# สถานะนิวเคลียร์ในพลาสมาควาร์กกลูออนที่มีอันตรกิริยาอย่างแรงจากความสมมูลระหว่างทฤษฎีเกจและทฤษฎีความโน้มถ่วง

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# NUCLEAR STATES IN STRONGLY COUPLED QUARK-GLUON PLASMA FROM GAUGE-GRAVITY CORRESPONDENCE

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A Dissertation Submitted in Partial Fulfillment of the Requirements
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ควาร์ก ปฏิควาร์ก และกลูออนในเฟสไม่กักขังไม่จำเป็นต้องอยู่ในสถานะยึดเหนี่ยวไม่มีสี พวกมันก่อให้เกิดสถานะยึดเหนี่ยวนิวเคลียร์แบบไม่ซิงเกล็ตเชิงรงค์ได้(หรือแบบเอ็กโซติก) เช่นเดียวกับสถานะยึดเหนี่ยวซิงเกล็ตเชิงรงค์ ในพลาสมาควาร์กกลูออนคู่ควบแรง (sQGP) ที่ความหนาแน่นจำนวนแบรีออนมากพอ เราศึกษาสสารซึ่งคือ sQGP ที่บรรจุด้วยสถานะนิวเคลียร์โดยแบบจำลองทวิภาพที่เรียกกันว่าแบบจำลองซาไก-ซูกิโมโตะ ซึ่งเป็นทวิภาพกับทฤษฎี QCD แบบจำนวนสี $N_c$ มาก เราเสนอแบบจำลองทวิภาพโน้มถ่วงของสถานะนิวเคลียร์เอ็กโซติกในพื้นหลังแบบไม่กักขังของแบบจำลองซาไก-ซูกิโมโตะ ที่ซึ่งค่าองศาเสรีทางรสถูกปิด นอกจากนี้เราคำนวณ และเปรียบเทียบพลังงานยึดเหนี่ยว และความยาวกั้นขวางของสถานะนิวเคลียร์เหล่านี้ เพื่อเป็นการวิเคราะห์ความเสถียรของพวกมัน ผลที่ได้บ่งชี้ว่าพวกมันเสถียรน้อยกว่าพวกฮาดรอนปกติ จากนั้นโดยการเปิดองศาเสรีทางรสเราเสาะค้นความเป็นไปได้ของการมีอยู่ซึ่งสสารมัลติควาร์กอันเป็นเฟสทางอุณพลศาสตร์ที่แสดงถึง sQGP ที่เต็มไปด้วยสถานะนิวเคลียร์เอ็กโซติกในบริเวณหนึ่งในแผนภูมิเฟส สสารนี้ถูกพบว่าเสถียรทางอุณหพลศาสตร์กว่าเฟสสุญญากาศและพลาสมาควาร์กกลูออนที่มีสมมาตรไครัล ($\chi$ S-QGP) ในแผนภูมิเฟสที่ช่วงค่าศักย์เคมีมากและอุณหภูมิต่ำถึงแม้ว่ามันจะเสถียรน้อยกว่าสสารนิวเคลียร์ธรรมดาก็ตาม นอกจากนี้ ความสัมพันธ์ทางอุณหพลศาสตร์ก็ได้ถูกคำนวณหา ความสัมพันธ์ระหว่างความดันกับความหนาแน่นจำนวนแบรีออนถูกพบว่าเป็นแบบ $P \sim d^2$ สำหรับความหนาแน่นต่ำ และ $P \sim d^{7/5}$ สำหรับความหนาแน่นสูง ค่าความหนาแน่นเอนโทรปีถูกพบว่าขึ้นกับอุณหภูมิเป็นไปตาม $s \sim T^5$ สำหรับสสารนิวเคลียร์ปกติ แต่เป็นแบบ $s \sim T$ สำหรับสสารนิวเคลียร์เอ็กโซติก

ภาควิชา.............ฟิสิกส์...........................ลายมือชื่อนิสิต..........................................................
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EKAPONG HIRUNSIRISAWAT : NUCLEAR STATES IN STRONGLY COUPLED QUARK-GLUON PLASMA FROM GAUGE-GRAVITY CORRESPONDENCE. THESIS ADVISOR : ASST. PROF. AUTTAKIT CHATRABHUTI, Ph.D., 101 pp.

In a deconfined phase, quarks, anti-quarks and gluons are not necessary to be in a colorless bound state. They can form various color non-singlet (or *exotic* for short) nuclear bound states as well as color singlet bound states in the strongly coupled quark-gluon plasma (sQGP) given that baryon number density is sufficiently large. We study the matter of sQGP mainly occupied by these nuclear states via the so-called Sakai-Sugimoto model, which is a gravity dual model of large $N_c$ QCD. Note that $N_c$ denotes the number of colours. We propose the gravity dual models of exotic nuclear states in the deconfining background of the Sakai-Sugimoto model with flavour degrees of freedom turned off. Moreover, we calculate and compare binding energies and screening lengths of these nuclear states as an analysis of their stabilities. The results indicate that these nuclear states are less stable than normal hadrons. Then, turning on the flavour degrees of freedom, we explore the possibilities of the existence of the multiquark matter, which is a thermodynamic phase representing sQGP mainly filled with exotic nuclear states, to be present in a certain region of phase diagram. This matter is found to be more stable thermodynamically than the vacuum and chiral-symmetric quark-gluon plasma phases ($\chi$S-QGP) in the region in phase diagram of high chemical potential and low temperature, although it is less stable than the normal nuclear matter. The thermodynamic relations of this matter are also determined. The relations of pressure versus baryon number density are found to be $P \sim d^2$ for small number density, and $P \sim d^{7/5}$ for large number density. The entropy density is found to be proportional to the temperature as $s \sim T^5$ for the normal nuclear matter, but $s \sim T$ for the exotic nuclear matter.

Department:.........Physics.........Student's Signature ........................

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## INTRODUCTION

String theory is supposed to be the best candidate on the quantum theory of gravity nowadays. Its developments provide the hopes of unifying all fundamental interactions of nature. Since the string scale is very far from accessible by terrestrial accelerators and astrophysical sources, the theory cannot be applied to any phenomena, and as a result it is likely impossible that the theory could be directly verified by any future accelerators. This would have been the case if there had been no sufficient progress of the studies of D-branes, the extended object in superstring theory on which open strings can end. Namely, it turned out that the explorations of the physics of D-branes in superstring theory lead to a new tool, so-called gauge-gravity correspondence, for studying the phenomena in our real world.

Since its discovery by Juan Maldacena in the last decade of the twentieth century [1], the conjectured equivalence between a superstring theory in the Anti-de Sitter (AdS) space and a supersymmetric conformal gauge field theory (CFT), so-called AdS/CFT correspondence, and other following gauge-gravity dualities have been widely applied to describe the nuclear phenomena in the strong coupling regime of the Quantum Chromodynamics (QCD). These strongly coupled phenomena are, for example, in the areas of the hadronic physics and the strongly coupled quark-gluon plasma (sQGP).

Remarkably, the viewpoints on this topic of research are different between the string theorists and the particle phenomenologists. On the string theorists' view, the application of the dualities to the strongly coupled phenomena is the challenges leading to the theories' developments. The phenomenologists, however, tries to employ these dualities, and also engineering, as important tools for obtaining the physical insights of the nature of QCD. So far, these correspondence theories have succeeded in predicting the physical properties consistent with the experimental results. Among them, the prediction using AdS/CFT that the shear viscosity-to-entropy density ratio in sQGP surprisingly agreed with the experimental results of the collider at the Brookheaven

National Laboratory (BNL) [2]. This was just as a milestone of the gauge-gravity dualities which brought the astonishing interests to the physics community at the beginning of the 21st century.

In this thesis, we explore about the properties of nuclear states in the sQGP via gauge/gravity duality as in the phenomenologists' way. Some gravity dual models of the colour non-singlet nuclear bound states which might be possible in the sQGP are proposed. We investigate the stability of these bound states by calculating in the framework of the gravity side of the duality. Moreover, we look into thermodynamical properties of these exotic nuclear matter. These involve several areas of study in both sides of the duality. Therefore, it is worth to survey some subjects and to introduce definitions of a number of terms in this introductory Chapter. By selection, we start to introduce gauge/gravity duality in section 1.1, and then quark-gluon plasma and QCD phase transition in section 1.2. We move on giving an introduction on colour non-singlet nuclear bound states in section 1.3. In this section, we do not only discuss how we are interested in this kind of nuclear states, but we also emphasize our research questions and statement of the problem. Lastly, we close this Chapter with outline of the remaining content of this thesis.

## 1.1 Gauge/gravity duality

Guided by black hole physics in string theory, Maldacena conjectured that there are two equivalent descriptions of a stack of superimposed large $N_c$ D3-branes in 9+1 dimensional spacetime. This equivalence is between type IIB superstring theory in $AdS_5 \times S^5$ space and $\mathcal{N} = 4$ supersymmetric Yang-Mills theory with gauge group SU($N_c$). This is so-called the Anti-de Sitter/Conformal Field Theory (AdS/CFT) correspondence [1].

Remarkably, the AdS/CFT correspondence is the first successful realization of the holographic principle [3, 4], which is a speculative idea that the information of quantum gravity in a higher dimensional spacetime could be completely encoded into its "hologram", the theory living in the boundary of that spacetime; the super Yang-Mills theory is the superconformal field theory living in the 3+1 dimensional Minkowski spacetime which is the boundary of five dimensional anti-de Sitter space, in which the superstring theory lives. Even though this holographic duality between these two theories is just a conjecture, the matching of symmetry groups between them supports their equivalence. The global symmetry SO(4, 2) of the superconformal theory matches with the isometry group SO(4, 2) of the $AdS_5$ space. And the SU(4) R-symmetry on the gauge theory side is isomorphic to the isometry group SO(6) of the space $S^5$. Moreover, the duality is supported with the holographic dictionary. This dictionary is the map between the observable quantities of two theory sides. For example, the AdS/CFT correspondence owns a field-operator map which can relate gauge-invariant operators of the $\mathcal{N} = 4$ super Yang-Mills theory in a particular irreducible representation of SU(4) to supergravity fields in the same representation [5, 6].

The AdS/CFT correspondence was found very useful in dealing with the non-perturbative non-Abelian gauge theories. This results from the observation that a stack of $N_c$ D3-branes could be described equivalently by the superstring theory in small curvature limit $(R/l_s)^{-1} \ll 1$ and the super Yang-Mills gauge theory in large 't Hooft coupling limit $g^2_{\mathrm{YM}} N_c \gg 1$, where $R$, $l_s$ and $g_{\mathrm{YM}}$ denote curvature radius, string length and Yang-Mills coupling, respectively. Also, the equivalence should hold for vice versa. This illustrates that the intractable stronlgly coupled supersymmetric non-Abelian gauge theory could be described by the straightforward decoupled supergravity theory.

In this thesis, we exploit this advantage of the gauge/gravity duality to study the strongly interacting matter in QCD. We are interested in the holographic nuclear states in the strongly coupled quark-gluon plasma whose many aspects need to be explored. As will be mentioned in section 1.3, both colour singlet and non-singlet nuclear bound states could exist in the sQGP at the temperature just above the $T_{\mathrm{deconf}}$. In this regime, the chiral symmetry could still not be restored, but broken, and could contribute the extra generated mass of quarks in nuclear bound states. Consequently, it is the mass spectra of the nuclear bound states in the hadronic and exotic (colour non-singlet) forms that are interesting to be explored. Furthermore, the thermodynamic and hydrodynamic properties of sQGP are possible to be affected by the existence of these nuclear states. Once the knowledge of these properties is established, we can have better predictions of the probe for signature of the occurrence of the quark-gluon plasma in the relativistic heavy ion experiments.

However, the AdS/CFT correspondence needs to be modified in many ways in order to be suitable tool in studying the nonperturbative QCD phenomena in our real world. This is due to the incompatibilities between this kind of gauge/gravity duality and QCD. For example, QCD cannot be in the same fashion as the $\mathcal{N} = 4$ super Yang-Mills theory which is the conformal and supersymmetric field theory. QCD owns the energy scale called the QCD scale of about 200 MeV due to the dimensional transmutation, therefore it is not conformal. Additionally, it cannot be the supersymmetric theory; there is no superpartner found in the strong interaction in our real world. Furthermore, AdS/CFT lacks the flavor degrees of freedom corresponding to that of dynamical quarks. Actually, the efforts in engineering the duality theories to support the aim of studying nonperturbative QCD have been progressing so much since the Maldacena's discovery. The holographic models from these efforts are so-called AdS/QCD.

There are two complementary classes of AdS/QCD models: top-down models and bottom-up models.

- **Top-down models** are the gauge/gravity duality models rooted from engineering D-branes and strings in ten dimensional spacetime of string theory. The advantage of this kind of models is that two sides of theories are often well understood. In top-down approach, string theorists have engineered a D-branes configuration in such a way that low-energy spectrum of open string fluctuations has the field theoretic interpretations corresponding to the QCD-like theory. These models of confining gauge theories with known supergravity duals includes, for instance, the $\mathcal{N} = 1^*$ theory of Polchinski and Strassler [7], the Klebanov-Strassler cascading gauge theory [8], the $N_c$ D3-branes intersecting with the $N_f$ flavor D7-branes of Kruczenski et al. [9], and the $N_c$ D4-branes intersecting with the $N_f$ flavor D8-branes and $N_f$ anti-D8-branes of Sakai and Sugimoto [10, 11]. Among these, Sakai-Sugimoto model is so far the example of the gauge/string duality which is most closely related to QCD.

- **Bottom-up models** are obtained from constructing the gravity models with an extra dimension whose properties are consistent with the phenomenological features. This approach has the benefit that there is more freedom to build in properties of QCD. The bottom-up AdS/QCD models are often been applied for light hadronic resonances in QCD. This originates from the observation that the Kaluza-Klein modes of fields in an extra dimension can be identified with the radial excitations of hadrons in a confining gauge theory. Some examples of the bottom-up AdS/QCD models includes the hard wall model [12, 13] and the soft wall model [14].

In the present thesis, we will choose to use the Sakai-Sugimoto model. Apart from the behavior of confinement, the Sakai-Sugimoto model owns the chiral symmetry breaking/restoration mechanism governed by the dynamics of $N_f$ flavor D8-branes and $N_f$ anti-D8-branes. This transition can occur at a particular finite temperature above the gluon-deconfining temperature by the finite temperature Sakai-Sugimoto model [10, 11]. Using this model, we will explore the properties of hadrons and exotic bound states in the sQGP.

## 1.2 Quark-gluon plasma and QCD phase transition

At low energy, only hadrons can be observed. Due to the large coupling of the strong interaction on large distance scale, the genuine constituents of the nuclear matter are confined within the baryons and mesons. They can be explored only with a high energy probe, e.g. in the Deep Inelastic Scattering (DIS) experiments. When the energy scale involved is sufficiently large, roughly few hundred GeVs, the interaction among quarks and gluons become perturbatively weak, the phenomenon known as the asymptotic freedom. The quarks and gluons subsequently become "deconfined" from the confinement of the strong interaction.

It was expected that there exists a hot and dense stage at a few $10^{-5}$ seconds after the Big Bang which is occupied mostly by quarks and gluons. This phase of matter could also be produced in the "little bang" immediately after the collisions of relativistic heavy ions. The collisions took place in colliders such as the Super Proton Synchrotron (SPS) of CERN, the Relativistic Heavy Ion Collider (RHIC) of BNL, and the latest Large Hadron Collider (LHC) of CERN. These experiments exhibited some signatures of the appearances of the so-called quark-gluon plasma (QGP). Intriguingly, series of experimental results from RHIC suggests that the produced QGP is strongly coupled (sQGP) [15, 16, 17, 18]. This has been confirmed recently by the results of ALICE and ATLAS, at CERN [19, 20]

Additionally, the quark-gluon plasma could be generated in the cold and extremely dense situation such as at the core of a compact star, e.g. the neutron star. Also, the quark-gluon plasma with high baryon density is planned to be explored at the proposed Facility for Antiproton and Ion Research (FAIR) of the GSI Helmholtz Centre for Heavy Ion Research GmbH.

The quark-gluon plasma is the phase in which the majority of the constituents becomes effectively free quarks and gluons. To describe this kind of plasma, the characteristics of the theory governing the interactions among quarks and gluons and some critical phenomena need to be discussed beforehand.

Quantum Chromodynamics (QCD) was established to be the theory that describes the strong nuclear interactions. This renormalizable non-Abelian gauge field theory with SU(3) gauge symmetry possesses a number of noteworthy features.

At zero temperature, the strength of interactions among quarks and gluons, determined by the effective coupling constant $\alpha_s$, varies with the distance scales or the transferred momenta $q$. The theory predicts that $\alpha_s(q^2)$ decreases logarithmically. As a result, quarks and gluons appear to be weakly coupled at short distances or large momenta $q$ while there exist quark confinement and chiral symmetry breaking at large distances or small momenta. Additionally, the QCD vacuum at low energies is characterized by the so-called vacuum condensates such as the quark condensate $\langle\bar{\psi}\psi\rangle$ (responsible for the breaking of chiral and conformal symmetry) and the gluon condensate $\langle\alpha_s G_{\mu\nu}G^{\mu\nu}\rangle$ (responsible for the breaking of scale invariance of QCD by quantum effects).

At finite temperature, the effective coupling constant $\alpha_s(T)$ falls logarithmically with increasing temperature. Consequently, quarks and gluons can only feel weak-coupling interactions from others in shorter ranges as the temperature increases. The long-range interactions are dynamically screened. This portrays the two extreme situations of very low temperatures and very high temperatures such that there should be a transition from the strongly coupled nuclear matter at low temperatures to the QCD phase so-called the quark-gluon plasma (QGP) at high temperatures. The latter, described by finite temperature perturbative QCD, exhibits neither confinement nor chiral symmetry breaking.

It is speculated that there are two kinds of phase transitions occurring: the (gluon) confinement/deconfinement phase transition, and the chiral symmetry breaking/chiral symmetry restoration phase transition ($\chi$SB/$\chi$S). Nevertheless, these critical behaviors are not well-established nowadays. Particularly, while the $\chi$SB/$\chi$S phase transition is considered based on the $\mathrm{SU}(N_f)_\mathrm{L} \times \mathrm{SU}(N_f)_\mathrm{R}$ chiral symmetry with discontinuity of the quark condensate, serving as the order parameter, the confinement/deconfinement transition has yet to be known whether it is continuous or discontinuous. Actually, the expectation value of the Polyakov loop can serve as the order parameter for deconfinement phase transition, associated with the breaking of the center group $Z(N_c)$ symmetry, in the pure gauge theories, i.e. without the light flavor degrees of freedom, so that there can exist the phase transition with discontinuity in the spirit of Landau theory of phase transition. In the theories with dynamical quarks like QCD, such an order parameter is however ambiguous. [21]. In spite of these, there are several attempts to explore the properties and the critical temperatures of both kinds of phase transitions based on various models through the numerical simulations of lattice gauge theory. Those models are different depending on, for example, the number of colours and flavor degrees of freedom.

There may be a question whether these two QCD phase transitions have one or two critical points. In other words, the critical temperatures are the same or different. Indeed, based on the studies by the numerical lattice QCD, both scenarios occurs depending on models. For example, lattice results for quarks in the fundamental representation of the gauge group exhibit that the chiral phase transition temperature $T_{\rm chiral}$ is approximately the same as the deconfining phase transition temperature $T_{\rm deconf}$ [22]. However, lattice results for quarks in the adjoint representation show that $T_{\rm chiral}$ is about eight times of $T_{\rm deconf}$ [23]. If this is the case, we can have the intermediate phase of gluon deconfinement but chiral-symmetry breaking, hence the strongly coupled quark-gluon plasma with the chiral symmetry broken. As this critical puzzle still remains, it is interesting to explore the structure of QCD phases around the boundary between the strongly interacting nuclear matter and strongly coupled QGP. In this region of QCD phase diagram, a very rich structure appears.

A general picture of the deconfinement process of quarks and gluons within hadrons is currently incomplete at the most. Naively, from argument of the RGE (Renormalization Group Equation) running of the beta function, effectively free quarks and gluons are expected to appear at high energies and/or temperatures. Transition from non-perturbative phase of nuclear matter to the perturbative regime where the perturbative QCD is reliable is explored most successfully in the lattice approach. Lattice studies of the QCD predicts the deconfinement temperature around 175 MeV [24]. Nuclear matter at such temperature would undergo a phase transition into a deconfined phase. Most bound states of light quarks would melt down at this temperature leaving free quarks and gluons in the sQGP. Remarkably, the mesonic states of heavy quarks (e.g. charmonium) in the nuclear matter at such high temperature tend to persist melting at least until $1.5T_{\rm deconf}$ [25, 26, 27] due to the remaining screened Coulomb-type binding potential between quark and antiquark. Multiquark states such as baryons can also exist in the QGP up to certain temperatures provided that the baryonic charge density is sufficiently large.

## 1.3 Colour non-singlet multiquark bound states

In addition to baryons and mesons, the possibility of multiquark states were recognized by Gell-Mann since the proposal of the quark model. $Q^m\bar{Q}^n$-multiquark $(n+m>3)$ such as the tetraquark and dibaryon were proposed since 1977 by Jaffe [28, 29, 30] using the MIT bag model. There are theoretical models of colour-singlet multiquarks using interactions of various origins, *e.g.* chromomagnetism, flux tube confinement, hadronic molecules. Recently, there were some positive results on searches for multiquark states. For example, the four-quark state called Z(4430) was reported to be found at KEKB by the Belle collaboration [31]. Moreover, the discovery of three exotic sub-atomic particles, labeled by $\mathrm{Z_1}(4051)$, $\mathrm{Z_2}(4248)$, and $\mathrm{Y_b}(10890)$, was announced by the same team. Remark that the signal of finding $\mathrm{Y_b}(10890)$ might be the first clear example of an exotic hybrid particle. It contains a bottom quark, an anti-bottom quark, and an excited gluon.

In the confined phase, only colour singlet states can exist as free particle due to the confinement. Above the deconfinement temperature, quarks and gluons with colour charges can propagate with more freedom in the plasma. It is therefore possible that the coloured multiquark states such as diquarks could also exist in the deconfined nuclear medium. Similar to the mesonic states of the heavy quarks, these multiquarks could persist melting up to relatively high temperature above the deconfinement temperature. We can expect the multiquarks to be abundant in the nuclear matter when the density is large up to temperature well above the deconfinement temperature. Consequently, it is interesting to investigate the physical properties of the multiquarks as well as their thermodynamical phase diagram in details. Unfortunately, perturbative QCD based on quarks and gluons is not reliable during the deconfinement phase transition. Lattice QCD is applicable only when the baryon density is small.

The fact that the QGP is strongly coupled near-and-above the deconfinement temperature $T_c$ suggests the possibility of the existence of exotic bound states with colour degrees of freedom in the deconfined QGP[1]. Recall that an interaction between two heavy quarks in the confined phase at $0<T<T_c$, can be described empirically by the screened Cornell potential[2]

[1]For convenience, we use $T_c$ as the deconfinement temperature, instead of $T_{\mathrm{deconf}}$, later on.

[2]The potential between quark and anti-quark in a quarkonium had been studied in Lattice and

$$V_{Q\bar{Q}}(r,T) = \sigma r \left[ \frac{1 - e^{-M_D(T)r}}{M_D(T)r} \right] - \frac{\alpha}{r} \left[ e^{-M_D(T)r} \right], \tag{I.2}$$

where $M_D$ is the Debye screening mass (or the inverse of length) depending on $T$ and $\alpha$ is the effective coupling. The first part represents the (colour-screened) confining force due to QCD string with the effective string tension $\sigma$; it is around 0.20 (GeV)$^2$ as suggested by the lattice studies. The second part represents the effective (colour-screened) Coulomb potential due to transverse string oscillation. By definition, the effective string tension $\sigma$ vanishes at $T > T_c$. As a result, only the screened Coulomb part contributes to the interaction between quarks but within the range of screening length $M_D^{-1}$. Yet, as suggested by [34], a short string-like configuration of colour fields at low $T$ becomes longer strings at-and-near $T_c$ which contribute to the binding between quarks and gluons. Therefore, the bound states of gluons and quarks can exist in both colour singlet and colour non-singlet forms in the sQGP.

The studies of the multibody bound states in the sQGP were initiated by Shuryak and colleagues [35, 36, 34]. Based on the studies in [34], three proposed multibody bound states: (i) diquark or "polymer-chain" ($\bar{q}gg \ldots gq$) (ii) baryons ($qqq$) (iii) closed (3-)chains of gluons (ggg) seem to exist only for $T = (1 - 1.5)T_c$ (or $T$ is in the temperature range from $T_c$ to $1.5T_c$). Importantly, the existence of these bound states could affect the thermodynamical and hydrodynamical properties of the sQGP. This indicates that the knowledge of these bound states might be a key to understand the quark-gluon plasma in the strongly coupled regime, i.e. at the temperature not far from the deconfinement temperature.

It is challenging to apply gauge/gravity duality to study this issue of strongly coupled QCD. On the one hand, it might give us some insights about this matter. On the other hand, this can be a test on how powerful the gauge/gravity duality is for exploring this strongly coupled QCD phenomena. Actually, there are several applications of gauge/gravity duality to study this area and they have been found to succeed in providing the results consistent with the experimental results.

The holographic models of colour singlet baryon was originally investigated by Witten, Gross and Ooguri [37, 38]. In the $AdS_5 \times S^5$ background, a D5-brane wrapped

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spectroscopic studies [32, 33]. These results suggest that the potential can take the form

$$V(r) = \sigma r - \frac{\alpha}{r}, \tag{I.1}$$

where the first term exhibits the confining force between quark and anti-quark, and the second term exhibits an effective Coulomb potential. This is generally known as the "Cornell potential".

around the subspace $S^5$ with $N_c$ strings attached is proposed to be a dual description of a baryon. A holographic dual of a $k$-quark ($k < N_c$) with colour degrees of freedom is discussed in [39] (see also [40]) for the supersymmetric background. There is a number of interesting articles investigating various possibilities of the multiquarks in both confined and deconfined medium, some of them consider deformed baryon vertex [41, 42, 43, 44, 45, 46, 47, 48, 49]. In this thesis, we use a simplified configuration with only one point-like baryon vertex to describe a variety of classes of the multiquarks with and without the colour degrees of freedom. Note that these configurations of holographic multiquarks are based on our work in [41].

With the proposal of these multiquarks, we attempt to address several issues of the properties of the matter. Firstly, we try to investigate the stability of their holographic configurations. This can be done for an individual bound state by comparing energies; the bound states with lowest energy tend to be more stable than others. In addition, we can examine the stability, in macroscopic point of view, of the quark-gluon plasma mostly occupied by these bound states, rather than a single multiquark. For this purpose, the stability in thermodynamical aspect have to be studied. We can also raise a question about how these bound states affect the sQGP. This is the same question as what the equations of state and the thermodynamic relations of this matter are.

