CHAPTER 3

Optical waveguide

The purpose of this chapter is to provide a qualitative understanding of the optically confining mechanism occurring in a laser diode. There are two things required to operate a laser : (i) a gain medium that can amplify the electromagnetic radiation propagating inside it and provide the spontaneous emission and (ii) a feedback mechanism that confine the electromagnetic field through the well-defined optical modes. The optical feedback is obtained by using the cleaved facets that form a Fabry-Perot cavity and mode confinement is achieved through dielectric waveguiding which concerns to two optical modes : *Transverse-Electric (TE)* or *Transverse-Magnetic (TM)*.

3.1 Dielectric waveguide

The successful operation of laser requires that the generated optical field should remain confined in the vicinity of the gain region. In DH laser diode, the optical confinement occurs by virtue of a fortunate coincidence. The active layer with a smaller bandgap also has a higher refractive index compared with that of the surrounding cladding layers. Because of the index difference, the active layer in effect acts as a dielectric waveguide. The physical mechanism behind the confinement is total internal refraction as illustrated in Fig. 3.1. When a ray traveling at an angle θ (measured from the interface normal) hits the interface, it is reflected back if the angle θ exceeds the critical angle θ_c given by

$$\theta_c = sin^{-1}\frac{n_1}{n_2} \tag{3.1}$$

where n_1 and n_2 are the refractive indices of the cladding and active layer respectively. Thus, rays traveling nearly parallel to the interface are trapped and constitute the waveguide mode. This mechanism is often referred to as ***index-guiding***. A more detailed discussion of waveguide modes requires the use of Maxwell's equations and is given in next sections.

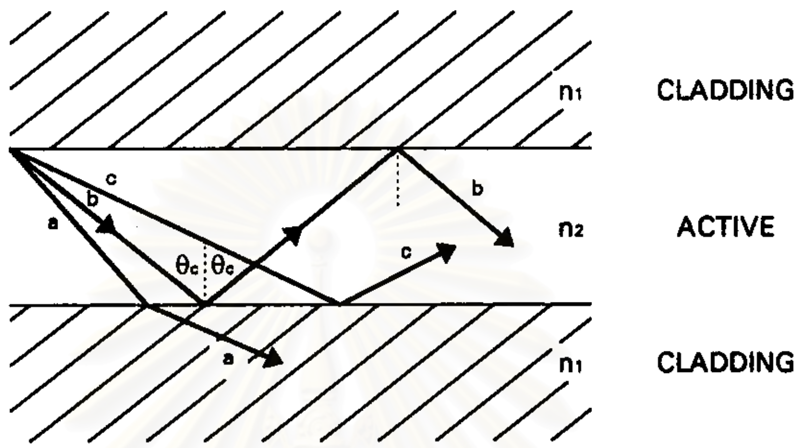


Fig. 3.1 Dielectric waveguiding in a heterostructure laser diode. The relatively higher refractive index ($n_2 > n_1$) of the active layer allows total internal reflection to occur at the two interfaces for angle such that $\theta \geq \theta_c$ as shown in ray b and c.

3.2 Waveguide modes

In a heterostructure laser, the emitted light has finite transverse dimensions, since it should be confined in the vicinity of the this active region, which provides gain for stimulated emission. The output is in the form of narrow beam with an elliptic cross section. Depending on the laser structure, the field distribution across the beam can take certain well-defined forms, often referred to as the *laser modes*. Mathematically, a laser mode is the specific solution of the time-independent Maxwell's wave equation that satisfies all the boundary conditions imposed by the laser structure. In the general multimode case, the optical field is denoted by E_{pqm} , where the subscript m denoted the l*ongitudinal* or *axial* modes. The subscripts p and q take integer values; they stand for the *transverse* and *lateral* modes specifying the field distribution in the direction perpendicular and parallel to the junction plane, respectively. An understanding of the number of allowed modes and the resulting field distributions is essential for their control, since it is often desirable to design laser diode that emit light predominantly in a single mode. Further, important laser characteristics such as the near- and far-field widths depend on details of the laser modes. For laser diode, the far-field distribution plays an important role as it controls the amount of power coupled into a fiber optic.

