuel or no cooking
$\mathbf{x}_{(5)}$	Proportion of flush latrine or bath flush and squat latrine
$\mathbf{x}_{(6)}$	Proportion of bottle drink water or treated tap drink water
$\mathbf{x}_{(7)}$	Proportion of owned LCD / LED / PLASMA
$\mathbf{x}_{(8)}$	Proportion of owned Video / VCD / DVD Player
$\mathbf{x}_{(9)}$	Proportion of owned mobile phone
$\mathbf{x}_{(10)}$	Proportion of owned home computer
$\mathbf{x}_{(11)}$	Proportion of owned refrigerator
$\mathbf{x}_{(12)}$	Proportion of owned microwave oven
$\mathbf{x}_{(13)}$	Proportion of owned washing machine
$\mathbf{x}_{(14)}$	Proportion of owned air conditioner
$\mathbf{x}_{(15)}$	Proportion of owned automobile

Table 4.1: The list of area level auxiliary variables.

For the selection of appropriate variables in the area-level model, we select the auxiliary variables that are related to the welfare variable using the forward and backward selections with Akaike Information Criterion (AIC) technique to select the most appropriate model.

We first apply the forward selection to the Socio-Economic Survey data with the auxiliary variables in Table 4.1, we get the following results.

Step	Add	Model	AIC
1	No	$\bar{Y}_U \sim 1$	-216
2	$\mathbf{x}_{(7)}$	$\bar{Y}_U \sim \mathbf{x}_{(7)}$	-312
3	$\mathbf{x}_{(3)}$	$\bar{Y}_U \sim \mathbf{x}_{(7)} + \mathbf{x}_{(3)}$	-340
4	$\mathbf{x}_{(9)}$	$\bar{Y}_U \sim \mathbf{x}_{(7)} + \mathbf{x}_{(3)} + \mathbf{x}_{(9)}$	-361
5	$\mathbf{x}_{(15)}$	$\bar{Y}_U \sim \mathbf{x}_{(7)} + \mathbf{x}_{(3)} + \mathbf{x}_{(9)} + \mathbf{x}_{(15)}$	-368
6	$\mathbf{x}_{(5)}$	$\bar{Y}_U \sim \mathbf{x}_{(7)} + \mathbf{x}_{(3)} + \mathbf{x}_{(9)} + \mathbf{x}_{(15)} + \mathbf{x}_{(5)}$	-370
7	$\mathbf{x}_{(2)}$	$\bar{Y}_U \sim \mathbf{x}_{(7)} + \mathbf{x}_{(3)} + \mathbf{x}_{(9)} + \mathbf{x}_{(15)} + \mathbf{x}_{(5)} + \mathbf{x}_{(2)}$	-379
8	$\mathbf{x}_{(12)}$	$\bar{Y}_U \sim \mathbf{x}_{(7)} + \mathbf{x}_{(3)} + \mathbf{x}_{(9)} + \mathbf{x}_{(15)} + \mathbf{x}_{(5)} + \mathbf{x}_{(2)} + \mathbf{x}_{(12)}$	-379
	$\mathbf{x}_{(13)}$	$\bar{Y}_U \sim \mathbf{x}_{(7)} + \mathbf{x}_{(3)} + \mathbf{x}_{(9)} + \mathbf{x}_{(15)} + \mathbf{x}_{(5)} + \mathbf{x}_{(2)} + \mathbf{x}_{(13)}$	-379
	$\mathbf{x}_{(6)}$	$\bar{Y}_U \sim \mathbf{x}_{(7)} + \mathbf{x}_{(3)} + \mathbf{x}_{(9)} + \mathbf{x}_{(15)} + \mathbf{x}_{(5)} + \mathbf{x}_{(2)} + \mathbf{x}_{(6)}$	-378
	$\mathbf{x}_{(4)}$	$\bar{Y}_U \sim \mathbf{x}_{(7)} + \mathbf{x}_{(3)} + \mathbf{x}_{(9)} + \mathbf{x}_{(15)} + \mathbf{x}_{(5)} + \mathbf{x}_{(2)} + \mathbf{x}_{(4)}$	-378
	$\mathbf{x}_{(14)}$	$\bar{Y}_U \sim \mathbf{x}_{(7)} + \mathbf{x}_{(3)} + \mathbf{x}_{(9)} + \mathbf{x}_{(15)} + \mathbf{x}_{(5)} + \mathbf{x}_{(2)} + \mathbf{x}_{(14)}$	-378
	$\mathbf{x}_{(10)}$	$\bar{Y}_U \sim \mathbf{x}_{(7)} + \mathbf{x}_{(3)} + \mathbf{x}_{(9)} + \mathbf{x}_{(15)} + \mathbf{x}_{(5)} + \mathbf{x}_{(2)} + \mathbf{x}_{(10)}$	-378
	$\mathbf{x}_{(1)}$	$\bar{Y}_U \sim \mathbf{x}_{(7)} + \mathbf{x}_{(3)} + \mathbf{x}_{(9)} + \mathbf{x}_{(15)} + \mathbf{x}_{(5)} + \mathbf{x}_{(2)} + \mathbf{x}_{(1)}$	-377
	$\mathbf{x}_{(8)}$	$\bar{Y}_U \sim \mathbf{x}_{(7)} + \mathbf{x}_{(3)} + \mathbf{x}_{(9)} + \mathbf{x}_{(15)} + \mathbf{x}_{(5)} + \mathbf{x}_{(2)} + \mathbf{x}_{(8)}$	-377
	$\mathbf{x}_{(11)}$	$\bar{Y}_U \sim \mathbf{x}_{(7)} + \mathbf{x}_{(3)} + \mathbf{x}_{(9)} + \mathbf{x}_{(15)} + \mathbf{x}_{(5)} + \mathbf{x}_{(2)} + \mathbf{x}_{(11)}$	-377
End	The final model is add $\mathbf{x}_{(2)}, \mathbf{x}_{(3)}, \mathbf{x}_{(5)}, \mathbf{x}_{(7)}, \mathbf{x}_{(9)}$ and $\mathbf{x}_{(15)}$.		

Table 4.2: The variable selection by using a forward selection for area level model.

We see that the AICs in Step 7 and Step 8 are quite close, so we need to make sure whether those variables should be added into the model by applying the analysis of variance (ANOVA) test as the following.

Initial Model: $\bar{Y}_U \sim \mathbf{x}_{(2)} + \mathbf{x}_{(3)} + \mathbf{x}_{(5)} + \mathbf{x}_{(7)} + \mathbf{x}_{(9)} + \mathbf{x}_{(15)}$

Null hypothesis: Initial model

Alternative hypothesis: Initial model with adding each of the following variables

Add	Res.Df	RSS	Df	Sum of Square	F	P-value
No	70	0.46892				
$\mathbf{x}_{(12)}$	69	0.45614	1	0.012775	0.19324	0.1690
$\mathbf{x}_{(13)}$	69	0.45783	1	0.011084	1.6705	0.2005
$\mathbf{x}_{(6)}$	69	0.45965	1	0.0092633	1.3905	0.2424
$\mathbf{x}_{(4)}$	69	0.46215	1	0.0067657	1.0101	0.3184
$\mathbf{x}_{(14)}$	69	0.46308	1	0.0058354	0.8695	0.3543
$\mathbf{x}_{(10)}$	69	0.46892	1	0.0045062	0.6695	0.4160
$\mathbf{x}_{(1)}$	69	0.46547	1	0.003451	0.5116	0.4769
$\mathbf{x}_{(8)}$	69	0.46658	1	0.0023427	0.3464	0.5581
$\mathbf{x}_{(11)}$	69	0.46880	1	0.00012095	0.0178	0.8942

Table 4.3: ANOVA Table for test area auxiliary variables.

Table 4.3 shows that the P-values of all cases are greater than the significance level of 0.05. Then, we do not reject the null hypothesis. Therefore, the listed variables should not be included in the model. Hence, the final model from the forward selection is $\bar{Y}_U \sim \mathbf{x}_{(2)} + \mathbf{x}_{(3)} + \mathbf{x}_{(5)} + \mathbf{x}_{(7)} + \mathbf{x}_{(9)} + \mathbf{x}_{(15)}$.

Moreover, we also apply the backward selection technique to the area variable selection as shown in Table 4.4.

