วิธีอิงการแยกของเบนเดอร์สสำหรับการวางแผนขยายระบบส่งไฟฟ้าแบบหลายชั้น
ที่มีข้อบังคับด้านความมั่นคง

นายสมภพ อัษฎมงคล

วิทยานิพนธ์นี้เป็นส่วนหนึ่งของการศึกษาตามหลักสูตรปริญญาวิศวกรรมศาสตรดุษฎีบัณฑิต
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BENDERS DECOMPOSITION BASED METHOD FOR MULTISTAGE TRANSMISSION EXPANSION PLANNING WITH SECURITY CONSTRAINTS

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วิทยานิพนธ์นี้ นำเสนอวิธีการแก้ปัญหาสำหรับการวางแผนขยายระบบส่งไฟฟ้าแบบหลายชั้นที่มีข้อบังคับด้านความมั่นคง โดยอาศัยวิธีการแยกของเบนเดอร์สและอาศัยแบบจำลองกระแสสลับในการจำลองปัญหาการวางแผนขยายระบบส่งไฟฟ้าแทนที่แบบจำลองกระแสตรงซึ่งมักถูกนำมาใช้ในงานวิจัยที่ผ่านมา โดยตัวปัญหาจะถูกแยกออกเป็นปัญหาการลงทุนและปัญหาปฏิบัติการ วิธีการที่นำเสนอสามารถหาผลตอบแผนงานขยายระบบส่งไฟฟ้าที่เหมาะสมได้จากการแก้ปัญหาทั้งสองประเภทแบบวนซ้ำ นอกจากนี้ ในปัญหาการวางแผนดังกล่าว ยังมีการพิจารณาต้นทุนของการผลิตไฟฟ้าด้วย ทำให้แผนงานที่ได้จากวิธีการที่นำเสนอสามารถลดความสูญเสียของกำลังไฟฟ้าจริงและความคับคั่งในระบบส่งไฟฟ้าลงได้ ผลการทดสอบกับระบบไฟฟ้า 6 บัส ระบบไฟฟ้า IEEE 24 บัส และระบบไฟฟ้าภาคตะวันออกเฉียงเหนือของประเทศไทย แสดงให้เห็นว่า วิธีการที่นำเสนอมีศักยภาพเพียงพอ ในการนำไปประยุกต์ใช้สำหรับกระบวนการวางแผนขยายระบบส่งไฟฟ้าในทางปฏิบัติได้อย่างมีประสิทธิภาพ

ภาควิชา วิศวกรรมไฟฟ้า ลายมือชื่อนิสิต ....

สาขาวิชา วิศวกรรมไฟฟ้า ลายมือชื่อ อ.ที่ปรึกษาวิทยานิพนธ์หลัก ....

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SOMPHOP ASADAMONGKOL : BENDERS DECOMPOSITION BASED METHOD FOR MULTISTAGE TRANSMISSION EXPANSION PLANNING WITH SECURITY CONSTRAINTS. ADVISOR : PROFESSOR BUNDHIT EUA-ARPORN, Ph.D., 163 pp.

This dissertation proposes a Benders decomposition based method for solving a multistage transmission expansion planning (TEP) problem with security constraints. An AC model is used to develop the TEP problem instead of a DC model, which is generally used in previous research works. With the proposed method, the TEP problem is decomposed into an investment and operation problems. An optimum plan can be obtained by iteratively solving both problems. In addition, the generation cost is taken into account, which results in active power loss decrease and transmission congestion alleviation. The proposed method has been tested with 6-bus Garver, IEEE 24-bus and 75-bus northeastern Thailand systems to demonstrate its capability of solving various sizes of the problems. With the obtained results, it shows that the proposed method can be applied in actual transmission system planning.

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List of Abbreviations

Indices

l Index of branches,

p Index of paths,

t Index of stages in planning period, and

s Index of scenarios.

Sets

$\mathcal{C}$ Index set of all candidate branches,

$\mathcal{C}_p$ Index set of candidate branches in path p,

$\mathcal{C}_{\mathrm{a}}$ Index set of candidate branches which are available for stringing the additional circuits after it is constructed,

$\mathcal{C}_{\mathrm{s}}$ Index set of candidate branches which will be constructed by stringing on the towers of other candidate branches,

$\mathcal{P}$ Index set of paths, and

$\mathcal{U}$ Index set of infeasible operation problems.

Constants parameters

$\mathbf{c_b}$ Vector in $\mathbb{R}^{\boldsymbol{nc}}$ representing construction cost of candidate branches; in US$,

$\mathbf{c_g}$ Vector in $\mathbb{R}^{\boldsymbol{ng}}$ representing unit cost of active power generation; in US$/p.u.,

$\mathbf{c_r}$, $\mathbf{c_c}$ Vectors in $\mathbb{R}^{nb}$ representing the installation costs of reactors and capacitors respectively; in US$/p.u.,

$\mathbf{c_f}$ Vector in $\mathbb{R}^{\boldsymbol{nf}}$ representing installation cost of UPFCs; in US$,

$\mathbf{v^{min}}$, $\mathbf{v^{max}}$ Vectors in $\mathbb{R}^{nb}$ representing minimum and maximum limits of voltage magnitude; in p.u.,

$\mathbf{p_g^{min}}$, $\mathbf{p_g^{max}}$ Vectors in $\mathbb{R}^{ng}$ representing minimum and maximum limits of active power generation; in p.u.,

$\mathbf{q_g^{min}}$, $\mathbf{q_g^{max}}$	Vectors in $\mathbb{R}^{ng}$ representing minimum and maximum limits of reactive power generation; in p.u.,
$\mathbf{i_{be}^{max}}$	Vector in $\mathbb{R}^{ne}$ representing maximum current limit of existing branches; in p.u.,
$\mathbf{i_{bc}^{max}}$	Vector in $\mathbb{R}^{nc}$ representing maximum current limit of candidate branches; in p.u.,
$\mathbf{i_{se}^{max}}$, $\mathbf{i_{sh}^{max}}$	Vectors in $\mathbb{R}^{nf}$ representing maximum current limits of series and shunt converters; in p.u.,
$\mathbf{p_{dc}^{max}}$	Vector in $\mathbb{R}^{nf}$ representing capacity of DC links; in p.u.,
$\mathbf{d_r^{max}}$, $\mathbf{d_c^{max}}$	Vectors in $\mathbb{R}^{nb}$ representing maximum capacities of installed reactors and capacitors; in p.u.,
$\mathbf{d_r^0}$, $\mathbf{d_c^0}$	Vectors in $\mathbb{R}^{nb}$ representing capacities of existing installed reactors and capacitors; in p.u.,
$\mathbf{p_d}$, $\mathbf{q_d}$	Vectors in $\mathbb{R}^{nb}$ representing active and reactive power demand; in p.u.,
$\mathbf{r}$	Vector in $\mathbb{R}^{nr}$ representing the right-hand side of constraint relating to existing branches for TEP problem using DC model,
$\mathbf{w}$	Vector in $\mathbb{R}^{nw}$ representing the right-hand side of constraint relating to candidate branches for TEP problem using DC model,
$\mathbf{e}_{\boldsymbol{n}}$	Vector in $\mathbb{R}^{\boldsymbol{n}}$ consisting of all 1s,
$\mathbf{0}_{v\times w}$	Matrix in $\mathbb{R}^{v\times w}$ consisting of all 0s,
$\mathbf{A_g}$	Generator-bus incidence matrix in $\mathbb{R}^{ng\times nb}$,
$\mathbf{A_{be}}$	Existing branch-bus incidence matrix in $\mathbb{R}^{ne\times nb}$,
$\mathbf{A_{bc}}$	Candidate branch-bus incidence matrix in $\mathbb{R}^{nc\times nb}$,
$\mathbf{A_{bef}}$, $\mathbf{A_{bet}}$	Existing branch-bus incidence matrices in $\mathbb{R}^{ne\times nb}$ regarding as ***from bus*** and ***to bus*** respectively,
$\mathbf{A_{bcf}}$, $\mathbf{A_{bct}}$	Candidate branch-bus incidence matrices in $\mathbb{R}^{nc\times nb}$ regarding as ***from bus*** and ***to bus*** respectively,
$\mathbf{A_{ub}}$	UPFC-bus incidence matrix in $\mathbb{R}^{nf\times nb}$,
$\mathbf{A_{up}}$	UPFC-candidate path incidence matrix in $\mathbb{R}^{nfc\times np}$,
$\mathbf{U_f}$	Matrix in $\mathbb{R}^{nfc\times nf}$ representing UPFCs connected to candidate lines,
$\mathbf{B_e}$	Diagonal matrix in $\mathbb{R}^{ne\times ne}$ of which the elements represent susceptance of existing branches; in p.u.,

$\mathbf{B_c}$	Diagonal matrix in $\mathbb{R}^{nc\times nc}$ of which the elements represent susceptance of candidate branches; in p.u.,
$\mathbf{M}$	Diagonal matrix in $\mathbb{R}^{nc\times nc}$ of which the elements represent disjunctive parameters,
$\mathbf{T}$	Candidate path-branch relational matrix in $\mathbb{R}^{np\times nc}$,
$\mathbf{N}$	Constraint matrix representing the stringing of additional circuits on the existing towers,
$\mathbf{E}^{(t)}$	Matrix in $\mathbb{R}^{nc\times(nc.ns)}$ used to transform the single stage parameter to the multistage parameter,
$\mathbf{F}$	Constraint matrix of candidate branch in $\mathbb{R}^{nw\times nc}$ concerning with the investment variable,
$\mathbf{D}$	Constraint matrix of existing branch in $\mathbb{R}^{nr\times m}$ concerning with the operation variable,
$\mathbf{G}$	Constraint matrix of candidate branch in $\mathbb{R}^{nw\times m}$ concerning with the operation variable,
$\mathbf{H_a}$	Constraint matrix in $\mathbb{R}^{nc\times ma}$ of TEP problem using AC model,
$\mathbf{H_{fa}}$	Constraint matrix in $\mathbb{R}^{nc\times mf}$ concerning with transmission expansion plan of TEP problem with UPFC application,
$\mathbf{H_{ff}}$	Constraint matrix in $\mathbb{R}^{nf\times mf}$ concerning with UPFC installation of TEP problem with UPFC application,
$\mathbf{R_q}$	Diagonal matrix in $\mathbb{R}^{nb\times nb}$ of which the elements represent the ratios between reactive and active power demand,
ng	Number of generators,
nb	Number of buses,
ne	Number of existing branches,
nc	Number of candidate branches,
np	Number of paths,
nf	Number of UPFCs,
nfe	Number of UPFCs connected to existing lines,
nfc	Number of UPFCs connected to candidate lines,

ns	Number of stages,
nv	Number of scenarios,
ny	Number of years for each stage,
nl	Expected life time of transmission system equipments; in year,
nr	Number of constraints related to the existing branches of TEP problem using DC model,
nw	Number of constraints related to the candidate branches of TEP problem using DC model,
m	Number of variables in operation problem formulated by using DC model,
ma	Number of variables in operation problem formulated by using AC model,
mf	Number of variables in operation problem of TEP with UPFC application,
neq	Number of equality constraints of TEP problem using AC model,
nin	Number of inequality constraints of TEP problem using AC model,
r	Interest rate; in % per year,
g	Demand growth rate; in % per year,
$c_{\text{inv},t}$	Investment cost of stage t; in US$,
$c_{\text{opr},t}$	Operating cost for the representative year of stage t; in US$,
sv_t	Salvage value of the transmission equipment installed at stage t; in US$,
$pv_{\text{inv},t}$	Present value of investment cost of stage t less the corresponding salvage value; in US$,
$pv_{\text{op},t}$	Present value of operating cost of stage t; in US$,
IVF,t	Factor for converting the investment cost of stage t to the present value,
OPF,t	Factor for converting the operating cost of stage t to the present value,
$c_{\text{op,min}}$	Minimum generation cost without consideration of transmission constraints; in US$,
σ_{min}	Minimum eigenvalue of reduced Jacobian matrix,
VS_{lim}	Voltage stability margin,
k	Iteration counter,
α	Parameter of cut modification, and

ϵ Tolerance of termination criterion.

Variables

$\mathbf{x}$ Investment variable in $\{0,1\}^{nc}$ (binary variable) representing a decision on the selection of candidate branches into the investment plan, i.e. $x_l = 1$ if the candidate branch l is selected,

$\mathbf{x_f}$ Investment variable in $\{0,1\}^{nf}$ representing a decision on the installation of UPFCs,

$\mathbf{y}$ Operation variable in $\mathbb{R}^m$ of TEP problem using DC model,

$\mathbf{y_a}$ Operation variable in $\mathbb{R}^{ma}$ of TEP problem using AC model,

$\mathbf{y_f}$ Operation variable in $\mathbb{R}^{mf}$ of TEP problem with UPFC application,

$\mathbf{p_g}$ Variable in $\mathbb{R}^{ng}$ representing active power generation; in p.u.,

$\mathbf{q_g}$ Variable in $\mathbb{R}^{ng}$ representing reactive power generation; in p.u.,

$\mathbf{v}$ Variable in $\mathbb{R}^{nb}$ representing voltage magnitude; in p.u.,

$\boldsymbol{\delta}$ Variable in $\mathbb{R}^{nb}$ representing voltage angle; in radian,

$\mathbf{d_r}, \mathbf{d_c}$ Variables in $\mathbb{R}^{nb}$ representing the capacities of installed reactors and capacitors; in p.u.,

$\mathbf{p_s}$ Variable in $\mathbb{R}^{nb}$ representing active power demand shedding; in p.u.,

$\mathbf{u}$ Variable in $\mathbb{R}^{nc}$ representing dummy variables according to the investment plan obtained from solving the investment problem,

$\mathbf{u_f}$ Variable in $\mathbb{R}^{nf}$ representing dummy variables according to the UPFC installation obtained from solving the investment problem,

$\mathbf{p_{be}}$ Vector in $\mathbb{R}^{ne}$ representing active power flow in existing branches; in p.u.,

$\mathbf{p_{bc}}$ Vector in $\mathbb{R}^{nc}$ representing active power flow in candidate branches; in p.u.,

$\boldsymbol{\lambda}_\mathbf{r}$ Vector in $\mathbb{R}^{nr}$ representing Lagrange multipliers corresponding to the constraints of existing branches of operation problem formulated by using DC model,

$\boldsymbol{\lambda}_\mathbf{w}$ Vector in $\mathbb{R}^{nw}$ representing Lagrange multipliers corresponding to the constraints of candidate branches of operation problem formulated by using DC model,

$\boldsymbol{\mu}_\mathbf{r}$ Vector in $\mathbb{R}^{nr}$ representing Lagrange multipliers corresponding to the constraints of existing branches of feasibility problem formulated by using DC model,

$\boldsymbol{\mu}_{\mathbf{w}}$ Vector in $\mathbb{R}^{nw}$ representing Lagrange multipliers corresponding to the constraints of candidate branches of feasibility problem formulated by using DC model,

$\boldsymbol{\lambda}_{\mathbf{H}}$ Vector in $\mathbb{R}^{nc}$ representing Lagrange multipliers corresponding to the linear constraints of operation problem formulated by using AC model,

$\boldsymbol{\lambda}_{\mathbf{G}}$ Vector in $\mathbb{R}^{nc}$ representing Lagrange multipliers corresponding to the nonlinear constraints of operation problem formulated by using AC model,

$\boldsymbol{\lambda}_{\mathbf{Ha}}$ Vector in $\mathbb{R}^{nc}$ representing Lagrange multipliers corresponding to the linear constraints, concerning with investment plan, of operation problem of TEP with UPFC application,

$\boldsymbol{\lambda}_{\mathbf{Hf}}$ Vector in $\mathbb{R}^{nf}$ representing Lagrange multipliers corresponding to the linear constraints, concerning with UPFC installation, of operation problem of TEP with UPFC application,

$\boldsymbol{\mu}_{\mathbf{H}}$ Vector in $\mathbb{R}^{nc}$ representing Lagrange multipliers corresponding to the linear constraints of feasibility problem formulated by using AC model,

$\boldsymbol{\mu}_{\mathbf{Ge}}$ Vector in $\mathbb{R}^{nc}$ representing Lagrange multipliers corresponding to the nonlinear equality constraints of feasibility problem formulated by using AC model,

$\boldsymbol{\mu}_{\mathbf{Gi}}$ Vector in $\mathbb{R}^{nc}$ representing Lagrange multipliers corresponding to the nonlinear inequality constraints of feasibility problem formulated by using AC model,

$\boldsymbol{\mu}_{\mathbf{Ha}}$ Vector in $\mathbb{R}^{nc}$ representing Lagrange multipliers corresponding to the linear constraints, concerning with investment plan, of feasibility problem of TEP with UPFC application, and

$\boldsymbol{\mu}_{\mathbf{Hf}}$ Vector in $\mathbb{R}^{nf}$ representing Lagrange multipliers corresponding to the linear constraints, concerning with UPFC installation, of feasibility problem of TEP with UPFC application.

Functions

$\mathbf{P_{inj}}$ Function of active power injected into buses, returning value in $\mathbb{R}^{nb}$; in p.u.,

$\mathbf{Q_{inj}}$ Function of reactive power injected into buses, returning value in $\mathbb{R}^{nb}$; in p.u.,

$\mathbf{p_{bef}}, \mathbf{p_{bet}}$ Functions of active power flowing through existing branches regarding as ***from bus*** and ***to bus*** respectively, returning value in $\mathbb{R}^{ne}$; in p.u.,

$\mathbf{p_{bcf}}, \mathbf{p_{bct}}$ Functions of active power flowing through candidate branches regarding as ***from bus*** and ***to bus*** respectively, returning value in $\mathbb{R}^{nc}$; in p.u.,

$\mathbf{q_{bef}}, \mathbf{q_{bet}}$ Functions of reactive power flowing through existing branches regarding as ***from bus*** and ***to bus*** respectively, returning value in $\mathbb{R}^{ne}$; in p.u.,

$\mathbf{q_{bcf}}, \mathbf{q_{bct}}$ Functions of reactive power flowing through candidate branches regarding as ***from bus*** and ***to bus*** respectively, returning value in $\mathbb{R}^{nc}$; in p.u.,

$\mathbf{i_{bef}}, \mathbf{i_{bet}}$ Functions of current flowing through existing branches regarding as ***from bus*** and ***to bus*** respectively, returning value in $\mathbb{R}^{ne}$; in p.u.,

$\mathbf{i_{bcf}}, \mathbf{i_{bct}}$ Functions of current flowing through candidate branches regarding as ***from bus*** and ***to bus*** respectively, returning value in $\mathbb{R}^{nc}$; in p.u.,

$\mathbf{i_{se}}, \mathbf{i_{sh}}$ Functions of current flowing through series and shunt converters, returning value in $\mathbb{R}^{nf}$; in p.u.,

$\mathbf{p_{se}}, \mathbf{p_{sh}}$ Functions of active power flowing through series and shunt converters, returning value in $\mathbb{R}^{nf}$; in p.u.,

$\mathbf{q_{cmp}}$ Functions of reactive power supplied from reactive power compensation devices, returning value in $\mathbb{R}^{nb}$; in p.u.,

$\mathbf{G_{eq}}$ Equality constraint of TEP problem using AC model, returning value in $\mathbb{R}^{neq}$,

$\mathbf{G_{in}}$ Inequality constraint of TEP problem using AC model, returning value in $\mathbb{R}^{nin}$,

$\mathbf{G_{a}}$ Constraint of TEP problem using AC model, returning value in $\mathbb{R}^{2neq+nin}$,

L_{ac} Lagrange function of the operation problem formulated by using AC model, and

L_{fe} Lagrange function of the feasibility problem formulated by using AC model.

Symbols

$(\,.\,)^{(t)}$ Constants or variables (.) of stage t,

$(\,.\,)^{(s)}$ Constants or variables (.) of scenario s,

$(\,.\,)^{(t,s)}$ Constants or variables (.) of stage t, scenario s, and

$(\,.\,)^{(t,s,k)}$ Constants or variables (.) of stage t, scenario s in iteration k.

Abbreviations

AC	Alternating current,
BD	Benders decomposition,
CHA	Constructive heuristic algorithm,
DC	Direct current,
FACTS	Flexible AC transmission systems,
GA	Genetic algorithm,
GBD	Generalized Benders decomposition,
KCL	Kirchhoff's current law,
KVL	Kirchhoff's voltage law,
LP	Linear programming,
MILP	Mixed integer linear programming,
MINLP	Mixed integer nonlinear programming,
MIP	Mixed integer programming,
NLP	Nonlinear programming,
NP	Nondeterministic polynomial,
TEP	Transmission expansion planning, and
UPFC	Unified power flow controller.

Problems

STEP-DC	Single stage TEP problem using DC model without security constraint,
STEP-DC-NSEC	Single stage TEP problem using DC model with N-1 security constraints,
STEP-AC	Single stage TEP problem using AC model without security constraint,
MTEP-DC	Multistage TEP problem using DC model without security constraint,

MTEP-AC-NSEC Multistage TEP problem using AC model with N-1 security constraints,

MTEP-AC-NSEC-VSTAB Multistage TEP problem using AC model with N-1 security and voltage stability constraints, and

STEP-UPFC Single stage TEP problem using AC model with UPFC application.

CHAPTER I

INTRODUCTION

Transmission expansion planning (TEP) is a process of determining an optimal transmission expansion plan which ensures electricity demand can be served throughout a planning period [**1**]. In general, system planners conduct in connection with generation expansion planning to serve the increase of demand. An obtained plan from the TEP process is generally a minimum cost plan complying with defined planning criteria.

A conventional method for solving the TEP problem is normally based on the comparison of alternative plans. A set of alternatives of the transmission plans covering a defined planning period is generally chosen from a feasible solution space based on experience of planner with the aid of power system analysis tools. Then, the least cost plan is selected by comparing the cost streams in the planning period [**2**–**4**]. The advantage of this method is that it is easy to implement whilst the results can be acceptable in case the power system is not too much complex. However, in the case of an actual large scale power system, the feasible solution space may be extremely large. Therefore, it is a very difficult task to chose the best alternative set from the solution space by the planners.

Consequently, from the theoretical point of view, the simplified conventional method may not be appropriate for solving the TEP. Mathematically, TEP is a mixed integer programming (MIP) of which the integer variables represent the decision on the selection of new transmission lines and transformers into the plan. The constraints can be divided into two categories. The first one consists of planning criteria depending on operation limits, e.g. generation limits, thermal limits, voltage magnitude limits, etc. The second one consists of the constraints according to electrical circuit theory, i.e. Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL). From the computational point of view, the MIP is in the class of nondeterministic polynomial (NP) problems [**5**].

Several research in solving the TEP aim to overcome the computational difficulty. There are two aspects of the TEP problem which can be noticed from the previous research, comprising algorithm development and problem formulation complexity.

From the algorithm development aspect, the methods for solving TEP problem can be classified into three categories [**6**], i.e. mathematics, heuristics and metaheuristics. The mathematical based methods [**7**–**11**] rigorously solve the problem by applying optimization techniques which are generally based on branch and bound algorithms [**5**, **12**, **13**]. Employing the mathematical based methods to solve the TEP in an unsophisticated manner usually encounters the problem about computational burden, especially in case of large scale problems. With application of decomposition based techniques, e.g. Benders decomposition [**14**], outer approximation [**15**], computational burden can be considerably reduced as shown for an example in Ref. [**11**]. It should be noted that as long as the model is convex, the mathematical based methods, i.e. the general branch and bound and decomposition based methods, can return a global optimal solution. However, in case the model is nonconvex, the global optimality cannot be guaranteed [**16**]. Several methods have been developed to improve the solution quality. In some cases, the global optimality of the solution is guaranteed [**17**–**19**]. However, the computational burden is extremely high. Therefore, they can be applied to only small scale problems.

On the other hand, the heuristic based methods [**20**–**24**] utilize some guidelines for searching solutions. In general, they are derived from sensitivity indices regarding potential of candidates to alleviate the violation of operating limits. Even though the computational burden is low, the obtained solutions are usually trapped at local optima. The complexity of the heuristic methods may range from a simple greedy algorithm [**25**] to a more sophisticated method, e.g. local branching [**26**]. In general, the heuristic methods are appropriate for practical TEP problems of which only suboptimal solutions is sufficient for using in planning activities. The metaheuristic methods [**27**] employ intelligent search techniques. The computation time is usually high and, in some cases, may be higher than the mathematical based methods. In fact, the metaheuristic methods are appropriate for nonconvex problems since they have mechanisms to escape the local optima. Nevertheless, the solutions are not guaranteed to be a global optimum. Examples of the metaheuristic methods are genetic algorithm (GA) [**28**, **29**], tabu search [**30**], simulated annealing [**31**], etc. From all the above methods, it should be noted that there is no best method which is suitable for all types of the TEP problems. Selection of a suitable method is mainly based on size of the problem as well as the problem formulation complexity.

The formulation of TEP problem should be reflected actual practices. However, due to computational performance of the current developed algorithms, compromise between computational time and the problem formulation complexity is inevitable. In general, the problem formulation complexity of the TEP can be classified into three main issues as follows:

(a) model used in the power flow equation, i.e. DC or AC model,

(b) the number of stages in planning period, and

(c) planning criteria taken into account, e.g. transmission line thermal limits, voltage magnitude limits, N-1 security constraints, etc.

In case of the power flow equations, most of research works apply a DC model [**32**] rather than an AC model, since the TEP with the AC model is a mixed integer nonlinear programming (MINLP) which is a very complicated problem. However, the problem becomes simpler if the DC model is applied instead. For a basic DC model [**33**], there are multiplication terms between the integer variables and voltage angle variables, resulting in an MINLP problem. There are various models developed based on the basic DC model aiming to transform the problem to a mixed integer linear programming (MILP) which is easier to be solved than the MINLP, i.e. transportation [**10**], hybrid [**32**], and disjunctive models [**32**, **34**]. In general, the transportation and hybrid models are rough models since the KVL constraints are neglected in the problem formulation. On contrary, all the constraints of DC power flow are taken into account in the disjunctive model.

In practice, solving the TEP based on the DC model may not be acceptable for transmission system planning activity of electric utilities. With the DC model, an investment plan solution may be easily obtained. However, it has to be revised by the planner before making decision for the final plan with following reasons.

(a) The obtained investment plan may be infeasible since the KVL constraints are relaxed with the linear equations.

(b) Voltage magnitude limits and voltage stability constraints are not taken into account.

(c) Power loss cannot be evaluated.

(d) Installation plan of reactive power compensation device cannot be directly obtained.

(e) Benefit of FACTS device installation cannot be clearly shown.

Currently, there are fairly limited number of TEP research based on an AC model [**35**,**36**]. A constructive heuristic algorithm (CHA) is employed in Ref. [**35**] by starting with an infeasible solution, and continuously adding candidate branches into a plan, based on sensitivity indices, until the updated solution is feasible. As stated in Ref. [**35**], the CHA does not always find the optimal expansion plan. In Ref. [**36**], GA is applied to solve TEP using an AC model. However, it is known that the computational burden of GA is very high especially for a large scale TEP problem. Therefore, a trade-off between the computational time and the solution quality has to be considered.

Considering the number of stages in planning period, one can classify TEP into two categories [**6**, **37**], i.e. single stage planning and multistage planning. In the single stage planning [**32**], the planning interval is considered as a single period of time, i.e. there is only one stage in the planning period. Therefore, all of transmission lines under the investment plan are assumed to be constructed at the same time, i.e. at the beginning of the considered period. Generally, the problem concerns only where to construct new candidate lines. Consequently, the single stage planning may not be appropriate for a long term TEP according to economical aspect, since it does not take into account the time value of the money. In case of the multistage planning [**37–39**], the planning period is divided into several stages, and each stage has its owned corresponding plan. The multistage planning problem concerns with questions about when and where to construct new transmission lines. The plan in the current stage depends on the plans in the previous stages. Therefore, the multistage planning is much more complicate than the single stage planning.

From the aspect of planning criteria, there are various types of the criteria which can be incorporated into the TEP problem. In general, they are treated as additional sets of constraints which result in a more complicated problem. For example, a system consists of N branches, therefore the size of the TEP problem with consideration of N-1 security is at least increased to $N+1$ times from the size of the TEP problem without consideration of N-1 security. Therefore, most of the research on TEP usually consider only basic planning criteria , i.e. transmission line thermal limits, generation limits, without taking into account contingency analysis. There are fairly limited number of research works which take the N-1 security constraints into account. Examples of the previous works can be found in Refs. [**40–42**]. In Ref. [**40**], security cuts are iteratively added to a basic model which does not include security constraints. The concept of cuts generation is based on a transportation model. Therefore the obtained solution may not comply with Kirchhoff's voltage law (KVL). In Ref. [**41**], Chu-Beasley GA is employed to demonstrate its application on various sizes of test systems. Ref. [**42**] proposes a modified heuristic method which is normally used to solve a linear programming problem. However, it should be noted that the problem formulation of TEP with N-1 security in Refs. [**40**–**42**] are based on DC model.

From the above-mentioned characteristics of the TEP problem, one can summarize the complication of the TEP problem for each issue in the problem formulation as below.

(a) **Model used in the power flow equation** : The TEP problem based AC model is a very complicated problem, since both integer programming and nonlinear programming are involved in the main problem.

(b) **The number of stages in the planning period** : The number of integer variables and constraints of the TEP problem will be linearly increased with the number of stages.

(c) **N-1 security** : The number of constraints of the TEP problem will be linearly increased with the number of branches in the system.

1.1 Problem Statement

To simultaneously cope with all the above issues, i.e. the problem formulation based on AC model, the multistage planning, and the N-1 security constraints, of a large scale TEP problem has never been addressed in previous research works. Since the formulated problem is very complicated, employing general algorithms to solve the problem always encounters the computational difficulty. Consequently, a sophisticate method for solving this kind of problem will be developed in this dissertation.

1.2 Contribution

Transmission expansion plan also has impact on both electricity cost and system security. In general planning, a criteria has to be defined. The selected transmission plan has to comply with those criteria. Regarding electricity cost, it is mainly affected by the investment cost and the operating cost. The investment cost obviously depends on the expansion plan, whereas the operating cost is affected by the power loss and the transmission congestion which are actually effected by the expansion plan. Consequently, inappropriate selection of the transmission plan may lead to an overinvestment causing high electricity tariff. This problem motivates the development of methods to find the optimal transmission expansion plan in a considered planning period.

This dissertation proposes a method for solving the multistage TEP based on AC model with security constraints, aiming to answer the aforementioned problem for large scale power systems. Consequently, the developed method can be applied to actual transmission expansion planning activities.

1.3 Scope of Work and Limitations

The details of scope and limitation of the dissertation are listed below.

(a) Develop a method of which the framework is based on the generalized Benders decomposition (GBD) [**43**] for solving the multistage TEP based on AC model with security constraints. The main TEP problem is divided into investment and operation problems. The investment problem is MILP concerning with the process for searching the investment plan whereas the operation problem formulation depends on the model used in the power flow equation, planning stages, and defined planning criteria.

(b) Illustrate the benefit of TEP using AC model in case of voltage stability constraint and FACTS device installation.

(c) In the case of the formulation based on the AC model, the global optimality of solution is not guaranteed. However, the developed method attempts to find a good quality local optimum solution.

(d) Only construction costs of transmission lines are treated as nonlinear functions with respect to the number of circuits. For the generation costs, the linearities of the cost functions are assumed.

(e) Only deterministic analysis is taken into account in this dissertation.

1.4 Dissertation Outline

The dissertation is composed of seven chapters. In the next chapter, the basic concept of TEP and mathematical background will be provided. Chapter 3 presents various formulations of the TEP problems and the solving methodologies. In Chapter 4, supplementary methods are introduced in order to improve the performance of the proposed methods and the solution quality. The next chapter illustrates the further application of the proposed methods in voltage stability constraint and FACTS device installation. Then, chapter 6 shows the test results of the proposed methods. Finally, Chapter 7 draws conclusions and future extension for this dissertation.

CHAPTER II

BASIC BACKGROUNDS

In this chapter, basic concept of the TEP and mathematical background are reviewed. In the first section, a conventional method for solving the TEP problem, covering single stage and multistage TEP will be introduced. Then mathematical backgrounds, i.e. Benders decomposition and generalized Benders decomposition, used in this dissertation will be presented.

2.1 Basic Concept of TEP

2.1.1 Conventional Method for Solving TEP

TEP is generally conducted based on experiences of system planners. The method is generally based on least cost solution techniques [**2**–**4**]. A set of alternatives of long term expansion plans in the planning period is chosen from the set of all feasible plans. The number of alternatives should be reasonable for manual implementation in the next steps. The computational tools employed in the process is only power system analysis softwares based on the Newton-Raphson algorithm [**44**] for solving a set of nonlinear power flow equations. Then an alternative plan is chosen by planners based on the experience and results from power flow solutions. The process is performed in an iterative manner. By starting from the base case of the considered scenario, if the system is not feasible, i.e. planning criteria are not satisfied, candidate transmissions or transformers are selected into the considered plan. The power flow equation is solved again to verify the feasibility. The process is performed until the candidates provides the feasibility.

It should be noted that the number of considered scenarios may be reasonably high in practice. Since each scenario is connected with corresponding conditions of the system for each stage in planning period, the number of scenarios will increase if there are many stages in the planning period. In addition, if the N-1 security constraints are taken into account, the planners have to analyze the scenario for each contingency. In this case, the contingency selection technique [**1**,**45**] may be applied to reduce the number of the considered scenarios. The process for selection of the alternative plans is illustrated in Figure 2.1

After feasible alternative plans are obtained, their cost streams throughout the planning period are compared with each other. Time value of the money should be taken into account according to economic aspect. Then the least cost plan is selected.

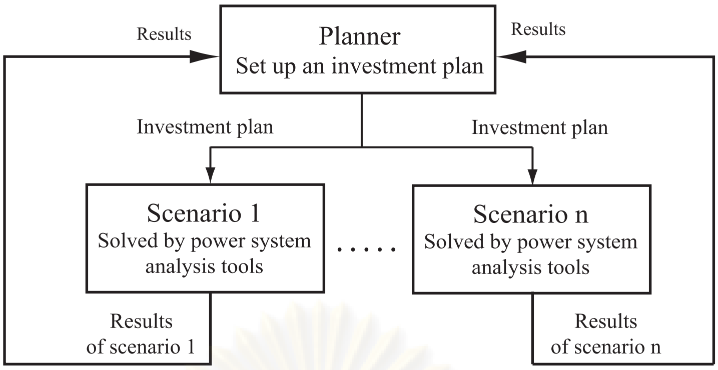

Figure 2.1 Conventional method for solving TEP

2.1.2 Single Stage TEP Problem Using Disjunctive Model

The single stage TEP can be firstly formulated by using a disjunctive model [**11**, **34**] of which the power flow equations are based on a DC power flow as follows:

$$\min \ \mathbf{c}_{\mathbf{b}}^{\mathrm{T}}\mathbf{x} \tag{2.1}$$

subject to

$$\mathbf{A}_{\mathbf{g}}^{\mathrm{T}}\mathbf{p}_{\mathbf{g}} - \mathbf{A}_{\mathbf{be}}^{\mathrm{T}}\mathbf{p}_{\mathbf{be}} - \mathbf{A}_{\mathbf{bc}}^{\mathrm{T}}\mathbf{p}_{\mathbf{bc}} = \mathbf{p}_{\mathbf{d}} \tag{2.2}$$

$$\mathbf{p}_{\mathbf{be}} - \mathbf{B}_{\mathbf{e}}\mathbf{A}_{\mathbf{be}}\boldsymbol{\delta} = \mathbf{0} \tag{2.3}$$

$$-\mathbf{M}\left(\mathbf{e}_{nc} - \mathbf{x}\right) \leq \mathbf{p}_{\mathbf{bc}} - \mathbf{B}_{\mathbf{c}}\mathbf{A}_{\mathbf{bc}}\boldsymbol{\delta} \leq \mathbf{M}\left(\mathbf{e}_{nc} - \mathbf{x}\right) \tag{2.4}$$

$$\mathbf{p}_{\mathbf{g}}^{\mathbf{min}} \leq \mathbf{p}_{\mathbf{g}} \leq \mathbf{p}_{\mathbf{g}}^{\mathbf{max}} \tag{2.5}$$

$$-\mathbf{p}_{\mathbf{be}}^{\mathbf{max}} \leq \mathbf{p}_{\mathbf{be}} \leq \mathbf{p}_{\mathbf{be}}^{\mathbf{max}} \tag{2.6}$$

$$-\mathbf{P}_{\mathbf{bc}}^{\mathbf{max}}\mathbf{x} \leq \mathbf{p}_{\mathbf{bc}} \leq \mathbf{P}_{\mathbf{bc}}^{\mathbf{max}}\mathbf{x} \tag{2.7}$$

$$\mathbf{x} \in \{0, 1\}^{nc}, \quad \boldsymbol{\delta} \in \mathbb{R}^{nb}, \quad \mathbf{p}_{\mathbf{g}} \in \mathbb{R}^{ng}, \quad \mathbf{p}_{\mathbf{be}} \in \mathbb{R}^{ne}, \quad \mathbf{p}_{\mathbf{bc}} \in \mathbb{R}^{nc}$$

It is clearly seen that the objective function shown by (2.1) concerns with the investment cost only. All the constraints in the described formulation comply with the DC power flow model as well as operation limits. Considering the constraints (2.4) and (2.7), one can express the constraints corresponding to the candidate branch l which directly connects from bus i to bus j as follows:

$$|p_l - B_l(\delta_i - \delta_j)| \leq M_l(1 - x_l) \tag{2.8}$$

$$|p_l| \leq p_l^{\max} x_l \tag{2.9}$$

It should be noticed that if the candidate branch l is selected for the plan, i.e. $x_l = 1$, the constraint (2.8) will comply with the KVL. On the other hand, if the candidate branch l is not selected, i.e. $x_l = 0$, p_l will be zero by the constraint (2.9). In addition, δ_i and δ_j are not controlled by the constraint (2.8) due to the large value of M_l. The proper value of M_l can be determined by the method presented in [**11**].

The advantage of the disjunctive model over the other models is that the formulated problem from the disjunctive model is MILP which is easy to be solved by current existing solvers [**46**,**47**]. In addition, the disjunctive model can handle the nonlinearity of the investment cost with respect to the number of circuits as will be described in the next chapter.

2.1.3 Multistage TEP

In general, a multistage TEP can be considered as a sequence of the single stage TEPs. The planning period and the investment plan corresponding to each stage of the multistage TEP is demonstrated in Figure 2.2. In this figure, it is assumed that the planning period is nine years, which is divided into three stages. The plan is carried out at the beginning of each stage in order that the increased demand in the corresponding stage can be served. The plans obtained from the previous stages have to be perceived at the current stage. It is noted that the demand used in the problem formulation for each stage can be determined from the forecasted peak value over the stage.

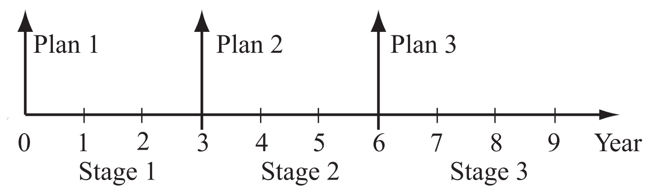

Figure 2.2 Planning period of multistage TEP

Formulation of the multistage TEP can be found in [**37**–**39**]. Generally, the formulation of the multistage TEP is based on the formulation of the single stage TEP. The derived multistage model will be inherited all properties and efficiencies from the single stage model. Consequently, formulating the multistage TEP based on the basic DC model using the integer decision variables [**38**] will cause the problem to become MINLP. On the other hand, the problem will be MILP, if the disjunctive model is applied to the formulation of the multistage TEP problem [**39**].

2.1.4 Concept of Cost in Planning Period

In general, the concept presented in this subsection can be adopted for both single stage and multistage TEP problems. However, to generalize this concept, the case of multistage TEP problem is explained.

The cash flow diagrams of the investment and operating costs in the planning period are shown in Figures 2.3(a) and 2.3(b)

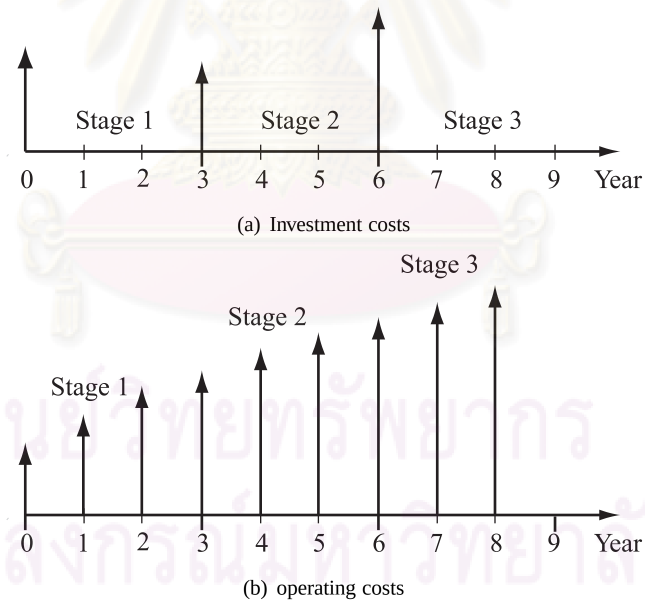

Figure 2.3 Cash flow diagram of costs in planning period

In the case of the investment cost, since expected life time of the transmission system equipments installed in each stage is usually longer than the considered planning period, salvage values of these equipments should be taken into account at the end of the planning period to reflect the utilization of the equipments as shown in Figure 2.4.

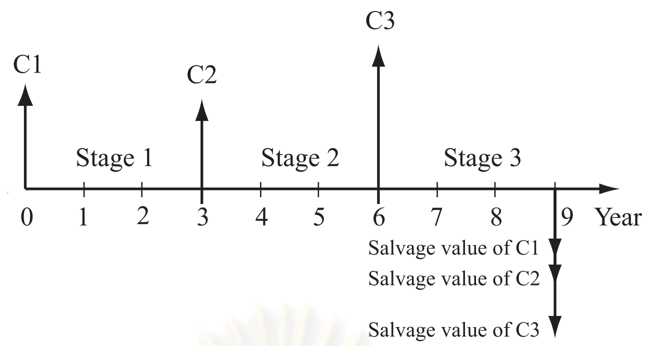

Figure 2.4 Cash flow diagram of investment costs with salvage values

The salvage value at the end of period can be estimated by a straight line method [**4**]. It is assumed that the value of the equipment is zero when it was operated until its life time. Therefore, the salvage value of the equipment installed at stage t, can be calculated by (2.10).

$$sv_t = c_{\text{inv},t}\left(\frac{nl - ny\,(ns - t + 1)}{nl}\right) \tag{2.10}$$

Therefore, the present value of the investment cost of stage t less its salvage value can be calculated by (2.11).

$$pv_{\text{inv},t} = c_{\text{inv},t}\left(\frac{1}{(1+r)^{ny(t-1)}} - \frac{nl - ny\,(ns - t + 1)}{nl\,(1+r)^{ny.ns}}\right) \tag{2.11}$$

In the case of the operating cost, it is assumed that the cost for each year increases by the same rate as the demand growth. In addition, if the demand monotonously increases over the planning period, the peak demand of the last year for each stage will be used as a representative value of the demand of that stage. Therefore, the present value of the operating cost for stage t can be calculated by (2.12).

$$\begin{aligned} pv_{\text{op},t} &= \frac{c_{\text{opr},t}}{(1+r)^{ny(t-1)}\,(1+g)^{(ny-1)}}\left(1 + \left(\frac{1+g}{1+r}\right) + \ldots + \left(\frac{1+g}{1+r}\right)^{ny-1}\right) \\ &= c_{\text{opr},t}\frac{(1+r)^{ny} - (1+g)^{ny}}{(r-g)\,(1+g)^{(ny-1)}\,(1+r)^{ny.t-1}} \end{aligned} \tag{2.12}$$

Now one can define factors, IVF_t to convert the investment cost for stage t to its present value taking into account the salvage value at horizon year, and OPF_t to convert the operating cost of the representative year of stage t, i.e. the year when the peak demand occurs in that stage, to the present value of opeartion cost for stage t as described by (2.13) and (2.14).

$$IVF_t = \frac{1}{(1+r)^{ny.(t-1)}} - \frac{nl - ny\,(ns - t + 1)}{nl\,(1+r)^{ny.ns}} \tag{2.13}$$

$$OPF_t = \begin{cases} \dfrac{(1+r)^{ny} - (1+g)^{ny}}{(r-g)\,(1+g)^{(ny-1)}\,(1+r)^{ny.t-1}} & , r \neq g \\ \dfrac{ny}{(1+g)^{(ny-1)}\,(1+r)^{ny(t-1)}} & , r = g \end{cases} \tag{2.14}$$

2.2 Mathematical Backgrounds

Since the generalized Benders decomposition (GBD) is used as a key framework of the decomposition based method proposed in this dissertation, two essential mathematical decomposition methods will be presented in this section. Initially, Benders decomposition (BD) which is the preliminary version of the GBD usually applied to MILP solving is introduced. Then the basic concept of GBD is presented. It should be noted that the notations of constants, variables and sets are defined separately for describing the backgrounds in this section only. There is no meaning related to the notation used in the other sections.

2.2.1 Benders Decomposition

Benders decomposition (BD) was proposed by J. F. Benders [**14**]. It is appropriate for solving the problem composed of two complicated subproblems. In this section, the BD is introduced with the application of solving an MILP.

Considering an MILP problem in a particular form as (2.15)–(2.17).

$$\min \left(\mathbf{c_1^T x_1} + \mathbf{c_2^T x_2} \right) \tag{2.15}$$

subject to

$$\mathbf{A_1 x_1} \leq \mathbf{b_1} \tag{2.16}$$

$$\mathbf{B_1 x_1} + \mathbf{B_2 x_2} \leq \mathbf{b_2} \tag{2.17}$$

$$\mathbf{x_1} \in \mathbb{Z}^{n1} \qquad , \mathbf{x_2} \in \mathbb{R}^{n2}$$

where $\mathbf{c_1} \in \mathbb{R}^{n1}$, $\mathbf{c_2} \in \mathbb{R}^{n2}$, $\mathbf{A_1} \in \mathbb{R}^{m1 \times n1}$, $\mathbf{B_1} \in \mathbb{R}^{m2 \times n1}$, $\mathbf{B_2} \in \mathbb{R}^{m2 \times n2}$, $\mathbf{b_1} \in \mathbb{R}^{m1}$, and $\mathbf{b_2} \in \mathbb{R}^{m2}$.

The above problem can be expressed in the equivalent form as (2.18)–(2.21).

$$\min z \tag{2.18}$$

subject to

$$-z + \mathbf{c_1^T x_1} + \mathbf{c_2^T x_2} \leq 0 \tag{2.19}$$

$$\mathbf{A_1 x_1} \leq \mathbf{b_1} \tag{2.20}$$

$$\mathbf{B_1 x_1} + \mathbf{B_2 x_2} \leq \mathbf{b_2} \tag{2.21}$$

$$\mathbf{x_1} \in \mathbb{Z}^{n1}, \qquad \mathbf{x_2} \in \mathbb{R}^{n2}, \quad z \in \mathbb{R}$$

In the process of BD, the problem is partitioned into two subproblems, i.e. master problem and slave problem. The master problem is an MILP concerning with the variables z and $\mathbf{x_1}$, while the slave problem is a linear programming (LP) dealing with the variables $\mathbf{x_2}$ only.

The master problem can be initially defined as (2.22) and (2.23).

Master Problem:

$$\min z \tag{2.22}$$

subject to

$$\mathbf{A_1 x_1} \leq \mathbf{b_1} \tag{2.23}$$

$$\mathbf{x_1} \in \mathbb{Z}^{n1}, z \in \mathbb{R}$$

After solving the master problem, if the problem is infeasible, it is implied that the main problem defined by (2.18)–(2.21) is also infeasible, otherwise z and $\mathbf{x}_1$ can be obtained. Supposing $(\bar{z}, \bar{\mathbf{x}}_\mathbf{1})$ is the minimizer of the master problem, one can find $\mathbf{x_2}$ by solving the slave problem defined as (2.24) and (2.25). In addition, the slave problem can also be defined in a dual form as (2.26) and (2.27).