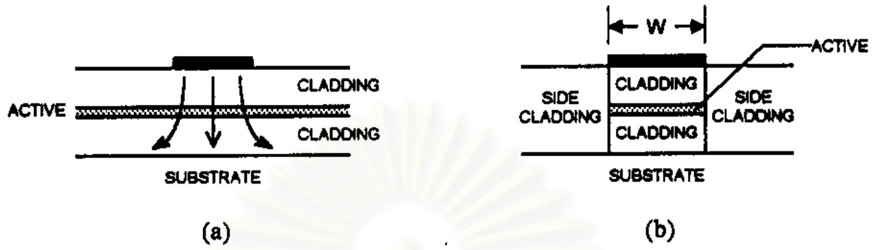


Fig. 3.2 Cross-section schematic of (a) gain-guided and (b) index-guided devices.

In transverse heterostructure, the field confinement in the transverse direction occurs through index-guiding described in the previous section. However, the field confinement in lateral direction is not always due to index-guiding. Laser diode can be classified as gain-guided and index-guided depending on whether it is the lateral variation of the optical gain or the refractive index that confines the mode. Fig. 3.2 shows schematically the both kinds of devices. Historically, gain-guided devices based on the strip geometry were developed first in view of their ease of fabrication. However, such devices have a number of undesirable characteristics that become worse as the laser wavelength increases. Index guiding is therefore almost invariably used in most practical laser diodes. Both gain-guided and index-guided structures and performances have been already discussed in the previous chapter.

3.3 Effective index approximation

The mathematical description of the laser modes is based on the time-independent Maxwell's wave equation

$$\nabla^2\mathbf{E} + \varepsilon(x,y)\,k_o^2\,\mathbf{E} = 0 \qquad (3.2)$$

where the x axis is parallel and the y axis is perpendicular to the junction plane shown in Fig. 3.3 and $k_o = \omega/c = 2\pi/\lambda$ is the vacuum wave number. The dielectric constant

ε may vary with x and y but is assumed to be independent of z , the direction of field propagated along the cavity length. For some laser diodes (e.g. distributed feedback (DFB) lasers), ε varies with z. In normal cases, such variations are small enough that they can be ignored in the discussion of the waveguide modes. The spatially varying dielectric constant in generally of the form,

$$\varepsilon(x,y) = \varepsilon_j(x) \tag{3.3}$$

where the subscript j numbers various layers in a heterostructure laser. To account for the absorption, the dielectric constant ε_j is a complex number in each layer. Further, as in the references [8,54], within the active layer it also varies with external pumping.

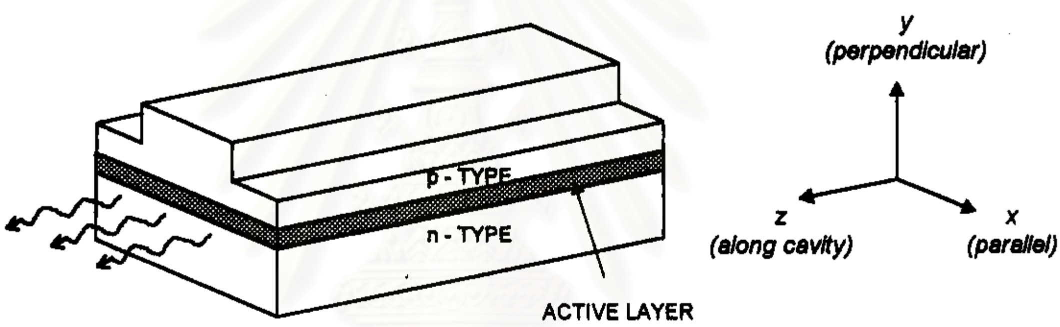


Fig. 3.3 Clarification of the x, y and z axis used in the effective refractive index approximation.

To obtain an exact solution of Eq.(3.2) is a difficult task. It is essential to make certain simplifying assumptions whose nature and validity vary from device to device. For index guided laser such as BH, the effect of gain or loss on the passive-cavity modes can be often ignored. The resulting rectangular waveguide problem can then be solved to obtain transverse and lateral modes supported by the device. This approach is, however, not suitable for gain-guided lasers where the lateral modes arise precisely because of the active-region gain.