Step	Remove	Model	AIC
1	No	$\bar{Y}_U \sim \mathbf{x}_{(1)} + \mathbf{x}_{(2)} + \mathbf{x}_{(3)} + \mathbf{x}_{(4)} + \mathbf{x}_{(5)} + \mathbf{x}_{(6)} + \mathbf{x}_{(7)} + \mathbf{x}_{(8)} + \mathbf{x}_{(9)} + \mathbf{x}_{(10)} + \mathbf{x}_{(11)} + \mathbf{x}_{(12)} + \mathbf{x}_{(13)} + \mathbf{x}_{(14)} + \mathbf{x}_{(15)}$	-371
2	$\mathbf{x}_{(11)}$	$\bar{Y}_U \sim \mathbf{x}_{(1)} + \mathbf{x}_{(2)} + \mathbf{x}_{(3)} + \mathbf{x}_{(4)} + \mathbf{x}_{(5)} + \mathbf{x}_{(6)} + \mathbf{x}_{(7)} + \mathbf{x}_{(8)} + \mathbf{x}_{(9)} + \mathbf{x}_{(10)} + \mathbf{x}_{(12)} + \mathbf{x}_{(13)} + \mathbf{x}_{(14)} + \mathbf{x}_{(15)}$	-373
3	$\mathbf{x}_{(10)}$	$\bar{Y}_U \sim \mathbf{x}_{(1)} + \mathbf{x}_{(2)} + \mathbf{x}_{(3)} + \mathbf{x}_{(4)} + \mathbf{x}_{(5)} + \mathbf{x}_{(6)} + \mathbf{x}_{(7)} + \mathbf{x}_{(8)} + \mathbf{x}_{(9)} + \mathbf{x}_{(12)} + \mathbf{x}_{(13)} + \mathbf{x}_{(14)} + \mathbf{x}_{(15)}$	-375
4	$\mathbf{x}_{(8)}$	$\bar{Y}_U \sim \mathbf{x}_{(1)} + \mathbf{x}_{(2)} + \mathbf{x}_{(3)} + \mathbf{x}_{(4)} + \mathbf{x}_{(5)} + \mathbf{x}_{(6)} + \mathbf{x}_{(7)} + \mathbf{x}_{(9)} + \mathbf{x}_{(12)} + \mathbf{x}_{(13)} + \mathbf{x}_{(14)} + \mathbf{x}_{(15)}$	-376
5	$\mathbf{x}_{(14)}$	$\bar{Y}_U \sim \mathbf{x}_{(1)} + \mathbf{x}_{(2)} + \mathbf{x}_{(3)} + \mathbf{x}_{(4)} + \mathbf{x}_{(5)} + \mathbf{x}_{(6)} + \mathbf{x}_{(7)} + \mathbf{x}_{(9)} + \mathbf{x}_{(12)} + \mathbf{x}_{(13)} + \mathbf{x}_{(15)}$	-378
6	$\mathbf{x}_{(1)}$	$\bar{Y}_U \sim \mathbf{x}_{(2)} + \mathbf{x}_{(3)} + \mathbf{x}_{(4)} + \mathbf{x}_{(5)} + \mathbf{x}_{(6)} + \mathbf{x}_{(7)} + \mathbf{x}_{(9)} + \mathbf{x}_{(12)} + \mathbf{x}_{(13)} + \mathbf{x}_{(15)}$	-379
7	$\mathbf{x}_{(6)}$	$\bar{Y}_U \sim \mathbf{x}_{(2)} + \mathbf{x}_{(3)} + \mathbf{x}_{(4)} + \mathbf{x}_{(5)} + \mathbf{x}_{(7)} + \mathbf{x}_{(9)} + \mathbf{x}_{(12)} + \mathbf{x}_{(13)} + \mathbf{x}_{(15)}$	-380
8	$\mathbf{x}_{(4)}$	$\bar{Y}_U \sim \mathbf{x}_{(2)} + \mathbf{x}_{(3)} + \mathbf{x}_{(5)} + \mathbf{x}_{(7)} + \mathbf{x}_{(9)} + \mathbf{x}_{(12)} + \mathbf{x}_{(13)} + \mathbf{x}_{(15)}$	-380
	$\mathbf{x}_{(15)}$	$\bar{Y}_U \sim \mathbf{x}_{(2)} + \mathbf{x}_{(3)} + \mathbf{x}_{(4)} + \mathbf{x}_{(5)} + \mathbf{x}_{(7)} + \mathbf{x}_{(9)} + \mathbf{x}_{(12)} + \mathbf{x}_{(13)}$	-379
	$\mathbf{x}_{(13)}$	$\bar{Y}_U \sim \mathbf{x}_{(2)} + \mathbf{x}_{(3)} + \mathbf{x}_{(4)} + \mathbf{x}_{(5)} + \mathbf{x}_{(7)} + \mathbf{x}_{(9)} + \mathbf{x}_{(12)} + \mathbf{x}_{(15)}$	-379
	$\mathbf{x}_{(5)}$	$\bar{Y}_U \sim \mathbf{x}_{(2)} + \mathbf{x}_{(3)} + \mathbf{x}_{(4)} + \mathbf{x}_{(7)} + \mathbf{x}_{(9)} + \mathbf{x}_{(12)} + \mathbf{x}_{(13)} + \mathbf{x}_{(15)}$	-379
	$\mathbf{x}_{(12)}$	$\bar{Y}_U \sim \mathbf{x}_{(2)} + \mathbf{x}_{(3)} + \mathbf{x}_{(4)} + \mathbf{x}_{(5)} + \mathbf{x}_{(7)} + \mathbf{x}_{(9)} + \mathbf{x}_{(13)} + \mathbf{x}_{(15)}$	-378
	$\mathbf{x}_{(7)}$	$\bar{Y}_U \sim \mathbf{x}_{(2)} + \mathbf{x}_{(3)} + \mathbf{x}_{(4)} + \mathbf{x}_{(5)} + \mathbf{x}_{(9)} + \mathbf{x}_{(12)} + \mathbf{x}_{(13)} + \mathbf{x}_{(15)}$	-370
	$\mathbf{x}_{(2)}$	$\bar{Y}_U \sim \mathbf{x}_{(3)} + \mathbf{x}_{(4)} + \mathbf{x}_{(5)} + \mathbf{x}_{(7)} + \mathbf{x}_{(9)} + \mathbf{x}_{(12)} + \mathbf{x}_{(13)} + \mathbf{x}_{(15)}$	-369
	$\mathbf{x}_{(9)}$	$\bar{Y}_U \sim \mathbf{x}_{(2)} + \mathbf{x}_{(3)} + \mathbf{x}_{(4)} + \mathbf{x}_{(5)} + \mathbf{x}_{(7)} + \mathbf{x}_{(12)} + \mathbf{x}_{(13)} + \mathbf{x}_{(15)}$	-367

End: The included auxiliary variables are $\mathbf{x}_{(2)}, \mathbf{x}_{(3)}, \mathbf{x}_{(4)}, \mathbf{x}_{(5)}, \mathbf{x}_{(7)}, \mathbf{x}_{(9)}, \mathbf{x}_{(12)}, \mathbf{x}_{(13)}, \mathbf{x}_{(15)}$.

Table 4.4: The variable selection by using a backward selection for area level model.

From Table 4.4, the included auxiliary variables from the backward selection by AIC technique are $\mathbf{x}_{(2)}$, $\mathbf{x}_{(3)}$, $\mathbf{x}_{(4)}$, $\mathbf{x}_{(5)}$, $\mathbf{x}_{(7)}$, $\mathbf{x}_{(9)}$, $\mathbf{x}_{(12)}$, $\mathbf{x}_{(13)}$ and $\mathbf{x}_{(15)}$. For variable selections from the forward and backward selections, the different variables are $\mathbf{x}_{(4)}$, $\mathbf{x}_{(12)}$ and $\mathbf{x}_{(13)}$. To finalize the model, we apply the ANOVA test to choose the appropriate model for the area auxiliary variables as follows.

The null hypothesis is Model 1 (no $\mathbf{x}_{(4)}$, $\mathbf{x}_{(12)}$ and $\mathbf{x}_{(13)}$):

$$\bar{Y}_U \sim \mathbf{x}_{(2)} + \mathbf{x}_{(3)} + \mathbf{x}_{(5)} + \mathbf{x}_{(7)} + \mathbf{x}_{(9)} + \mathbf{x}_{(15)}.$$

The alternative hypothesis is Model 2 (with $\mathbf{x}_{(4)}$, $\mathbf{x}_{(12)}$ and $\mathbf{x}_{(13)}$):

$$\bar{Y}_U \sim \mathbf{x}_{(2)} + \mathbf{x}_{(3)} + \mathbf{x}_{(4)} + \mathbf{x}_{(5)} + \mathbf{x}_{(7)} + \mathbf{x}_{(9)} + \mathbf{x}_{(12)} + \mathbf{x}_{(13)} + \mathbf{x}_{(15)}.$$

Model	Res. Df	RSS	Df	Sum of Square	F	P-value
1	70	0.46892				
2	67	0.42737	3	0.041545	2.171	0.09957

Table 4.5: ANOVA table for the variables $\mathbf{x}_{(4)}$, $\mathbf{x}_{(12)}$ and $\mathbf{x}_{(13)}$.