<u>Slave Problem</u>: (Primal Form)

$$\min \mathbf{c_2^T x_2} \tag{2.24}$$

subject to

$$\mathbf{B_2 x_2} \leq \mathbf{b_2} - \mathbf{B_1 \bar{x}_1} \tag{2.25}$$
$$\mathbf{x_2} \in \mathbb{R}^{n2}$$

<u>Slave Problem</u>: (Dual Form)

$$\max \left(\mathbf{b_2} - \mathbf{B_1 \bar{x}_1}\right)^{\mathrm{T}} \boldsymbol{\lambda} \tag{2.26}$$

subject to

$$\mathbf{B_2^T} \boldsymbol{\lambda} = \mathbf{c_2} \tag{2.27}$$
$$\boldsymbol{\lambda} \leq \mathbf{0}$$
$$\boldsymbol{\lambda} \in \mathbb{R}^{m2}$$

It should be noted that after solving the slave problem, two conditions can take place as follows:

C1: The minimizer, $\mathbf{\bar{x}_2}$ of the primal slave problem can be found. In this case, the constraint (2.19) has to be verified. If the constraint (2.19) is satisfied in a situation where $-\bar{z}+\mathbf{c_1^T \bar{x}_1}+\mathbf{c_2^T \bar{x}_2} = 0$, the minimum solution of the main problem (2.15) is attained at $(\mathbf{\bar{x}_1}, \mathbf{\bar{x}_2})$, then the process is terminated. On the other hand, if the constraint (2.19) is not satisfied, i.e. $-\bar{z} + \mathbf{c_1^T \bar{x}_1} + \mathbf{c_2^T \bar{x}_2} > 0$, the constraint (2.28) is established and added into the master problem.

$$-z + \mathbf{c_1^T x_1} + \boldsymbol{\bar{\lambda}}^{\mathrm{T}} \mathbf{b_2} - \left(\boldsymbol{\bar{\lambda}}^{\mathrm{T}} \mathbf{B_1}\right) \mathbf{x_1} \leq 0 \tag{2.28}$$

where $\boldsymbol{\bar{\lambda}}$ is Lagrange multiplier corresponding to the the constraints (2.25) of the primal slave problem which also be directly determined by the minimizer of the dual slave problem.

C2: The primal slave problem is infeasible. This condition implies that the value of $\bar{\mathbf{x}}_\mathbf{1}$ is not suitable. Therefore, the constraint (2.25) is not satisfied. From the duality theory [**48**, **49**], the dual slave problem is unbounded. In addition, the direction of unbounded ray, $\bar{\boldsymbol{\mu}}$ can be employed in generating the constraint (2.29) which is subsequently added into the master problem.

$$\bar{\boldsymbol{\mu}}^{\mathrm{T}}\mathbf{b_2} - \left(\bar{\boldsymbol{\mu}}^{\mathrm{T}}\mathbf{B_1}\right)\mathbf{x_1} \leq 0 \tag{2.29}$$

After adding either (2.28) or (2.29) into the master problem, the master problem will be solved again. If the problem is infeasible, the main problem is also infeasible, otherwise the new minimizer, $(\bar{z},\, \bar{\mathbf{x}}_\mathbf{1})$ is obtained. With this minimizer, the slave problem will be redefined and solved again. The process is performed in this iterative manner until it is terminated.

At the beginning of the process, the value of $\bar{z}$, which is obtained from solving the master problem without additional constraints, always tends toward the negative infinity. This value should be increased when the constraint (2.28) is consecutively added into the master problem. During the process of BD, the value of $\bar{z}$ is always less than the value of $\mathbf{c}_\mathbf{1}^{\mathrm{T}}\bar{\mathbf{x}}_\mathbf{1} + \mathbf{c}_\mathbf{2}^{\mathrm{T}}\bar{\mathbf{x}}_\mathbf{2}$. Consequently, the value of $\bar{z}$ is called the lower bound, LBD_k, whereas the value of $\mathbf{c}_1^{\mathrm{T}}\bar{\mathbf{x}}_\mathbf{1} + \mathbf{c}_\mathbf{2}^{\mathrm{T}}\bar{\mathbf{x}}_\mathbf{2}$ is used to define the upper bound, UBD_k by (2.30).

$$UBD_k = \min\left\{UBD_{k-1},\, \mathbf{c}_1^{\mathrm{T}}\bar{\mathbf{x}}_1 + \mathbf{c}_2^{\mathrm{T}}\bar{\mathbf{x}}_2\right\} \tag{2.30}$$

where k is the iteration counter.

The process will be terminated when the gap between the upper bound and the lower bound is less than a defined tolerance, ϵ. Therefore, the termination criterion is defined as (2.31).

$$\left|\frac{UBD_k - LBD_k}{LBD_k}\right| < \epsilon \tag{2.31}$$

It should be noted that the constraints (2.16) and (2.17) are always satisfied with the current values of $\bar{\mathbf{x}}_\mathbf{1}$ and $\bar{\mathbf{x}}_\mathbf{2}$ on the condition C1. The constraint (2.28) is sequentially added into the master problem in order that the value of $\bar{\mathbf{x}}_\mathbf{1}$will be adjusted to the minimizer of the main problem, of which the objective function defined by (2.15), when the master problem is iteratively solved. For this reason, the constraint (2.28) will be called the optimality cut.

In case the constraint (2.29) is added to the master problem, it is aimed to prevent the infeasibility of the primal slave problem. Therefore, it will be called the feasibility cut.

2.2.2 Generalized Benders Decomposition

Since the slave problem in the BD procedure can be only an LP problem. A. M. Geoffrion proposed a generalized version of the BD, i.e. generalized Benders decomposition (GBD) [**43**, **50**, **51**], to handle the slave problem which can be a nonlinear programming (NLP) problem. Actually, the concept of GBD is similar to the one of BD. However, the method for establishing the optimality and feasibility cuts may be more complicated. In this subsection, the concept of GBD will be presented together with the application of the MINLP solving.

An MINLP problem to be solved by the GBD can be expressed as follows:

$$\min f(\mathbf{x_1}, \mathbf{x_2}) \tag{2.32}$$

subject to

$$\mathbf{H}\left(\mathbf{x_1}, \mathbf{x_2}\right) = \mathbf{0} \tag{2.33}$$

$$\mathbf{G}\left(\mathbf{x_1}, \mathbf{x_2}\right) \leq \mathbf{0} \tag{2.34}$$

$$\mathbf{x_1} \in \mathbb{Z}^{n1} \cap \mathcal{X}_1, \quad \mathcal{X}_1 \subset \mathbb{R}^{n1}$$

$$\mathbf{x_2} \in \mathcal{X}_2, \quad \mathcal{X}_2 \subset \mathbb{R}^{n2}$$

where

$f : \mathbb{R}^{n1} \times \mathbb{R}^{n2} \rightarrow \mathbb{R}$ and $\mathbf{G} : \mathbb{R}^{n1} \times \mathbb{R}^{n2} \rightarrow \mathbb{R}^{m}$ are convex functions when the variable $\mathbf{x_1}$ is fixed.

$\mathbf{H} : \mathbb{R}^{n1} \times \mathbb{R}^{n2} \rightarrow \mathbb{R}^{p}$ is a linear function when the variable $\mathbf{x_1}$ is fixed.

As the case of the BD, the master problem concerning with $\mathbf{x_1}$ can be initially defined as (2.35).

Master Problem:

$$\min z \tag{2.35}$$

$$\mathbf{x_1} \in \mathbb{Z}^{n1} \cap \mathcal{X}_1, \quad \mathcal{X}_1 \subset \mathbb{R}^{n1}, \quad z \in \mathbb{R}$$

It should be noticed that if the set $\mathcal{X}_1$ can be described by linear constraints, the master problem will be MILP problem. In addition, solving the master problem in the first iteration will return the value of z tending toward the negative infinity as well as any value of $\mathbf{x_1}$ in $\mathbb{Z}^{n1} \cap \mathcal{X}_1$.

After obtaining the minimizer $(\bar{z},\, \bar{\mathbf{x}}_\mathbf{1})$ from solving the master problem, the slave problem, which is parameterized by this minimizer, can be defined as (2.36)

Slave Problem:

$$\min f(\bar{\mathbf{x}}_\mathbf{1}, \mathbf{x_2}) \tag{2.36}$$

subject to

$$\mathbf{H}\left(\bar{\mathbf{x}}_\mathbf{1}, \mathbf{x_2}\right) = \mathbf{0} \tag{2.37}$$

$$\mathbf{G}\left(\bar{\mathbf{x}}_\mathbf{1}, \mathbf{x_2}\right) \leq \mathbf{0} \tag{2.38}$$

$$\mathbf{x_2} \in \mathcal{X}_2, \quad \mathcal{X}_2 \subset \mathbb{R}^{n2}$$

Two conditions may occur after solving the slave problem as follows:

C1: The problem is feasible, therefore the minimizer, $\bar{\mathbf{x}}_\mathbf{2}$ can be found. However it may not be the minimizer of the main problem defined by (2.32)–(2.34) since the slave problem is parameterized by $\bar{\mathbf{x}}_\mathbf{1}$ which may not also be the minimizer of the main problem. To verify whether $(\bar{\mathbf{x}}_\mathbf{1},\, \bar{\mathbf{x}}_\mathbf{2})$ is the minimizer of the main problem, the value of lower bound, LBD_k defined by $\bar{z}$ is compared with the value of upper bound, UBD_k defined by (2.39).

$$UBD_k = \min\left\{UBD_{k-1},\; f(\bar{\mathbf{x}}_\mathbf{1}, \bar{\mathbf{x}}_\mathbf{2})\right\} \tag{2.39}$$

If the lower bound is close to the upper bound, i.e. (2.31) is satisfied, $(\bar{\mathbf{x}}_\mathbf{1},\, \bar{\mathbf{x}}_\mathbf{2})$ will be the minimizer of the main problem, therefore the process is terminated, otherwise the the constraint called optimality cut defined in (2.40) is generated and added into the master problem. After that the master problem is solved again.

$$-z + \min_{\mathbf{x_2} \in \mathcal{X}_2} L_{\mathrm{s}}\left(\mathbf{x_1}, \mathbf{x_2}, \bar{\boldsymbol{\lambda}}_{\mathbf{l}}, \bar{\boldsymbol{\lambda}}_{\mathbf{n}}\right) \leq 0, \qquad \bar{\boldsymbol{\lambda}}_{\mathbf{n}} \geq \mathbf{0} \tag{2.40}$$

where $L_{\mathrm{s}}\left(\mathbf{x_1}, \mathbf{x_2}, \boldsymbol{\lambda}_{\mathbf{l}}, \boldsymbol{\lambda}_{\mathbf{n}}\right)$ is Lagrange function according to the slave problem defined by (2.41)

$$L_{\mathrm{s}}\left(\mathbf{x_1}, \mathbf{x_2}, \boldsymbol{\lambda}_{\mathbf{l}}, \boldsymbol{\lambda}_{\mathbf{n}}\right) = f\left(\mathbf{x_1}, \mathbf{x_2}\right) + \boldsymbol{\lambda}_{\mathbf{l}}^{\mathrm{T}} \mathbf{H}\left(\mathbf{x_1}, \mathbf{x_2}\right) + \boldsymbol{\lambda}_{\mathbf{n}}^{\mathrm{T}} \mathbf{G}\left(\mathbf{x_1}, \mathbf{x_2}\right) \tag{2.41}$$

and $\bar{\boldsymbol{\lambda}}_{\mathbf{l}}$, $\bar{\boldsymbol{\lambda}}_{\mathbf{n}}$ are the Lagrange multipliers according to (2.37) and (2.38) respectively. It should be noted that $\bar{\boldsymbol{\lambda}}_{\mathbf{l}}$, $\bar{\boldsymbol{\lambda}}_{\mathbf{n}}$ can be obtained from solving the slave problem. From the optimization theory [**50**, **51**], $\bar{\boldsymbol{\lambda}}_{\mathbf{n}}$ is always greater than or equal to zero.

C2: The problem is infeasible. Since the dual form a general NLP problem cannot be derived in an explicit form, this condition is treated by solving the feasibility problem defined by (2.42)–(2.45) instead of solving the dual slave problem to obtain the direction of unbounded ray as the process of the BD.

$$\min \mathbf{e}_m^{\mathrm{T}} \boldsymbol{\alpha} \tag{2.42}$$

subject to

$$\mathbf{H}\left(\bar{\mathbf{x}}_{\mathbf{1}}, \mathbf{x_2}\right) = \mathbf{0} \tag{2.43}$$

$$\mathbf{G}\left(\bar{\mathbf{x}}_{\mathbf{1}}, \mathbf{x_2}\right) \leq \boldsymbol{\alpha} \tag{2.44}$$

$$\boldsymbol{\alpha} \geq \mathbf{0} \tag{2.45}$$

$$\boldsymbol{\alpha} \in \mathbb{R}^m, \quad \mathbf{x_2} \in \mathcal{X}_2, \quad \mathcal{X}_2 \subset \mathbb{R}^{n2}$$

It should be noted that the above feasibility problem employs a 1-norm minimization of the constraint violations. Generally, the feasibility problem can be defined by other formations, e.g. ∞-norm minimization of the constraint violations, the formulations related to the physical aspect of the problem to which the GBD is applied.

After solving the feasibility problem, the obtained Lagrange multipliers corresponding to (2.43) and (2.44), i.e. $\bar{\boldsymbol{\lambda}}_{\mathbf{fl}}$ and $\bar{\boldsymbol{\lambda}}_{\mathbf{fn}}$, will be used in generating the feasibility cut defined by (2.46). Then the cut is added into the master problem before it is solved again.

$$\min_{\mathbf{x_2} \in \mathcal{X}_2} L_{\mathrm{f}}\left(\mathbf{x_1}, \mathbf{x_2}, \bar{\boldsymbol{\lambda}}_{\mathbf{fl}}, \bar{\boldsymbol{\lambda}}_{\mathbf{fn}}\right) \leq 0, \qquad \bar{\boldsymbol{\lambda}}_{\mathbf{fn}} \geq \mathbf{0} \tag{2.46}$$

where $L_{\mathrm{f}}\left(\mathbf{x_1}, \mathbf{x_2}, \boldsymbol{\lambda}_{\mathbf{fl}}, \boldsymbol{\lambda}_{\mathbf{fn}}\right)$ is defined as below.

$$L_{\mathrm{f}}\left(\mathbf{x_1}, \mathbf{x_2}, \boldsymbol{\lambda}_{\mathbf{fl}}, \boldsymbol{\lambda}_{\mathbf{fn}}\right) = \boldsymbol{\lambda}_{\mathbf{fl}}^{\mathrm{T}} \mathbf{H}\left(\mathbf{x_1}, \mathbf{x_2}\right) + \boldsymbol{\lambda}_{\mathbf{fn}}^{\mathrm{T}} \mathbf{G}\left(\mathbf{x_1}, \mathbf{x_2}\right) \tag{2.47}$$

The new minimizer obtained from solving the master problem will be used in redefining the slave problem, and the above-mentioned process is performed again until the lower bound, LBD_k is close to the upper bound, UBD_k which is verified by (2.31).

One can see that the terms $\min\limits_{\mathbf{x_2}} L_{\mathrm{s}}\left(\mathbf{x_1}, \mathbf{x_2}, \bar{\boldsymbol{\lambda}}_{\mathbf{l}}, \bar{\boldsymbol{\lambda}}_{\mathbf{n}}\right)$ in the optimality cut and $\min\limits_{\mathbf{x_2}} L_{\mathrm{f}}\left(\mathbf{x_1}, \mathbf{x_2}, \bar{\boldsymbol{\lambda}}_{\mathbf{fl}}, \bar{\boldsymbol{\lambda}}_{\mathbf{fn}}\right)$ in the feasibility cut depend on $\mathbf{x_1}$ only, since $\bar{\boldsymbol{\lambda}}_{\mathbf{l}}$, $\bar{\boldsymbol{\lambda}}_{\mathbf{n}}$, $\bar{\boldsymbol{\lambda}}_{\mathbf{fl}}$ and $\bar{\boldsymbol{\lambda}}_{\mathbf{fn}}$ are the constants obtained from solving the slave problem and feasibility problem. In addition, $\mathbf{x_2}$ has to vanish after minimization over the space of $\mathcal{X}_2$. However, the explicit form of (2.40) and (2.46) can be derived only in some cases, e.g. $\mathbf{x_1}$ and $\mathbf{x_2}$ are linearly separated in $f\left(\mathbf{x_1},\, \mathbf{x_2}\right)$, $\mathbf{H}\left(\mathbf{x_1},\, \mathbf{x_2}\right)$ and $\mathbf{G}\left(\mathbf{x_1},\, \mathbf{x_2}\right)$.

CHAPTER III

PROPOSED FORMULATIONS AND METHODOLOGY

In this chapter, a basic framework of the decomposition based method for solving the TEP problem is proposed, including single stage TEP problem based on a DC model, single stage TEP problem with N-1 security constraints, multistage TEP problem, etc. The formulation of the TEP problems in this chapter will be proposed in a structure complied with the Benders decomposition based methods.

3.1 Basic Framework

From the concepts of the BD and GBD presented in the previous chapter, it is clearly seen that a complicated MIP problems can be solved by decomposing the main problem into the master problem and the slave problem. Then, the two subproblems can be solved separately in an iterative manner. The master problem deals with integer variables whereas the slave problem is parameterized by integer variables, obtained from solution of the master problem. The information regarding the optimality and feasibility of the main problem represented as a cut is sent from the slave problem to the master problem in order that the new integer solution will change towards the optimum solution of the main problem.

In this TEP, the master problem refers to an investment problem, while the slave problem refers to an operation problem. Generally, the investment problem is an MILP of which the minimizer represents the investment plan. From the obtained investment plan, the operation problem can be defined according to defined scenarios of the power system. Therefore, there may be several operation problems according to the defined number of scenarios. For example, in case of the single stage TEP with consideration of N-1 security constraint, the number of operation problems should be equal to the number of the contingencies to be taken into account. In case of the multistage TEP without consideration of the security constraint, the number of operation problems is equal to the number of stages in the planning period.

If a DC model is employed in the problem formulation, the operation problem will be an LP problem. However, if the AC model is used, the operation problem will be an NLP problem. The constraints of the operation problem deal with planning criteria and the law of electric circuit theory, i.e. KCL and KVL. Generally, the objective function is an operating cost. The operation variables consist of voltage magnitude, voltage angle, active power generation, reactive power generation, and reactive power compensation device.

The results obtained from the operation problems provide signals of the optimality and feasibility of the investment plan. The information is sent to the investment problem in order to modify the investment plan in the next iteration. The basic framework of the decomposition based method can be illustrated in Figure 3.1

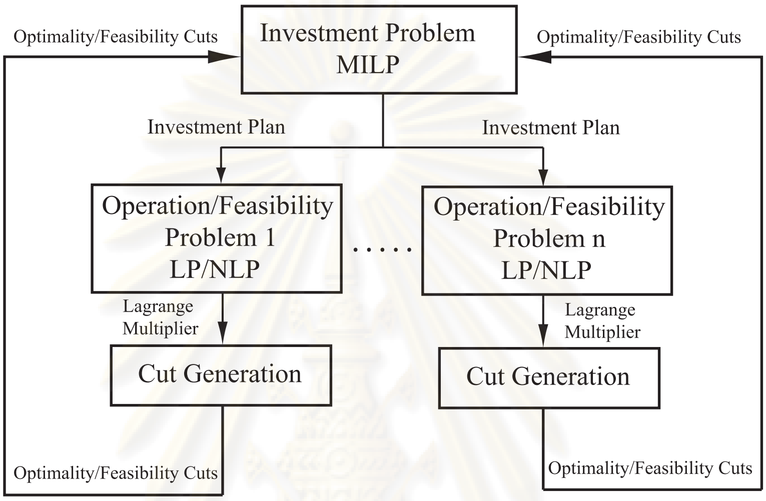

Figure 3.1 Basic framework of the decomposition based method

It should be noted that basic framework of the decomposition based method is similar to a selecting process for alternative plans in a conventional method shown in Figure 2.1. It can be seen that the decision on the plan selection of the planners in conventional TEP can be directly compared with the solving process of the investment problem under the proposed framework. The obtained results from an operation problem can be viewed as the results from power system analysis in a conventional TEP. However, in the decomposition based method, the results are sent to the investment problem as the cuts. Therefore, the proposed method is more systematic than the conventional TEP method.

In addition, the operation problems corresponding to the considered scenarios can be solved independently. Therefore, parallel processing techniques can be easily applied to this framework. However, this issue is not in the scope of the dissertation.

In the following sections, the methods for solving TEP problem are developed based on the concept presented in this section. There are three key issues which should be considered as follows:

(a) Investment problem formulation,

(b) Operation problems formulation, and

(c) Method for generating cuts.

3.2 Handling of Nonlinearity of Investment Cost

The investment plan is expressed as a vector, $\mathbf{x}$ of which the elements represent the circuits of candidate branches selected into the plan. Generally, there are two kinds of representation of the selected circuits, i.e. integer and binary representations. In case of the integer representation, each element corresponding to each candidate branch describes the number of selected circuits. For the case of binary representation, the value of each element indicates the decision on the selection of the candidate branch. i.e. $x_l = 1$, if the candidate branch l is selected into the investment plan.

Most of the TEP research works treat the investment cost as a linear function of the number of circuits. However, the investment cost is generally nonlinear with respect to the number of circuits in practical point of view. To handle this characteristic of the cost while the linearity of the cost function is preserved [**52**], the binary representation of the investment plan is applied. However, in this case, each element of the vector represents the candidate branch specified by a corresponding type. The types of the candidate branches are defined by the number of circuits. In addition, other parameters which affect the cost of the candidates, e.g. types of conductor, types of tower, etc., can be taken into account in the definition of the types of the candidate branches. An example of the candidate branches defined by the number of circuits is shown in Figure 3.2. In this figure, if there is one circuit connected between buses i and j, the candidate A will be selected. If there are two circuits connected between buses i and j, the candidate B will be selected, and so on. The cost of each candidate is defined according to the type of tower depending on the number of circuits.

From the above concept, one can define candidate paths on each bus pair. In actual situation, for the case of transmission line, the candidate paths are right-of-way which the utilities can provide for constructing transmission lines between two substations. In case of transformer, the candidate paths refer to available spaces at substations for installation of transformers.

It should be noted that only one candidate branch can be selected on each candidate path.

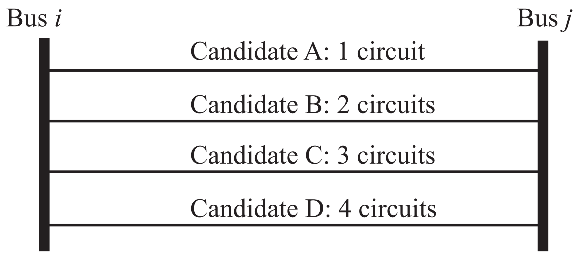

Figure 3.2 Example of candidate branch definition

Therefore, the constraint (3.1) has to be added into the problem.

$$\sum_{l \in \mathcal{C}_p} x_l \leq 1, \qquad p \in \mathcal{P} \tag{3.1}$$

where $\mathcal{C}_p$ is an index set of candidate branches in candidate path p.

Apart from the general types of the candidate lines defined according to the tower types, there are another type of transmission line construction, i.e. stringing the additional circuits on the existing tower constructed in the previous stages. This type of construction will also be considered in this dissertation. However, some constraints have to be involved in the problem.

Without loss of generalization, it is assumed that there is no existing tower available for stringing the additional circuits before the considered planning period.

Now one can define subsets of $\mathcal{C}$ as follows:

$\mathcal{C}_{\mathrm{a}}$ is an index set of candidate branches which are available for stringing the additional circuits after it is constructed, and

$\mathcal{C}_{\mathrm{s}}$ is an index set of candidate branches which will be constructed by stringing on the towers of the other candidate branches.

For all $l_a \in \mathcal{C}_{\mathrm{a}}$, there is a corresponding candidate branch which may be strung on the same tower after the branch l_a is constructed. Therefore, one can define a function $s : \mathcal{C}_{\mathrm{a}} \rightarrow \mathcal{C}_{\mathrm{s}}$ as (3.2).

$$s\left(l_a\right) = l_s, \qquad \text{if } l_s \text{ is strung on the tower of } l_a \tag{3.2}$$

By taking into account the stringing of additional circuits on the existing towers, the constraint (3.1) should be modified to be (3.3)

$$\sum_{l \in \mathcal{C}_p \backslash \mathcal{C}_{\mathrm{s}}} x_l \leq 1, \qquad p \in \mathcal{P} \tag{3.3}$$

In addition, the candidate branches, $l_s \in \mathcal{C}_{\mathrm{s}}$ cannot be selected if the candidate branches, $l_a \in \mathcal{C}_{\mathrm{a}}$ is not selected. This condition is described by (3.4).

$$-x_{l_a} + x_{l_s} \leq 0, \qquad \forall\, l_a \in \mathcal{C}_{\mathrm{a}}, \quad l_s = s\,(l_a) \tag{3.4}$$

The constraints (3.3) and (3.4) can be expressed in a matrix form as (3.5) and (3.6).

$$\mathbf{Tx} \leq \mathbf{e}_{np} \tag{3.5}$$

$$\mathbf{Nx} \leq \mathbf{0} \tag{3.6}$$

where

$$T_{p,l} = \begin{cases} 1 & \text{, if } l \in \mathcal{C}_p \backslash \mathcal{C}_{\mathrm{s}} \\ 0 & \text{, otherwise} \end{cases}$$

$$N_{i,j} = \begin{cases} -1 & \text{, if } j = i \\ 1 & \text{, if } j = s(i) \\ 0 & \text{, otherwise} \end{cases}$$

It is clearly seen that the additional circuits on the existing towers provides the benefit in case of multistage TEP. Therefore, the constraints (3.6) will be taken into account for the only case of multistage TEP.

3.3 Single Stage TEP Using DC Model

3.3.1 Basic Formulation

Considering the formulation of single stage TEP problem using DC model (STEP-DC) presented in Section 2.1.2, it can be seen that generation cost is not taken into account in the formulation. Since, the unit cost of the power plant is not equal to each other, minimizing only the investment cost may affect the congestion problems of transmission system [**53**]. Therefore, the generation cost should be taken into account in the TEP problem as below.

$$\min\left(IVF_1\mathbf{c}_{\mathbf{b}}^{\mathrm{T}}\mathbf{x} + OPF_1\mathbf{c}_{\mathbf{g}}^{\mathrm{T}}\mathbf{p}_{\mathbf{g}}\right) \tag{3.7}$$

subject to

$$\mathbf{T}\mathbf{x} \leq \mathbf{e}_{np} \tag{3.8}$$

$$\mathbf{A}_{\mathbf{g}}^{\mathrm{T}}\mathbf{p}_{\mathbf{g}} - \mathbf{A}_{\mathbf{be}}^{\mathrm{T}}\mathbf{p}_{\mathbf{be}} - \mathbf{A}_{\mathbf{bc}}^{\mathrm{T}}\mathbf{p}_{\mathbf{bc}} = \mathbf{p}_{\mathbf{d}} \tag{3.9}$$

$$\mathbf{p}_{\mathbf{be}} - \mathbf{B}_{\mathbf{e}}\mathbf{A}_{\mathbf{be}}\boldsymbol{\delta} = \mathbf{0} \tag{3.10}$$

$$-\mathbf{M}\left(\mathbf{e}_{nc} - \mathbf{x}\right) \leq \mathbf{p}_{\mathbf{bc}} - \mathbf{B}_{\mathbf{c}}\mathbf{A}_{\mathbf{bc}}\boldsymbol{\delta} \leq \mathbf{M}\left(\mathbf{e}_{nc} - \mathbf{x}\right) \tag{3.11}$$

$$\mathbf{p}_{\mathbf{g}}^{\mathbf{min}} \leq \mathbf{p}_{\mathbf{g}} \leq \mathbf{p}_{\mathbf{g}}^{\mathbf{max}} \tag{3.12}$$

$$-\mathbf{p}_{\mathbf{be}}^{\mathbf{max}} \leq \mathbf{p}_{\mathbf{be}} \leq \mathbf{p}_{\mathbf{be}}^{\mathbf{max}} \tag{3.13}$$

$$-\mathbf{P}_{\mathbf{bc}}^{\mathbf{max}}\mathbf{x} \leq \mathbf{p}_{\mathbf{bc}} \leq \mathbf{P}_{\mathbf{bc}}^{\mathbf{max}}\mathbf{x} \tag{3.14}$$

$$\mathbf{x} \in \{0,\, 1\}^{nc}, \quad \boldsymbol{\delta} \in \mathbb{R}^{nb},\, \mathbf{p}_{\mathbf{g}} \in \mathbb{R}^{ng},\, \mathbf{p}_{\mathbf{be}} \in \mathbb{R}^{ne},\, \mathbf{p}_{\mathbf{bc}} \in \mathbb{R}^{nc}$$

Constraints (3.9)–(3.14) are the same as (2.2)–(2.7), while constraint (3.8) relates to the concept of candidate branch selection as described by (3.5). The constants IVF_1 and OPF_1 can be calculated by (2.13) and (2.14) respectively. It should be noted that in case of the STEP-DC prpblem, the number of stage, ns is equal to one.

3.3.2 Investment Problem Formulation

Considering the formulation defined by (3.7)–(3.14), by application of the BD described in Section 2.2.1, the investment problem can be defined as (3.15).

$$\min z \tag{3.15}$$

subject to

$$\mathbf{T}\mathbf{x} \leq \mathbf{e}_{np} \tag{3.16}$$

$$z \geq 0 \tag{3.17}$$

$$z \in \mathbb{R}, \qquad \mathbf{x} \in \{0,\,1\}^{nc}$$

It can be seen that at the beginning of the process, the investment problem is unbounded. Therefore, the value of z tends toward the negative infinity. To bound the problem and lift up the lower bound to an appropriate value, constraint (3.18) should be added into the investment problem.

$$-z + IVF_1\mathbf{c}_\mathbf{b}^\mathrm{T}\mathbf{x} + OPF_1 c_{\text{op,min}} \leq 0 \tag{3.18}$$

where $c_{\text{op,min}}$ is a minimum generation cost which can be found by a generation dispatch techniques without consideration of transmission constraints [**45**].

3.3.3 Operation Problem Formulation

After solving the investment problem defined as (3.15)–(3.18), the minimizer, $\bar{\mathbf{x}}$ can be obtained. Then the operation problem can be defined as (3.19)–(3.21).

$$\min\left(OPF_1\mathbf{c}_\mathbf{op}^\mathrm{T}\mathbf{y}\right) \tag{3.19}$$

subject to

$$\mathbf{D}\,\mathbf{y} \leq \mathbf{r} \tag{3.20}$$

$$\mathbf{G}\,\mathbf{y} \leq \mathbf{w} - \mathbf{F}\,\bar{\mathbf{x}} \tag{3.21}$$

$$\mathbf{y} \in \mathbb{R}^m, \qquad m = nb + ng + ne + nc$$

where $\mathbf{y}$ is an operation vector defined as (3.22).

$$\mathbf{y} = \begin{bmatrix} \boldsymbol{\delta}^\mathrm{T} & \mathbf{p}_\mathbf{g}^\mathrm{T} & \mathbf{p}_\mathbf{be}^\mathrm{T} & \mathbf{p}_\mathbf{bc}^\mathrm{T} \end{bmatrix}^\mathrm{T} \tag{3.22}$$

$$\mathbf{c}_\mathbf{op} = \begin{bmatrix} \mathbf{0}_{nb}^\mathrm{T} & \mathbf{c}_\mathbf{g}^\mathrm{T} & \mathbf{0}_{ne}^\mathrm{T} & \mathbf{0}_{nc}^\mathrm{T} \end{bmatrix}^\mathrm{T} \tag{3.23}$$

It should be noted that $\mathbf{D}$, $\mathbf{F}$, $\mathbf{G}$, $\mathbf{r}$, and $\mathbf{w}$ can be directly derived from (3.9)–(3.14) by rearrangement of the constraints.

3.3.4 Cut Generation

After solving the investment problem, one can obtain $\bar{\mathbf{x}}$ and $\bar{z}$ which is referred to the lower bound, LBD_k. Now the operation problem can be defined and solved. If the operation problem is feasible, the minimizer, $\bar{\mathbf{y}}$ and Lagrange multiplier, $\bar{\boldsymbol{\lambda}}$ can be obtained. The upper bound, UBD_k is defined by (3.24).

$$UBD_k = \min\left\{UBD_{k-1}, IVF_1\mathbf{c}_{\mathbf{b}}^{\mathrm{T}}\bar{\mathbf{x}} + OPF_1\mathbf{c}_{\mathbf{op}}^{\mathrm{T}}\bar{\mathbf{y}}\right\} \tag{3.24}$$

If the termination criterion defined in (3.25) is not satisfied, an optimality cut will be established according to the BD's method as (3.26).

$$\left|\frac{UBD_k - LBD_k}{LBD_k}\right| < \epsilon \tag{3.25}$$

$$-z + IVF_1\mathbf{c}_{\mathbf{b}}^{\mathrm{T}}\mathbf{x} + \bar{\boldsymbol{\lambda}}_{\mathbf{r}}^{\mathrm{T}}\mathbf{r} + \bar{\boldsymbol{\lambda}}_{\mathbf{w}}^{\mathrm{T}}\mathbf{w} - \bar{\boldsymbol{\lambda}}_{\mathbf{w}}^{\mathrm{T}}\mathbf{F}\mathbf{x} \leq 0 \tag{3.26}$$

where $\bar{\boldsymbol{\lambda}}_{\mathbf{r}}$ and $\bar{\boldsymbol{\lambda}}_{\mathbf{w}}$ are Lagrange multipliers according to the constraints (3.20) and (3.21) respectively.

On the other hand, if the operation problems is infeasible, the feasibility cut will be generated based on the direction of unbounded ray obtained from solving the dual form of the operation problem as described in Section 2.2.1. However, it may be inconvenient to obtain the unbounded ray from solving the operation problem, since some LP solvers do not provide it when they recognize that the problem is unbounded.

To create the feasibility cut without the unbounded ray, a feasibility problem defined as (3.27) is proposed.

$$\min \beta \tag{3.27}$$

subject to

$$\mathbf{D}\,\mathbf{y} - \beta\,\mathbf{e}_{nr} \leq \mathbf{r} \tag{3.28}$$

$$\mathbf{G}\,\mathbf{y} - \beta\,\mathbf{e}_{nw} \leq \mathbf{w} - \mathbf{F}\,\bar{\mathbf{x}} \tag{3.29}$$

$$\mathbf{y} \in \mathbb{R}^m, \quad \beta \geq 0, \qquad \beta \in \mathbb{R}$$

where $\mathbf{e}_{nr}$ and $\mathbf{e}_{nw}$ are vectors consisting of all 1s. The number of elements of $\mathbf{e}_{nr}$ and $\mathbf{e}_{nw}$ are equal to the number of row of $\mathbf{D}$ and $\mathbf{G}$ respectively.

After solving the feasibility problems, the feasibility cuts can be obtained by (3.30).

$$\bar{\boldsymbol{\mu}}_{\mathbf{r}}^{\mathrm{T}}\mathbf{r} + \bar{\boldsymbol{\mu}}_{\mathbf{w}}^{\mathrm{T}}\mathbf{w} - \bar{\boldsymbol{\mu}}_{\mathbf{w}}^{\mathrm{T}}\mathbf{F}\mathbf{x} \leq 0 \tag{3.30}$$

where $\bar{\boldsymbol{\mu}}_{\mathbf{r}}$ and $\bar{\boldsymbol{\mu}}_{\mathbf{w}}$ are Lagrange multipliers according to the constraints (3.28) and (3.29) respectively.

3.3.5 Computational Procedure

From the concepts described in the previous subsections, computational steps of the decomposition based method for solving the STEP-DC problem can be listed below.

Step 0: Set the iteration counter, k to one, and UBD_0 to infinity. Define the value of ϵ.

Step 1: Solve the investment problem defined as (3.15)–(3.18). Set the value of the lower bound, LBD_k to $\bar{z}$.

Step 2: With the minimizer, $\bar{\mathbf{x}}$ obtained from Step 1, define and solve the operation problem. If the problem is feasible, the minimizer, $\bar{\mathbf{y}}$ and Lagrange multiplier, $\bar{\boldsymbol{\lambda}}_{\mathbf{r}}, \bar{\boldsymbol{\lambda}}_{\mathbf{w}}$ can be obtained. Then the value of upper bound is updated by (3.24). If the termination criterion defined as (3.25) is satisfied, the process is terminated, and $(\bar{\mathbf{x}}, \bar{\mathbf{y}})$ is the solution of the STEP-DC problem, otherwise the optimality cut defined as (3.26) will be created.

In case the operation problem is infeasible, the feasibility cuts defined as (3.30) will be created.

Step 3: After obtaining either optimality or feasibility cut from Step 2, add it into the investment problem, increase the iteration counter by one, i.e. $k = k + 1$, and go to Step 1.

3.4 Single Stage TEP Using DC Model with N-1 Security

3.4.1 Basic Formulation

The formulation of the single stage TEP using DC model with consideration of N-1 security constraints (STEP-DC-NSEC) can be directly extended from the STEP-DC by taking into account the operation constraints according to the considered outage contingency as (3.31)–(3.38).

$$\min\left(IVF_1\mathbf{c}_{\mathbf{b}}^{\mathrm{T}}\mathbf{x} + OPF_1\mathbf{c}_{\mathbf{g}}^{\mathrm{T}}\mathbf{p}_{\mathbf{g}}^{(0)}\right) \tag{3.31}$$

subject to

$$\mathbf{T}\mathbf{x} \leq \mathbf{e}_{np} \tag{3.32}$$

$$\mathbf{A}_{\mathbf{g}}^{\mathrm{T}}\mathbf{p}_{\mathbf{g}}^{(s)} - \mathbf{A}_{\mathbf{be}}^{\mathrm{T}}\mathbf{p}_{\mathbf{be}}^{(s)} - \mathbf{A}_{\mathbf{bc}}^{\mathrm{T}}\mathbf{p}_{\mathbf{bc}}^{(s)} = \mathbf{p}_{\mathbf{d}} \tag{3.33}$$

$$\mathbf{p}_{\mathbf{be}}^{(s)} - \mathbf{B}_{\mathbf{e}}^{(s)}\mathbf{A}_{\mathbf{be}}\boldsymbol{\delta}^{(s)} = \mathbf{0} \tag{3.34}$$

$$-\mathbf{M}^{(s)}\left(\mathbf{e}_{nc} - \mathbf{x}\right) \leq \mathbf{p}_{\mathbf{bc}}^{(s)} - \mathbf{B}_{\mathbf{c}}^{(s)}\mathbf{A}_{\mathbf{bc}}\boldsymbol{\delta}^{(s)} \leq \mathbf{M}^{(s)}\left(\mathbf{e}_{nc} - \mathbf{x}\right) \tag{3.35}$$

$$\mathbf{p}_{\mathbf{g}}^{\mathbf{min}} \leq \mathbf{p}_{\mathbf{g}}^{(s)} \leq \mathbf{p}_{\mathbf{g}}^{\mathbf{max}} \tag{3.36}$$

$$-\mathbf{p}_{\mathbf{be}}^{\mathbf{max}} \leq \mathbf{p}_{\mathbf{be}}^{(s)} \leq \mathbf{p}_{\mathbf{be}}^{\mathbf{max}} \tag{3.37}$$

$$-\mathbf{P}_{\mathbf{bc}}^{\mathbf{max}}\mathbf{x} \leq \mathbf{p}_{\mathbf{bc}}^{(s)} \leq \mathbf{P}_{\mathbf{bc}}^{\mathbf{max}}\mathbf{x} \tag{3.38}$$

$$\mathbf{x} \in \{0,\,1\}^{nc}, \quad \boldsymbol{\delta}^{(s)} \in \mathbb{R}^{nb},\, \mathbf{p}_{\mathbf{g}}^{(s)} \in \mathbb{R}^{ng},\, \mathbf{p}_{\mathbf{be}}^{(s)} \in \mathbb{R}^{ne},\, \mathbf{p}_{\mathbf{bc}}^{(s)} \in \mathbb{R}^{nc}$$

$$s = 0,\, 1,\, \ldots,\, nv$$

The constraint (3.32) refers to the concept of candidate branch selection as described by (3.5). The constraints (3.33)–(3.38) are stated for $s = 0,\, 1,\, \ldots,\, nv$, where nv is the number of considered outage contingencies. In addition, $\boldsymbol{\delta}^{(s)}$, $\mathbf{p}_{\mathbf{g}}^{(s)}$, $\mathbf{p}_{\mathbf{be}}^{(s)}$, and $\mathbf{p}_{\mathbf{bc}}^{(s)}$ are the operation variables according to scenario of outage contingency s.

From the above formulation, there are some remarks as follows:

- The generation costs in all outage scenarios are approximated to be equal to the cost in the base case scenario, i.e. no contingency, since those costs cannot be determined from the deterministic point of view.

- The matrices $\mathbf{B}_{\mathbf{e}}^{(s)}$, $\mathbf{B}_{\mathbf{c}}^{(s)}$, and $\mathbf{M}^{(s)}$ are changed according to the outage contingencies.

It can be seen that (3.31)–(3.38) can be expressed in a matrix form as follows:

$$\min\left(IVF_1\mathbf{c}_{\mathbf{b}}^{\mathrm{T}}\mathbf{x} + OPF_1\mathbf{c}_{\mathbf{op}}^{\mathrm{T}}\mathbf{y}^{(0)}\right) \tag{3.39}$$

subject to

$$\mathbf{T}\mathbf{x} \le \mathbf{e}_{np} \tag{3.40}$$

$$\mathbf{D}^{(s)}\mathbf{y}^{(s)} \le \mathbf{r}^{(s)}, \qquad s = 0,\, 1,\, \ldots,\, nv \tag{3.41}$$

$$\mathbf{F}^{(s)}\mathbf{x} + \mathbf{G}^{(s)}\mathbf{y}^{(s)} \le \mathbf{w}^{(s)}, \qquad s = 0,\, 1,\, \ldots,\, nv \tag{3.42}$$

$$\mathbf{x} \in \{0,\, 1\}^{nc},\ \mathbf{y}^{(s)} \in \mathbb{R}^{m}, \qquad m = nb + ng + ne + nc$$

where $\mathbf{y}^{(s)}$ is an operation vector for scenario s defined as (3.43).

$$\mathbf{y}^{(s)} = \begin{bmatrix} \boldsymbol{\delta}^{(s)\mathrm{T}} & \mathbf{p}_{\mathbf{g}}^{(s)\mathrm{T}} & \mathbf{p}_{\mathbf{be}}^{(s)\mathrm{T}} & \mathbf{p}_{\mathbf{bc}}^{(s)\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \tag{3.43}$$

It should be noted that $\mathbf{D}^{(s)}$, $\mathbf{F}^{(s)}$, $\mathbf{G}^{(s)}$, $\mathbf{r}^{(s)}$, and $\mathbf{w}^{(s)}$ can be directly derived from (3.33)–(3.38).

In general, the number of outage scenarios should be equal to the number of branches in the system. In the case of large scale power systems, there may be several scenarios causing high computational burden. Therefore, the contingency selection technique [**1**,**45**] can be applied for reducing the number of scenarios.

3.4.2 Investment Problem Formulation

With the application of the decomposition based method, the investment problem of the single stage TEP with consideration of N-1 security constraints can be initialized in the same form as the case of the single stage TEP without consideration of security constraint. The formulation is restated below.

$$\min z \tag{3.44}$$

subject to

$$\mathbf{T}\mathbf{x} \leq \mathbf{e}_{np} \tag{3.45}$$

$$-z + IVF_1\mathbf{c}_{\mathbf{b}}^{\mathrm{T}}\mathbf{x} \leq -OPF_1 c_{\text{op,min}} \tag{3.46}$$

$$z \geq 0 \tag{3.47}$$

$$z \in \mathbb{R}, \qquad \mathbf{x} \in \{0,\, 1\}^{nc}$$

3.4.3 Operation Problem Formulation

The operation problem corresponding to the outage scenario t can be defined as (3.48)–(3.50).

$$\min \left(OPF_1 \mathbf{c}_{\mathbf{op}}^{(s)\mathrm{T}} \mathbf{y}^{(s)} \right) \tag{3.48}$$

subject to

$$\mathbf{D}^{(s)}\mathbf{y}^{(s)} \leq \mathbf{r}^{(s)} \tag{3.49}$$

$$\mathbf{G}^{(s)}\mathbf{y}^{(s)} \leq \mathbf{w}^{(s)} - \mathbf{F}^{(s)}\bar{\mathbf{x}} \tag{3.50}$$

$$\mathbf{y}^{(s)} \in \mathbb{R}^{m}, \qquad m = nb + ng + ne + nc$$

where

$$\mathbf{c}_{\mathbf{op}}^{(s)} = \begin{cases} \mathbf{c}_{\mathbf{op}} & \text{, if } s = 0 \\ \mathbf{0} & \text{, otherwise} \end{cases} \tag{3.51}$$

3.4.4 Cut Generation

Since the value of objective function of each operation problem corresponding to the outage scenario is always zero, the information obtained from solving those operation problems does not signal the optimality of the main problem. Consequently, there is only one optimality cut generated from solving the operation problems relating to the base case scenario. The optimality cut can be defined as (3.52).

$$-z + IVF_1\mathbf{c}_\mathbf{b}^\mathrm{T}\mathbf{x} + \bar{\boldsymbol{\lambda}}_\mathbf{r}^{(0)\mathrm{T}}\mathbf{r}^{(0)} + \bar{\boldsymbol{\lambda}}_\mathbf{w}^{(0)\mathrm{T}}\mathbf{w}^{(0)} - \bar{\boldsymbol{\lambda}}_\mathbf{w}^{(0)\mathrm{T}}\mathbf{F}^{(0)}\mathbf{x} \leq 0 \tag{3.52}$$

For the feasibility cuts, they can be created after solving the feasibility problems, which corresponds to the operation problem s can be expressed as (3.53)–(3.55).

$$\min \beta^{(s)} \tag{3.53}$$

subject to

$$\mathbf{D}^{(s)}\mathbf{y}^{(s)} - \beta^{(s)}\mathbf{e}_{nr} \leq \mathbf{r}^{(s)} \tag{3.54}$$

$$\mathbf{G}^{(s)}\mathbf{y}^{(s)} - \beta^{(s)}\mathbf{e}_{nw} \leq \mathbf{w}^{(s)} - \mathbf{F}^{(s)}\bar{\mathbf{x}} \tag{3.55}$$

$$\mathbf{y}^{(s)} \in \mathbb{R}^m, \quad \beta^{(s)} \geq 0, \quad \beta^{(s)} \in \mathbb{R}$$

Let $\mathcal{U}$ is an index set of infeasible operation problems. For each infeasible operation problem, the feasibility problem is defined. After all feasibility problems are solved, the obtained Lagrange multipliers, $\bar{\boldsymbol{\mu}}_\mathbf{r}^{(s)}$ and $\bar{\boldsymbol{\mu}}_\mathbf{w}^{(s)}, s \in \mathcal{U}$ are used in generation the feasibility cuts as (3.56).

$$\bar{\boldsymbol{\mu}}_\mathbf{r}^{(s)\mathrm{T}}\mathbf{r}^{(s)} + \bar{\boldsymbol{\mu}}_\mathbf{w}^{(s)\mathrm{T}}\mathbf{w}^{(s)} - \bar{\boldsymbol{\mu}}_\mathbf{w}^{(s)\mathrm{T}}\mathbf{F}^{(s)}\mathbf{x} \leq 0, \qquad s \in \mathcal{U} \tag{3.56}$$

3.4.5 Computational Procedure

The computational procedure can be summarized below.

Step 0: Set the iteration counter, k to one, and UBD_0 to infinity. Define the value of ϵ.

Step 1: Solve the investment problem defined as (3.44)–(3.47). Set the value of the lower bound, LBD_k to $\bar{z}$.

Step 2: From the minimizer, with $\bar{\mathbf{x}}$ obtained from Step 1, define and solve the operation problems for all considered scenarios, i.e. $nv + 1$ problems. If all problems are feasible, the minimizers, $\bar{\mathbf{y}}^{(s)}$ and Lagrange multipliers, $\bar{\boldsymbol{\lambda}}_\mathbf{r}^{(s)}, \bar{\boldsymbol{\lambda}}_\mathbf{w}^{(s)}$ for $s = 0, 1, \ldots, nv$ can be obtained. Then the value of upper bound is updated by (3.57).

$$UBD_k = \min\left\{UBD_{k-1},\ IVF_1\mathbf{c}_{\mathbf{b}}^{\mathrm{T}}\bar{\mathbf{x}} + OPF_1\mathbf{c}_{\mathbf{op}}^{\mathrm{T}}\bar{\mathbf{y}}^{(0)}\right\} \tag{3.57}$$

If the termination criterion defined as (3.25) is satisfied, the process is terminated, and $\left(\bar{\mathbf{x}},\ \bar{\mathbf{y}}^{(0)},\ \ldots,\ \bar{\mathbf{y}}^{(nv)}\right)$ is the solution of the STEP-DC-NSEC problem, otherwise the optimality cuts defined as (3.52) will be created.

In case there are some infeasible operation problems, the feasibility cuts defined as (3.56) will be created.

Step 3: After obtaining either optimality or feasibility cut from Step 2, add it into the investment problem, increase the iteration counter by one, i.e. $k = k + 1$, and go to Step 1.