The present thesis is organized as follows. In Chapter II, we give the theoretical background necessary for understanding our issues. We start with introducing the AdS/CFT correspondence and discussing its general aspects. Then we describe how we can reach to the holographic QCD. The AdS/CFT correspondence is different from the real QCD in many aspects: it has the field theory side which is conformal and supersymmetric, it has zero temperature, and there is no fundamental degree of freedom corresponding to that of dynamical quarks. We will discuss the attempts of modifying this original version of the duality to obtain holographic large $N_c$ QCD with confining feature, supersymmetry broken, finite temperature, and the presence of fundamental matter. We move on introducing the Sakai-Sugimoto model which is one of the gauge/gravity duality mostly close to the real world QCD at low enough energies. Apart from the features as mentioned above, this holographic model of large $N_c$ QCD has the additional transition between the phase of chiral symmetry broken and that of chiral symmetry restoration which can be realized by a geometric mechanism of the embedding of its flavour branes. It is this framework in which we study the properties of the colour non-singlet multiquark matter for the hope that we obtain the results closely fit with the real world phenomena.

Chapter III is devoted to the holographic model of colour non-singlet multiquark states in the deconfined phase and the analysis of their stability. For simplicity, we explore their properties in the absence of the flavour degrees of freedom. We will turn on these in the studies of Chapter IV. In Chapter III, we start with reviewing the gravity dual model of a baryon vertex in the original AdS/CFT correspondence and then discuss how the object can be constructed in the background fields of the Sakai-Sugimoto model but, as mentined above, with the flavour degrees of freedom turned off. Next, we discuss about the proposed holographic multiquark bound states and argue why these exotic multiquarks can be present only in the deconfining background in which there exists the horizon at a small value of the radial direction. For investigating about their stabilities, we discuss the calculations of the binding energies and the screening lengths of these bound states. Finally, the calculations for investigating the dependence of these bound states on the free quark mass are reported.

In Chapter IV, we explore the possibilities of the existence of the multiquark matter in a certain region in phase diagram of the holographic QCD, and determine its thermodynamical properties. In this Chapter, the setup of the holographic model of multiquark is different from that in Chapter III because the fundamental degrees of freedom are turned on, and, importantly, our consideration are macroscopic, or equivalently the multiquark matter represents a thermodynamic phase, rather than an individual bound state. Precisely, the multiquark matter is the strongly coupled quark gluon plasma mainly filled with the multiquark bound states. We start Chapter IV with an introduction of baryons in the Sakai-Sugimoto model in many aspects, and phase transitions in the deconfining background. Then, we introduce the configuration of the multquark matter phase in which the embedding of the flavour D8- and $\overline{\text{D8}}$- branes needs to be carefully considered. We also comment on the approach to obtain the quantity in this holographic model corresponding to baryon number density and baryon chemical potential. Next, we continue to compare the grand canonical potential among different matter phases and determine the phase diagram. The presence of the multiquark matter phase in the phase diagram will shed light on how thermodynamic stable this matter is, relative to other phases. Furthermore, we determine thermodynamical properties. For this purpose, the calculations for thermodynamic relations are done in both analytical and numerical way.

In Chapter V, we summarise the results presented in this thesis. Some comments on the results and prospects for further researches are also discussed. We also include Appendix A which describes briefly how to determine the embedding of the flavour D8- and $\overline{\text{D8}}$-branes at the cusp for multiquark configuration. This will be helpful in the calculations in Chapter IV.

# Chapter II

## THEORETICAL BACKGROUND

In this thesis, we explore properties of nuclear states in holographic description. Because of this, our real world phenomena under consideration are interpreted in "holographic language". In this Chapter, we present some arguments to illustrate general aspects of gauge/gravity duality. Even though we do not work in holographic description of the original AdS/CFT correspondence, it is very helpful to discuss several features in this mostly well-known framework to understand the "holographic language". For this purpose, we discuss several aspects of the AdS/CFT correspondence in Section 2.1. However, this is not presented as a review. Instead, we give only the basic concepts relevant to this thesis.

In addition, the approaches to reach a holographic dual of our real world QCD are discussed in Section 2.2. These are the modifications of the AdS/CFT correspondence in various ways to get the features of QCD, including the confining behaviour, the finite temperature of field theory, the absence of supersymmetry, and dynamical quarks represented by fundamental representation of the flavour group. Lastly, we introduce the Sakai-Sugimoto model in Section 2.3. This is the gauge/string duality in which we study gravity dual model of the colour non-singlet nuclear states. The Sakai-Sugimoto model adopts the above mentioned features of QCD as well as the feature of chiral symmetry. the flavour branes.

## 2.1 Aspects of the AdS/CFT correspondence

As introduced in Section 1.1, the AdS/CFT correspondence is the duality between "the $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) gauge theory with gauge group SU($N_c$)" and "the type IIB superstring theory in $AdS_5 \times S^5$ spacetime." The former is the supersymmetric gauge theory in $(1+3)$ dimensional spacetime, whereas the latter is the quantum gravity theory living in $(1+9)$ dimensional spacetime. Although this equivalence between two theories of different number of dimensions has been merely the conjecture, there are several evidences, sometimes just clues, to support it. Here, we would like to give the arguments involving these in order to emphasize a number of important and nice aspects of this duality, rather than review the whole story of the AdS/CFT correspondence. Note that there are many good reviews on this topic, for example [50, 51, 52, 53, 54, 55, 56, 57].

### 2.1.1 Two descriptions of a stack of $N_c$ D3-branes

In non-perturbative level, string theory does not possess only closed strings, but it also has a new sector of open strings due to the presence of a solitonic object so-called D$p$-brane. The letter 'D' in D$p$-brane stands for the Dirichlet boundary condition of open strings; open strings must satisfy this boundary condition such that their endpoints lie on D$p$-branes. Moreover, a D$p$-brane is an extended object sweeps out (p+1)-dimensional worldvolume in spacetime. Suppose that a D$p$-brane extend in the $X^\mu = (X^0, X^1, \ldots, X^p)$, the transverse directions are labelled as $Y^i = (X^{p+1}, \ldots, X^9)$. By quantization, closed strings lying in all directions of ten dimensions of spacetime has the spectrum corresponding to dynamical fluctuations of the spacetime, while open strings lying in $(p+1)$-dimensional world-volume has the quantized modes as the spectrum corresponding to fluctuations of the D$p$-branes.

The massless open string spectrum along a stack of $N$ coincident D$p$-branes gives rise to a non-Abelian gauge theory of the gauge group U($N$). This can be seen by considering what is so-called Chan-Paton degrees of freedom on the endpoints of strings. Each of the two endpoints of an open string has $N$ possibilities depending on which two branes they end on. Consequently, a vector mode of open strings contains two indices of Chan-Paton degrees of freedom. This is a gauge field transforming as the adjoint in the gauge group U($N$). Note that the gauge group turns out to Abelian, i.e. U(1), in the case of single D$p$-brane. Moreover, the open string modes are massless because there is no separation between D-branes. In the case of a set of D$p$-branes with a separation between some of them, the gauge fields can have a mass given by the tension of the string linking between separated branes multiples with the distance between these branes, i.e. $m = r/2\pi\alpha'$.

The equivalence as stated in the AdS/CFT correspondence between type IIB superstring in anti-de Sitter spacetime and $\mathcal{N} = 4$ supersymmetric non-Abelian gauge field theory with conformal symmetry indeed originates from matching two different descriptions of the system of a stack of $N_c$ D3-branes. The AdS/CFT correspondence has been found very useful in dealing with nonperturbative QCD. This results from the observation that a stack of $N_c$ D3-branes could be described equivalently by the supergravity theory in small curvature limit and the super Yang-Mills gauge theory in large 't Hooft coupling limit. And the equivalence should hold for vice versa. This relation between gauge and gravitational theory could be said as a strong-weak coupling duality, although this kind of duality is usually used for two theories of the same number of dimensions. In the AdS/CFT, the strong-weak coupling duality is as follows: $\lambda \to 1/\lambda$, where $\lambda$ is so-called 't Hooft coupling. Importantly, this transformation is not just to inverse the coupling but also interchange between closed and open string descriptions of the system of D-branes, as will be seen below.

Before discussing about these two descriptions of a system of D-branes, let us spend a space for introducing the controlling parameters on each side of the AdS/CFT correspondence. The gauge theory, $\mathcal{N} = 4$ super Yang-Mills gauge theory, is controlled by 't Hooft coupling $\lambda \equiv g_{\rm YM}^2 N_c$ and the number of gauge group $N_c$. Initiated by 't Hooft [58], a large $N_c$ Yang-Mills theory is obtained by treating the number $N_c$ of the colour gauge group SU($N_c$) as a paremeter and to be large. Expanding physical quantities in $1/N_c^2$ and $\lambda$, the theory has some natures in common with string theory. This might be a clue of the connection between string theory and QCD before the epoch of gauge/gravity duality. The reason for rearranging the amplitude in term of the 't Hooft coupling $\lambda$, instead of the original Yang-Mills coupling $g_{\rm YM}$, in this large $N_c$ non-Abelian gauge theory is that the theory is much more simple and the mathematical formula of its scattering amplitude is more systematic. By drawing the Feynman diagram in double-line notation and keeping the 't Hooft coupling fixed, the diagrams are organized by their topology. The vacuum-to-vacuum amplitude of this pure gauge theory can be expanded in $1/N_c^2$ and $\lambda$ as

$$\mathcal{A} = \sum_{g=0}^{\infty} N_c^{2-2g} \sum_{n=0}^{\infty} c_{g,n} \lambda^n = N_c^2 f_0(\lambda) + f_1(\lambda) + \frac{1}{N_c^2} f_2(\lambda) + \cdots , \tag{2.1}$$

where $g$ denotes the genus of the diagram, $c_{g,n}$ is a constant at a fixed $g$ and $n$, and $f_g$, $g = 0, 1, \ldots$ are functions of the 't Hooft coupling at fixed order of $1/N_c^2$ expansion. Note that the diagram is planar for $g = 0$; there is no double-line crossing over another and the corresponding compact Riemann surface has the topology of sphere of genus zero. The diagram becomes non-planar for $g = 1, 2, \ldots$; the corresponding compact surface has the topology of genus $1, 2, \ldots$ In the limit $N_c \to \infty$, the $N_c^2$ term ($g = 0$) dominates in the the formula (2.1) such that the amplitude can written as a combination of all possible planar diagrams. In other words, the amplitude is mainly contributed by $N_c^2 f_0(\lambda)$, where $f_0(\lambda) = c_{0,0} + c_{0,1}\lambda^1 + c_{0,2}\lambda^2 + \cdots$. We would like to emphasize here that a large $N_c$ non-Abelian gauge theory is tractable as long as $\lambda \ll 1$, because it is in a weak coupling regime. On the contrary, the theory is non-perturbative in the opposite limit. These are the same for the case of the $\mathcal{N} = 4$ super Yang-Mills gauge theory.

On the other hand, the gravity side of the AdS/CFT correspondence, which is a maximally symmetric spacetime, is controlled by the (ten-dimensional) Newton constant $G$ and the string scale $l_s$ in units of the curvature radius $R$. As will be discussed in detail later, these relate to the parameter on the gauge theory side, $\lambda$ and $N_c$, as follows:

$$G \sim \frac{1}{N_c^2} \quad , \quad l_s^2 \sim \frac{1}{\sqrt{\lambda}}, \tag{2.2}$$

where the curvature radius $R$ is set to be one here. Interestingly, the $N_c \to \infty$ limit implies that all string loop effects are neglected due to the fact that $G \propto g_s^2$, i.e. $g_s \sim 1/N_c$, such that any quantum effects are suppressed. Hence, the theory is classical supergravity. Moreover, closed strings on the gravity side can have higher modes of fluctuations corresponding to the terms in $\alpha'$-expansion in string theory. Note that $\alpha' = l_s^2$. From (2.2), the $\lambda \gg 1$ limit means that any effects of string fluctuations are suppressed, hence only massless fields survive. Noting that the $\alpha'$-expansion corresponds to the expansion in $1/\sqrt{\lambda}$, we can see that the theory on the gravity side is tractable in the $\lambda \to \infty$ limit, since the curvature radius $R$ is much greater than the string scale. On the contrary, it turns out to be uncontrollable in the opposite limit.

### On the gauge theory side

The gauge theory side of the AdS/CFT correspondence arises from the open string description of $N_c$ D3-branes in type IIB superstring theory[1]. The gauge theory arising from this open string description is tractable once 't Hooft coupling $\lambda \ll 1$ such that the D3-branes is just a defect of spacetime, i.e. a zero-thickness hyperplane in 10-dimensional flat Minkowski spacetime. The open string degrees of freedom on these branes are described by a Dirac-Born-Infeld (DBI) action in four-dimensional world-

[1] D$p$-branes in type IIA superstring theory are stable for $p = 0, 2, 4, 6, 8$, while D$p$-branes in type IIB superstring theory are stable for $p = 1, 3, 5, 7$ [59]. Note that D9-branes can also exit in type IIB superstring theory once additional consistency conditions are imposed.

volume of D3-branes[2]

$$S_{\mathrm{D3}} = -T_{\mathrm{D3}} \int d^4x \sqrt{-\det(\eta_{\mu\nu} + \alpha'^2 F_{\mu\nu})}, \tag{2.4}$$

where the D3-brane tension, or its mass per unit spatial volume, can be written as

$$T_{\mathrm{D3}} = \frac{1}{(2\pi)^3 g_s l_s^4}. \tag{2.5}$$

Note that we have used the fact that the D3-branes are embedded in 10 dimensional Minkowski spacetime such that the induced metric describing the embedding of D3-branes is simply $\eta_{\mu\nu}$, $\mu, \nu = 0, 1, 2, 3$. At the lowest order in derivatives, this action reduces to $\mathcal{N} = 4$ super Yang-Mills action in which all fields are nothing but the quantized modes of open strings on the D3-branes world-volume.

The massless field content filling in the (1+3) dimensional supermultiplet consists of a gauge field $A_\mu$, $\mu = 0, 1, 2, 3$, which is a singlet of SU(4) global R-symmetry group, six scalar fields $\phi^i$, $i = 1, 2, 3, \ldots, 6$, in the **6** of SU(4), and four Weyl fermions $\psi_a$, $a = 1, 2, 3, 4$, in the **4** of SU(4). These fields are in the adjoint representation of the gauge group SU($N_c$). For illustration, we show here the bosonic part of the Lagrangian density of $\mathcal{N} = 4$ super Yang-Mills theory as

$$\mathcal{L} \sim -\frac{1}{g_{\mathrm{YM}}^2} \mathrm{tr} \left( \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} D^\mu \phi^i D_\mu \phi^j + \left[\phi^i, \phi^j\right] \right). \tag{2.6}$$

Note that the Yang-Mills coupling $g_{\mathrm{YM}}$ can be identified with string coupling $g_s$ as follows:

$$g_{\mathrm{YM}}^2 = 4\pi g_s, \tag{2.7}$$

since $g_s^{-1}$ from $T_{\mathrm{D3}}$ is indeed the prefactor of (2.6) as obtained by expanding (2.4) in $\alpha'$. Recall that $\lambda$ is define by $g_{\mathrm{YM}}^2$, the above relation implies that we can also write the 't Hooft coupling in term of the string coupling as

$$\lambda = 4\pi g_s. \tag{2.8}$$

Moreover, as mentioned above, $A_\mu$, $\phi^i$, (and also $\psi_a$) transform in the adjoint representation of SU($N_c$), thus they are the SU($N_c$) matrix valued fields in (2.6). As a result, the trace operator acts on SU($N_c$) matrices of these fields. Remarkably, the

[2]Here, the form of the tension of D$p$-brane $T_{\mathrm{D}p}$ in the DBI action

$$S_{\mathrm{DBI}} = -T_{\mathrm{D}p} \int d^{p+1}x \mathrm{e}^{-\Phi} \sqrt{-\det(g_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})}, \tag{2.3}$$

has incorporate $\mathrm{e}^{-\Phi_0} = g_s^{-1}$, where $\Phi_0$ is the vacuum expectation value of dilaton field, whereas fluctuation of the dilaton field $\Phi = 0$ for the present case.

beta function of this $\mathcal{N} = 4$ super Yang-Mills theory vanishes exactly, due to the large number of supersymmetries. Hence, it is a conformal field theory.

At the low energy limit, the interactions between open string degrees of freedom are suppressed by $\alpha' E^2$. Furthermore, the coupling between closed string modes with each other is suppressed by $GE^8$, hence gravity is infrared-free. This is also hold for the coupling between closed and open strings. Consequently, open strings on the D3-branes world-volume decouple from closed string in the background, and we can summarize that a system of $N_c$ coincident D3-branes with $\lambda = g_s N_c \ll 1$ and at low energies is well described by $\mathcal{N} = 4$ super Yang-Mills theory with gauge group SU($N_c$) in (1+3) dimensional Minkowski spacetime, which decouples from ten dimensional free gravity. The only interacting sector of this description is the former.

**On the gravity side**

In large 't Hooft limit, $\lambda \gg 1$, the backreaction of a stack of $N_c$ D3-branes cannot be neglected. Recall that $\lambda = R^4/l_s^4$, this limit implies that the characteristic scale of the spacetime, i.e. the curvature radius, is much larger than the scale of closed string. In other words, the spacetime is weakly curved. Since the source of gravity is $N_c$ coincident D3-branes, it acts as a point mass of $M \sim N_c T_{\mathrm{D3}}$ in six transverse directions. This spacetime metric can be obtained by solving the supergravity equations of motion. Its line-element takes the form

$$ds^2 = H^{-1/2}\left(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + H^{1/2}\left(dr^2 + r^2 d\Omega_5^2\right), \qquad (2.9)$$

where the first term consists of coordinates in the directions parallel to the world-volume of D3-branes, whereas coordinates in the second term are in six transverse directions whose the coordinate $r$ represents the separation in radial direction, transverse to D3-branes, from the horizon of spacetime. Remark that the spatial dimensions at a certain value of radial coordinate $r$ should have topology of $\mathbb{R}^3 \times S^5$. For an extremal 3-branes solution, the function $H(r)$ and the radius of curvature $R$ are given by

$$H(r) = 1 + \frac{R^4}{r^4}, \qquad (2.10)$$

$$R^4 = 4\pi g_s N_c l_s^4. \qquad (2.11)$$

Note that the parameter $N_c$ appears in the background of type IIB supergravity as the flux of the five-form Ramond-Ramond field strength on the $S^5$,

$$\int_{S^5} F_5 = N_c. \qquad (2.12)$$

The geometry described by (2.9) depends on only the radial coordinate $r$ of the transverse directions. To get some insights, it is useful to consider the case of $r \gg R$. At this asymptotic point, the gravity is weak such that the warp factor $H(r) \simeq 1$ with the small correction

$$\frac{R^4}{r^4} = \frac{4\pi N_c g_s l_s^4}{r^4} \sim \frac{GM}{r^4}, \tag{2.13}$$

where we have used (2.11) for the first equality, and we have used the fact that the gravitational effect of $N_c$ D3-branes is similar to a point particle with the mass $M \sim N_c T_{\mathrm{D3}}$ and using (2.5), and the Newton's gravitational constant in 10 dimensions is given by

$$G = (2\pi)^7 g_s^2 l_s^8, \tag{2.14}$$

for the second proportionality. Manifestly, the term $R^4/l_s^4$ can be interpreted as the gravitational potential due to a point mass $M$ in six spatial dimensions.

Now, let us consider the situation of strong gravity $r \ll R$. In this limit, the warp factor $H(r)$ is approximately $R^4/r^4$. As a result, the metric (2.9) becomes

$$\begin{aligned} ds^2 &= \frac{r^2}{R^2}\left(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + \frac{R^2}{r^2}\left(dr^2 + r^2 d\Omega_5^2\right) && (2.15)\\ &= ds^2_{AdS_5} + R^2 d\Omega_5^2, && (2.16) \end{aligned}$$

where

$$ds^2_{AdS_5} = \frac{r^2}{R^2}\left(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + \frac{R^2}{r^2} dr^2. \tag{2.17}$$

The last expression describe the geometry of the five dimensional anti-de Sitter spacetime, which is a maximally symmetric spacetime with the negative cosmological constant $\Lambda = -6/R^2$ and negative curvature, i.e. $-12/R^2$. The $AdS_5$ spacetime can be written in another form by substituting $r = R^2/z$, such that

$$ds^2_{AdS_5} = \frac{R^2}{z^2}\left(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + dz^2\right). \tag{2.18}$$

This is the anti-de Sitter geometry in the Poincaré patch coordinate which manifests conformal symmetry of the geometry. This is also useful to consider holographic renormalization group flow, as will be seen below.

As a summary, the near-horizon spacetime ($r \to 0$) of the extremal 3-branes solution has 'throat' geometry of the form $AdS_5 \times S^5$, while the spacetime becomes flat, ten dimensional Minkowski spacetime, near the boundary of spacetime ($r \to \infty$). This is the closed string description of $N_c$ D3-branes which corresponds to the spacetime geometry in which only closed strings propagate. There is no open strings in this description.

Importantly, the formula (2.8), (2.11), and (2.14) lead to two relations:

$$\frac{G}{R^8} = \frac{\pi^4}{2N_c^2}, \qquad \frac{R^4}{l_s^4} = \lambda = g_{\rm YM}^2 N_c, \tag{2.19}$$

which are the full formula of (2.2) that we used to discuss earlier. Since $G \sim l_p^8$, with $l_p$ the Planck length, these relations imply

$$\frac{l_p^8}{R^8} \propto \frac{1}{N_c^2}, \qquad \frac{l_s^2}{R^2} \propto \frac{1}{\sqrt{\lambda}}. \tag{2.20}$$

In general, type IIB superstring theory on $AdS_5 \times S^5$ is complicated, and there is no systematic treatment available in the present. As mentioned earlier in this Section, the limit $N_c \to \infty$ and $\lambda \to \infty$ renders drastically simplified. It can be approximated by classical supergravity since, as implied by the first relation of (2.20), the Planck length is much smaller than the characteristic scale $R$ of the space $AdS_5$ and $S^5$. Furthermore, as implied by the second relation of (2.20), $l_s \ll R$ in this limit resulting in a weakly curved spacetime.

At low energy limit, there are two distinct sets of closed string excitations which include those propagating in the asymptotic region, called here as the Minkowski region, and those propagating in the throat region. These are focused in the viewpoint of an observer at the asymptotic region in order to shed light on how these closed string modes in two regions are decoupled. The closed strings propagating in the Minkowski region itself are massless at low energies. Moreover, closed string modes are decoupled to each other because their interactions depends on the coupling proportional to $GE^8$. On the other hand, the closed strings in the throat region have to propagates from that region to the Minkowski region so that they get a great amount of redshift. They need to climb up the gravitational potential. The deeper in the throat a closed string excitation is , the more redshifted it get. As a result, closed strings of an arbitrarily high proper energy in the throat have a very low energy as seen by an observer in the Minkowski region. This indicates that closed string modes in the near-horizon geometry, both massless and massive, decouple from the modes in the asymptotic flat region. Hence, type IIB string theory in $AdS_5 \times S^5$ is only the interacting sector of this description, and it decouples from ten dimensional free gravity. Note although the closed string modes in $AdS_5 \times S^5$ can be both massless and massive, massive modes are suppressed as the power of $1/\sqrt{\lambda}$ in $\alpha'$-expansion. In large 't Hooft limit, $\lambda \to \infty$, any stringy effects associated with string fluctuations can be neglected, hence only massless string modes survive.

We can compare this closed string description with what we discussed in the above part on the gauge theory side. By matching the interacting sector of each of these description, we may deduce from this what is stated in the AdS/CFT correspondence: the equivalence between the $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) gauge theory with gauge group SU($N_c$) and the type IIB supergravity theory in $AdS_5 \times S^5$ spacetime. Inferred from the arguments in both the closed string and open string descriptions of the system of D3-branes, the strongly coupled (equivalently, $\lambda \gg 1$) $\mathcal{N} = 4$ super Yang-Mills theory is dual to the weakly curved (equivalently, $\lambda = R^4/l_s^4 \gg 1$) type IIB supergravity in $AdS_5 \times S^5$. And vice versa in the opposite limit.

Before closing this Section, it is worth to discuss about another aspect of the AdS/CFT correspondence. This is the matching of symmetries between two sides of the correspondence, which is an evidence that support this equivalence. As mentioned above, the beta function of the coupling $g_{\rm YM}$ in the gauge theory side vanishes to all orders in perturbation theory, i.e. $\beta(g_{\rm YM}) = 0$. The conformal symmetry of the $\mathcal{N} = 4$ super Yang-Mills gauge theory is represented as the global group SO(4,2). Nicely, this matches with the isometry group SO(4,2) of the $AdS_5$ space. Furthermore, the global R-symmetry of the super Yang-MIlls theory here is described by the SU(4) group. This is isomorphic to the isometry group SO(6) of the $S^5$. Therefore, the SO(4,2) spacetime symmetry group and the global SU(4) R-symmetry group of this conformal field theory is identical to the isometry group of the $AdS_5 \times S^5$ spacetime. However, these are the symmetry groups in bosonic part of either side. Considering the full supersymmetric theories, the global symmetry $\mathrm{SU}(2,2|\,4)$ matches on both sides of the correspondence (see, for example, [50, 51]).

In addition, the two theories on both sides of the AdS/CFT correspondence have the discrete symmetry $\mathrm{SL}(2,\mathbb{Z})$ [60, 61, 62]. Apart from the matching between the $g_{\rm YM}$ and $g_s$ as shown in (2.7), the theta angle coupling $\theta$ in $\mathcal{N} = 4$ super Yang-Mills theory can be matched with the expectation value of the Ramond-Ramond scalar (or axion) $\chi$. Therefore, we can write a complex parameter

$$\tau \equiv \frac{4\pi i}{g_{\rm YM}^2} + \frac{\theta}{2\pi} = \frac{i}{g_s} + \frac{\chi}{2\pi}. \tag{2.21}$$

Precisely, the first term of the above expression is the matching between the dilaton expectation value $e^{\Phi_0} = 1/g_s$ on the gravity side and $1/g_{\rm YM}^2$ in the gauge theory side, whereas the second term is the matching between the axion expectation value $\chi$ on the gravity side and the $\theta$ angle coupling on the gauge theory side. $\mathcal{N} = 4$ super Yang-Mills theory has Montonen-Olive or S-duality symmetry under which the conplex constant $\tau$ transforms as $\tau \to (a\tau + b)/(c\tau + d)$, where $a, b, c, d$ are integers with $ad - bc = 1$, the Möbius transformations in SL(2,$\mathbb{Z}$). On the closed string description of $N_c$ D3-branes, this symmetry is a global discrete symmetry of type IIB string theory, under which the constant $\tau$ in term the dilaton and axion expectation values transforms in the same fashion as in the super Yang-Mills theory. Therefore, the SU(2,$\mathbb{Z}$) duality symmetry is present in both sides of the AdS/CFT correspondence.

### 2.1.2 Geometrizing the RG flow and UV/IR connection

It is quite mysterious at first glance when one knows that these two different theories are equivalent. However, there might be some clues about this before the discovery of Maldacena in 1997 [1]. The discussions about these might render the statement of the AdS/CFT correspondence more sensible. One of them may be the subject of renormalization group (RG) flow[3]. As pioneered by Kadanoff, Wilson and others in 1960's (see e.g. [63, 64]), a good description of a system of many-body or a quantum field theory is to express it in the scale picture. Namely, the equations governing physics of the system are organized in term of length (or energy) scales. We will denote the length scale of the system as $z$. The capability to describe the system in this way results from the fact that degrees of freedom at widely separated scales are largely decoupled from each other.