Since the P-value is 0.09957, which is greater than the significance level of 0.05, we do not reject the model under null hypothesis from forward selection. Hence, the final model is

$$\bar{Y}_U \sim \mathbf{x}_{(2)} + \mathbf{x}_{(3)} + \mathbf{x}_{(5)} + \mathbf{x}_{(7)} + \mathbf{x}_{(9)} + \mathbf{x}_{(15)}.$$

Let $\mathbf{x}_d = (1, x_{d,(2)}, x_{d,(3)}, x_{d,(5)}, x_{d,(7)}, x_{d,(9)}, x_{d,(15)})'$ be the vector of included auxiliary variables with the intercept term for province d ($d = 1, \dots, 77$). For illustration, the vector of included auxiliary variables of Bangkok ($d = 1$) is

$$\mathbf{x}_1 = (1, x_{1,(2)}, x_{1,(3)}, x_{1,(5)}, x_{1,(7)}, x_{1,(9)}, x_{1,(15)})' = \begin{pmatrix} 1 \\ 0.8678 \\ 0.5156 \\ \vdots \\ 0.3509 \end{pmatrix}_{7 \times 1} .$$

Doing the same processes for all provinces, the included auxiliary variables \mathbf{x} with the intercept term for Thailand are shown as follows.

$$\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{77}) = \begin{pmatrix} 1 & 1 & \dots & 1 \\ 0.8678 & 0.8925 & \dots & 0.8602 \\ 0.5156 & 0.4247 & \dots & 0.7933 \\ \vdots & \vdots & \ddots & \vdots \\ 0.3509 & 0.1989 & \dots & 0.2992 \end{pmatrix}_{7 \times 77} . \quad (4.1)$$

We now obtain the matrix of auxiliary variables \mathbf{x} that is available to be used in the model.

4.2 Methodology

This section starts with a discussion of the constrained Bayes method and then the constrained Empirical Bayes and constrained Hierarchical Bayes methods.

4.2.1 Constrained Bayes (CB) method

Although the Bayesian model is widely used in many applications such as income, expenditure, poverty, risk or even in game theory. Louis (1984) shows that the usual Bayes has a limitation that the sampling variability of Bayes estimates is always smaller than the posterior expectation sample variance. For this reason, he proposed the new method known as the constrained Bayes (CB) method. Louis shows the limitation of the usual Bayes in the following Proposition 4.2.1.

Proposition 4.2.1 (Louis (1984)). Let $\boldsymbol{\theta} = (\theta_1, \dots, \theta_D)'$ be the vector of population means for expenditure variable of D provinces and $\boldsymbol{\theta}^B(\mathbf{Y}) = (\theta_1^B(\mathbf{Y}), \dots, \theta_D^B(\mathbf{Y}))'$ be the vector of Bayes estimates of the parameter $\boldsymbol{\theta}$ on the sample means \mathbf{Y} . The limitation of the usual Bayes is shown as follows:

$$E \left[\sum_{d=1}^D (\theta_d - \bar{\theta})^2 | \mathbf{Y} \right] > \sum_{d=1}^D \{ \theta_d^B(\mathbf{Y}) - \bar{\theta}^B(\mathbf{Y}) \}^2. \quad (4.2)$$

In practice, this sampling variability of Bayes estimates and this posterior expected sample variance should be equal. This clearly shows the limitation of usual Bayes estimates to estimate the true variation of parameter $\boldsymbol{\theta}$. The constrained Bayes (CB) $\boldsymbol{\theta}^{CB} = (\theta_1^{CB}, \dots, \theta_D^{CB})^T$ estimate of $\boldsymbol{\theta}$, where $\boldsymbol{\theta}^{CB}$ minimizes

$$E \left[\sum_{d=1}^D (\theta_d - t_d)^2 | \mathbf{Y} \right], \quad (4.3)$$

where $\mathbf{t}(\mathbf{Y}) = (t_1, \dots, t_D)^T$ is the estimate of $\boldsymbol{\theta}$, subject to the constraints

$$(a) \quad E(\bar{\theta}|\mathbf{Y}) = D^{-1} \sum_{d=1}^D t_d(\mathbf{Y}) = \bar{t}(\mathbf{Y}),$$

$$(b) \quad E \left[\sum_{d=1}^D (\theta_d - \bar{\theta})^2 | \mathbf{Y} \right] = \sum_{d=1}^D (t_d(\mathbf{Y}) - \bar{t}(\mathbf{Y}))^2.$$

From the limitation of the usual Bayes presented in Proposition 4.2.1, it illustrates that the Bayes estimate satisfies only the constrained (a) but not for (b). For this reason, the constrained Bayes was proposed to solve the limitation of the usual Bayes.

Theorem 4.2.1 (Ghosh (1992)). The constrained Bayes (CB) estimate, $\boldsymbol{\theta}^{CB}(\mathbf{Y}) = (\theta_1^{CB}(\mathbf{Y}), \dots, \theta_D^{CB}(\mathbf{Y}))^T$, can be stated as:

$$\theta_d^{CB}(\mathbf{Y}) = a_B \theta_d^B(\mathbf{Y}) + (1 - a_B) \bar{\theta}^B(\mathbf{Y}), \quad d = 1, \dots, D,$$

where $a_B \equiv a_B(\mathbf{Y}) = \left[1 + \frac{H_1(\mathbf{Y})}{H_2(\mathbf{Y})} \right]^{1/2},$ (4.4)

such that $H_1(\mathbf{Y}) = \text{tr}[V(\boldsymbol{\theta} - \bar{\boldsymbol{\theta}}\mathbf{1}_D | \mathbf{Y})] = \text{tr}[(\mathbf{I}_D - D^{-1}\mathbf{J}_D)V(\boldsymbol{\theta} | \mathbf{Y})],$ (4.5)

and $H_2(\mathbf{Y}) = \sum_{d=1}^D (\theta_d^B(\mathbf{Y}) - \bar{\theta}^B(\mathbf{Y}))^2.$ (4.6)

4.2.2 Constrained Empirical Bayes (CEB) method

This section applies the constrained Bayes concept to the Empirical Bayes method, called the constrained Empirical Bayes (CEB) method. Next, we consider the normal area-level model as follows:

$$\begin{aligned} (i) \quad Y_d | \theta_d, \sigma_d^2, \boldsymbol{\beta} &\sim N(\theta_d, \sigma_d^2), \quad d = 1, \dots, D, \\ (ii) \quad \theta_d | \boldsymbol{\beta} &\sim N(\mathbf{x}'_d \boldsymbol{\beta}, A), \end{aligned} \tag{4.7}$$

where \mathbf{x}_d is a $p \times 1$ vector of area level auxiliary variables for province d shown in (4.1), $\boldsymbol{\beta}$ is the $p \times 1$ vector of regression coefficients, σ_d^2 is the known sampling variance and A is the unknown variance of area random effect needs to be estimated. Even more, the distribution of the likelihood $(\theta_d | Y_d)$ is shown as in Lemma 4.2.2.

Lemma 4.2.2. If $Y_d | \theta_d, \sigma_d^2, \boldsymbol{\beta} \sim N(\theta_d, \sigma_d^2)$ and $\theta_d | \boldsymbol{\beta} \sim N(\mathbf{x}'_d \boldsymbol{\beta}, A)$, where $d = 1, \dots, D$, then $\theta_d | Y_d \sim N(\mu_d, AB_d)$ where $\mu_d = (1 - B_d)Y_d + B_d \mathbf{x}'_d \boldsymbol{\beta}$ and $B_d = \frac{\sigma_d^2}{A + \sigma_d^2}$.