An alternative approach is based on the *effective index approximation* [5-10]. Instead of solving the two-dimension wave equation, the problem is split into two one-dimensional parts whose solutions are relatively easy to obtain. Such an approach is partially successful for both gain-guided and index-guided lasers. At the same time, it is helpful for a physical understanding of the guiding mechanism. The physical motivation behind the effective index approximation is that often the dielectric constant $\varepsilon(x,y)$ varies slowly in the lateral x direction compared to its variation in the transverse y direction. To a good approximation, the slab-waveguide problem in the y direction can be solved for each x and the resulting solution can then be used to account for the lateral variation. The electric field in Eq.(3.2) is thus approximated by

$$\mathbf{E} \cong \hat{\mathbf{e}}\, E_y(y;x)\, E_x(x)\, e^{i\beta z} \tag{3.4}$$

where β is the propagation constant of the mode and $\hat{\mathbf{e}}$ is the unit vector in the direction along which the mode is polarized. On substituting Eq.(3.4) in Eq.(3.2) we obtain

$$\frac{1}{E_x}\frac{\partial^2 E_x}{\partial x^2} + \frac{1}{E_y}\frac{\partial^2 E_y}{\partial y^2} + [\, \varepsilon(x,y)\, k_o^2 - \beta^2 \,]\, E_y = 0 . \tag{3.5}$$

In the effective index approximation, the transverse field distribution $E_y(y;x)$ is obtain first by solving

$$\frac{\partial^2 E_y}{\partial y^2} + [\, \varepsilon(x,y)\, k_o^2 - \beta_{eff}^2(x) \,]\, E_y = 0 \tag{3.6}$$

where $\beta_{eff}(x)$ is the effective propagation constant for a fixed value of x. the lateral field distribution $E_x(x)$ is then obtained by solving

$$\frac{\partial^2 E_x}{\partial x^2} + [\, \beta_{eff}^2(x) - \beta^2 \,]\, E_x = 0 . \tag{3.7}$$

For a given laser structure Eqs. (3.6) and (3.7) can be used to obtain the transverse and lateral modes, respectively. Since $\varepsilon(x,y)$ is generally complex, $\beta_{eff}(x)$ is also complex. The complex refractive effective index is defined as

$$n_{eff}(x) = \beta_{eff}(x) / k_0 . \tag{3.8}$$

Equation (3.6) is a one-dimensional eigenvalue equations and can be solved using the method developed for dielectric slab waveguides. Although it is in general possible to include the gain or loss occurring in each layer, the resulting analysis is cumbersome. A simpler approach is to treat the effect of gain or loss as a small perturbation to the eigenvalue problem. This is justified for heterostructure lasers since the mode confinement in the y direction occurs mainly because of the index step at the heterostructure interfaces. The dielectric constant $\varepsilon(x,y)$ is of the form

$$\varepsilon(x,y) = n_b^2(y) + \Delta\varepsilon(x,y) \tag{3.9}$$

where n_b is the background (real) refractive index, constant for each layer. The small perturbation $|\Delta\varepsilon| \ll n_b^2$ includes the loss and the contribution of external pumping. If we use the first-order perturbation theory [55,56], the eigenvalue given by n_{eff} becomes

$$n_{eff}(x) = n_e(x) + \Delta n_e(x) \tag{3.10}$$

where $n_e(x)$ is obtained by solving the unperturbed eigenvalue equation

$$\frac{\partial^2 E_y}{\partial y^2} + k_o^2 [\, n_b^2(y) - n_e^2(x) \,] E_y = 0 \tag{3.11}$$

and the perturbation Δn_e is obtained using

$$2\, n_e\, \Delta n_e = \frac{\int_{-\infty}^{\infty} \Delta\varepsilon(x,y)\, E_y^2(y;x)\, dy}{\int_{-\infty}^{\infty} E_y^2(y;x)\, dy} \quad . \tag{3.12}$$