3.5 Multistage TEP Using DC Model

3.5.1 Basic Formulation

The formulation of the multistage TEP using DC model without consideration of security constraint (MTEP-DC) can be developed based on the formulation of the STEP-DC described in Section 3.3. For the multistage TEP comprising ns stages, the problem can be expressed as below [**39**].

$$\min\left\{\sum_{t=1}^{ns}\left(IVF_t\mathbf{c}_{\mathbf{b}}^{\mathrm{T}}\mathbf{x}^{(t)} + OPF_t\mathbf{c}_{\mathbf{g}}^{\mathrm{T}}\mathbf{p}_{\mathbf{g}}^{(t)}\right)\right\} \tag{3.58}$$

subject to

$$\sum_{t=1}^{ns}\mathbf{T}\mathbf{x}^{(t)} \leq \mathbf{e}_{np} \tag{3.59}$$

$$\sum_{h=1}^{t}\mathbf{N}\mathbf{x}^{(h)} \leq \mathbf{0} \tag{3.60}$$

$$\mathbf{A}_{\mathbf{g}}^{\mathrm{T}}\mathbf{p}_{\mathbf{g}}^{(t)} - \mathbf{A}_{\mathbf{be}}^{\mathrm{T}}\mathbf{p}_{\mathbf{be}}^{(t)} - \mathbf{A}_{\mathbf{bc}}^{\mathrm{T}}\mathbf{p}_{\mathbf{bc}}^{(t)} = \mathbf{p}_{\mathbf{d}}^{(t)} \tag{3.61}$$

$$\mathbf{p}_{\mathbf{be}}^{(t)} - \mathbf{B}_{\mathbf{e}}\mathbf{A}_{\mathbf{be}}\boldsymbol{\delta}^{(t)} = \mathbf{0} \tag{3.62}$$

$$-\mathbf{M}\left(\mathbf{e}_{nc} - \sum_{h=1}^{t}\mathbf{x}^{(h)}\right) \leq \mathbf{p}_{\mathbf{bc}}^{(t)} - \mathbf{B}_{\mathbf{c}}\mathbf{A}_{\mathbf{bc}}\boldsymbol{\delta}^{(t)} \leq \mathbf{M}\left(\mathbf{e}_{nc} - \sum_{h=1}^{t}\mathbf{x}^{(h)}\right) \tag{3.63}$$

$$\mathbf{p}_{\mathbf{g}}^{\mathbf{min},(t)} \leq \mathbf{p}_{\mathbf{g}}^{(t)} \leq \mathbf{p}_{\mathbf{g}}^{\mathbf{max},(t)} \tag{3.64}$$

$$-\mathbf{p}_{\mathbf{be}}^{\mathbf{max}} \leq \mathbf{p}_{\mathbf{be}}^{(t)} \leq \mathbf{p}_{\mathbf{be}}^{\mathbf{max}} \tag{3.65}$$

$$-\mathbf{P}_{\mathbf{bc}}^{\mathbf{max}} \left(\sum_{h=1}^{t} \mathbf{x}^{(h)} \right) \leq \mathbf{p}_{\mathbf{bc}}^{(t)} \leq \mathbf{P}_{\mathbf{bc}}^{\mathbf{max}} \left(\sum_{h=1}^{t} \mathbf{x}^{(h)} \right) \tag{3.66}$$

The constraints (3.60)–(3.66) are stated for $t = 1, \ldots, ns$.

$$\mathbf{x}^{(t)} \in \{0, 1\}^{nc}, \quad \boldsymbol{\delta}^{(t)} \in \mathbb{R}^{nb}, \mathbf{p}_{\mathbf{g}}^{(t)} \in \mathbb{R}^{ng}, \mathbf{p}_{\mathbf{be}}^{(t)} \in \mathbb{R}^{ne}, \mathbf{p}_{\mathbf{bc}}^{(t)} \in \mathbb{R}^{nc}$$
$$t = 1, \ldots, ns$$

The variables and constants with the superscript (t) refer to those variables and constants in the stage t.

There are four key points which are different from the formulation of the STEP-DC problem as described below.

- All variables in the formulation of the STEP-DC are extended for all other stages. In addition, the power demand and generation capacity should be varied according to load forecast and generation expansion plan.
- Constraints (3.63) and (3.66) are modified from the constraints (3.11) and (3.14) in order that the current stage will recognize the plans in the previous stages.
- Constraint (3.59) ensures that only one candidate branch in each path can be selected into the plan for only one stage.
- Constraint (3.60) is stated for each stage. Therefore, the additional circuit can be strung on the corresponding tower constructed in the previous stages.

For simplicity of describing the proposed method, (3.58)–(3.66) should be rearranged and expressed in a matrix form as follows:

$$\min \left(\mathbf{c}_{\mathbf{bm}}^{\mathrm{T}} \mathbf{x}_{\mathbf{m}} + \sum_{t=1}^{ns} \mathbf{c}_{\mathbf{om}}^{(t)\mathrm{T}} \mathbf{y}^{(t)} \right) \tag{3.67}$$

subject to

$$\mathbf{T}_{\mathbf{m}} \mathbf{x}_{\mathbf{m}} \leq \mathbf{e}_{np} \tag{3.68}$$

$$\mathbf{N_m x_m} \leq \mathbf{0} \tag{3.69}$$

$$\mathbf{D}^{(t)}\mathbf{y}^{(t)} \leq \mathbf{r}^{(t)}, \qquad t = 1,\, \ldots,\, ns \tag{3.70}$$

$$\mathbf{F}_\mathbf{m}^{(t)}\mathbf{x_m} + \mathbf{G}^{(t)}\mathbf{y}^{(t)} \leq \mathbf{w}^{(t)}, \qquad t = 1,\, \ldots,\, ns \tag{3.71}$$

$$\mathbf{x_m} \in \{0,\, 1\}^{nc.ns},\, \mathbf{y}^{(t)} \in \mathbb{R}^m, \qquad m = nb + ng + ne + nc$$

where $\mathbf{x_m}$ is a vector representing a long term investment plan, and $\mathbf{y}^{(t)}$ is an operation vector for stage t defined as below.

$$\mathbf{x_m} = \left[\begin{array}{cccc} \mathbf{x}^{(1)\mathrm{T}} & \mathbf{x}^{(2)\mathrm{T}} & \ldots & \mathbf{x}^{(ns)\mathrm{T}} \end{array}\right]^\mathrm{T} \tag{3.72}$$

$$\mathbf{y}^{(t)} = \left[\begin{array}{cccc} \boldsymbol{\delta}^{(t)\mathrm{T}} & \mathbf{p}_\mathbf{g}^{(t)\mathrm{T}} & \mathbf{p}_\mathbf{be}^{(t)\mathrm{T}} & \mathbf{p}_\mathbf{bc}^{(t)\mathrm{T}} \end{array}\right]^\mathrm{T} \tag{3.73}$$

$$\mathbf{T_m} = \mathbf{TE}^{(ns)} \tag{3.74}$$

$$\mathbf{N_m} = \left[\begin{array}{cccc} \mathbf{N}_\mathbf{m}^{(1)\mathrm{T}} & \mathbf{N}_\mathbf{m}^{(2)\mathrm{T}} & \ldots & \mathbf{N}_\mathbf{m}^{(ns)\mathrm{T}} \end{array}\right]^\mathrm{T} \tag{3.75}$$

$$\mathbf{N}_\mathbf{m}^{(t)} = \mathbf{NE}^{(t)} \tag{3.76}$$

$\mathbf{E}^{(t)}$ is a matrix transforming the single stage parameter into the multistage parameter for stage t defined as (3.77).

$$\mathbf{E}^{(t)} = \Big[\; \overbrace{\mathbf{I}_{nc} \;\; \ldots \;\; \mathbf{I}_{nc}}^{t \text{ submatrices}} \;\; \overbrace{\mathbf{0}_{nc\times nc} \;\; \ldots \;\; \mathbf{0}_{nc\times nc}}^{ns-t \text{ submatrices}} \;\Big] \tag{3.77}$$

$$\mathbf{c_{bm}} = \left[\begin{array}{cccc} IVF_1\mathbf{c}_\mathbf{b}^\mathrm{T} & IVF_2\mathbf{c}_\mathbf{b}^\mathrm{T} & \ldots & IVF_{ns}\mathbf{c}_\mathbf{b}^\mathrm{T} \end{array}\right]^\mathrm{T} \tag{3.78}$$

$$\mathbf{c}_\mathbf{om}^{(t)} = OPF_t\mathbf{c_{op}} \tag{3.79}$$

Constraint (3.70) corresponds to constraints (3.61), (3.62), (3.64), and (3.65) respectively, while constraint (3.71) corresponds to constraints (3.63) and (3.66). In addition, $\mathbf{D}^{(t)}$, $\mathbf{F}_\mathbf{m}^{(t)}$, $\mathbf{G}^{(t)}$, $\mathbf{r}^{(t)}$, and $\mathbf{w}^{(t)}$ can be directly derived from (3.61)–(3.66), e.g.

$$\mathbf{F}_\mathbf{m}^{(t)} = \mathbf{FE}^{(t)} \tag{3.80}$$

3.5.2 Investment Problem Formulation

Investment problem of the multistage TEP can be initially defined as (3.81)–(3.85).

$$\min\ z \tag{3.81}$$

subject to

$$\mathbf{T_m}\mathbf{x_m} \leq \mathbf{e}_{np} \tag{3.82}$$

$$\mathbf{N_m}\mathbf{x_m} \leq \mathbf{0} \tag{3.83}$$

$$-z + \mathbf{c}_{\mathbf{bm}}^{\mathrm{T}}\mathbf{x_m} \leq -\sum_{t=1}^{ns} OPF_t c_{\text{op,min}}^{(t)} \tag{3.84}$$

$$z \geq 0 \tag{3.85}$$

$$z \in \mathbb{R}, \quad \mathbf{x_m} \in \{0,\, 1\}^{nc.ns}$$

where $c_{\text{op,min}}^{(t)}$ is a minimum of geneartion cost of stage t.

3.5.3 Operation Problem Formulation

For the multistage TEP of which the planning period divided into ns stages, there are ns operation problems. Each operation problem corresponds to each stage. For the stage t, the operation problem can be defined as (3.86)–(3.88).

$$\min \mathbf{c}_{\mathbf{om}}^{(t)\mathrm{T}}\mathbf{y}^{(t)} \tag{3.86}$$

subject to

$$\mathbf{D}^{(t)}\mathbf{y}^{(t)} \leq \mathbf{r}^{(t)} \tag{3.87}$$

$$\mathbf{G}^{(t)}\mathbf{y}^{(t)} \leq \mathbf{w}^{(t)} - \mathbf{F}_{\mathbf{m}}^{(t)}\bar{\mathbf{x}}_{\mathbf{m}} \tag{3.88}$$

$$\mathbf{y}^{(t)} \in \mathbb{R}^m, \qquad m = nb + ng + ne + nc$$

3.5.4 Cut Generation

After solving the operation problems for every ns problems, if all problems are feasible, Lagrange multipliers can be obtained. In addition, if the termination criterion defined as (3.25) is not satisfied, an optimality cut will be established according to the BD's method as below.

$$-z + \mathbf{c}_{\mathbf{bm}}^{\mathrm{T}}\mathbf{x}_{\mathbf{m}} + \sum_{t=1}^{ns}\left(\bar{\boldsymbol{\lambda}}_{\mathbf{r}}^{(t)\mathrm{T}}\mathbf{r}^{(t)} + \bar{\boldsymbol{\lambda}}_{\mathbf{w}}^{(t)\mathrm{T}}\mathbf{w}^{(t)} - \bar{\boldsymbol{\lambda}}_{\mathbf{w}}^{(t)\mathrm{T}}\mathbf{F}_{\mathbf{m}}^{(t)}\mathbf{x}_{\mathbf{m}}\right) \leq 0 \tag{3.89}$$

where $\bar{\boldsymbol{\lambda}}_{\mathbf{r}}^{(t)}$, $\bar{\boldsymbol{\lambda}}_{\mathbf{w}}^{(t)}, t = 1, \ldots, ns$ are Lagrange multipliers of the operation problem t, according to the constraints (3.87) and (3.88) respectively.

On the other hand, if some operation problems are infeasible, the feasibility cuts will be generated based on the solution obtained from solving feasibility problems defined as (3.90)–(3.92).

$$\min \beta^{(t)} \tag{3.90}$$

subject to

$$\mathbf{D}^{(t)}\mathbf{y}^{(t)} - \beta^{(t)}\mathbf{e}_{nr} \leq \mathbf{r}^{(t)} \tag{3.91}$$

$$\mathbf{G}^{(t)}\mathbf{y}^{(t)} - \beta^{(t)}\mathbf{e}_{nw} \leq \mathbf{w}^{(t)} - \mathbf{F}_{\mathbf{m}}^{(t)}\bar{\mathbf{x}}_{\mathbf{m}} \tag{3.92}$$

$$\mathbf{y}^{(t)} \in \mathbb{R}^m, \quad \beta^{(t)} \geq 0, \quad \beta^{(t)} \in \mathbb{R}$$

The feasibility problems are defined according to the infeasible operation problems. After solving those feasibility problems, the feasibility cuts can be obtained by

$$\bar{\boldsymbol{\mu}}_{\mathbf{r}}^{(t)\mathrm{T}}\mathbf{r}^{(t)} + \bar{\boldsymbol{\mu}}_{\mathbf{w}}^{(t)\mathrm{T}}\mathbf{w}^{(t)} - \bar{\boldsymbol{\mu}}_{\mathbf{w}}^{(t)\mathrm{T}}\mathbf{F}_{\mathbf{m}}^{(t)}\mathbf{x}_{\mathbf{m}} \leq 0, \quad t \in \mathcal{U} \tag{3.93}$$

where $\mathcal{U}$ is an index set of infeasible operation problems, $\bar{\boldsymbol{\mu}}_{\mathbf{r}}^{(t)}$ and $\bar{\boldsymbol{\mu}}_{\mathbf{w}}^{(t)}$ are Lagrange multipliers of the feasibility problem t, according to the constraints (3.91) and (3.92) respectively.

3.5.5 Computational Procedure

From the concepts described in the previous subsections, one can summarize the step of computation as below.

Step 0: Set the iteration counter, k to one, and UBD_0 to infinity. Define the value of ϵ.

Step 1: Solve the investment problem defined as (3.81)–(3.85). Set the value of the lower bound, LBD_k to $\bar{z}$.

Step 2: From the minimizer, with $\bar{\mathbf{x}}_{\mathbf{m}}$ obtained from Step 1, define and solve the operation problems for all stage, i.e. ns problems. If all problems are feasible, the minimizers, $\bar{\mathbf{y}}_{\mathbf{m}}^{(t)}$ and Lagrange multipliers, $\bar{\boldsymbol{\lambda}}_{\mathbf{r}}^{(t)}$, $\bar{\boldsymbol{\lambda}}_{\mathbf{w}}^{(t)}$ for $t = 1, \ldots, ns$ can be obtained. Then the value of upper bound is updated by (3.94).

$$UBD_k = \min \left\{ UBD_{k-1},\, \mathbf{c}_{\mathbf{bm}}^{\mathrm{T}} \bar{\mathbf{x}}_{\mathbf{m}} + \sum_{t=1}^{ns} \mathbf{c}_{\mathbf{om}}^{(t)\mathrm{T}} \bar{\mathbf{y}}^{(t)} \right\} \tag{3.94}$$

If the termination criterion defined as (3.25) is satisfied, the process is terminated, and $\left(\bar{\mathbf{x}}_{\mathbf{m}},\, \bar{\mathbf{y}}^{(1)},\, \ldots,\, \bar{\mathbf{y}}^{(ns)}\right)$ is the solution of the MTEP-DC problem, otherwise the optimality cut defined as (3.89) will be created.

In case there are some infeasible operation problems, the feasibility cuts defined as (3.93) will be created.

Step 3: After obtaining either optimality or feasibility cut from Step 2, add it into the investment problem, increase the iteration counter by one, i.e. $k = k + 1$, and go to Step 1.

3.6 Single Stage TEP Using AC Model

In the case of the STEP-AC problem, the additional constraints, i.e. voltage limits and reactive power limits, are involved in the problem. The current limits of transmission lines and transformers at both terminals are taken into account. In addition, the installation of reactive power compensation device to alleviate the voltage violation is also considered.

3.6.1 Basic Formulation

The single stage TEP problem using AC model without consideration of security constraint (STEP-AC) problem can be expressed as (3.95)–(3.107).

$$\min \left(IVF_1 \left(\mathbf{c_b^T x} + \mathbf{c_r^T d_r} + \mathbf{c_c^T d_c} \right) + OPF_1 \mathbf{c_g^T p_g} \right) \tag{3.95}$$

subject to

$$\mathbf{Tx} \leq \mathbf{e}_{np} \tag{3.96}$$

$$\mathbf{P_{inj}} \left(\mathbf{v}, \boldsymbol{\delta}, \mathbf{p_g}, \mathbf{x} \right) = \mathbf{0} \tag{3.97}$$

$$\mathbf{Q_{inj}} \left(\mathbf{v}, \boldsymbol{\delta}, \mathbf{q_g}, \mathbf{d_r}, \mathbf{d_c}, \mathbf{x} \right) = \mathbf{0} \tag{3.98}$$

$$\mathbf{v^{min}} \leq \mathbf{v} \leq \mathbf{v^{max}} \tag{3.99}$$

$$\mathbf{p_g^{min}} \leq \mathbf{p_g} \leq \mathbf{p_g^{max}} \tag{3.100}$$

$$\mathbf{q_g^{min}} \leq \mathbf{q_g} \leq \mathbf{q_g^{max}} \tag{3.101}$$

$$\mathbf{i_{bef}^2} \left(\mathbf{v}, \boldsymbol{\delta} \right) \leq \left(\mathbf{i_{be}^{max}} \right)^2 \tag{3.102}$$

$$\mathbf{i_{bet}^2} \left(\mathbf{v}, \boldsymbol{\delta} \right) \leq \left(\mathbf{i_{be}^{max}} \right)^2 \tag{3.103}$$

$$\mathbf{i_{bcf}^2} \left(\mathbf{v}, \boldsymbol{\delta}, \mathbf{x} \right) \leq \left(\mathbf{i_{bc}^{max}} \circ \mathbf{x} \right)^2 \tag{3.104}$$

$$\mathbf{i_{bct}^2} \left(\mathbf{v}, \boldsymbol{\delta}, \mathbf{x} \right) \leq \left(\mathbf{i_{bc}^{max}} \circ \mathbf{x} \right)^2 \tag{3.105}$$

$$\mathbf{0} \leq \mathbf{d_r} \leq \mathbf{d_r^{max}} \tag{3.106}$$

$$\mathbf{0} \leq \mathbf{d_c} \leq \mathbf{d_c^{max}} \tag{3.107}$$

$$\mathbf{x} \in \{0,\, 1\}^{nc}$$

$$\mathbf{v},\, \boldsymbol{\delta},\, \mathbf{d_r},\, \mathbf{d_c} \in \mathbb{R}^{nb}, \quad \mathbf{p_g},\, \mathbf{q_g} \in \mathbb{R}^{ng}$$

where

$$\begin{aligned} \mathbf{P_{inj}} \left(\mathbf{v}, \boldsymbol{\delta}, \mathbf{p_g}, \mathbf{x} \right) = \; & \mathbf{A_g^T p_g} - \mathbf{A_{bef}^T p_{bef}} \left(\mathbf{v}, \boldsymbol{\delta} \right) - \mathbf{A_{bet}^T p_{bet}} \left(\mathbf{v}, \boldsymbol{\delta} \right) \\ & - \mathbf{A_{bcf}^T p_{bcf}} \left(\mathbf{v}, \boldsymbol{\delta}, \mathbf{x} \right) - \mathbf{A_{bct}^T p_{bct}} \left(\mathbf{v}, \boldsymbol{\delta}, \mathbf{x} \right) - \mathbf{p_d} \end{aligned} \tag{3.108}$$

$$\begin{aligned}\mathbf{Q_{inj}}\left(\mathbf{v},\boldsymbol{\delta},\mathbf{q_g},\mathbf{d_r},\mathbf{d_c},\mathbf{x}\right) = \quad & \mathbf{A}_{\mathbf{g}}^{\mathrm{T}}\mathbf{q_g} - \mathbf{A}_{\mathbf{bef}}^{\mathrm{T}}\mathbf{q_{bef}}\left(\mathbf{v},\boldsymbol{\delta}\right) - \mathbf{A}_{\mathbf{bet}}^{\mathrm{T}}\mathbf{q_{bet}}\left(\mathbf{v},\boldsymbol{\delta}\right) \\ & - \mathbf{A}_{\mathbf{bcf}}^{\mathrm{T}}\mathbf{q_{bcf}}\left(\mathbf{v},\boldsymbol{\delta},\mathbf{x}\right) - \mathbf{A}_{\mathbf{bct}}^{\mathrm{T}}\mathbf{q_{bct}}\left(\mathbf{v},\boldsymbol{\delta},\mathbf{x}\right) \\ & + \mathbf{q_{cmp}}\left(\mathbf{v},\mathbf{d_r},\mathbf{d_c}\right) - \mathbf{q_d}\end{aligned} \tag{3.109}$$

The branch can be represented as a nominal-π model shown in Figure 3.3.

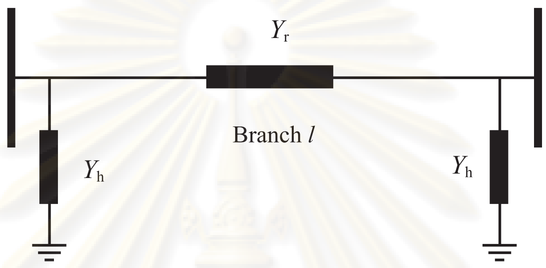

Figure 3.3 Nominal-π model of branch

The l^{th} elements of $\mathbf{p_{bef}}$, $\mathbf{p_{bet}}$, $\mathbf{q_{bef}}$, $\mathbf{q_{bet}}$, $\mathbf{i}_{\mathbf{bef}}^2$, and $\mathbf{i}_{\mathbf{bet}}^2$ corresponding to the existing branch l can be derived as below.

$$I_{\text{bef},l} = \left(V_{\text{ef},l} - V_{\text{et},l}\right)Y_{\text{r},l} + V_{\text{ef},l}Y_{\text{h},l} \tag{3.110}$$

$$S_{\text{bef},l} = V_{\text{ef},l}I_{\text{bef},l}^{*} \tag{3.111}$$

$$p_{\text{bef},l} = \operatorname{Re}\left\{S_{\text{bef},l}\right\} \tag{3.112}$$

$$q_{\text{bef},l} = \operatorname{Im}\left\{S_{\text{bef},l}\right\} \tag{3.113}$$

$$i_{\text{bef},l}^{2} = I_{\text{bef},l}I_{\text{bef},l}^{*} \tag{3.114}$$

$$I_{\text{bet},l} = \left(V_{\text{et},l} - V_{\text{ef},l}\right)Y_{\text{r},l} + V_{\text{et},l}Y_{\text{h},l} \tag{3.115}$$

$$S_{\text{bet},l} = V_{\text{et},l}I_{\text{bet},l}^{*} \tag{3.116}$$

$$p_{\text{bet},l} = \operatorname{Re}\left\{S_{\text{bet},l}\right\} \tag{3.117}$$

$$q_{\text{bet},l} = \operatorname{Im}\left\{S_{\text{bet},l}\right\} \tag{3.118}$$

$$i_{\text{bet},l}^{2} = I_{\text{bet},l}I_{\text{bet},l}^{*} \tag{3.119}$$

where

$$V_{\text{ef},l} = v_{\text{ef},l}\angle\delta_{\text{ef},l} \tag{3.120}$$

$$V_{\mathrm{et},l} = v_{\mathrm{et},l} \angle \delta_{\mathrm{et},l} \tag{3.121}$$

$$\mathbf{v_{ef}} = \mathbf{A_{bef}} \mathbf{v} \tag{3.122}$$

$$\boldsymbol{\delta}_{\mathbf{ef}} = \mathbf{A_{bef}} \boldsymbol{\delta} \tag{3.123}$$

$$\mathbf{v_{et}} = \mathbf{A_{bet}} \mathbf{v} \tag{3.124}$$

$$\boldsymbol{\delta}_{\mathbf{et}} = \mathbf{A_{bet}} \boldsymbol{\delta} \tag{3.125}$$

In the same manner as the existing branch, the l^{th} elements of $\mathbf{p_{bcf}}$, $\mathbf{p_{bct}}$, $\mathbf{q_{bcf}}$, $\mathbf{q_{bct}}$, $\mathbf{i^2_{bcf}}$, and $\mathbf{i^2_{bct}}$ corresponding to the candidate branch l can be derived as below.

$$I_{\mathrm{bcf},l} = \left(V_{\mathrm{cf},l} - V_{\mathrm{ct},l}\right) Y_{\mathrm{r},l} + V_{\mathrm{cf},l} Y_{\mathrm{h},l} \tag{3.126}$$

$$S_{\mathrm{bcf},l} = V_{\mathrm{cf},l} I^*_{\mathrm{bcf},l} \tag{3.127}$$

$$p_{\mathrm{bcf},l} = \mathrm{Re}\left\{S_{\mathrm{bcf},l}\right\} \tag{3.128}$$

$$q_{\mathrm{bcf},l} = \mathrm{Im}\left\{S_{\mathrm{bcf},l}\right\} \tag{3.129}$$

$$i^2_{\mathrm{bcf},l} = I_{\mathrm{bcf},l} I^*_{\mathrm{bcf},l} \tag{3.130}$$

$$I_{\mathrm{bct},l} = \left(V_{\mathrm{ct},l} - V_{\mathrm{cf},l}\right) Y_{\mathrm{r},l} + V_{\mathrm{ct},l} Y_{\mathrm{h},l} \tag{3.131}$$

$$S_{\mathrm{bct},l} = V_{\mathrm{ct},l} I^*_{\mathrm{bct},l} \tag{3.132}$$

$$p_{\mathrm{bct},l} = \mathrm{Re}\left\{S_{\mathrm{bct},l}\right\} \tag{3.133}$$

$$q_{\mathrm{bct},l} = \mathrm{Im}\left\{S_{\mathrm{bct},l}\right\} \tag{3.134}$$

$$i^2_{\mathrm{bct},l} = I_{\mathrm{bct},l} I^*_{\mathrm{bct},l} \tag{3.135}$$

where

$$V_{\mathrm{cf},l} = v_{\mathrm{cf},l} \angle \delta_{\mathrm{cf},l} \tag{3.136}$$

$$V_{\mathrm{ct},l} = v_{\mathrm{ct},l} \angle \delta_{\mathrm{ct},l} \tag{3.137}$$

$$\mathbf{v_{cf}} = \mathbf{A_{bcf}} \mathbf{v} \tag{3.138}$$

$$\boldsymbol{\delta}_{\mathbf{cf}} = \mathbf{A_{bcf}} \boldsymbol{\delta} \tag{3.139}$$

$$\mathbf{v_{ct}} = \mathbf{A_{bct}} \mathbf{v} \tag{3.140}$$

$$\boldsymbol{\delta}_{\mathbf{ct}} = \mathbf{A_{bct}} \boldsymbol{\delta} \tag{3.141}$$

The i^{th} element of $\mathbf{q_{cmp}}$ representing the reactive power injected from the device at bus i can be expressed as (3.142).

$$q_{\text{cmp},i} = v_i^2 d_{\text{c},i} - v_i^2 d_{\text{r},i} + v_i^2 d_{\text{c},i}^0 - v_i^2 d_{\text{r},i}^0 \tag{3.142}$$

where $d_{\text{c},i}^0$ and $d_{\text{r},i}^0$ are the capacities of the existing capacitor and reactor installed at bus i.

It should be noted that the term $\mathbf{i_{bc}^{max}} \circ \mathbf{x}$ in (3.104) and (3.105) is the elementwise product of $\mathbf{i_{bc}^{max}}$and $\mathbf{x}$. In addition, $(\,.\,)^2$ in (3.102)–(3.105) refers to the elementwise square of $(\,.\,)$.

For simplicity in describing the decomposition based method in the next sections, (3.95)–(3.107) will be reformulated as (3.143)–(3.146).

$$\min \left(IVF_1 \left(\mathbf{c_b^T x} + \mathbf{c_r^T d_r} + \mathbf{c_c^T d_c} \right) + OPF_1 \mathbf{c_g^T p_g} \right) \tag{3.143}$$

subject to

$$\mathbf{Tx} \le \mathbf{e}_{np} \tag{3.144}$$

$$\mathbf{G_{eq}}\left(\mathbf{x}, \mathbf{v}, \boldsymbol{\delta}, \mathbf{p_g}, \mathbf{q_g}, \mathbf{d_r}, \mathbf{d_c}\right) = \mathbf{0} \tag{3.145}$$

$$\mathbf{G_{in}}\left(\mathbf{x}, \mathbf{v}, \boldsymbol{\delta}, \mathbf{p_g}, \mathbf{q_g}, \mathbf{d_r}, \mathbf{d_c}\right) \le \mathbf{0} \tag{3.146}$$

$$\mathbf{x} \in \{0,\, 1\}^{nc}$$

$$\mathbf{v},\, \boldsymbol{\delta},\, \mathbf{d_r},\, \mathbf{d_c} \in \mathbb{R}^{nb}, \quad \mathbf{p_g},\, \mathbf{q_g} \in \mathbb{R}^{ng}$$

Since an equality constraint can be expressed in two inequality constraints, the constraints (3.145) and (3.146) can be expressed as (3.147).

$$\mathbf{G_a}\left(\mathbf{x}, \mathbf{v}, \boldsymbol{\delta}, \mathbf{p_g}, \mathbf{q_g}, \mathbf{d_r}, \mathbf{d_c}\right) \le \mathbf{0} \tag{3.147}$$

In general, $\mathbf{d_r}$ and $\mathbf{d_c}$ are not continuous variables, since the compensation devices are usually manufactured with standard capacities. However, the steps of the standard capacities are not too large. In addition, the installation costs of the reactive power compensation devices are very less than the construction costs of transmission lines and generation cost. Therefore, $\mathbf{d_r}$ and $\mathbf{d_c}$ will be treated as continuous variables in the dissertation.

3.6.2 Investment Problem Formulation

As other formulations developed in the previous sections, the investment problem can be initially defined as (3.148)–(3.151).

$$\min z \tag{3.148}$$

subject to

$$\mathbf{T}\mathbf{x} \leq \mathbf{e}_{np} \tag{3.149}$$

$$-z + IVF_1 \mathbf{c}_\mathbf{b}^\mathrm{T} \mathbf{x} \leq -OPF_1 c_{\text{op,min}} \tag{3.150}$$

$$z \geq 0 \tag{3.151}$$

$$z \in \mathbb{R}, \quad \mathbf{x} \in \{0,\, 1\}^{nc}$$

It should be noted that the investment problem of the STEP-AC is similar to the one of the STEP-DC. Therefore, the benefit of the information obtained from solving the STEP-DC problem can be gained by initializing the investment problem of the STEP-AC with the investment problem of the STEP-DC after finishing the procedure for solving the STEP-DC problem.

3.6.3 Operation Problem Formulation

A key point in the operation problem formulation is the linear separation of the investment variable and operation variables. This property is necessary and recommended in Ref. [**43**] in order that explicit form of the cut can be expressed. To achieve this property, a dummy variable, $\mathbf{u}$ is introduced into the problem. Therefore, the operation problem can be defined as (3.152)–(3.154).

$$\min \left(IVF_1 \left(\mathbf{c}_\mathbf{r}^\mathrm{T} \mathbf{d}_\mathbf{r} + \mathbf{c}_\mathbf{c}^\mathrm{T} \mathbf{d}_\mathbf{c} \right) + OPF_1 \mathbf{c}_\mathbf{g}^\mathrm{T} \mathbf{p}_\mathbf{g} \right) \tag{3.152}$$

subject to

$$\mathbf{u} = \bar{\mathbf{x}} \tag{3.153}$$

$$\mathbf{G_a}\left(\mathbf{u},\mathbf{v},\boldsymbol{\delta},\mathbf{p_g},\mathbf{q_g},\mathbf{d_r},\mathbf{d_c}\right)\leq\mathbf{0} \tag{3.154}$$
$$\mathbf{u}\in\mathbb{R}^{nc},\quad \mathbf{v},\,\boldsymbol{\delta},\,\mathbf{d_r},\,\mathbf{d_c}\in\mathbb{R}^{nb},\quad \mathbf{p_g},\,\mathbf{q_g}\in\mathbb{R}^{ng}$$

Now one can define an operation variable, $\mathbf{y_a}$ as described below.

$$\mathbf{y_a}=\left[\begin{array}{ccccccc}\mathbf{u}^{\mathrm{T}} & \mathbf{v}^{\mathrm{T}} & \boldsymbol{\delta}^{\mathrm{T}} & \mathbf{p_g^{\mathrm{T}}} & \mathbf{q_g^{\mathrm{T}}} & \mathbf{d_r^{\mathrm{T}}} & \mathbf{d_c^{\mathrm{T}}}\end{array}\right]^{\mathrm{T}} \tag{3.155}$$

Therefore, the operation problem can be expressed in a compact form as (3.156)–(3.158).

$$\min\left(\mathbf{c_{ac}^{\mathrm{T}}}\mathbf{y_a}\right) \tag{3.156}$$

subject to

$$\mathbf{H_a}\mathbf{y_a}=\bar{\mathbf{x}} \tag{3.157}$$
$$\mathbf{G_a}\left(\mathbf{y_a}\right)\leq\mathbf{0} \tag{3.158}$$
$$\mathbf{y_a}\in\mathbb{R}^{ma},\quad ma=nc+4nb+2ng$$

where

$$\mathbf{c_{ac}}=\left[\begin{array}{ccccccc}\mathbf{0}_{nc}^{\mathrm{T}} & \mathbf{0}_{nb}^{\mathrm{T}} & \mathbf{0}_{nb}^{\mathrm{T}} & OPF_1\mathbf{c_g^{\mathrm{T}}} & \mathbf{0}_{ng}^{\mathrm{T}} & IVF_1\mathbf{c_r^{\mathrm{T}}} & IVF_1\mathbf{c_c^{\mathrm{T}}}\end{array}\right]^{\mathrm{T}} \tag{3.159}$$

and

$$\mathbf{H_a}=\left[\begin{array}{cc}\mathbf{I}_{nc} & \mathbf{0}_{nc\times(4nb+2ng)}\end{array}\right] \tag{3.160}$$

Now the investment variable is linearly separated from the opeartion variable. The explicit forms of cuts can be derived in the next section.

3.6.4 Cut Generation

The concept of cut generation presented in this section is based on the GBD method which requires the convexity of the operation problem. However, the operation problem of the STEP-AC problem is always nonconvex. Therefore, the global optimality of the obtained plan is not guaranteed by the proposed method.

With an investment plan of which $\bar{\mathbf{x}}$ obtained from solving the investment problem, if all operation problems are feasible and the termination criterion defined as (3.25) is not satisfied, the optimality cut will be created by (3.161).

$$-z + IVF_1 \mathbf{c}_{\mathbf{b}}^{\mathrm{T}} \mathbf{x} + \min_{\mathbf{y_a}} L_{\text{ac}} \left(\mathbf{x}, \mathbf{y_a}, \bar{\boldsymbol{\lambda}}_{\mathbf{H}}, \bar{\boldsymbol{\lambda}}_{\mathbf{G}} \right) \leq 0 \tag{3.161}$$

where

$L_{\text{ac}} \left(\mathbf{x}, \mathbf{y_a}, \boldsymbol{\lambda}_{\mathbf{H}}, \boldsymbol{\lambda}_{\mathbf{G}} \right)$ is Lagrange function defined by (3.162),

$\bar{\boldsymbol{\lambda}}_{\mathbf{H}}$ and $\bar{\boldsymbol{\lambda}}_{\mathbf{G}}$ are the obtained Lagrange multipliers according to (3.157) and (3.158) respectively.

$$L_{\text{ac}} \left(\mathbf{x}, \mathbf{y_a}, \boldsymbol{\lambda}_{\mathbf{H}}, \boldsymbol{\lambda}_{\mathbf{G}} \right) = \mathbf{c}_{\mathbf{ac}}^{\mathrm{T}} \mathbf{y_a} + \boldsymbol{\lambda}_{\mathbf{H}}^{\mathrm{T}} \left(\mathbf{H_a} \mathbf{y_a} - \mathbf{x} \right) + \boldsymbol{\lambda}_{\mathbf{G}}^{\mathrm{T}} \mathbf{G_a} \left(\mathbf{y_a} \right) \tag{3.162}$$

Since

$$\begin{aligned}
\min_{\mathbf{y_a}} L_{\text{ac}} \left(\mathbf{x}, \mathbf{y_a}, \bar{\boldsymbol{\lambda}}_{\mathbf{H}}, \bar{\boldsymbol{\lambda}}_{\mathbf{G}} \right) &= \min_{\mathbf{y_a}} \left(\mathbf{c}_{\mathbf{ac}}^{\mathrm{T}} \mathbf{y_a} + \bar{\boldsymbol{\lambda}}_{\mathbf{H}}^{\mathrm{T}} \left(\mathbf{H_a} \mathbf{y_a} - \mathbf{x} \right) + \bar{\boldsymbol{\lambda}}_{\mathbf{G}}^{\mathrm{T}} \mathbf{G} \left(\mathbf{y_a} \right) \right) \\
&= \min_{\mathbf{y_a}} \left(\mathbf{c}_{\mathbf{ac}}^{\mathrm{T}} \mathbf{y_a} + \bar{\boldsymbol{\lambda}}_{\mathbf{H}}^{\mathrm{T}} \left(\mathbf{H_a} \mathbf{y_a} - \bar{\mathbf{x}} \right) + \bar{\boldsymbol{\lambda}}_{\mathbf{G}}^{\mathrm{T}} \mathbf{G} \left(\mathbf{y_a} \right) \right) \\
&\quad + \bar{\boldsymbol{\lambda}}_{\mathbf{H}}^{\mathrm{T}} \left(\bar{\mathbf{x}} - \mathbf{x} \right) \\
&= \bar{L}_{\text{ac}} + \bar{\boldsymbol{\lambda}}_{\mathbf{H}}^{\mathrm{T}} \left(\bar{\mathbf{x}} - \mathbf{x} \right)
\end{aligned} \tag{3.163}$$

where

$$\bar{L}_{\text{ac}} = \min_{\mathbf{y_a}} \left(\mathbf{c}_{\mathbf{ac}}^{\mathrm{T}} \mathbf{y_a} + \bar{\boldsymbol{\lambda}}_{\mathbf{H}}^{\mathrm{T}} \left(\mathbf{H_a} \mathbf{y_a} - \bar{\mathbf{x}} \right) + \bar{\boldsymbol{\lambda}}_{\mathbf{G}}^{\mathrm{T}} \mathbf{G} \left(\mathbf{y_a} \right) \right) \tag{3.164}$$

It is not necessary to evaluate the value of $\bar{L}_{\text{ac}}$, since from the strong duality [**50**],

$$\begin{aligned}
\bar{L}_{\text{ac}} &= \min_{\mathbf{y_a}} \left(\mathbf{c}_{\mathbf{ac}}^{\mathrm{T}} \mathbf{y_a} + \bar{\boldsymbol{\lambda}}_{\mathbf{H}}^{\mathrm{T}} \left(\mathbf{H_a} \mathbf{y_a} - \bar{\mathbf{x}} \right) + \bar{\boldsymbol{\lambda}}_{\mathbf{G}}^{\mathrm{T}} \mathbf{G} \left(\mathbf{y_a} \right) \right) \\
&= \mathbf{c}_{\mathbf{ac}}^{\mathrm{T}} \bar{\mathbf{y}}_{\mathbf{a}}
\end{aligned} \tag{3.165}$$

where $\bar{\mathbf{y}}_{\mathbf{a}}$ is a minimizer obtained from solving the operation problem. In general, $\bar{L}_{\text{ac}}$ is equal to the minimum value of the objective function resulting from solving the operation problem.

From (3.161), (3.163) and (3.165), the explicit form of optimality cut can be expressed as (3.166).

$$-z + IVF_1 \mathbf{c}_{\mathbf{b}}^{\mathrm{T}} \mathbf{x} + \mathbf{c}_{\mathbf{ac}}^{\mathrm{T}} \bar{\mathbf{y}}_{\mathbf{a}} + \bar{\boldsymbol{\lambda}}_{\mathbf{H}}^{\mathrm{T}} \left(\bar{\mathbf{x}} - \mathbf{x}\right) \leq 0 \qquad (3.166)$$

In the same manner, an explicit form of the feasibility cut can be obtained by solving the feasibility problem, which will be defined as (3.167)–(3.170) when the operation problem is infeasible.

$$\min \mathbf{e}_{nb}^{\mathrm{T}} \mathbf{p}_{\mathbf{s}} \qquad (3.167)$$

subject to

$$\mathbf{H}_{\mathbf{a}} \mathbf{y}_{\mathbf{a}} = \bar{\mathbf{x}} \qquad (3.168)$$

$$\mathbf{G}_{\mathbf{eq}} \left(\mathbf{y}_{\mathbf{a}}\right) + \mathbf{S} \mathbf{p}_{\mathbf{s}} = \mathbf{0} \qquad (3.169)$$

$$\mathbf{G}_{\mathbf{in}} \left(\mathbf{y}_{\mathbf{a}}\right) \leq \mathbf{0} \qquad (3.170)$$

$$\mathbf{y}_{\mathbf{a}} \in \mathbb{R}^{ma}, \quad \mathbf{p}_{\mathbf{s}} \in \mathbb{R}^{nb}$$

where

$$\mathbf{S} = \begin{bmatrix} \mathbf{I}_{nb} \\ \mathbf{R}_{\mathbf{q}} \end{bmatrix} \qquad (3.171)$$

$\mathbf{R}_{\mathbf{q}}$ is a diagonal matrix of which the elements represent the ratios between reactive and active power demand.

Actually, the feasible problem is the minimization problem of load shedding. After solving the feasible problem, the Lagrange multipliers can be obtained. Then the feasibility cut is generated by (3.172).

$$\min_{\mathbf{y}_{\mathbf{a}}} L_{\text{fe}} \left(\mathbf{x}, \mathbf{y}_{\mathbf{a}}, \bar{\boldsymbol{\mu}}_{\mathbf{H}}, \bar{\boldsymbol{\mu}}_{\mathbf{Ge}}, \bar{\boldsymbol{\mu}}_{\mathbf{Gi}}\right) \leq 0 \qquad (3.172)$$

where

$L_{\text{fe}} \left(\mathbf{x}, \mathbf{y}_{\mathbf{a}}, \boldsymbol{\mu}_{\mathbf{H}}, \boldsymbol{\mu}_{\mathbf{Ge}}, \boldsymbol{\mu}_{\mathbf{Gi}}\right)$ is defined as (3.173), and

$\bar{\boldsymbol{\mu}}_{\mathbf{H}}$, $\bar{\boldsymbol{\mu}}_{\mathbf{Ge}}$ and $\bar{\boldsymbol{\mu}}_{\mathbf{Gi}}$ are the obtained Lagrange multipliers according to (3.168), (3.169) and (3.170) respectively.

$$L_{\text{fe}}\left(\mathbf{x},\mathbf{y_a},\boldsymbol{\mu}_\mathbf{H},\boldsymbol{\mu}_\mathbf{Ge},\boldsymbol{\mu}_\mathbf{Gi}\right)=\mathbf{e}_{nb}^{\mathrm{T}}\mathbf{p_s}+\boldsymbol{\mu}_\mathbf{H}^{\mathrm{T}}\left(\mathbf{H_a}\mathbf{y_a}-\mathbf{x}\right) \\ +\boldsymbol{\mu}_\mathbf{Ge}^{\mathrm{T}}\left(\mathbf{G_{eq}}\left(\mathbf{y_a}\right)+\mathbf{S}\mathbf{p_s}\right)+\boldsymbol{\mu}_\mathbf{Gi}^{\mathrm{T}}\mathbf{G_{in}}\left(\mathbf{y_a}\right) \tag{3.173}$$

Therefore, the explicit form of the feasibility cut can be expressed as (3.174).

$$\mathbf{e}_{nb}^{\mathrm{T}}\bar{\mathbf{p}}_\mathbf{s}+\bar{\boldsymbol{\mu}}_\mathbf{H}^{\mathrm{T}}\left(\bar{\mathbf{x}}-\mathbf{x}\right)\leq 0 \tag{3.174}$$

where $\bar{\mathbf{p}}_\mathbf{s}$ can be obtained from solving the feasibility problem.

3.6.5 Computational Procedure

From the concept described in the previous subsections, the computational procedure can be summarized below.

Step 0: Set the iteration counter, k to one, and UBD_0 to infinity. Define the value of ϵ.

Step 1: Solve the investment problem. Set the value of the lower bound, LBD_k to $\bar{z}$.

Step 2: From the minimizer, with $\bar{\mathbf{x}}$ obtained from Step 1, define and solve the operation problem. If the problem is feasible, the minimizer, $\bar{\mathbf{y}}_\mathbf{a}$ and Lagrange multiplier, $\bar{\boldsymbol{\lambda}}_\mathbf{H}$ can be obtained. Then the value of upper bound is updated by (3.175).

$$UBD_k=\min\left\{UBD_{k-1},\, IVF_1\mathbf{c}_\mathbf{b}^{\mathrm{T}}\bar{\mathbf{x}}+\mathbf{c}_\mathbf{ac}^{\mathrm{T}}\bar{\mathbf{y}}_\mathbf{a}\right\} \tag{3.175}$$

If the termination criterion defined as (3.25) is satisfied, the process is terminated, and $(\bar{\mathbf{x}},\,\bar{\mathbf{y}}_\mathbf{a})$ will be the solution of the STEP-AC problem, otherwise the optimality cut defined as (3.166) will be created.

In case the operation problem is infeasible, the feasibility cut defined as (3.174) will be created.

Step 3: After obtaining either optimality or feasibility cut from Step 2, add it into the investment problem, increase the iteration counter by one, i.e. $k=k+1$, and go to Step 1.

3.7 Multistage Stage TEP Using AC Model with N-1 Security

From the TEP formulations presented in the previous sections, the concept of multistage TEP, N-1 security constraints, and TEP based on AC model will be integrated into the complete formulation in this section.

It should be noted that for the case of the multistage TEP, the reactive power devices which can be operated in each stage have to depend on the installation capacities in the previous stages. Taking into account the correlation of the reactive power installation plan in each stage may cause the problem to be more complicated. Fortunately, it is well-known from practical experiences that the cost of installation of the reactive power compensation devices is much less than the cost of transmission line investment and the operating cost. Therefore, in the formulation of the multistage TEP problem using AC model with consideration of N-1 security constraint (MTEP-AC-NSEC), the cost of installation of the devices will not involve in the objective function. However, the capacities of operating devices are still the operation variables to be considered in the problem.

In addition, after the TEP problem has been solved, the investment plan can be obtained. The operation problem for each stage in each scenario will be solved again with taking account of the cost of reactive power device. Therefore, the optimum location and capacity for installation of the reactive power devices in each stage can be determined.