From Lemma (4.2.2), we obtain the Bayes's estimate $\theta_d^B(\mathbf{Y})$ of the population mean θ_d as

$$\theta_d^B(\mathbf{Y}) = E(\theta_d | Y_d) = (1 - B_d)Y_d + B_d \mathbf{x}'_d \boldsymbol{\beta} \quad \text{where} \quad B_d = \frac{\sigma_d^2}{A + \sigma_d^2}.$$

The idea of the Empirical Bayes (EB) is replacing the unknown parameter by an estimate. Hence, replacing $\boldsymbol{\beta}$ by $\hat{\boldsymbol{\beta}}$ and A by \hat{A} we obtain $\hat{B}_d = \frac{\sigma_d^2}{\hat{A} + \sigma_d^2}$.

This leads to the Empirical Bayes (EB) estimate of the population mean θ_d as

$$\theta_d^{EB}(\mathbf{Y}) = (1 - \hat{B}_d)Y_d + \hat{B}_d \mathbf{x}'_d \hat{\boldsymbol{\beta}}. \tag{4.8}$$

Substitute $\theta_d^{EB}(\mathbf{Y})$ into constrained Bayes model stated in Theorem 4.2.1, therefore the constrained Empirical Bayes estimate of θ_d is

$$\theta_d^{CEB}(\mathbf{Y}) = a_{EB}\theta_d^{EB}(\mathbf{Y}) + (1 - a_{EB})\bar{\theta}^{EB}(\mathbf{Y}), \quad d = 1, \dots, D, \quad (4.9)$$

where a_{EB} is the a_B where B is replaced by \hat{B} .

4.2.2.1 Algorithm and Process for CEB method

The algorithm of the CEB method is explained in Algorithm 8 as follows.

Algorithm 8 Algorithm for the constrained Empirical Bayes (CEB) method.

1: Obtain the EB estimate of θ_d from the model (4.7):

$$\begin{aligned} (i) & Y_d | \theta_d, \sigma_d^2, \boldsymbol{\beta} \sim N(\theta_d, \sigma_d^2), \quad d = 1, \dots, D, \\ (ii) & \theta_d | \boldsymbol{\beta} \sim N(\mathbf{x}'_d \boldsymbol{\beta}, A). \end{aligned}$$

Output: The estimate $(\boldsymbol{\theta}^{EB})$ of $\boldsymbol{\theta}$ and the estimate (\hat{A}) of A .

2: Calculate $H_1(\mathbf{Y})$ in (4.5),

$$H_1(\mathbf{Y}) = \text{tr}[(\mathbf{I}_D - D^{-1}\mathbf{J}_D)V(\boldsymbol{\theta}|\mathbf{Y})].$$

3: Replace $\boldsymbol{\theta}^B$ by $\boldsymbol{\theta}^{EB}$ in (4.6), then calculate $H_2(\mathbf{Y})$ from

$$H_2(\mathbf{Y}) = \sum_{d=1}^D (\theta_d^{EB}(\mathbf{Y}) - \bar{\theta}^{EB}(\mathbf{Y}))^2.$$

4: Calculate a_{EB} by substituting $H_1(\mathbf{Y})$ from Step 2 and $H_2(\mathbf{Y})$ from Step 3 into (4.4).

5: Calculate θ_d^{CEB} in (4.9), therefore we obtain the CEB estimate as

$$\theta_d^{CEB}(\mathbf{Y}) = a_{EB}\theta_d^{EB}(\mathbf{Y}) + (1 - a_{EB})\bar{\theta}^{EB}(\mathbf{Y}) \quad \text{for } d = 1, \dots, D.$$

Following Algorithm 8, we obtain the results described as follows. Let $\mathbf{Y} = (Y_1, \dots, Y_{77})'$ be the 77-vector of sample means of log transformation of the average monthly expenditures per capita for all 77 provinces in Thailand. For the auxiliary variables \mathbf{x} .

Under the SRS sampling, we can calculate the sampling variance σ_d^2 as

$$\sigma_d^2 = \frac{S_d^2}{n_d} \left(1 - \frac{n_d}{N_d}\right), \quad (4.10)$$

where S_d^2 is the variance of the area population with population size N_d and sample size n_d .

Follow the Algorithm 8, fit the area-level model (4.7) as

$$\begin{aligned} (i) \quad & Y_d | \theta_d, \sigma_d^2, \boldsymbol{\beta} \sim N(\theta_d, \sigma_d^2), \quad d = 1, \dots, D, \\ (ii) \quad & \theta_d | \boldsymbol{\beta} \sim N(\mathbf{x}'_d \boldsymbol{\beta}, A), \end{aligned}$$

where \mathbf{x}_d is a 7×1 vector of area level auxiliary variables, $\boldsymbol{\beta}$ is the 7×1 vector of coefficient regression parameters, σ_d^2 is the known sampling variance shown in (4.10) and A is the variance of area random effect.

Having fitted the model, we obtain Empirical Bayes estimates of parameters as follows.

$$\boldsymbol{\theta}^{EB} = \begin{pmatrix} 9.380 \\ 9.264 \\ 9.328 \\ \vdots \\ 8.501 \end{pmatrix}, \quad \text{and} \quad \hat{A} = 0.0072.$$

From Lemma 4.2.2, substituting the parameter estimate A by \hat{A} to $V(\theta_d|Y_d)$, we obtain $V(\theta_d|Y_d) = 0.0072\hat{B}_d$ where $\hat{B}_d = \left(\frac{\sigma_d^2}{0.0072+\sigma_d^2}\right)$. For all provinces, $H_1(\mathbf{Y})$ is obtained by substituting this $V(\boldsymbol{\theta}|\mathbf{Y})$ into (4.5) as follows.

$$H_1(\mathbf{Y}) = tr [(\mathbf{I}_{77} - (77)^{-1}\mathbf{J}_{77})V(\boldsymbol{\theta}|\mathbf{Y})] = 0.1005.$$

For $H_2(\mathbf{Y})$, substituting $\theta_d^B(\mathbf{Y})$ by $\theta_d^{EB}(\mathbf{Y})$ into (4.6), obtain $H_2(\mathbf{Y})$ as follows

$$H_2(\mathbf{Y}) = \sum_{d=1}^D (\theta_d^{EB}(\mathbf{Y}) - \bar{\theta}^{EB}(\mathbf{Y}))^2 = 4.5517.$$

Consequently, the estimate $a_{EB}(\mathbf{Y})$ is

$$a_{EB}(\mathbf{Y}) = \left[1 + \frac{H_1(\mathbf{Y})}{H_2(\mathbf{Y})}\right]^{1/2} = 1.0110.$$

Finally, substituting a_{EB} , θ_d^{EB} , and $\bar{\theta}^{EB}$ into the constrained Empirical Bayes model (4.9) we obtain θ_d^{CEB} . Apply the same process for all provinces, and construct the CEB estimate by $\boldsymbol{\theta}^{CEB} = (\theta_1^{CEB}, \dots, \theta_{77}^{CEB})$. For model evaluation, the same process is applied for $L = 100$ times. Therefore, the Empirical Bayes and the constrained Empirical Bayes were denoted by $\boldsymbol{\theta}^{EB(l)}$ and $\boldsymbol{\theta}^{CEB(l)}$ for $l = 1, \dots, 100$.

4.2.2.2 Result for CEB method

To diagnose the performance of the concept of the constrained Bayes that add the condition (b) saying that posterior expected sample variance and sample variance should be equal. That is,

$$E \left[\sum_{d=1}^D (\theta_d - \bar{\theta})^2 | \mathbf{Y} \right] = \sum_{d=1}^D (t_d(\mathbf{Y}) - \bar{t}(\mathbf{Y}))^2.$$

Adding constraint would reduce the difference between the left-hand side and the right-hand side of (b). That is, we consider

$$E \left[\sum_{d=1}^D (\theta_d - \bar{\theta})^2 | \mathbf{Y} \right] - \sum_{d=1}^D (\theta_d^{EB}(\mathbf{Y}) - \bar{\theta}^{EB}(\mathbf{Y}))^2$$

comparing with

$$E \left[\sum_{d=1}^D (\theta_d - \bar{\theta})^2 | \mathbf{Y} \right] - \sum_{d=1}^D (\theta_d^{CEB}(\mathbf{Y}) - \bar{\theta}^{CEB}(\mathbf{Y}))^2.$$

Figure 4.1 shows comparisons of the differences between sample variance and posterior expected sample variance for the EB and CEB methods.