Consider a quantum field theory in $d$-dimensional Minkowski spacetime $(t, \vec{x})$ and assume that it is defined with a short-distance cutoff, let us say $z = \epsilon$. At a particular low energy scale, or equivalently $z \gg \epsilon$, we can involve just an effective theory at length scale $z$, which can be obtained by integrating out all degrees of freedom at the scales smaller than $z$. Importantly, physical quantities under consideration of this length scale $z$ is obtained by using renormalization scheme on the bare theory defined at the scale $\epsilon$. This procedure provides a renormalization group (RG) flow through the differential equation which is so-called the renormalization group equation (RGE). This equation informs us how the running of coupling behaves as the energy scale change. Note that the flow from low to high energy scales corresponds to the change of $z$ from large to small value. The RG flow gives rise to a continuous family of effective theories in $d$-dimensional Minkowski spacetime, each defined by the length scale $z$. Because of the continuity of $z$, this might be realized as a $(d+1)$-dimensional theory in which the RG scale $z$ is treated as a spatial coordinate and a slice of a certain value of $z$ is the quantum field theory in $d$-dimensional Minkowski

[3]Note that we mainly follow [57] for the content in this subsection.

As discussed in last subsection, the radial coordinate of the spacetime background can be interpreted as the RG scale. Because of this, the cutoff of the flavour branes at $r = L$ means that the open string degrees of freedom on the flavour branes have the lower bound in energy. This is indeed the mass gap for the degrees of freedom of the fundamental matter. Since the cutoff $r = L$ is proportional to quark mass $m_{\mathrm{q}}$ as shown in (2.29), it is natural to guess that this mass gap is linear in $m_{\mathrm{q}}$. However, this mass gap is not exactly the quark mass but the low-lying mode of meson spectrum which could be obtained from the fluctuations of the fields, which are the normalizable modes, on the probe D7-branes. By analysis of meson spectrum, the mass gap is found to be in the order of $m_{\mathrm{q}}/\sqrt{g_s N_c}$. This indicates that the mesons are much lighter than the quarks in the limit $\lambda \gg 1$. It is important to note that the presence of the mass gap here does not mean this theory is confining.

As will be seen later, the confining gauge theory can be realized as the spacetime with a smooth cutoff at a particular value of radial coordinate near the origin of spacetime, and it gives rise to the mass gap of the adjoint field.

## 2.2 The confining theory and nonzero temperature

In recent years, the AdS/CFT correspondence has attracted interests in its applicability to the phenomenological studies of non-perturbative QCD. However, this correspondence cannot provide the gravity dual of the large $N_c$ QCD. As its name suggests, the AdS/CFT have the gauge theory side which is conformal, differing from the confining behaviour of the real-world QCD. There has been many attempts to engineer the holographic model whose the confining feature is taken into account [68, 7, 8, 69].

One natural way is to consider a stack of $N_c$ D4-branes, in Type IIA string theory, whose the world-volume possesses one compact spatial direction [68]. In the near-horizon metric of a near-extremal D4-brane, the compactified spatial circle shrinks to zero size at some finite value of the radial direction representing a smooth cut-off of the spacetime. This feature can provide us with the confining spacetime background in which the potential between a holographic quark-antiquark bound state is mainly contributed by the tension of string lying along the smooth cutoff of the spacetime, or the "hard-wall". Consequently, the potential is linearly proportional to the separation between two ends of the string. This corresponds to the confining potential of interactions between quark and antiquark in the dual gauge theory. In other words, this spacetime background with smooth cutoff is dual to the confining gauge theory.

To consider finite temperature, the time coordinate is Wick-rotated, and the asymptotic circumference of the Euclidean time circle equals to the inverse of the temperature, $T^{-1}$. Consequently, the confining spacetime background at finite temperature has two compact directions. The metric of the geometry then can be written as

$$ds^2 = \left(\frac{u}{R_{\rm D4}}\right)^{\frac{3}{2}} \left[\delta_{ij}dx^i dx^j + d\theta_1^2 + f(u)d\theta_2^2\right] + \left(\frac{R_{\rm D4}}{u}\right)^{\frac{3}{2}} \left[\frac{du^2}{f(u)} + u^2 d\Omega_4\right], \quad (2.32)$$

where $\theta_1$ is the Euclidean time with temperature dependent period $\delta\theta_1 = \beta \equiv T^{-1}$, $\theta_2$ is the compact spatial circle with period $\delta\theta_2 \equiv \frac{4\pi}{3}\frac{R^{3/2}}{u_\Lambda^{1/2}}$, and $f(u) \equiv 1 - \left(\frac{u_\Lambda}{u}\right)^3$. Notice that $f(u)$ equals to zero for $u = u_\Lambda$ but equals to one as $u$ approaches infinity. This $f(u)$ factor renders the $\theta_2 - u$ subspace a cigar-like shape, while the $\theta_1 - u$ subspace has a cylindrical shape. However, there is an alternative supergravity solution whose the time and the compact spatial coordinates exchange the role. That is, $\theta_1$ is the compact spatial coordinate with fixed circumference, $\theta_2$ is the Euclidean time with period $\delta\theta_2 = \beta = \frac{4\pi}{3}\frac{R^{3/2}}{u_T^{1/2}}$, and $f(u) \equiv 1 - \left(\frac{u_T}{u}\right)^3$. In other words, there are two geometries which can be the supergravity solution. The comparison of the free energy between these two competing geometries tells us about the deconfinement phase transition in the gauge theory side. It is important to emphasize that the asymptotic circumference of the time-circle can be variable depending on the temperature, namely $\delta\theta_1 = T^{-1}$, while the $\theta_2$-circle has a fixed circumference. As a result, the phase transition occurs once the asymptotic circumferences of the two circles become the same in both geometries such that they have the same value of free energy. This gives rise to the deconfinement transition line in the $T - \mu$ phase diagram of the holographic nuclear matter [68, 70]. For a concise review, see [54].

## 2.3 The Sakai-Sugimoto Model

More realistic holographic dual of the large $N_c$ QCD is the Sakai-Sugimoto (SS) model [10, 11]. The brane construction of the SS model is a stack of $N_c$ D4 branes intersecting with $N_f$ D8- and $N_f$ anti-D8- branes, where $N_f \ll N_c$ such that the presence of the probe branes D8/anti-D8-branes does not affect the D4-background. This probe limit corresponds to the quenched approximation in the lattice QCD.

Stack of $N_f$ D8 and $\overline{\rm D8}$ branes are introduced as the flavour branes. They are located at separation distance $L_0$ along the compactified $x_4$ direction at the boundary $u \to \infty$. Open-string excitation with one end on the flavour branes behave like a

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| $N_c$ D4 | o | x | x | x | o | | | | | |
| $N_f$ D8($\overline{\text{D8}}$) | o | x | x | x | | x | x | x | x | x |

Table 2.3: The brane construction of the Sakai-Sugimoto model is composed of $N_c$ D4-branes and $N_f$ D8-branes. The 'x' sign denotes the coordinates in infinitely extending directions of the D-brane world-volume, and the 'o' sign denotes the coordinates in compact directions of the D-brane world-volume.

chiral "quark". In the setup where D8 and $\overline{\text{D8}}$ are parallel in the $(x_4, u)$ projection, each chiral excitation on each stack of branes transform independently, therefore the theory has a chiral symmetry. For the setup where D8 and $\overline{\text{D8}}$ connect, forming a U-shape or a V-shape configuration in the $(x_4, u)$ projection, chiral symmetry is broken.

To obtain a SUSY broken QCD at low energy, the boundary conditions of the superpartners in the $x_4$ direction are chosen so that the zero$^{\text{th}}$ modes vanish (Scherk-Schwarz mechanism). For energies below the first KK modes, the gauge theory therefore contains only gluons and chiral quarks. If the number of the stack of D4-branes source $N_c$ is chosen to be 3, this low-energy gauge theory will look exactly like QCD. The brane construction of the SS model is shown in Table 2.3. Note that the 'x' sign signifies that the coordinate is occupied by an infinite extending direction of the D-brane world-volume and the 'o' sign means that the coordinate is occupied by a compact direction of the D-brane world-volume. This holographic model is a QCD-like theory in many aspects, which are listed below.

1. It is non-supersymmetric resulting from the anti-periodicity for superpartners around the $x^4$ circle.

2. It has the confining behaviour and the deconfinement phase transition in the same way as mentioned above. In the confined phase, the $x^4$ coordinate is the cigar-like compact direction and $x^0$ (the Euclidean time) is the cylindrical compact direction. In the deconfined phase the two coordinates exchange their roles. To summarize, the coordinates $\theta_1$ and $\theta_2$ in Eqn. (2.32) can be specified in the confined and deconfined phase as shown in Table 2.4.

3. It has dynamical quarks, though only massless, resulting from the presence of the flavour branes.

| | confined phase ($T < T_{\text{deconf}}$) | deconfined phase ($T > T_{\text{deconf}}$) |
|---|---|---|
| $\theta_1$ | $x^0$ | $x^4$ |
| $\theta_2$ | $x^4$ | $x^0$ |
| $f(u)$ | $1 - \left(\frac{u_\Lambda}{u}\right)^3$ | $1 - \left(\frac{u_T}{u}\right)^3$ |

Table 2.4: Geometric assignment of the compactified bulk coordinates in the Sakai-Sugimoto model

4. The phases of chiral symmetry breaking and chiral symmetric quark-gluon plasma ($\chi S$-QGP) can be realized. There exist two configurations of the flavour D8- and anti-D8-branes, both satisfy the equation of motion. One is the connected configuration of the D8- and anti-D8-branes representing the chiral symmetry breaking phase. Another is the parallel configuration of the D8- and anti-D8-branes lying along the radial direction of the bulk spacetime representing the chiral symmetric phase. Note that $T_{\text{chiral}} = T_{\text{deconf}}$ when the separation between the D8- and anti-D8-branes $L_0 \gtrsim 0.97R$; $R \equiv$ the radius of the $x_4$ circle, while $T_{\text{deconf}} < T_{\text{chiral}}$ when $L_0 \lesssim 0.97R$ [70].

Since the SS model is the holographic model which gives exactly the particle content of the QCD at low energy, we will consider the holographic multiquarks in the deconfined SS model. The idea is to construct a gravity dual of the 5-dimensional gauge theory with chiral fermions which gives approximately the 4-dimensional QCD at low energy. The supersymmetry of the dual gauge theory in the string construction is broken at the position of the flavour branes used to introduce the chiral fermions. To construct the SS model, stack of D4-branes is used as the source to generate a curved background of the type IIA string theory. After taking the near-horizon limit and adding a black hole horizon, we arrive at the following background metric

$$ds^2 = \left(\frac{u}{R_{D4}}\right)^{3/2}\left(f(u)dt^2 + \delta_{ij}dx^i dx^j + dx_4{}^2\right) + \left(\frac{R_{D4}}{u}\right)^{3/2}\left(u^2 d\Omega_4^2 + \frac{du^2}{f(u)}\right) \quad (2.33)$$

$$F_{(4)} = \frac{2\pi N_c}{V_4}\epsilon_4, \qquad e^{\phi} = g_s\left(\frac{u}{R_{D4}}\right)^{3/4}, \qquad R_{D4}^3 \equiv \pi g_s N_c l_s^3,$$

where $f(u) \equiv 1 - u_T^3/u^3$, $u_T = 16\pi^2 R_{\text{D4}}^3 T^2/9$. Note that the compact $x_4$ coordinate ($x_4$ transverse to the probe D8 branes), with arbitrary periodicity $2\pi R$, never shrinks to zero. The volume of the unit four-sphere $\Omega_4$ is denoted by $V_4$ and the corresponding volume 4-form by $\epsilon_4$. $F_{(4)}$ is the 4-form field strength, $l_s$ is the string length and $g_s$ is the string coupling. The dilaton in this background has $u$-dependence and its value changes along the radial direction $u$. This is a crucial difference in comparison to the AdS-Schwarzschild metric case where dilaton contribution is constant.

In the Sakai-Sugimoto model of D4-D8 branes construction, the D4-brane wrapping the $S^4$ is used as the baryon vertex. Remarkably, it was found that the baryon can also be realized as an instanton in the bulk of $N_c$ D4-brane induced background spacetime, corresponding to baryon in the Skyrme model on the gauge theory side. This instanton can be described in term of the Chern-Simons action in the bulk. Therefore, these two pictures of baryon are equivalent.

Before closing this Section, we would like to point out a weak point of the Sakai-Sugimoto model for a holographic model of QCD. As shown in Table 2.3, there is no direction that transverse to both the D4-branes and the D8 ($\overline{\text{D8}}$)-branes. This indicates that the separation between these two stacks of branes on the open string description is zero, hence quark fields are massless, in contrast to those in the setup of D3/D7. Consequently, we can have only massless quarks in the Sakai-Sugimoto model. However, there are many attempts to introduce mass to the quarks in the models such as those discussed in [71, 72, 73, 74, 75, 76, 77, 78, 79, 80].

# Chapter III

## HOLOGRAPHIC MULTIQUARKS

This Chapter devotes to the extensions of the gravity dual model of colour-singlet baryons. We propose these models in the absence of flavour degrees of freedom. These are the models of coloured states without dynamical quarks. In other words, we are interested in the multiquark vertex in this Chapter. In section 3.1, we introduce the previous works on the attempts to construct a gravity dual model of the baryon vertex in the AdS/CFT correspondence. Next, we discuss basic concepts of colour non-singlet nuclear states in section 3.2. Then, gravity dual models of non-singlet multiquark vertex in the deconfined phase are proposed and discussed in section 3.3. Possible classes of holographic multiquark bound states are explored with the restriction that the configurations satisfy the force balance conditions. As a result, the conditions for stable configurations of holographic multiquarks are calculated. In section 3.4, we calculate the binding energies and the screening lengths in order to compare the relative stability among these different classes of multiquark bound states. Finally, in section 3.5, we explore about the dependence of the binding energy of the multiquark bound state on the free quark mass corresponding to the position of of the probe D-branes at which the strings end in near-boundary region.

## 3.1 Introduction

As discussed in Chapter II, gauge/gravity duality gives a powerful tool to study strongly coupled quark-gluon plasma due to the fact that it can be an embodiment of the so-called strong-weak coupling duality [81]. Namely, the strongly coupled gauge theories could be described by a weakly coupled gravitational theory. This allows us to perform analytic calculations in the strong coupling regimes which are very difficult to implement for the real QCD even for lattice calculations.

There has been many attempts to construct model to explore the hadronic physics through this new holographic tool. Despite describing only QCD-like theory, the gauge/gravity duality tends to give the results of analytic calculations which show that the hadrons predicted by the duality behaving in many ways similarly to the hadrons in QCD (see [10, 82, 83, 84], for example). One of the simplest constructions of hadronic states is that of mesons. The heavy quark sources, massive objects in the fundamental representation of SU($N_c$), were introduced into the AdS/CFT correspondence as the elementary strings stretched from a probe (*i.e.* non-back reacting) D3-brane at large radii in anti-de Sitter space to the $N_c$ D3-branes at the origin [85, 86]. Note that the term "probe" in any probe D$p$-brane means that this extended object does not affect the spacetime background. The endpoint of the string at the boundary carries fundamental charge. Two such strings with different orientations, going up to and going down from the boundary, can represent a quark and an anti-quark of which the interaction energy is related to the area law of a Wilson loop in the field theory [85]. Considering the expectation value of the Wilson loop operator, the mesonic state can be realized as a string hanging from the boundary of spacetime to the bulk. In this picture, the static potential between quark-anti-quark pair is proportional to the Nambu-Goto action of this hanging string subtracted by the Nambu-Goto action of the free strings, up to the integration over time direction.

It is worth to remark that these particles are in a pure gauge theory without dynamical quark fields. The constructions in $AdS_5 \times S^5$ corresponds to the bound states composed of only the adjoint matter and the external quarks. An elementary string added to the gravity dual model give rise to a heavy charged particle, representing external quark which is static. Even though this transforms fundamentally in the gauge group SU($N_c$), it cannot have any dynamics; hence it cannot contribute as the dynamical quark field appearing in QCD. In other words, the supergravity picture of the AdS/CFT correspondence have, comparing with QCD, only gluon dynamics but no dynamics from the fundamental quark fields. Consequently, we can sometimes call the mesonic state, mentioned above, as the mesonic vertex due to the fact that it is the same as the QCD meson in the adjoint matter sector, but not in the fundamental sector.

As mentioned in Chapter II, the fundamental degrees of freedom can be simply added into the supergravity picture through introducing the flavour branes [67]. In a gauge theory with fundamental matter, light mesons can be realized in gravity picture as the quantized modes of the fluctuations of the flavour D-branes [82], whereas the heavy-light mesons and heavy mesons are the strings hanging from flavour branes (see a review, [87]). The quark matter is represented by the strings extending from the flavour branes to the horizon of the space-time.

Baryonic states can also be realized in gravity picture as a combination of strings and a probe D-brane. Without the fundamental degrees of freedom, this can however be called the baryon vertex. In the presence of the fundamental degrees of freedom, the corresponding configuration is developed to be the baryon vertex attached with strings extending to the flavour branes. Actually, the strings tend to shrink to zero length, hence the vertex become a probe D-brane embedded within the flavour D-branes behaving as an instanton. We will discuss these in detail in the following section.

### 3.1.1 Baryon vertex in gauge-gravity correspondence

The string/brane realization of baryons was firstly addressed by Witten [37] and Gross and Ooguri [38]. In QCD, the baryon is a colour singlet bound state of three quarks which form the totally antisymmetric representation of the colour gauge group SU(3). This should be the case in gauge/gravity duality but with the gauge group SU($N_c$), where the number of colours $N_c$ is large, instead. In the gravity picture, we expect that there are $N_c$ elementary type IIB superstrings in the bulk with their ends go to the boundary of the space-time background in a similar way as the case of meson. These endpoints of elementary strings on the boundary possess the external charges. Equivalently, the corresponding particles are in the fundamental representation of the gauge group SU($N_c$). These endpoints therefore represent the external quarks in $\mathcal{N} = 4$ super Yang-Mills theory.

Nevertheless, one difficulty arise relating to where another end of each string should end. How does the combination of these strings end naturally in the bulk? Witten proposed that there should be a probe D5-brane wrapped around five-sphere at which the endpoints of these strings locate in the bulk [37]. We describe here the reason for introducing a wrapped D5-brane, as the key in the construction of a baryon vertex in string theory.

It has been known that the Ramond-Ramond self-dual form fields, five-form for $AdS_5 \times S^5$ (but four-form for the space-time background of the Sakai-Sugimoto model which we will discuss in the next subsection) integrating around the compact dimensions of the spacetime background provide us with $N_c$ units of five-form flux on S$^5$:

$$\int_{S^5} \frac{G_5}{2\pi} = N_c, \tag{3.1}$$

where $G_5$ is the five-form. A probe D5-brane has a U(1) gauge field $\hat{A}$ on its world-volume. This gauge field couples to the five-form flux as follows,

$$\int_{S^5 \times R} a \wedge \frac{G_5}{2\pi} \tag{3.2}$$

Consequently, this Ramond-Ramond five-form field $G_5$ contributes $N_c$ units of U(1) charge in the bulk.

One more point that should be emphasized is that the total charge of a U(1) gauge field must vanish in a closed space [37], like the $S^5$ space. As a result, there must be some sources to cancel completely $N_c$ units of U(1) charge of the wrapped D5-brane. It is coincident that the collection of endpoints of $N_c$ strings oriented up to the boundary contribute $-N_c$ units of charge. Because of these, we can claim that the wrapped $D5$-brane is the place in the bulk at which these string should end. Remarkably, this is also a natural way to have the stable combination of these strings since U(1) charges vanish in the bulk.

In addition, the baryonic states in SU($N_c$) gauge theory are gauge-invariant combination of $N_c$ quarks which are totally anti-symmetric under permutations. This reflects that each of quark sources should behave as fermions. In other words, the complete anti-symmetricity results from the fact that all constituents of the bound state are fermionic. This suggests that all of $N_c$ fundamental strings linking the probe D5-brane wrapped around the $S^5$ to the boundary necessarily behave as fermions.

How could we admit of this feature into $N_c$ strings? Fortunately, it was found [88, 89] that the ground state of a string stretching between linked D-branes is fermionic and nondegenerate. It is also pointed out in [37] that one can obtain the same result, i.e. the ground state of the strings behave as fermions, locally as long as the strings connect any transverse D-branes whose total dimensions add up to eight. Considering a string connecting these two branes, given the boundary conditions at the two ends, its ground state energy is positive in the Neveu-Schwarz (NS) sector while it is zero in the Ramond (R) sector. These cause the ground state of a string connecting these branes is fermionic. Moreover, the ground state of each of these strings is nondegenerate since there is no fermion zero mode in the Ramond sector.

To be precise, it is better to identify the world-volume of the transverse D-branes connecting by the strings. The world-volume of a wrapped D5-brane is $\mathbf{R} \times \mathbf{S}^5$, where $\mathbf{R}$ is a timelike curve in $AdS_5$. Obviously, we require a probe D3-brane locating near the boundary of $AdS_5$ space as another brane transverse to a wrapped five-brane so that total dimensions of these two branes add up to eight. Instead of the boundary of the $AdS_5$ space, the probe D3-brane become a boundary condition of $N_c$ strings. For convenience, we consider the Euclidean version of the $AdS_5$ space whose the boundary is $\mathbf{R} \times \mathbf{S}^3$, where $\mathbf{R}$ is the "time" direction. The probe D3-brane located around there have the world-volume of the same topology. Remarkably, a probe D3-brane near the boundary of the spacetime as a boundary condition of strings has been used in [85, 86].

The (3-5) fundamental strings connecting these two transverse branes have totally eight coordinates with Neumann-Dirichlet (ND) and Dirichlet-Neumann (DN) types of boundary conditions, i.e. in the $X^1, X^2, X^3$ and $X^5, X^6, X^7, X^8, X^9$ directions, respectively. Later on, we denote the total number of DN and ND directions by $\#_{\mathrm{ND}}$, hence $\#_{\mathrm{ND}} = 8$ here. There are also one Dirichlet-Dirichlet (DD) coordinate $X^4$, and one Neumann-Neumann (NN) coordinate $X^0$ (see Table 3.1.1). The oscillator excitations of $X^0$ and $X^4$ and their fermionic partner can be eliminated by the super Virasoro constraints [89]. The ground state energies are different between Ramond and Neveu-Schwarz sectors. Note that these two sectors of open strings correspond to two periodicity for world-sheet fermions. Importantly, the ND (DN) boundary condition interchanges between the mode expansion in NS sector and R sector in that DN direction. Namely, it changes the NS sector from expanding in half-integer modes to integer modes, whereas it changes the R sector from expanding in integer modes to half-integer modes. The string excitations in these eight directions (DN and ND) have vanishing zero-point energy in the Ramond sector as in the case of DD and NN boundary conditions, since bosons and fermions with the same periodicity cancel. In the NS sector, the ground state energy is (see e.g. [90])

$$(8 - \#_{\mathrm{ND}}) \left( -\frac{1}{24} - \frac{1}{48} \right) + \#_{\mathrm{ND}} \left( \frac{1}{24} + \frac{1}{48} \right) = -\frac{1}{2} + \frac{\#_{\mathrm{ND}}}{8}. \tag{3.3}$$

Substituting $\#_{\mathrm{ND}} = 8$, the ground state energy of strings in DN and ND directions in the NS sector is $1/2$. Consequently, an open string with these boundary conditions behaves as a fermion.

Therefore, the gravity dual model of the baryon vertex in AdS/CFT correspondence can be represented by the probe D5-brane wrapped around the five-sphere attached with $N_c$ strings extending to the probe D3-brane near the boundary of the spacetime, where these strings are totally antisymmetric under permutations. This construction represents the gauge invariant combination of $N_c$ external quarks which is the totally antisymmetric representation of the gauge group $\mathrm{SU}(N_c)$. It is important to emphasize that this construction is based on the assumption that the baryon

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| a wrapped D5 | x | | | | | x | x | x | x | x |
| | (N) | (D) | (D) | (D) | (D) | (N) | (N) | (N) | (N) | (N) |
| a probe D3 | x | x | x | x | | | | | | |
| | (N) | (N) | (N) | (N) | (D) | (D) | (D) | (D) | (D) | (D) |

Table 3.1: Two transverse branes at which the elementary strings end are a probe D5-brane wrapped around five-sphere and a probe D3-brane located near the $AdS$ boundary. The world-volume of the wrapped D5-brane, $\mathbf{R} \times \mathbf{S}^5$, occupies in the directions 0th and 5th to 9th, while the world-volume of the probe D3-brane, $\mathbf{R} \times \mathbf{S}^3$, occupies in the directions 0th-3rd. Note that we also specify the types of the string boundary condition for each spacetime direction. (D) denotes the Dirichlet type of boundary condition and (N) denotes the Neumann type.

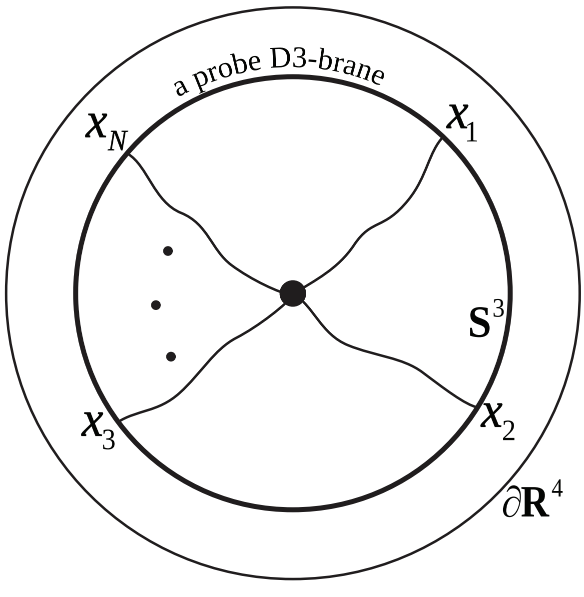


Figure 3.1: A construction of the baryon vertex in AdS/CFT correspondence is composed of a probe D5-brane wrapped five-sphere located in the bulk, a probe D3-brane, transverse to the D5-brane, near the $AdS$ boundary, and $N_c$ strings connecting these two branes which behave as fermions such that the baryon vertex is completely anti-symmetric under permutations among these strings.

vertex configuration have the point-like shape. Some non-trivial configurations of the baryon vertex have been investigated in [91, 92, 93].

Note that all of the $N_c$ strings have the orientation going out from the wrapped D5-brane to the D3-brane around the boundary. Each of them contributes the U(1) charge of -1 at the wrapped D5-brane in the bulk. Combination of $N_c$ strings therefore give the $-N_c$ unit of charge and cancel with $+N_c$ unit of U(1) charge of the wrapping brane. In the same way, if we choose that the orientations of all string is in the direction of going from the D3-brane near the boundary into the wrapped D5-brane, they would contribute $+N_c$ unit of U(1) charge. To cancel the charges, we need the anti-D5-brane wrapped around $S^5$ such that we have $-N_c$ unit of the charge.

Generally, the supergravity solutions of a stack of $N_c$ D$p$-branes includes the presence of the $(8-p)$-form field strength, and the baryon vertex has to be constructed from a probe D$(8-p)$-brane wrapped around the subspace $S^{8-p}$ of the background spacetime. The U(1) gauge field on the world-volume of the D$(8-p)$-brane couples with the antisymmetric $(8-p)$-form field strength $G_{(8-p)}$ and induce $N_c$ units of U(1) charge upon the wrapped D$(8-p)$-brane, canceled completely with the charges brought about by $N_c$ strings. Obviously, the above discussed construction of the baryon vertex in AdS/CFT correspondence can be obtained by substituting $p = 3$ since the background fields are induced from the presence of a stack of $N_c$ D$p$-branes where $p = 3$.