Since $\Delta\varepsilon(x,y)$ is constant within each layer, Eq.(3.12) can be simplified to become

$$\Delta n_e(x) = \frac{1}{2n_e} \sum_j \Gamma_j(x)\, \Delta\varepsilon_j(x) \tag{3.13}$$

where the sum is over the number of layers, $\Delta\varepsilon_j$ is the dielectric perturbation of jth layer, and

$$\Gamma_j(x) = \frac{\int_{d_j} E_y^2(y;x)\, dy}{\int_{-\infty}^{\infty} E_y^2(y;x)\, dy} \tag{3.14}$$

is the fraction of the mode intensity contained in that layer. For the active layer, Γ_j is referred to as the *confinement* or *filling factor* since it indicates the extent to which the mode is confined to the active region. Both Γ_j and the effective index n_e vary with x if the active layer is not laterally uniform in thickness. This is the case, for example, the BH, CSP, CDH and RW devices discussed in chapter 2.

3.4 Transverse mode

The transverse modes are obtained by solving Eq.(3.11) and depend on the thickness and refractive indices of the various layers used to fabricate a laser diode. The number of layers to be considered depends on the specific laser structure, and it is often necessary to consider all layers for a reasonably accurate description of the transverse modes. Slab-waveguide model used for studying is found to support two sets of the modes, the TE and TM modes, which are distinguished on the basis of their polarization. For TE modes, the electric field E is polarized along the heterojunction plane, i.e., the polarization vector in Eq.(3.4) is along the x axis. For TM modes, it is the magnetic field H that is polarized along x axis.

In both cases, the same equation (3.11) can be used to obtain the field distributions. The boundary condition that the tangential component of the electric and magnetic fields be continuous (E_y and dE_y/dy matching) at every dielectric interface. The other boundary condition for the guided modes is that the field E_y should vanish as y tends to infinity. It should be pointed out that a slab waveguide also supports unguided modes, that so-called radiation modes, for which the latter boundary condition does not apply. For a laser diode, these radiation modes are not of significant interest and would not be considered. However, their inclusion is crucial if an arbitrary optical field is expanded in terms of a complete set of waveguide modes.

As discussed before, the analysis depends on the laser structure containing various thickness and refractive indices. For the transverse mode, we will consider the

DH structure for example. The other structures corresponding to ridge waveguide analysis will be examined in the next chapter.

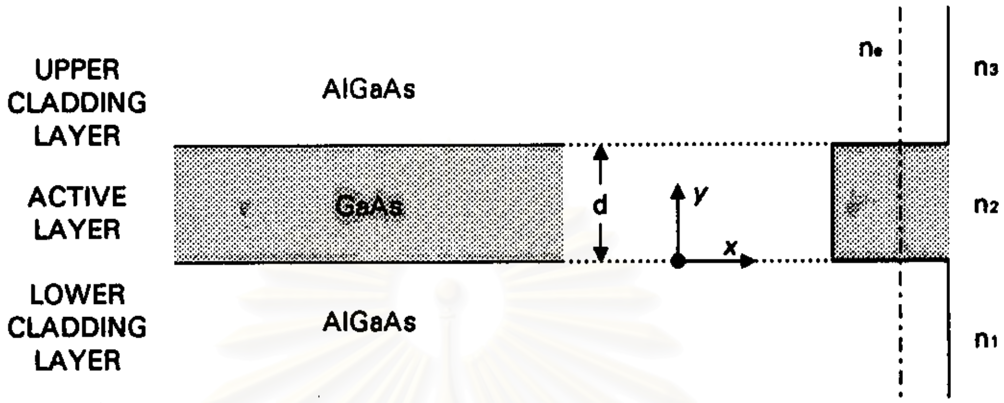


Fig. 3.4 Schematic and index profile of a DH structure. Shaded area designates the waveguiding region.

The DH is a basic structure involved in dielectric waveguiding which may be understood using a three-layer slab waveguide shown schematically in Fig. 3.4. The active layer of thickness d is surrounded on both sides by cladding layers. If the cladding layers are sufficiently thick such that the mode is largely confined within the three layers, the remaining layers can be ignored.