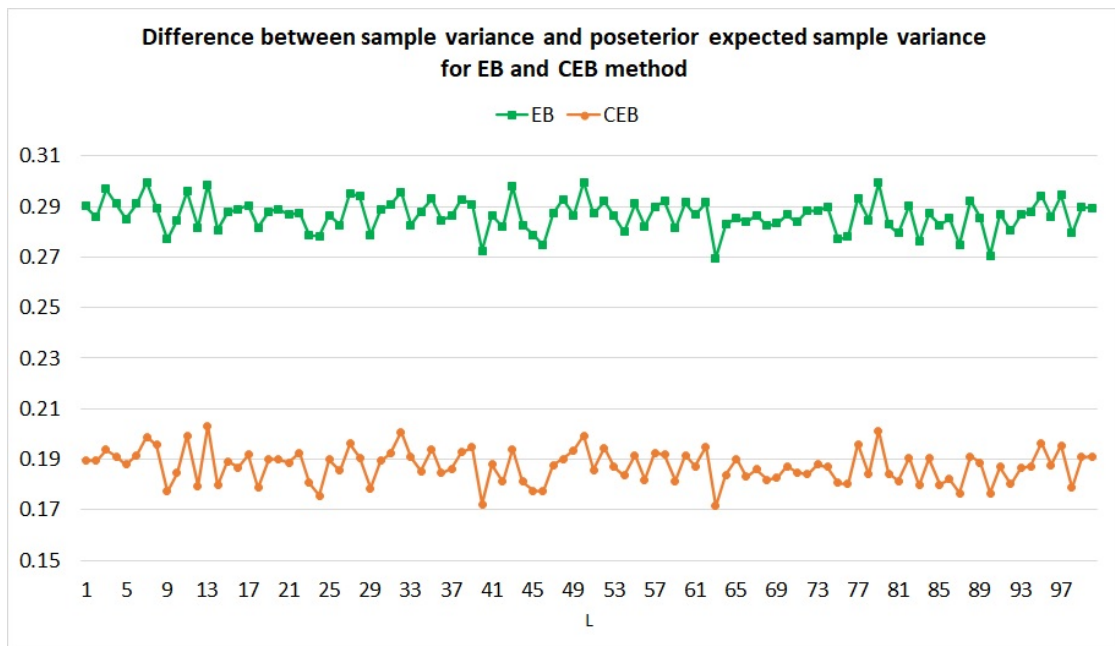


Figure 4.1: The comparisons of the differences between sample variance and poseterior expected sample variance for the EB and CEB methods.

From Figure 4.1, we can see that the differences between sample variance and posterior expected sample variance of the EB are higher than the CEB for all l ($l = 1, \dots, 100$). That is, the sample variance of the CEB close to the posterior expected sample variance than the EB method. That is the constrained Bayes can improve the limitation of the usual Bayes.

4.2.3 Constrained Hierarchical Bayes (CHB) method

In this section, we apply the constrained Bayes concept to the Hierarchical Bayes, called constrained Hierarchical Bayes (CHB) method. We consider the Hierarchical area-level model as follow:

$$\begin{aligned}
 (i) \quad & Y_d | \theta_d, \boldsymbol{\beta} \sim N(\theta_d, \sigma_d^2), \quad d = 1, \dots, D, \\
 (ii) \quad & \theta_d | \boldsymbol{\beta} \sim N(\mathbf{x}'_d \boldsymbol{\beta}, A), \\
 (iii) \quad & f(A) \propto \frac{1}{A}.
 \end{aligned}
 \tag{4.11}$$

where \mathbf{x}_d is a $p \times 1$ vector of area level auxiliary variables for province d shown in (4.1), $\boldsymbol{\beta}$ is the $p \times 1$ vector of regression coefficients, σ_d^2 is the known sampling variance and A is the unknown variance of area random effect needs to be estimated.

The CHB estimator (θ_d^{CHB}) is obtained by replacing θ_d^B by θ_d^{HB} into the constrained Bayes model in Theorem (4.2.1) for $d = 1, \dots, D$. Hence, the CHB estimator is obtained by minimizing the posterior squared error $E \left[\sum_{d=1}^D (\theta_d - t_d)^2 | \mathbf{Y} \right]$ subject to

$$\begin{aligned}
 (c1) \quad & t_d = \theta_d^{HB} \\
 (c2) \quad & \sum_{d=1}^D (t_d(\mathbf{Y}) - \bar{t}(\mathbf{Y}))^2 = E \left[\sum_{d=1}^D (\theta_d - \bar{\theta})^2 | \mathbf{Y} \right].
 \end{aligned}$$

Therefore, the constrained Hierarchical Bayes estimate of θ_d is

$$\theta_d^{CHB} = a_{HB}\theta_d^{HB} + (1 - a_{HB})\bar{\theta}^{HB},$$

$$\text{where } a_{HB} \equiv a_{HB}(\mathbf{Y}) = \left[1 + \frac{H_1(\mathbf{Y})}{H_2(\mathbf{Y})}\right]^{1/2}, \quad (4.12)$$

$$\text{such that } H_1(\mathbf{Y}) = \text{tr}[(\mathbf{I}_D - D^{-1}\mathbf{J}_D)V(\boldsymbol{\theta}|\mathbf{Y})], \quad (4.13)$$

$$\text{and } H_2(\mathbf{Y}) = \sum_{d=1}^D (\theta_d^{HB}(\mathbf{Y}) - \bar{\theta}^{HB}(\mathbf{Y}))^2. \quad (4.14)$$

4.2.3.1 Algorithm and Process for CHB method

The algorithm of the CHB method is explained as follows.

Algorithm 9 Algorithm for the constrained Hierarchical Bayes (CHB) method.

1: Obtain the HB estimate of θ_d from the model as follows.

$$\begin{aligned} (i) & Y_d|\theta_d, \boldsymbol{\beta} \sim N(\theta_d, \sigma_d^2), \quad d = 1, \dots, D, \\ (ii) & \theta_d|\boldsymbol{\beta} \sim N(\mathbf{x}'_d\boldsymbol{\beta}, A), \quad d = 1, \dots, D, \\ (iii) & f(A) \propto \frac{1}{A}. \end{aligned}$$

Remark: The uniform informative prior on (0,1) was used.

Output: The estimate $\boldsymbol{\theta}^{HB}$ of $\boldsymbol{\theta}$ and the posterior variance \hat{A} .

2: Calculate $H_1(\mathbf{Y})$ in (4.13),

$$H_1(\mathbf{Y}) = \text{tr}[(\mathbf{I}_D - D^{-1}\mathbf{J}_D)V(\boldsymbol{\theta}|\mathbf{Y})].$$

3: Replace $\boldsymbol{\theta}^B$ by $\boldsymbol{\theta}^{HB}$ in (4.14), then calculate $H_2(\mathbf{Y})$ from

$$H_2(\mathbf{Y}) = \sum_{d=1}^D (\theta_d^{HB}(\mathbf{Y}) - \bar{\theta}^{HB}(\mathbf{Y}))^2.$$

4: Calculate a_{HB} by substituting $H_1(\mathbf{Y})$ from Step 2 and $H_2(\mathbf{Y})$ from Step 3 into (4.12).

5: Calculate θ_d^{CHB} in (4.2.3), therefore we obtain the CHB estimate as

$$\theta_d^{CHB}(\mathbf{Y}) = a_{HB}\theta_d^{HB}(\mathbf{Y}) + (1 - a_{HB})\bar{\theta}^{HB}(\mathbf{Y}) \quad \text{for } d = 1, \dots, D.$$

Follow Algorithm 9, the set of sample means (\mathbf{Y}) of log-transformation of the average monthly expenditures per capita and the auxiliary variables (\mathbf{x}) for all 77 provinces in Thailand is described in the CEB method.