In the theory with fundamental degrees of freedom, it is natural to replace the probe D3 branes as boundary conditions of $N_c$ strings, in the model of baryon vertex, with the flavour branes. As introduced in Chapter II, it is the probe D8-branes and anti-D8-branes in the Sakai-Sugimoto model. These branes do not only give rise to the fundamental degrees of freedom, but also the phase structure of the fundamental matter. We are interested in exploring the properties of holographic hadronic states in this model because it is one of the closest cousin of our real-world QCD, and some results obtained from the model have been found to be notable for comparing with the low-energy QCD.

In the rest of this Chapter, we will explore about the stability of the baryon vertex and the multiquark vertex in the background fields corresponding to the Sakai-Sugimoto model except that we ignore the effects of the presence of the flavour branes. Namely, these are the background fields induced by a stack of coincident $N_c$ D4-branes with one spatial world-volume direction compactified in $S^1$. The considerations in this simplified version of the Sakai-Sugimoto model can, at least, provide us with a clarified picture of baryons and multiquarks, though just as vertex with only the external quarks, in the supergravity description.

In the supergravity description of a stack of $N_c$ D4-branes, there is a Ramond-Ramond 4-form field strength. Since the four-sphere is a subspace of this $N_c$ D4-branes induced spacetime background, the baryon vertex can be given by a probe D4-brane wrapped around $S^4$ attached with $N_c$ strings extending to a probe brane near the boundary of spacetime. As to be discussed in the next Chapter, in the Sakai-Sugimoto model, the wrapped D4-brane imbedded in D8-brane, whose $N_c$ strings tends to have zero length, give rise to the instanton number which corresponds to the baryon number obtained from the Skyrme model in the gauge theory side.

## 3.2 Multiquarks in the deconfined phase

In the deconfined phase, coloured states of a number of quarks and antiquarks can exist in the medium as long as it is more energetically favoured than free quarks and antiquarks. However, this is not the case in the confined phase; the only allowed bound states are those with colour singlet combinations of quarks. We will see later in this section whether a colour non-singlet bound state of quarks can exist or not in the supergravity picture depend on the presence of the horizon in the spacetime background.

Remarkably, the orientation of strings plays the important role in the consideration. In a conventional baryon vertex in the gravity picture, we choose the orientation of all strings going up to the boundary to represent the baryon within which each valence quark contributing one of different positive colour charges. This should not be confused with flavor charges and electric charges of which each quark contribute different values. In the real world baryons such as proton and neutron, the colour charges are red, green, and blue which are three different colour charges of each quarks in the SU(3) invariant bound state. Obviously, each quark contribute equally one unit of colour charge but different to each other. They are not anti-colour such as anti-red (or cyan), anti-green (or magenta) and anti-blue (or yellow). In the gravity picture, these all have the orientation going out to the boundary.

In the gravity dual models of multiquarks, we will discuss coloured bound states of quarks with not only colour but also anti-colour. As mentioned above, the bound states in the deconfined phase can have colour and thus can have more varieties than the situation in the confined phase. In the gravity picture, the spacetime background corresponding to the deconfined phase has the horizon into which one end of "quark" strings can hide. For example, a string can either start from the baryon vertex and go radially to the horizon of the background spacetime or it can come from the horizon and end at the baryon vertex. We will call this string configuration which is allowed in the deconfined phase as the "radial string". A free radial string configuration lying along the radial coordinate, given by the classical solution of the Nambu-Goto action [94], can represent a free (anti)quark state in the QGP medium.

In $\mathcal{N} = 4$ supersymmetric Yang-Mills theory, baryon configurations can be stable in the confining background [39]. The binding energy of a baryon is found to be linear in $N_c$ and in the size of the baryon on the boundary. Remarkably, it is possible to have stable configurations for baryons which are made of $k$ quarks, or "k-baryon", if $5N_c/8 < k \leq N$ in the finite temperature $\mathcal{N}_{SUSY} = 4$ Super Yang-Mills theory [39]. Such a configuration is made of baryon vertex with $k$ strings extending up to the boundary and the rest $N_c - k$ strings stretched down to the horizon. With the presence of these radial strings, the baryonic states are not colour singlet and transform as $\frac{N_c!}{k!(N_c-k)!}$ representation under $SU(N_c)$ gauge group. In a confining theory we do not expect to find such a bound state. It was also proposed in [43, 42] that the $k < N_c$ bound states can only exist in a deconfined phase. Note that we will include baryons as a special case of the multiquark states in this thesis.

Some attempts have been made in constructing holographic description of these exotic multi-quarks bound states [43, 42, 47, 46]. The author in [46] considered exotic multi-quark configurations in holographic picture formed by combining two or more baryon vertices together. Nevertheless, the gravity dual model of an exotic baryon can be constructed from a single baryon vertex. This is expected to be more energetically preferred.

In general, there are infinite ways of combining strings with the probe D-brane vertex such that the U(1) charges vanish at the vertex. Hence, the total number of strings attached to the baryon vertex need not to be equal to $N_c$. In the following section, some possible classes of multi-quark bound states are introduced. The combinations of strings are not arbitrary, but they are restricted by the force balance conditions at the vertex. As mentioned above, these constructions are proposed in the background of the black $N_c$ D4-branes corresponding to the flavour-less version of the Sakai-Sugimoto model.

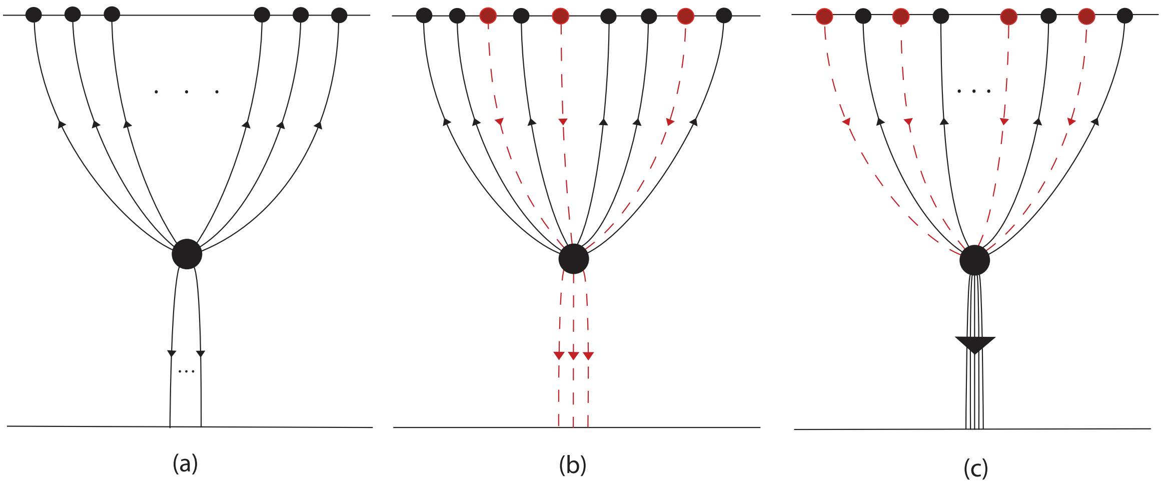


Figure 3.2: The gravity dual configurations of the hypothetical exotic states (a) $k$-baryon with the number of hanging strings $k_{\mathrm{h}} = k < N_c$ and the number of radial strings $k_{\mathrm{r}} = N_c - k$. (b) $(N_c + \bar{k})$-baryon with $k_{\mathrm{h}} = N_c + \bar{k}$ and $k_{\mathrm{r}} = \bar{k}$. (c) $j$-mesonance with $k_{\mathrm{h}} = 2j$ and $k_{\mathrm{r}} = N_c$.

## 3.3 Some classes of multi-quark states

Based on the assumption of string charge cancelation at the vertex, three classes of exotic multiquarks are proposed in [41]. Namely, they are $k$-baryons, $(N_c + \bar{k})$ -baryon and $j$-mesonance (strongly coupled bunch of mesons), corresponding to diquark, some exotic baryons such as pentaquark, and a bunch of mesons, respectively. We parameterize $k_h$ as the number of hanging strings which extends from the vertex to the boundary, and $k_r$ as the number of radial strings extending from the vertex to the horizon. Figure 3.2 shows three classes of the holographic multiquarks.

In the deconfined phase of QGP, it is possible to have $k_{\mathrm{h}}$ strings hanging from the spacetime boundary down to the baryon vertex and another $k_{\mathrm{r}}$ strings stretching radially from the baryon vertex down to the horizon. The total number $k_{\mathrm{h}} + k_{\mathrm{r}} = N_c$ is the charge conservation constraint on the configuration. This configuration is known as "***k*-baryon**" [39].

Another possible configuration is composed of $N_c$ strings with the going-up orientation and $\bar{k}$ strings with the going-down orientation hanging down to the vertex from the probe branes at the boundary of the spacetime. To conserve the charge in the bulk, $\bar{k}$ strings with the going-down orientation need to be added at the vertex and extend from the vertex down to the horizon of spacetime. We call this configuration "**$(N_c + \bar{k})$-baryon**".

An even more interesting configuration allowed in the deconfined phase is when there are $j$ pairs of quark and antiquark strings hanging from the probe branes at the boundary of spacetime down to the vertex. Again, to conserve the charge, we need $N_c$ radial strings stretching from the vertex down to the horizon. Obviously, this configuration can decay into $j$ mesons when it is less energetically favoured. Therefore we will call this state, a "**$j$-mesonance**", representing a binding state of $j$ mesons in the QGP.

In summary, the charge conservation constraint for each case can be expressed as the following.

For $k$-baryon,

$$k_{\mathrm{h}} + k_{\mathrm{r}} = N_c; \quad k_{\mathrm{h}} = k. \tag{3.4}$$

For $(N_c + \bar{k})$-baryon,

$$k_{\mathrm{h}} - k_{\mathrm{r}} = N_c; \quad k_{\mathrm{h}} = N_c + \bar{k}. \tag{3.5}$$

For $j$-mesonance,

$$k_{\mathrm{h}} = 2j; \qquad k_{\mathrm{r}} = N_c. \tag{3.6}$$

Note that $k_{\mathrm{h}}$ is the number of strings hanging from the boundary down to the baryon vertex and $k_{\mathrm{r}}$ is the number of radial strings extending from the vertex down to the horizon. Note that the value of $\bar{k}$ and $j$ can be as large as $N_c \times N_f$ in the case with $N_f$ flavour degrees of freedom. However, we will simply take this number to be large, ignoring the upper bound on $\bar{k}$ and $j$. Each configuration of exotic baryons is illustrated in Fig. 3.2.

### 3.3.1 Force balance conditions

We will consider the force balance condition for each string-brane realization of exotic configuration of multi-quarks in the deconfined phase. As will be seen later, this is the equilibrium condition for the existence of the multiquark states. Assume the vertex to be a point at the cusp position $u_c$ that does not receive any distortion from the attached strings. The distortion of the baryon vertex due to the attached strings is discussed in detail in [92, 93].

Again, the calculation will be performed in the gravity background similar to the Sakai-Sugimoto model [10]. Even though the chiral symmetry restoration can be addressed within this model, the consideration of this chiral-transition aspect is postponed to the next Chapter. Here, we focus on the high temperature phase where quarks and antiquarks are effectively free in the absence of the linear confining potential. The positions of D8/$\overline{\text{D8}}$ will be taken to be large and we will approximate it to be infinity in this section as well as in the discussion of binding energy and screening length in Section 3.4. Analysis in this heavy-quark limit provides us with valuable physical understanding of certain essential features of the exotic states.

Generalized results for a near-horizon background metric of the D$p$-branes solution and its dependence on positions of the probe branes will be given in section 3.5.

Even in the deconfined phase, quarks and antiquarks feel effective (screened) potential from other constituents. Therefore, a number of population of them will exist in various forms of bound states, some of which are exotic in the sense that they cannot be formed in the confined phase at low temperature.

As shown in Chapter II, the deconfining background of fields in the Sakai-Sugimoto model take the form

$$ds^2 = \left(\frac{u}{R_{D4}}\right)^{3/2} \left(f(u)dt^2 + \delta_{ij}dx^i dx^j + dx_4{}^2\right) + \left(\frac{R_{D4}}{u}\right)^{3/2} \left(u^2 d\Omega_4^2 + \frac{du^2}{f(u)}\right)$$

$$F_{(4)} = \frac{2\pi N}{V_4}\epsilon_4, \qquad e^{\phi} = g_s \left(\frac{u}{R_{D4}}\right)^{3/4}, \qquad R_{D4}^3 \equiv \pi g_s N l_s^3,$$

where $f(u) \equiv 1 - u_T^3/u^3$, $u_T = 16\pi^2 R_{\rm D4}^3 T^2/9$. Note that the compactified $x_4$ coordinate, transverse to the probe D8 branes), with arbitrary periodicity $2\pi R$. It never shrinks to zero in the deconfined phase. The volume of the unit four-sphere $\Omega_4$ is denoted by $V_4$ and the corresponding volume 4-form by $\epsilon_4$. $F_{(4)}$ is the 4-form field strength, $l_s$ is the string length and $g_s$ is the string coupling.

Remark that the deconfining temperature $T_c$ is the value at which the circumferences of time-circle and $x_4$-circle have the same value, i.e. $\beta = 2\pi R$. As a result, we have $T_c = 1/2\pi R$. For convenience, we set here

$$T_c = 0.025, \tag{3.7}$$

which implies that the $x_4$-circle has radius $R = 6.3662$. Moreover, we set $R_{D4} = 1$, so that the horizon is at $u_T \simeq 0.011$.

The action of the baryon configuration is given by

$$S = S_{\rm D4} + k_{\rm h} S_{\rm F1} + k_{\rm r} \tilde{S}_{\rm F1}, \tag{3.8}$$

where $S_{D4}$ represents the action of the D4-brane. $S_{F1}$ is the action of a stretched string from the boundary down to the baryon vertex and $\tilde{S}_{F1}$ is the action of a radial string hanging from the baryon vertex down to the horizon. Recall that $S_{D4}$ can be obtained from the Dirac-Born-Infeld action which is

$$S_{DBI} = \int dx^0 d\xi^p T_p$$

where the D$p$-brane tension is

$$T_p = \left(e^{-\phi}(2\pi)^p \alpha'^{(p+1)/2}\right)^{-1} \sqrt{-det(g)}$$

By integrating out, these actions take the form

$$\begin{aligned} S_{\rm D4} &= \frac{\tau N u_c \sqrt{f(u_c)}}{6\pi\alpha'}, && (3.9) \\ S_{\rm F1} &= \frac{\tau}{2\pi\alpha'} \int_0^L d\sigma \sqrt{u'^2 + f(u)\left(\frac{u}{R}\right)^3} && (3.10) \\ \tilde{S}_{\rm F1} &= \frac{\tau}{2\pi\alpha'}(u_c - u_{\rm T}), && (3.11) \end{aligned}$$

where $\tau$ is the total time over which we evaluate the action and $u_c$ is the position where the D4-brane vertex is located.

Because of spherical symmetry of the configuration in the $(x_1, x_2, x_3)$ subspace, the action is sensitive to only the variation in the holographic direction $u$. The variation of the action gives the volume term as well as the surface term. The equation of motion is obtained by requiring that the volume term and surface term vanishes separately. The volume term gives the Euler-Lagrange equation which determines the shape of the hanging strings. On the other hand, the surface term provides the equilibrium condition of the configuration at the tip $u_c$ under the variation in the $u$ direction, i.e. the force balance condition at the cusp.

As an approximation, we assume the baryon vertex to be a point (not being distorted by the connecting strings) located at a fixed value of $u = u_c$ as in [39]. Under this assumption, the surface terms provide additional *zero-force condition* on the configuration,

$$\frac{N}{3} G_0(x) - k_{\rm h} B + k_{\rm r} = 0 \qquad (3.12)$$

where

$$G_0(x) \equiv \frac{1 + \frac{x^3}{2}}{\sqrt{1 - x^3}}, \quad x \equiv \frac{u_T}{u_c} < 1, \text{ and } \quad B \equiv \frac{u'_c}{\sqrt{u_c'^2 + f(u_c)(\frac{u_c}{R_{\rm D4}})^3}}. \qquad (3.13)$$

Notice that these conditions occur at the location of the vertex at $u = u_c$, at which there exists the balance between the pull-up force (toward the direction of increasing $u$) due to the tension of hanging strings and the pull-down force due to the "weight" of D4-brane plus the tension of radial strings.

Actually, this is not exactly the *weight* in the usual sense since the direct gravitational force on D-brane is already balanced by the force from the RR-flux, but it is the force originated from minimization of self-energy due to the brane tension caused by the background metric and the gauge interaction. This is very similar to the self-energy of a spring under gravity where the spring potential energy changes with the tidal force from gravity in the background. The DBI action of the D4$\sim$ $u_c\sqrt{f(u_c)}$ which is positive for $u_c > u_T$ and becomes zero (minimum) at $u_c = u_T$ and thus it represents the "weight" on D4 towards the horizon.

Since $B$ is always less than one, we obtain the equilibrium condition

$$k_{\rm h} \geq \frac{N}{3} G_0(x) + k_{\rm r}, \tag{3.14}$$

which expresses the lower bound of the number of hanging strings. In other words, the number of hanging strings cannot be less than this critical value, otherwise the no-force condition is not satisfied. The equality of (3.14) is held only when all hanging strings are stretched straight, otherwise we require more hanging strings to balance the pull-down force. Let us now consider each class of the multi-quark states.

In the case of $k$**-baryon**, plugging the condition (3.4) into (3.14), we obtain

$$k_{\rm h} = k \geq \frac{N}{6}\left(G_0(x) + 3\right). \tag{3.15}$$

Apart from the lower bound, we also have the upper bound, $k \leq N$, therefore $G_0(x)$ cannot be larger than 3, resulting in

$$x \lesssim 0.922. \tag{3.16}$$

Notice that this restriction on $x$ is a result from the conditions of the force balance and conservation of string charges. This shows that there is an upper-bound on the temperature, over which the horizon is too near to the point vertex that the pull-down force always overcomes the pull-up one.

In the case of $(N_c+\bar{k})$**-baryon**, in the same way as the preceding case, plugging the condition of charge conservation (3.5) into (3.14), we have the following condition,

$$k_{\rm h} = N_c + \bar{k} \geq \frac{N_c}{3} G_0(x) + \bar{k}.$$

Unlike the case of $k$-baryon, the upper-bound of the number of hanging strings does not exist. However, we still obtain the same condition $G_0(x) \leq 3$, hence $x \lesssim 0.922$.

Finally, in the case of $j$**-mesonance**, similarly, Eqn. (3.6) results in

$$j \geq \frac{N}{6}\left(G_0(x) + 3\right). \tag{3.17}$$

The lower-bound of the value of $j$ is $2N/3$ at zero temperature ($x = 0$) and it will be larger as the temperature grows. Nevertheless, the upper-bound of the limit on $j$ does not exist.

Finally, we would like to comment on the limits on the value of $k, \bar{k}, j$ when the temperature is zero. In terms of $m \equiv 7 - p$ (of the spacetime background generated by D$p$-branes), the condition (3.14) becomes

$$k_h \geq \frac{N_c}{m} + k_r \tag{3.18}$$

which leads to

$$\frac{k}{N_c}, \frac{j}{N_c} \geq \frac{m+1}{2m}, \tag{3.19}$$

and no conditions on $\bar{k}$. This critical numbers are $5/8, 2/3$ for $m = 4, 3$ (the AdS-Schwarzschild and Sakai-Sugimoto model) respectively. It is an interesting coincidence that the critical numbers are the same for both $k$-baryon and $j$-mesonance. Remarkably, it turns out that the condition (3.18) does not give any constraint on the $(N_c+\bar{k})$ configuration.

## 3.4 Binding energy and the screening length

Theoretically, all of these bound states are allowed to exist. However, a question arises which multiquark state is more stable than another. This can be addressed by considering the binding energies of each class of the multiquarks. Naturally, the binding energy of each of these holographic bound states is the total energy of the configuration subtracted by the energy of the free quarks. Similar to the calculation of Wilson loop in [85], the binding energy in the large $N_c$ limit could be estimated to be the total classical action divided by $\tau$.

In this section we will calculate the binding energies of the $k$-baryon, $(N_c + \bar{k})$-baryon, and $j$-mesonance in the deconfined phase. These binding energies are taken to be the differences between the total energies of each configuration and the corresponding energies of the free strings configuration which represents the free quarks and/or antiquarks state. The number of free strings in the free quarks state is determined solely by the total number of strings hanging from the boundary, $k_h$.

The total energy is given by $E = S/\tau$ of the corresponding action $S$ for each configuration. The solution or the shape of the hanging strings can be obtained by using the Nambu-Goto action from Eqn. (3.11), the regulated energy of the hanging strings (subtracted by energy of the free quarks) is

$$E_{\text{F1}} = \frac{1}{2\pi}\int_0^L d\sigma \sqrt{u'^2 + \left(\frac{u}{R_{\text{D4}}}\right)^3 f(u)} - \frac{1}{2\pi}\int_{u_T}^{\infty} du. \tag{3.20}$$

Due to the no-force condition in the surface term, we impose Eqn. (3.12) and Eqn. (3.13), or

$$u_c'^{\,2} = \frac{f(u_c)B^2}{1-B^2}\left(\frac{u_c}{R_{\text{D4}}}\right)^3 \tag{3.21}$$

where the tension of each hanging string at $u_c$ is constrained by

$$B = B(k_{\text{h}}, k_{\text{r}}, x) = \frac{N}{3k_{\text{h}}}G_0(x) + \frac{k_{\text{r}}}{k_{\text{h}}}. \tag{3.22}$$

Since the Lagrangian $\mathcal{L}$ does not depend on $\sigma$ explicitly, the conserved Hamiltonian can be defined to be

$$\mathcal{H} \equiv \mathcal{L} - u'\frac{\partial \mathcal{L}}{\partial u'} = \text{const}, \tag{3.23}$$

leading to

$$\frac{f(u_c)(\frac{u_c}{R_{\text{D4}}})^3}{\sqrt{u_c'^{\,2} + f(u_c)(\frac{u_c}{R_{\text{D4}}})^3}} = \frac{f(u)(\frac{u}{R_{\text{D4}}})^3}{\sqrt{u'^2 + f(u)(\frac{u}{R_{\text{D4}}})^3}}. \tag{3.24}$$

Then substituting Eqn. (3.21) into this equation, we obtain

$$u'^2 = \frac{f(u)^2(\frac{u}{R_{\text{D4}}})^6}{f(u_c)(\frac{u_c}{R_{\text{D4}}})^3(1-B^2)} - f(u)\left(\frac{u}{R_{\text{D4}}}\right)^3. \tag{3.25}$$

This gives the size (radius) of the baryon as seen on the gauge theory side,

$$L = \frac{R_{\text{D4}}^{3/2}}{u_c^{1/2}}\int_1^{\infty} dy \sqrt{\frac{(1-x^3)(1-B^2)}{(y^3-x^3)(y^3-x^3-(1-x^3)(1-B^2))}}. \tag{3.26}$$

Note that $u_c \approx \frac{R_{\text{D4}}^3}{L^2}$ at the leading order.

Using Eqn. (3.25) and let $y \equiv u/u_c$, the regulated binding energy now becomes

$$E_{\text{F1}} = \frac{u_c}{2\pi}\left\{\int_1^{\infty} dy \left[\sqrt{\frac{y^3-x^3}{(y^3-x^3)-(1-x^3)(1-B^2)}} - 1\right] - (1-x)\right\}. \tag{3.27}$$

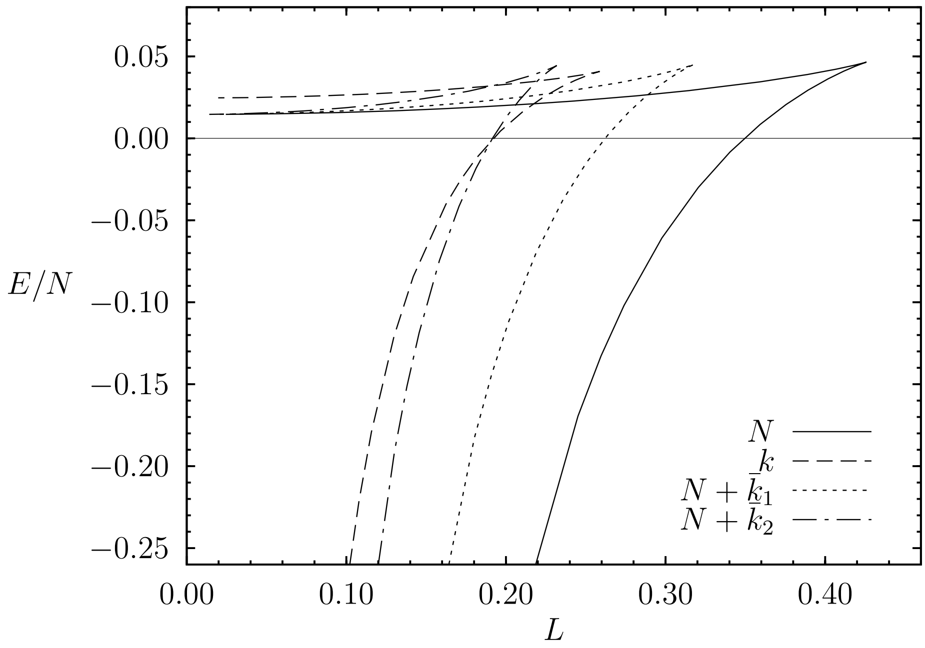


Figure 3.3: Comparison of the potential per $N_c$ between $N_c$-baryon, $k$-baryon, and $(N_c+\bar{k})$-baryon for $k/N_c = 0.8, \bar{k}_1/N_c = 2/3, \bar{k}_2/N_c = 2$ at temperature $T = 0.25$ (or $T = 10\,T_c$, see (3.7)).

Hence, we obtain the total energy of the configurations as

$$E = \frac{Nu_T}{2\pi}\left(\frac{\sqrt{1-x^3}}{3x} + \left(\frac{k_{\rm h}}{N}\right)\frac{\mathcal{E}}{x} + \left(\frac{k_{\rm r}}{N}\right)\frac{1-x}{x}\right) \quad (3.28)$$

$$\sim \frac{N^2}{L^2} \quad (3.29)$$

where $\mathcal{E}$ represents the terms within the brace of (3.27).

To obtain the relations between the total energy of the configurations $E(x)$ and $L(x)$, we eliminate the parameter $x = u_T/u_c$. By numerical calculations, the results are shown in Fig. 3.3 and Fig. 3.4. The binding energy of $N_c$-baryon is the deepest, suggesting that it is the most tightly bound state. For $(N_c+\bar{k})$-baryon, increasing $\bar{k}$ makes the binding energy smaller and the bound state is less tightly bound. The case of $j$-mesonance is quite similar. Generically, a $j$-mesonance has shallower binding potential than the total energy of $j$ mesons. However, as $j$ grows, the difference gets smaller and smaller.