3.7.1 Basic Formulation

The formulation of MTEP-AC-NSEC problem can be expressed as (3.176)–(3.190).

$$\min \left\{ \sum_{t=1}^{ns} \left(IVF_t \mathbf{c}_{\mathbf{b}}^{\mathrm{T}} \mathbf{x}^{(t)} + OPF_t \mathbf{c}_{\mathbf{g}}^{\mathrm{T}} \mathbf{p}_{\mathbf{g}}^{(t,0)} \right) \right\} \tag{3.176}$$

subject to

$$\sum_{t=1}^{ns} \mathbf{T}\mathbf{x}^{(t)} \leq \mathbf{e}_{np} \tag{3.177}$$

$$\sum_{h=1}^{t} \mathbf{N}\mathbf{x}^{(h)} \leq \mathbf{0} \tag{3.178}$$

$$\mathbf{x}_{\mathbf{c}}^{(t)} = \sum_{h=1}^{t} \mathbf{x}^{(h)} \tag{3.179}$$

$$\mathbf{P}_{\mathbf{inj}}^{(t,s)}\left(\mathbf{v}^{(t,s)},\boldsymbol{\delta}^{(t,s)},\mathbf{p}_{\mathbf{g}}^{(t,s)},\mathbf{x}_{\mathbf{c}}^{(t)}\right)=\mathbf{0} \tag{3.180}$$

$$\mathbf{Q}_{\mathbf{inj}}^{(t,s)}\left(\mathbf{v}^{(t,s)},\boldsymbol{\delta}^{(t,s)},\mathbf{q}_{\mathbf{g}}^{(t,s)},\mathbf{d}_{\mathbf{r}}^{(t,s)},\mathbf{d}_{\mathbf{c}}^{(t,s)},\mathbf{x}_{\mathbf{c}}^{(t)}\right)=\mathbf{0} \tag{3.181}$$

$$\mathbf{v}^{\mathbf{min}}\le\mathbf{v}^{(t,s)}\le\mathbf{v}^{\mathbf{max}} \tag{3.182}$$

$$\mathbf{p}_{\mathbf{g}}^{\mathbf{min},(t)}\le\mathbf{p}_{\mathbf{g}}^{(t,s)}\le\mathbf{p}_{\mathbf{g}}^{\mathbf{max},(t)} \tag{3.183}$$

$$\mathbf{q}_{\mathbf{g}}^{\mathbf{min},(t)}\le\mathbf{q}_{\mathbf{g}}^{(t,s)}\le\mathbf{q}_{\mathbf{g}}^{\mathbf{max},(t)} \tag{3.184}$$

$$\mathbf{i}_{\mathbf{bef}}^{(t,s)\,2}\left(\mathbf{v}^{(t,s)},\boldsymbol{\delta}^{(t,s)}\right)\le(\mathbf{i}_{\mathbf{be}}^{\mathbf{max}})^2 \tag{3.185}$$

$$\mathbf{i}_{\mathbf{bet}}^{(t,s)\,2}\left(\mathbf{v}^{(t,s)},\boldsymbol{\delta}^{(t,s)}\right)\le(\mathbf{i}_{\mathbf{be}}^{\mathbf{max}})^2 \tag{3.186}$$

$$\mathbf{i}_{\mathbf{bcf}}^{(t,s)\,2}\left(\mathbf{v}^{(t,s)},\boldsymbol{\delta}^{(t,s)},\mathbf{x}_{\mathbf{c}}^{(t)}\right)\le\left(\mathbf{i}_{\mathbf{bc}}^{\mathbf{max}}\circ\mathbf{x}_{\mathbf{c}}^{(t)}\right)^2 \tag{3.187}$$

$$\mathbf{i}_{\mathbf{bct}}^{(t,s)\,2}\left(\mathbf{v}^{(t,s)},\boldsymbol{\delta}^{(t,s)},\mathbf{x}_{\mathbf{c}}^{(t)}\right)\le\left(\mathbf{i}_{\mathbf{bc}}^{\mathbf{max}}\circ\mathbf{x}_{\mathbf{c}}^{(t)}\right)^2 \tag{3.188}$$

$$\mathbf{0}\le\mathbf{d}_{\mathbf{r}}^{(t,s)}\le\mathbf{d}_{\mathbf{r}}^{\mathbf{max}} \tag{3.189}$$

$$\mathbf{0}\le\mathbf{d}_{\mathbf{c}}^{(t,s)}\le\mathbf{d}_{\mathbf{c}}^{\mathbf{max}} \tag{3.190}$$

The constraints (3.178)–(3.190) are stated for $t=1,\,\ldots,\,ns$ and $s=1,\,\ldots,\,nv$.

$$\begin{aligned}
&\mathbf{x}^{(t)},\,\mathbf{x}_{\mathbf{c}}^{(t)}\in\{0,\,1\}^{nc}\\
&\mathbf{v}^{(t,s)},\,\boldsymbol{\delta}^{(t,s)},\,\mathbf{d}_{\mathbf{r}}^{(t,s)},\,\mathbf{d}_{\mathbf{c}}^{(t,s)}\in\mathbb{R}^{nb},\quad \mathbf{p}_{\mathbf{g}}^{(t,s)},\,\mathbf{q}_{\mathbf{g}}^{(t,s)}\in\mathbb{R}^{ng}\\
&t=1,\,\ldots,\,ns,\quad s=0,\,\ldots,\,nv
\end{aligned}$$

The functions $\mathbf{P}_{\mathbf{inj}}^{(t,s)}$, $\mathbf{Q}_{\mathbf{inj}}^{(t,s)}$, $\mathbf{i}_{\mathbf{bef}}^{(t,s)2}$, $\mathbf{i}_{\mathbf{bet}}^{(t,s)2}$, $\mathbf{i}_{\mathbf{bcf}}^{(t,s)2}$, and $\mathbf{i}_{\mathbf{bct}}^{(t,s)2}$ can be defined by (3.108), (3.109), (3.114), (3.119), (3.130) and (3.135) according to the changed network parameters depending on stage t and scenario s.

It should be emphasized that only the operating cost of the base case is taken into account in the objective function since the cost of other scenarios cannot be determined from the deterministic point of view.

In the same manner of Section 3.6, the constraints (3.180)–(3.190) for stage t, scenario s can be expressed as (3.191).

$$\mathbf{G}_{\mathbf{a}}^{(t,s)}\left(\mathbf{x}_{\mathbf{c}}^{(t)},\mathbf{v}^{(t,s)},\boldsymbol{\delta}^{(t,s)},\mathbf{p}_{\mathbf{g}}^{(t,s)},\mathbf{q}_{\mathbf{g}}^{(t,s)},\mathbf{d}_{\mathbf{r}}^{(t,s)},\mathbf{d}_{\mathbf{c}}^{(t,s)}\right)\le\mathbf{0} \tag{3.191}$$

It can be seen that the formulation of MTEP-AC-NSEC problem is extremely complicated. For example, in the case of a medium scale problem, i.e. three-stage problem of IEEE-24 bus test system, consisting of 24 buses, 38 existing branches, 12 power plants, and 82 right-of-ways of which two types of candidates can be selected to be constructed, one can see that the MINLP problem consists of 73,572 variables and 392,278 constraints. The computational time and the amount of storage will be enormous. Therefore, the decomposition based method will be used for this problem.

3.7.2 Investment Problem Formulation

The investment problem of MTEP-AC-NSEC can be defined as (3.192)–(3.196).

$$\min z \tag{3.192}$$

subject to

$$\mathbf{T_m x_m} \leq \mathbf{e}_{np} \tag{3.193}$$

$$\mathbf{N_m x_m} \leq \mathbf{0} \tag{3.194}$$

$$-z + \mathbf{c}_{\mathbf{bm}}^{\mathrm{T}} \mathbf{x_m} \leq -\sum_{t=1}^{ns} OPF_t c_{\text{op,min}}^{(t)} \tag{3.195}$$

$$z \geq 0 \tag{3.196}$$

$$z \in \mathbb{R}, \quad \mathbf{x_m} \in \{0,\, 1\}^{nc.ns}$$

where $c_{\text{op,min}}^{(t)}$ is a minimum of geneartion cost of stage t, $\mathbf{x_m}$, $\mathbf{T_m}$, $\mathbf{N_m}$, and $\mathbf{c_{bm}}$ are defined as (3.72), (3.74), (3.75), and (3.78) in Section 3.5.

3.7.3 Operation Problem Formulation

Given the long term investment plan representing by $\bar{\mathbf{x}}_\mathbf{m}$, one can define the operation problem corresponding to stage t, scenario s as (3.197)–(3.199).

$$\min \mathbf{c}_{\mathbf{ac}}^{(t,s)\mathrm{T}} \mathbf{y}_{\mathbf{a}}^{(t,s)} \tag{3.197}$$

subject to

$$\mathbf{u}^{(t)} = \mathbf{E}^{(t)}\bar{\mathbf{x}}_\mathbf{m} \tag{3.198}$$

$$\mathbf{G}_\mathbf{a}^{(t,s)}\left(\mathbf{u}^{(t)}, \mathbf{v}^{(t,s)}, \boldsymbol{\delta}^{(t,s)}, \mathbf{p}_\mathbf{g}^{(t,s)}, \mathbf{q}_\mathbf{g}^{(t,s)}, \mathbf{d}_\mathbf{r}^{(t,s)}, \mathbf{d}_\mathbf{c}^{(t,s)}\right) \leq \mathbf{0} \tag{3.199}$$

$$\mathbf{u}^{(t)} \in \mathbb{R}^{nc}, \quad \mathbf{v}^{(t,s)},\, \boldsymbol{\delta}^{(t,s)},\, \mathbf{d}_\mathbf{r}^{(t,s)},\, \mathbf{d}_\mathbf{c}^{(t,s)} \in \mathbb{R}^{nb}, \quad \mathbf{p}_\mathbf{g}^{(t,s)},\, \mathbf{q}_\mathbf{g}^{(t,s)} \in \mathbb{R}^{ng}$$

The operation variable for stage t, scenario s is defined as below.

$$\mathbf{y}_\mathbf{a}^{(t,s)} = \begin{bmatrix} \mathbf{u}^{(t)\mathrm{T}} & \mathbf{v}^{(t,s)\mathrm{T}} & \boldsymbol{\delta}^{(t,s)\mathrm{T}} & \mathbf{p}_\mathbf{g}^{(t,s)\mathrm{T}} & \mathbf{q}_\mathbf{g}^{(t,s)\mathrm{T}} & \mathbf{d}_\mathbf{r}^{(t,s)\mathrm{T}} & \mathbf{d}_\mathbf{c}^{(t,s)\mathrm{T}} \end{bmatrix}^\mathrm{T} \tag{3.200}$$

and

$$\mathbf{c}_\mathbf{ac}^{(t,s)} = \begin{cases} \begin{bmatrix} \mathbf{0}_{nc}^\mathrm{T} & \mathbf{0}_{nb}^\mathrm{T} & \mathbf{0}_{nb}^\mathrm{T} & OPF_t\mathbf{c}_\mathbf{g}^\mathrm{T} & \mathbf{0}_{ng}^\mathrm{T} & \mathbf{0}_{2nb}^\mathrm{T} \end{bmatrix}^\mathrm{T} & \text{, if } s = 0 \\ \mathbf{0} & \text{, otherwise} \end{cases} \tag{3.201}$$

In a compact form, the operation problem can be stated as (3.202)–(3.204).

$$\min \mathbf{c}_\mathbf{ac}^{(t,s)\mathrm{T}}\mathbf{y}_\mathbf{a}^{(t,s)} \tag{3.202}$$

subject to

$$\mathbf{H}_\mathbf{a}\mathbf{y}_\mathbf{a}^{(t,s)} = \mathbf{E}^{(t)}\bar{\mathbf{x}}_\mathbf{m} \tag{3.203}$$

$$\mathbf{G}_\mathbf{a}^{(t,s)}\left(\mathbf{y}_\mathbf{a}^{(t,s)}\right) \leq \mathbf{0} \tag{3.204}$$

$$\mathbf{y}_\mathbf{a}^{(t,s)} \in \mathbb{R}^{ma}, \quad ma = nc + 4nb + 2ng$$

where $\mathbf{H}_\mathbf{a}$ is defined as (3.160).

3.7.4 Cut Generation

It can be obviously seen that there are several operation problems for the MTEP-AC-NSEC problem. In addition, the formulation of each operation problem is similar to the one of STEP-AC problem. Therefore, the process of cut generation of the MTEP-AC-NSEC problem can be performed in a similar manner of the one of STEP-AC problem described in Section 3.6.4.

With an investment plan, i.e. $\bar{\mathbf{x}}$, if all operation problems are feasible and the termination criterion defined as (3.25) is not satisfied, the optimality cut will be created by (3.205).

$$-z + \mathbf{c}_{\mathbf{bm}}^{\mathrm{T}}\mathbf{x}_{\mathbf{m}} + \sum_{t=1}^{ns}\left(\min_{\mathbf{y}_{\mathbf{a}}^{(t,0)}} L_{\mathrm{ac}}^{(t,0)}\left(\mathbf{x}_{\mathbf{m}}, \mathbf{y}_{\mathbf{a}}^{(t,0)}, \bar{\boldsymbol{\lambda}}_{\mathbf{H}}^{(t,0)}, \bar{\boldsymbol{\lambda}}_{\mathbf{G}}^{(t,0)}\right)\right) \leq 0 \tag{3.205}$$

where

$L_{\mathrm{ac}}^{(t,0)}\left(\mathbf{x}_{\mathbf{m}}, \mathbf{y}_{\mathbf{a}}^{(t,0)}, \boldsymbol{\lambda}_{\mathbf{H}}^{(t,0)}, \boldsymbol{\lambda}_{\mathbf{G}}^{(t,0)}\right)$ is Lagrange function corresponding to the operation problem for stage t, base case scenario.

$\bar{\boldsymbol{\lambda}}_{\mathbf{H}}^{(t,0)}$ and $\bar{\boldsymbol{\lambda}}_{\mathbf{G}}^{(t,0)}$ are the obtained Lagrange multipliers according to (3.203) and (3.204) respectively.

In the same manner of STEP-AC problem, the explicit form of (3.205) can be expressed as (3.206).

$$-z + \mathbf{c}_{\mathbf{bm}}^{\mathrm{T}}\mathbf{x}_{\mathbf{m}} + \sum_{t=1}^{ns}\left(\mathbf{c}_{\mathbf{ac}}^{(t,0)\mathrm{T}}\bar{\mathbf{y}}_{\mathbf{a}}^{(t,0)} + \bar{\boldsymbol{\lambda}}_{\mathbf{H}}^{(t,0)\mathrm{T}}\left(\bar{\mathbf{x}}_{\mathbf{m}} - \mathbf{x}_{\mathbf{m}}\right)\right) \leq 0 \tag{3.206}$$

In the case of the feasibility cut, one can obtain the explicit form by solving the feasibility problem, which will be defined as (3.207)–(3.210), for each scenario in which the system is infeasible.

$$\min \mathbf{e}_{nb}^{\mathrm{T}}\mathbf{p}_{\mathbf{s}}^{(t,s)} \tag{3.207}$$

subject to

$$\mathbf{H}_{\mathbf{a}}\mathbf{y}_{\mathbf{a}}^{(t,s)} = \mathbf{E}^{(t)}\bar{\mathbf{x}}_{\mathbf{m}} \tag{3.208}$$

$$\mathbf{G}_{\mathbf{eq}}^{(t,s)}\left(\mathbf{y}_{\mathbf{a}}^{(t,s)}\right) + \mathbf{S}^{(t)}\mathbf{p}_{\mathbf{s}}^{(t,s)} = \mathbf{0} \tag{3.209}$$

$$\mathbf{G}_{\mathbf{in}}^{(t,s)}\left(\mathbf{y}_{\mathbf{a}}^{(t,s)}\right) \leq \mathbf{0} \tag{3.210}$$

$$\mathbf{y}_{\mathbf{a}}^{(t,s)} \in \mathbb{R}^{ma}, \quad \mathbf{p}_{\mathbf{s}}^{(t,s)} \in \mathbb{R}^{nb}$$

where

$$\mathbf{S}^{(t)} = \begin{bmatrix} \mathbf{I}_{nb} \\ \mathbf{R}_{\mathbf{q}}^{(t)} \end{bmatrix} \tag{3.211}$$

$\mathbf{R}_{\mathbf{q}}^{(t)}$ is a diagonal matrix of which the elements represent the ratios between reactive and active power demand for stage t.

Let $\mathcal{U}^{(s)}$, $s = 0,\, 1,\, \ldots,\, nv$, is an index set of stages in which the system for the scenario s is infeasible. After all feasibility problems corresponding to all stages in the set $\mathcal{U}^{(s)}$ are solved, the obtained Lagrange multipliers, $\bar{\boldsymbol{\mu}}_{\mathbf{H}}^{(t,s)}$, $t \in \mathcal{U}^{(s)}$, $s = 0,\, 1,\, \ldots,\, nv$, are used in generation of the feasibility cuts as (3.212).

$$\min_{\mathbf{y}_{\mathbf{a}}^{(t,s)}} L_{\text{fe}}\left(\mathbf{x}_{\mathbf{m}}, \mathbf{y}_{\mathbf{a}}^{(t,s)}, \bar{\boldsymbol{\mu}}_{\mathbf{H}}^{(t,s)}, \bar{\boldsymbol{\mu}}_{\mathbf{Ge}}^{(t,s)}, \bar{\boldsymbol{\mu}}_{\mathbf{Gi}}^{(t,s)}\right) \leq 0, \qquad (3.212)$$
$$t \in \mathcal{U}^{(s)}, \quad \mathcal{U}^{(s)} \neq \emptyset, \quad s = 0,\, 1, \ldots, nv$$

where

$L_{\text{fe}}\left(\mathbf{x}_{\mathbf{m}}, \mathbf{y}_{\mathbf{a}}^{(t,s)}, \bar{\boldsymbol{\mu}}_{\mathbf{H}}^{(t,s)}, \bar{\boldsymbol{\mu}}_{\mathbf{Ge}}^{(t,s)}, \bar{\boldsymbol{\mu}}_{\mathbf{Gi}}^{(t,s)}\right)$ is a Lagrange function corresponding to the feasibility problem for stage t, scenario s, and

$\bar{\boldsymbol{\mu}}_{\mathbf{H}}$, $\bar{\boldsymbol{\mu}}_{\mathbf{Ge}}$ and $\bar{\boldsymbol{\mu}}_{\mathbf{Gi}}$ are the obtained Lagrange multipliers according to (3.208), (3.209) and (3.210) respectively.

Therefore, the explicit form of the feasibility cut can be expressed as (3.213).

$$\mathbf{e}_{nb}^{\mathrm{T}} \bar{\mathbf{p}}_{\mathbf{s}}^{(t,s)} + \bar{\boldsymbol{\mu}}_{\mathbf{H}}^{(t,s)\mathrm{T}} \left(\bar{\mathbf{x}}_{\mathbf{m}} - \mathbf{x}_{\mathbf{m}}\right) \leq 0 \qquad (3.213)$$
$$t \in \mathcal{U}^{(s)}, \quad \mathcal{U}^{(s)} \neq \emptyset, \quad s = 0,\, 1, \ldots, nv$$

where $\bar{\mathbf{p}}_{\mathbf{s}}^{(t,s)}$ can be obtained from solving the feasibility problem for stage t, scenario s.

It should be noted that the number of feasibility cuts is extremely high, possibly causing difficulty in solving the investment problem, especially when the iteration number is large. Therefore, the cuts for each scenario will be aggregated as (3.214).

$$\sum_{t \in \mathcal{U}^{(s)}} \left(\mathbf{e}_{nb}^{\mathrm{T}} \bar{\mathbf{p}}_{\mathbf{s}}^{(t,s)} + \bar{\boldsymbol{\mu}}_{\mathbf{H}}^{(t,s)\mathrm{T}} \left(\bar{\mathbf{x}}_{\mathbf{m}} - \mathbf{x}_{\mathbf{m}}\right)\right) \leq 0 \qquad (3.214)$$
$$\mathcal{U}^{(s)} \neq \emptyset, \quad s = 0,\, 1, \ldots, nv$$

3.7.5 Computational Procedure

The computational procedure for solving MTEP-AC-NSEC problem can be summarized as below.

Step 0: Set the iteration counter, k to one, and UBD_0 to infinity. Define the value of ϵ.

Step 1: Solve the investment problem. Set the value of the lower bound, LBD_k to $\bar{z}$.

Step 2: From the minimizer, with $\bar{\mathbf{x}}_{\mathbf{m}}$ obtained from Step 1, define and solve the operation problems for all stages and scenarios. If the problems are all feasible, the minimizer, $\bar{\mathbf{y}}_{\mathbf{a}}^{(t,0)}$ and Lagrange multiplier, $\bar{\boldsymbol{\lambda}}_{\mathbf{H}}^{(t,0)}$, $t = 1, \ldots, ns$ can be obtained. Then the value of the upper bound is updated by (3.215).

$$UBD_k = \min\left\{UBD_{k-1},\, \mathbf{c}_{\mathbf{bm}}^{\mathrm{T}}\bar{\mathbf{x}}_{\mathbf{m}} + \sum_{t=1}^{ns}\left(\mathbf{c}_{\mathbf{ac}}^{(t,0)\mathrm{T}}\bar{\mathbf{y}}_{\mathbf{a}}^{(t,0)}\right)\right\} \tag{3.215}$$

If the termination criterion defined as (3.25) is satisfied, the process is terminated, and $\left(\bar{\mathbf{x}}_{\mathbf{m}},\, \bar{\mathbf{y}}_{\mathbf{a}}^{(t,s)}\right)$, $t = 1, \ldots, ns$; $s = 0, 1, \ldots, nv$ will be the solution of the MTEP-AC-NSEC problem, otherwise the optimality cut defined as (3.206) will be created.

In case some operation problems are infeasible, the feasibility cuts defined as (3.214) will be created.

Step 3: After obtaining either optimality or feasibility cut from Step 2, add it into the investment problem, increase the iteration counter by one, i.e. $k = k + 1$, and go to Step 1.

3.8 Conclusion

This chapter has described the proposed method for solving both single stage and multistage TEP. The formulation has been developed to tackle the following problems, i.e.

(a) Single stage TEP problem based on the DC model,

(b) Single stage TEP problem based on the DC model with N-1 security constraints,

(c) Multistage TEP problem based on the DC model,

(d) Single stage TEP problem based on the AC model, and

(e) Multistage TEP problem based on the AC model with N-1 security constraints.

In solving a practical TEP, the computational time may be highly required. However, it may be yield a global optimum solution due to its nonconvexity. Improvement on both computational time and solution quality of the proposed method will be presented in the next chapter.

CHAPTER IV

IMPROVEMENT OF DEVELOPED METHODOLOGY

In general, the method proposed in the previous chapter is the basic framework which can be used for solving the most comprehensive TEP, i.e. multistage transmission expansion planning problem with N-1 security constraints. However, in the case of application for large scale multistage TEP problems, the proposed method should be further improved to increase its calculation performance together with acceptable quality of the obtained solutions.

Regarding performance, it can be seen that the main burden of the calculation procedure is the computational time in solving the investment problem. To reduce this burden, a local search technique will be applied. Even though the local search is a simple heuristic method, with a good initial solution and a suitable defined neighborhood, it can find a good quality local optimum solution with less computational time than other methods [**54**]. Therefore, it will be applied in this dissertation. However, it will be used only in the intermediate iterations. For the final iteration, a complete search space will be performed.

Another concern is the quality of the obtained solution due to nonconvexity of the operation problem. Since the GBD method assumes the operation problem to be convex to achieve a global optimum solution. Therefore, in case of the TEP based on AC model, the proposed method will return only a local optimum plan which cannot guarantee the global optimum from mathematical point of view. The main reason is that the cuts added into the investment problem may exclude some feasible plans, which may be of better quality. Therefore, the cut should be modified in order that they will expand the feasible region of the investment problem. This technique will be performed by automatic procedure in this dissertation.

The first section of this chapter will present the local search technique which can be applied to solve the large scale problem proposed in the previous chapter. In the next section, the cut modification technique will be introduced.

4.1 Local Search Application

The local search technique [**54**, **55**] is a simple heuristic method. It is normally applied in various applications. In this dissertation, the local search is applied to solve the investment problem of MTEP-AC-NSEC problem. This technique is based on the concept that the minimizer obtained from solving the investment problem in each iteration, which is not the final iteration, is only used to create cuts from the operation problem. Therefore, it is not necessary to obtain the global minimizer from the investment problem. However, the quality of the local minimizer should be fairly good since it will impact the quality of the cuts, which consequently has the effect on speed of convergence of the GBD process.

4.1.1 Basic Concept

The local search is performed in a searching subspace which guarantees that a good quality local minimizer of the investment problem can be found. To achieve this objective, the investment problem of the MTEP-AC-NSEC problem will be simplified to the single stage problem considering the whole planning period as a single period. In addition, the nonlinearity of investment cost function is neglected in the simplified problem. The solution obtained from solving the simplified problem gives the information concerning with the path needed in the investment. From this information, the original investment problem is solved to gain the benefit of the multistage planning. However, in this case, the local search is performed, i.e. only the plans consisting of the candidates in those paths are searched.

After obtaining the local minimizer of the investment problem, the operation problems is defined and then the cuts are created as the process described in the previous chapter. The local search is performed in every iteration until the termination criterion (3.25) is satisfied. After that the investment problem will be solved completely to find the global minimizer.

4.1.2 Mathematical Formulation

The investment problem of the MTEP-AC-NSEC problem can be expressed as (4.1)–(4.3).

$$\min z \tag{4.1}$$

subject to

$$\mathbf{C}\mathbf{x_m} + \mathbf{f}z \leq \mathbf{t} \tag{4.2}$$

$$z \geq 0 \tag{4.3}$$

$$z \in \mathbb{R}, \quad \mathbf{x_m} \in \{0,\, 1\}^{nc.ns}$$

where $\mathbf{C}$ is composed of the initial constraints of the investment problem defined as (3.193)–(3.195) including the added cuts, $\mathbf{t}$ is the right-hand side of the constraints, and $\mathbf{f}$ is defined as below.

$$f_i = \begin{cases} -1 & \text{, if row } i \text{ of } \mathbf{C} \text{ corresponds to the optimality cut and} \\ & \text{ the constraint (3.195)} \\ 0 & \text{, otherwise} \end{cases} \tag{4.4}$$

To determine potential paths, the investment problem of MTEP-AC-NSEC will be simplified to a single stage problem by considering the whole planning interval as a single period. Mathematically, the candidates are constrained to be selected in the only first stage. Therefore, the columns of $\mathbf{C}$ corresponding to the second stage to the last stage will be neglected. The simplified investment problem can expressed as below.

$$\min z \tag{4.5}$$

subject to

$$\mathbf{C}\mathbf{E}^{(1)\mathrm{T}}\mathbf{x}^{(1)} + \mathbf{f}z \leq \mathbf{t} \tag{4.6}$$

$$z \geq 0 \tag{4.7}$$

$$z \in \mathbb{R}, \quad \mathbf{x}^{(1)} \in \{0,\, 1\}^{nc}$$

Now the number of the binary variables is reduced. In addition, there are various types of the candidates in each path. Even though the number of circuits and the towers used for the construction of the candidates in each path may be different, the electrical parameters for each circuit can be assumed to be the same value if they are strung with the same type of conductor. To further reduce the burden of the investment problem for determining the potential paths, the set of all candidates, $\mathcal{C}$ will be partitioned into subsets. Each subset contains the candidates in the same path which have the identical parameters for each circuit. Then, the representative candidate is selected for each subset. Suppose the number of the representative candidate is nt, the index set of the representative, $\mathcal{R}$ can be defined as below.

$$\mathcal{R} = \{1, \ldots, nt\} \tag{4.8}$$

Therefore, the subset which contains the representative i is referred to $\mathcal{T}_i, i \in \mathcal{R}$.

After that the matrix, $\mathbf{K} \in \mathbb{R}^{nt \times nc}$ can be defined as below.

$$K_{i,j} = \begin{cases} 1 & \text{, if the candidate } j \text{ is the representative } i \\ 0 & \text{, otherwise} \end{cases}$$

Now one can define the simplified investment problem as below.

$$\min z \tag{4.9}$$

subject to

$$\mathbf{C_r x_r} + \mathbf{f} z \leq \mathbf{t} \tag{4.10}$$

$$\mathbf{0} \leq \mathbf{x_r} \leq \mathbf{x_r^{max}} \tag{4.11}$$

$$z \geq 0 \tag{4.12}$$

$$z \in \mathbb{R}, \quad \mathbf{x_r} \in \mathbb{Z}^{nt}$$

where

$$\mathbf{C_r} = \mathbf{C}\mathbf{E}^{(1)\mathrm{T}}\mathbf{K}^{\mathrm{T}} \tag{4.13}$$

$$(x_{\mathrm{r}}^{\mathrm{max}})_i = \max_{j \in \mathcal{T}_i} \{n_j\}, \qquad i \in \mathcal{R} \tag{4.14}$$

$$\mathbf{x}^{(1)} = \mathbf{K}^{\mathrm{T}}\mathbf{x_r} \tag{4.15}$$

n_j is the number of circuits of the candidate j.

After solving the problem (4.9)–(4.12), the minimizer, $\bar{\mathbf{x}}_\mathbf{r}$ indicates the potential representative candidates which can be used to determine the potential paths.

Let $\mathcal{P}_\mathrm{t} \subset \mathcal{P}$ is an index set of the potential paths obtained from solving the simplified investment problem defined as (4.9)–(4.12). Now the complete investment problem defined as (4.1)–(4.3) is solved with the restriction of search space on the potential paths. Mathematically, the constraint (4.16) is added into the investment problem.

$$(x_m)_i = 0 \tag{4.16}$$

where $i = 1, \ldots, nc.ns$, $(x_m)_i$ is the decision variable corresponding to the candidate branch in path $p \in \mathcal{P} \backslash \mathcal{P}_{\mathrm{t}}$.

The minimizer obtained from solving the above problem will then be used in the cut generation process.

It should be noted that there are two MILP problems which have to be solved in the proposed local search procedure. However, the computational burden is less than that of solving the original investment problem, since the first MILP problem is the simplified investment problem and the second one is the investment problem with restricted search space.

4.2 Modification of Cuts to Handle Nonconvexity

Since the operation problems of MTEP-AC-NSEC are nonconvex, the global optimality condition of the GBD cannot be held [**43**, **50**]. It seems that this is an inevitable characteristics of the TEP problem when the AC model is applied. To overcome this difficulty without relaxing the model of TEP problem, global optimization algorithms [**19**, **56**] should be employed. Most of the global optimizations of MINLP problems apply spatial branch and bound techniques. For example, in Ref. [**19**], the feasible region is recursively partitioned into subregion. Then, a convex underestimation [**57**, **58**] for the objective function and the constraints is applied. After that the convex underestimated problem can be defined for each subregion, and the problem can be solved by convex optimization algorithms. The partition of region will be proceeded until the solution of convex underestimated problem is equal to the solution of the original problem in the domain restricted to the considered region. Then the global optimum for this subregion is obtained. The process of the spatial branch and bound controls the region partitioning and selects the best global optimum from the ones obtained in the partitioned subregions. For more details, one can refer to Ref. [**19**, **57**–**59**].

It should be note that there are several MINLP problems defined according to the partitioned subregions of which the number may be high in the case of large dimensional search space. Therefore, the algorithms based on this concept usually suffer from a curse of dimensionality which limits they to small scale problems.

In a practical TEP, a good quality local optimum plan might be expected. Therefore, the developed methods for solving the TEP problem should have some mechanisms to deal with the nonconvexity of the problem in order that one of good quality plan can be obtained. In this dissertation, the modification of cuts is considered for handling the nonconvexity.

4.2.1 Basic Concept

In the GBD process, the master problem, i.e. the investment problem, acts as an estimator of the optimum solution by using cuts sent from the operation problems. The feasibility cuts contain information used to correct the current solution in order that it should be feasible in the next iteration, whereas the optimality cuts contain the information for updating the current solution to the better one, i.e. lower the value of the objective function. In case of a convex problem, the solution is guaranteed to be converged to the global optimum by coordinating the feasibility and optimality cuts. However, in case of a nonconvex problem, the cuts may overestimate the infeasibility obtained from solving the feasibility problem, causing the global optimum solution, including some of local optimum ones, are neglected from the feasible region. Therefore, the global optimality cannot be guaranteed.

This characteristic can be illustrated by Figure 4.1, in the case of the convex problem. It should be noted that this illustration is only an analogy of the TEP problem. The investment plan is depicted as the point on the horizontal axis. The left vertical axis indicates the infeasibility (solid curve) obtained from solving the feasibility problem, while the right vertical axis indicates the total cost (dashed curve), i.e. the investment and operation costs, which can be determined from any feasible plans, located on the right of point E. From the figure, one can see that when the level of investment is increased, the infeasibility will be lower since the violation of planning criteria can be alleviated and the total cost will be decreased since the transmission congestion and active power loss are reduced. However, in the case of the total cost, if the level of investment is greater than the level specified at point L, it will be increased due to the overinvestment.

At point A, the problem is infeasible. Therefore, the feasibility cut is created, and then it estimates the investment plan expected that the problem should be feasible at the point B. Since the problem is strictly convex, the cut always underestimates the infeasibility. Therefore, the problem is still infeasible, and the feasibility cut is generated again to estimate the feasible plan to point D, but still infeasible. However, after the process is performed in this iterative manner, the investment plan will be converged to point E, which is a feasible plan.

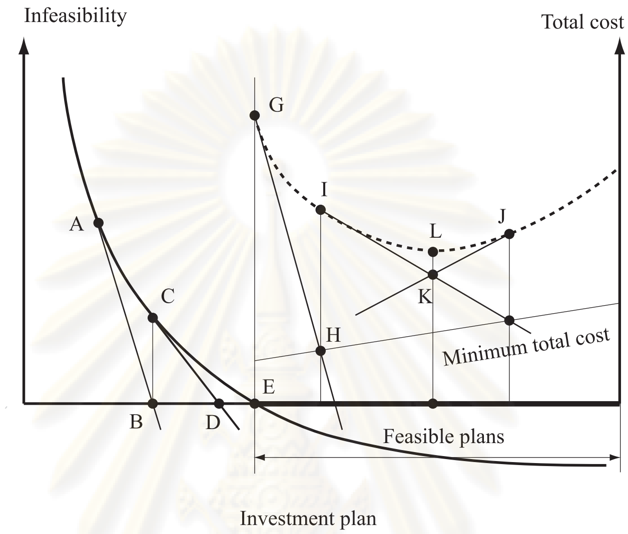

Figure 4.1 Underestimation of cut for convex problem

After the feasible plan is found, the operation problem can be solved. Then, the total cost depicted by point G in the figure can be obtained. After that the optimality cut is created to estimate the lower total cost. A feasible plan together with estimated total cost at point H can be obtained. As in the case of the feasibility cut, the optimality cut always underestimates the total cost for a convex problem. Therefore, after solving the operation problem, the total cost is actually at point I. By performing the process in this manner, the total cost will finally be converged to the optimum solution at point L.

On the other hand, in case of the nonconvex problem, the feasibility cut may overestimate the infeasibility obtained from solving the feasibility problem, leaving some feasible plans out of consideration, as illustrated in Figure 4.2. When the feasibility cut is generated at point A, it estimates that the plans located between point E and point B are infeasible. Therefore, the global optimal plan (point D) is excluded from the set of the feasible plans. At point B, the operation problem is defined and solved. The total cost can be obtained at point C. After that the optimality cut is created to estimate the lower total cost. However, point B is on the bound of estimated feasible plans. Therefore, the next estimated total cost is still at point C.

Then the process is terminated since the lower bound, i.e. the estimated total cost obtained from investment problem, is equal to the upper bound, i.e. the actual total cost. The solution is located at point C which is a local optimum solution.

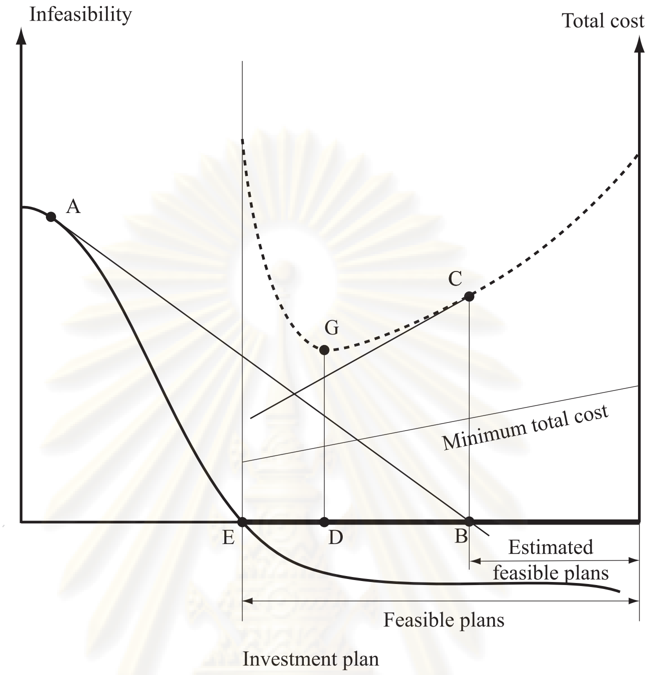

Figure 4.2 Overestimation of cut for nonconvex problem

The overestimation of cut illustrated in Figure 4.2 is pointed out in the only case of the feasibility cut. However, it can also occur in the case of the optimality cut.

To handle this characteristic of MTEP-AC-NSEC problem, the cuts should be modified in order that it should estimate the infeasibility and the total cost to be lower than the actual values. In general, this approach is similar to the one applied in the damped Newton method [**60**]. This concept can be illustrated in Figure 4.3. One can see that the cuts are dropped. Therefore, the feasibility cut generated at point A estimates that the plan at point H is feasible. However, after solving the feasibility problem, it is found that the plan at point H is actually infeasible. Therefore the dropped feasibility cut is created again. Now it estimates that the plan at point I is feasible. After this process is iteratively performed, the feasible plan will be converged to point E.

When the feasible plan is found, the operation problem can be defined and solved. The total cost can be obtained at point J. After that the optimality cut is generated to estimate the lower total cost which is at point K. However, when solving the operation problem, the actual total cost is located at point L. Therefore, the optimality cut is created again. Finally, the solution is converged at point G, which is the global optimum solution.

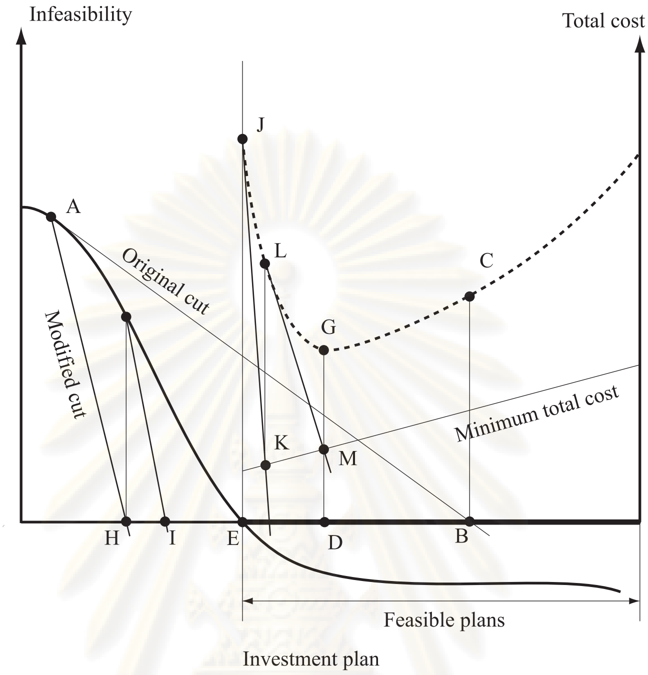

Figure 4.3 Modification of cuts

The optimality cuts can also be modified as in the case of the feasibility cuts. However, Figure 4.3 illustrates only the modification of the feasibility cuts.

4.2.2 Mathematical Formulation

From Section 3.7.4, the feasibility cut is generated when the investment plan obtained from solving the investment problem causes the power system is infeasible. The explicit form of cut can be written by (4.17).

$$\sum_{t \in \mathcal{U}^{(s)}} \left(\mathbf{e}_{nb}^{\mathrm{T}} \bar{\mathbf{p}}_{\mathbf{s}}^{(t,s)} + \bar{\boldsymbol{\mu}}_{\mathbf{H}}^{(t,s)\mathrm{T}} \left(\bar{\mathbf{x}}_{\mathbf{m}} - \mathbf{x}_{\mathbf{m}} \right) \right) \leq 0 \tag{4.17}$$
$$\mathcal{U}^{(s)} \neq \emptyset, \quad s = 0, 1, \ldots, nv$$

where $\mathcal{U}^{(s)}$, $s = 0, 1, \ldots, nv$, is an index set of stages in which the system for the scenario s is infeasible.

To modify the cuts based on the concept presented in the previous subsection, the Lagrange multipliers are multiplied by a constant as below.

$$\sum_{t \in \mathcal{U}^{(s)}} \left(\mathbf{e}_{nb}^{\mathrm{T}} \bar{\mathbf{p}}_{\mathbf{s}}^{(t,s)} + \left(\frac{1}{1-\alpha} \right) \bar{\boldsymbol{\mu}}_{\mathbf{H}}^{(t,s)\mathrm{T}} \left(\bar{\mathbf{x}}_{\mathbf{m}} - \mathbf{x}_{\mathbf{m}} \right) \right) \leq 0 \tag{4.18}$$

$$\mathcal{U}^{(s)} \neq \emptyset, \quad s = 0, 1, \ldots, nv$$

$$0 \leq \alpha < 1$$

where α is a parameter which controls the cut dropping. If $\alpha = 0$, the cuts will not be modified. When the value α is increased, the degree of modification is increased.

In the same manner of the feasibility cut modification, the modified optimality cut can be expressed as (4.19).

$$-z + \mathbf{c}_{\mathbf{bm}}^{\mathrm{T}} \mathbf{x}_{\mathbf{m}} + \sum_{t=1}^{ns} \left(\mathbf{c}_{\mathbf{ac}}^{(t,0)\mathrm{T}} \bar{\mathbf{y}}_{\mathbf{a}}^{(t,0)} + \left(\frac{1}{1-\alpha} \right) \bar{\boldsymbol{\lambda}}_{\mathbf{H}}^{(t,0)\mathrm{T}} \left(\bar{\mathbf{x}}_{\mathbf{m}} - \mathbf{x}_{\mathbf{m}} \right) \right) \leq 0 \tag{4.19}$$

$$0 \leq \alpha < 1$$

With the proposed method for cut modification, it is expected that the quality of the obtained local optimum plan will be improved. However, the step size of the search process in the GBD framework may be damped, causing higher number of iterations. Therefore, the value of α should be selected according to the compromising between the solution quality and the computational time which should be based on the judgment of the users.

However, the value of α can be automatically changed to the value depending on each cut when the algorithm finds that the defined value is not large enough. There are three situations in which the algorithm will change the value of α as follows:

(a) The feasible plans found so far are cut from the set of feasible plan estimated by the current feasibility cuts.

(b) The total costs of the feasible plans found so far are overestimated by the current optimality cuts.

(c) The investment problem is infeasible

In the first situation, each feasibility cut, which corresponds to each scenario, generated in each iteration will be verified by (4.20).

$$\sum_{t\in\mathcal{U}^{(s)}} \left(\mathbf{e}_{nb}^{\mathrm{T}} \bar{\mathbf{p}}_{\mathbf{s}}^{(t,s)} + \left(\frac{1}{1-\alpha}\right) \bar{\boldsymbol{\mu}}_{\mathbf{H}}^{(t,s)\mathrm{T}} \left(\bar{\mathbf{x}}_{\mathbf{m}} - \hat{\mathbf{x}}_{\mathbf{m}}\right) \right) \leq 0 \tag{4.20}$$

$$\forall\, \hat{\mathbf{x}}_{\mathbf{m}} \in \mathcal{F}, \qquad s = 0,\, 1,\, \ldots,\, nv, \quad \mathcal{U}^{(s)} \notin \emptyset$$

where $\mathcal{F}$ is a set of all feasible plans found so far. It should be noted that $\mathcal{F}$ is updated for every iteration.

If (4.20) is not satisfied for some scenario s, the value of α corresponding to these scenario should be modified in order that (4.20) is met as below.

$$\alpha^{(s)} \geq \max_{\hat{\mathbf{x}}_{\mathbf{m}}\in\mathcal{F}} \left\{ \frac{\sum_{t\in\mathcal{U}^{(s)}} \bar{\boldsymbol{\mu}}_{\mathbf{H}}^{(t,s)\mathrm{T}} \left(\bar{\mathbf{x}}_{\mathbf{m}} - \hat{\mathbf{x}}_{\mathbf{m}}\right)}{\sum_{t\in\mathcal{U}^{(s)}} \mathbf{e}_{nb}^{\mathrm{T}} \bar{\mathbf{p}}_{\mathbf{s}}^{(t,s)}} + 1 \right\} \tag{4.21}$$

Since $0 \leq \alpha < 1$, the inequality (4.21) is insisted that

$$\sum_{t\in\mathcal{U}^{(s)}} \bar{\boldsymbol{\mu}}_{\mathbf{H}}^{(t,s)\mathrm{T}} \left(\bar{\mathbf{x}}_{\mathbf{m}} - \hat{\mathbf{x}}_{\mathbf{m}}\right) < 0, \quad \forall\, \hat{\mathbf{x}}_{\mathbf{m}} \in \mathcal{F} \tag{4.22}$$

In case the inequality (4.22) is not satisfied for some cuts, these cut should be discarded. Therefore, they will be replaced by the constraint (4.23) to cut the current plan, i.e. $\bar{\mathbf{x}}_{\mathbf{m}}$, which is an infeasible plan, from the set of feasible plans.

$$\sum_{j\in\mathcal{B}\backslash\bar{\mathcal{S}}} \left(x_m\right)_j + \sum_{j\in\bar{\mathcal{S}}} \Big(1 - \left(x_m\right)_j\Big) \geq 1 \tag{4.23}$$

where $\mathcal{B} = \{1,\, \ldots,\, nc.ns\}$, and $\bar{\mathcal{S}} = \Big\{ j \in \mathcal{B},\, \left(\bar{x}_m\right)_j = 1 \Big\}$.

After the feasibility cuts are modified, they will be added to the investment problem.

The cut modification in the second situation is similar to the first situation, however the optimality cuts is considered instead of the feasibility cuts. In this case, the current optimality cut will be verified by (4.24).

$$-\hat{z} + \mathbf{c}_{\mathbf{bm}}^{\mathrm{T}}\hat{\mathbf{x}}_{\mathbf{m}} + \sum_{t=1}^{ns}\left(\mathbf{c}_{\mathbf{ac}}^{(t,0)\mathrm{T}}\bar{\mathbf{y}}_{\mathbf{a}}^{(t,0)} + \left(\frac{1}{1-\alpha}\right)\bar{\boldsymbol{\lambda}}_{\mathbf{H}}^{(t,0)\mathrm{T}}\left(\bar{\mathbf{x}}_{\mathbf{m}} - \hat{\mathbf{x}}_{\mathbf{m}}\right)\right) \leq 0 \qquad (4.24)$$
$$\forall\ \hat{\mathbf{x}}_{\mathbf{m}} \in \mathcal{F}$$

where $\hat{z}$ is the minimizer of the investment problem corresponding to $\hat{\mathbf{x}}_{\mathbf{m}}$.

If (4.24) is not satisfied, the value of α will be modified as below.

$$\alpha \geq \max_{\hat{\mathbf{x}}_{\mathbf{m}}\in\mathcal{F}}\left\{\frac{\sum_{t=1}^{ns}\bar{\lambda}_{\mathbf{H}}^{(t,0)\mathrm{T}}\left(\bar{\mathbf{x}}_{\mathbf{m}} - \hat{\mathbf{x}}_{\mathbf{m}}\right)}{-\hat{z} + \mathbf{c}_{\mathbf{bm}}^{\mathrm{T}}\hat{\mathbf{x}}_{\mathbf{m}} + \sum\limits_{t=1}^{ns}\mathbf{c}_{\mathbf{ac}}^{(t,0)\mathrm{T}}\bar{\mathbf{y}}_{\mathbf{a}}^{(t,0)}} + 1\right\} \qquad (4.25)$$

It should be noted that before the value of α will be modified, the following conditions should be checked.

$$\sum_{t\in\mathcal{U}^{(s)}}\bar{\lambda}_{\mathbf{H}}^{(t,s)\mathrm{T}}\left(\bar{\mathbf{x}}_{\mathbf{m}} - \hat{\mathbf{x}}_{\mathbf{m}}\right) < 0, \quad \forall\,\hat{\mathbf{x}}_{\mathbf{m}} \in \mathcal{F} \qquad (4.26)$$

$$-\hat{z} + \mathbf{c}_{\mathbf{bm}}^{\mathrm{T}}\hat{\mathbf{x}}_{\mathbf{m}} + \sum_{t\in\mathcal{U}}\mathbf{c}_{\mathbf{ac}}^{(t,0)\mathrm{T}}\bar{\mathbf{y}}_{\mathbf{a}}^{(t,0)} > 0, \quad \forall\,\hat{\mathbf{x}}_{\mathbf{m}} \in \mathcal{F} \qquad (4.27)$$

Since $0 \leq \alpha < 1$, if the condition (4.26) or (4.27) is not satisfied, the value of α cannot be modified. This situation indicates that the cut generated at the current feasible plan, $\bar{\mathbf{x}}_{\mathbf{m}}$ conflicts with the feasible plan, $\hat{\mathbf{x}}_{\mathbf{m}}$ obtained from the previous iterations. Therefore, the total cost of the current feasible plan, $\bar{\mathbf{x}}_{\mathbf{m}}$ will be compared with that of the plan, $\hat{\mathbf{x}}_{\mathbf{m}}$. If the total cost of the current feasible plan is lower, the cut will not be modified and normally added to the investment problem in the next step, otherwise the cut will be discarded and the constraint (4.23) will be added to the investment problem.

However, if there is no the feasible plan found until the current iteration, the situation (c), i.e. the investment problem is infeasible, can occur. It should be noted that this is the worse case which does not frequently occur, however the method for handling this situation is developed in this dissertation.

To describe the proposed method, the investment problem will be expressed as (4.28)–(4.31).

$$\min z \tag{4.28}$$

subject to

$$\mathbf{T_m x_m} \leq \mathbf{e}_{np} \tag{4.29}$$

$$\mathbf{N_m x_m} \leq \mathbf{0} \tag{4.30}$$

$$-z + \mathbf{c}_{\mathbf{bm}}^{\mathrm{T}} \mathbf{x_m} \leq -\sum_{t=1}^{ns} OPF_t c_{\text{op,min}}^{(t)} \tag{4.31}$$

$$\mathbf{C_f x_m} \leq \mathbf{t_f} \tag{4.32}$$

$$z \geq 0 \tag{4.33}$$

$$z \in \mathbb{R}, \quad \mathbf{x_m} \in \{0,\, 1\}^{nc.ns}$$

where (4.29)–(4.31) are initial constraints of the investment problem, and (4.32) represents the feasibility cuts.