Having fitted the Hierarchical area-level model (4.11) with a uniform prior on A , generated chains of length 10000, discarding the first 5000 as a burn-in with 2 Markov chains and 2 thinning rate. That is, $M = 5000$ is the number of bootstraps. We obtain Hierarchical Bayes estimates for $M = 5000$. For illustration, the HB estimates ($\boldsymbol{\theta}^{HB}$) and posterior variance (\hat{A}) for the first bootstrap are

$$\boldsymbol{\theta}^{HB} = \begin{pmatrix} 9.236 \\ 9.215 \\ 9.315 \\ \vdots \\ 8.636 \end{pmatrix}, \quad \text{and} \quad \hat{A} = 0.00894.$$

From Lemma 4.2.2, substituting the parameter estimate A by \hat{A} to $V(\theta_d|Y_d)$, obtain $V(\theta_d|Y_d) = 0.00894\hat{B}_d$ where $\hat{B}_d = \left(\frac{\sigma_d^2}{0.00894 + \sigma_d^2}\right)$. For all provinces, $H_1(\mathbf{Y})$ obtain by substituting this $V(\boldsymbol{\theta}|\mathbf{Y})$ into (4.5) as follows.

$$H_1(\mathbf{Y}) = \text{tr} [(\mathbf{I}_{77} - (77)^{-1}\mathbf{J}_{77})V(\boldsymbol{\theta}|\mathbf{Y})] = 0.1044.$$

For $H_2(\mathbf{Y})$, substituting $\theta_d^B(\mathbf{Y})$ by $\theta_d^{EB}(\mathbf{Y})$ into (4.6), obtain $H_2(\mathbf{Y})$ as follows

$$H_2(\mathbf{Y}) = \sum_{d=1}^D (\theta_d^{HB}(\mathbf{Y}) - \bar{\theta}^{HB}(\mathbf{Y}))^2 = 3.8781.$$

Consequently, the estimate $a_{HB}(\mathbf{Y})$ is

$$a_{HB}(\mathbf{Y}) = \left[1 + \frac{H_1(\mathbf{Y})}{H_2(\mathbf{Y})}\right]^{1/2} = 1.0134.$$

Finally, substituting a_{HB} , θ_d^{HB} and $\bar{\theta}^{HB}$ into the constrained Hierarchical Bayes model (4.2.3), we obtain θ_d^{CHB} . Apply the same process for all provinces, and construct the CHB estimate by $\boldsymbol{\theta}^{CHB} = (\theta_1^{CHB}, \dots, \theta_{77}^{CHB})$. For the remaining bootstrap ($m = 2, \dots, 5000$), doing these processes and then take the average will obtain the new $\boldsymbol{\theta}^{CB} = (\theta_1^{CB}, \dots, \theta_{77}^{CB})$ and $\boldsymbol{\theta}^{CHB} = (\theta_1^{CHB}, \dots, \theta_{77}^{CHB})$. For model evaluation, the same process is applied for $L = 100$ times. Therefore, the Hierarchical Bayes and the constrained Hierarchical Bayes were denoted by $\boldsymbol{\theta}^{HB(l)}$ and $\boldsymbol{\theta}^{CHB(l)}$ for $l = 1, \dots, 100$.

4.2.3.2 Result for CHB method

To diagnose the performance of the concept of constrained Bayes, now we consider

$$E \left[\sum_{d=1}^D (\theta_d - \bar{\theta})^2 | \mathbf{Y} \right] - \sum_{d=1}^D (\theta_d^{HB}(\mathbf{Y}) - \bar{\theta}^{HB}(\mathbf{Y}))^2$$

comparing with

$$E \left[\sum_{d=1}^D (\theta_d - \bar{\theta})^2 | \mathbf{Y} \right] - \sum_{d=1}^D (\theta_d^{CHB}(\mathbf{Y}) - \bar{\theta}^{CHB}(\mathbf{Y}))^2.$$

Figure 4.2 shows comparisons of the differences between sample variance and posterior expected sample variance for the HB and CHB methods.

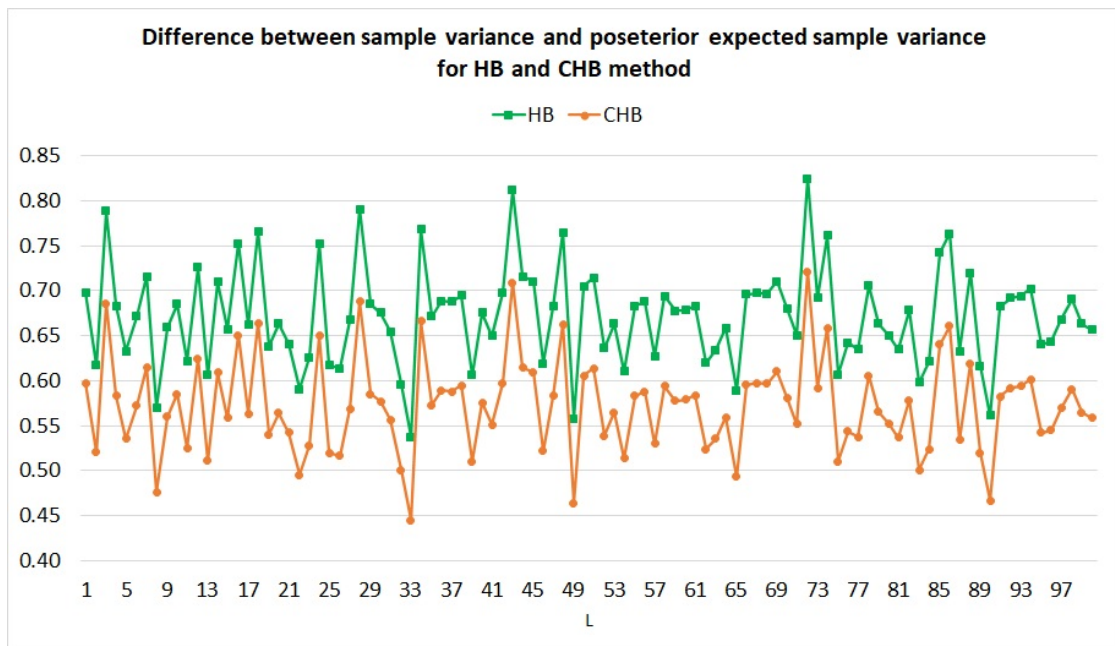


Figure 4.2: The comparisons of the differences between sample variance and posterior expected sample variance for the HB and CHB methods.

From Figure 4.2, we can see that the differences between sample variance and posterior expected sample variance of the HB is higher than the CHB for all l ($l = 1, \dots, 100$). That is the sample variance of the CHB close to the posterior expected sample variance than the HB method. This proves the advantages of constrained Bayes estimation.

4.3 Result of Bayesian models for expenditure variable

Since the concept of constrained Bayes was not proposed to improve, it was present to deal with the limitation of the usual Bayes. Therefore, Kim and Kim (2013) presented the Index criteria for matching the first and second moments as follows:

$$\text{Index} = w \left[E(\bar{\theta}|\mathbf{Y}) - D^{-1} \sum_{d=1}^D t_d(\mathbf{Y}) \right]^2 + (1-w) \left| E \left[\sum_{d=1}^D (\theta_d - \bar{\theta})^2 | \mathbf{Y} \right] - \sum_{d=1}^D [t_d(\mathbf{Y}) - \bar{t}(\mathbf{Y})]^2 \right|$$

where w is the weight for each moments, D is the number of areas, $\boldsymbol{\theta}$ is the vector of area means of welfare and $\mathbf{t}(\mathbf{Y})$ is all estimates of $\boldsymbol{\theta}$. The Index criteria of Kim and Kim (2013) is used to verify that the constrained Bayes can deal with conditions (a) and (b) better than the usual Bayes. They considered the difference of the condition (a) and the difference of condition (b) and then created as an Index by adding weight to the criteria, which is w . If the constrained Bayes is actually better than usual Bayes, the Index should close to zero because the conditions (a) and (b) should be zero.

Figure 4.3 shows the comparison plot of the Index for the EB and CEB estimates for 100 times and Figure 4.4 shows the comparison of for the Index for the HB and CHB estimates.

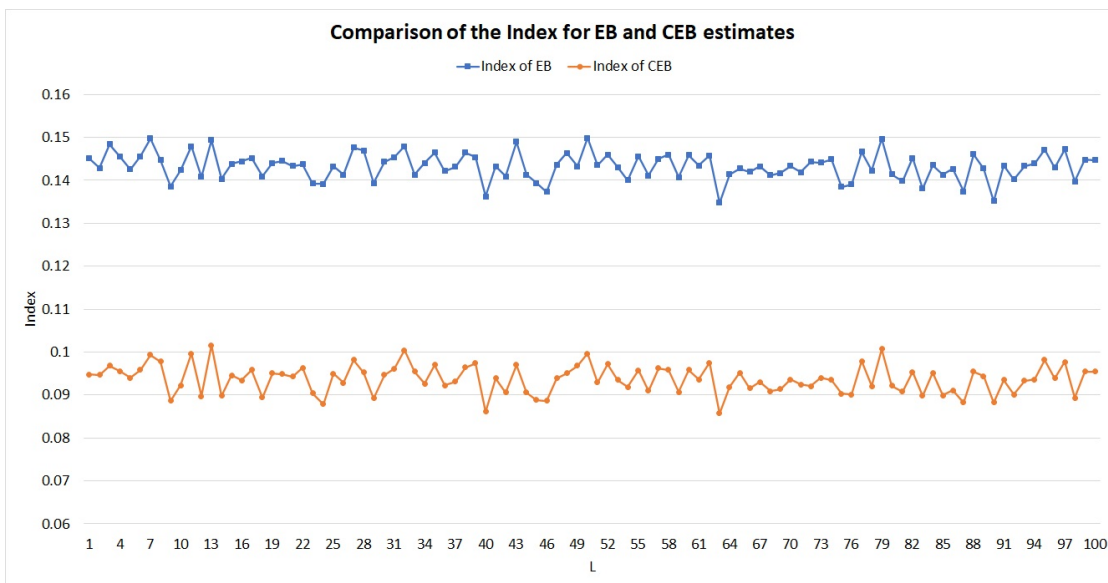


Figure 4.3: The comparison plot of the Index for the EB and CEB estimator for 100 times.