The screening radius or screening length of exotic multi-quark states is defined to be the value of radius $L^*$ at which the binding energy becomes zero from negative values at smaller distances. This screening radius is therefore one-half of the usual

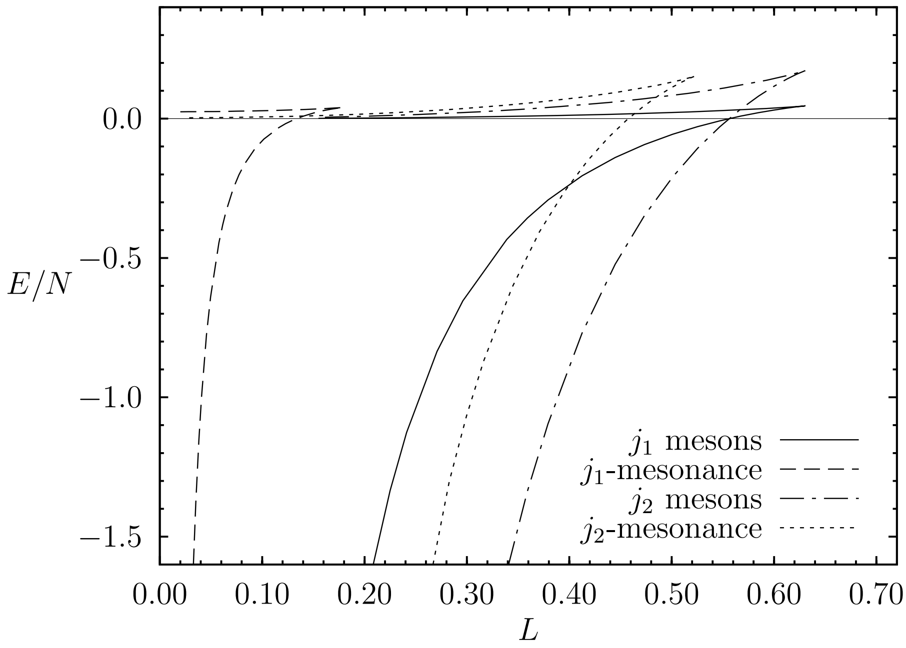


Figure 3.4: Comparison of the potential per $N_c$ between $j$-mesonance and $j$ mesons for $j_1/N_c = 0.8, j_2/N_c = 3$ at temperature $T = 0.25$ (or $T = 10\,T_c$).

definition of screening length in the discussion of mesonic state where it is defined as the zero-potential distance between quark and antiquark.

Numerical results suggest that the screening length of baryons and mesonance decrease as the temperature increases, i.e. $L^* \sim 1/T$ for a fixed value of $k, \bar{k}, j$ as is shown in Fig. 3.5-3.7. This is the generic form for the screening length in both the AdS-Schwarzschild and Sakai-Sugimoto models because it is the quantity which does not depend on the 't Hooft coupling at the leading order [95]. It is also an increasing function of $k$ and $j$. Interestingly, $(N_c + \bar{k})$-baryon has the opposite tendency with the screening length decreases as $\bar{k}$ grows. On the other hand, the screening length of $j$-mesonance has a saturation value $L^*_{j-mesonance} \to L^*_{meson}$ as $j \to \infty$.

## 3.5 Dependence on the free quark mass

In this section, we will study dependence of the binding potential on the position of the probe branes. This is useful when position of the probe branes are at finite distance from the black hole horizon and the corresponding quarks have finite mass. For example, the probe branes are D8 and $\overline{\text{D8}}$ flavour branes in the Sakai-Sugimoto model.

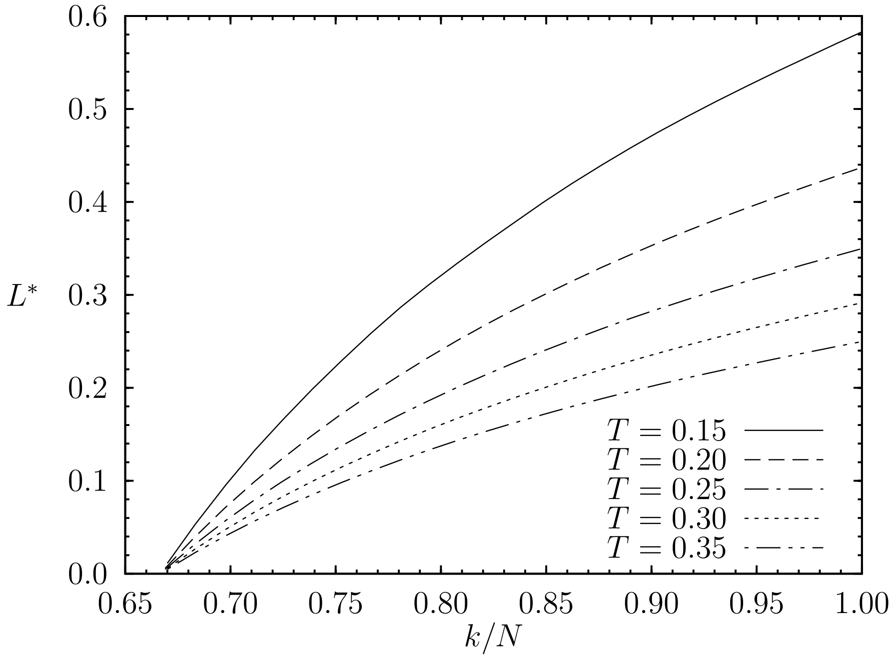


Figure 3.5: Screening length with respect to $k$ for the temperatures in $0.15 - 0.35$ range (or $6\,T_c - 14\,T_c$).

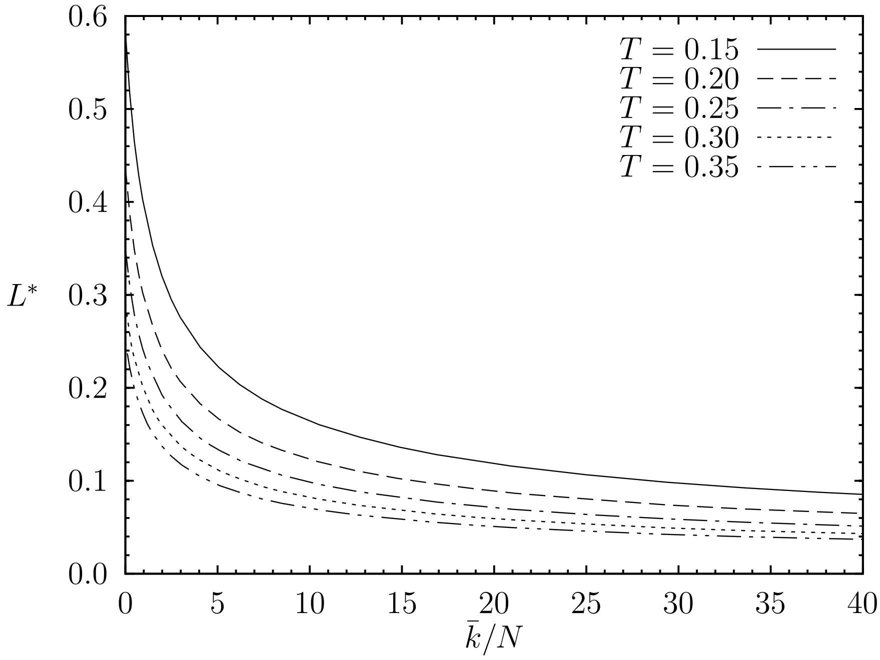


Figure 3.6: Screening length with respect to $\bar{k}$ for the temperatures in $0.15 - 0.35$ range (or $6\,T_c - 14\,T_c$).

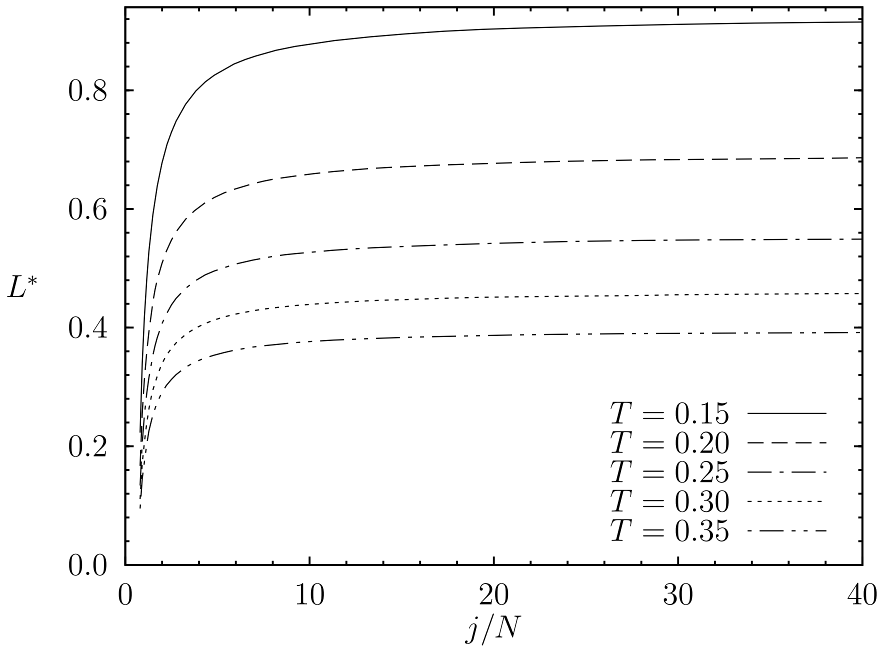


Figure 3.7: Screening length with respect to $j$ for the temperatures in $0.15 - 0.35$ range (or $6\,T_c - 14\,T_c$).

The calculation of binding energy as a function of the radius $L$ of the multi-quark states in the previous sections can be generalized to the case where the background metric is generated by a stack of D$p$-branes as the following. Start with the energy of a hanging fundamental string with $n = 7 - p$,

$$E_{F1} = \frac{u_c}{2\pi}\left\{ \int_1^\infty dy \left[ \sqrt{\frac{y^n - x^n}{(y^n - x^n) - (1 - x^n)(1 - A(n)^2)}} - 1 \right] - (1 - x) \right\} \tag{3.30}$$

and the radius,

$$L = \frac{R^{n/2}}{u_c{}^{(n-2)/2}} \int_1^\infty dy \sqrt{\frac{(1 - x^n)(1 - A(n)^2)}{(y^n - x^n)(y^n - x^n - (1 - x^n)(1 - A(n)^2))}}. \tag{3.31}$$

The total regulated binding energy of the configuration then becomes

$$E_{tot} = \frac{N_c u_h}{2\pi}\left\{ \frac{\sqrt{1 - x^n}}{nx} + \left(\frac{k_{\rm h}}{N_c}\right)\frac{\mathcal{E}}{x} + \left(\frac{k_{\rm r}}{N_c}\right)\frac{1 - x}{x} \right\} \tag{3.32}$$

where

$$\mathcal{E} = \int_1^\infty dy \left[ \sqrt{\frac{y^n - x^n}{(y^n - x^n) - (1 - x^n)(1 - A(n)^2)}} - 1 \right] - (1 - x), \tag{3.33}$$

and

$$A(n) = \frac{u_c'}{\sqrt{{u_c'}^2 + f(u_c)(\frac{u_c}{R_{\mathrm{D}p}})^n}} = \frac{N_c}{nk_{\mathrm{h}}}\left(\frac{1+\frac{n-2}{2}x^n}{\sqrt{1-x^n}}\right) + \frac{k_{\mathrm{r}}}{k_{\mathrm{h}}}. \quad (3.34)$$

The parameter $x$ is again given by

$$x = \frac{u_T(n)}{u_c}, \quad u_T(n=3,4) = \frac{16}{9}\pi^2 T^2, \pi T. \quad (3.35)$$

Note that the case $n = 3$ and $n = 4$ corresponds to the case of Sakai-Sugimoto and AdS-Schwarzschild gravity dual model respectively. As mentioned before, we have set the curvature radius to be unity for both the Sakai-Sugimoto model (i.e $R_{D4} = 1$) and the AdS-Schwarzschild model (i.e. $R_{D3} = 1$).

Introduction of quark masses into the configuration can be done by terminating hanging strings at certain radial distance $u_{max} < \infty$. In other words, the total binding energy of the bound state of finite mass $E_{F1}$(finite mass) can be obtained by taking the integration of hang strings, see (3.30), from $u_c$ to $u_{max}$, rather than from $u_c$ to $\infty$ as before. Recall that the universal behaviour of heavy-quark potential comes from the limit $u_{max} \to \infty$, i.e. $E_{\mathrm{F1}}(u_{max} \to \infty)$. This is the binding potential of a bound state whose its constituents have infinite mass. Here, we are interested in the mass dependent part of the binding potential which is the difference of $E_{\mathrm{F1}}$(finite mass) from $E_{\mathrm{F1}}(u_{max} \to \infty)$. Because of this, we split the total binding potential of the string, $E_{\mathrm{F1}}$(finite mass), into two parts. The first part is the binding potential in the $u_{max} \to \infty$ limit and the second part is the mass dependent potential, i.e.

$$E_{\mathrm{F1}}(\text{finite mass}) = E_{\mathrm{F1}}(u_{max} \to \infty) + E_{\mathrm{F1}}(u_{max}), \quad (3.36)$$

where $E_{\mathrm{F1}}(u_{max})$ is the mass dependent part of the binding potential. Note that the relation between free quark mass and $u_{max}$ is $m = u_{max}/2\pi$.

The mass dependent part of the binding potential can be expressed as

$$\begin{aligned} E_{\mathrm{F1}}(u_{max}) &= -\frac{u_c}{2\pi}\int_{u_{max}/u_c}^{\infty} dy \\ &\quad \left[\sqrt{\frac{y^n - x^n}{(y^n - x^n) - (1-x^n)(1-A(n)^2)}} - 1\right] \quad (3.37) \\ &= -\frac{u_{max}(1-A(n)^2)}{4\pi(n-1)}\left(\frac{u_c^n - u_T^n}{u_{max}^n}\right) + O(u_{max}^{1-2n}). \quad (3.38) \end{aligned}$$

Eliminate $u_c$ by using

$$L = \frac{R^{n/2}}{{u_c}^{(n-2)/2}}\int_1^{u_{max}/u_c} dy \sqrt{\frac{(1-x^n)(1-A(n)^2)}{(y^n-x^n)(y^n-x^n-(1-x^n)(1-A(n)^2))}}. \quad (3.39)$$

The result involves complicated functions of $A$ which can be cast in the following form,

$$E_{\rm F1}(u_{max}) \;\sim\; -u_{max}^{1-n}\left(R^{n^2/(n-2)}f_1(A)+u_T^n f_2(A)\right), \tag{3.40}$$

where $f_{1,2}(A)$ are some functions of $A$.

Interestingly, the mass dependence of multiquark potentials has the form similar to the mass dependence of mesonic state $\sim m^{1-n}$ in [94]. This is natural due to the fact that most of the mass of constituent quarks come from the tail part of strings which extend to the large-$u$ region. The mass dependence of the binding potential at the leading order is therefore determined only by the contribution of the hanging strings from the large-$u$ region. As long as the background spacetime of the gravity dual is asymptotically similar to the background considered here in the large-$u$ limit, we would expect the same mass dependence as the form we have obtained in this section.

# Chapter IV

## MULTIQUARK MATTER AND ITS THERMODYNAMICAL PROPERTIES

In this Chapter, we explore properties of the multiquark matter phase, especially its existence in phase diagram. After the Introduction in section 4.1, we describe the holographic setup of (exotic) nuclear matter, and determine the embedding of the flavour branes satisfying the force-balance condition at the cusp. Then, we comment about baryon number density and baryon chemical potential. As mentioned above, the former relates to the instanton number in a subspace of the D8-$\overline{\text{D8}}$-branes world-volume. On the other hand, the latter needs to be introduced once using the grand canonical ensemble. These are in the section 4.2. Then, in section 4.3, we investigate the possibility that the (exotic) nuclear matter can be thermodynamically preferred than others. The comparison between colour non-singlet multiquark matter and normal baryon matter is also done. As a result, we obtain phase diagram in the same way as [96] except that the colour non-singlet multiquark matter is taken into account here. The results and discussions of these issues are brought from our works in [41] and [97].

We end this Chapter with delving into the thermodynamic relations in the multiquark matter phase, in section 4.4. We work out for this purpose with both analytic and numerical calculations. Note that these are based on our work in [98].

## 4.1 Introduction

### 4.1.1 5-dimensional YM-CS theory from D8-branes action

The construction in supergravity picture of the Sakai-Sugimoto model is composed of $N_f$ flavour D8-branes and $N_f$ $\overline{\text{D8}}$-branes in the background of a stack of $N_c$ D4-branes with one spatial direction, $x_4$, compactifying on $S^1$ of radius $R = M_{KK}^{-1}$. Note that

$N_f \ll N_c$ such that there is no back reaction due to the flavour D8- and $\overline{\text{D8}}$-branes on the $N_c$ D4-branes induced background. As discussed in Chapter II, the holographic dual of this D4-D8 model is the four-dimensional U($N_c$) QCD with $N_f$ massless quarks at low energies, $E \lesssim M_{KK}$, hence in a strong coupling regime.

Interestingly, the relevant phase transitions inherent in this holographic model of QCD includes the confinement/deconfinement phase transition and the chiral symmetry breaking/restoration transition. The latter can be realized as the transition between the connected configuration of $N_f$ D8- and $\overline{\text{D8}}$-branes and the parallel configuration of $N_f$ D8- and $\overline{\text{D8}}$-branes at a certain temperature greater than that of the former[1] and at a certain baryon chemical potential[2]. This results from that the gauge symmetry on the world-volume of parallel D8-$\overline{\text{D8}}$ pairs, $\text{U}(N_f)_{\text{D8}} \times \text{U}(N_f)_{\overline{\text{D8}}}$, can be interpreted as the global symmetry $\text{U}(N_f)_L \times \text{U}(N_f)_R$ under which massless quarks of each chirality (either left or right) transform separately. Remarkably, left (right) quark fields correspond to the modes of open strings connecting D4-branes and D8 ($\overline{\text{D8}}$)-branes, namely 4-8 (4-$\bar{8}$) strings and 8-4 ($\bar{8}$-4) strings. This chiral symmetry turns out to be broken once a $\text{U}(N_f)_L \times \text{U}(N_f)_R$ chiral symmetry is spontaneously broken to a diagonal $\text{U}(N_f)_{\text{V}}$ through the transition from a parallel configuration to a connected one of D8- and $\overline{\text{D8}}$-branes. Consequently, the open string modes representing quark fields transform under $\text{U}(N_f)_{\text{V}}$ gauge group in the world-volume of connected D8-$\overline{\text{D8}}$-branes.

The 8-8 open string modes in D4-D8 model can be interpreted as the mesons. While a heavy meson has, as introduced in last Chapter, the construction of an open string hanging from the connected D8-$\overline{\text{D8}}$-branes, a light meson have the gravity dual as a quantized mode of vector fields or scalar fields (that is a fluctuation from the embedding of probe D8-branes) on the D8-branes world-volume[3]. The tower of these open string modes provides us with the spectrum of mesons. This is in a similar manner as the glueball spectrum represented by the tower of quantized modes of closed string (or graviton) in the bulk [103]. There are an infinite number of mesons

[1]Note that the temperature of the chiral symmetry breaking/restoration phase transition is greater than the deconfinement temperature as long as $L \gtrsim 0.97 \times R$, where $L$ is the asymptotic separation between D8-branes and $\overline{\text{D8}}$-branes and $R$ the radius of the compactified-$x_4$ circle at the spacetime boundary [70].

[2]In this thesis, we work in the grand canonical ensemble so that the thermodynamic variable involving baryonic degrees of freedom is the baryon chemical potential, rather than the baryon number density.

[3]A heavy-light meson can also be realized. But we need two branes at which two ends of the string locate are in different values in the holographic radial direction [99, 100, 101, 102].

lying in the spectrum, which includes, for example, pseudo-scalar meson (pion or $\pi$), vector meson ($\rho$ meson), axial vector meson ($a_1$ meson), etc. These mesons can be written in the chiral Lagragian by considering the effective theory on the world-volume of D8(and $\overline{\text{D8}}$)-branes. Noticeably, a pion field, represented by a pseudo-scalar mode of 8-8 strings, appears to be the Numbu-Goldstone mode when the chiral symmetry is broken, indicating that the breaking of chiral symmetry is spontaneous [10].

A theory of mesons can be obtained from the low energy effective theory of the open strings on the D8-branes which is the holographic description of a nine-dimensional U($N_f$) gauge theory. The fields (or fluctuations) on the world-volume of D8-branes are supposed to be small such that we obtain the Yang-Mills action by expanding the DBI action of the D8-branes up to the second-order of the fields. Higher-order terms of this expansion include couplings between mesons. Since the nine-dimensional D8-branes world-volume includes $S^4$ as its subspace, there is an SO(5) isometry corresponding to the rotations on this subspace. The field contents in QCD, gluons and quarks, should be invariant under SO(5) transformation, therefore our considerations are restricted to the SO(5) invariant sector. The U($N_f$) gauge field $\mathcal{A}$ on the D8-branes world-volume has thus components in $0, 1, 2, 3, z$ directions, where $z$ is the coordinate in the direction along which the connected D8-$\overline{\text{D8}}$-branes lie. Remark that the flavour D8-$\overline{\text{D8}}$-branes end at the boundary with $z \to \pm\infty$. Namely, $\mathcal{A} = \mathcal{A}_\alpha dx^\alpha = \mathcal{A}_\mu dx^\mu + \mathcal{A}_z dz$, ($\alpha = 0, 1, 2, 3, z$), and its field strength is $\mathcal{F} = \frac{1}{2}\mathcal{F}_{\alpha\beta} dx^\alpha \wedge dx^\beta = d\mathcal{A} + i\mathcal{A} \wedge \mathcal{A}$. Note that the U($N_f$) gauge field $\mathcal{A}$ can be decomposed into SU($N_f$) gauge field $A$ and U(1) vector $\hat{A}$ as

$$\mathcal{A} = A + \frac{1}{\sqrt{2N_f}}\hat{A} = A^a T^a + \frac{1}{\sqrt{2N_f}}\hat{A}, \tag{4.1}$$

where $T^a (a = 1, 2, \ldots, N_f^2 - 1)$ are the generators of SU($N_f$) with

$$\mathrm{tr}(T^a T^b) = \frac{1}{2}\delta^{ab}. \tag{4.2}$$

Considering only the SO(5) invariant sector, the D8-branes action placed in the D4-branes induced background can be integrated out of the coordinates of $S^4$. The effective theory thus becomes 5-dimensional. Namely, it is a five-dimensional U($N_f$)

Yang-Mills theory (YM) with Chern-Simons (CS) coupling as described in[4] [10]

$$S_{\rm 5d} \simeq S_{\rm YM} + S_{\rm CS}, \tag{4.3}$$

$$S_{\rm YM} = -\kappa \int d^4xdz\ tr\left[\frac{1}{2}h(z)\mathcal{F}_{\mu\nu}^2 + k(z)\mathcal{F}_{\mu z}^2\right], \tag{4.4}$$

$$S_{\rm CS} = \frac{N_c}{24\pi^2}\int_{M^4\times\mathbb{R}} \omega_5(\mathcal{A}). \tag{4.5}$$

Note that the warp factors $h(z)$ and $k(z)$ in the Yang-Mills action are given by $h(z) = (1+z^2)^{-1/3}$ and $k(z) = 1 + z^2$. The constant $\kappa = a\lambda N_c$, where $a \equiv 1/216\pi^3$, and the CS 5-form $\omega_5(\mathcal{A})$ is defined by

$$\omega_5(\mathcal{A}) = {\rm tr}\left(\mathcal{A}\mathcal{F}^2 - \frac{i}{2}\mathcal{A}^3\mathcal{F} - \frac{1}{10}\mathcal{A}^5\right). \tag{4.6}$$

This five-dimensional action is supposed to describe an effective theory of mesons as mentioned above. An infinite number of vector mesons and axial-vector mesons as well as the massless pion made of the Kaluza-Klein (KK) modes around a circle in the direction $x^4$. The mass scale of the model is given by the Kaluza-Klein mass parameter $M_{KK}$. It is the only dimensionful parameter of the model. However, it is not explicitly shown in the above action because $M_{KK}$ is set to be one for convenience[5].

### 4.1.2 Baryon in different aspects

In the same way as the pion effective theory, the pseudo-scalar mode of gauge field on the D8-branes world-volume, representing pion field $\Pi(x^\mu)$, can be written in the form of $U(N_f)$-valued field

$$U(x^\mu) \equiv e^{2i\Pi(x^\mu)/f_\pi}. \tag{4.7}$$

We can choose an appropriate gauge choice, namely $\mathcal{A}_z = 0$ gauge, such that the gauge field in the component $\mu = 0,1,2,3$ takes the form at the boundary as $\mathcal{A}_\mu(x^\mu, z\to\infty) = iU^{-1}\partial_\mu U$ and $\mathcal{A}_\mu(x^\mu, z\to-\infty) = 0$. As shown in [10], this leads to the Yang-Mills action in the form of the Skyrme model [106, 107]

$$S_{\rm YM} = \int d^4x\ \left(\frac{f_\pi^2}{4}{\rm tr}(U^{-1}\partial_\mu U)^2 + \frac{1}{32e_S^2}{\rm tr}\left[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U\right]^2 + \cdots\right), \tag{4.8}$$

[4] Here, we follow the notation in [104]. It is also helpful to see Chapter 15 of [105] for a review.

[5] In [10, 11], the parameters $M_{KK}$ and $\kappa$ are chosen as $M_{KK} = 949$ MeV and $\kappa = 0.00745$, by fitting the model with the experimental values of the $\rho$ meson mass and the pion decay constant $f_\pi \simeq 92.4$ MeV.

where '$\cdots$' are terms including the vector mesons and axial-vector mesons. Here, the pion decay constant $f_\pi$ and a dimensionless parameter $e_S$ takes the form

$$\begin{aligned} f_\pi^2 &= \frac{4}{\pi}\kappa M_{KK}^2, \\ e_S^2 &= \kappa \int dz h(z)(1-\psi_0^2)^2 \simeq 2.51\kappa. \end{aligned}$$

Remarkably, we obtain the action (4.8) describing the Skyrme model from the effective theory on the the D8-branes world-volume. The Skyrme model is the model in the effective theory in which pions act as the mean field [106, 107]. The pion field configuration described by the Skyrme model has the solitonic behaviour, and thus a Skyrmion can be interpreted as a baryon. An interesting point is that the Skyrmion emerge as a fermion in the background of bosonic fields, i.e. pseudo-scalar pions.

Atiyah and Manton [108] have found that computing the holonomy of Yang-Mills instantons gives good approximation to static Skyrmion solutions of the Skyrme model. This indicates that Skyrmions have something in common with Yang-Mills instantons. Surprisingly, this is also the case in the holographic description. In addition to the Skyrmion emerged from the consideration of the low-energy effective action of the D8-branes, baryons can also be considered as instanton in the Sakai-Sugimoto model. As an instanton, the holographic baryon can also be given by a D4-brane, wrapped around the $S^4$, located within D8 branes. This relates to the baryon vertex in the case of no flavour branes shown in Chaper III. In the Sakai-Sugimoto model with the presence of flavour branes, the baryon is realized by the probe D4-brane wrapped around the $S^4$ attached with $N_c$ strings extending to the flavour D8- and $\overline{\text{D8}}$-branes. This D4-brane turns out to be embedded into D8-branes since the equation of motion determine that the hanging $N_c$ strings tend to have zero length. Consequently, the D4-brane is embedded within the connected D8-$\overline{\text{D8}}$-branes. Since D$p$-brane embedded within D$(p+4)$-branes are equivalent to the gauge configurations in the D$(p+4)$-brane world-volume gauge theory with non-trivial instanton number in the 4-dimensional space transverse to the D$p$-brane [109]. Therefore, a wrapped D4-brane within D8-branes act as an instanton in five-dimensional Yang-MIlls theory which is localized in spatial four dimensions parametrized by $(x^1, x^2, x^3, z)$.