A general solution of Eq.(3.11) is of the form

$$E_y(y) = \begin{cases} A_1 e^{-\gamma_1 y} + A_2 e^{\gamma_1 y} & , y \leq 0 \\ B_1 \cos(\kappa y) + B_2 \sin(\kappa y) & , 0 \leq y \leq d \\ C_1 e^{-\gamma_2 y} + C_2 e^{\gamma_2 y} & , d \leq y \end{cases} \tag{3.15}$$

where

$$\gamma_1 = k_o (n_e^2 - n_1^2)^{1/2} \tag{3.16}$$

$$\kappa = k_o (n_2^2 - n_e^2)^{1/2} \tag{3.17}$$

$$\gamma_2 = k_o (n_e^2 - n_3^2)^{1/2} \tag{3.18}$$

and *n1*, *n2* and *n3* are the material refractive indices for the lower cladding, active and upper cladding layers, respectively, with $n2 > n1$ and $n3$.

The boundary condition which $E_y(y \rightarrow \pm\infty) = 0$ reveals that

$$A_1 e^{\infty} + A_2 e^{-\infty} = 0 \tag{3.19}$$

$$C_1 e^{-\infty} + C_2 e^{\infty} = 0, \tag{3.20}$$

thus, A_2 and C_1 must be set to zero

$$A_1 = 0 \tag{3.21}$$

$$C_2 = 0 \tag{3.22}$$

The continuity of E_y and dE_y/dy at $y = 0$ requires that

$$B_1 = C_1 \tag{3.23}$$

$$\kappa B_1 = \gamma_1 C_1 \tag{3.24}$$

and also at $y = d$ requires that

$$A_2 = B_1 \cos(\kappa d) + B_2 \sin(\kappa d) \tag{3.25}$$

$$-\gamma_1 A_2 = -\kappa B_1 \sin(\kappa d) + \kappa B_2 \cos(\kappa d) \;. \tag{3.26}$$

From these four equation (3.23) - (3.26), we obtain the eigenvalue equation

$$\begin{bmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & \kappa & -\gamma_2 \\ 1 & -\cos(\kappa d) & -\sin(\kappa d) & 0 \\ -\gamma_1 & \kappa \sin(\kappa d) & -\kappa \cos(k d) & 0 \end{bmatrix} \begin{bmatrix} A_2 \\ B_1 \\ B_2 \\ C_1 \end{bmatrix} = 0 \tag{3.27}$$

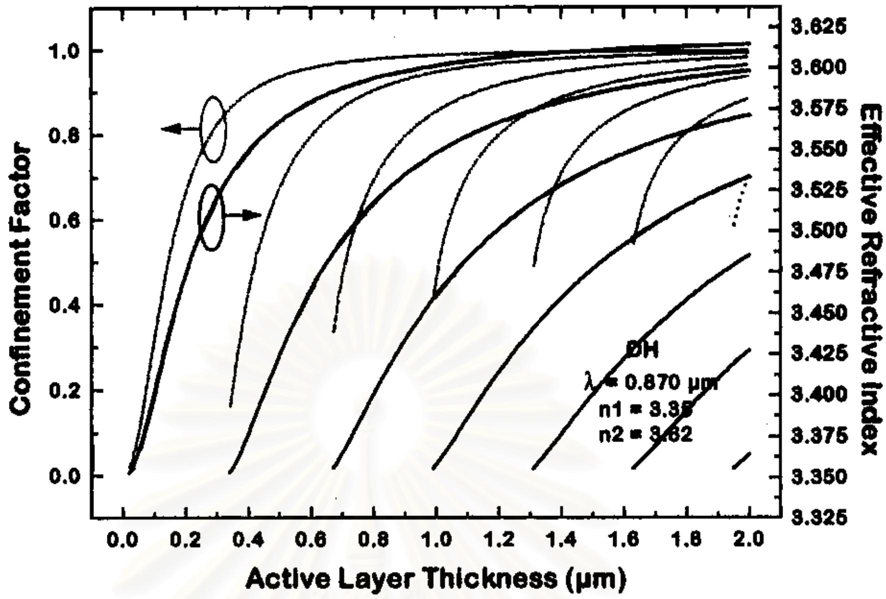
whose solutions yield the effective mode index (n_e). In general, multiple solutions are possible corresponding to different TE modes. For all guided modes, the inequality $n2 > ne > n1$ and $n3$ is satisfied.