Considering the problem (4.28)–(4.33), one can see that the problem can be infeasible by the only constraint (4.32) since the constraints (4.29)–(4.31) are the initial constraints. Therefore, when the investment problem is infeasible, the problem (4.34)–(4.40) is defined and solved.

$$\min \zeta \tag{4.34}$$

subject to

$$\mathbf{T_m x_m} \leq \mathbf{e}_{np} \tag{4.35}$$

$$\mathbf{N_m x_m} \leq \mathbf{0} \tag{4.36}$$

$$-z + \mathbf{c}_{\mathbf{bm}}^{\mathrm{T}} \mathbf{x_m} \leq -\sum_{t=1}^{ns} OPF_t c_{\text{op,min}}^{(t)} \tag{4.37}$$

$$\mathbf{C_f x_m} - \mathbf{e}_{nf} \zeta \leq \mathbf{t_f} \tag{4.38}$$

$$z \geq 0 \tag{4.39}$$

$$\zeta \geq 0 \tag{4.40}$$

$$z,\, \zeta \in \mathbb{R}, \quad \mathbf{x_m} \in \{0,\, 1\}^{nc.ns}$$

The minimizer, $\mathbf{x}_{\mathbf{m}}^{*}$ obtained from solving the problem (4.34)–(4.40) will be used for the modification of the feasibility cuts in the investment problem as described below.

All feasibility cuts added to the investment problem until the iteration K can be written as (4.41).

$$\sum_{t\in\mathcal{U}^{(s,k)}}\left(\mathbf{e}_{nb}^{\mathrm{T}}\bar{\mathbf{p}}_{\mathbf{s}}^{(t,s,k)}+\left(\frac{1}{1-\alpha}\right)\bar{\boldsymbol{\mu}}_{\mathbf{H}}^{(t,s,k)\mathrm{T}}\left(\bar{\mathbf{x}}_{\mathbf{m}}^{(k)}-\mathbf{x}_{\mathbf{m}}\right)\right)\leq 0 \tag{4.41}$$
$$k=1,\ldots,K,\qquad s=0,1,\ldots,nv,\quad \mathcal{U}^{(s,k)}\neq\emptyset$$

The feasibility cuts will be modified on the assumption that the minimizer, $\mathbf{x}_{\mathbf{m}}^{*}$, of the problem (4.34)–(4.37) might be a feasible solution. Therefore, all the cuts will be verified at $\mathbf{x}_{\mathbf{m}}^{*}$ by (4.42).

$$\sum_{t\in\mathcal{U}^{(s,k)}}\left(\mathbf{e}_{nb}^{\mathrm{T}}\bar{\mathbf{p}}_{\mathbf{s}}^{(t,s,k)}+\left(\frac{1}{1-\alpha}\right)\bar{\boldsymbol{\mu}}_{\mathbf{H}}^{(t,s,k)\mathrm{T}}\left(\bar{\mathbf{x}}_{\mathbf{m}}^{(k)}-\mathbf{x}_{\mathbf{m}}^{*}\right)\right)\leq 0 \tag{4.42}$$
$$k=1,\ldots,K,\qquad s=0,1,\ldots,nv,\quad \mathcal{U}^{(s,k)}\neq\emptyset$$

If some feasibility cuts are not satisfied. The value of α corresponding to these cuts should be changed by (4.43).

$$\alpha^{(s,k)}\geq\frac{\sum_{t\in\mathcal{U}^{(s,k)}}\bar{\boldsymbol{\mu}}_{\mathbf{H}}^{(t,s,k)\mathrm{T}}\left(\bar{\mathbf{x}}_{\mathbf{m}}^{(k)}-\mathbf{x}_{\mathbf{m}}^{*}\right)}{\sum_{t\in\mathcal{U}^{(s,k)}}\mathbf{e}_{nb}^{\mathrm{T}}\bar{\mathbf{p}}_{\mathbf{s}}^{(t,s,k)}}+1 \tag{4.43}$$

where k and s are the iteration number and the scenario according to the modified cut.

Since $0\leq\alpha<1$, (4.44) will also be verified before the value of α is changed by (4.43).

$$\sum_{t\in\mathcal{U}^{(s,k)}}\bar{\boldsymbol{\mu}}_{\mathbf{H}}^{(t,s,k)\mathrm{T}}\left(\bar{\mathbf{x}}_{\mathbf{m}}^{(k)}-\mathbf{x}_{\mathbf{m}}^{*}\right)<0 \tag{4.44}$$

If (4.44) is not satisfied for some cuts, these cut will be discarded, otherwise the constraint (4.45) will be added to the investment problem.

$$\sum_{j\in\mathcal{B}\setminus\bar{\mathcal{S}}^{(k)}}(x_m)_j+\sum_{j\in\bar{\mathcal{S}}^{(k)}}\left(1-(x_m)_j\right)\geq 1 \tag{4.45}$$

where $\mathcal{B} = \{1, \ldots, nc.ns\}$, and $\bar{\mathcal{S}}^{(k)} = \left\{ j \in \mathcal{B}, \left(\bar{x}_m^{(k)}\right)_j = 1 \right\}$, k is the iteration number according to the modified cut.

After all feasibility cuts are modified, the investment problem is solved again.

It should be noted that the modified values of α for all situations should be strictly satisfied (4.21), (4.25) and (4.43), i.e. the values should be greater than the right-hand side terms with some tolerances.

4.3 Complete Procedure for Solving MTEP-AC-NSEC Problem

When integrating the developed methods in this chapter to the main framework proposed in the previous chapter. The complete procedure for solving MTEP-AC-NSEC problem can be summarized as below.

Step 0: Initialization

Set the iteration counter, k to one, UBD_0 to infinity, and $\mathcal{F} = \emptyset$. Define the value of α and ϵ. Initialize the investment problem by (3.192)–(3.196).

Step 1: Solving of the investment problem

Step 1.1: (***Local search***) Solve the simplified investment problem (4.9)–(4.12) to obtain the set of potential paths, $\mathcal{P}_{\mathrm{t}}$. If the problem is feasible, define the constraint (4.16), add it to the investment problem and go to Step 1.2, otherwise go to Step 1.3.

Step 1.2: Solve the investment problem by the general MILP solver. Set the value of the lower bound, LBD_k to $\bar{z}$. If the local search is currently applied, delete the constraint (4.16) which has been added to the investment problem in Step 1.1, and go to Step 2.

Step 1.3: (***Modification of cuts in case of infeasible problem***) Solve the problem define as (4.34)–(4.40) to obtain $\mathbf{x}_{\mathbf{m}}^{*}$. Then verify all feasibility cuts already added to the investment problem by (4.42). For each unsatisfied cut, the condition (4.44) is checked. If it is true for the considered cut, the value of α of this cut will be modified according to (4.43) and go to Step 1.1, otherwise the cut is discarded and the constraint (4.45) is added to the investment problem and go to Step 1.1.

Step 2: Solving of the feasibility and operation problem

From the minimizer, with $\bar{\mathbf{x}}_{\mathbf{m}}$ obtained from Step 1, define and solve the operation problems for all stages and scenarios. If the problems are all feasible, the minimizer, $\bar{\mathbf{y}}_{\mathbf{a}}^{(t,0)}$ and Lagrange multiplier, $\bar{\boldsymbol{\lambda}}_{\mathbf{H}}^{(t,0)}$, $t = 1, \ldots, ns$ can be obtained. Then the value of the upper bound is updated by (4.46).

$$UBD_k = \min \left\{ UBD_{k-1},\, \mathbf{c}_{\mathbf{bm}}^{\mathrm{T}} \bar{\mathbf{x}}_{\mathbf{m}} + \sum_{t=1}^{ns} \left(\mathbf{c}_{\mathbf{ac}}^{(t,0)\mathrm{T}} \bar{\mathbf{y}}_{\mathbf{a}}^{(t,0)} \right) \right\} \tag{4.46}$$

After that the minimizer, $\bar{\mathbf{x}}_{\mathbf{m}}$ is added to the set $\mathcal{F}$. In addition, if the termination criterion defined as (3.25) is satisfied, go to Step 5, otherwise the optimality cut defined as (4.19) will be created and go to Step 3.

In case some operation problems are infeasible, the feasibility cuts defined as (4.18) will be created and go to Step 3.

Step 3: Modification of cuts

For the feasibility cuts, verify each cut obtained from Step 2 by (4.20). If it is not satisfied for some feasibility cuts, (4.22) is checked. If (4.22) is satisfied, the values of α of these cuts are modified according to (4.21), otherwise the feasibility cuts is replaced by (4.23).

For the optimality cuts, the cut is verified by (4.24). If it is not satisfied, the conditions stated in (4.26) and (4.27) are also checked. In case both conditions are true, the values of α of the cut is modified according to (4.25), otherwise the total cost of the current feasible plan, i.e. $\mathbf{c}_{\mathbf{bm}}^{\mathrm{T}} \bar{\mathbf{x}}_{\mathbf{m}} + \sum_{t=1}^{ns} \left(\mathbf{c}_{\mathbf{ac}}^{(t,0)\mathrm{T}} \bar{\mathbf{y}}_{\mathbf{a}}^{(t,0)} \right)$ will be compared with the total cost of the plan, $\hat{\mathbf{x}}_{\mathbf{m}} \in \mathcal{F}$. If the total cost of the current feasible plan is lower, the cut will not be modified, otherwise the cut will be replaced by the constraint (4.23).

Step 4: Adding the cuts to the investment problem

After obtaining either optimality or feasibility cut in Step 3, add it into the investment problem, increase the iteration counter by one, i.e. $k = k + 1$. If the local search is currently employed for solving the investment problem, go to Step 1.1, otherwise go to Step 1.2.

Step 5: Termination

If the local search is used to solve the investment problem, go to Step 1.2 (change to solving the investment problem in the complete search space), otherwise the process is terminated, and $\left(\bar{\mathbf{x}}_{\mathbf{m}},\, \bar{\mathbf{y}}_{\mathbf{a}}^{(t,s)}\right)$, $t = 1, \ldots, ns$ and $s = 0,\, 1, \ldots, nv$ is the solution of the MTEP-AC-NSEC problem.

Figure 4.4 shows the flow diagram of the complete procedure for solving the MTEP-AC-NSEC problem.

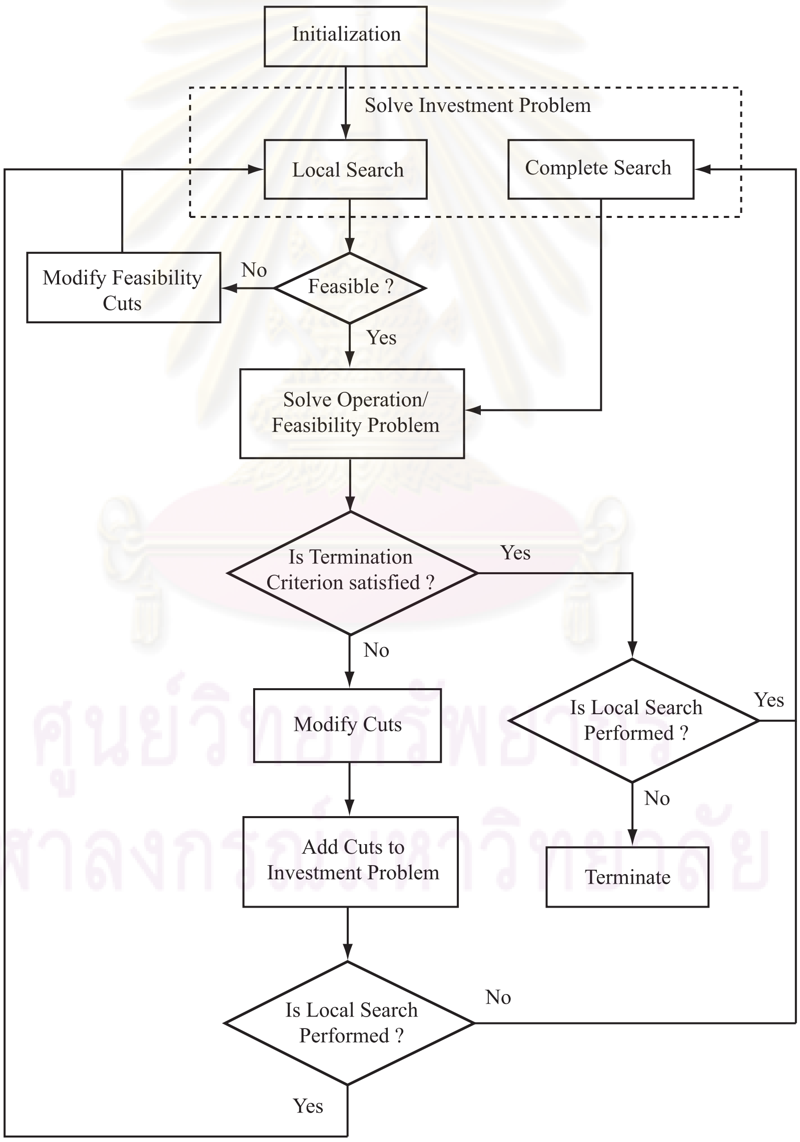

Figure 4.4 Diagram of complete procedure for solving MTEP-AC-NSEC problem

4.4 Conclusion

This chapter has presented the methods for improving the performance and solution quality of the main framework proposed in Chapter 3. The first method is the local search applied in solving the investment problem to reduce the computational burden, while the second one is the modification of cuts, which is used to handle the nonconvexity of the TEP problem.

By integrating the method developed in this chapter to the main framework for solving MTEP-AC-NSEC problem, the complete procedure for solving the multistage TEP problem based on the AC model with N-1 security constraints can be obtained. In general, the proposed framework can be applied in the actual transmission planning activities. However, in some special cases, additional issues should also be taken into account. The applications of the proposed method in additional aspects will be presented in the next chapter.

CHAPTER V

TEP WITH VOLTAGE STABILITY AND FACTS APPLICATIONS

In this chapter, the proposed method will be further developed to take into account the applications in power system planning and operation. Since the advantage of the AC model over the DC model in the TEP application has never been illustrated in previous research works, this chapter will demonstrate its ability to tackle problems which DC model cannot solve. The first application demonstrates the TEP problem with voltage stability constraints, whereas the second one is the TEP problem with consideration of FACTS devices.

It should be emphasized that the applications presented in this chapter are only the preliminary studies. Aim of the presentation in this chapter is to show that the proposed method is not limited only to the general aspects considered in Chapter 3.

5.1 TEP Problem with Voltage Stability Constraint

The TEP problem is an optimization problem. Therefore, it is aimed to minimize the total cost in a considered planning period. In case of an AC model, the process may choose to install reactive power compensation devices instead of constructing new transmission lines, if the compensation devices can alleviate the violation of planning criteria. It should be noted that cost of the compensation devices is much less than the cost of transmission lines. Therefore, it is possible that the amount of installed capacitors in the system is excessive, of which the condition may cause voltage instability [**61**, **62**] which can lead to blackout in the system [**63**, **64**]. To deal with this problem, voltage stability constraints should be taken into account. In most cases, it is considered as a voltage stability margin, since the voltage may be stable when the system is operated within operation limits.

It should be noted that there is no previous TEP work considering the voltage stability constraints, since the formulations are mostly limited to a DC model which cannot handle this problem. By applying the proposed formulation of the MTEP-AC-NSEC problem, the voltage stability constraint can be easily integrated into the problem. In general, there are two kinds of voltage stability analysis. The first one is dynamic analysis which is based on the time-domain simulation [**65**]. The second one is static analysis, e.g. V-Q sensitivity [**65**, **66**], modal analysis [**61**, **67**], continuation power flow [**65**]. In this dissertation, the modal analysis is used to derive the voltage stability constraint added to the TEP problem. For more details of the modal analysis, it can be referred to Refs. [**61**, **67**].

5.1.1 Mathematical Formulation

In modal analysis, the degree of voltage stability can be measured by considering the value of the eigenvalue of the reduced Jacobian matrix of the system as described below.

Let $\mathbf{J}$ is the Jacobian matrix of the system expressed as (5.1).

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_1 & \mathbf{J}_2 \\ \mathbf{J}_3 & \mathbf{J}_4 \end{bmatrix} \tag{5.1}$$

The reduced Jacobian matrix, $\mathbf{J}_\mathrm{R}$ can be derived by (5.2).

$$\mathbf{J}_\mathrm{R} = \mathbf{J}_4 - \mathbf{J}_3\mathbf{J}_1^{-1}\mathbf{J}_2 \tag{5.2}$$

Now one can obtain the relation between the variation of bus voltages, $\Delta\mathbf{V}$ and the variations of reactive power injected to buses, $\Delta\mathbf{Q}$ as (5.3).

$$\Delta\mathbf{V} = \mathbf{J}_\mathrm{R}^{-1}\Delta\mathbf{Q} \tag{5.3}$$

Since $\mathbf{J}_\mathrm{R}^{-1}$ can be written as (5.4).

$$\mathbf{J}_\mathrm{R}^{-1} = \boldsymbol{\xi}\boldsymbol{\Sigma}^{-1}\boldsymbol{\eta} \tag{5.4}$$

$$\mathbf{v} = \boldsymbol{\Sigma}^{-1}\mathbf{q} \tag{5.5}$$

where

$\boldsymbol{\xi}$ is the matrix of which each column corresponds to the right eigenvector of $\mathbf{J}_\mathrm{R}$,
$\boldsymbol{\eta}$ is the matrix of which each row corresponds to the left eigenvector of $\mathbf{J}_\mathrm{R}$,
$\boldsymbol{\Sigma}$ is the diagonal eigenvalue matrix of $\mathbf{J}_\mathrm{R}$,
$\mathbf{v} = \boldsymbol{\xi}^{-1}\Delta\mathbf{V} = \boldsymbol{\eta}\Delta\mathbf{V}$ is the vector of variation of modal voltages, and
$\mathbf{q} = \boldsymbol{\eta}\Delta\mathbf{Q}$ is the vector of variation of modal reactive power.

Relation between the variation of modal voltage and modal reactive power for mode i can be written as (5.6).

$$v_i = \frac{q_i}{\sigma_i} \tag{5.6}$$

where σ_i is the corresponding eigenvalue.

It can be seen that if $\sigma_i < 0$, the modal voltage will change to opposite direction of the change of modal reactive power. Therefore, the voltage is unstable. In addition, if the $\sigma_i = 0$, the voltage will collapse since the small change of the modal reactive power causes the infinite change of the modal voltage.

Consequently, the value of minimum eigenvalue of $\mathbf{J}_{\mathrm{R}}$ can be used to measure voltage stability. In application to TEP, it will be constrained to be the voltage stability margin as described by (5.7).

$$\sigma_{\min} \geq VS_{\lim} \tag{5.7}$$

where $\sigma_{\min}$ is the minimum eigenvalue of $\mathbf{J}_{\mathrm{R}}$, and $VS_{\lim}$ is the defined voltage stability margin.

It should be noted that the value of the minimum eigenvalue of $\mathbf{J}_{\mathrm{R}}$indicates a relative measure of the proximity to voltage instability. In practice, the value of $VS_{\lim}$ can be defined according to the suggestion of past experience of the system operators.

The constraint (5.7) can be easily integrated into a computational procedure for solving the MTEP-AC-NSEC problem proposed in Section 4.3 , by solving the feasibility problem concerned with the voltage stability defined as (5.8)–(5.12), after verifying that all operation problems are feasible in Step 2.

$$\min \beta^{(t,s)} \tag{5.8}$$

subject to

$$\mathbf{H_a y_a}^{(t,s)} = \mathbf{E}^{(t)} \bar{\mathbf{x}}_\mathbf{m} \tag{5.9}$$

$$\mathbf{G}_\mathbf{a}^{(t,s)} \left(\mathbf{y}_\mathbf{a}^{(t,s)} \right) \leq \mathbf{0} \tag{5.10}$$

$$\sigma_{\min}\left(\mathbf{y}_{\mathbf{a}}^{(t,s)}\right) + \beta^{(t,s)} \geq VS_{\lim} \tag{5.11}$$

$$\beta^{(t,s)} \geq 0 \tag{5.12}$$

$$\beta^{(t,s)} \in \mathbb{R}, \quad \mathbf{y}_{\mathbf{a}}^{(t,s)} \in \mathbb{R}^{ma}, \quad ma = nc + 4nb + 2ng$$

For each scenario s, the set $\mathcal{U}^{(s)}$ is initialized by an empty set. Then, the voltage stability of the system in scenarios s for each stage is verified by comparing the minimum eigenvalue of the reduced Jacobian of the system, $\sigma_{\min}^{(t,s)}$ to the defined voltage stability margin, $VS_{\lim}$. If $\sigma_{\min}^{(t,s)} < VS_{\lim}$, the problem (5.8)–(5.12) is defined and solved for that stage in scenario s. The index of the stage is added to the set $\mathcal{U}^{(s)}$.

After all the scenarios are verified, feasibility cuts for voltage stability will be created in the same manner of the feasibility cut generation presented in Section 3.7.4, i.e.

$$\sum_{t\in\mathcal{U}^{(s)}} \left(\bar{\beta}^{(t,s)} + \bar{\boldsymbol{\mu}}_{\mathbf{H}}^{(t,s)\mathrm{T}} \left(\bar{\mathbf{x}}_{\mathbf{m}} - \mathbf{x}_{\mathbf{m}}\right)\right) \leq 0 \tag{5.13}$$

$$\mathcal{U}^{(s)} \neq \emptyset, \quad s = 0,\, 1, \ldots, nv$$

where $\bar{\beta}^{(t,s)}$ is the solution obtained from solving the problem (5.8)–(5.12) for stage t, scenario s. After that the cuts are added to the investment problem, and the subsequent steps are performed in general.

From computational aspect, there are two key issues which should be considered in solving the problem (5.8)–(5.12) as follows:

(a) The explicit form of $\sigma_{\min}\left(\,\cdot\,\right)$ cannot be obtained, therefore, the numerical techniques, e.g. finite differencing, automatic differentiation [**68**], should be adopted for calculating the gradient and Hessian of the constraint (5.11). However, it is emphasized that these techniques should be employed only for the constraint (5.11) to reduce the unnecessary computational burden in the calculation.

(b) The function $\sigma_{\min}\left(\,\cdot\,\right)$ is very complicated causing the difficulty in solving the problem (5.8)–(5.12). Therefore, an initial feasible solution should be provided for the optimization procedure. Since the problem is defined after the corresponding operation problem is feasible, the feasible solution can be determined as $\left(\bar{\mathbf{y}}_{\mathbf{a}}^{(t,s)}, \hat{\beta}^{(t,s)}\right)$, where $\bar{\mathbf{y}}_{\mathbf{a}}^{(t,s)}$ is the minimizer of the operation problem of stage t, scenario s, and $\hat{\beta}^{(t,s)}$ can be defined according to (5.14).

$$\hat{\beta}^{(t,s)} \geq VS_{\lim} - \sigma_{\min}^{(t,s)} \tag{5.14}$$

5.2 TEP Problem with FACTS Device Installation

In this section, the application of TEP problem will be extended to the installation of FACTS devices. Only the unified power flow controller (UPFC) is considered in this dissertation. However, other types of FACTS devices, e.g. HVDC, STATCOM, etc., can be incorporated into the TEP problem in the same manner presented in this section.

UPFC is a device which can simultaneously control voltage magnitude at a local bus, and power flow in a transmission line [**69**]. In case of voltage control, the benefit to a power system in the viewpoint of steady state analysis is similar to other reactive power compensation devices. However, its characteristic is usually better than that of the shunt capacitor since its reactive power supply can be controlled. In case of power flow control, the power transfer capability of a considered transmission line can be increased. Therefore, the power flows in overloaded transmission lines may be reduced, and the system performance can be improved to meet the defined planning criteria. For more details of the basic operation of the UPFC, one can refer to [**69**–**72**].

Consequently, the installation of the UPFC can defer the construction of new transmission lines. However, the benefit from improving the system performance and installation cost should be compared with the ones obtained from construction of the new transmission lines. To evaluate this comparison, the UPFC installation should be incorporated into the TEP problem.

5.2.1 Power Flow Equation with UPFC

The operating diagram of the UPFC can be illustrated in Figure 5.1 [**70**,**71**]. In this figure, the UPFC is connected between bus f and transmission line l. Therefore, the voltage at bus f and the power flow in transmission line l can be controlled. It can be seen that key components of the UPFC comprise two converters, i.e. series and shunt converters, connected together by a DC link. In addition, the active power is transfered between both converters by the DC link.

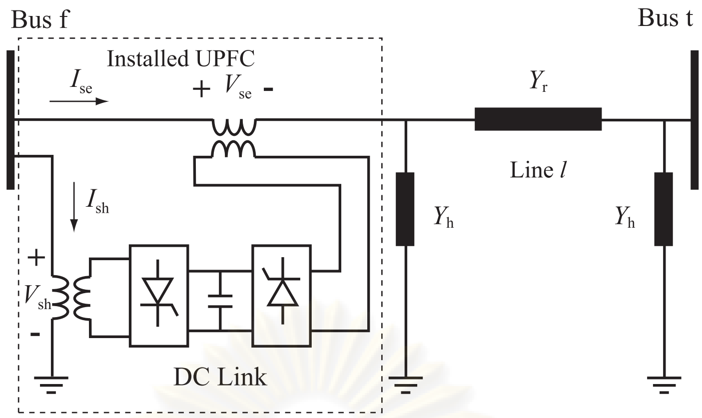

Figure 5.1 Operating diagram of UPFC

Equivalent circuit of the UPFC can be shown in Figure 5.2 [**70**, **71**]. The power equation of the transmission line with UPFC can be developed. In the TEP application, the power equation should depend on variable, x_{f}, which corresponds to the decision on UPFC installation, i.e. $x_{\mathrm{f}} = 1$ if the considered UPFC is selected.

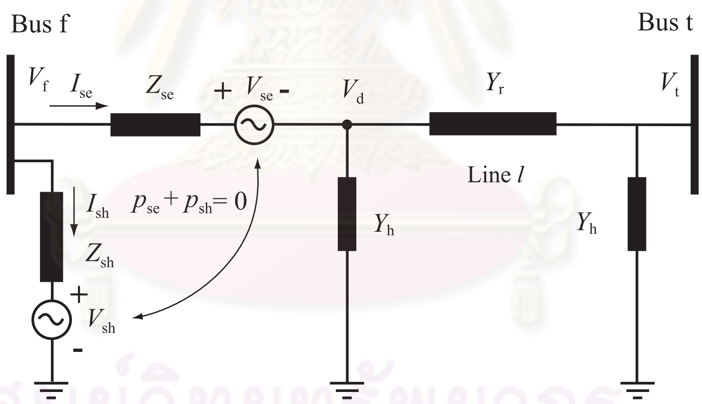

Figure 5.2 Equivalent circuit of UPFC

Given phasor voltages at ***from bus***, V_{f} and ***to bus***, V_{t}, and phasor voltages of the series converter, V_{se} and shunt converter, V_{sh}, one can obtain the current in the transmission line regarding as ***from bus***, which is equal to the current through the series converter, I_{se}, as follows:

By applying KCL at the node of the transmission line connected to the UPFC, one can obtain (5.15).

$$I_{\mathrm{se}} - V_{\mathrm{d}} Y_{\mathrm{h}} - (V_{\mathrm{d}} - V_{\mathrm{t}})\, Y_{\mathrm{r}} = 0 \tag{5.15}$$

Since

$$V_{\mathrm{d}} = V_{\mathrm{f}} - I_{\mathrm{se}} Z_{\mathrm{se}} - V_{\mathrm{se}} \tag{5.16}$$

From (5.15) and (5.16), one can obtain (5.17).

$$I_{\mathrm{se}} + I_{\mathrm{se}} Z_{\mathrm{se}} Y_{\mathrm{h}} + I_{\mathrm{se}} Z_{\mathrm{se}} Y_{\mathrm{r}} = (V_{\mathrm{f}} - V_{\mathrm{t}})\, Y_{\mathrm{r}} + V_{\mathrm{f}} Y_{\mathrm{h}} - V_{\mathrm{se}} Y_{\mathrm{h}} - V_{\mathrm{se}} Y_{\mathrm{r}} \tag{5.17}$$

Since current through the transmission line without the UPFC installation regarding as ***from bus*** can be expressed as (5.18)

$$I_{\mathrm{f}} = (V_{\mathrm{f}} - V_{\mathrm{t}})\, Y_{\mathrm{r}} + V_{\mathrm{f}} Y_{\mathrm{h}} \tag{5.18}$$

Therefore, the current in the transmission line with the UPFC installation regarding as ***from bus*** can be written as (5.19).

$$I_{\mathrm{f}}^{u} = I_{\mathrm{se}} = \frac{I_{\mathrm{f}} - V_{\mathrm{se}}\,(Y_{\mathrm{h}} + Y_{\mathrm{r}})}{1 + Z_{\mathrm{se}}\,(Y_{\mathrm{h}} + Y_{\mathrm{r}})} \tag{5.19}$$

It can be found that the decision variable of the UPFC, i.e. x_{f}, can be integrated into (5.19) in order that the current in the transmission line will be controlled by x_{f} according to the installation of the UPFC as shown below.

$$I_{\mathrm{f}}^{u} = \frac{I_{\mathrm{f}} - x_{\mathrm{f}} V_{\mathrm{se}}\,(Y_{\mathrm{h}} + Y_{\mathrm{r}})}{1 + x_{\mathrm{f}} Z_{\mathrm{se}}\,(Y_{\mathrm{h}} + Y_{\mathrm{r}})} \tag{5.20}$$

In addition, the current in the transmission line regarding as ***to bus*** can be expressed as

$$I_{\mathrm{t}}^{u} = (V_{\mathrm{t}} - V_{\mathrm{d}})\, Y_{\mathrm{r}} + V_{\mathrm{t}} Y_{\mathrm{h}}$$

When x_{f} is integrated into (5.16), one can obtain (5.21).

$$V_{\mathrm{d}} = V_{\mathrm{f}} - x_{\mathrm{f}}\,(I_{\mathrm{se}} Z_{\mathrm{se}} + V_{\mathrm{se}}) \tag{5.21}$$

Therefore,

$$
\begin{aligned}
I_{\mathrm{t}}^{u} &= \left(V_{\mathrm{t}} - V_{\mathrm{f}}\right) Y_{\mathrm{r}} + V_{\mathrm{t}} Y_{\mathrm{h}} + x_{\mathrm{f}} \left(I_{\mathrm{f}}^{u} Z_{\mathrm{se}} Y_{\mathrm{r}} + V_{\mathrm{se}} Y_{\mathrm{r}}\right) \\
&= I_{\mathrm{t}} + x_{\mathrm{f}} \left(I_{\mathrm{f}}^{u} Z_{\mathrm{se}} Y_{\mathrm{r}} + V_{\mathrm{se}} Y_{\mathrm{r}}\right) \qquad (5.22)
\end{aligned}
$$

where I_{t} is the current in the transmission line regarding as ***to bus*** when the UPFC is not installed.

In case of shunt converter, the current through the shunt converter can be expressed by (5.23)

$$I_{\mathrm{sh}} = \frac{x_{\mathrm{f}} \left(V_{\mathrm{f}} - V_{\mathrm{sh}}\right)}{Z_{\mathrm{sh}}} \qquad (5.23)$$

5.2.2 Basic Formulation of TEP with UPFC

The formulation of the single stage TEP problem with the UPFC installation (STEP-UPFC) can be derived from the formulation of the STEP-AC problem in Section 3.6 by including some constraints according to the UPFC operation as below.

$$\min \left(IVF_1 \left(\mathbf{c}_{\mathbf{b}}^{\mathrm{T}} \mathbf{x} + \mathbf{c}_{\mathbf{f}}^{\mathrm{T}} \mathbf{x}_{\mathbf{f}} \right) + OPF_1 \mathbf{c}_{\mathbf{g}}^{\mathrm{T}} \mathbf{p}_{\mathbf{g}} \right) \qquad (5.24)$$

subject to

$$
\begin{aligned}
&\mathbf{Tx} \le \mathbf{e}_{np} && (5.25)\\
&\mathbf{Nx} \le \mathbf{0} && (5.26)\\
&\mathbf{U_f x_f} - \mathbf{A_{up} T x} \le \mathbf{0} && (5.27)\\
&\mathbf{P_{inj}}\left(\mathbf{v}, \boldsymbol{\delta}, \mathbf{p_g}, \mathbf{v_{se}}, \boldsymbol{\delta}_{\mathbf{se}}, \mathbf{v_{sh}}, \boldsymbol{\delta}_{\mathbf{sh}}, \mathbf{x}, \mathbf{x_f}\right) = \mathbf{0} && (5.28)\\
&\mathbf{Q_{inj}}\left(\mathbf{v}, \boldsymbol{\delta}, \mathbf{q_g}, \mathbf{d_r}, \mathbf{d_c}, \mathbf{v_{se}}, \boldsymbol{\delta}_{\mathbf{se}}, \mathbf{v_{sh}}, \boldsymbol{\delta}_{\mathbf{sh}}, \mathbf{x}, \mathbf{x_f}\right) = \mathbf{0} && (5.29)\\
&\mathbf{v^{min}} \le \mathbf{v} \le \mathbf{v^{max}} && (5.30)\\
&\mathbf{p_g^{min}} \le \mathbf{p_g} \le \mathbf{p_g^{max}} && (5.31)\\
&\mathbf{q_g^{min}} \le \mathbf{q_g} \le \mathbf{q_g^{max}} && (5.32)\\
&\mathbf{i_{bef}^2}\left(\mathbf{v}, \boldsymbol{\delta}, \mathbf{v_{se}}, \boldsymbol{\delta}_{\mathbf{se}}, \mathbf{x_f}\right) \le \left(\mathbf{i_{be}^{max}}\right)^2 && (5.33)\\
&\mathbf{i_{bet}^2}\left(\mathbf{v}, \boldsymbol{\delta}, \mathbf{v_{se}}, \boldsymbol{\delta}_{\mathbf{se}}, \mathbf{x_f}\right) \le \left(\mathbf{i_{be}^{max}}\right)^2 && (5.34)
\end{aligned}
$$

$$\mathbf{i}_{\mathbf{bcf}}^{2}\left(\mathbf{v},\boldsymbol{\delta},\mathbf{v}_{\mathbf{se}},\boldsymbol{\delta}_{\mathbf{se}},\mathbf{x},\mathbf{x}_{\mathbf{f}}\right)\leq\left(\mathbf{i}_{\mathbf{bc}}^{\mathbf{max}}\circ\mathbf{x}\right)^{2} \tag{5.35}$$

$$\mathbf{i}_{\mathbf{bct}}^{2}\left(\mathbf{v},\boldsymbol{\delta},\mathbf{v}_{\mathbf{se}},\boldsymbol{\delta}_{\mathbf{se}},\mathbf{x},\mathbf{x}_{\mathbf{f}}\right)\leq\left(\mathbf{i}_{\mathbf{bc}}^{\mathbf{max}}\circ\mathbf{x}\right)^{2} \tag{5.36}$$

$$\mathbf{0}\leq\mathbf{d}_{\mathbf{r}}\leq\mathbf{d}_{\mathbf{r}}^{\mathbf{max}} \tag{5.37}$$

$$\mathbf{0}\leq\mathbf{d}_{\mathbf{c}}\leq\mathbf{d}_{\mathbf{c}}^{\mathbf{max}} \tag{5.38}$$

$$\mathbf{v}_{\mathbf{se}}^{\mathbf{min}}\leq\mathbf{v}_{\mathbf{se}}\leq\mathbf{v}_{\mathbf{se}}^{\mathbf{max}} \tag{5.39}$$

$$\mathbf{v}_{\mathbf{sh}}^{\mathbf{min}}\leq\mathbf{v}_{\mathbf{sh}}\leq\mathbf{v}_{\mathbf{sh}}^{\mathbf{max}} \tag{5.40}$$

$$\mathbf{i}_{\mathbf{se}}^{2}\left(\mathbf{v},\boldsymbol{\delta},\mathbf{v}_{\mathbf{se}},\boldsymbol{\delta}_{\mathbf{se}},\mathbf{x},\mathbf{x}_{\mathbf{f}}\right)\leq\left(\mathbf{i}_{\mathbf{se}}^{\mathbf{max}}\circ\mathbf{x}_{\mathbf{f}}\right)^{2} \tag{5.41}$$

$$\mathbf{i}_{\mathbf{sh}}^{2}\left(\mathbf{v},\boldsymbol{\delta},\mathbf{v}_{\mathbf{sh}},\boldsymbol{\delta}_{\mathbf{sh}},\mathbf{x}_{\mathbf{f}}\right)\leq\left(\mathbf{i}_{\mathbf{sh}}^{\mathbf{max}}\circ\mathbf{x}_{\mathbf{f}}\right)^{2} \tag{5.42}$$

$$\left|\mathbf{p}_{\mathbf{sh}}\left(\mathbf{v},\boldsymbol{\delta},\mathbf{v}_{\mathbf{sh}},\boldsymbol{\delta}_{\mathbf{sh}},\mathbf{x}_{\mathbf{f}}\right)\right|\leq\mathbf{p}_{\mathbf{dc}}^{\max} \tag{5.43}$$

$$\mathbf{p}_{\mathbf{sh}}\left(\mathbf{v},\boldsymbol{\delta},\mathbf{v}_{\mathbf{sh}},\boldsymbol{\delta}_{\mathbf{sh}},\mathbf{x}_{\mathbf{f}}\right)+\mathbf{p}_{\mathbf{se}}\left(\mathbf{v},\boldsymbol{\delta},\mathbf{v}_{\mathbf{se}},\boldsymbol{\delta}_{\mathbf{se}},\mathbf{x},\mathbf{x}_{\mathbf{f}}\right)=\mathbf{0} \tag{5.44}$$

$$\mathbf{x}\in\{0,\,1\}^{nc},\quad \mathbf{x}_{\mathbf{f}}\in\{0,\,1\}^{nf}$$

$$\mathbf{v},\,\boldsymbol{\delta},\,\mathbf{d}_{\mathbf{r}},\,\mathbf{d}_{\mathbf{c}}\in\mathbb{R}^{nb},\quad \mathbf{p}_{\mathbf{g}},\,\mathbf{q}_{\mathbf{g}}\in\mathbb{R}^{ng}$$

$$\mathbf{v}_{\mathbf{se}},\,\boldsymbol{\delta}_{\mathbf{se}},\,\mathbf{v}_{\mathbf{sh}},\,\boldsymbol{\delta}_{\mathbf{sh}}\in\mathbb{R}^{nf}$$

where

$$\begin{aligned}\mathbf{P}_{\mathbf{inj}}=\ &\mathbf{A}_{\mathbf{g}}^{\mathrm{T}}\mathbf{p}_{\mathbf{g}}-\mathbf{A}_{\mathbf{bef}}^{\mathrm{T}}\mathbf{p}_{\mathbf{bef}}-\mathbf{A}_{\mathbf{bet}}^{\mathrm{T}}\mathbf{p}_{\mathbf{bet}}\\&-\mathbf{A}_{\mathbf{bcf}}^{\mathrm{T}}\mathbf{p}_{\mathbf{bcf}}-\mathbf{A}_{\mathbf{bct}}^{\mathrm{T}}\mathbf{p}_{\mathbf{bct}}-\mathbf{A}_{\mathbf{ub}}^{\mathrm{T}}\mathbf{p}_{\mathbf{shb}}-\mathbf{p}_{\mathbf{d}}\end{aligned} \tag{5.45}$$

$$\begin{aligned}\mathbf{Q}_{\mathbf{inj}}=\ &\mathbf{A}_{\mathbf{g}}^{\mathrm{T}}\mathbf{q}_{\mathbf{g}}-\mathbf{A}_{\mathbf{bef}}^{\mathrm{T}}\mathbf{q}_{\mathbf{bef}}-\mathbf{A}_{\mathbf{bet}}^{\mathrm{T}}\mathbf{q}_{\mathbf{bet}}\\&-\mathbf{A}_{\mathbf{bcf}}^{\mathrm{T}}\mathbf{q}_{\mathbf{bcf}}-\mathbf{A}_{\mathbf{bct}}^{\mathrm{T}}\mathbf{q}_{\mathbf{bct}}-\mathbf{A}_{\mathbf{ub}}^{\mathrm{T}}\mathbf{q}_{\mathbf{shb}}+\mathbf{q}_{\mathbf{cmp}}-\mathbf{q}_{\mathbf{d}}\end{aligned} \tag{5.46}$$

From the power flow equation presented in Section 5.2.1, one can obtain the l^{th} elements of $\mathbf{p}_{\mathbf{bef}}$, $\mathbf{p}_{\mathbf{bet}}$, $\mathbf{q}_{\mathbf{bef}}$, $\mathbf{q}_{\mathbf{bet}}$, $\mathbf{i}_{\mathbf{bef}}^{2}$, and $\mathbf{i}_{\mathbf{bet}}^{2}$ corresponding to the existing line l on which the UPFC f can be installed as shown below.

Let $\mathbf{V}_{\mathbf{ef}}$ and $\mathbf{V}_{\mathbf{et}}$ be vectors of phasor voltages, regarding as ***from bus*** and ***to bus*** respectively.

$$V_{\text{ef},l}=v_{\text{ef},l}\angle\delta_{\text{ef},l} \tag{5.47}$$

$$V_{\text{et},l} = v_{\text{et},l} \angle \delta_{\text{et},l} \tag{5.48}$$

where

$$\mathbf{v_{ef}} = \mathbf{A_{bef}} \mathbf{v} \tag{5.49}$$

$$\boldsymbol{\delta}_{\mathbf{ef}} = \mathbf{A_{bef}} \boldsymbol{\delta} \tag{5.50}$$

$$\mathbf{v_{et}} = \mathbf{A_{bet}} \mathbf{v} \tag{5.51}$$

$$\boldsymbol{\delta}_{\mathbf{et}} = \mathbf{A_{bet}} \boldsymbol{\delta} \tag{5.52}$$

$\mathbf{V_{se}}$ and $\mathbf{V_{sh}}$ are vectors of phasor voltages of the series and shunt converters. The f^{th} element corresponding to the UPFC f can be expressed as shown below.

$$V_{\text{se},f} = v_{\text{se},f} \angle \delta_{\text{se},f} \tag{5.53}$$

$$V_{\text{sh},f} = v_{\text{sh},f} \angle \delta_{\text{sh},f} \tag{5.54}$$

Now one can obtain the following quantities.

$$I_{\text{bef},l} = \frac{\left(\left(V_{\text{ef},l} - V_{\text{et},l}\right) Y_{\text{r},l} + V_{\text{ef},l} Y_{\text{h},l} - x_f V_{\text{se},f} \left(Y_{\text{r},l} + Y_{\text{h},l}\right)\right)}{\left(1 + x_f Z_{\text{se},f} \left(Y_{\text{r},l} + Y_{\text{h},l}\right)\right)} \tag{5.55}$$

$$S_{\text{bef},l} = V_{\text{ef},l} I^*_{\text{bef},l} \tag{5.56}$$

$$p_{\text{bef},l} = \text{Re}\left\{S_{\text{bef},l}\right\} \tag{5.57}$$

$$q_{\text{bef},l} = \text{Im}\left\{S_{\text{bef},l}\right\} \tag{5.58}$$

$$i^2_{\text{bef},l} = I_{\text{bef},l} I^*_{\text{bef},l} \tag{5.59}$$

$$I_{\text{bet},l} = \left(V_{\text{et},l} - V_{\text{ef},l}\right) Y_{\text{r},l} + V_{\text{et},l} Y_{\text{h},l} + x_f \left(I_{\text{bef},l} Z_{\text{se},f} Y_{\text{r},l} + V_{\text{se},f} Y_{\text{r},l}\right) \tag{5.60}$$

$$S_{\text{bet},l} = V_{\text{et},l} I^*_{\text{bet},l} \tag{5.61}$$

$$p_{\text{bet},l} = \text{Re}\left\{S_{\text{bet},l}\right\} \tag{5.62}$$

$$q_{\text{bet},l} = \text{Im}\left\{S_{\text{bet},l}\right\} \tag{5.63}$$

$$i^2_{\text{bet},l} = I_{\text{bet},l} I^*_{\text{bet},l} \tag{5.64}$$

In case of the candidate line l, the corresponding l^{th} elements of $\mathbf{p_{bcf}}$, $\mathbf{p_{bct}}$, $\mathbf{q_{bcf}}$, $\mathbf{q_{bct}}$, $\mathbf{i}^2_{\mathbf{bcf}}$, and $\mathbf{i}^2_{\mathbf{bct}}$ can be shown below.

$$I_{\text{bcf},l} = x_l \left(\frac{\left(\left(V_{\text{cf},l} - V_{\text{ct},l}\right) Y_{\text{r},l} + V_{\text{cf},l} Y_{\text{h},l} - x_f V_{\text{se},f} \left(Y_{\text{r},l} + Y_{\text{h},l}\right)\right)}{\left(1 + x_f Z_{\text{se},f} \left(Y_{\text{r},l} + Y_{\text{h},l}\right)\right)} \right) \tag{5.65}$$

$$S_{\text{bcf},l} = V_{\text{cf},l} I^*_{\text{bcf},l} \tag{5.66}$$

$$p_{\text{bcf},l} = \text{Re}\left\{S_{\text{bcf},l}\right\} \tag{5.67}$$

$$q_{\text{bcf},l} = \text{Im}\left\{S_{\text{bcf},l}\right\} \tag{5.68}$$

$$i^2_{\text{bcf},l} = I_{\text{bcf},l} I^*_{\text{bcf},l} \tag{5.69}$$

$$I_{\text{bct},l} = x_l \left(\left(V_{\text{ct},l} - V_{\text{cf},l}\right) Y_{\text{r},l} + V_{\text{ct},l} Y_{\text{h},l} + x_f \left(I_{\text{bcf},l} Z_{\text{se},f} Y_{\text{r},l} + V_{\text{se},f} Y_{\text{r},l}\right)\right) \tag{5.70}$$

$$S_{\text{bct},l} = V_{\text{ct},l} I^*_{\text{bct},l} \tag{5.71}$$

$$p_{\text{bct},l} = \text{Re}\left\{S_{\text{bct},l}\right\} \tag{5.72}$$

$$q_{\text{bct},l} = \text{Im}\left\{S_{\text{bct},l}\right\} \tag{5.73}$$

$$i^2_{\text{bct},l} = I_{\text{bct},l} I^*_{\text{bct},l} \tag{5.74}$$

where

$$V_{\text{cf},l} = v_{\text{cf},l} \angle \delta_{\text{cf},l} \tag{5.75}$$

$$V_{\text{ct},l} = v_{\text{ct},l} \angle \delta_{\text{ct},l} \tag{5.76}$$

$$\mathbf{v_{cf}} = \mathbf{A_{bcf}} \mathbf{v} \tag{5.77}$$

$$\boldsymbol{\delta}_{\mathbf{cf}} = \mathbf{A_{bcf}} \boldsymbol{\delta} \tag{5.78}$$

$$\mathbf{v_{ct}} = \mathbf{A_{bct}} \mathbf{v} \tag{5.79}$$

$$\boldsymbol{\delta}_{\mathbf{ct}} = \mathbf{A_{bct}} \boldsymbol{\delta} \tag{5.80}$$

Now one can derive the current and power flows through the UPFC f installed on the line l, i.e. the f^{th} of $\mathbf{i}^2_{\mathbf{se}}$, $\mathbf{i}^2_{\mathbf{sh}}$, $\mathbf{p_{se}}$, $\mathbf{p_{sh}}$, $\mathbf{p_{shb}}$ and $\mathbf{q_{shb}}$, as follows:

$$I_{\text{se},f} = \begin{cases} x_f I_{\text{bef},l} & \text{, if UPFC } f \text{ is connected to existing line } l \\ x_f I_{\text{bcf},l} & \text{, if UPFC } f \text{ is connected to candidate line } l \end{cases} \tag{5.81}$$

$$S_{\text{se},f} = V_{\text{se},f} I^*_{\text{se},f} \tag{5.82}$$

$$p_{\text{se},f} = \text{Re}\left\{S_{\text{se},f}\right\} \tag{5.83}$$

$$i^2_{\text{se},f} = I_{\text{se},f} I^*_{\text{se},f} \tag{5.84}$$

$$I_{\text{sh},f} = \frac{x_f\left(V_{\text{b},f} - V_{\text{sh},f}\right)}{Z_{\text{sh},f}} \tag{5.85}$$

$$S_{\text{sh},f} = V_{\text{sh},f} I^*_{\text{sh},f} \tag{5.86}$$

$$p_{\text{sh},f} = \text{Re}\left\{S_{\text{sh},f}\right\} \tag{5.87}$$

$$i^2_{\text{sh},f} = I_{\text{sh},f} I^*_{\text{sh},f} \tag{5.88}$$

$$S_{\text{shb},f} = V_{\text{b},f} I^*_{\text{sh},f} \tag{5.89}$$

$$p_{\text{shb},f} = \text{Re}\left\{S_{\text{shb},f}\right\} \tag{5.90}$$

$$q_{\text{shb},f} = \text{Im}\left\{S_{\text{shb},f}\right\} \tag{5.91}$$

$$V_{\text{b},f} = \begin{cases} V_{\text{ef},l} & \text{, if UPFC } f \text{ is connected to existing line } l \\ V_{\text{cf},l} & \text{, if UPFC } f \text{ is connected to candidate line } l \end{cases} \tag{5.92}$$

Comparing the STEP-UPFC problem defined as (5.24)–(5.44) to the STEP-AC problem presented in Section 3.6, one can see that the constraints (5.27), (5.39)–(5.44) are added to the problem. In addition, the variable $\mathbf{x_f}$ represents the decision on the installation of UPFC. It is noted that, in the proposed formulation, $\mathbf{x_f}$ should be arranged according to (5.93).

$$\mathbf{x_f}^{\text{T}} = \begin{bmatrix} \mathbf{x_{fe}}^{\text{T}} & \mathbf{x_{fc}}^{\text{T}} \end{bmatrix} \tag{5.93}$$

where $\mathbf{x_{fe}}$ is the decision variable corresponding to the UPFCs connected to existing lines, and $\mathbf{x_{fc}}$ is the decision variable corresponding to the UPFCs connected to candidate lines. Therefore, the index set of UPFCs can be expressed by $\{1, \ldots, nfe, nfe+1, \ldots, nfc\}$, where nfe is the number of UPFCs connected to the existing branches, and nfc is the number of UPFCs connected to the candidate branches.