These instantons localize in spatial four dimensions and has instanton number equal to the number of wrapped D4-branes, that is conserved in time direction. Therefore, it behaves as point-like particles which can be interpreted as baryons.

Baryon number charge in Skyrme model can be shown to be equivalent with instanton number from wrapped D4-brane embedded within the flavour D8-branes.

Let's consider a static configuration of the gauge field on the flavour D8-branes. The number $n$ of the wrapped D4-branes is related to the instanton number on $B \simeq \mathbb{R}^4$, parameterized by $(x^1, x^2, x^3, z)$, as follows

$$n = \frac{1}{8\pi^2} \int_B \text{tr}(\mathcal{F} \wedge \mathcal{F}). \tag{4.9}$$

Note that this expression is present within the Chern-Simons term of D8-brane action.

Let us consider in the aspect of a Skyrmion. The pion field $U(x^\mu)$ determines a map from the $\partial\mathbb{R}^4 \simeq \mathbf{S}^3_\infty$ into $\text{U}(N_f)$, classified by the winding number in the third homotopy group $\pi_3(U(N_f)) \simeq \mathbb{Z}$. This integral number, which is in $\mathbb{Z}$, is interpreted as the baryon number in the Skyrme model. By using $\text{tr}(\mathcal{F} \wedge \mathcal{F}) = d\omega_3(\mathcal{A})$, where $\omega_3(\mathcal{A})$ is the Chern-Simons 3-form, we write the above expression (4.9) of the instanton number as

$$\frac{1}{8\pi^2} \int_B \text{tr}\mathcal{F}^2 = \frac{1}{8\pi^2} \int_{\partial B \simeq \mathbf{S}^3} \omega_3(\mathcal{A})\Big|_{z=\infty} = -\frac{1}{24\pi^2} \int_{\mathbf{S}^3} \text{tr}(U^{-1}dU)^3 \tag{4.10}$$

Note that Stoke's theorem has been used for the first equality and we have used the gauge choice $\mathcal{A}_z$, as mentioned above, such that $\mathcal{A}_\mu(x^\mu, z \to \infty) = iU^{-1}\partial_\mu U$ while $\mathcal{A}_\mu(x^\mu, z \to -\infty) = 0$ [10]. Remarkably, the last expression coincide with the baryon number charge in the Skyrme model. Intriguingly, two seemingly different pictures of baryons, i.e. (1) the Skyrmion in the pion effective theory and (2) the bound state of quarks corresponding to the wrapped D4-brane attached with $N_c$ strings, are connected in the holographic description.

### 4.1.3 Phase transitions in the deconfining background

In the non-antipodal Sakai-Sugimoto model, the holographic plasma can have two distinctive phase transitions; a deconfinement and the chiral symmetry restoration [70]. The deconfinement could occur at lower temperature than the chiral symmetry restoration. For the temperature in-between the two transitions, quarks and gluons are deconfined from the confining flux tube but still interact strongly among each other through the remaining screened Coulomb-type $SU(N_c)$ potential. Therefore it is possible to have the multiquark phase in the temperature range between that of the deconfinement and the chiral phase transition. This is consistent, at least in a qualitative way, with the studies of the *multibody* bound states in the sQCD in the framework of the real QCD [34].

To actually understand the physics of deconfined QGP, it is thus crucial to investigate the thermodynamical properties of the holographic multiquark phase. In order to extract the thermodynamic potential from the gravity dual model, the path integral approach in quantum gravity [110] has been used. In this technique, the time direction is circled with period $\beta = 1/T$ in the same manner as the thermal circle in the finite temperature quantum field theory. As discussed in [111] based on the early works [112, 113], the grand canonical potential, or the Gibbs free energy, $\Omega(T, \mu)$ has the leading contribution from the classical Euclidean action of the bulk theory in the grand canonical ensemble, i.e. $\Omega(T, \mu) \sim S^{\text{on-shell}}_{\text{bulk}}$. Similarly, the Helmholtz free energy $F(T, n_b)$ has the leading contribution from the Legendre transform with respect to the baryonic charge of the classical Euclidean action, i.e. $F(T, n_b) \sim \tilde{S}^{\text{on-shell}}_{\text{bulk}}$ in the canonical ensemble. If we are interested in the situation of non-fixed baryon number density but fixed chemical potential, the relevant thermodynamic potential is the grand canonical potential. Holographic phase transition at finite chemical potential is firstly studied in [114].

The deconfinement phase transition can be realized as the Hawking-Page transition due to the competition between the action of the background geometry corresponding to the confined phase and the action of the background corresponding to the deconfined phase [68]. Intriguingly, whereas the deconfining spacetime geometry action (scales as $N_c^2$) dominates the action of the fundamental matter sector (scales as $N_c N_f$), the dominating part can be ignored in the consideration of the holographic phase transition in the deconfined phase. Above the deconfinement, the multiquarks phase competes with the vacuum phase and the chiral-symmetric quark-gluon plasma.

It is interesting to study colour non-singlet multiquark bound states which are possible to exist in a deconfined phase as discussed in Chapter III. In the Sakai-Sugimoto model, the configuration representing a multiquark should be the modification of holographic baryon. One possibility corresponding to the multiquark bound state in the case of no flavour branes, discussed in last Chapter, is that there are a number of radial strings attached to a D4-brane within D8-$\overline{\text{D8}}$-branes and extending to the horizon of the spacetime.

Note that the radial strings can have orientation either going down to the horizon or going up from the horizon. The configuration with the going-down radial strings correspond to $k$-baryon, discussed in last Chapter, since the end of each of these strings yields a unit of negative string charge resulting in the net U(1) charge of the configuration on the flavour D8-branes world-volume less than $N_c$. Obviously, this corresponds to a colour non-singlet bound state with the number of quarks less than $N_c$. On the other hand, the configuration of the going-up radial strings corresponds to

$(N_c+k)$-baryon, i.e. the multiquark bound state with the number of quarks more than that of a normal colour-singlet baryon by the number $k$. Remarkably, this is not the same, though similar, as $(N_c+\bar{k})$-baryon, shown in last Chapter. Unfortunately, the configuration corresponding to the $j$-mesonance cannot exist, as we will see later that it is unstable under density fluctuations. Recall that a $j$-mesonance has the number of radial string as $N_c$. The analysis of the instability issue under density fluctuations indicate that we cannot have the number of radial strings more than about $0.3N_c$.

Since the colour non-singlet multiquark matter can exist only in the deconfined phase, its grand canonical potential in the Sakai-Sugimoto model is $\beta$ times the combination of the classical action of the deconfining spacetime geometry and the configuration of flavour sector, which includes $N_f$ D8-$\overline{\text{D8}}$-branes, the probe D4-brane vertex and the radial strings. Note that the part of hanging strings, extending from the baryon vertex to the flavour branes, is neglected and we assume that there is no distortion of the vertex due to the connecting strings (such distortion is discussed in [92, 93]). As a result, the baryon vertex is embedded into the flavour branes and becomes an instanton on them.

This configuration of the multiquark matter phase was proposed in our work [41] to address the issue of its existence in thermodynamical point of view. This can be done by exploring about which region in the phase diagram that the multiquark could exist. Since the colour non-singlet multiquark bound states can exist only in the deconfining background, we will consider the phase transition line between different matter phases in the deconfined phase.

The Sakai-Sugimoto model has an interesting phase structure, especially in the deconfined phase. Finite baryon density in the Sakai-Sugimoto model has been studied in [115, 116] and extended to the full parameter space in [96]. There are several kind of matter in the deconfined phase considered in [96]. These matter phases can be listed in the following.

- **Vacuum phase** This phase is the phase of matter with zero baryon number density. Consequently, the vacuum matter is occupied with only mesons, which all of them have zero baryon number charge. The configuration of this matter is the connected $N_f$ D8-$\overline{\text{D8}}$-branes on which there is nothing as a source. The gauge fields on the world-volume of this configuration have the discrete modes corresponding to mesons.

- **Nuclear matter phase** Unlike the vacuum phase, this phase has finite value of baryon number density. This is due to the presence of colour singlet bound states, namely baryons. Certainly, the nuclear matter is also occupied with mesons. This phase can be represented by the configuration of connected D8-$\overline{\text{D8}}$-branes sourced by the D4-brane wrapped around the four-sphere, or the Chern-Simons term of the D8-branes action.

- **Quark matter phase** The phase of quark matter can be represented by strings stretched from the flavour D8-branes down to the horizon without the presence of the D4-brane, i.e. the Chern-Simons term is turned off, on the world-volume of the D8-branes.

- **Chiral-symmetric quark-gluon plasma phase** All of the above mentioned phases, i.e. vacuum, nuclear matter and quark matter, has the connected configuration of D8-branes and $\overline{\text{D8}}$-branes, indicating that they are the phases with broken chiral symmetry. It is worth to remark that the embedding of the flavour branes in these phases play a role of a "thin air" of flavour degrees of freedom in the projection on the coordinates in radial direction and $x^\mu$, $\mu = 0, 1, 2, 3$. This "thin air" is equipped with the fields transforming under the gauge group $U(N_f)$. As so-called in the area of gauge/string duality, this is the Minkowski embedding. The chiral-symmetric quark-gluon plasma ($\chi$S-QGP) is another kind from these. Instead of the Minkowski embedding, its configuration is composed of the parallel D8-branes and $\overline{\text{D8}}$-branes lying along the radial direction of spacetime into the horizon. This kind of embedding is so-called the black hole embedding. While the modes of gauge fields in the Minkowski embedding of flavour branes are in the discrete spectrum indicating the existence of mesons, the spectrum turns out to be continuous in the black hole embedding indicating that all mesons are completely melted [117]. As a consequence, the configuration of $\chi$S-QGP can has no any bound states, even mesons[6], and we can say that quarks and gluons are completely free in this phase. Moreover, this phase exist in the range of temperature that is high enough such that the chiral symmetry is restored.

These phases of matter of the Sakai-Sugimoto model are compared in the grand canonical ensemble. Thus, the baryon chemical potential $\mu$ has to be introduced in the holographic description. This will be discussed in next Section. The authors

[6]Currently, there is no consensus on whether some mesons can remain at the temperature above that of the chiral symmetry restoration in our real world QCD.

of [96] have shown that the phase diagram in the $(T-\mu)$-plane consists of three regions including vacuum phase, nuclear matter phase and $\chi$S-QGP with first-order and second-order phase transition lines between these phases.

A matter phase can be the ground state of QCD as long as its grand canonical potential is less than all others under consideration. The first-order phase transition can be found when there is a change of the phase of the least value of the grand canonical potential $\Omega$. By comparing this quantity, the first derivative of $\Omega$ with respect to $\mu$ is discontinuous at the transition point, hence a first-order phase transition. This kind of transition from one matter to another is in an abrupt way. Given that the baryon number density $n_4 = \frac{\partial\Omega}{\partial\mu}$, it thus changes discontinuously at the moment of a phase change of this type. At the transition, two phases cannot be in the same state of matter even though they are in equilibrium. This is also found in the transition between vacuum and $\chi$S-QGP phases, and between nuclear matter and $\chi$S-QGP phases [96]. On the other hand, the first-order derivative $\frac{\partial\Omega}{\partial\mu}$ is not discontinuous in a second-order phase transition, but it is so for the second-order derivative $\frac{\partial^2\Omega}{\partial\mu^2}$. The baryon number density $n_4$, relating to $\Omega$ as defined above, is an order parameter for the present investigation which is continuous across the transition point at a certain value of $\mu$. It has zero value in one phase for the state of disorder, and becomes nonzero representing the onset of order in another phase at a particular value of baryon chemical potential $\mu_{\text{onset}}$. This occurs in the holographic set-up of [96] for the transition from the vacuum phase to the nuclear matter phase. However, the vacuum-nuclear matter phase transition might be of first-order when one includes the interactions between instatons on the D8-branes world-volume, corresponding to baryons. In a second-order phase transition, the phase matter changes in a smooth way, in the same way as the phase transition between liquid and vapour, and two phases can have the same state of matter at the transition point as well as an equilibrium.

Additionally, quark matter has been found [96] to be unstable to baryon number density fluctuations. The stability issue in thermodynamics can be considered through entropy of the system. In equilibrium, the entropy tends to be maximum and cannot increase further. However, once there is a fluctuation perturbing the system in equilibrium, this can either increase or decrease the entropy of the system whereas the entropy of the system combined with that of the environment increases, or at least not change, following the second law of thermodynamics. Basically, a system can be said to be stable under any perturbation if that does not change state of the system away from (and not go back again to) the stable point where the entropy is already maximum and cannot increase further. In other words, the condition for stable equilibrium is that any change of the state of the system due to a fluctuation should lead to a decrease in entropy [118]. Consequently, one of the stability conditions is that the isothermal compressibility $\kappa_T > 0$ [119]. Since $\kappa_T$ is proportional to $\frac{\partial n_4}{\partial \mu}$, a thermodynamic system is stable under number density fluctuations as long as $\frac{\partial n_4}{\partial \mu}$ is more than zero. Nevertheless, the quark matter phase has

$$\frac{\partial n_4}{\partial \mu} < 0, \tag{4.11}$$

indicating that the quark matter cannot exist because it is unstable under baryon number density fluctuations $\delta n_4$.

Interestingly, the multiquark phase is found that it cannot have too many radial strings. If the number of radial strings is more than a certain value, about $0.3N_c$, the multiquark matter tends to be unstable under density fluctuations in some regions in phase diagram [41]. When the number of radial strings increase, the multiquark matter deviates more from the colour-singlet being, and it tends to become the matter which is close to being the quark matter. This implies that the multiquark configuration appear as the state of matter between the nuclear matter phase and the quark matter phase.

The multiquark matter can be thought of as a generalization of the nuclear phase; the nuclear matter is the multiquark matter with zero number of radial string. In this thesis, we use the term the "(exotic) nuclear phase" for the matter occupied with colour singlet baryons and, also, the matter occupied with the non-singlet bound states. However, we do not consider the matter occupied with the mixing between these nuclear states in this thesis. The configurations of flavour branes and strings of different phases which is compared in the phase diagram are shown in the Fig. 4.1 [41].

## 4.2 Multiquark matter phase

A natural question to ask is whether we have a phase where exotic multiquark states are preferred over normal nuclear matter, vacuum, and chiral-symmetric quark-gluon plasma phases. To calculate the phase diagram involving multiquark matter, it is necessary to consider the contribution from D8- and $\overline{\text{D8}}$-branes in the Sakai-Sugimoto model attached with radial strings and D4-branes. We will assume that the characteristic distance between D8 and $\overline{\text{D8}}$ in $x^4$ direction at the boundary of spacetime,

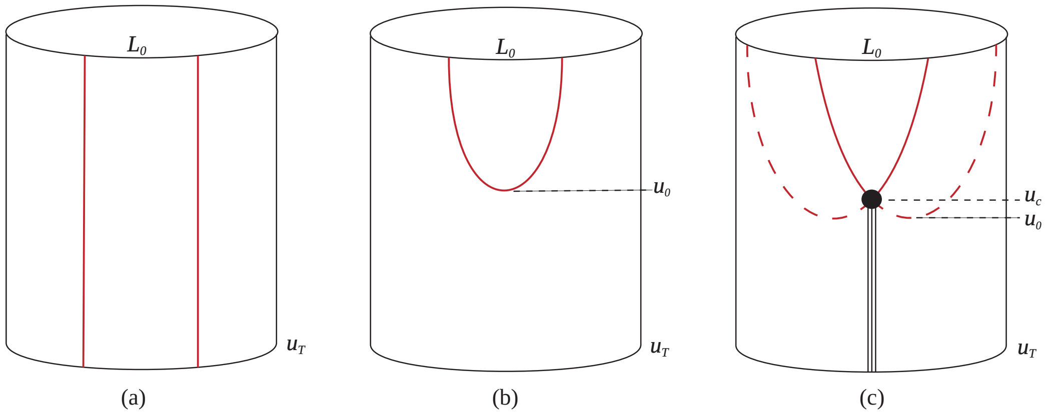


Figure 4.1: Configurations of $\chi$S-QGP (a), vacuum (b), and exotic nuclear phase (c) in $x^4 - u$ projection. The red lines represent the embedding of the flavour D8- and $\overline{\text{D8}}$-branes. The black straight lines is the radial strings attached to the D4-brane wrapped around $S^4$ (shown as a black dot) embedded within the connected D8-$\overline{\text{D8}}$-branes at the value of radius coordinate $u_c$ extending to the horizon of spacetime at $u_T$.

$u \to \infty$, is $L_0$. The relevant scales of the model therefore depend on the horizon $u_T$ as well as $L_0$.

When there is no radial string pulling the vertex down towards the horizon, it was demonstrated in [92, 93] by numerical method that the vertex is pulled all the way up to the position of the flavour branes if the temperature is not very high. Addition of radial strings to the vertex would pull the vertex and the flavour branes towards the horizon. As temperature rises, the radial strings pull the vertex down with stronger force since they are closer to the horizon. It is possible that the vertex then starts to separate from the flavour branes and we might need to consider the configuration where the vertex and flavour branes are separated. However, we can see that the difference between the two configurations should be relatively small, namely, only the force conditions will be slightly different. As a result, we can approximate the situation by considering the configuration where the vertex is not separated from the flavour branes. It will be assumed that the D4-brane vertex is always in the flavour branes for the discussion in this section. Moreover, the vertex will be treated as a static configuration and any distortion caused by the strings attached to it will be ignored.

The calculations presented in this section are adapted from [96] except that we add radial strings hanging from the vertex down to the horizon for the consideration of (exotic) nuclear phase. We also use position of the D4, $u_c$, instead of $u_0$ (where

$x_4'(u_0) \to \infty$) in our calculation concerning the exotics. This approach allows us to deal with the contribution from radial strings more conveniently. As shown in Fig. 4.1, the vacuum phase with broken chiral symmetry corresponds to the configuration where D8 and $\overline{\text{D8}}$ are connected into a curve in the $x_4 - u$ projection. The chiral-symmetric phase of quark-gluon plasma ($\chi$S-QGP) corresponds to the configuration with the parallel D8 and $\overline{\text{D8}}$ stretching from the spacetime boundary down to the horizon. Finally, the nuclear (including exotic) phase corresponds to the configuration where the D4 vertex is located at the D8-$\overline{\text{D8}}$ curve, pulling it down towards the horizon by its "weight" in the background. Each vertex has radial strings attached to it, pulling it further towards the horizon. Remark that the chiral symmetry is broken in this phase.

### 4.2.1 The embedding of the flavour D8-branes

Under above assumptions, the contribution from the strings hanging down from the spacetime boundary to the vertex is negligible. The only contribution of strings is from the radial strings extending from the vertex to the horizon. The total action of the configuration is given by

$$S_{\text{total}} = S_{\text{D8}} + S_{\text{D4}} + \tilde{S}_{\text{F1}}, \tag{4.12}$$

where $S_{\text{D8}}$ is the Dirac-Born-Infeld (DBI) action of the $N_f$ connected D8-$\overline{\text{D8}}$-branes, $S_{\text{D4}}$ the action of the D4-brane wrapped around $S^4$, and $\tilde{S}_{\text{F1}}$ the action of radial strings.

Firstly, we introduce the action of sources on the flavour D8-branes, i.e. $S_{\text{D4}}$ and $\tilde{S}_{\text{F1}}$. Assume that there are the number of the D4-branes, localized around the connected point between D8-branes and $\overline{\text{D8}}$-branes, in a unit volume of subspace $(x^1, x^2, x^3)$ as $n_4$. In other words, $n_4$ represents the number density of D4-branes. As will be seen in next subsection, the parameter $n_4$ is the instanton number density, hence the baryon number density. Then, in the deconfining background

$$ds^2 = \left(\frac{u}{R_{D4}}\right)^{3/2} \left(f(u)dt^2 + \delta_{ij}dx^i dx^j + dx_4{}^2\right) + \left(\frac{R_{D4}}{u}\right)^{3/2} \left(u^2 d\Omega_4^2 + \frac{du^2}{f(u)}\right), \tag{4.13}$$

the DBI action of the D4-branes of the number density $n_4$ is given by [96]

$$S_{\text{D4}} = \frac{n_4 V_3 \mu_4}{R^3} \int d\Omega_4 d\tau \; e^{-\Phi} \sqrt{\det g_{MN}} \tag{4.14}$$

$$= \frac{1}{3} \mathcal{N} u_c \sqrt{f(u_c)} d. \tag{4.15}$$

where $d$ can be thought of as the rescaled baryon number density, relating to $n_4$ as will be seen in (4.53), even though it is actually the constant of motion which describe the electric displacement[7] in the direction $u$ in the D8-branes world-volume. In addition, the constant $\mathcal{N}$ is a constant which will be defined following (4.20). Now, we comes to the action of radial strings. Let us define the number of radial strings in unit of $N_c$ per unit volume of subspace $(x^1, x^2, x^3)$ as $n_s$. We can write the Numbu-Goto action of the radial strings of the fractional number density $n_s$ as

$$\tilde{S}_{\mathrm{F1}} = \mathcal{N} n_s (u_c - u_T) d. \tag{4.16}$$

The number of radial strings $n_s$ represents the number of strings extending down from D4-branes to the horizon in unit of $N_c$. For $k$, $(N+k)$-baryon and $j$-mesonance, the values of $n_s$ are $1 - k/N, k/N, 1$ respectively. For a given value of the asymptotic separation of the D8- and $\overline{\mathrm{D8}}$-branes $L_0$, increasing the number of strings $n_s$ results in D4-D8 configuration being pulled down more towards the horizon.

Generically, the DBI action of D8-branes with the presence of $\mathrm{U}(N_f)$ gauge fields $\mathcal{A}$ is given by

$$S_{\mathrm{D8}} = -\mu_8 \int d^9 X e^{-\Phi} \mathrm{tr} \sqrt{-\det(g_{MN} + 2\pi\alpha' \mathcal{F}_{MN})} \tag{4.17}$$

where the field strength

$$\mathcal{F} = d\mathcal{A} + i\mathcal{A} \wedge \mathcal{A}. \tag{4.18}$$

is of the gauge group $\mathrm{U}(N_f)$ which corresponds to the global flavour group $\mathrm{U}(N_f)$ [10], $\mu_8$ is the tension of D8-branes, and $\Phi$ is the dilaton field. Note that $g_{MN}$ denotes the induced metric of the D8-branes world-volume which is the pullback of the metric of the deconfining background, as shown in (4.13), and $\mathcal{F}_{MN}$ denotes the field strength tensor of the gauge group $\mathrm{U}(N_f)$ living in the $N_f$ flavour D8-branes.

The $\mathrm{U}(N_f)$ gauge field $\mathcal{A}$ can be decomposed into the $\mathrm{SU}(N_f)$ part $A$ and the diagonal $\mathrm{U}(1)_{\mathrm{V}}$ part $\hat{A}$ as shown in (4.1). For our setup, we turn on only the time component of the diagonal $\mathrm{U}(1)_{\mathrm{V}}$ part in order to introduce finite baryon number density, or equivalently finite baryon chemical potential, into the model. From these together with the deconfining spacetime metric (4.13), the D8-branes action (4.17) takes the form

$$S_{\mathrm{D8}} = \mathcal{N} \int du\, u^4 \sqrt{f(u)(x_4'(u))^2 + \frac{1 - (\hat{a}_0'(u))^2}{u^3}} \tag{4.19}$$

[7] We use the term 'electric' here and later on in the sense that it involves a U(1) gauge field living in the D-branes world-volume.

where the constant $\mathcal{N}$ scales linearly with $N_f$ as

$$\mathcal{N} = \frac{\mu_8 \tau N_f \Omega_4 V_3 R^5}{g_s}, \tag{4.20}$$

and the gauge field of the $U(1)$ diagonal subgroup $\hat{A}$ has been rescaled to $\hat{a}$ as follows,

$$\hat{a} \;=\; \frac{2\pi\alpha' \hat{A}}{R\sqrt{2N_f}}. \tag{4.21}$$

Since the D8-branes action of the form (4.19) does not depend on both $\hat{a}_0(u)$ and $x_4'(u)$ explicitly, we can obtain the constants of motion by varying the action with respect to either of them. Precisely, given that $\mathcal{L}_{\rm D8}$ is the Lagrangian density of the action, we obtain

$$\frac{\partial \mathcal{L}_{\rm D8}}{\partial \hat{a}_0'(u)} = \text{const.} \quad \text{and} \quad \frac{\partial \mathcal{L}_{\rm D8}}{\partial x_4'(u)} = \text{const.} \tag{4.22}$$

That is, they do not depend on the radial coordinate $u$. Varying the action with respect to $\hat{a}_0(u)$ therefore leads to a constant of motion

$$d \;=\; d(u) \tag{4.23}$$

$$\equiv \frac{u\hat{a}_0'(u)}{\sqrt{f(u)(x_4'(u))^2 + u^{-3}(1-(\hat{a}_0'(u))^2)}}, \tag{4.24}$$

where the function $d(u)$ has been defined as follows

$$d(u) \equiv -\frac{1}{\mathcal{N}}\frac{\partial \mathcal{L}_{\rm D8}}{\partial \hat{a}_0'(u)} \sim \frac{\delta S_{\rm D8}}{\delta \hat{F}_{0u}}, \tag{4.25}$$

where $\hat{F}$ is the field strength tensor of the diagonal U(1) part due to turning on $\hat{A}_0$. The quantity $d(u)$ is thus the (rescaled) electric displacement field along the holographic direction $u$. We will see in next subsection that the constant $d$ can be interpreted as the baryon number density sourced by D4-branes once we introduce the Chern-Simon action of the gauge field. In the confined phase, the only possible source for $d$ is D4-branes, each wrapped on $S^4$, in D8-branes would-volume. In the deconfined phase, either D4-branes or strings extending from D8-branes down to the horizon can serve as the source for $d$. Here, in the study of exotic multiquark matter, we consider the case where *both* D4-brane and strings are present as the sources. This possibility was not investigated in [96].

On the other hand, varying the D8-branes action with respect to $x_4'(u)$ leads to the constant of motion

$$C \;=\; C(u) \tag{4.26}$$

$$\equiv \frac{u^4 f(u) x_4'(u)}{\sqrt{f(u)(x_4'(u))^2 + \frac{1-(\hat{a}_0'(u))^2}{u^3}}}. \tag{4.27}$$

Rearranging (4.24), we can write

$$(\hat{a}_0'(u))^2 = \frac{d^2}{u^2}\left[\frac{f(u)(x_4'(u))^2 + \frac{1}{u^3}}{1+\frac{d^2}{u^5}}\right]. \tag{4.28}$$

Substituting this into (4.27), we obtain

$$C = \frac{u^4 f(u) x_4'(u)\sqrt{1+\frac{d^2}{u^5}}}{\sqrt{f(u)(x_4'(u))^2 + \frac{1}{u^3}}}, \tag{4.29}$$

such that the function $x_4(u)$ which describes the shape of embedding of D8-branes obeys the following equation

$$(x_4'(u))^2 = \frac{1}{u^3 f(u)}\left[\frac{f(u)(u^8 + u^3 d^2)}{C^2} - 1\right]^{-1}. \tag{4.30}$$

At large $u$, we can approximate that

$$x_4(u) \approx \frac{L_0}{2} - \frac{2}{9}\frac{C}{u^{9/2}}, \tag{4.31}$$

where $L_0$ is the separation between D8 and $\overline{\text{D8}}$ branes at $u \to \infty$ defined by

$$L_0 \equiv 2\int_{u_c}^{\infty} x_4'(u) du. \tag{4.32}$$

Note that we set $L_0 = 1$ to allow the possibility of the chiral symmetry restoration as separate phase transition from the deconfinement; these two kinds of phase transition are separated once $L_0 \lesssim 0.97 \times R$; where $R$ is the radius of the circle $x_4$ at $u \to \infty$ [70].