The TM modes are obtained using the same procedure. The only difference lies in the application of boundary conditions. The continuity of the tangential component of the electric field (E_z) requires that $n_j^{-2}(dE_y/dy)$ be continuous across the heterostructure interface at $y = 0$ and $y = d$. Here n_j is either $n1$, $n2$ or $n3$ depending on the side from which the interface is approached. The origin of this difference between TE and TM modes can be traced back to Maxwell's equation. In the heterostructure laser diodes, the TE modes are generally favored over the TM modes since the facet reflectivity is higher for TE modes [10]. In the further discussion we therefore consider only the TE modes.

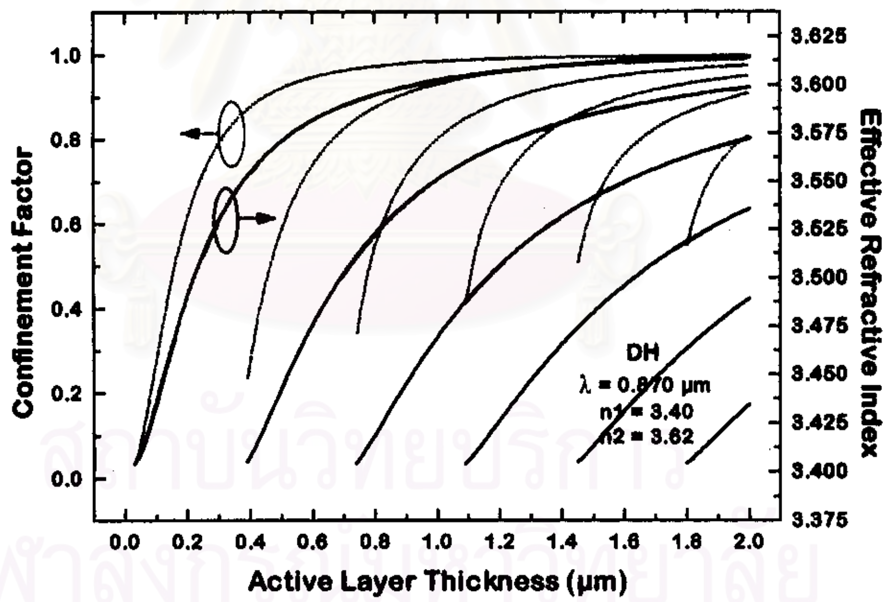
As mentioned before, a quantity that plays an important role for heterostructure laser is the *transverse confinement factor* (Γ_T) because it represents the fraction of the mode energy within the active layer that is available for interaction with the injected charged carriers. Using $E_y(y)$ from Eq.(3.15) in

$$\Gamma_T = \frac{\int_0^d E_y^2(y)\, dy}{\int_{-\infty}^{\infty} E_y^2(y)\, dy} \qquad (3.28)$$

Fig. 3.5 shows the simulation results of effective refractive index (n_e) and transverse confinement factor (Γ_T) as the functions of the active layer thickness (d) for the 0.870 μm GaAs lasers with $Al_xGa_{1-x}As$ cladding layers. By varying the Al-content (x) in the ternary compound, the bulk refractive indices of both cladding layers (n_1 and n_3)can be set to 3.35, 3.40, 3.45 and 3.50 corresponding to Figs. 3.5a, 3.5b, 3.5c and 3.5d, while the active-layer index is 3.62 constantly. In these four graphs, both effective index and confinement factor increase with active layer thickness and tend to steady. It should be noted that there is a fact investigated from the graph comparison. The higher optical mode occurs when active layer thickness increases, the maximum thickness maintaining fundamental mode are 0.30, 0.40, 0.45 and 0.45 μm according to the cladding layer indices 3.35, 3.40, 3.45 and 3.50, respectively. In the other words, less index difference between active and cladding layers (n_2-n_1) produces higher mode discrimination which a fundamental mode operation and increased confinement factor can be achieved at the larger active layer thickness.