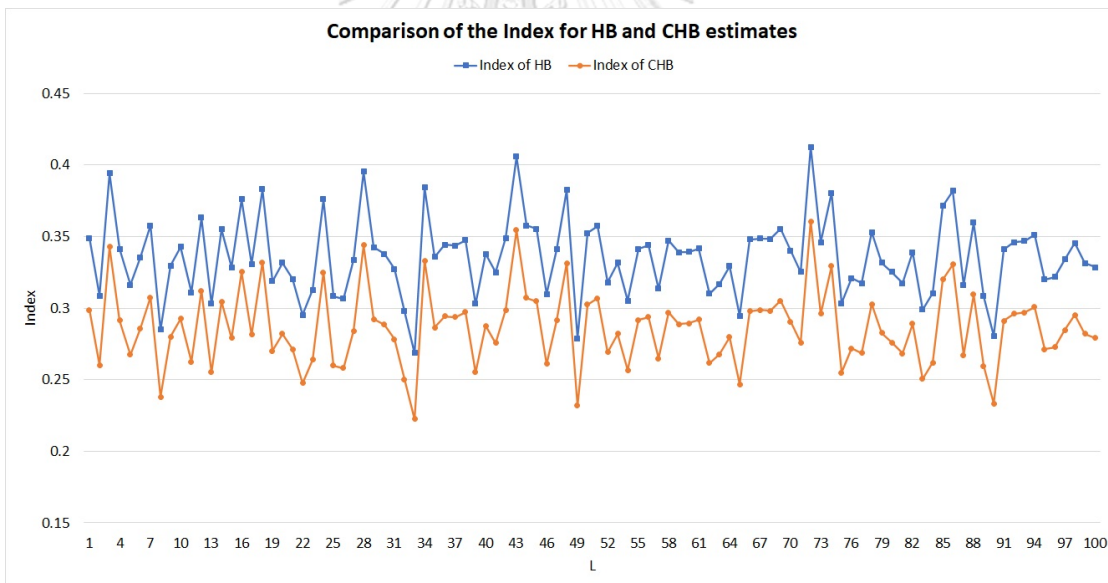


Figure 4.4: The comparison plot of the Index for the HB and CHB estimator for 100 times.

From Figure 4.3, the Index of EB estimator is approximately 0.09 while the CEB is approximately 0.04, which is closer to zero than the EB estimator. That is the constrained Bayes can deal with conditions (a) and (b) better than the usual Bayes. The same notice can be found for the CHB estimator, that is the Index of CHB is smaller than the HB, as shown in Figure 4.4. In summary, the constrained Bayes can deal with the limitation of the usual Bayes.

According to Index criteria, the best method should give the Index close to zero. From Figure 4.3 and Figure 4.4, we can see that the Index of the EB and CEB methods are smaller than the HB and CHB methods for all $L = 100$. Hence, the CEB method is better than CHB method for area-level model of the Socio-Economic Survey in 2017.



CHAPTER V

DISCUSSIONS

This chapter consists of two sections, a discussion of the Bayesian models for poverty indicators at unit level and a discussion of the Bayesian models for expenditure variable at area-level.

5.1 Discussion of the Bayesian Models for Poverty Indicators at Unit Level

Chapter 3 presents unit level models, which are the original ELL, Empirical Bayes (EB) and Hierarchical Bayes (HB) methods, which are applied to the three types of FGT poverty indicators. We focus on the FGT poverty indicators on the logarithm of the average monthly total expenditure per capita and the auxiliary variables in Table 3.8. For the best method, the estimate of the FGT poverty indicator should be as close as possible to the true FGT. Therefore, absolute biases and mean square errors were used for study the performance of the three methods.

As shown in Chapter 3, we can conclude as follows:

- Figure 3.12 - Figure 3.14 show the percentages of absolute biases for the poverty incidence, poverty gap and poverty severity for all 77 provinces. In summary, the absolute biases for the original ELL method in most areas are higher than HB and EB methods. The same notice can be seen in Figure 3.15 - Figure 3.17 for the mean square errors.
- For the average across areas of absolute biases over 77 provinces in Table 3.16, we can conclude that the HB method performs better than the original

ELL and EB methods for the poverty gap and poverty severity while the poverty incidence of the EB method is better than other models.

- For the average across areas of mean square errors over 77 provinces in Table 3.17, the HB method performs better than the original ELL and EB methods for all of the FGT poverty indicators.

Conclusively, the Hierarchical Bayes (HB) method is the most appropriate method while the original ELL method performs poorly for the FGT poverty indicators for the Household Socio-Economic Survey (SES) in 2017.

5.2 Discussion of the Bayesian Models for Expenditure Variable at Area-Level

Chapter 4 presents the constrained Bayes method for area-level model. The constrained Bayes has drawn interests because the usual Bayes has a limitations that the sampling variability is always smaller than the posterior variance. However, this should be equal in practice. This leads to the concept of constrained Bayes by adding the constraint that sampling variability should be equal to the posterior expected sample variance. Furthermore, the constrained Bayes were applied to the Empirical Bayes and Hierarchical Bayes, also known as the constrained Empirical Bayes (CEB) and constrained Hierarchical Bayes (CHB). The Household Socio-Economics Survey (SES) in 2017 was applied to this study in terms of the area-level model. The criteria used to compare the performance of constrained Bayes is Index criteria, which proposed to considering the added constraint.

As shown in Chapter 4, we can conclude as follows:

- Figure 4.1 shows that the difference between sample variance and posterior expected sample variance of the CEB method is smaller than of the EB method. That is the sample variance of the CEB close to the posterior expected sample variance than the sample variance of the EB method.

- Figure 4.2 shows that the difference between sample variance and posterior expected sample variance of the CHB method is smaller than of the HB method. That is the sample variance of the CHB close to the posterior expected sample variance than the sample variance of the HB method.
- Figure 4.3 shows the Index of the CEB estimator is closer to zero than the EB estimator. That is the constrained Bayes can deal with the limitation of the usual Bayes. The same notice can be found for the CHB method, that is the Index of the CHB is smaller than the Index of the HB method, as shown in Figure 4.4.
- Figure 4.3 and Figure 4.4 show that the Index of the EB and CEB methods are smaller than the HB and CHB methods. Hence, the CEB method is better than the CHB method for area-level model of the Socio-Economic Survey in 2017.

Overall, it shows the advantage of the constrained Bayes that can solve the limitation of the usual Bayes for the Household Socio-Economic Survey (SES) in 2017.

REFERENCES

- [1] H. Akaike. Information theory and an extension of the maximum likelihood principle. In *Proceedings of the 2nd international symposium on information*, pages 267–281, 1973.
- [2] F. Ballini, G. Betti, S. Carrette, and L. Neri. Poverty and inequality mapping in the commonwealth of dominica. *Estudios Económicos*, pages 123–162, 2009.
- [3] G. E. Battese, R. M. Harter, and W. A. Fuller. An error-components model for prediction of county crop areas using survey and satellite data. *Journal of the American Statistical Association*, 83(401):28–36, 1988.
- [4] C. Elbers, J. O. Lanjouw, and P. Lanjouw. Micro level estimation of poverty and inequality. *Econometrica*, 71(1):355–364, 2013.
- [5] J. Foster, J. Greer, and E. Thorbecke. A class of decomposable poverty measures. *Econometrica*, 52(3):761–766, 1984.
- [6] A. Gelman. Objections to bayesian statistics. *Bayesian Analysis*, 3(3):445–449, 2008.
- [7] M. Ghosh. Constrained bayes estimation with applications. *Journal of the American Statistical Association*, 87(418):533–540, 1992.
- [8] A. J. Healy and S. Jitsuchon. Finding the poor in thailand. *Econometrica*, 18(5):739–759, 2007.
- [9] A. J. Healy, S. Hitsuchon, and Y. Vajaragupta. Spatially disaggregated estimates of poverty and inequality in thailand. *Massachusetts Institute of Technology and Thailand Development Research Institute*, 2003.