The constraint (5.27) implies that the UPFC connected to the candidate line can be selected for installation if some candidate lines in the corresponding path are selected. The matrix $\mathbf{A_{up}}$, representing the relation between the UPFC connected to the candidate line and the corresponding path of that candidate line, can be expressed as (5.94).

$$(A_{\text{up}})_{i,p} = \begin{cases} 1 & \text{, if UPFC } nfe+i \text{ can be installed in path } p \\ 0 & \text{, otherwise} \end{cases} \tag{5.94}$$

In addition, the matrix $\mathbf{U_f}$ can be expressed by (5.95).

$$\mathbf{U_f} = \left[\begin{array}{cc} \mathbf{0}_{nfc \times nfe} & \mathbf{I}_{nfc} \end{array} \right] \tag{5.95}$$

The constraints (5.39) and (5.40) are voltage limits of the series and shunt converters, whereas the constraints (5.41) and (5.42) are current limits of the series and shunt converters. The constraint (5.43) concerns about the power limit of DC power exchange between both converters, while the constraint (5.44) is the power balance equation of the converters.

The investment and the operation problems of the STEP-UPFC can be defined in the same manner as those of the STEP-AC. However, the number of variables in both the investment problem and the operation problem are increased. The formulation of the investment problem is shown as (5.96)–(5.101).

$$\min z \tag{5.96}$$

subject to

$$\mathbf{Tx} \leq \mathbf{e}_{np} \tag{5.97}$$

$$\mathbf{Nx} \leq \mathbf{0} \tag{5.98}$$

$$\mathbf{U_f x_f} - \mathbf{A_{up} T x} \leq \mathbf{0} \tag{5.99}$$

$$-z + IVF_1 \left(\mathbf{c_b^T x} + \mathbf{c_f^T x_f} \right) \leq -OPF_1 c_{\text{op,min}} \tag{5.100}$$

$$z \geq 0 \tag{5.101}$$

$$z \in \mathbb{R}, \quad \mathbf{x} \in \{0,\, 1\}^{nc}, \quad \mathbf{x_f} \in \{0,\, 1\}^{nf}$$

In addition, the operation problem can be formulated as (5.102)–(5.105).

$$\min\left(\mathbf{c}_{\mathbf{fac}}^{\mathrm{T}}\mathbf{y}_{\mathbf{f}}\right) \tag{5.102}$$

subject to

$$\mathbf{H}_{\mathbf{fa}}\mathbf{y}_{\mathbf{f}} = \bar{\mathbf{x}} \tag{5.103}$$

$$\mathbf{H}_{\mathbf{ff}}\mathbf{y}_{\mathbf{f}} = \bar{\mathbf{x}}_{\mathbf{f}} \tag{5.104}$$

$$\mathbf{G}_{\mathbf{f}}\left(\mathbf{y}_{\mathbf{f}}\right) \leq \mathbf{0} \tag{5.105}$$

$$\mathbf{y}_{\mathbf{f}} \in \mathbb{R}^{mf}, \quad mf = nc + 5nf + 4nb + 2ng$$

where $\mathbf{G}_{\mathbf{f}}\left(\mathbf{y}_{\mathbf{f}}\right)$ can be derived from the constraints (5.28)–(5.44).

$$\mathbf{c}_{\mathbf{fac}} = \left[\begin{array}{ccccccc} \mathbf{0}_{nc}^{\mathrm{T}} & \mathbf{0}_{5nf}^{\mathrm{T}} & \mathbf{0}_{nb}^{\mathrm{T}} & \mathbf{0}_{nb}^{\mathrm{T}} & OPF_1\mathbf{c}_{\mathbf{g}}^{\mathrm{T}} & \mathbf{0}_{ng}^{\mathrm{T}} & \mathbf{0}_{2nb}^{\mathrm{T}} \end{array}\right]^{\mathrm{T}} \tag{5.106}$$

$$\mathbf{y}_{\mathbf{f}} = \left[\begin{array}{cccccccccccc} \mathbf{u}^{\mathrm{T}} & \mathbf{u}_{\mathbf{f}}^{\mathrm{T}} & \mathbf{v}_{\mathbf{se}}^{\mathrm{T}} & \boldsymbol{\delta}_{\mathbf{se}}^{\mathrm{T}} & \mathbf{v}_{\mathbf{sh}}^{\mathrm{T}} & \boldsymbol{\delta}_{\mathbf{sh}}^{\mathrm{T}} & \mathbf{v}^{\mathrm{T}} & \boldsymbol{\delta}^{\mathrm{T}} & \mathbf{p}_{\mathbf{g}}^{\mathrm{T}} & \mathbf{q}_{\mathbf{g}}^{\mathrm{T}} & \mathbf{d}_{\mathbf{r}}^{\mathrm{T}} & \mathbf{d}_{\mathbf{c}}^{\mathrm{T}} \end{array}\right]^{\mathrm{T}} \tag{5.107}$$

$$\mathbf{H}_{\mathbf{fa}} = \left[\begin{array}{cc} \mathbf{I}_{nc} & \mathbf{0}_{nc\times(5nf+4nb+2ng)} \end{array}\right] \tag{5.108}$$

$$\mathbf{H}_{\mathbf{ff}} = \left[\begin{array}{ccc} \mathbf{0}_{nf\times nc} & \mathbf{I}_{nf} & \mathbf{0}_{nf\times(4nf+4nb+2ng)} \end{array}\right] \tag{5.109}$$

In case the operation problem is infeasible, the feasibility problem will be defined as (5.110)–(5.114).

$$\min \mathbf{e}_{nb}^{\mathrm{T}}\mathbf{p}_{\mathbf{s}} \tag{5.110}$$

subject to

$$\mathbf{H}_{\mathbf{fa}}\mathbf{y}_{\mathbf{f}} = \bar{\mathbf{x}} \tag{5.111}$$

$$\mathbf{H}_{\mathbf{ff}}\mathbf{y}_{\mathbf{f}} = \bar{\mathbf{x}}_{\mathbf{f}} \tag{5.112}$$

$$\mathbf{G}_{\mathbf{feq}}\left(\mathbf{y}_{\mathbf{f}}\right) + \mathbf{S}\mathbf{p}_{\mathbf{s}} = \mathbf{0} \tag{5.113}$$

$$\mathbf{G}_{\mathbf{fin}}\left(\mathbf{y}_{\mathbf{f}}\right) \leq \mathbf{0} \tag{5.114}$$

$$\mathbf{y}_{\mathbf{f}} \in \mathbb{R}^{mf}, \quad \mathbf{p}_{\mathbf{s}} \in \mathbb{R}^{nb}$$

where $\mathbf{S}$ is defined as (3.171).

The optimality and feasibility cuts can be derived according to the GBD based method similar to the one of the STEP-AC problem. The explicit form of the optimality and feasibility cuts can be expressed as (5.115) and (5.116).

$$-z + IVF_1\left(\mathbf{c}_\mathbf{b}^\mathrm{T}\mathbf{x} + \mathbf{c}_\mathbf{f}^\mathrm{T}\mathbf{x}_\mathbf{f}\right) + \mathbf{c}_\mathbf{fac}^\mathrm{T}\bar{\mathbf{y}}_\mathbf{f} + \bar{\boldsymbol{\lambda}}_\mathbf{Ha}^\mathrm{T}\left(\bar{\mathbf{x}} - \mathbf{x}\right) + \bar{\boldsymbol{\lambda}}_\mathbf{Hf}^\mathrm{T}\left(\bar{\mathbf{x}}_\mathbf{f} - \mathbf{x}_\mathbf{f}\right) \leq 0 \tag{5.115}$$

$$\mathbf{e}_{nb}^\mathrm{T}\bar{\mathbf{p}}_\mathbf{s} + \bar{\boldsymbol{\mu}}_\mathbf{Ha}^\mathrm{T}\left(\bar{\mathbf{x}} - \mathbf{x}\right) + \bar{\boldsymbol{\mu}}_\mathbf{Hf}^\mathrm{T}\left(\bar{\mathbf{x}}_\mathbf{f} - \mathbf{x}_\mathbf{f}\right) \leq 0 \tag{5.116}$$

where

$(\bar{\mathbf{x}},\ \bar{\mathbf{x}}_\mathbf{f})$ is the minimizer obtained from solving the investment problem,

$\bar{\mathbf{y}}_\mathbf{f}$ and $\bar{\mathbf{p}}_\mathbf{s}$ are the minimizer of the operation problem and the feasibility problem,

$\bar{\boldsymbol{\lambda}}_\mathbf{Ha}$ and $\bar{\boldsymbol{\lambda}}_\mathbf{Hf}$ are the Lagrange multipliers of the operation problem according to the constraints (5.103) and (5.104).

$\bar{\boldsymbol{\mu}}_\mathbf{Ha}$ and $\bar{\boldsymbol{\mu}}_\mathbf{Hf}$ are the Lagrange multipliers of the feasibility problem according to the constraints (5.111) and (5.112).

The procedure for solving the STEP-UPFC can be performed in the same manner as the one of the STEP-AC presented in Section 3.6.5.

In case of the multistage TEP problem with N-1 security constraints, the method for solving the problem with FACTS device application can be developed from the one presented in this section by applying the formulation proposed in Section 3.7.

5.3 Conclusion

The proposed method has been further developed for TEP problem to include voltage stability and FACTS device of which the DC model cannot solve for. The proposed formulation can be applied using the framework of decomposed based method proposed in the previous chapters. The first application is the TEP problem with voltage stability constraint, whereas the second one is the TEP problem with consideration of FACTS device. In the second application, although, the formulation is presented in only the case of UPFC, it can be applied to the cases of other FACTS devices. Numerical results on test systems will be shown in the next chapter.

CHAPTER VI

TEST RESULTS

In this chapter, the decomposition based method for solving the TEP problems proposed in this dissertation will be tested with six-bus Garver [**35**], IEEE-24 bus [**35**] and 75-bus northeastern Thailand systems. The objective of the tests is to illustrate the capability of the decomposed formulations to cope with transmission expansion planning. Figures 6.1 and 6.2 show basic configurations of the Garver and IEEE-24 bus system. Detailed data of the test systems is shown in the appendix. It should be noted that this test data is generally used for a single stage TEP problem in previous TEP research works. Therefore, in the case of the multistage TEP problems, the power demand and the generation capacity shown in the appendix will be used as the values of the first stage. For the next stages, the power demand and the generation capacity will be defined by scaling the corresponding values in the first stage by the growth rates specified in the tests.

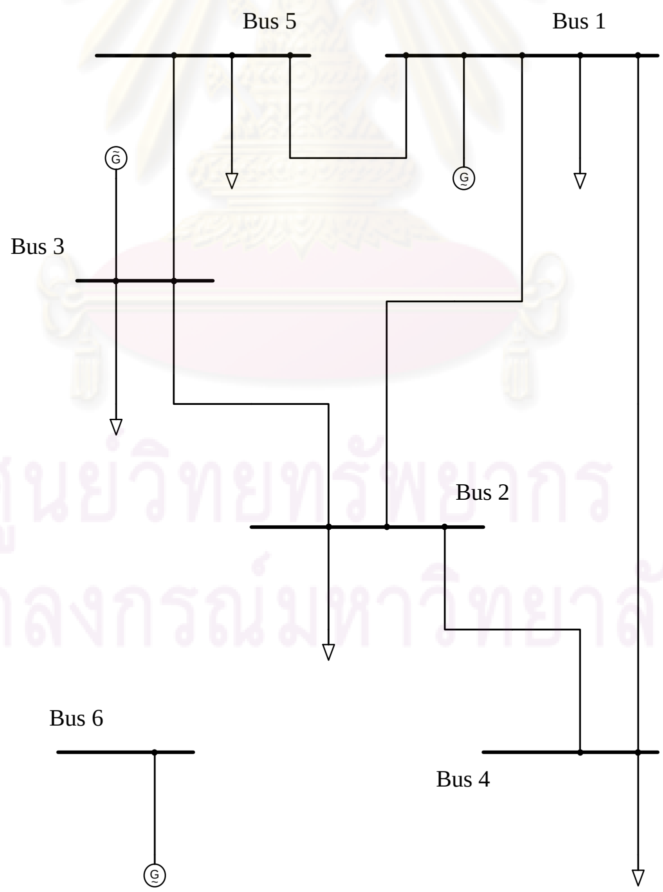

Figure 6.1 Basic configuration of Garver system

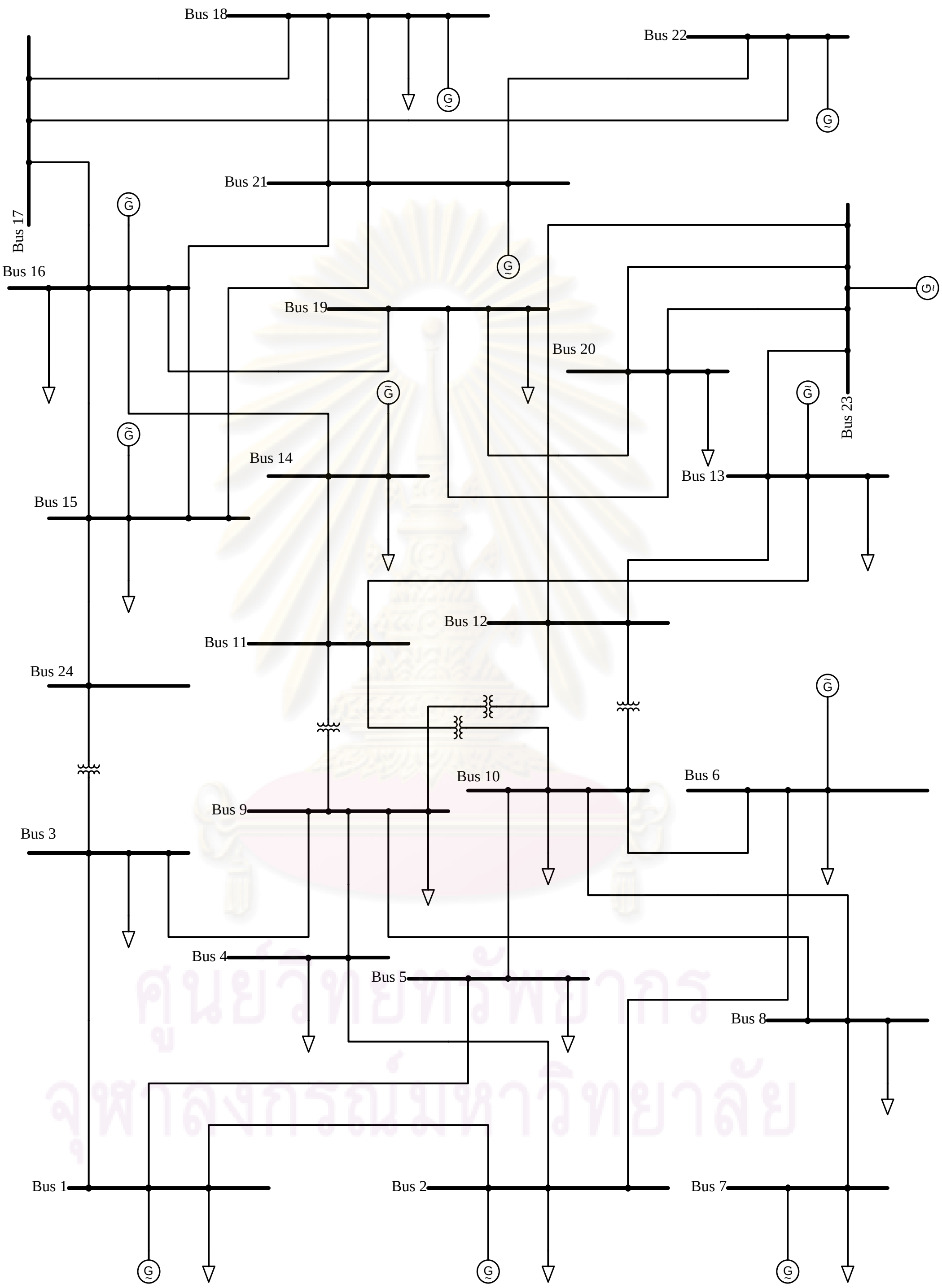

Figure 6.2 Basic configuration of IEEE-24 bus system

The life-time of all transmission equipments is assumed to be 25 years, whereas the interest rate is 10 % per year. For the case of the single stage TEP, it is assumed that the length of planning period is one year. In the case of the multistage TEP, the length of the planning period is nine years, of which the period is divided into three stages, spanning for three years each. The generation costs defined in the appendix are expressed in US$/KWh. Therefore, the generated active power has to be multiplied by the plant factor before calculating the operation cost for each stage. It is assumed that the plant factor is 60 %.

The maximum and minimum limits of the voltage magnitude are set at 1.05 p.u. and 0.95 p.u., respectively. The current limits of transmission lines and transformers are calculated from the thermal limits, $s_{ij}^{\max}$ expressed in the appendix.

The value of ϵ in the termination criterion of GBD is set at 10^{-4} for all tests. TOMLAB [73, 74] running on MATLAB is used as an optimization tool. For MILP and LP problems, CPLEX [75], which is based on the branch and cut algorithm [5, 76], is employed as a solver. For the cases of MINLP and NLP problems, KNITRO [77, 78] and SNOPT [79, 80], which is based on the sequential quadratic programming (SQP), are used to solve the problems respectively.

6.1 Single Stage TEP Using DC Model

Objective

The objectives of this test are listed below.

(a) To compare the capability of the decomposition based method with the direct method when solving the STEP-DC problem.

(b) To show the benefit from cost saving when the operating cost is taken into account in the TEP problem.

Details of test

The STEP-DC problem is formulated in a decomposition structure and solved by the proposed method of which the computational procedure described in Section 3.3.5. The results for the Garver system compared with the ones obtained from directly solving the problems formulated by the basic formulation presented in Section 3.3.1 are shown in Table 6.1. The investment plans obtained from both methods are the same. The detail is listed in Table 6.2.

For the IEEE-24 bus system, result comparison is shown in Table 6.3. The investment plans obtained from both methods are also the same. The detail is listed in Table 6.4.

Table 6.1 Results of STEP-DC for Garver system

Result	Decomposition method	Direct method
Investment cost (10^6 US$)	65.0	65.0
Operating cost (10^6 US$)	334.3	334.3
Computational time (sec.)	0.06	0.05
Number of iterations	3	–

Table 6.2 Detail of investment plan of STEP-DC for Garver system

From	To	Number of circuits	Cost (10^6 US$)
2	6	2	45.0
3	5	1	20.0
Total			65.0

Table 6.3 Results of STEP-DC for IEEE-24 bus system

Result	Decomposition method	Direct method
Investment cost (10^6 US$)	106.0	106.0
Operating cost (10^6 US$)	1,682.9	1,682.9
Computational time (sec.)	0.18	0.18
Number of iterations	5	–

To demonstrate the advantage of the proposed model over the original disjunctive model proposed in Ref. [**34**], the operating costs are all neglected in solving the STEP-DC problem. The obtained investment plan is shown in Table 6.5 which is different from the one shown in

Table 6.4 Detail of investment plan of STEP-DC for IEEE-24 bus system

From	To	Number of circuits	Cost (10^6 US$)
6	10	1	16.0
14	16	1	54.0
16	17	1	36.0
Total			106.0

Table 6.4. It should be noted that this plan is same as the one reported in Ref. [**36**] which is also neglected the operating cost in the formulation.

Table 6.5 Investment plan of STEP-DC for IEEE-24 bus system when neglecting operating cost in TEP

From	To	Number of circuits	Cost (10^6 US$)
6	10	1	16.0
7	8	2	24.0
Investment cost			40.0
Operating cost			1893.7

For the northeastern Thailand system, it is found that the system satisfies the defined planning criteria when the problem is formulated by using the DC model without N-1 security constraints. Therefore, it is not tested in the case of STEP-DC problem.

Discussion

From the results, it can be seen that, for a small system, the performance of the decomposition method is comparable with the one of the direct solving method. Since the MILP problems are not complicated, the direct method can solve the problem efficiently. Therefore, the efficiency of the proposed method is not evident.

When the operating cost is taken into account in the TEP problem, the transmission congestion can be alleviated. Therefore, the operating cost for the IEEE-24 bus system can be decreased by 11 % compared to the one when the operating cost is neglected in solving the TEP problem. One can see that some transmission routes to be selected as the obtained plans in both cases are different. Even though the investment cost in the case of neglecting the operating cost is lower, the total cost is higher.

6.2 Single Stage TEP Using DC Model with N-1 Security Constraints

Objective

The objective of this test is to compare the capability of the decomposition based method with the direct method when solving the STEP-DC-NSEC problem.

Details of test

The STEP-DC-NSEC problem is solved by the procedure of decomposition based method described in Section 3.4.5. The results for the Garver system, IEEE-24 bus system and northeastern Thailand system compared with the ones obtained from the direct solving method based on the basic formulation presented in Section 3.4.1 are shown in Tables 6.6, 6.8 and 6.10 respectively.

The investment plans obtained from both methods are the same for all test systems, whereas the details are listed in Tables 6.7, 6.9 and 6.11.

Table 6.6 Results of STEP-DC-NSEC for Garver system

Result	Decomposition method	Direct method
Investment cost (10^6 US$)	115.0	115.0
Operating cost (10^6 US$)	334.3	334.3
Computational time (sec.)	1.45	809.10
Number of iterations	15	–

Discussion

From the results, the advantage of the decomposition based method over the direct method is clearly shown. The reason is that the difficulty of the STEP-DC-NSEC problem

Table 6.7 Detail of investment plan of STEP-DC-NSEC for Garver system

From	To	Number of circuits	Cost (10^6 US$)
2	3	1	20.0
3	5	1	20.0
4	6	1	30.0
4	6	2	45.0
Total			115.0

Table 6.8 Results of STEP-DC-NSEC for IEEE-24 bus system

Result	Decomposition method	Direct method
Investment cost (10^6 US$)	238.0	238.0
Operating cost (10^6 US$)	1682.9	1682.9
Computational time (sec.)	468	24,179
Number of iterations	109	–

Table 6.9 Detail of investment plan of STEP-DC-NSEC for IEEE-24 bus system

From	To	Number of circuits	Cost (10^6 US$)
3	9	1	31.0
4	9	1	27.0
6	10	2	24.0
7	8	1	16.0
10	12	1	50.0
14	16	1	54.0
16	17	1	36.0
Total			238.0

is mainly caused by the high number of constraints corresponding to each scenario. Therefore, decomposition based method can extremely reduce the computational burden.

Table 6.10 Results of STEP-DC-NSEC for northeastern Thailand system

Result	Decomposition method	Direct method
Investment cost (10^6 US$)	179.2	179.2
Operating cost (10^6 US$)	630.3	630.3
Computational time (sec.)	325	57,725
Number of iterations	27	–

Table 6.11 Detail of investment plan of STEP-DC-NSEC for northeastern Thailand system

From	To	Number of circuits	Cost (10^6 US$)
6	11	1	25.6
7	38	1	18.5
10	49	1	21.3
16	47	1	19.5
17	69	1	24.2
18	20	2	9.2
18	20	2	9.2
24	37	1	32.2
56	61	1	19.5
Total			179.2

Considering the obtained plans, one can investigate that the levels of transmission investment increase from the plans obtained from solving the STEP-DC problem for both Garver and IEEE-24 bus systems. In addition, some transmission lines in the plan of the STEP-DC problem are not necessary in the plan of the STEP-DC-NSEC problem, e.g. the transmission line 2–6 in the case of Garver system. Therefore, determining the transmission plan of the STEP-DC-NSEC problem by using the plan of the STEP-DC problem as a basic configuration may lead to a local optimum plan, even though the problem is formulated by the DC model, which is a convex problem.

6.3 Multistage TEP Using DC Model

Objective

The objectives of this test are listed below.

(a) To compare the capability of the decomposition based method to the direct solving method when apply to the MTEP-DC problem.

(b) To show the advantage of the multistage planning over the single stage planning.

Details of test

The planning period is divided into three stages, of which each interval spans for three years. It is assumed that the power demand and the generation capacity monotonously increase throughout the planning period. Therefore, the plan established at the beginning of each stage must be able to serve the demand at the end of stage. For this reason, the power demand and the generation capacities at year 3, 6 and 9 will be used as the representative values of the first, second, and third stages respectively.

In the first stage, the power demand and the installed generation capacity are defined according to the data shown in the appendix. For the next two stages, the demand is assumed to grow by 8 %, 4 % and 6 % per year for the Garver system, the IEEE-24 bus system and the northeastern Thailand system, respectively. The increase of demand and generation capacity from the data in the appendix for the second and third stages are summarized in Table 6.12.

Apart from the comparison between the decomposition based method and the direct method as the previous tests, the advantage of the multistage TEP over the single stage TEP will be shown in this test. It should be noted that the concept of the multistage planning is relevant to the economic aspect about the using of resource in the planning period. Consequently, to demonstrate this advantage, the resource should be limited. In the case of the TEP, the transmission paths are deficient resource, especially in the urban area. Therefore, the transmission paths will be limited to two paths for the Garver system and one path for the IEEE-24 bus and northeastern Thailand system.

Comparison of the results obtained from the decomposition based method and the direct

Table 6.12 Assumption of power demand growths and generation capacities

Description	Second stage	Third stage
Garver system		
Demand	26.0 %	58.7 %
Generation	25.0 %	50.0 %
IEEE-24 bus system		
Demand	12.5 %	26.5 %
Generation	15.0 %	25.0 %
Northeastern Thailand system		
Demand	19.1 %	41.9 %
Generation	20.0 %	40.0 %

method are shown in Tables 6.13, 6.15 and 6.17. The transmission plans obtained from both methods are the same plan which are shown in Tables 6.14, 6.16 and 6.18 compared with the plan obtained from consecutively solving the single stage TEP problem of each stage, in which the base configuration of the network obtained from the plan of the previous stage. In the tables, MTEP-DC refers to solving the multistage TEP by the decomposition based method and the direct method which are used the multistage formulation, while CSTEP-DC refers to the solving the single stage TEP consecutively.

Table 6.13 Results of MTEP-DC for Garver system

Result	Decomposition method	Direct method
Investment cost [1,2] $(10^6\ \text{US\$})$	78.3	78.3
Operating cost [1] $(10^6\ \text{US\$})$	2,420.1	2,420.1
Computational time (sec.)	11.7	5.4
Number of iterations	29	–

[1] Net present value at 10 % interest rate
[2] Less the salvage value at the end of planning period

Discussion

From the results, the computational time in solving the problem of the decomposition based method is greater than that of the direct method. The reason is that in the case of

Table 6.14 Detail of investment plan of multistage TEP for Garver system

Stage	MTEP-DC		CSTEP-DC	
	From–To	Number	From–To	Number
First stage	2–6	2	2–6	2
	3–5	1	3–5	1
Second stage	2–3	1	2–6	1
	4–6	2	3–5 [1]	1
Third stage	3–5 [1]	1	2–3	1
			4–6	2
Investment cost [2, 3] (10^6 US$)	78.3		79.5	
Operating cost [2] (10^6 US$)	2,420.1		2,420.1	

[1] Stringing the second circuit on the tower constructed in the first stage
[2] Net present value at 10 % interest rate
[3] Less the salvage value at the end of planning period

Table 6.15 Results of MTEP-DC for IEEE-24 bus system

Result	Decomposition method	Direct method
Investment cost [1, 2] (10^6 US$)	170.4	170.4
Operating cost [1] (10^6 US$)	11,255.6	11,255.6
Computational time (sec.)	3,318	234
Number of iterations	207	–

[1] Net present value at 10 % interest rate
[2] Less the salvage value at the end of planning period

multistage TEP, the difficulty of the problem is the complication of the investment problem of which the number of integer variables is high. It can be noticed that the structure of the MTEP-DC problem is similar to the one of the STEP-DC problem. However, the number of integer variables and the number of constraints of the MTEP-DC problem are increased ns times from the ones of the STEP-DC problem. The increase of the number of integer variables causes the investment problem to be more complicated, while the increase of the number of constraints causes the number of the operation problems increases linearly. Since the investment problem is an MILP which is in class of nondeterministic polynomial (NP) problems, the increasing of its complexity is not linear with respect to the number of stages as the case of the operation problem which is solved by the decomposition approach. Therefore, the complexity

Table 6.16 Detail of investment plan of multistage TEP for IEEE-24 bus system

Stage	MTEP-DC		CSTEP-DC	
	From–To	Number	From–To	Number
First stage	6–10	1	6–10	1
	14–16	1	14–16	1
	16–17	1	16–17	1
Second stage	1–5	1	1–5	1
	13–23	1	13–23	1
	15–21	1	16–17 [1]	1
			17–18	1
Third stage	7–8	1	7–8	1
			15–21	1
Investment cost [2,3] (10^6 US$)	170.4		175.1	
Operating cost [2] (10^6 US$)	11,255.6		11,255.6	

[1] Stringing the second circuit on the tower constructed in the first stage
[2] Net present value at 10 % interest rate
[3] Less the salvage value at the end of planning period

Table 6.17 Results of MTEP-DC for northeastern Thailand system

Result	Decomposition method	Direct method
Investment cost [1,2] (10^6 US$)	21.6	21.6
Operating cost [1] (10^6 US$)	4,392.7	4,392.7
Computational time (sec.)	2,335	219
Number of iterations	96	–

[1] Net present value at 10 % interest rate
[2] Less the salvage value at the end of planning period

of the TEP problem due to the integer variables cannot be reduced by using the decomposition based method.

From above-mentioned reason, the method for increasing the performance in solving the investment problem, e.g. the local search presented in Section 4.1, should be developed when taking into account the N-1 security in the multistage TEP problem.

Apart from the computational aspect, the benefit of the multistage planning is shown. It

Table 6.18 Detail of investment plan of multistage TEP for northeastern Thailand system

Stage	MTEP-DC		CSTEP-DC	
	From–To	Number	From–To	Number
First stage	–		–	
Second stage	1–10	1	1–10	1
	2–3	1	2–3	1
	18–20	1	18–20	1
	20–25	1	20–25	1
Third stage	–		–	
Investment cost [1,2] (10^6 US$)	21.6		21.6	
Operating cost [1] (10^6 US$)	4,392.7		4,392.7	

[1] Net present value at 10 % interest rate
[2] Less the salvage value at the end of planning period

can obviously seen from Tables 6.14 and 6.16 that the investment costs obtained from solving the multistage TEP problems of the Garver system and IEEE-24 bus system by the proposed formulation are lower than the ones obtained from consecutively solving the single stage TEP problems.

For the northeastern Thailand system, the benefit of the multistage TEP cannot be evident in this test. The reason is that the systems in the first and second stages actually meet the defined planning criteria for the TEP based on the DC model without N-1 security constraints, since the test system is modified from the actual system which is analyzed based on the AC model, and mostly satisfied N-1 planning criteria. The transmission lines added in the second stage are only used to reduce the transmission congestion. Both MTEP-DC method and CSTEP-DC method search for the best plan that can greatly reduce the congestion in the second stage because the congestion should be reduced since the early stage. Therefore, they obtain the same plan, which is the best plan. In addition, this plan causes the system in the third stage is feasible, and the transmission congestion does not exist.

6.4 Single Stage TEP Using AC Model

Objective

The objectives of the test are listed below.

(a) To compare the capability of the decomposition based method to the direct method when solving the STEP-AC problem.

(b) To show the advantage of the AC model over the DC model in the TEP problem formulation.

(c) To compare the results of STEP-AC problem obtained from the proposed method to previous works.

Details of test

The STEP-AC problem is solved by the proposed decomposition based method of which the computational procedure described in Section 3.6.5. The investment problem is initialized by the one of STEP-DC.

The result compared with the one obtained from the direct method for the Garver system is shown in Table 6.19. It is found that both methods provide the same plan, with detail shown in Table 6.20. The power flow diagram of the Garver system after solving the STEP-AC problem is shown in Figure 6.3.

Table 6.19 Results of STEP-AC for Garver system

Result	Decomposition method	Direct method
Investment cost (10^6 US$)	95.0	95.0
Installation cost of reactive power (10^6 US$)	1.5	1.5
Operating cost (10^6 US$)	342.5	342.5
Computational time (sec.)	1.14	40.38
Number of iterations	16	–

For the IEEE-24 bus system, the direct method cannot find the optimum solution. The solver reports the problem is infeasible. It should be noted that this result can occur in any cases

Table 6.20 Detail of investment plan of STEP-AC for Garver system

From	To	Number of circuits	Cost (10^6 US$)
2	6	1	30.0
3	5	1	20.0
4	6	2	45.0
Total			95.0

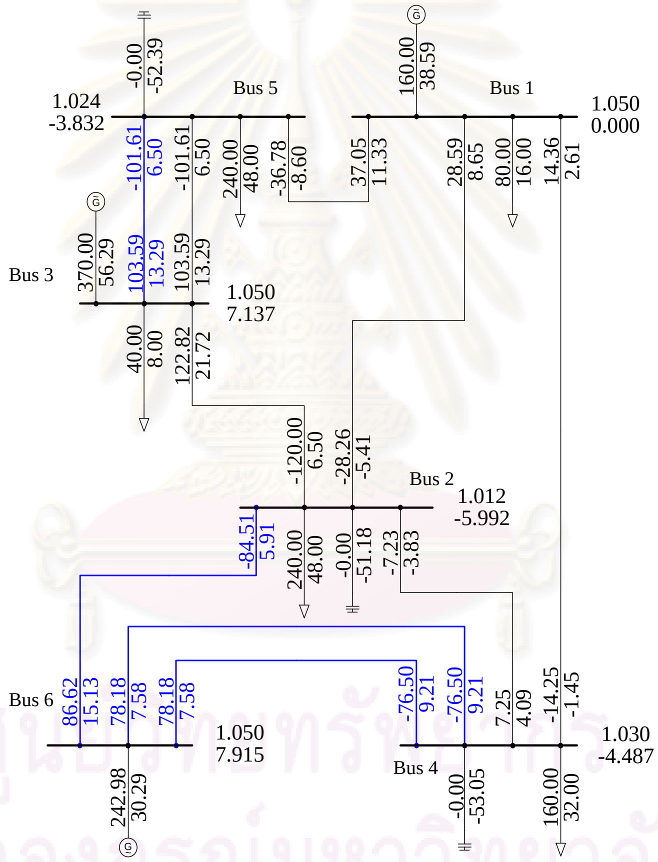

Figure 6.3 Power flow diagram of STEP-AC problem for Garver system

when solving nonconvex problems by solvers based on the convex optimization. The result of STEP-AC problem for the IEEE-24 bus system is shown in Tables 6.21 and 6.22. The power flow diagram is shown in Figure 6.4.

In case of the northeastern Thailand system, the system meets the defined planning criteria for STEP-AC problem with installation of the reactive power compensation devices. The

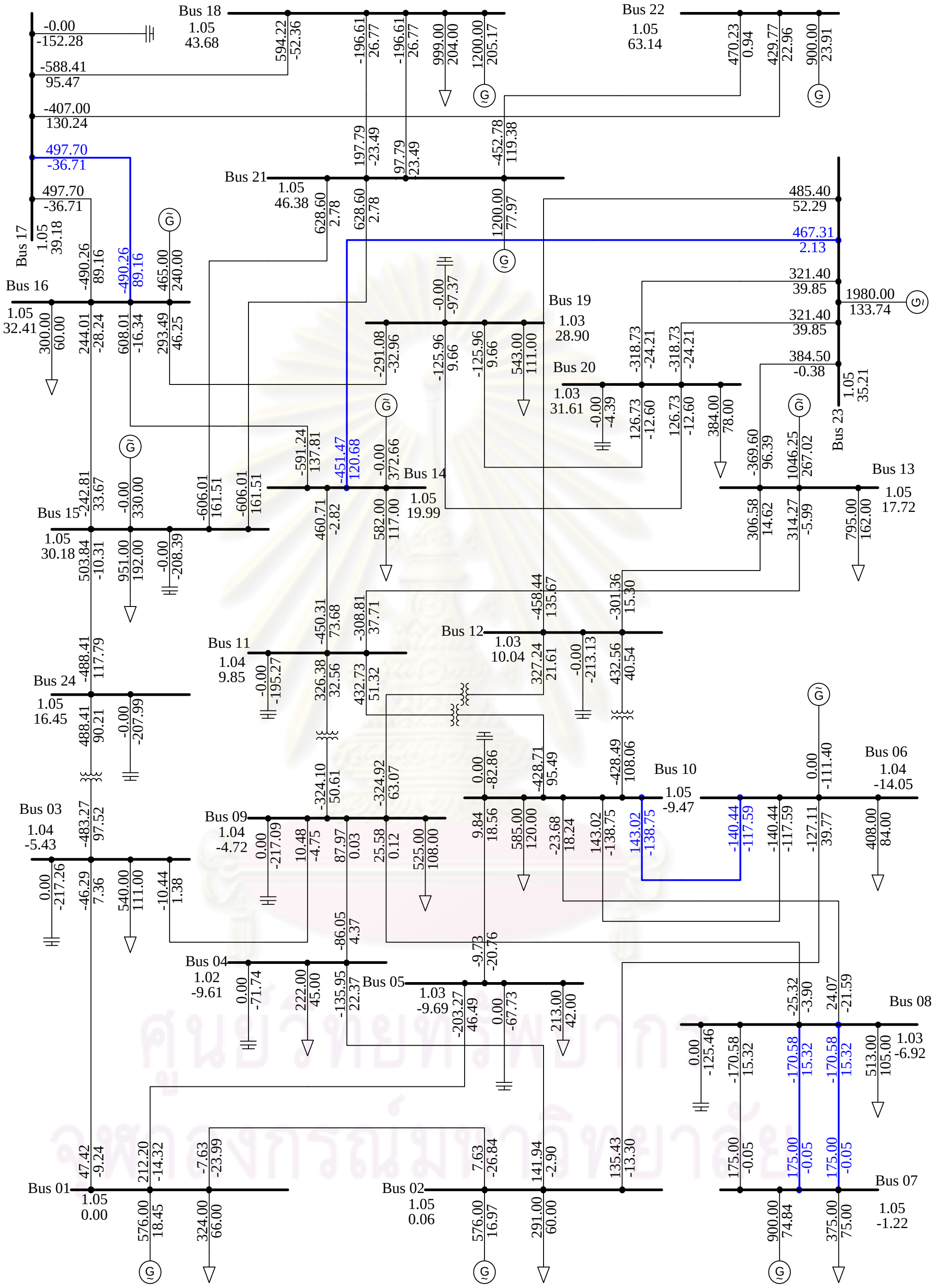

Figure 6.4 Power flow diagram of STEP-AC problem for IEEE-24 bus system

Table 6.21 Results of STEP-AC for IEEE-24 bus system

Result	Decomposition method	Direct method
Investment cost (10^6 US$)	162.0	N/A
Installation cost of reactive power (10^6 US$)	17.5	N/A
Operating cost (10^6 US$)	1,806.0	N/A
Computational time (sec.)	20.5	N/A
Number of iterations	76	–

Table 6.22 Detail of investment plan of STEP-AC for IEEE-24 bus system

From	To	Number of circuits	Cost (10^6 US$)
6	10	1	16.0
7	8	2	24.0
14	23	1	86.0
16	17	1	36.0
Total			162.0

additional transmission lines are not required. Therefore, it will not be considered in this test.

The proposed method is also compared to the methods for solving the TEP problem with AC model proposed in previous works [**35**, **36**] which do not take into account the operating cost and the nonlinearity of investment cost function. Therefore, STEP-AC problem is solved by the decomposition based method by neglecting the operation cost and the nonlinearity of investment cost function. The obtained plans for both systems are the same as the best ones obtained from Refs [**35**, **36**]. The details of plans are shown in Tables 6.23 and 6.24.

Discussion

Based on results comparison, it is clearly seen that the decomposition based method is superior to the direct method from both the efficiency and the solution quality aspects.

For the Garver system, the advantage of the AC model over the DC model in the appli-

Table 6.23 Investment plan of STEP-AC for Garver system when neglecting operating cost and nonlinearity of investment cost function

From	To	Number of circuits	Cost (10^6 US$)
2	6	2	60.0
3	5	1	20.0
4	6	1	30.0
Total			110.0

Table 6.24 Investment plan of STEP-AC for IEEE-24 bus system when neglecting operating cost and nonlinearity of investment cost function

From	To	Number of circuits	Cost (10^6 US$)
6	10	1	16.0
7	8	2	32.0
Total			48.0

cation of the TEP is also clearly shown. It should be noted that the plan obtained from solving STEP-DC problem cannot provide a feasible solution for the AC model. Therefore, additional transmission lines, i.e. two circuits of line 4–6, are required. In addition, the transmission congestion is also alleviated.

In the case of the IEEE-24 bus system, the plan obtained from solving the STEP-DC is feasible for the STEP-AC by installing reactive power devices. Transmission lines 7–8 and 14–23 are also added to reduce the active power loss and the transmission congestion, while transmission line 14–16, which is in the plan of STEP-DC problem, is not in the plan of STEP-AC problem. This numerical result shows that the plan obtained from STEP-DC problem is not necessary to be in the plan obtained from STEP-AC problem, even though, the investment problem of the STEP-AC problem is initialized by the one obtained from solving STEP-DC problem.

In addition, the results obtained from the proposed method can be comparable to the best ones found in the previous works of TEP based on the AC model which apply a constructive heuristic algorithm [**35**] and a genetic algorithm [**36**] to solve the problem. However, the results can be compared in only case of neglecting the operating cost and the nonlinearity of investment cost function.

6.5 Multistage TEP Using AC model with N-1 Security Constraints

Objective

The objective of this test is to show the capability of the proposed method for solving the MTEP-AC-NSEC problem, ultimately applied in actual transmission planning.

Garver system

The MTEP-AC-NSEC is formulated in the decomposed structure as proposed in Sections 3.7.2–3.7.5. In addition, the methods for improving the performance and solution quality, i.e. the local search technique and the modification of cuts, presented in Chapter 4 are adopted in this test. The parameter for cut modification, α, is set at 0.5.

The results are shown in Table 6.25. The proposed procedure is performed and finished in 112 seconds with 50 iterations.

The power flow diagram in each stage is shown in Figures 6.5, 6.6 and 6.7. The examples of the single outage contingencies are shown in Figures 6.8, 6.9 and 6.10. It should be noted that the 27.11 MVAr capacitor is installed at bus 1 in the first stage. However, it is out-of-service in the base case. From the numerical results, it will be operated when the line 2–3 is tripped.

In comparison, the MTEP-AC-NSEC problem for this Garver system is also formulated by the basic formulation presented in Section 3.7.1 and solved by the direct method. However, the solver cannot find the optimum solution within 24 hours, which is set as computational time limitation. Therefore, the problem is reduced by neglecting the N-1 security constraints. After 5 hours, the incumbent of integer solution cannot be found. In addition, the lower bound of the branch and bound process, i.e. the higher solution of NLP problems solved so far, is greater than 2,612 million US$. This situation indicates that even though the optimum solution can be found, its value will not be lower than 2,612 million US$. Comparing the results obtained from the proposed method, one can see that the total of investment and operation costs is 2,589.4

Table 6.25 Detail of investment plan of MTEP-AC-NSEC for Garver system

From	To	Number of circuits	Cost (10^6 US$)
First stage			
2	6	2	45.0
3	5	2	30.0
4	6	2	45.0
Second stage			
2	3	1	20.0
4	6	2	45.0
Third stage			
1	5	2	30.0
2	3	1 [1]	16.0
Investment cost [2, 3]			124.1
Installation cost of capacitors [2, 3]			1.3
Operating cost [2]			2,465.3

[1] Stringing the second circuit on the tower constructed in the second stage
[2] Net present value at 10 % interest rate
[3] Less the salvage value at the end of planning period

US$. Therefore, it can be concluded that the proposed method can attain the plan better than the one (if exist) obtained from the direct method even though the N-1 security constraints are taken into account.

IEEE-24 bus

The MTEP-AC-NSEC is formulated in the decomposed structure as proposed in Sections 3.7.2–3.7.5. The methods for improving the performance and solution quality are also applied. The parameter for cut modification, α, is set at 0.5.

The results are shown in Table 6.26. After performing 163 iterations, the proposed procedure is finished within 15,489 seconds. The details of the installation of reactive power devices are shown in Table 6.27. In the table, ‘CAP.’ is refers to the capacitor, and ‘REAC.’ is refers to the reactor.

The power flow diagram for each stage is shown in Figures 6.11, 6.12 and 6.13. The examples of the single outage contingencies can be shown in Figures 6.14, 6.15, 6.16.