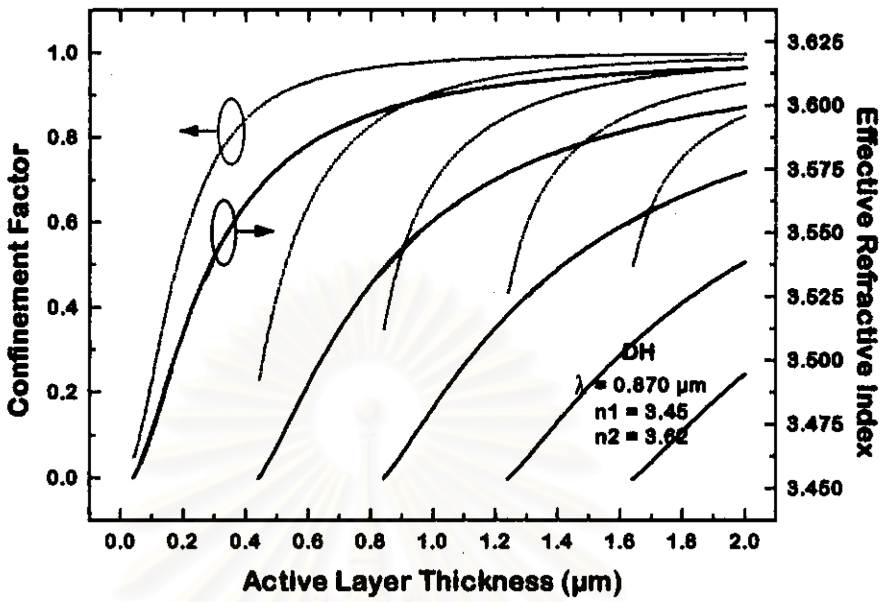


(a)



(b)

Fig. 3.5 Simulation results of transverse confinement factor (Γ_T) and effective refractive index (n_e) as the functions of the active layer thickness (d) for 0.870 μm GaAs DH lasers with varied cladding-layer indices (a) 3.35, (b) 3.40, (c) 3.45 and (d) 3.50. The solid and dotted line are represented to effective refractive index and confinement factor, respectively.



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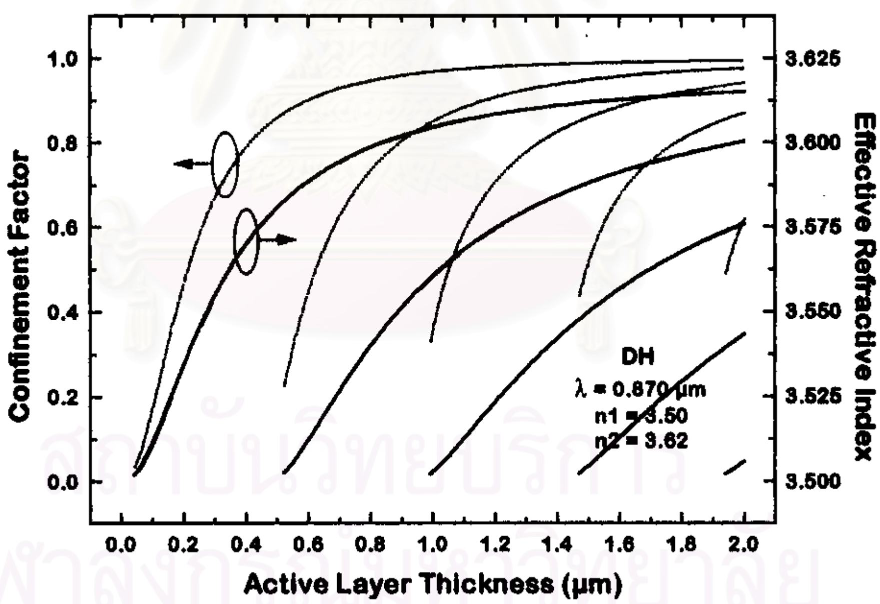

(d)

Fig. 3.5 (cont.) Simulation results of transverse confinement factor (Γ_T) and effective refractive index (n_e) as the functions of the active layer thickness (d) for 0.870 μm GaAs DH lasers with varied cladding-layer indices (a) 3.35, (b) 3.40, (c) 3.45 and (d) 3.50. The solid and dotted line are represented to effective refractive index and confinement factor, respectively.