The parameter $C$ can be thought of as the curvature of the D8-$\overline{\text{D8}}$ branes around the cusp. It becomes zero when the flavour embedding is in the parallel configuration representing the chiral-symmetric QGP. According to [120], this means that it can be used as an order parameter of the nuclear matter/$\chi$S-QGP phase transition.

So far, we have left the constant $C$ to be an unknown. Recall from (4.29) that $C$ is constant for arbitrary $u$ in the range $u_c \leq u < \infty$. We can have the condition at the cusp $u_c$:

$$C = \frac{u_c^4 f(u_c) x_4'(u_c)\sqrt{1+\frac{d^2}{u_c^5}}}{\sqrt{f(u_c)(x_4'(u_c))^2 + \frac{1}{u_c^3}}}. \tag{4.33}$$

We need to determine $x_4'(u_c)$ in order to obtain $C$ written in term of $u_c$ explicitly. For this purpose, we consider the force condition at the cusp $u_c$, since $x_4'(u_c)$ is the quantity which relates to how much the cusp of the flavour branes at $u_c$ are pulled up by the flavour branes and pulled down by the D4-brane vertex and radial strings, whereas the net force at that point is zero.

By fixing values of $d, T, L_0$, we can obtain the force balance condition at the cusp $u_c$ by extermize the total action

$$\tilde{S}_{\text{total}} = \tilde{S}_{\text{D8}} + S_{\text{source}}(u_c, d, T) \tag{4.34}$$

with respect to $u_c$ [96], namely

$$\frac{\partial \tilde{S}_{\text{total}}}{\partial u_c} = \frac{\partial(\tilde{S}_{D8} + S_{D4} + \tilde{S}_{F1})}{\partial u_c} = 0, \tag{4.35}$$

where the source term in the multiquark configuration is the combination of the D4-brane vertex and radial strings, i.e. $S_{\text{source}} = S_{\text{D4}} + \tilde{S}_{\text{F1}}$. As shown in Appendix A and firstly presented by [96], the force balance condition balance condition at the cusp $u_c$ is

$$\tilde{\mathcal{L}}_{\text{D8}}(u_c) - x_4'(u_c) \frac{\delta \tilde{S}_{\text{D8}}}{\delta x_4'}\bigg|_{u_c} = \frac{\partial S_{\text{source}}}{\partial u_c}, \tag{4.36}$$

or

$$x_4'(u_c) = \left(\tilde{\mathcal{L}}_{\text{D8}}(u_c) - \frac{\partial S_{\text{source}}}{\partial u_c}\right) \Bigg/ \frac{\delta \tilde{S}_{\text{D8}}}{\delta x_4'}\bigg|_{u_c}, \tag{4.37}$$

We have to clarify the formula of the Legendre transformed action of D8-branes $\tilde{S}_{\text{D8}}$ for obtaining the explicit dependence of $x_4$ on $u_c$, whereas the action of the D4-brane wrapped around the four-sphere $S_{\text{D4}}$ is given by (4.15), and the action of the radial strings $\tilde{S}_{\text{F1}}$ by (4.16). The Legendre transform of the D8-branes action from the dependence on $\hat{a}_0'$ to the dependence on $d$ is given by

$$\tilde{S}_{\text{D8}} = S_{\text{D8}} + \mathcal{N} \int_{u_c}^{\infty} du\, d(u) \hat{a}_0'(u). \tag{4.38}$$

Using (4.28) and (4.19) into (4.38), we obtain

$$\begin{aligned} \tilde{S}_{\text{D8}} &= \mathcal{N} \int_{u_c}^{\infty} du\, u^4 \left[ \sqrt{\frac{f(u)(x_4'(u))^2 + \frac{1}{u^3}}{1 + \frac{d^2}{u^5}}} + \frac{d^2}{u^5} \sqrt{\frac{f(u)(x_4'(u))^2 + \frac{1}{u^3}}{1 + \frac{d^2}{u^5}}} \right] && (4.39) \\ &= \mathcal{N} \int_{u_c}^{\infty} du\, u^4 \sqrt{\left(f(u)(x_4'(u))^2 + \frac{1}{u^3}\right)\left(1 + \frac{d^2}{u^5}\right)}. && (4.40) \end{aligned}$$

Remarkably, it can be shown that $S_{\text{D8}}$ can be written in the form of the electric displacement $d$ as follows,

$$S_{\text{D8}} = \mathcal{N} \int_{u_c}^{\infty} du\, u^4 \sqrt{\frac{f(u)(x_4'(u))^2 + \frac{1}{u^3}}{1 + \frac{d^2}{u^5}}}, \tag{4.41}$$

whose the Lagrangian density is different from that of $\tilde{S}_{D8}$ by the factor $\left(1+\frac{d^2}{u^5}\right)^{-1}$. This form of action (4.41) will used to determine the grand canonical potential later.

Substituting (4.15), (4.16), and (4.40) into (4.37), we obtain

$$(x_4'(u_c))^2 = \frac{1}{f(u_c)u_c^3}\left[\frac{9}{d^2}\frac{f(u_c)(u_c^5+d^2)}{\left(1+\frac{1}{2}\left(\frac{u_T}{u_c}\right)^3+3n_s\sqrt{f(u_c)}\right)^2}-1\right], \tag{4.42}$$

and, substituting this into (4.37), the constant $C$ thus takes the form

$$C = {u_c}^3 f(u_c)\left[\left({u_c}^5+d^2\right)-\frac{d^2\eta_c^2(T,n_s)}{9f(u_c)}\right], \tag{4.43}$$

where

$$\eta_c(T,n_s) \equiv 1+\frac{1}{2}\left(\frac{u_T}{u_c}\right)^3+3n_s\sqrt{f(u_c)}. \tag{4.44}$$

Therefore, the embedding of the flavour branes in the (exotic) nuclear matter is described by (4.30) and (4.43), given that the separation between D8-branes and $\overline{\text{D8}}$-branes at the boundary ($u\to\infty$) $L_0$ is set to one.

### 4.2.2 A comment on baryon number density and baryon chemical potential

Before going further to determine the phase diagram, let us comment about the electric displacement $d$. It has been shown in [96] that $d$ is related to the baryon number density. The baryon number density corresponds to the number density of instantons, $n_4$, on the D8-branes. It also contributes to the Chern-Simons (CS) action of the flavour branes [10].

Beginning with the D8-brane CS term [90]

$$S_{D8}^{\text{CS}} = \frac{\mu_8}{3!}\int_{R^4\times R_+\times S^4} C_3\wedge \text{Tr}(2\pi\alpha'\mathcal{F})^3. \tag{4.45}$$

By following Appendix A of [10], it is convenient to rescale the Ramond-Ramond (RR) field such that

$$S_{D8}^{\text{CS}} = \frac{1}{48\pi^3}\int_{R^4\times R_+\times S^4} C_3\wedge \text{Tr}\mathcal{F}^3 \tag{4.46}$$

$$= \frac{1}{48\pi^3}\int_{R^4\times R_+\times S^4} F_4\,\omega_5(\mathcal{A}), \tag{4.47}$$

where the last expression is obtained through integration by parts. $F_4 = dC_3$ is the RR 4-form field strength and $\omega_5(\mathcal{A})$ is the CS 5-form:

$$\omega_5(\mathcal{A}) = \mathrm{Tr}\left(\mathcal{A}\mathcal{F}^2 - \frac{1}{2}\mathcal{A}^3\mathcal{F} + \frac{1}{10}\mathcal{A}^5\right), \tag{4.48}$$

satisfying $d\omega_5 = \mathrm{Tr}\mathcal{F}^3$. Using the fact that integrating the $F_4$ flux over the $S^4$ in the $N_c$ D4-branes background gives

$$\frac{1}{2\pi}\int_{S^4} F_4 = N_c, \tag{4.49}$$

and the relevant term is only the first term in the CS 5-form, Eqn. (4.48), once turning on only the time-component of the diagonal $\mathrm{U}(1)_V$ field, we obtain

$$S_{D8}^{\mathrm{CS}} = \frac{N_c}{24\pi^2}\int_{R^4\times R_+} \frac{1}{\sqrt{2N_f}}\hat{\mathcal{A}}_0 \wedge \mathrm{Tr}(\mathcal{F}\wedge\mathcal{F}). \tag{4.50}$$

Assuming a uniform distribution of D4-branes in $\mathbb{R}^3$ at $u = u_c$, we have [109]

$$\frac{1}{8\pi^2}\mathrm{Tr}(\mathcal{F}\wedge\mathcal{F}) = R^{-3} n_4 \delta(u-u_c) d^3\mathbf{x}\, du, \tag{4.51}$$

where $n_4$ is defined to be the (dimensionless) number density of the wrapped D4-branes, or equivalently the number density of instantons, at $u = u_c$. From the viewpoint that the low-energy effective theory on the D8-brane includes the Skyrme model [10], it is natural to interpret $n_4$ as the baryon number density.

Using (4.21), (4.50) and (4.51), we obtain

$$S_{D8}^{\mathrm{CS}} = \frac{n_4 N_c \beta V_3}{2\pi\alpha' R^2}\int_{u_c}^{\infty} du\, \hat{a}_0(u)\, \delta(u-u_c). \tag{4.52}$$

From both the DBI and CS parts of D8-branes action, the equation of motion with respect to the $U(1)$ gauge field gives [96]

$$n_4 = \frac{2\pi\alpha' R^2 \mathcal{N}}{\beta V_3 N_c} d. \tag{4.53}$$

Note that this reflects the one-dimensional electrostatic effect in which the electric point charges are put at $u_c$, generating constant electric field in the holographic direction.

Let's introduce the chemical potential. It is the non-normalizable modes of the $\mathrm{U(1)_V}$ gauge field in the bulk. It can be thought of as the conjugate of the source corresponding to the value of bulk field (in non-normalizable mode) at the boundary of the spacetime background.

Phase transition for a system where the number of particles varies is most conveniently described by the grand canonical ensemble. As discussed in Chapter II, the normalized grand canonical potential per unit volume of subspace $(x^1, x^2, x^3)$, or grand canonical potential density, of each phase can be defined using the on-shell action of the D8-branes as [112, 113]

$$\Omega(\mu) = \frac{1}{\mathcal{N}} S_{D8}[x_4(u), \hat{a}_0(u)]_{cl}. \tag{4.54}$$

Since the D8-brane action diverges from the limit $u \to \infty$ of the integration, the grand canonical potential density needs to be regulated by subtracting with the grand canonical potential density of the vacuum phase at the same temperature.

Apart from the grand canonical potential density, the chemical potential also needs to be holographically identified in the dual bulk theory. To this end, the time component of the $U(1)_V$ gauge field $\hat{A}_0$ is taken into account. From the field/operator matching scheme, a bulk field evaluated at $u \to \infty$, i.e. the boundary of the spacetime background, plays a role as the source of the dual operator in the generating function of correlation functions in quantum field theory. In other words, this non-normalizable mode of the bulk field is dual to the coefficient of the field operator. Since the chemical potential is the coefficient of the charge density operator term, it can be holographically identified as $\hat{A}_0(\infty)$. By rescaling for convenience, we can write the dimensionless chemical potential as

$$\mu = \hat{a}_0(\infty), \tag{4.55}$$

Similarly, the baryon number density in our normalization is given by

$$n_b = -\frac{\partial \Omega(T, \mu)}{\partial \mu} = d, \tag{4.56}$$

even though the true baryon number density is $n_4$ defined in (4.53). Consequently, $d$ can then be used to denote the baryon number density.

Since the free energy in the canonical ensemble is the combination of the on-shell Legendre-transformed D8-brane action and the source-term, it is convenient to obtain $\mu$ through

$$\mu = \frac{\partial \mathcal{F}_E(T, d)}{\partial d}, \tag{4.57}$$

where the free energy density is holographically defined as the Legendre-transformed D8-branes action plus the source terms [112, 113],

$$\mathcal{F}_E(T, d) = \frac{1}{\mathcal{N}} \left( \tilde{S}_{D8}[T, x_4(u), d(u)]_{\text{on-shell}} + S_{\text{source}}(T, d, u_c) \right). \tag{4.58}$$

Recall that the Legendre-transformed action $\tilde{S}_{D8}|^{\text{on-shell}}$ is in the form of (4.40). The chemical potential can then be written as

$$\mu = \frac{1}{\mathcal{N}}\Bigg\{\int_{u_c}^{\infty} du\left(\frac{\delta\tilde{S}_{D8}}{\delta d(u)}+\frac{\delta\tilde{S}_{D8}}{\delta x_4'}\frac{\partial x_4'}{\partial d}\right)\Bigg|_{T,L_0,u_c}^{\text{on-shell}} \tag{4.59}$$

$$+\frac{\partial u_c}{\partial d}\Bigg|_{T,L_0}\left(\frac{\partial\tilde{S}_{D8}}{\partial u_c}+\frac{\partial S_{\text{source}}}{\partial u_c}\right)\Bigg|_{d,T,L_0}^{\text{on-shell}}+\frac{\partial S_{\text{source}}}{\partial d}\Bigg|_{T,L_0,u_c}\Bigg\}.$$

The second, third and forth terms drop out. It is clear from (4.35) corresponding to the equilibrium at the cusp that the third and fourth terms vanish. For the second term, it is because $\delta\tilde{S}_{\text{D8}}/\delta x_4'(u)$ is constant as can be seen from (4.40) that $\tilde{S}_{\text{D8}}$ depends only on $x_4'$. Integrating over the remaining gives $\partial L_0/\partial d$ which is zero due to the scale fixing condition $L_0 = 1$. Hence we obtain

$$\mu = \int_{uc}^{\infty} du\; \hat{a}_0'(u) + \frac{1}{\mathcal{N}}\frac{\partial S_{\text{source}}}{\partial d}\Bigg|_{T,L_0,u_c}, \tag{4.60}$$

where the second term on the RHS of the equation can be called as $\mu_{\text{source}}$. The chemical potential due to the source terms can be obtained by using (4.15) and (4.16) such that

$$\mu_{\text{source}} = \frac{1}{3}u_c\sqrt{f(u_c)} + n_s(u_c - u_T). \tag{4.61}$$

## 4.3 Phase Diagram

Now, we are ready to express the grand canonical potential density for the multiquark (baryon corresponds to $n_s = 0$) phase. Using (4.54), (4.19), (4.24), (4.30), we obtain the formulae of the grand canonical potential density for the multiquark matter. The chemical potential can be calculated from (4.60) by eliminating $\hat{a}_0'$ via (4.24) and substituting (4.30). The grand canonical potential density and the baryon chemical potential of the phases can be expressed as in the following,

nuclear (including exotics) phase :

$$\Omega_{nuc} = \int_{u_c}^{\infty} du\left[1-\frac{C^2}{f(u)(u^8+u^3d^2)}\right]^{-1/2}\frac{u^5}{\sqrt{u^5+d^2}}, \tag{4.62}$$

$$\mu_{nuc} = \int_{u_c}^{\infty} du\left[1-\frac{C^2}{f(u)(u^8+u^3d^2)}\right]^{-1/2}\frac{d}{\sqrt{u^5+d^2}} + \frac{1}{3}u_c\sqrt{f(u_c)}+n_s(u_c-u_T). \tag{4.63}$$

Recall that $C$ depends on $u_c$, $d$, $T$ and $n_s$ according to the (4.43) and (4.44). The last two terms in the (4.63) come from the derivative of the source-term action with respect to $d$.

There are at least other two phases that compete with the multiquark phase: the vacuum phase and the chiral-symmetric QGP phase. From the above formula of $\Omega_{nuc}$ and $\mu_{nuc}$, we can obtain the grand canonical potential density and the chemical potential of the vacuum simply by (i.) setting $d = 0$, (ii.) dropping the source terms in (4.63), (iii.) changing the lower bound of integration from $u_c$ to $u_0$, and (iv.) replacing $C$ by the constant of motion in the vacuum configuration, from $\delta S_{D8}/\delta x_4'$, $C_0 = f(u_0)u_0^8$. Thus we obtain

vacuum phase, $d = 0$:

$$\Omega_{vac} = \int_{u_0}^{\infty} du \left[1 - \frac{C_0^2}{f(u)u^8}\right]^{-1/2} u^{5/2}, \tag{4.64}$$

and we can notice that the baryon chemical potential is absent in this case since the density $d = 0$ corresponding to $\mu < \mu_{\text{onset}}$. Similarly, the grand canonical potential density and the chemical potential of the $\chi$S-QGP phase can be obtained by setting $x_4'(u) = 0$, reflecting its parallel configuration, and turning off the source terms. That is, setting $C = 0$ in (4.62) and (4.63), changing lower bound of integration to $u_T$ and dropping the source terms in (4.63) give

$\chi$S-QGP phase, $x_4'(u) = 0$:

$$\Omega_{qgp} = \int_{u_T}^{\infty} du \frac{u^5}{\sqrt{u^5 + d^2}}, \tag{4.65}$$

$$\mu_{qgp} = \int_{u_T}^{\infty} du \frac{d}{\sqrt{u^5 + d^2}}, \tag{4.66}$$

At fixed temperature $T$ and chemical potential $\mu$, a first order phase transition line between phase 1 and 2 is obtained when $\Omega_1 = \Omega_2, \mu_1 = \mu_2 = \mu$. Transitions between vacuum $\leftrightarrow$ $\chi$S-QGP and $\chi$S-QGP $\leftrightarrow$ nuclear phases are of this kind. On the other hand, phase transition between nuclear $\leftrightarrow$ vacuum is second order in nature, at least for this case when there is no interaction between each D4. The second order phase transition line occurs when

$$\frac{\partial \mu}{\partial d} = \frac{\partial^2 \mathcal{F}_E}{\partial d^2} \tag{4.67}$$

has discontinuity at $d = 0$.

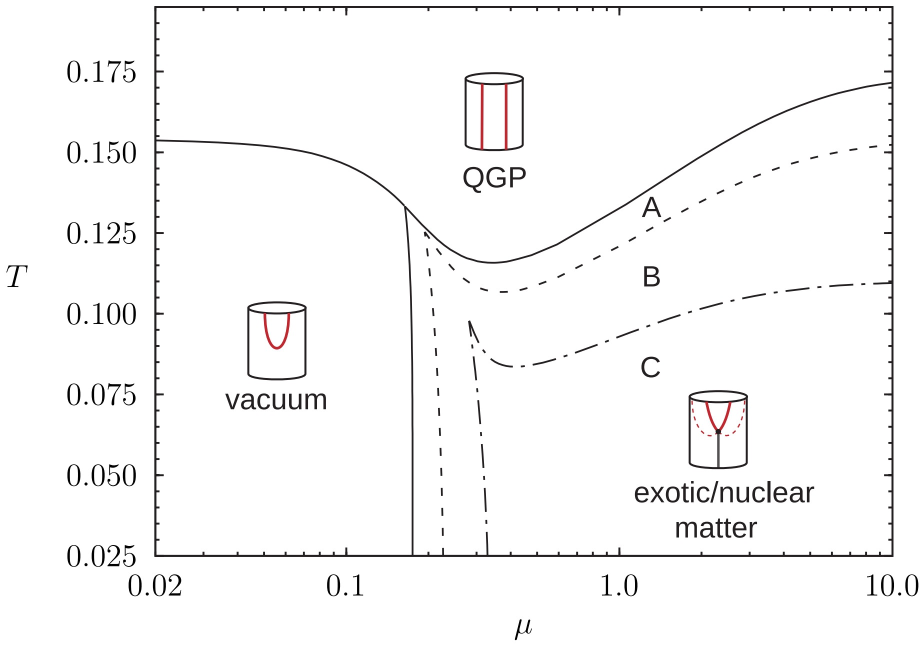


Figure 4.2: The phase diagram of exotic nuclear matters above the deconfinement temperature. Nuclear phase including exotics is shown as the region on the lower right corner where it is divided into 3 parts for representative purpose. $A, B, C$ represents the region where exotic baryon phase with $n_s = 0$ ($N_c$-baryon)$, 0.1, 0.3$ is preferred over vacuum and $\chi$S-QGP respectively.

In the Sakai-Sugimoto model, there is a phase transition temperature above which gluons become deconfined. However, it does not necessarily imply that quarks and antiquarks are totally free. When the baryon chemical potential is sufficiently high, baryons can exist even when the temperature is higher than the deconfinement temperature [96]. Only when the temperature increases further to another critical temperature, all bound states are completely dissolved and it remains only free quarks and gluons. In the phase above this critical temperature, the chiral symmetry is also restored. We also see this behavior in the phase diagram in Figure 4.2 where we ignore the confined region at low temperature and present only the deconfined part of the phase diagram.

The phase diagram of vacuum with broken chiral symmetry, $\chi$S-QGP and phase of nuclear including exotic multiquark states is shown in Figure 4.2. The phase diagram involving vacuum and $\chi$S-QGP phases was first obtained in [116] and the full phase diagram without the exotics was obtained in [96]. Since the strings pull down the D4-D8 configuration towards the horizon, the configuration with $n_s > 0$ is less stable than the normal $N_c$-baryon ($n_s = 0$). This is shown in Fig. 4.2 where the region of $n_s > 0$ nuclear phase $(B, C)$ is smaller than the region of $N_c$-baryon phase $(A)$. They are actually less stable than the $N_c$-baryon since the grand canonical potential density $\Omega_{n_s>0}(T, \mu) > \Omega_{n_s=0}(T, \mu)$ for $0 < n_s < 0.5$. Above $n_s > 0.3$, the exotic phase becomes unstable to density fluctuations ($\frac{\partial \mu}{\partial d} < 0$) at high temperatures in certain range of $d$ but still remains stable in a region of parameter space [41]. Numerical studies reveal that for approximately $n_s > 0.5$, the multiquark states become unstable thermodynamically with respect to density fluctuations for most of the temperatures.

Addition of radial strings introduces extra source of the baryonic chemical potential. We can see from Fig. 4.2 that the value of $\mu_{\text{onset}}$ for the exotic nuclear phase increases with the value of $n_s$. Nevertheless, once emerged (i.e. $\mu > \mu_{\text{onset}}$), the exotic phases are more stable than the vacuum at any temperature, but less stable than $\chi$S-QGP at sufficiently high temperatures above which chiral symmetry is restored.

## 4.4 Thermodynamical properties

From last section, the multiquark matter tends to exist in a region of intermediate temperature between $T_{\text{deconf.}}$ and $T_{\chi\text{SB}/\chi\text{S}}$ and of high enough baryon chemical potential $\mu > \mu_{\text{onset}}$ in the phase diagram even though it seems to coexist with the normal baryon matter. It is interesting to explore the thermodynamical properties of this exotic phase and investigate how the deviation of the colour-singlet being, represented by $n_s$, affects thermodynamic relations. In this section, we will determine some thermodynamic relations of the matter which can tell us about thermodynamical properties. Since the formula of the grand canonical potential and the baryon chemical potential from last section are quite complicated, in order to analytically calculate the thermodynamic relations, we need to do some approximations. However, this gives us some insight on the thermodynamical properties of the multiquark matter. The results of these analytic calculations are also confirmed by our numerical studies. We found that there are phase transition at a certain value of density and the dependence of thermodynamic relations on the number density of radial strings $n_s$. The results and discussions of this section are based on our works in [98] and [97].

### 4.4.1 Analytical studies of thermodynamic relations with some approximations

Thermodynamical properties of the nuclear/exotic matter phase can be described by the equation of state. First, we will investigate the relations between the pressure and the number density. From previous section, the grand potential density and the chemical potential of the nuclear/exotic matters are given by (4.62) and (4.63), respectively.

Since the differential of the grand potential $G_\Omega$ can be written as

$$dG_\Omega = -PdV - SdT - Nd\mu \tag{4.68}$$

where the state parameters describing the system $P$, $V$, $S$, $T$, $N$ are the pressure, volume, entropy, temperature, and the total number of particles of the system respectively. Since the change in volume is not our concern, we define the volume density of $G_\Omega$, $S$ and $N$ to be $\Omega$, $s$ and $d$, respectively. Therefore, we have, at a particular $T$ and $\mu$,

$$P = -G_\Omega/V \equiv -\Omega(T,\mu). \tag{4.69}$$

By assuming that the multi-quark states are spatially uniform, we obtain

$$d = \frac{\partial P}{\partial \mu}(T,\mu). \tag{4.70}$$

Using the chain rule, we obatin

$$\left.\frac{\partial P}{\partial d}\right|_T = \left.\frac{\partial \mu}{\partial d}\right|_T d, \tag{4.71}$$

so that

$$P(d,T,n_s) = \mu(d,T,n_s)\ d - \int_0^d \mu(d',T,n_s)\ \mathrm{d}(d'), \tag{4.72}$$

where we have assumed that the regulated pressure is zero when there is no nuclear matter, i.e. $d = 0$.

For convenience, we will write a function at $u = u_c$ ($u = u_0$) as that function with the subscript 'c' ('0') in the following calculations. For example, the function $f_c \equiv f(u_c)$, $f_0 \equiv f(u_0)$, $\eta_c = \eta(u_c)$, and $\eta_0 = \eta(u_0)$.