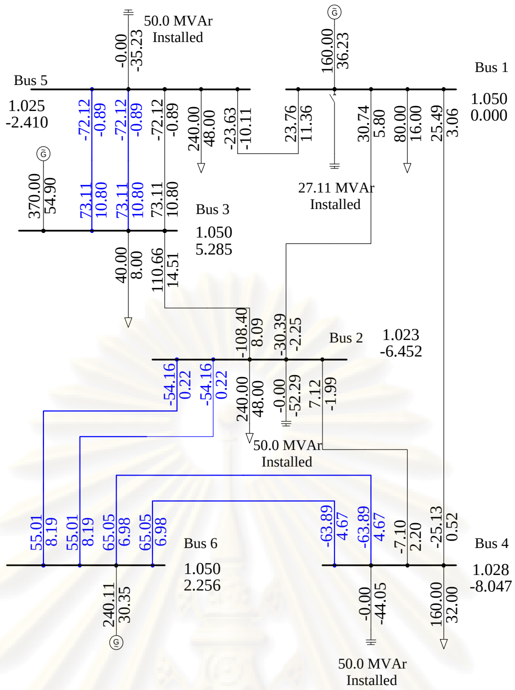

Figure 6.5 Power flow diagram of the first stage for Garver system

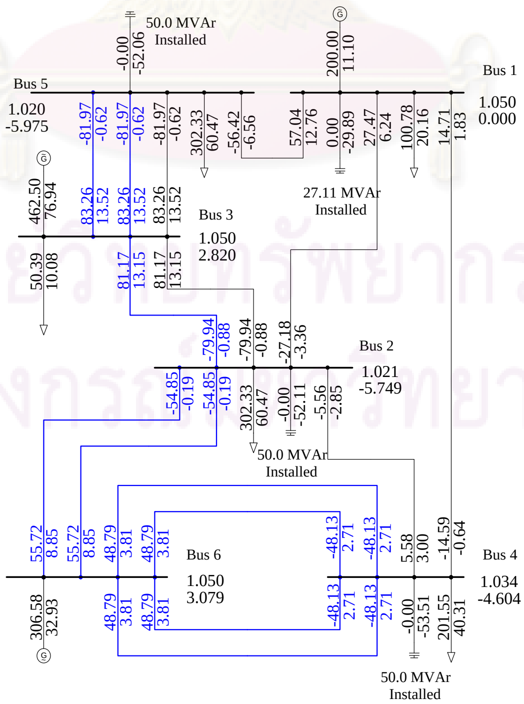

Figure 6.6 Power flow diagram of the second stage for Garver system

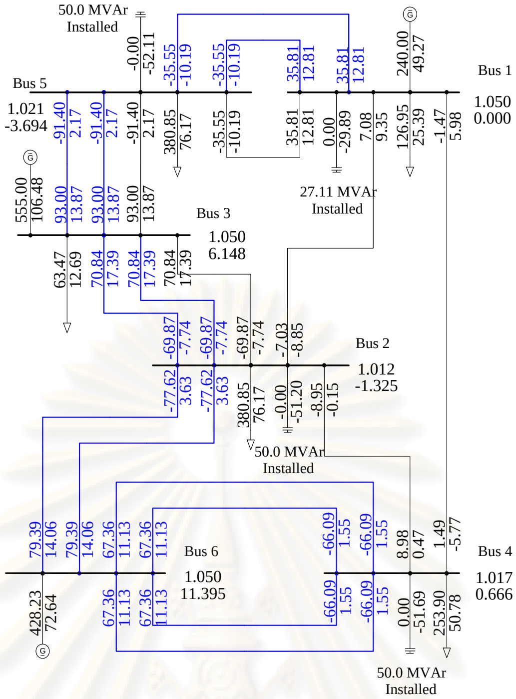

Figure 6.7 Power flow diagram of the third stage for Garver system

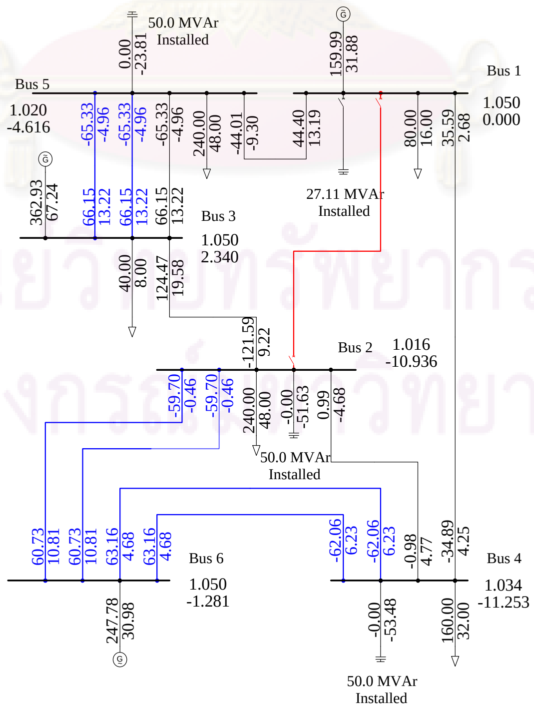

Figure 6.8 Power flow diagram of the first stage for Garver system with line 1–2 tripped

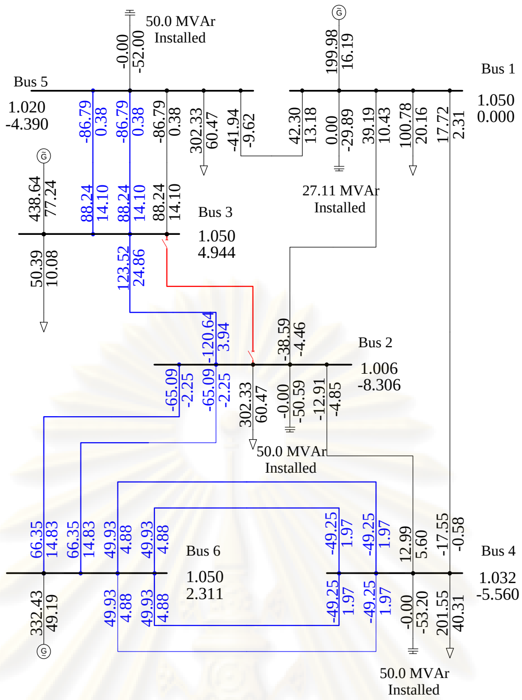

Figure 6.9 Power flow diagram of the second stage for Garver system with line 2–3 tripped

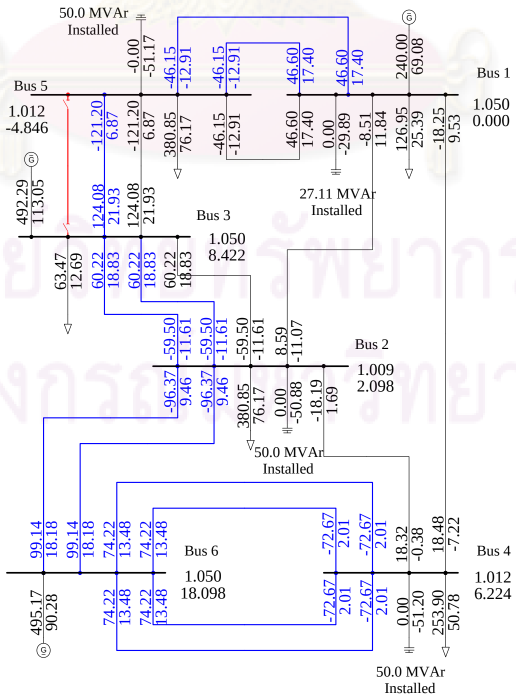

Figure 6.10 Power flow diagram of the third stage for Garver system with line 3–5 tripped

Table 6.26 Detail of investment plan of MTEP-AC-NSEC for IEEE-24 bus system

From	To	Number of circuits	Cost (10^6 US$)
First stage			
1	5	2	33.0
2	4	1	33.0
3	24	1	50.0
6	10	1	16.0
6	10	2	24.0
7	8	2	24.0
7	8	2	24.0
14	23	2	129.0
15	21	2	102.0
15	24	1	72.0
Second stage			
10	11	1	50.0
16	17	2	54.0
Third stage			
11	14	1	58.0
Investment cost [1, 2]			425.1
Installation cost of capacitors [1, 2]			18.0
Operating cost [1]			11,940.1

[1] Net present value at 10 % interest rate
[2] Less the salvage value at the end of planning period

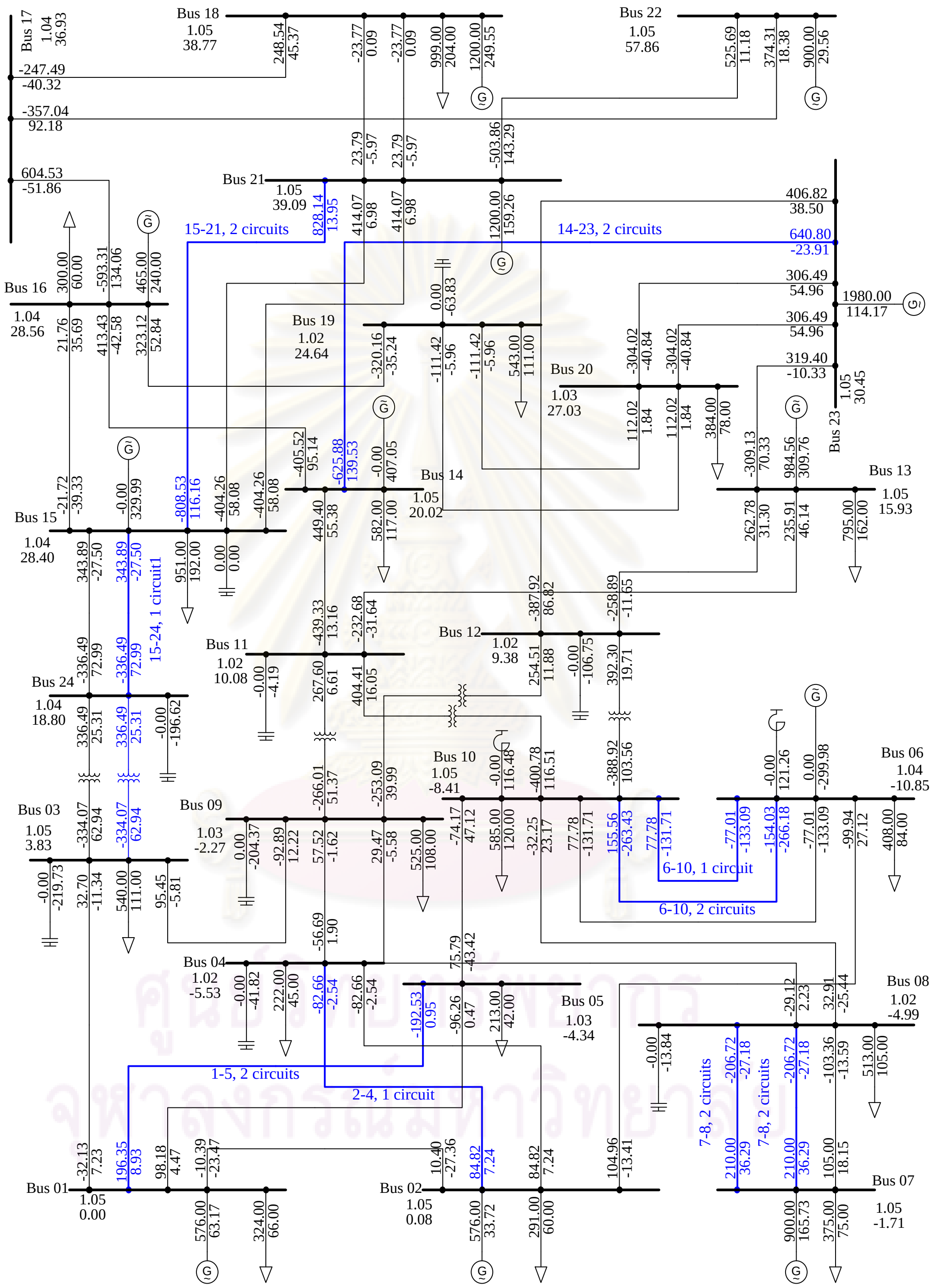

Figure 6.11 Power flow diagram of the first stage for IEEE-24 bus system

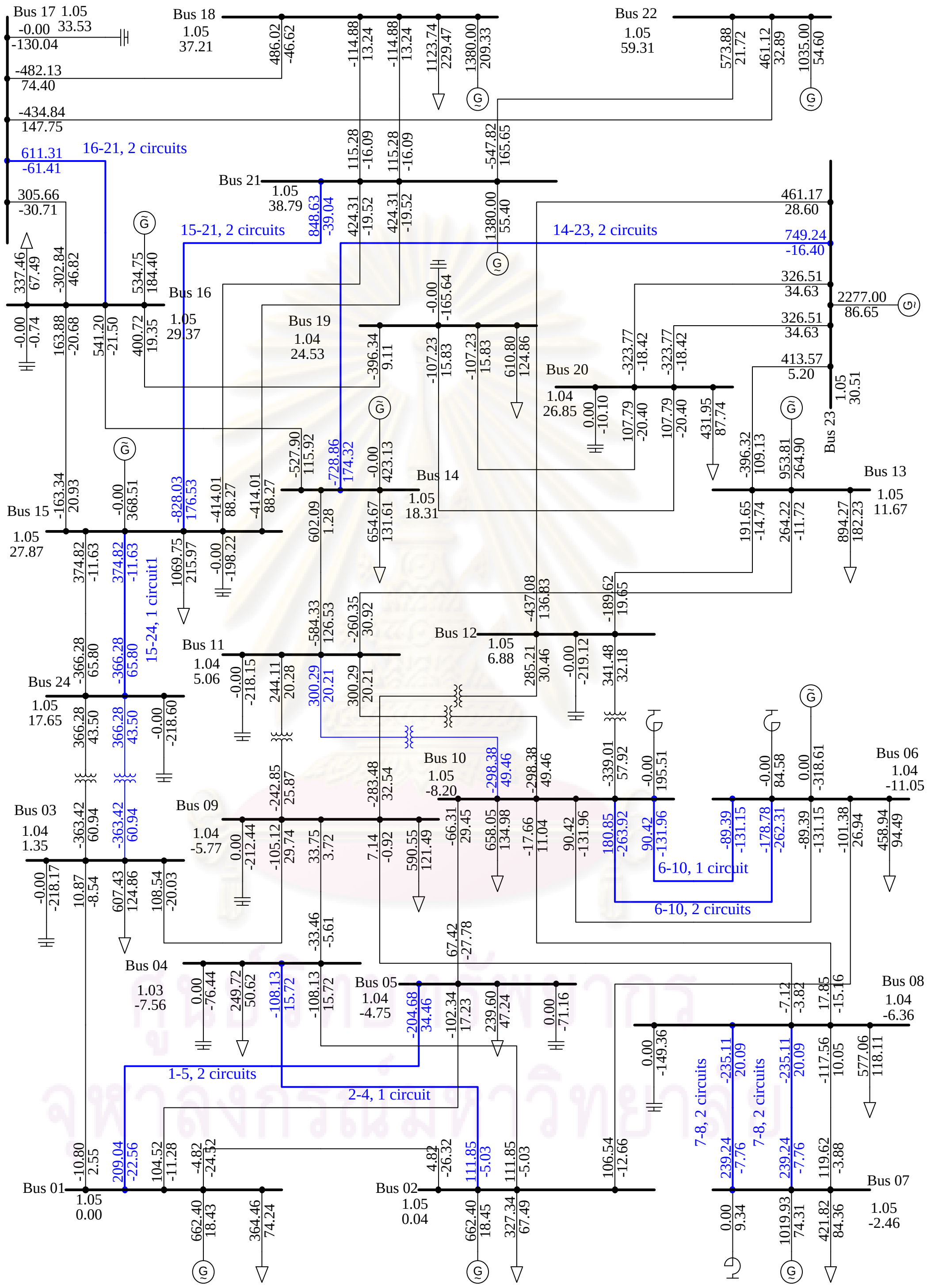

Figure 6.12 Power flow diagram of the second stage for IEEE-24 bus system

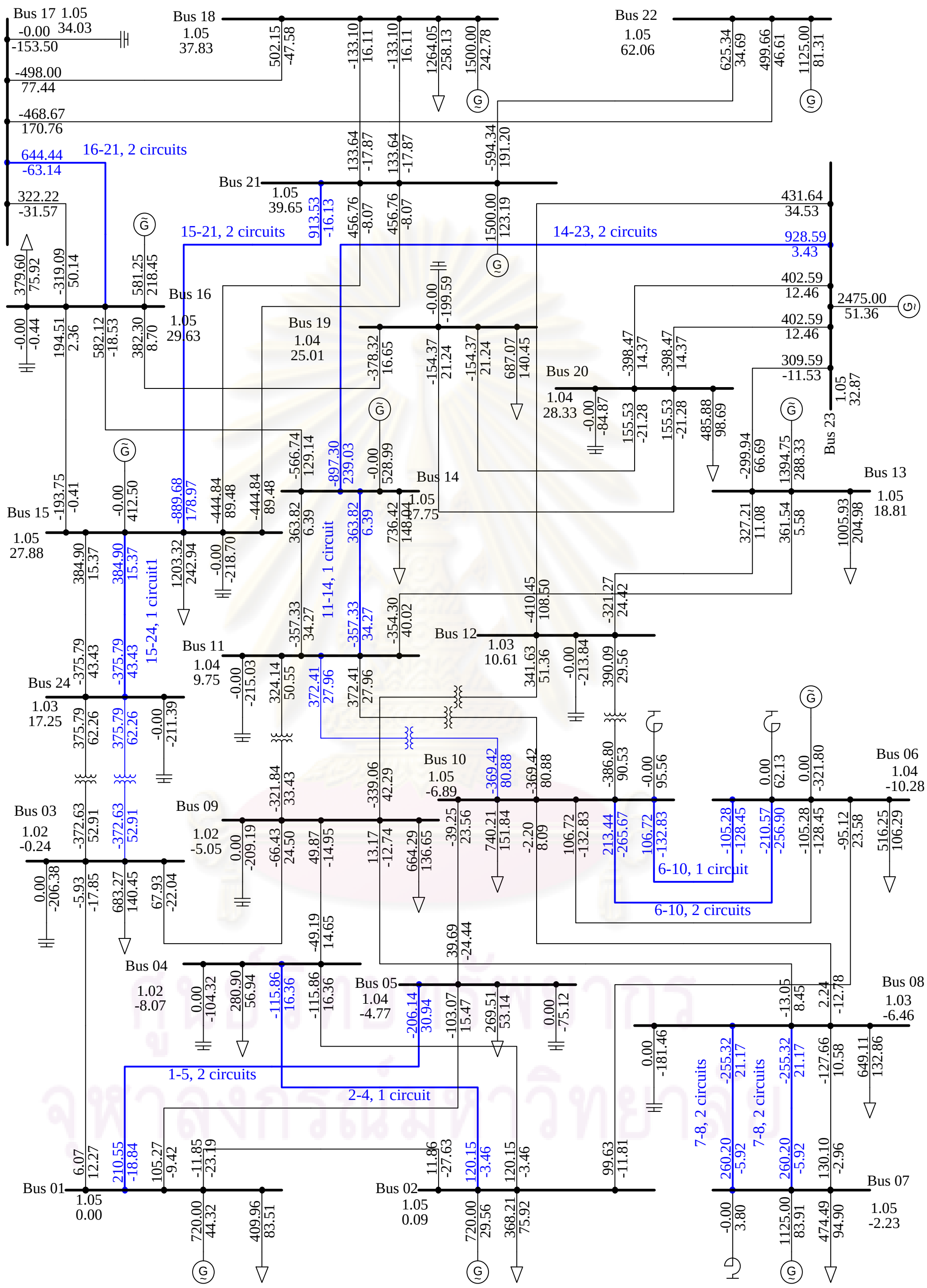

Figure 6.13 Power flow diagram of the third stage for IEEE-24 bus system

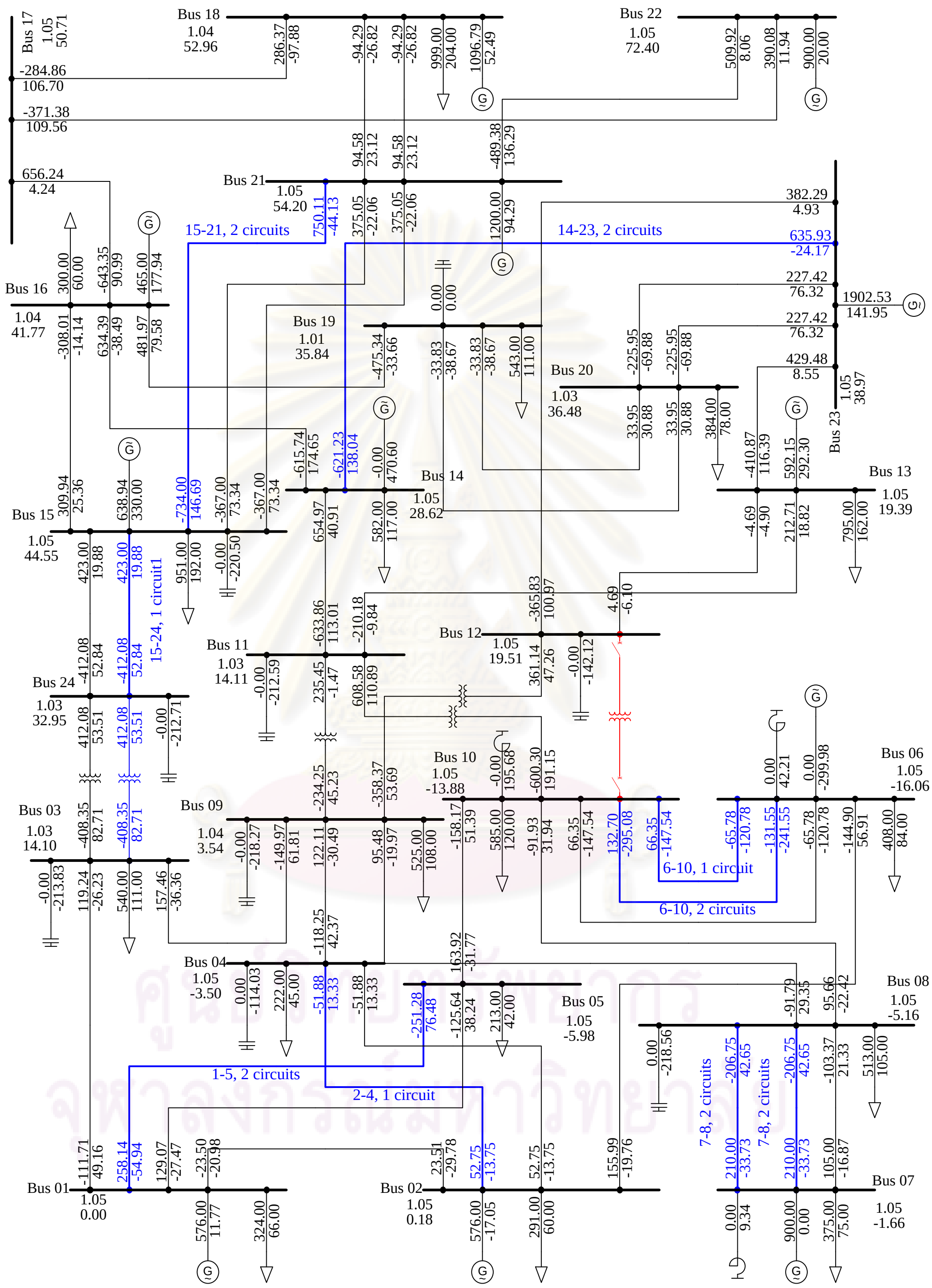

Figure 6.14 Power flow diagram of the first stage for IEEE-24 bus system with transformer 12–10 tripped

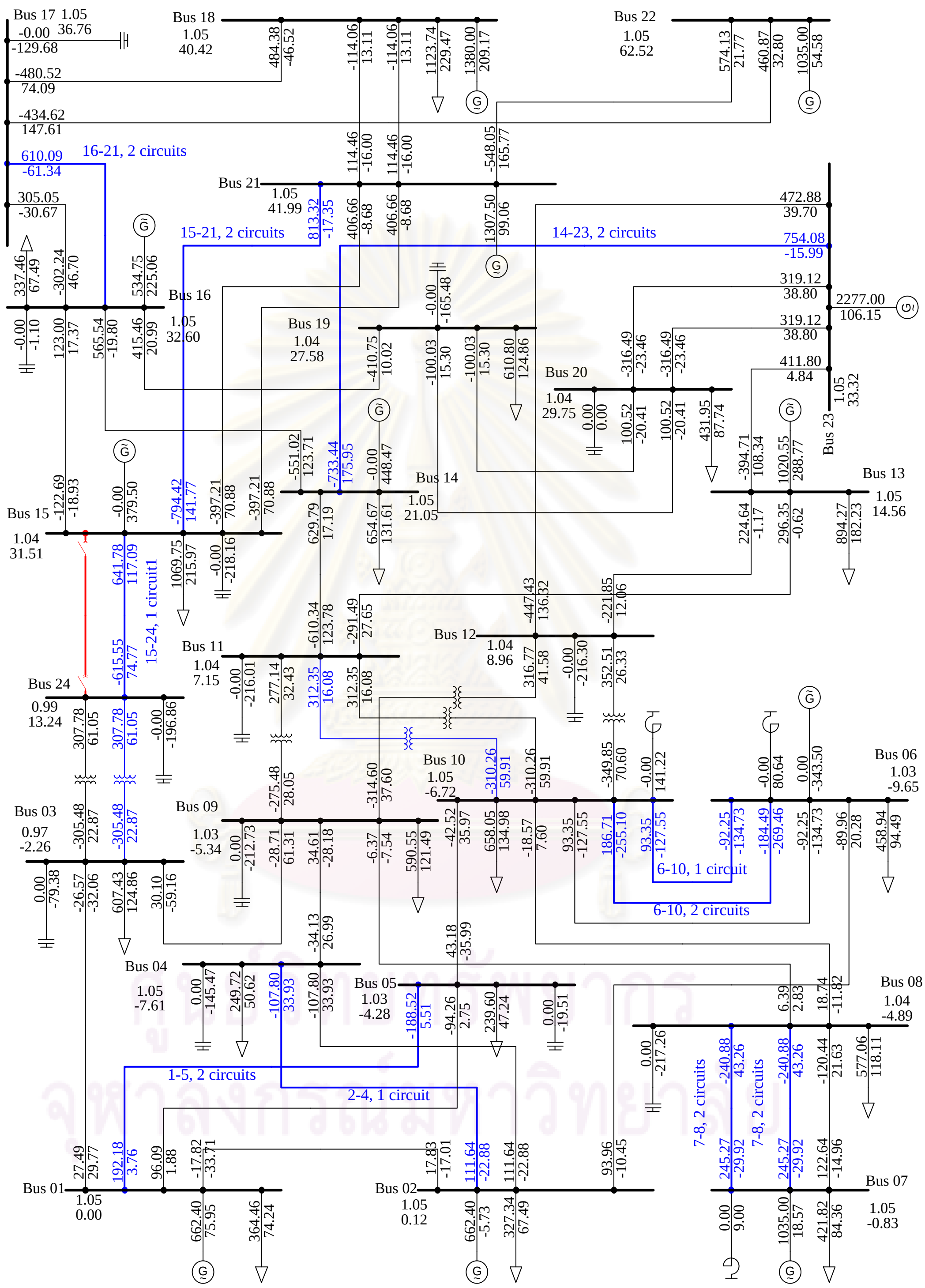

Figure 6.15 Power flow diagram of the second stage for IEEE-24 bus system with line 15–24 tripped

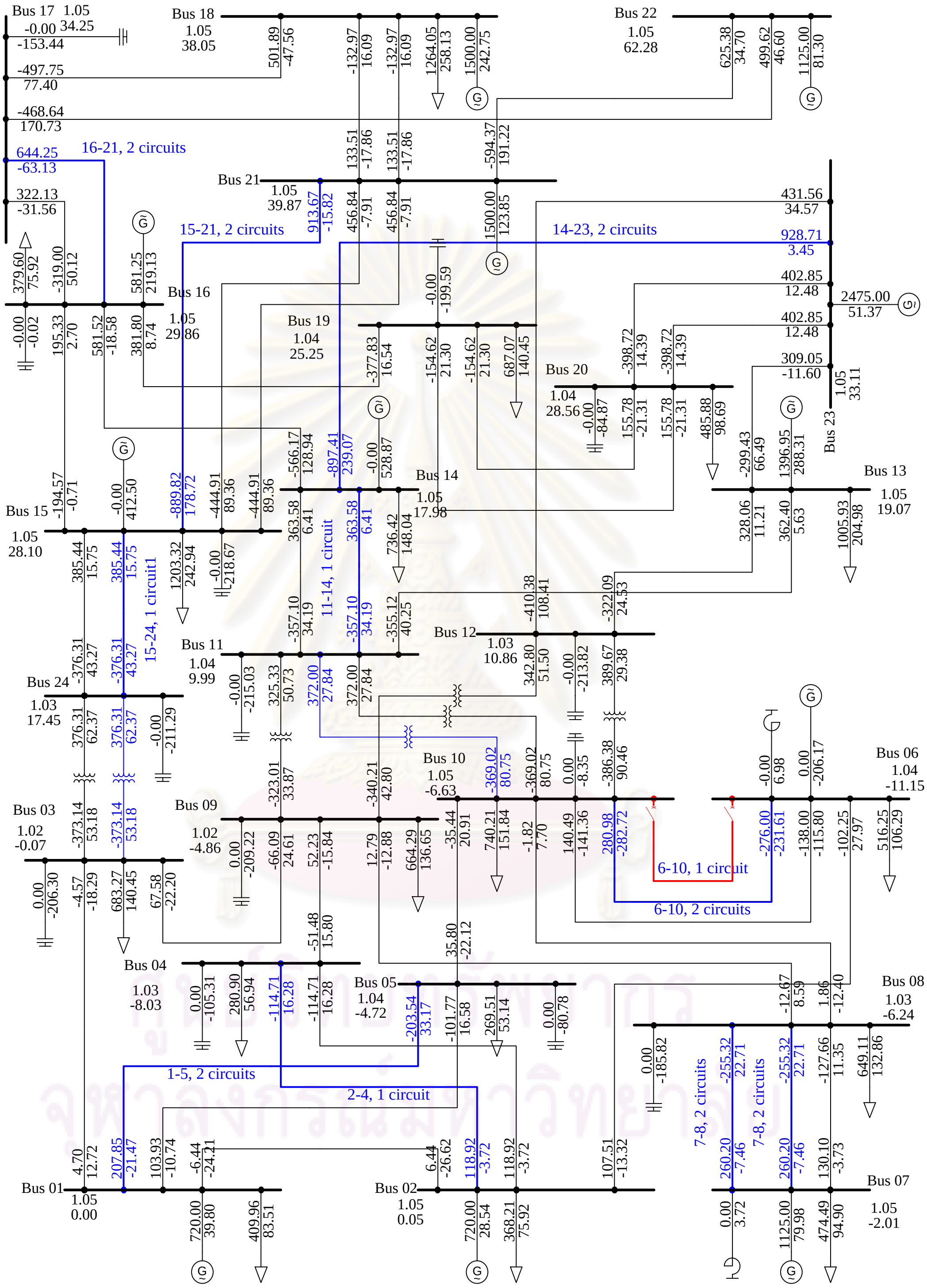

Figure 6.16 Power flow diagram of the third stage for IEEE-24 bus system with line 6–10 tripped

Table 6.27 Detail of installation of reactive power device for IEEE-24 bus system

Bus	First Stage		Second stage		Third Stage	
	CAP.	REAC.	CAP.	REAC.	CAP.	REAC.
3	200.0	–	–	–	–	–
4	106.8	–	25.1	–	36.3	–
5	113.6	–	–	–	–	–
6	–	120.2	–	–	–	–
7	–	8.5	–	–	–	–
8	200.0	–	–	–	–	–
9	200.0	–	–	–	–	–
10	–	177.5	–	22.5	7.6	–
11	200.0	–	–	–	–	–
12	200.0	–	–	–	–	–
15	200.0	–	–	–	–	–
16	1.6	–	–	–	–	–
17	200.0	–	–	–	–	–
19	153.9	–	30.6	–	15.5	–
20	9.4	–	69.1	–	121.5	157.4
24	200.0	–	–	–	–	–
Total	1,985.4	306.2	124.9	22.5	180.8	157.4

Northeastern Thailand system

The problem is formulated in the decomposed structure as proposed in Sections 3.7.2–3.7.5 with the methods for improving the performance and solution quality as proposed in Chapter 4. The parameter for cut modification, α, is set at 0.5.

After performing 158 iterations, the proposed procedure is finished within 18,220 seconds. The details of the investment plan are shown in Table 6.28.

The voltages at buses for the base case in each stage are shown in Table 6.29. The generated power and controlled voltages are shown in Table 6.30. In addition, the reactive power devices installed at buses are shown in Table 6.31.

Table 6.28 Detail of investment plan of MTEP-AC-NSEC for northeastern Thailand system

From	To	Number of circuits	Cost (10^6 US$)
First stage			
6	11	1	25.6
7	38	1	18.5
10	49	1	21.3
17	69	1	24.2
18	20	1	6.1
20	25	1	25.4
24	37	1	32.2
47	50	1	18.4
56	61	1	19.5
Second stage			
2	3	1	18.8
4	72	2	50.8
37	71	1	22.5
Third stage			
3	4	2	8.6
11	12	1	18.8
18	20	1 [1]	4.9
21	51	1	64.8
Investment cost [2, 3]			197.4
Installation cost of capacitors [2, 3]			27.8
Operating cost [2]			4,722.4

[1] Stringing the second circuit on the tower constructed in the first stage
[2] Net present value at 10 % interest rate
[3] Less the salvage value at the end of planning period

Table 6.29 Bus voltage of northeastern Thailand system

Bus	First Stage		Second stage		Third Stage	
	p.u.	deg.	p.u.	deg.	p.u.	deg.
1	0.99	–39.92	1.01	–48.02	1.01	–49.62
2	1.05	–26.36	1.05	–32.42	1.05	–32.41
3	1.04	–32.26	1.05	–37.55	1.05	–38.71
4	1.04	–33.42	1.05	–39.14	1.05	–39.83
5	1.02	–44.37	1.03	–52.59	1.04	–54.12
6	0.99	–30.96	1.02	–37.47	1.03	–38.40
7	0.99	–48.34	1.00	–52.62	1.00	–55.96
8	1.02	–32.78	1.03	–39.48	1.03	–41.26
9	1.00	–45.34	1.01	–50.89	1.01	–53.95
10	1.01	–34.44	1.02	–41.71	1.01	–44.70
11	1.02	–27.58	1.04	–33.44	1.05	–33.64
12	1.05	–23.86	1.05	–29.06	1.05	–30.39
13	1.05	–18.87	1.05	–22.57	1.05	–21.41
14	1.05	–23.15	1.05	–27.73	1.05	–27.26
15	1.05	–33.84	1.05	–39.98	1.05	–39.67
16	1.01	–40.26	1.02	–47.75	1.03	–48.92
17	1.01	–45.67	1.01	–54.09	1.01	–56.16
18	1.03	–32.65	1.04	–39.21	1.05	–40.20
19	1.05	–28.99	1.05	–35.29	1.05	–35.64
20	1.02	–33.48	1.04	–40.21	1.05	–41.01
21	1.05	–28.55	1.05	–34.79	1.05	–35.06
22	1.03	–28.27	1.03	–34.09	1.03	–36.91
23	1.05	0.00	1.05	0.00	1.05	0.00
24	0.97	–55.83	0.98	–58.63	0.98	–63.45
25	1.01	–39.60	1.02	–47.17	1.02	–48.39
26	0.98	–48.51	1.00	–56.11	0.99	–59.41
27	0.98	–48.76	1.00	–56.34	0.99	–59.72
28	1.01	–43.73	1.03	–50.17	1.03	–52.78
29	1.02	–40.52	1.04	–46.39	1.04	–48.39
30	1.05	–23.16	1.05	–27.97	1.05	–30.52
31	1.04	–21.29	1.04	–25.70	1.03	–28.00

Table 6.29 Bus voltage of northeastern Thailand system–continued

Bus	First Stage		Second stage		Third Stage	
	p.u.	deg.	p.u.	deg.	p.u.	deg.
32	1.03	–14.67	1.03	–17.80	1.02	–19.08
33	1.05	–37.65	1.05	–41.85	1.05	–43.08
34	1.05	–38.64	1.05	–41.80	1.05	–43.23
35	1.02	–43.17	1.03	–49.61	1.03	–51.92
36	1.01	–44.14	1.02	–50.74	1.03	–53.22
37	0.98	–51.25	1.00	–53.37	1.00	–57.06
38	0.99	–48.21	1.00	–52.48	1.00	–55.79
39	1.00	–45.52	1.01	–49.37	1.00	–52.31
40	1.05	–15.91	1.05	–18.87	1.05	–20.31
41	1.05	–39.76	1.05	–46.81	1.05	–47.52
42	1.02	–44.89	1.02	–53.09	1.02	–54.91
43	1.01	–44.78	1.01	–50.24	1.02	–53.21
44	0.99	–47.79	1.00	–53.41	1.00	–56.68
45	1.00	–44.08	1.01	–52.63	1.01	–54.20
46	1.02	–29.19	1.04	–35.46	1.04	–36.50
47	1.01	–41.14	1.03	–49.13	1.04	–49.71
48	1.01	–43.65	1.02	–47.35	1.02	–49.92
49	1.00	–35.92	1.01	–43.44	1.00	–46.75
50	1.02	–40.04	1.03	–47.86	1.05	–48.26
51	1.02	–37.63	1.04	–45.23	1.05	–44.60
52	1.03	–36.25	1.04	–43.69	1.05	–43.37
53	1.01	–44.45	1.03	–50.72	1.03	–53.38
54	1.01	–43.46	1.03	–49.61	1.04	–52.11
55	1.02	–41.42	1.04	–47.34	1.04	–49.48
56	0.99	–40.66	1.00	–48.86	1.00	–50.59
57	1.00	–45.91	1.01	–54.43	1.01	–56.36
58	1.05	–18.53	1.05	–22.32	1.05	–24.22
59	1.01	–44.75	1.02	–53.05	1.02	–54.82
60	1.00	–43.58	1.02	–50.90	1.02	–53.01
61	0.99	–39.90	1.01	–48.00	1.01	–49.56
62	1.01	–38.45	1.03	–46.29	1.03	–47.11
63	1.03	–39.04	1.04	–44.60	1.04	–46.31

Table 6.29 Bus voltage of northeastern Thailand system–continued

Bus	First Stage		Second stage		Third Stage	
	p.u.	deg.	p.u.	deg.	p.u.	deg.
64	1.00	–44.81	1.01	–48.62	1.01	–51.41
65	0.99	–46.85	1.01	–53.96	1.01	–57.09
66	1.05	–34.50	1.05	–39.09	1.05	–39.87
67	1.05	–32.16	1.05	–37.43	1.05	–37.84
68	1.01	–44.59	1.02	–52.86	1.02	–54.57
69	1.01	–44.08	1.02	–52.24	1.02	–53.94
70	1.02	–40.87	1.03	–48.45	1.04	–49.57
71	1.00	–45.70	1.01	–49.65	1.01	–52.58
72	1.00	–45.94	1.02	–48.87	1.01	–51.57
73	1.02	–44.94	1.02	–53.09	1.02	–54.91
74	1.03	–41.93	1.04	–49.99	1.05	–50.66
75	1.02	–39.17	1.04	–46.93	1.05	–46.77

Table 6.30 Generated power and controlled voltage of northeastern Thailand system

Bus	First Stage		Second stage		Third Stage	
	MW	p.u.	MW	p.u.	MW	p.u.
2	920.0	1.05	1,104.0	1.05	1,288.0	1.05
4	35.0	1.04	42.0	1.05	49.0	1.05
12	0.0	1.05	0.0	1.05	0.0	1.05
13	50.0	1.05	60.0	1.05	68.3	1.05
14	60.0	1.05	72.0	1.05	84.0	1.05
15	165.0	1.05	198.0	1.05	231.0	1.05
21	0.0	1.05	0.0	1.05	241.2	1.05
23	1,183.8	1.05	1,429.2	1.05	1,522.3	1.05
33	65.5	1.05	60.0	1.05	74.1	1.05
34	114.0	1.05	136.8	1.05	159.6	1.05
35	8.0	1.02	9.6	1.03	11.2	1.03
40	82.8	1.05	103.3	1.05	120.4	1.05
41	85.0	1.05	102.0	1.05	119.0	1.05

Table 6.30 Generated power and controlled voltage of northeastern Thailand system–continued

Bus	First Stage		Second stage		Third Stage	
	MW	p.u.	MW	p.u.	MW	p.u.
58	120.0	1.05	144.0	1.05	168.0	1.05
59	50.0	1.01	60.0	1.02	70.0	1.02
66	290.0	1.05	348.0	1.05	406.0	1.05
67	35.0	1.05	42.0	1.05	49.0	1.05
Total	3,264.1	–	3,910.9	–	4,661.1	–

Table 6.31 Detail of installation of reactive power device for northeastern Thailand system

Bus	First Stage		Second stage		Third Stage	
	CAP.	REAC.	CAP.	REAC.	CAP.	REAC.
1	32.51	–	11.90	–	31.07	–
2	–	187.82	–	–	–	–
3	–	–	47.72	200.00	19.30	–
4	85.90	74.40	–	10.59	–	–
5	48.05	–	1.24	–	23.77	–
6	43.31	–	12.66	–	13.94	–
7	1.28	–	–	–	0.79	–
8	34.04	–	7.97	–	9.85	–
9	15.71	–	4.61	–	0.01	–
10	103.07	–	11.86	–	17.52	–
11	47.02	–	86.88	–	–	–
13	–	5.21	–	0.42	–	0.26
14	–	–	–	–	0.11	–
15	–	–	–	–	–	0.85
16	36.14	–	–	10.52	–	–
17	21.43	–	1.68	–	8.10	–
18	200.00	–	–	–	–	–
20	79.27	–	17.51	–	8.88	–
22	70.01	–	24.08	–	16.27	–

Table 6.31 Detail of installation of reactive power device for northeastern Thailand system–continued

Bus	First Stage		Second stage		Third Stage	
	CAP.	REAC.	CAP.	REAC.	CAP.	REAC.
24	57.90	–	8.20	–	14.47	–
25	66.16	–	0.93	48.07	13.82	–
26	16.49	–	–	–	5.41	–
27	31.92	–	–	–	23.80	–
28	67.54	–	–	–	55.24	–
29	91.51	–	–	–	–	4.91
30	100.09	–	38.12	–	61.69	–
31	200.00	–	–	–	–	–
32	200.00	–	–	–	–	–
35	3.00	–	–	–	–	–
36	2.11	–	–	–	–	–
37	51.63	–	–	–	20.93	–
38	20.63	–	3.80	–	10.07	–
39	66.55	–	7.31	–	20.35	–
42	–	1.33	–	–	–	–
43	12.23	–	9.24	–	4.86	–
44	59.97	–	1.55	–	24.85	–
45	54.26	–	–	–	19.25	–
46	22.63	–	4.12	89.18	6.94	–
47	20.38	–	0.91	–	–	–
48	5.82	–	7.80	–	11.49	–
49	22.95	–	0.78	–	6.27	–
50	66.81	–	98.87	–	–	–
51	19.15	–	–	–	–	–
52	53.95	–	61.54	–	–	–
53	67.29	–	–	–	27.92	–
54	–	–	–	–	57.90	–
55	47.23	–	–	–	21.37	24.88
56	11.73	–	–	–	2.83	–
57	54.69	–	6.05	–	21.25	–
59	4.19	–	–	–	3.00	–

Table 6.31 Detail of installation of reactive power device for northeastern Thailand system–continued

Bus	First Stage		Second stage		Third Stage	
	CAP.	REAC.	CAP.	REAC.	CAP.	REAC.
60	36.16	–	–	–	10.40	–
61	6.79	–	–	–	0.32	–
62	45.68	–	–	–	–	–
63	–	–	–	–	40.66	–
64	0.50	–	0.05	–	1.54	–
65	31.16	–	–	–	1.08	32.60
67	–	2.33	–	15.63	–	1.00
68	15.90	–	–	–	9.03	–
69	77.15	–	1.12	–	97.54	–
70	–	–	40.77	–	57.84	–
71	93.04	–	–	–	0.51	–
72	110.98	–	1.91	–	48.98	–
73	–	1.33	–	–	–	–
74	46.17	–	–	–	1.73	–
Total	2,780.05	272.43	521.19	374.41	852.96	64.50

Discussion

It is clearly shown that the proposed method can provide an optimum plan for all the MTEP-AC-NSEC problems. Considering the obtained plan, one can investigate the following characteristics.

(a) The level of investment is increased from the ones obtained from other TEP problems in all the previous tests, since this test includes all necessary constraints, i.e. AC model, multistage planning, and N-1 security constraints.

(b) The level of investment in the first stage is higher than ones of the subsequent stages. The reason arise from two issues. The first one concerns with the N-1 security constraints. Since the initial configurations of test systems do not meet the N-1 planning criteria, the transmission lines are mainly required to serve that criteria in the first stage. In case of the northeastern Thailand system, the initial configuration nearly satisfies the N-1 planning criteria. Therefore, the transmission lines constructed in the first stage are mostly of the parallel to the existing redial lines. The second reason relates to the transmission congestion. Since the operation cost is generally higher than the investment cost, the congestion should be relieved since the early stage. Therefore, transmission lines in the first stage are also used to reduce the congestion.

It is also found that solving the MTEP-AC-NSEC problem by direct method, i.e. without decomposition, is not appropriate, since the size of the problem is very large, resulting in a large scale nonconvex MINLP. The general methods based on convex optimization usually fail in solving this problem. In addition, performance of the computation may be very poor due to the large storage of the computational data.

6.6 Multistage TEP with N-1 Security and Voltage Stability Constraints

In this section, the application of the proposed method with voltage stability consideration which is introduced in Section 5.1 is illustrated. The objective is to show that the constraints which can be taken into account by the decomposition based approach are not limited to the ones proposed in Chapter 3.

The Garver system is used to illustrated this application. The operating cost is not taken into account. The multistage TEP problem with N-1 security and voltage stability constraints (MTEP-AC-NSEC-VSTAB) is formulated by the formulation proposed in Section 5.1 compared to the MTEP-AC-NSEC problem. Both problems are solved by the decomposition based method. The parameter of cut modification, α, is set to 0.5. The results are shown in Table 6.32.

In this test, the voltage stability margin, VS_{lim} is set at 10. By performing the voltage stability analysis of the system when the plan obtained from solving the MTEP-AC-NSEC problem is applied, it can be found that the minimum eigenvalue of the system is 5.36, which occurs in the first stage when the line 4–6 was tripped. Therefore, it is reasonable to solve the MTEP-AC-NSEC-VSTAB problem to obtain a new feasible plan which satisfied the voltage stability margin.

Comparisons of the V–P characteristics at bus 2 between the plan obtained by solving MTEP-AC-NSEC-VTAB problem and the plan obtained by solving MTEP-AC-NSEC problem

Table 6.32 Comparison of plans between MTEP-AC-NSEC-VSTAB and MTEP-AC-NSEC

Stage	MTEP-AC-NSEC-VSTAB		MTEP-AC-NSEC	
	From–To	Number	From–To	Number
First stage	2–3	2	2–6	1
	3–5	2	3–5	2
	4–6	2	4–6	1
	4–6	2	4–6	2
Second stage	2–6	1	2–6	1 [1]
Third stage	1–5	2	1–5	1
	2–6	1 [1]	2–6	1
Investment cost [2, 3] (10^6 US$)	132.5		118.2	

[1] Stringing the second circuit on the tower constructed in the previous stage
[2] Net present value at 10 % interest rate
[3] Less the salvage value at the end of planning period

for the base case in every stage are shown in Figures 6.17, 6.18 and 6.19.

Discussion

Considering the results, one can see that the investment cost of the plan obtained from solving the MTEP-AC-NSEC-VSTAB problem is higher than the investment cost of the plan obtained from solving the MTEP-AC-NSEC problem since the transmission reinforcement is increased in order to meet the specified voltage stability margin. Comparing the plan for the first stage, one can also found that the line 2–6 which is in the plan obtained from solving the MTEP-AC-NSEC problem is not in the plan obtained from solving the MTEP-AC-NSEC-VSTAB problem. Therefore, solving the TEP problem with voltage stability constraints by using the plan obtained from the TEP problem without voltage stability constraints as a base configuration cannot be obtained the optimal plan same as the one obtained from the proposed method.

In addition, the results of voltage stability analysis are also consistent with the V-P curves obtained from solving the continuation power flow which indicate that the instability point for every stage is farther when the the plan obtained from the MTEP-AC-NSEC-VSTAB is applied.

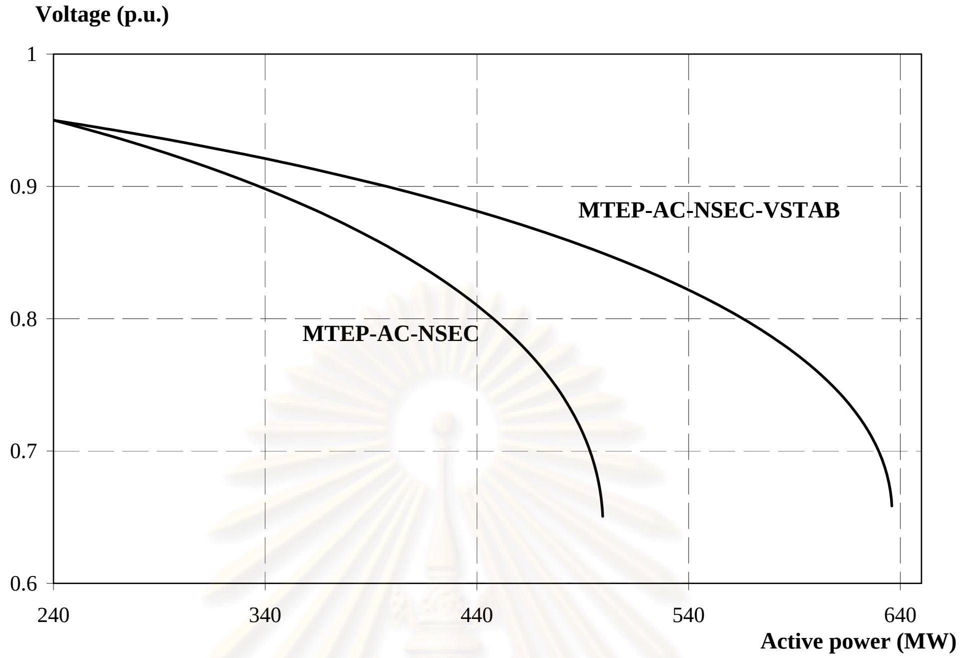

Figure 6.17 V–P curve of the first stage at bus 2

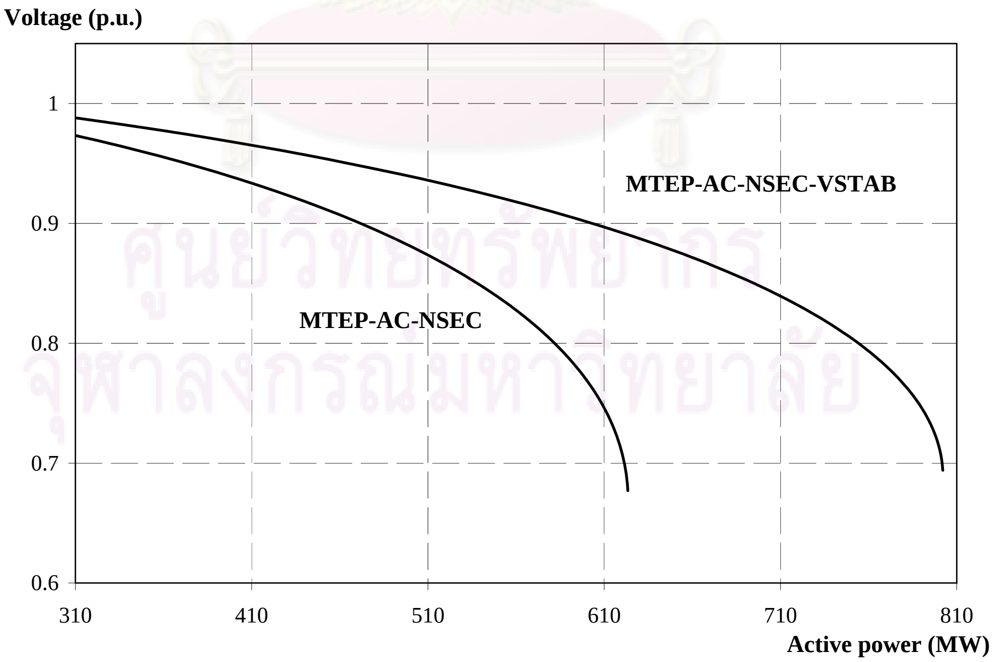

Figure 6.18 V–P curve of the second stage at bus 2

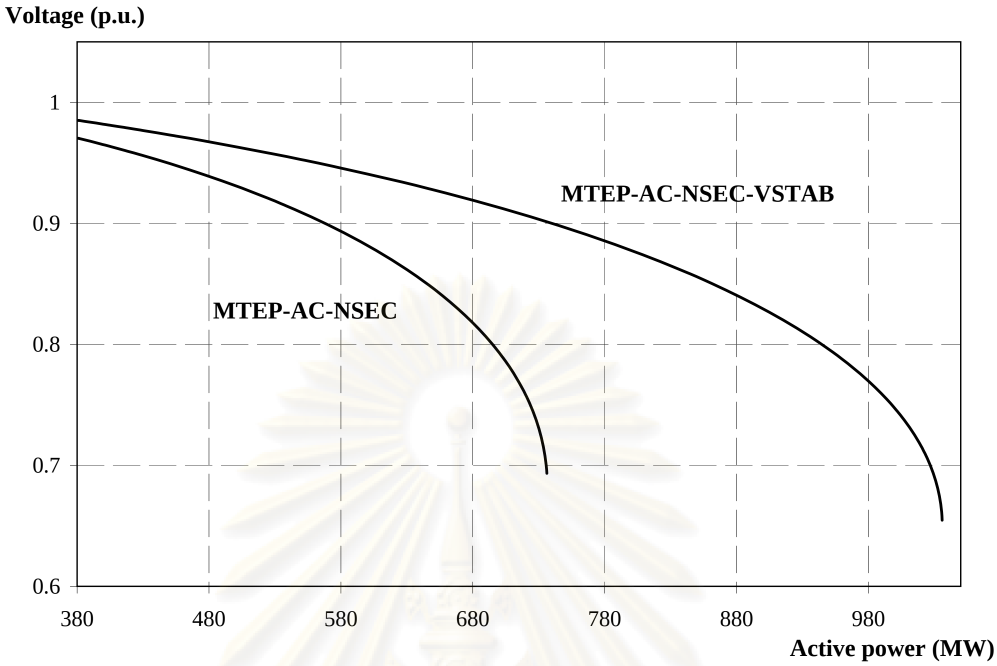

Figure 6.19 V–P curve of the third stage at bus 2

6.7 TEP with UPFC Installation

In this section, the UPFC installation will be taken into account in the TEP problem. The objective of this test is to demonstrate that the application of the proposed framework is not limited to the conventional method of power system development, i.e. construction of transmission lines and installation of transformers, but it can be extended to include modern devices, e.g. UPFC, which will be involved in the transmission planning activity in the future.