3.5 Lateral modes

The lateral modes are obtained by solving Eq.(3.7) which after using Eqs. (3.8) and (3.10) becomes

$$\frac{\partial^2 E_x}{\partial x^2} + k_o^2 \{ [n_e(x) + \Delta n_e(x)]^2 - \beta^2(x) \} E_x = 0 \qquad (3.29)$$

The lateral-mode behavior in laser diodes is different depending on whether gain guiding or index guiding is used to confine the lateral modes. In a gain-guided device, $n_e(x)$ is a constant not depending on lateral direction. By contrast, in a index-guided device, structural lateral variations are used to make n_e larger in a central region of width w illustrated in Fig. 3.2b. For the latter case, the slab-waveguide problem discussed in the previous section is solved separately in the two regions, and

$$n_e(x) = \begin{cases} n_e^{in} & , if\ |x| \leq w/2 \\ n_e^{out} & , otherwise \end{cases} \qquad (3.30)$$

where n_e^{in} and n_e^{out} are the effective indices corresponding to the central and out-of-central region, respectively. Their magnitude depends on the structural details, and the lateral index step

$$\Delta n_L = n_e^{in} - n_e^{out} \qquad (3.31)$$

determines the extent of index guiding. Whether the lateral mode is index-guided or gain-guided depends on the relative magnitudes of Δn_L and $\Delta n_e(x)$, and in general both should be considered.

3.5.1 Index-guided laser diodes

The devices with the index step $\Delta n_L \gg |\Delta n_e(x)|$ fall in the category of index-guided devices such as BH and RW laser diodes. In this case, the effect of gain can be treated as a small perturbation to the index-guided lateral mode and one can follow a perturbation procedure similar to that outlined in Eqs. (3.10) - (3.14). The mode-propagation constant is given by

$$\beta = k_o n_L + i\,\alpha_L/2 \tag{3.32}$$

whose n_L and α_L are the refractive index and the absorption coefficient of a lateral mode supported by the rectangular waveguide of width w and thickness d. The lateral modes are obtained by solving the symmetrical three-layer slab-waveguide problem

$$\frac{\partial^2 E_x}{\partial x^2} + k_o^2 [\, n_e^2(x) - n_L^2 \,] E_x = 0 . \tag{3.33}$$

The mode-absorption coefficient α_L is obtained using the first-order perturbation theory [10] and is given by

$$\alpha_L = \frac{ko}{n_L} \,\mathrm{Im}\, \frac{\int 2\, n_e(x)\, \Delta n_e(x)\, E_x^2(x)\, dx}{\int E_x^2(x)\, dx} \tag{3.34}$$

where $\Delta n_e(x)$ is given by Eq.(3.13).

By the same procedure as transverse mode, we can obtain numerical solutions of n_L and α_L from the eigenvalue equation. And the *lateral confinement factor* (Γ_L) can be considered from

$$\Gamma_L = \frac{\int_{-w/2}^{w/2} E_x^2(x)\, dx}{\int_{-\infty}^{\infty} E_x^2(x)\, dx} \tag{3.35}$$

Finally, we can represents the fraction of overall mode energy contained within active region, *overall confinement factor* (Γ), as following

$$\Gamma = \Gamma_L \Gamma_T \tag{3.36}$$

where Γ_L and Γ_T are the transverse and lateral confinement factor defined by Eqs. (3.28) and (3.35), respectively.

3.5.2 Gain-guided laser diodes

For gain-guided devices, the effective index n_e in Eq.(3.29) is constant along the lateral direction x, and the mode confinement occurs through $\Delta n_e(x)$ given by Eq.(3.13). In contrast to index-guided lasers where the index changes discontinuously [see Eq.(3.30)], $\Delta n_e(x)$ varies continuously.

The problem of lateral-mode determination for a gain-guided laser diodes is, however, exceedingly complex, and in general a numerical approach is necessary. This is so because stimulated emission couples the carrier-diffusion and the wave equations, and the two should be solved self-consistently for each value of the device current. A further complication is that the injected current density [$J(x)$] is itself laterally non-uniform because of current spreading in the bell-shape (see Fig. 2.3a). Moreover, the problem should includes Auger recombination for more precise determination.

This thesis does not show the details to solve gain-guided problem because its complexity and it is not the major mechanism in Ridge Waveguide which is the thesis's objective. Anyway, some literature [10,30] have shown several numerical models and various assumptions for the gain-guided laser diodes.