- [10] M. J. Kim and Y. H. Kim. Constrained bayes and empirical bayes estimator applications in insurance pricing. *Communications for Statistical Applications and Methods*, 20(4):321–327, 2013.
- [11] T. A. Louis. Estimating a population of parameter values using bayes and empirical bayes methods. *Journal of the American Statistical Association*, 79(386):393–398, 1984.
- [12] I. Molina and J. Rao. Small area estimation of poverty indicators. *Canadian Journal of Statistics*, 38(3):369–385, 2010.
- [13] I. Molina, B. Nandram, and J. N. K. Rao. Small area estimation of general parameters with application to poverty indicators: a hierarchical bayes approach. *The Annals of Applied Statistics*, 8(2):852–885, 2014.
- [14] L. Neri, F. Ballini, and G. Betti. Poverty and inequality mapping in transition countries. *Statistics in Transition*, pages 135–157, 2005.



APPENDIX

จุฬาลงกรณ์มหาวิทยาลัย
CHULALONGKORN UNIVERSITY

APPENDIX A : Poverty line 2017 for 77 provinces in Thailand

Province Index	Province Name	Poverty Line (Baht/capita/month)
1	Bangkok Metropolis	3165
2	Samut Prakan	3000
3	Nonthaburi	2872
4	Pathum thani	2865
5	Phra nakhon si ayutthaya	2774
6	Ang thong	2705
7	Lop buri	2778
8	Sing buri	2660
9	Chai nat	2857
10	Saraburi	2665
11	Chon buri	3148
12	Rayong	2980
13	Chanthaburi	2980
14	Trat	2822
15	Chachoengsao	2829
16	Prachin buri	2723
17	Nakhon nayok	2654
18	Sakaeo	2696
19	Nakhon ratchasima	2364
20	Buri ram	2421
21	Surin	2303
22	Si sa ket	2306
23	Ubon ratchathani	2361
24	Yasothon	2455
25	Chaiyaphum	2331
26	Amnat charoen	2410
27	Buengkal	2341
28	Nong bua lam phu	2395
29	Khon kaen	2594
30	Udon thani	2364

Continued on next page...

Table 1 – continued from previous page.

Province Index	Province Name	Poverty Line (Baht/capita/month)
31	Loei	2351
32	Nong khai	2350
33	Maha sarakham	2399
34	Roi et	2492
35	Kalasin	2443
36	Sakon nakhon	2457
37	Nakhon phanom	2402
38	Mukdahan	2490
39	Chiang mai	2583
40	Lamphun	2490
41	Lampang	2451
42	Uttaradit	2370
43	Phrae	2398
44	Nan	2331
45	Phayao	2489
46	Chiang rai	2552
47	Mae hong son	2320
48	Nakhon sawan	2399
49	Uthai thani	2324
50	Kamphang phet	2303
51	Tak	2345
52	Sukhothai	2311
53	Phitsanulok	2319
54	Phichit	2301
55	Phetchabun	2280
56	Ratchaburi	2740
57	Kanchanaburi	2801
58	Suphanburi	2772
59	Nakhon pathom	2834
60	Samut sakhon	3013

Continued on next page...

Table 1 – continued from previous page.

Province Index	Province Name	Poverty Line (Baht/capita/month)
61	Samut songkhram	2715
62	Phetchaburi	2733
63	Prachuap khiri khan	2711
64	Nakhon si thammarat	2687
65	Krabi	2649
66	Phangnga	2677
67	Phuket	3059
68	Surat thani	2870
69	Ranong	2908
70	Chumphon	2816
71	Songkhla	2880
72	Satun	2595
73	Trang	2798
74	Phatthalung	2865
75	Pattani	2505
76	Yala	2791
77	Naratiwat	2518

Table 1: Poverty line 2017 for 77 provinces in Thailand

APPENDIX B

Proof of Lemma 4.2.2

If $Y_d|\theta_d, \sigma_d^2, \boldsymbol{\beta} \sim N(\theta_d, \sigma_d^2)$ and $\theta_d|\boldsymbol{\beta} \sim N(\mathbf{x}'_d\boldsymbol{\beta}, A)$, where $d = 1, \dots, D$, then $\theta_d|Y_d \sim N(\mu_d, AB_d)$ where $\mu_d = (1 - B_d)Y_d + B_d\mathbf{x}'_d\boldsymbol{\beta}$ and $B_d = \frac{\sigma_d^2}{A+\sigma_d^2}$.

Proof. Follow the Bayes' Rule in Section 2.1.1, the conditional probability density function $f(\theta_d|Y_d)$ can be written as

$$\begin{aligned}
 f(\theta_d|Y_d) &= \frac{f(\theta_d)f(Y_d|\theta_d)}{f(Y_d)} \\
 &\propto f(\theta_d)f(Y_d|\theta_d) \\
 &= \frac{1}{\sqrt{2\pi A}} \exp\left\{-\frac{1}{2A}(\theta_d - \mathbf{x}'_d\boldsymbol{\beta})^2\right\} \times \frac{1}{\sqrt{2\pi\sigma_d^2}} \exp\left\{-\frac{1}{2\sigma_d^2}(Y_d - \theta_d)^2\right\} \\
 &= \frac{1}{2\pi\sqrt{A\sigma_d^2}} \exp\left\{-\frac{\theta_d^2 - 2\theta_d\mathbf{x}'_d\boldsymbol{\beta} + (\mathbf{x}'_d\boldsymbol{\beta})^2}{2A} - \frac{Y_d^2 - 2Y_d\theta_d + \theta_d^2}{2\sigma_d^2}\right\} \\
 &= \frac{1}{2\pi\sqrt{A\sigma_d^2}} \exp\left\{-\frac{-\theta_d^2\sigma_d^2 + 2\theta_d\mathbf{x}'_d\boldsymbol{\beta}\sigma_d^2 - (\mathbf{x}'_d\boldsymbol{\beta})^2\sigma_d^2 - Y_d^2A + 2Y_d\theta_dA - \theta_d^2A}{2A\sigma_d^2}\right\} \\
 &= \frac{1}{2\pi\sqrt{A\sigma_d^2}} \exp\left\{-\frac{-\theta_d^2(\sigma_d^2 + A) + 2\theta_d(\mathbf{x}'_d\boldsymbol{\beta}\sigma_d^2 + AY_d) - ((\mathbf{x}'_d\boldsymbol{\beta})^2\sigma_d^2 + AY_d^2)}{2A\sigma_d^2}\right\} \\
 &= \frac{1}{2\pi\sqrt{A\sigma_d^2}} \exp\left\{-\frac{-\theta_d^2 + 2\theta_d\frac{\mathbf{x}'_d\boldsymbol{\beta}\sigma_d^2 + AY_d}{A + \sigma_d^2} - \left(\frac{\mathbf{x}'_d\boldsymbol{\beta}\sigma_d^2 + AY_d}{A + \sigma_d^2}\right)^2}{\frac{2A\sigma_d^2}{A + \sigma_d^2}}\right\} \\
 &\quad \times \exp\left\{-\frac{(\mathbf{x}'_d\boldsymbol{\beta})^2\sigma_d^2 + AY_d^2}{A + \sigma_d^2}\right\} \\
 &\propto \exp\left\{-\frac{\left(\theta_d - \frac{\mathbf{x}'_d\boldsymbol{\beta}\sigma_d^2 + AY_d}{A + \sigma_d^2}\right)^2}{2 \times \frac{A\sigma_d^2}{A + \sigma_d^2}}\right\}.
 \end{aligned}$$

For $B_d = \frac{\sigma_d^2}{A+\sigma_d^2}$, letting $\mu_d = \frac{\mathbf{x}'_d\boldsymbol{\beta}\sigma_d^2 + AY_d}{A + \sigma_d^2} = (1 - B_d)Y_d + B_d\mathbf{x}'_d\boldsymbol{\beta}$.

Hence $f(\theta_d|Y_d) = \frac{1}{\sqrt{2\pi B_d}} \exp\left\{-\frac{(\theta_d - \mu_d)^2}{2AB_d}\right\}$.

We can conclude that $\theta_d|Y_d \sim N(\mu_d, AB_d)$.

□

BIOGRAPHY

Name Miss Sarasinee Somjettana

Date of Birth 12 September 1995

Place of Birth Surin, Thailand

Education B.S. (Applied Mathematics),
Chulalongkorn University, 2015