Since one of the major benefits of the UPFC is to increase the transfer capability of the transmission line, to demonstrate its application, the situation of the deficient in transmission paths is simulated. In this case, it is assumed that the transmission lines 2–6 and 4–6 of the Garver system cannot be constructed. Therefore, the generated power from the new power plant installed at bus 6 has to be transmitted to the other buses, i.e. buses 1, 3 and 5. In addition, the power demand at buses 2 and 4 cannot be received the power from bus 6. It should be noted that this scenario can occur in the actual situation in case the transmission paths 2–6 and 4–6 pass some forbidden areas. e.g. watershed area, reserved forest, etc.

The single stage TEP problem with N-1 security constraints and UPFC installation is formulated by the method proposed in Section 5.2 and solved by the decomposition based approach. The result compared with the one obtained from solving the single stage TEP problem with N-1 security constraints in which the UPFC installation is not taken into account is shown in Table 6.33.

Table 6.33 Comparison of plans between TEP with UPFC and TEP without UPFC

Result	TEP with UPFC			TEP without UPFC		
	From–To	Number	Cost (10^6 US$)	From–To	Number	Cost (10^6 US$)
Plan	2–3	2	30.0	2–3	1	20.0
	4–5 [1]	1	69.8	2–3	2	30.0
	5–6	2	91.5	2–4	2	60.0
	5–6	2	91.5	5–6	2	91.5
				5–6	2	91.5
Total cost (10^6 US$)			282.8			293.0

[1] Installing the UPFC at bus 4

The UPFC is installed at bus 4 connected to the transmission line 4–5. The power flow diagram of the Garver system when applying the obtained plan is shown in Figure 6.20.

From the power system analysis, it is found that the UPFC has the benefit to the power system operation in the case of contingencies. Based on the configuration shown in Figure 6.20, if the UPFC is not installed, the system cannot be operated within the defined operating limits in cases the lines 1–4, 2–3, 2–4 and 4–5 are tripped. When the UPFC is installed, the system meets the N-1 planning criteria. The examples of the power flow solutions of the contingency cases are shown in Figures 6.21 and 6.22. When the line 2–3 is tripped, the power exchange between the series and shunt converters is equal to 6.4 MW. In addition, when the line 4–5 is tripped, the series converter is also tripped, and the UPFC is operated in the mode of STATCOM.

Discussion

It can be seen from the obtained results that the proposed method for solving the TEP problem can be applied taking into account the UPFC device. From the power system analysis,

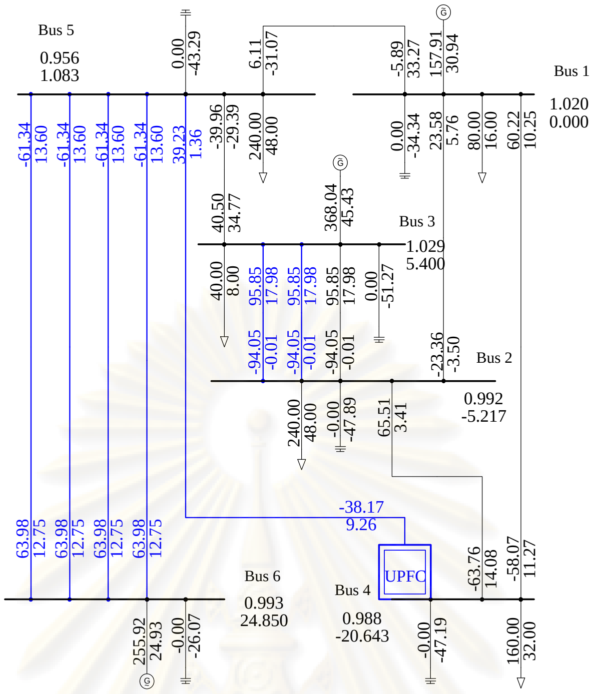

Figure 6.20 Power flow diagram with installed UPFC

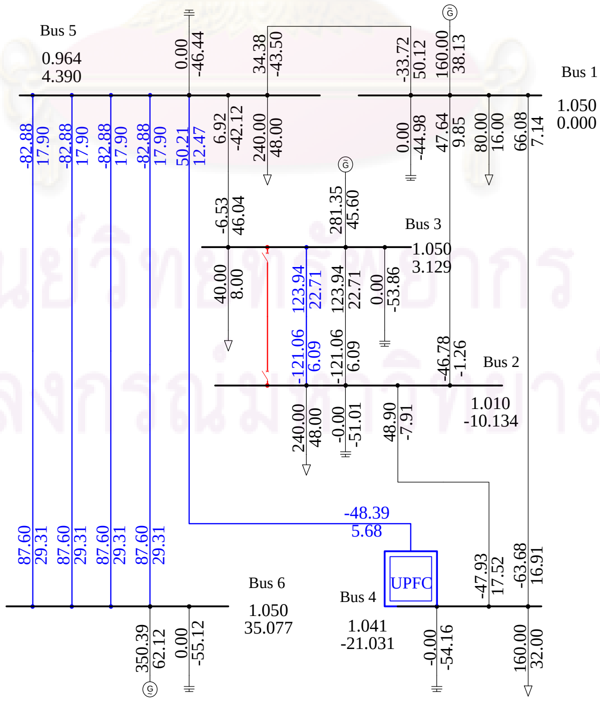

Figure 6.21 Power flow diagram with line 2–3 tripped

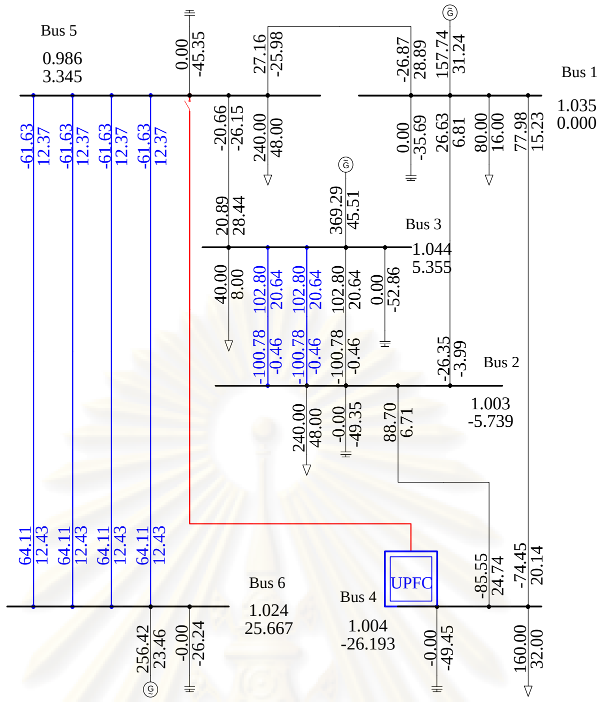

Figure 6.22 Power flow diagram with line 4–5 tripped

bus 4 is the load bus far from the power source. Without the UPFC installation, the demand at bus 4 should be supplied from bus 2 as shown in Table 6.33. However, bus 2 is not the source bus, and it also has a large amount of demand. Therefore, the transmission path 2–3 is more reinforced, resulting in high investment cost. Since bus 5 receives the power from bus 6 via the four-circuit transmission line, it is strong and able to transmit the power to other load buses. However, directly transmitting the power from bus 5 to bus 4 is infeasible in most contingency cases. For example, as shown in Figure 6.21, when the line 2–3 is tripped, the remaining circuits are overloaded. Therefore, the UPFC is utilized in this case to control the power flow in the line 4–5 so that the power flows in the lines 2–3 in both circuits are limited to their current limits.

CHAPTER VII

CONCLUSION

In this chapter, the advantage and disadvantage of the proposed method is summarized. In addition, the research works about the TEP which can be further developed are also introduced.

7.1 Dissertation Summary

This dissertation proposes a method for solving a multistage transmission expansion planning problem based on an AC model with N-1 security constraints (MTEP-AC-NSEC). The transmission congestion is also taken into account. It is noted that this problem is an actual problem in the TEP activity which has never been addressed in any previous research works. The problem formulation is proposed in a decomposed structure and then solved by the proposed method based on generalized Benders decomposition. In general, there are several formulations of TEP problem developed in this dissertation. The single stage TEP problem based on a DC model (STEP-DC) without security constraints [**34**] is used as a starting point. Then, subsequent formulations are developed from the formulation of the STEP-DC problem. For all of developed formulations, the decomposition concept is applied in solving those problems. Then, the complexity of the formulation is increased until the one of MTEP-AC-NSEC problem can be derived. The advantage of the decomposition based approach over the direct solving method is clearly shown when N-1 security constraints and AC model are included in the problem. However, in a simple problem, i.e. MTEP-DC, which deals with only the multistage planning, the decomposition based method is inferior to the direct solving method. To handle the difficulty when three aforementioned aspects, i.e. the multistage planning, the N-1 security constraints and the AC model, are simultaneously taken into account, the local search technique for solving the investment problem is developed to reduce the computational burden which arise from the multistage planning.

The solution quality are also taken into account in this dissertation. The cuts are modified in order that the set of feasible plans is underestimated by the modified cuts, and some good quality optimum plans are not be excluded from this feasible set. Even though the global optimality is not guaranteed, the proposed method attempts to find a good local optimum plan.

In addition, the applications of the decomposition based method are demonstrated in the cases of the TEP problem with voltage stability constraints and TEP problem with FACTS device installation. The proposed formulations of these problems can be exploited in the further research of these topics.

7.2 Advantage and Disadvantage

The advantage of the proposed method is the computational burden reduction derived from the decomposition concept. The results obtained from solving the convex problem, i.e. the problem based on the DC model, is guaranteed to be global optimum plan. For the nonconvex problem, i.e. the problem based on the AC model, the technique for improving the solution quality is developed. The obtained results for the nonconvex problem is same as the best ones found in previous research works for all common test systems.

Apart from the N-1 security constraints, the decomposition concept can be applied in case various load scenarios have to be taken into account in the TEP problem. The formulation of this problem can be developed in the same manner of the one in which the N-1 security constraints is taken into account.

One of the benefits which can be obviously seen from the proposed formulation concerns about the memory management of the computer running optimization solver. Since the operation problem for each scenario is solved separately from each other. The data of all operation problems are not necessary to be loaded simultaneously in case the memory resource is limited for a large scale problem. In addition, this concept can be extended to the case of parallel processing technique [**81**]. However, if all of operation problems can be solved simultaneously by distributed processing units, the computational time for solving the operation problem can be considerably reduced.

The disadvantage can be pointed out from two aspects. The first one concerns with the solution quality, i.e. the global optimality of the solution cannot be guaranteed in the case of problem based on the AC model. It is known that only one method to attain the global optimum of the nonconvex problem is to solve the problem by global optimization algorithm [**19**,**57**–**59**]. However, this method is not practical in the case of actual TEP problems since the algorithm will suffer from a curse of dimensionality. The second one relates to the computational aspect in case the TEP problem is infeasible. As stated in Chapter 4, the investment problem normally underestimates the set of feasible plan. Therefore, if the main problem is infeasible, the investment problem will estimate that it is feasible in the early iteration. However, when the iteration number increases, the infeasibility can be detected by the investment problem with the added cuts. The time spent to detect the infeasibility may be long, and in some cases, longer than that of the general optimization solver. Fortunately, this situation hardly occurs, since actual TEP problems are usually feasible.

7.3 Further Works

There are several topics of transmission expansion planning which are not comprehensively addressed in previous research works. Examples of them are listed below.

(a) Transmission expansion planning problem with additional constraints which have to be taken into account in some occasions, e.g. transient stability, short-circuit current limit, etc. The transient stability constraints are usually taken into account when transferring a large amount of power in the power purchasing project, while the short-circuit current constraints are normally considered in the transmission planning for the urban area.

(b) Transmission expansion planning in coordination with generation planning. The process of power system planning will be much more efficient when the generation planning is coordinated with the transmission expansion planning. When the transmission constraints are taken into account in the generation planning, a good quality generation plan will be attained. With the good quality generation plan, the good optimum transmission plan can be obtained.

(c) Transmission expansion planning with FACTS devices installation. With the versatile characteristics of the FACTS devices, i.e. UPFC, HVDC, STATCOM, etc., they will be completely involved in the power system development in the near future. Therefore, the method for solving the transmission expansion planning problem should be progressed to evaluated the benefit of the installation of FACTS devices. Even though the preliminary detail of the transmission expansion planning with the UPFC installation has been proposed in this dissertation, the comprehensive details can be studied in the further works.

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APPENDIX

APPENDIX

DATA OF TEST SYSTEMS

In this chapter, the detailed information of the Garver system, the IEEE-24 bus system and the northeastern Thailand system are presented. The active, reactive, and apparent power are expressed in MW, MVAr, and MVA, respectively. The columns p_d and q_d refer to the active and reactive power demand, $p_\mathrm{g}^\mathrm{max}$ and $p_\mathrm{g}^\mathrm{min}$ represent the maximum and minimum limits of the active power generation, $q_\mathrm{g}^\mathrm{max}$ and $q_\mathrm{g}^\mathrm{min}$ refer to the maximum and minimum limits of the reactive power generation.

The branch parameters, i.e. r_{ij}, x_{ij}, and b_{ij}, are expressed in per unit based on 100 MVA. The n^0 is the number of circuit of existing branches, n_p is the available number of transmission path connected between two buses and n_t is the maximum number of transformers which can be installed. The costs, c_b and c_g is expressed in million US$ and US$/KWh respectively.

There are three defined types of the transmission lines which can be constructed on each path as follows:

(a) ***Stringing the first circuit on double circuit tower***: The cost of construction is defined in the table.

(b) ***Double circuit tower***: The cost of construction is defined by multiplying the cost expresses in the table by 1.5.

(c) ***Stringing the second circuit on the existing tower***: The cost of construction is defined by multiplying the cost expresses in the table by 0.8.

For the transformer, the cost of installation is linear with respect to the number of installed units. The installation cost of both capacitors and reactors is set at 0.01 million US$/MVAr for every test system.

A.1 Garver system

The system consists of 6 buses and 6 transmission lines. There are 15 candidate paths. Bus and transmission line data are listed in Tables A.1 and A.2. The capacitors and reactors can be installed to compensate the reactive power at every bus. The maximum size of installation are 50 MVAr for both devices.

The UPFC data is shown in Table A.3. The installation cost is calculated by the formula presented in Ref. [**82**] which is interpolated from the Siemens AG Database. It is assumed that percent impedances and capacities of the series transformers are equal to those of the shunt transformers. In addition, the rated voltages at the high voltage side of the series transformers are equal to 20 % of system voltage. The capacities of DC link, series and shunt converters for each UPFC are equal to the capacity of transformer.

In Table A.3, z_f and s_f refer to the percent impedance and the capacity of the transformer in MVA; $v_{\text{se}}^{\min}$ and $v_{\text{se}}^{\max}$ are the minimum and maximum voltages of the series converter; $v_{\text{sh}}^{\min}$ and $v_{\text{sh}}^{\max}$ are the minimum and maximum voltages of the shunt converter; c_f is the installation cost.

Table A.1 Bus data of Garver system

Bus	Type	p_{d}	q_{d}	$p_{\text{g}}^{\max}$	$p_{\text{g}}^{\min}$	$q_{\text{g}}^{\max}$	$q_{\text{g}}^{\min}$	c_{g}
1	SL	80	16	160	0	48	–10	0.04
2	PQ	240	48	–	–	–	–	–
3	PV	40	8	370	0	101	–10	0.08
4	PQ	160	32	–	–	–	–	–
5	PQ	240	48	–	–	–	–	–
6	PV	–	–	610	0	183	–10	0.12

Table A.2 Transmission line data of Garver system

From	To	r_{ij}	x_{ij}	b_{ij}	$s_{ij}^{\max}$	c_{b}	n^0	n_{p}
1	2	0.040	0.40	0.00	120	40	1	4
1	3	0.038	0.38	0.00	120	38	–	4
1	4	0.060	0.60	0.00	100	60	1	4
1	5	0.020	0.20	0.00	120	20	1	4
1	6	0.068	0.68	0.00	90	68	–	4
2	3	0.020	0.20	0.00	120	20	1	4
2	4	0.040	0.40	0.00	120	40	1	4

Table A.2 Transmission line data of Garver system–continued

From	To	r_{ij}	x_{ij}	b_{ij}	$s_{ij}^{\max}$	c_{b}	n^0	n_{p}
2	5	0.031	0.31	0.00	120	31	–	4
2	6	0.030	0.30	0.00	120	30	–	4
3	4	0.059	0.59	0.00	120	59	–	4
3	5	0.020	0.20	0.00	120	20	1	4
3	6	0.048	0.48	0.00	120	48	–	4
4	5	0.063	0.63	0.00	95	63	–	4
4	6	0.030	0.30	0.00	120	30	–	4
5	6	0.061	0.61	0.00	98	61	–	4

Table A.3 UPFC data of Garver system

From	To	z_f	s_f	$v_{\text{se}}^{\min}$	$v_{\text{se}}^{\max}$	$v_{\text{sh}}^{\min}$	$v_{\text{sh}}^{\max}$	c_f
1	2	10	48.0	0.0	0.2	0.9	1.1	8.45
1	3	10	48.0	0.0	0.2	0.9	1.1	8.45
1	4	10	40.0	0.0	0.2	0.9	1.1	7.12
1	5	10	48.0	0.0	0.2	0.9	1.1	8.45
1	6	10	36.0	0.0	0.2	0.9	1.1	6.44
2	3	10	48.0	0.0	0.2	0.9	1.1	8.45
2	4	10	48.0	0.0	0.2	0.9	1.1	8.45
2	5	10	48.0	0.0	0.2	0.9	1.1	8.45
3	4	10	48.0	0.0	0.2	0.9	1.1	8.45
3	5	10	48.0	0.0	0.2	0.9	1.1	8.45
3	6	10	48.0	0.0	0.2	0.9	1.1	8.45
4	5	10	38.0	0.0	0.2	0.9	1.1	6.78
5	6	10	39.2	0.0	0.2	0.9	1.1	6.98

A.2 IEEE-24 bus system

The system consists of 24 buses, 33 circuits of transmission lines and 5 transformers. There are 41 candidate paths. Bus, transmission line and transformer data of the IEEE-24 bus system are listed in Tables A.4, A.5 and A.6. The capacitors and reactors can be installed to compensate the reactive power at every bus. The maximum size of installation are 200 MVAr for both devices.

Table A.4 Bus data of IEEE-24 bus system

Bus	Type	p_d	q_d	p_g^{max}	p_g^{min}	q_g^{max}	q_g^{min}	c_g
1	SL	324	66	576	0	240	–150	0.05
2	PV	291	60	576	0	240	–150	0.05
3	PQ	540	111	–	–	–	–	–
4	PQ	222	45	–	–	–	–	–
5	PQ	213	42	–	–	–	–	–
6	PV	408	84	0	0	0	–300	0.00
7	PV	375	75	900	0	540	0	0.08
8	PQ	513	105	–	–	–	–	–
9	PQ	525	108	–	–	–	–	–
10	PQ	585	120	–	–	–	–	–
11	PQ	–	–	–	–	–	–	–
12	PQ	–	–	–	–	–	–	–
13	PV	795	162	1773	0	720	0	0.08
14	PV	582	117	0	0	600	–150	0.00
15	PV	951	192	645	0	330	–150	0.08
16	PV	300	60	465	0	240	–150	0.03
17	PQ	–	–	–	–	–	–	–
18	PV	999	204	1200	0	600	–150	0.02
19	PQ	543	111	–	–	–	–	–
20	PQ	384	78	–	–	–	–	–
21	PV	–	–	1200	0	600	–150	0.02
22	PV	–	–	900	0	288	–180	0.01
23	PV	–	–	1980	0	930	–375	0.03
24	PQ	–	–	–	–	–	–	–

Table A.5 Transmission line data of IEEE-24 bus system

From	To	r_{ij}	x_{ij}	b_{ij}	$s_{ij}^{\max}$	c_{b}	n^0	n_{p}
1	2	0.0026	0.0139	0.4611	200	3	1	2
1	3	0.0546	0.2112	0.0572	220	55	1	2
1	5	0.0218	0.0845	0.0229	220	22	1	2
1	8	0.0348	0.1344	0.0000	220	35	–	2
2	4	0.0328	0.1267	0.0343	220	33	1	2
2	6	0.0497	0.1920	0.0520	220	50	1	2
2	8	0.0328	0.1267	0.0000	220	33	–	2
3	9	0.0308	0.1190	0.0322	220	31	1	2
4	9	0.0268	0.1037	0.0281	220	27	1	2
5	10	0.0228	0.0883	0.0239	220	23	1	2
6	7	0.0497	0.1920	0.0000	220	50	–	2
6	10	0.0139	0.0605	2.4590	200	16	1	2
7	8	0.0159	0.0614	0.0166	220	16	1	2
8	9	0.0427	0.1651	0.0447	220	43	1	2
8	10	0.0427	0.1651	0.0447	220	43	1	2
11	13	0.0061	0.0476	0.0999	625	66	1	2
11	14	0.0054	0.0418	0.0879	625	58	1	2
12	13	0.0061	0.0476	0.0999	625	66	1	2
12	23	0.0124	0.0966	0.2030	625	134	1	2
13	14	0.0057	0.0447	0.0000	625	62	–	2
13	23	0.0111	0.0865	0.1818	625	120	1	2
14	16	0.0050	0.0389	0.0818	625	54	1	2
14	23	0.0080	0.0620	0.0000	625	86	–	2
15	16	0.0022	0.0173	0.0364	625	24	1	2
15	21	0.0063	0.0490	0.1030	625	68	2	2
15	24	0.0067	0.0519	0.1091	625	72	1	2
16	17	0.0033	0.0259	0.0545	625	36	1	2
16	19	0.0030	0.0231	0.0485	625	32	1	2
16	23	0.0105	0.0822	0.0000	625	114	–	2
17	18	0.0018	0.0144	0.0303	625	20	1	2
17	22	0.0135	0.1053	0.2212	625	146	1	2
18	21	0.0033	0.0259	0.0545	625	36	2	2
19	20	0.0051	0.0396	0.0833	625	55	2	2

Table A.5 Transmission line data of IEEE-24 bus system–continued

From	To	r_{ij}	x_{ij}	b_{ij}	$s_{ij}^{\max}$	c_{b}	n^0	n_{p}
19	23	0.0078	0.0606	0.0000	625	84	–	2
20	23	0.0028	0.0216	0.0455	625	30	2	2
21	22	0.0087	0.0678	0.1424	625	94	1	2

Table A.6 Transformer data of IEEE-24 bus system

From	To	r_{ij}	x_{ij}	b_{ij}	$s_{ij}^{\max}$	c_{b}	n^0	n_{t}
3	24	0.0023	0.0839	0.0000	600	50	1	2
9	11	0.0023	0.0839	0.0000	600	50	1	2
9	12	0.0023	0.0839	0.0000	600	50	1	2
10	11	0.0023	0.0839	0.0000	600	50	1	2
10	12	0.0023	0.0839	0.0000	600	50	1	2

A.3 Northeastern Thailand system

The system consists of 75 buses, 129 transmission lines and 24 transformers. There are 119 candidate paths. Bus, transmission line and transformer data of the northeastern Thailands system are listed in Tables A.7, A.8 and A.9. The capacitors and reactors can be installed to compensate the reactive power at every bus. The maximum size of installation are 200 MVAr for both devices.

Table A.7 Bus data of northeastern Thailand system

Bus	Type	p_{d}	q_{d}	$p_{\mathrm{g}}^{\max}$	$p_{\mathrm{g}}^{\min}$	$q_{\mathrm{g}}^{\max}$	$q_{\mathrm{g}}^{\min}$	c_{g}
1	PQ	61.2	33.9	–	–	–	–	–
2	PV	21.6	13.4	920.0	0.0	446.4	–288.0	0.04
3	PQ	–	–	–	–	–	–	–
4	PV	90.9	50.3	35.0	0.0	18.0	–9.0	0.03
5	PQ	36.8	20.4	–	–	–	–	–
6	PQ	76.1	42.1	–	–	–	–	–

Table A.7 Bus data of northeastern Thailand system–continued

Bus	Type	p_{d}	q_{d}	$p_{\mathrm{g}}^{\max}$	$p_{\mathrm{g}}^{\min}$	$q_{\mathrm{g}}^{\max}$	$q_{\mathrm{g}}^{\min}$	c_{g}
7	PQ	3.5	1.9	–	–	–	–	–
8	PQ	32.2	17.8	–	–	–	–	–
9	PQ	19.5	10.8	–	–	–	–	–
10	PQ	62.7	34.7	–	–	–	–	–
11	PQ	106.9	59.1	–	–	–	–	–
12	PQ	–	–	880.0	0.0	542.9	–542.9	0.05
13	PV	0.6	0.4	50.0	0.0	23.3	–9.6	0.04
14	PV	42.3	23.4	60.0	0.0	28.4	–14.2	0.04
15	PV	1.2	0.7	165.0	0.0	72.0	–36.0	0.02
16	PQ	52.7	29.2	–	–	–	–	–
17	PQ	36.2	20.0	–	–	–	–	–
18	PQ	161.0	89.1	–	–	–	–	–
19	PQ	–	–	–	–	–	–	–
20	PQ	55.7	30.8	–	–	–	–	–
21	PQ	–	–	880.0	0.0	542.9	–542.9	0.05
22	PQ	90.4	50.0	–	–	–	–	–
23	SL	0.6	0.4	2300.0	0.0	1240.5	–1094.1	0.04
24	PQ	66.2	36.6	–	–	–	–	–
25	PQ	90.8	50.2	–	–	–	–	–
26	PQ	26.4	14.6	–	–	–	–	–
27	PQ	48.8	27.0	–	–	–	–	–
28	PQ	113.5	62.8	–	–	–	–	–
29	PQ	–	–	–	–	–	–	–
30	PQ	63.5	35.1	–	–	–	–	–
31	PQ	381.5	211.1	–	–	–	–	–
32	PQ	–	–	–	–	–	–	–
33	PV	–	–	72.0	0.0	39.7	–19.2	0.05
34	PV	1.2	0.7	114.0	0.0	59.5	–28.8	0.04
35	PV	0.6	0.4	8.0	0.0	4.3	0.0	0.05
36	PQ	6.1	3.4	–	–	–	–	–
37	PQ	42.1	23.3	–	–	–	–	–
38	PQ	34.0	18.8	–	–	–	–	–
39	PQ	112.2	62.1	–	–	–	–	–

Table A.7 Bus data of northeastern Thailand system–continued

Bus	Type	p_d	q_d	p_g^{max}	p_g^{min}	q_g^{max}	q_g^{min}	c_g
40	PQ	47.8	26.4	120.0	0.0	74.3	–74.3	0.04
41	PV	1.2	0.7	85.0	0.0	38.4	–26.4	0.05
42	PQ	–	–	–	–	–	–	–
43	PQ	17.9	9.9	–	–	–	–	–
44	PQ	90.1	49.8	–	–	–	–	–
45	PQ	83.0	45.9	–	–	–	–	–
46	PQ	24.6	13.6	–	–	–	–	–
47	PQ	35.5	19.6	–	–	–	–	–
48	PQ	–	–	–	–	–	–	–
49	PQ	38.3	21.2	–	–	–	–	–
50	PQ	61.8	34.2	–	–	–	–	–
51	PQ	–	–	–	–	–	–	–
52	PQ	–	–	–	–	–	–	–
53	PQ	45.4	25.1	–	–	–	–	–
54	PQ	–	–	–	–	–	–	–
55	PQ	–	–	–	–	–	–	–
56	PQ	21.9	12.1	–	–	–	–	–
57	PQ	92.6	51.2	–	–	–	–	–
58	PQ	40.2	22.3	120.0	0.0	74.3	–74.3	0.04
59	PV	52.2	28.9	50.0	0.0	22.3	–10.8	0.05
60	PQ	42.7	23.6	–	–	–	–	–
61	PQ	27.7	15.3	–	–	–	–	–
62	PQ	–	–	–	–	–	–	–
63	PQ	–	–	–	–	–	–	–
64	PQ	–	–	–	–	–	–	–
65	PQ	21.5	11.9	–	–	–	–	–
66	PV	1.2	0.7	290.0	0.0	156.0	–76.8	0.04
67	PV	0.6	0.4	35.0	0.0	15.5	–10.8	0.04
68	PQ	38.0	21.0	–	–	–	–	–
69	PQ	147.7	81.7	–	–	–	–	–
70	PQ	–	–	–	–	–	–	–
71	PQ	72.4	40.0	–	–	–	–	–
72	PQ	97.8	54.1	–	–	–	–	–

Table A.7 Bus data of northeastern Thailand system–continued

Bus	Type	p_{d}	q_{d}	$p_{\mathrm{g}}^{\max}$	$p_{\mathrm{g}}^{\min}$	$q_{\mathrm{g}}^{\max}$	$q_{\mathrm{g}}^{\min}$	c_{g}
73	PQ	–	–	–	–	–	–	–
74	PQ	48.3	26.7	–	–	–	–	–
75	PQ	–	–	–	–	–	–	–

Table A.8 Transmission line data of northeastern Thailand system

From	To	r_{ij}	x_{ij}	b_{ij}	$s_{ij}^{\max}$	c_{b}	n^0	n_{p}
1	10	0.0662	0.1404	0.0171	96.4	–	1	–
1	10	0.0289	0.1272	0.0191	162.9	20.6	–	2
1	45	0.0580	0.2550	0.0383	162.9	37.6	1	2
1	56	0.0298	0.1309	0.0196	162.9	21.0	–	2
1	61	0.0008	0.0033	0.0005	162.9	4.0	2	2
2	12	0.0129	0.0931	0.2044	429.4	72.9	–	2
2	19	0.0034	0.0244	0.0535	429.4	24.0	–	2
2	21	0.0015	0.0154	0.0628	858.9	–	2	–
2	21	0.0029	0.0211	0.0464	429.4	21.6	–	2
2	51	0.0129	0.0930	0.2043	429.4	72.9	–	2
2	52	0.0113	0.0812	0.1783	429.4	64.5	–	2
3	4	0.0019	0.0124	0.0032	325.9	–	2	–
3	4	0.0037	0.0163	0.0024	162.9	5.8	–	2
3	18	0.0213	0.0934	0.0140	162.9	16.0	–	2
3	20	0.0223	0.0980	0.0147	162.9	16.7	–	2
3	67	0.0117	0.0515	0.0077	162.9	10.5	–	2
4	18	0.0298	0.0836	0.0117	119.5	–	2	–
4	18	0.0184	0.0811	0.0122	162.9	14.4	–	2
4	20	0.0205	0.0901	0.0135	162.9	15.6	–	2
4	67	0.0623	0.0782	0.0083	67.1	–	1	–
4	67	0.0151	0.0663	0.0099	162.9	12.4	–	2
4	71	0.0991	0.2813	0.0387	119.5	–	2	–
4	71	0.0615	0.2705	0.0405	162.9	39.7	–	2
4	72	0.0832	0.2339	0.0328	325.9	–	2	–
4	72	0.0516	0.2269	0.0340	162.9	33.9	–	2

Table A.8 Transmission line data of northeastern Thailand system–continued

From	To	r_{ij}	x_{ij}	b_{ij}	$s_{ij}^{\max}$	c_{b}	n^0	n_{p}
5	26	0.2398	0.3017	0.0323	67.1	–	1	–
5	26	0.0583	0.2561	0.0384	162.9	37.8	–	2
5	68	0.0409	0.1799	0.0270	162.9	27.6	1	2
5	74	0.0495	0.1390	0.0195	119.5	–	2	–
5	74	0.0307	0.1348	0.0202	162.9	21.6	–	2
6	11	0.0376	0.1653	0.0248	162.9	25.6	1	2
7	38	0.0412	0.1212	0.0154	119.5	–	1	–
7	38	0.0255	0.1121	0.0168	162.9	18.5	–	2
8	18	0.0399	0.1119	0.0157	119.5	–	1	–
8	18	0.0247	0.1085	0.0163	162.9	18.1	–	2
8	20	0.0281	0.1234	0.0185	162.9	20.0	–	2
8	22	0.0975	0.2742	0.0385	119.5	–	1	–
8	22	0.0606	0.2662	0.0399	162.9	39.1	–	2
8	46	0.0235	0.1031	0.0155	162.9	17.3	–	2
9	43	0.0101	0.0300	0.0038	119.5	–	1	–
9	44	0.1106	0.3260	0.0417	119.5	–	1	–
9	44	0.0688	0.3024	0.0453	162.9	43.9	–	2
10	30	0.1579	0.3359	0.0411	96.4	–	1	–
10	30	0.0693	0.3047	0.0457	162.9	44.2	–	2
10	31	0.1389	0.3914	0.0551	119.5	–	2	–
10	31	0.0866	0.3806	0.0571	162.9	54.4	–	2
10	49	0.0489	0.1372	0.0192	119.5	–	1	–
10	49	0.0303	0.1331	0.0200	162.9	21.3	–	2
10	61	0.0315	0.1385	0.0208	162.9	22.1	–	2
11	46	0.0549	0.1616	0.0206	119.5	–	1	–
11	46	0.0340	0.1496	0.0224	162.9	23.5	–	2
12	19	0.0111	0.0803	0.1762	429.4	63.8	–	2
12	21	0.0097	0.0702	0.1546	429.4	–	2	–
12	21	0.0098	0.0703	0.1545	429.4	56.7	–	2
12	32	0.0101	0.0729	0.1605	429.4	58.7	2	2
13	14	0.0515	0.1523	0.0192	119.5	–	1	–
13	14	0.0319	0.1403	0.0210	162.9	22.3	–	2

Table A.8 Transmission line data of northeastern Thailand system–continued

From	To	r_{ij}	x_{ij}	b_{ij}	$s_{ij}^{\max}$	c_{b}	n^0	n_{p}
14	18	0.0872	0.2582	0.0326	119.5	–	1	–
14	18	0.0541	0.2380	0.0357	162.9	35.3	–	2
15	70	0.0225	0.1637	0.3661	429.4	–	2	–
16	20	0.0753	0.2113	0.0297	119.5	–	2	–
16	20	0.0467	0.2054	0.0308	162.9	31.0	–	2
16	25	0.1166	0.1464	0.0156	67.1	–	1	–
16	25	0.0283	0.1243	0.0186	162.9	20.2	–	2
16	47	0.0272	0.1195	0.0179	162.9	19.5	–	2
16	50	0.0382	0.1071	0.0150	119.5	–	2	–
16	50	0.0236	0.1039	0.0156	162.9	17.4	–	2
16	53	0.0775	0.3415	0.0513	162.9	49.2	1	2
16	60	0.0263	0.1158	0.0173	162.9	19.0	1	2
17	57	0.0354	0.1556	0.0233	162.9	24.3	–	2
17	69	0.0352	0.1547	0.0232	162.9	24.2	1	2
18	20	0.0071	0.0198	0.0028	119.5	–	2	–
18	20	0.0044	0.0192	0.0029	162.9	6.1	–	2
18	22	0.1365	0.3845	0.0541	119.5	–	1	–
18	22	0.0851	0.3739	0.0560	162.9	53.5	–	2
18	25	0.2222	0.2794	0.0299	67.1	–	1	–
18	25	0.0540	0.2372	0.0356	162.9	35.2	–	2
18	46	0.1890	0.2376	0.0254	67.1	–	1	–
18	46	0.0459	0.2016	0.0302	162.9	30.5	–	2
18	67	0.1410	0.1771	0.0189	67.1	–	1	–
18	67	0.0342	0.1503	0.0225	162.9	23.6	–	2
19	21	0.0007	0.0050	0.0110	429.4	–	2	–
19	21	0.0007	0.0050	0.0110	429.4	10.2	–	2
19	51	0.0106	0.0762	0.1672	429.4	60.9	–	2
19	52	0.0089	0.0639	0.1403	429.4	52.1	–	2
20	25	0.0372	0.1636	0.0245	162.9	25.4	–	2
20	67	0.0332	0.1461	0.0219	162.9	23.1	–	2
21	51	0.0113	0.0817	0.1794	429.4	64.8	–	2
21	52	0.0092	0.0663	0.1459	429.4	53.9	2	2

Table A.8 Transmission line data of northeastern Thailand system–continued

From	To	r_{ij}	x_{ij}	b_{ij}	$s_{ij}^{\max}$	c_{b}	n^0	n_{p}
22	30	0.0612	0.1719	0.0241	119.5	–	1	–
22	30	0.0379	0.1667	0.0250	162.9	25.8	–	2
22	31	0.0501	0.1407	0.0197	119.5	–	1	–
22	31	0.0310	0.1364	0.0204	162.9	21.8	–	2
23	32	0.0065	0.0468	0.1028	1503.1	–	2	–
23	32	0.0065	0.0468	0.1028	429.4	40.0	–	2
24	37	0.0787	0.2318	0.0296	119.5	–	1	–
24	37	0.0488	0.2147	0.0322	162.9	32.2	–	2
25	50	0.0986	0.1232	0.0133	67.1	–	1	–
25	50	0.0239	0.1050	0.0157	162.9	17.6	–	2
26	27	0.0055	0.0243	0.0036	162.9	6.8	1	2
26	65	0.0292	0.1282	0.0192	162.9	20.7	1	2
27	65	0.0226	0.0994	0.0149	162.9	16.8	1	2
28	54	0.1878	0.2095	0.0286	67.1	–	1	–
28	54	0.0456	0.2004	0.0300	162.9	30.3	–	2
28	65	0.0380	0.1670	0.0250	162.9	25.9	2	2
29	55	0.0086	0.0620	0.1370	429.4	50.9	1	2
29	63	0.0019	0.0136	0.0299	429.4	–	1	–
30	31	0.0074	0.0324	0.0048	162.9	7.9	2	2
30	46	0.2861	0.3603	0.0386	67.1	–	1	–
30	46	0.0696	0.3060	0.0459	162.9	44.4	–	2
31	58	0.0303	0.0872	0.0117	117.5	–	2	–
31	58	0.0187	0.0823	0.0123	162.9	14.6	–	2
33	34	0.0456	0.1349	0.0170	119.5	–	1	–
33	43	0.0862	0.2549	0.0321	119.5	–	1	–
34	48	0.0740	0.2080	0.0292	119.5	–	2	–
34	48	0.1892	0.2378	0.0254	67.1	–	1	–
36	44	0.0354	0.1557	0.0233	162.9	24.4	–	2
36	53	0.0164	0.0721	0.0108	162.9	13.2	1	2
36	54	0.0245	0.1078	0.0162	162.9	18.0	–	2
36	60	0.0330	0.1452	0.0217	162.9	22.9	1	2
37	71	0.0522	0.1534	0.0196	119.5	–	1	–

Table A.8 Transmission line data of northeastern Thailand system–continued

From	To	r_{ij}	x_{ij}	b_{ij}	$s_{ij}^{\max}$	c_{b}	n^0	n_{p}
37	71	0.0323	0.1420	0.0213	162.9	22.5	–	2
37	72	0.0388	0.1707	0.0256	162.9	26.4	1	2
38	44	0.0557	0.1656	0.0207	119.5	–	1	–
38	44	0.0345	0.1518	0.0228	162.9	23.8	–	2
38	71	0.0313	0.1374	0.0206	162.9	21.9	–	2
38	72	0.0390	0.1159	0.0145	119.5	–	1	–
38	72	0.0241	0.1061	0.0159	162.9	17.7	–	2
39	48	0.0261	0.0732	0.0103	119.5	–	2	–
39	64	0.0283	0.0355	0.0038	67.1	–	1	–
39	71	0.0475	0.1332	0.0187	119.5	–	1	–
39	71	0.1160	0.1456	0.0156	67.1	–	1	–
39	71	0.0294	0.1292	0.0194	162.9	20.8	–	2
39	72	0.0527	0.1480	0.0208	119.5	–	1	–
39	72	0.0327	0.1436	0.0215	162.9	22.7	–	2
40	58	0.0448	0.1290	0.0173	117.5	–	1	–
40	58	0.0277	0.1219	0.0183	162.9	19.8	–	2
41	59	0.0064	0.0283	0.0042	162.9	7.4	–	2
41	69	0.0717	0.2013	0.0283	119.5	–	2	–
41	69	0.0445	0.1954	0.0293	162.9	29.7	–	2
42	59	0.0608	0.1789	0.0228	119.5	–	1	–
42	73	0.0709	0.2087	0.0266	119.5	–	1	–
44	53	0.0746	0.1608	0.0190	96.4	–	1	–
44	53	0.0521	0.1535	0.0195	119.5	–	1	–
44	53	0.0326	0.1434	0.0215	162.9	22.7	–	2
44	54	0.0308	0.1354	0.0203	162.9	21.6	–	2
44	72	0.1454	0.3139	0.0372	96.4	–	1	–
44	72	0.0638	0.2803	0.0420	162.9	41.0	–	2
45	50	0.0576	0.2532	0.0380	162.9	37.4	2	2
47	50	0.0253	0.1110	0.0166	162.9	18.4	1	2
47	60	0.0317	0.1392	0.0209	162.9	22.2	–	2
48	64	0.0467	0.0586	0.0063	67.1	–	1	–
50	74	0.0701	0.1970	0.0276	119.5	–	2	–

Table A.8 Transmission line data of northeastern Thailand system–continued

From	To	r_{ij}	x_{ij}	b_{ij}	$s_{ij}^{\max}$	c_{b}	n^0	n_{p}
50	74	0.0435	0.1912	0.0287	162.9	29.1	–	2
51	52	0.0020	0.0144	0.0316	429.4	16.9	2	2
51	62	0.0145	0.1045	0.2295	429.4	81.1	–	2
51	75	0.0034	0.0360	0.1469	858.9	–	2	–
51	75	0.0069	0.0493	0.1084	429.4	41.8	–	2
52	62	0.0144	0.1038	0.2301	398.4	–	2	–
52	62	0.0145	0.1044	0.2293	429.4	81.0	–	2
52	75	0.0089	0.0644	0.1414	429.4	52.5	–	2
53	54	0.0056	0.0348	0.0102	325.9	–	2	–
53	54	0.0111	0.0489	0.0073	162.9	10.1	–	2
53	65	0.2780	0.2484	0.0246	48.2	–	1	–
53	65	0.0440	0.1933	0.0290	162.9	29.4	–	2
55	63	0.0075	0.0543	0.1200	429.4	–	1	–
56	61	0.0272	0.1196	0.0179	162.9	19.5	1	2
57	69	0.0608	0.1707	0.0240	119.5	–	2	–
57	69	0.0377	0.1657	0.0248	162.9	25.7	–	2
57	74	0.0897	0.2517	0.0354	119.5	–	2	–
57	74	0.0557	0.2447	0.0367	162.9	36.2	–	2
59	68	0.1640	0.1905	0.0239	67.1	–	2	–
59	68	0.0398	0.1748	0.0262	162.9	26.9	–	2
62	75	0.0137	0.0984	0.2160	429.4	76.7	–	2
63	66	0.0084	0.0605	0.1337	429.4	–	2	–
68	69	0.0302	0.0846	0.0119	119.5	–	2	–
68	69	0.0187	0.0821	0.0123	162.9	14.5	–	2
68	74	0.0965	0.2850	0.0362	119.5	–	1	–
68	74	0.0599	0.2635	0.0395	162.9	38.7	–	2
70	75	0.0114	0.0820	0.1801	429.4	65.1	–	2
71	72	0.0111	0.0321	0.0043	117.5	–	1	–
71	72	0.0069	0.0302	0.0045	162.9	7.6	–	2

Table A.9 Transformer data of northeastern Thailand system

From	To	r_{ij}	x_{ij}	b_{ij}	$s_{ij}^{\max}$	c_{b}	n^0	n_{t}
2	3	0.0000	0.0650	0.0000	200.0	–	1	–
2	3	0.0000	0.0621	0.0000	200.0	–	1	–
2	3	0.0000	0.0550	0.0000	300.0	18.8	–	2
11	12	0.0000	0.0692	0.0000	200.0	–	1	–
11	12	0.0000	0.0691	0.0000	200.0	–	1	–
11	12	0.0000	0.0550	0.0000	300.0	18.8	–	2
18	19	0.0000	0.0583	0.0000	200.0	–	1	–
18	19	0.0000	0.0583	0.0000	200.0	–	1	–
18	19	0.0000	0.0600	0.0000	200.0	–	1	–
18	19	0.0000	0.0550	0.0000	300.0	18.8	–	2
28	29	0.0000	0.0650	0.0000	200.0	–	1	–
28	29	0.0000	0.0650	0.0000	200.0	–	1	–
28	29	0.0000	0.0550	0.0000	300.0	18.8	–	2
31	32	0.0000	0.0550	0.0000	300.0	–	1	–
31	32	0.0000	0.0550	0.0000	300.0	–	1	–
31	32	0.0000	0.0550	0.0000	300.0	–	1	–
31	32	0.0000	0.0550	0.0000	300.0	18.8	–	2
35	36	0.0000	0.2338	0.0000	40.0	–	1	–
50	51	0.0000	0.0675	0.0000	200.0	–	1	–
50	51	0.0000	0.0675	0.0000	200.0	–	1	–
50	51	0.0000	0.0650	0.0000	200.0	–	1	–
50	51	0.0000	0.0550	0.0000	300.0	18.8	–	2
54	55	0.0000	0.0692	0.0000	200.0	–	1	–
54	55	0.0000	0.0692	0.0000	200.0	–	1	–
54	55	0.0000	0.0550	0.0000	300.0	18.8	–	2
61	62	0.0000	0.0650	0.0000	200.0	–	1	–
61	62	0.0000	0.0650	0.0000	200.0	–	1	–
61	62	0.0000	0.0550	0.0000	300.0	18.8	–	2
69	70	0.0000	0.0720	0.0000	200.0	–	1	–

Table A.9 Transformer data of northeastern Thailand system–continued

From	To	r_{ij}	x_{ij}	b_{ij}	$s_{ij}^{\max}$	c_{b}	n^0	n_{t}
69	70	0.0000	0.0720	0.0000	200.0	–	1	–
69	70	0.0000	0.0550	0.0000	300.0	18.8	–	2
74	75	0.0000	0.0650	0.0000	200.0	–	1	–
74	75	0.0000	0.0650	0.0000	200.0	–	1	–
74	75	0.0000	0.0550	0.0000	300.0	18.8	–	2

Biography

Somphop Asadamongkol was born in Bangkok, Thailand, on June 7, 1976. He received B.Eng. and M.Eng. degrees in electrical engineering from Chulalongkorn University, Thailand, in 1997 and 2007, respectively. He has joined System Planning Division, Electricity Generating Authority of Thailand (EGAT) since May 1997. Currently, he is Acting Head, Transmission and Substation Planning Section, Transmission System Planning Department, System Planning Division, EGAT, and working toward the Ph.D. degree in electrical engineering at Chulalongkorn University. His current research interests include power system planning, power system optimization and simulation.