In the limit of very small $d$, $u_c$ approaches $u_0$, $\eta_c$ becomes $\eta_0 + \mathcal{O}(d)$, where $\eta_0$ is defined to be $\eta_c$ with $u_c$ replaced by $u_0$. From (4.63), the baryon chemical potential can then be approximated to be

$$\mu - \mu_{\text{source}} \simeq d\left\{ \int_{u_c}^{\infty} du \left[ 1 - \frac{u_0^8 f_0}{f u^8} - \frac{f_0 u_0^3 \left(1 - \frac{\eta_0^2}{9f_0} - \frac{u_0^5}{u^5}\right) d^2}{f u^8} \right]^{-1/2} u^{-5/2} \left(1 - \frac{d^2}{2u^5}\right) \right\}, \tag{4.73}$$

where $\mu_{\text{source}} = \frac{1}{3} u_c \sqrt{f(u_c)} + n_s(u_c - u_T)$, and we have neglected the higher order terms of $d$. By using the binomial expansion, the above equation becomes

$$\begin{aligned} \mu - \mu_{\text{source}} &\simeq d\left\{ \int_{u_0}^{\infty} du \frac{u^{-5/2}}{\sqrt{1 - \frac{f_0 u_0^8}{f u^8}}} \left[ 1 + \left( \frac{f_0 u_0^3}{f u^8 - f_0 u_0^8} \left(1 - \frac{\eta_0^2}{9f_0} - \frac{u_0^5}{u^5}\right) - \frac{1}{u^5} \right) \frac{d^2}{2} \right] \right\} \\ &= \alpha_0 d - \beta_0(n_s) d^3, \end{aligned} \tag{4.74}$$

where we have defined

$$\alpha_0 \equiv \int_{u_0}^{\infty} du \frac{u^{-5/2}}{1 - \frac{f_0 u_0^8}{f u^8}}\ , \tag{4.75}$$

$$\beta_0(n_s) \equiv \int_{u_0}^{\infty} du \frac{u^{-5/2}}{2\sqrt{1 - \frac{f_0 u_0^8}{f u^8}}} \left( \frac{f_0 u_0^3}{f u^8 - f_0 u_0^8} \left(1 - \frac{\eta_0^2}{9f_0} - \frac{u_0^5}{u^5}\right) + \frac{1}{u^5} \right). \tag{4.76}$$

By substituting (4.74) into (4.72), we can determine the pressure in the limit of very small $d$ as

$$P \simeq \frac{\alpha_0}{2} d^2 - \frac{3\beta_0(n_s)}{4} d^4. \tag{4.77}$$

In the limit of very large $d$ and relatively small $T$,

$$\mu - \mu_{\text{source}} = \int_{u_c}^{\infty} du \left[1 - \frac{f_c u_c^3}{f u^3}\left(\frac{u_c^5 + d^2 - \frac{d^2\eta_c^2}{9f_c}}{u^5 + d^2}\right)\right]^{-1/2} \frac{d}{\sqrt{u^5+d^2}} \tag{4.78}$$

$$\approx \int_{u_c}^{\infty} du \frac{d}{\sqrt{u^5+d^2}} + \frac{1}{2}u_c^3 f_c d^2\left(1 - \frac{\eta_c^2}{9f_c}\right)\int_{u_c}^{\infty} du\, \frac{d}{f u^3 (u^5+d^2)^{3/2}} \tag{4.79}$$

$$\approx \frac{d^{2/5}}{5}\frac{\Gamma\left(\frac{1}{5}\right)\Gamma\left(\frac{3}{10}\right)}{\Gamma\left(\frac{1}{2}\right)} + \frac{u_c^3 f_c}{10}\left(1 - \frac{\eta_c^2}{9f_c}\right) d^{-4/5}\frac{\Gamma\left(-\frac{2}{5}\right)\Gamma\left(\frac{19}{10}\right)}{\Gamma\left(\frac{3}{2}\right)} \tag{4.80}$$

where we have used the fact that the lower limit of integration $u_c^5/d^2$ is approximately zero as $d$ is very large. Again by using (4.72), we obtain

$$P \simeq \frac{2}{35}\left(\frac{\Gamma\left(\frac{1}{5}\right)\Gamma\left(\frac{3}{10}\right)}{\Gamma\left(\frac{1}{2}\right)}\right) d^{7/5}. \tag{4.81}$$

Next we consider the entropy of the multiquarks phase. From the differential of the free energy,

$$dF_E = -PdV - SdT + \mu dN, \tag{4.82}$$

the entropy is given by

$$S = -\frac{\partial F_E}{\partial T}. \tag{4.83}$$

The entropy density can then be written as

$$s = -\frac{\partial \mathcal{F}_E}{\partial T}, \tag{4.84}$$

where $\mathcal{F}_E$ is the free energy density which relates to the grand potential density as $\mathcal{F}_E = \Omega + \mu d$. Since we have the pressure $P = -\Omega$, we can write

$$s = \frac{\partial P}{\partial T} - \left(\frac{\partial \mu}{\partial T}\right) d. \tag{4.85}$$

By using our approximations and noting that $\alpha_0, \beta_0$ is insensitive to temperature, the formula of the pressure ((4.77) for small $d$, and (4.81) for large $d$) and that of baryon chemical potential contributed only by D8-branes ((4.74) for small $d$, (4.80) for large $d$) are nearly independent on the temperature. As a result, the dominant contribution for the entropy density, in the form (4.85), comes only from $\mu_{\text{source}}$, thus

$$s \simeq -\left(\frac{\partial \mu_{\text{source}}}{\partial T}\right) d. \tag{4.86}$$

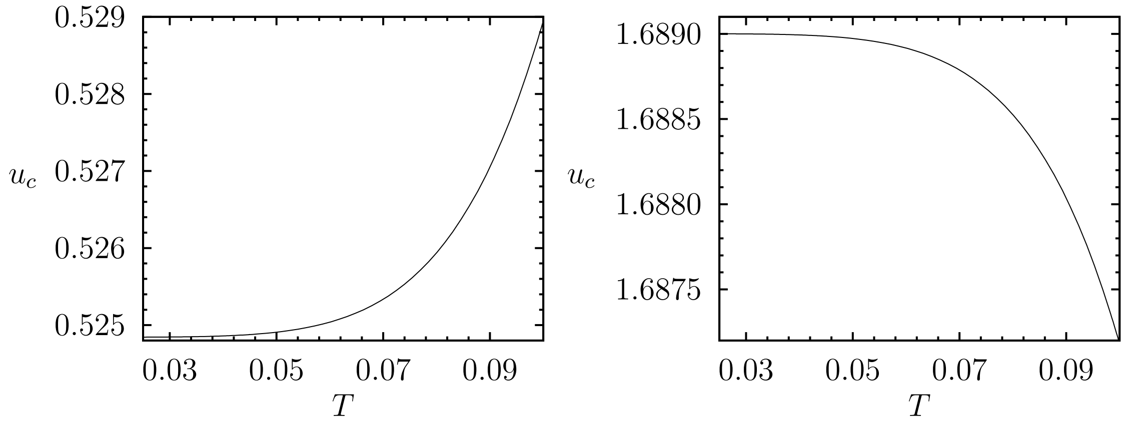


Figure 4.3: The graphs show the relations between $u_c$ and $T$ at small density (left) and at large density (right).

The baryon chemical potential from D8-branes is insensitive to the changes of temperature. This implies that the main contribution to the entropy density of the multiquark nuclear phase comes from the source term, namely vertices and strings.

Recall that $\mu_{\text{source}}$ is given by (4.61), we have

$$\frac{\partial \mu_{\text{source}}}{\partial T} = \frac{\partial}{\partial T}\left(\frac{1}{3}u_c\sqrt{f(u_c)} + n_s(u_c - u_T)\right). \tag{4.87}$$

Using the fact that $u_c$ is approximately constant with respect to the temperature in the range between the gluon deconfinement and the chiral symmetry restoration (see Fig. 4.3) and $u_T = 16\pi^2 R_{\text{D4}}^3 T^2/9 = 16\pi^2 T^2/9$ (setting $R_{\text{D4}} = 1$), the entropy density of the form (4.86) becomes

$$s \approx \frac{\left(\frac{16\pi^2}{9}\right)^3 T^5 d}{u_0^2\sqrt{1-\left(\frac{u_T}{u_0}\right)^3}} + n_s\frac{32\pi^2 T d}{9}. \tag{4.88}$$

Intriguingly, the above formula indicates that the entropy density is proportional to $T^5$ for small $n_s$. When $n_s$ gets larger (carrying colour charge), the entropy density becomes dominated by the colour term $s \propto n_s T$. This is confirmed numerically in section 4.4.2. It has been found that the entropy density of the $\chi$S-QGP scales as

$T^6$ [96] corresponding to the fluid of mostly free quarks and gluons. We can see that the effect of the colour charge of the multiquarks as quasi-particles is to make them less like free particles with the temperature dependence $\sim n_s T$, i.e. much less sensitive to the temperature.

It is interesting to compare the dependence of pressure on the number density, (4.77) and (4.81), to the confined case at zero temperature studied in [121]. The power-law relations for both small and large density of the confined and deconfined multiquark phases are in the same form (for $n_s = 0$). The reason is that the main contributions to the pressure for both phases are given by the D8-branes parts and they have similar dependence on the density for both phases. For the deconfined multiquark phase, the additional contributions from the source terms in (4.63), $\mu_{\text{source}}$, are mostly constant with respect to the density since $\mu_{\text{source}}$, following (4.61), depends implicitly on $d$ through $u_c$, which appears to be approximately independent of $d$ for both small and large $d$ limit. Consequently, when we substitute $\mu_{\text{source}}$ of the form (4.61) into (4.72), the source-term contributions cancel out and affect nothing on the pressure.

On the contrary, the entropy density for the deconfined phase is dominated by the contributions from the sources namely the vertex and strings. The contribution of the D8-branes is insensitive to the change of temperature and therefore does not affect the entropy density significantly. The additional source terms, however, depend on the temperature and thus contribute dominantly to the entropy density. Once the temperature rises beyond the gluon-deconfined temperature, entropy density will rise abruptly (for sufficiently large density $d$) and become sensitive to the temperature according to (4.88), due to the release of quarks from colourless confinement appearing as the sources. However, we will see later on using the numerical study in Section 4 that for low densities and for small $n_s$, the numerical value of the entropy density is yet relatively small.

### 4.4.2 Numerical studies of thermodynamic relations

From the analytic approximations in the previous subsection, we expect the pressure to appear as straight line in the logarithmic scale for small and large $d$ with the slope approximately 2 and 7/5 respectively. The relation between pressure and density of the multiquark from the full expressions can be plotted numerically as are shown in Fig. 4.4-4.6. The pressure is not really sensitive to any change of temperature and

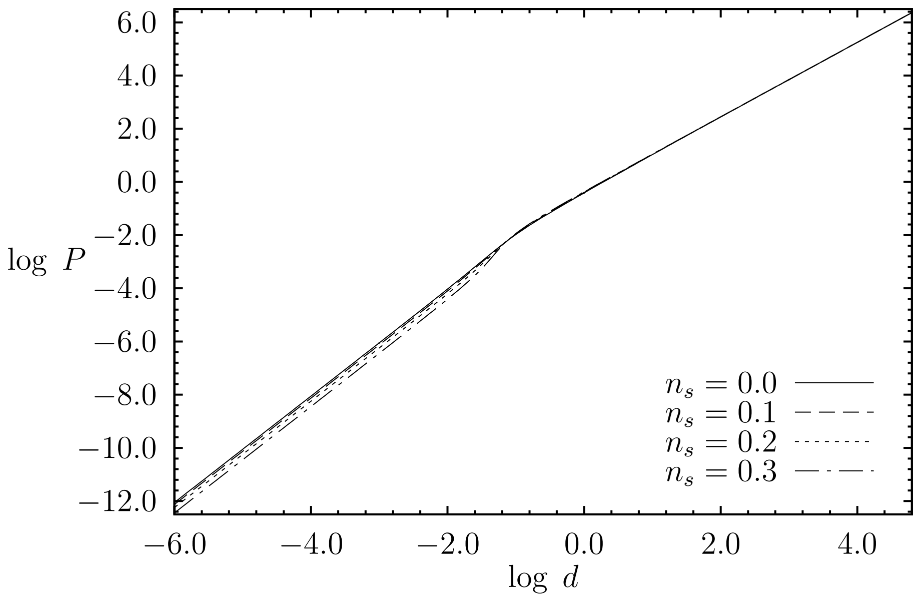


Figure 4.4: The relation between pressure and density in logarithmic scale at $T = 0.03$ for different values of $n_s$.

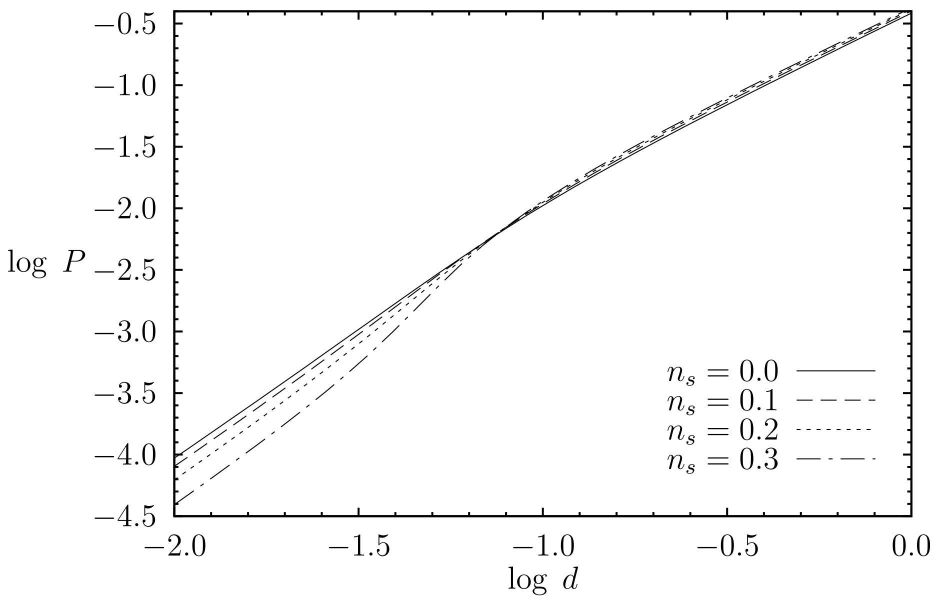


Figure 4.5: The relation of pressure versus density in logarithmic scale at $T = 0.03$, zoomed in around the transition region. Remark that the transition point from small density to large density is at $d \simeq 0.072$ for different $n_s$.

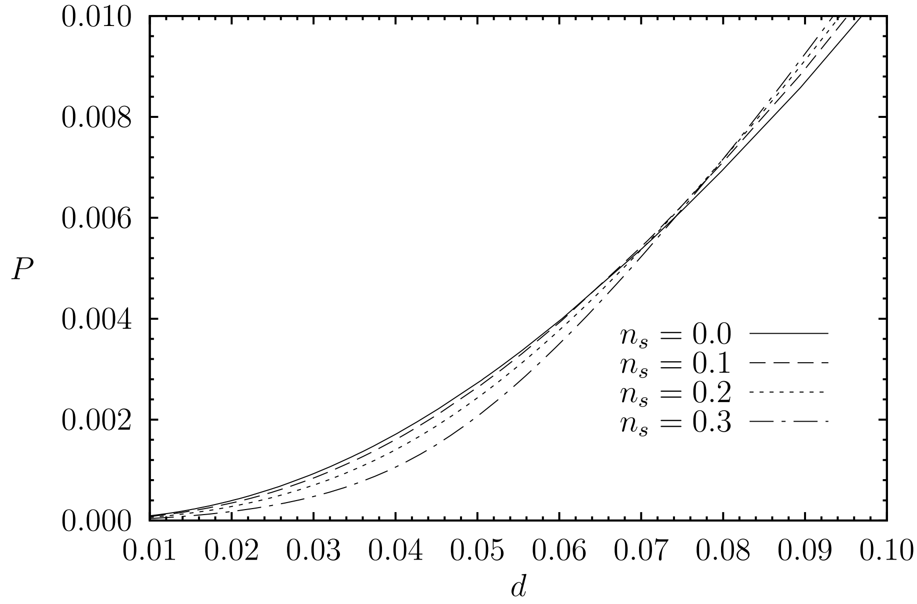


Figure 4.6: The relation between pressure and density in linear scale for different $n_s$.

we therefore present only the plots at $T = 0.03$. Remarkably, the transition from small to large $d$ is clearly visible in the logarithmic-scale plots. The transition occurs around $d_c \simeq 0.072$. Interestingly, as shown in Fig. 4.6, the multiquark with larger $n_s$ has lower pressure than one with smaller $n_s$ for $d < d_c$ whereas the opposite occurs for $d > d_c$. The dependence on $n_s$ remains to be seen for small $d$ as we can see from (4.77). For large $d$, the $n_s$-dependence is highly suppressed as predicted by (4.81).

The entropy density as a function of the temperature for various ranges of density is shown in Fig. 4.7. The temperature dependence for both small and large $d$ are the same, $\simeq T^5$ at the leading order. The $d$-dependence is linear and thus appears as separation of straight lines in the logarithmic-scale plot. For $n_s > 0$, we can see from (4.88) that the linear term in $T$ should become increasingly important. This is confirmed numerically as is shown in Fig. 4.7. The slope of the graph between the entropy density $s$ and $T$ in the double-log scale for $n_s = 0$ (the left plot) and $n_s = 0.3$ (the right plot) is approximately 5 and 1 respectively. Regardless of the temperature dependence, it should be noted that the numerical value of the entropy density for small densities and low $n_s$ in Fig. 4.7 is quite small.

Lastly, the relations between baryon number density and chemical potential are shown in Fig. 4.8. Temperature has very small effect on these curves and negligible for the range of temperature between the gluon deconfinement and the chiral-symmetry

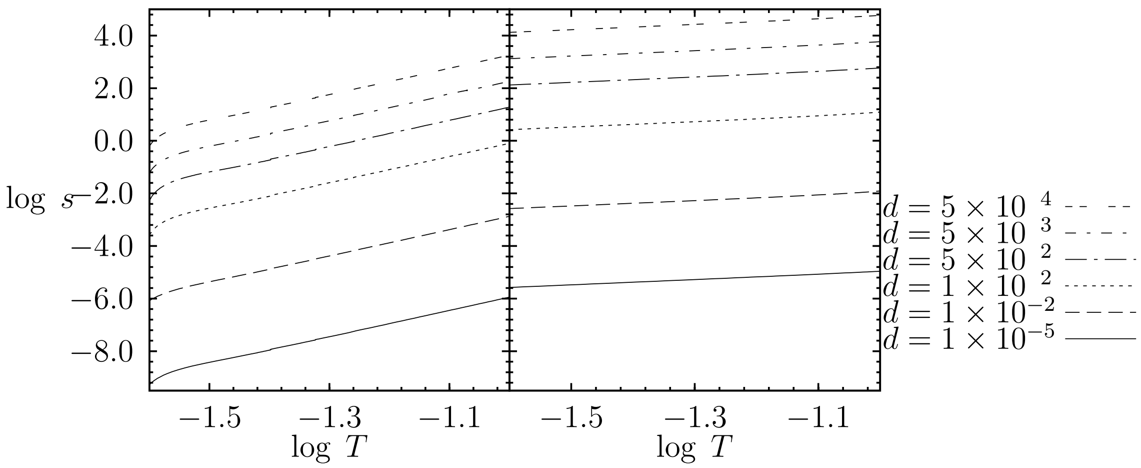


Figure 4.7: The relation of entropy versus temperature in logarithmic scale for $n_s = 0$ (left), $0.3$ (right). We can see that $s \sim T^5$ for $n_s = 0$ and $s \sim T$ for $n_s > 0$.

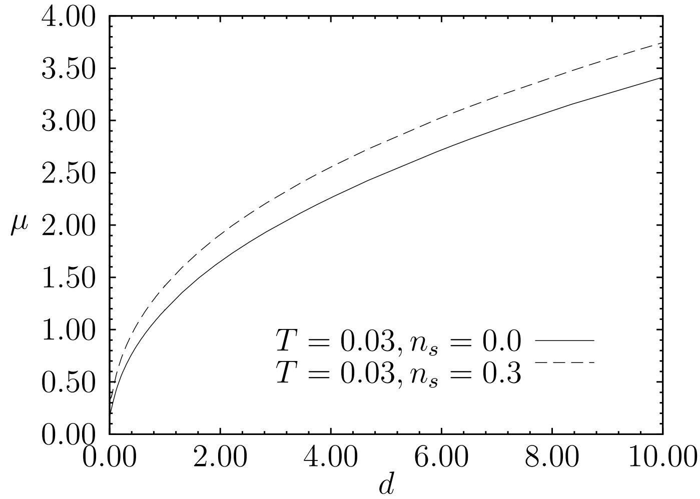


Figure 4.8: The baryon chemical potential versus number density in linear scale at $T = 0.03$.

restoration. The baryon chemical potential depends linearly on the number density for small $d$. For large $d$, the relation between the chemical potential and number density becomes $\mu \sim d^{2/5}$. The relation between $\mu$ and $d$ indicates that the majority of multiquark quasi-particles behave as fermions. This is due to the electric response of the DBI action [96]. Therefore, we can conclude that a strongly coupled quark-gluon plasma might be mainly occupied by multiquarks with each behaving as fermion, and the remainder with each behaving as boson.

# Chapter V

## CONCLUSION

Throughout this thesis, we have studied the stability of holographic colour non-singlet multiquark states in many aspects. Even though the background of the theory is gluon-deconfined, an amount of quarks remains to be in a bound state as a consequence of the interaction between quarks themselves appearing as in a Coulomb potential. It is important to emphasize that the colourless condition is not required in the deconfined phase. We propose the gravity dual models of the exotic multiquark in the absence of the flavour degrees of freedom. These include three classes of configurations, $k$-baryon, $(N_c + \bar{k})$-baryon, and $j$-mesonance. As expected, these configurations are found to be less energetically fovoured than the normal $N_c$-baryon. Despite this, these exotic bound states can in principle coexist with normal nuclear states, although they might be less abundant following to suppression by the Boltzmann factor $\exp\left(-|E|/k_{\mathrm{B}}T\right)$ due to their shallower binding potential relative to the colour singlet bound states[1]. Namely, the shallower (deeper) binding potential means the higher (lower) energy of the bound states. Therefore, the deeper the binding potential is, the harder the multiquark will melt in the thermal bath.

Moreover, we investigate their screening lengths. The dependence of the screening lengths of different kinds of exotic nuclear state on the parameters $k$, $\bar{k}$, $j$ are found following to the results shown in Fig. 3.5, 3.6, 3.7, respectively. The screening lengths of $k$-baryon and $j$-mesonance increase with the values of $k$ and $j$, whereas the screening length of $(N_c + \bar{k})$-baryon decreases as $\bar{k}$ increases. Interestingly, $j$-mesonance saturates the value of screening length, equal to the screening length of mesons, as $j \to \infty$. We also explore the dependence on the free quark mass $m$ of the binding potential of these exotic bound states. At the leading order, this is derived and found to scale as $m^{-2}$ for the multiquark configuration in the background corresponding to the Sakai-Sugimoto model with the absence of the flavour branes.

[1]More realistic statistical consideration for this system is more sophisticated. Here, we use the simple statistical consideration which treats these bound states as classical non-interacting objects. However, this should give a qualitative picture of relative amounts of these bound states.

Then, we move on studying the thermodynamical stability of the multiquark matter relative to other matter phases in the background of the Sakai-Sugimoto model equipped with the flavour degrees of freedom from the presence of the probe D8-$\overline{\text{D8}}$-branes. For this purpose, we explore about the possibility of its existence in phase diagram. In the background of the Sakai-Sugimoto model corresponding to the gluon deconfined phase, i.e. at temperature $T > T_{\text{deconf.}}$, the multiquark matter is proposed to be the configuration of connected flavour D8- and $\overline{\text{D8}}$-branes attached with D4-branes wrapped around the four-sphere and radial strings. Note that we assume here that the D4-vertex is pulled up away to embed within the flavour D8-branes. The number density of D4-branes $n_4$ in the flavour branes corresponds to the baryon number density, whereas the number density of radial strings $n_s$ contributes the number density of colour charges appearing in the gluon deconfined plasma in the field theory picture.

To obtain phase diagram, the grand canonical potential density is identified with the on-shell action of D8-branes. In the phase diagram, the multiquark matter is found to be more thermodynamically favoured than the $\chi$S-QGP and vacuum phases in a certain region of high baryon chemical potential and of the temperature range $T_{\text{deconf.}} < T < T_{\chi\text{SB}/\chi\text{S}}$). Nevertheless, the colour non-singlet nuclear matter phase of the radial string number density $n_s > 0$ are found to be less stable than the normal baryon phase of $n_s = 0$ in that region of the state parameters. The phase diagram is shown in Fig. 4.3. This results from that it costs more energy contributed by the configuration of the larger world-volume of the embedding of the flavour $N_f$ D8-branes. The reason of this is that the radial strings attached to the D4-branes pull the D4-D8 configuration down closer to the horizon, such that the D8-branes world-volume get larger. Consequently, the colour singlet nuclear matter is the ground state of (holographic) QCD in that region in phase diagram, rather than the multiquark matter. In this region of phase diagram, the nuclear matter is however possible to be the mixing between the colour singlet and colour non-singlet nuclear states. The coexistence of these should affect the thermodynamical properties, hence the existence of the colour non-singlet nuclear states might be inferred from some experimental results of heavy-ion collisions in the future.

Our consideration in the grand canonical ensemble suggests that we cannot have the multiquark configuration with too large value of the number of radial strings $n_s$. The stability issue under the fluctuations of density can be addressed by the quantity $\partial\mu/\partial d$. As we discussed in section 4.1, the quark matter is found to be unstable under density fluctuations due to $\partial\mu/\partial d < 0$ [96]. This is also the case for the exotic multiquark matter of the number density of colour charges (in the unit of $N_c$) $n_s > 0.5$. More precisely, the multiquark matter starts to be unstable under density fluctuations in some regions of phase diagram when $n_s > 0.3$. This result also suggests that the configuration corresponding to $j$-mesonance is unstable in this way. Moreover, this result is natural if we think that the configuration of the multiquark matter is intermediate between the colour-singlet nuclear matter and the quark matter. This can be seen from the construction of the gravity dual model of these matter.

In theoretical aspect, it is interesting to explore further the solitonic configuration of the gauge fields living in the world-volume of D8-branes corresponding to the multiquark configuration, whereas the solitonic gauge field configuration in the form of Skyrmion have been connected to the holographic baryon, equivalently a wrapped D4-brane within D8-branes. We expect it to be a soliton which is a modification of Skyrmion. Moreover, the studies can be extended to the multiquark configuration with the D4-vertex allowed to be distorted. This possibility has been explored for the bound states both in confined phase and in defonfined phase [92, 93, 42, 43]. Furthermore, the multiquark matter phase can be studied in the situation that there is an interaction between exotic baryons, equivalently an interaction between instatons. The author in [122, 123] has studied the nuclear force among baryon states through the analysis of closely separated instantons.

In Chapter IV, we demonstrate that the equation of state of the multiquark nuclear matter can be approximated by two different power-laws in the small and large density region. The calculations to determine these thermodynamic relations are done in both analytical and numerical ways. Both ways of calculations provides us the results which are consistent with each other. The relations of pressure versus baryon number density are found to be $P \sim d^2$ for small number density, and $P \sim d^{7/5}$ for large number density. The effect of colour charges is to reduce the pressure for small baryon number density, while to increase the pressure for large baryon number density. There is a transition between small density and large density at the value of the baryon number density $d = 0.072$. The entropy density is found to proportional to the temperature as $s \sim T^5$ for the normal nuclear matter with no colour charges, but $s \sim n_s T$ for the exotic nuclear matter with the number density of colour charges $n_s > 0$. The multiquark matter with colour charges has the entropy larger than that of the colour singlet nuclear matter but it depends less sensitively on the temperature. This indicates that the multiquark bound states in the deconfined phase tend to behave like quasi-particles with the entropy density $s$ being less sensitive to the temperature

Table 5.1: Summary table of the phases in the deconfined Sakai-Sugimoto model.

| The phase | vacuum | multiquark | $\chi$S-QGP |
|---|---|---|---|
| region in parameter space | $d = 0$ (i.e. $\mu < \mu_{\text{source}}$) | $d > 0$ (i.e. $\mu \geq \mu_{\text{source}}$) $0 \leq n_s \lesssim 0.3$ | $d > 0$ |
| preferred at | low $\mu$, low $T$ | high $\mu$, low $T$ | high $T$ |
| important properties | occupied by mesons | mixing of different $n_s$-multiquarks | free quarks and gluons |

than the gas of mostly free glons and quarks in the chiral-symmetric quark-gluon plasma phase.

As a summary, the main results of this thesis, namely the properties of the (exotic) nuclear matter, in comparison with other phases are shown in Table 5.1.

Even though it is not shown in the present thesis, our work of [98] applies the thermodynamic relations, derived from our model in the framework of gauge/string duality, to study further on the gravitational stabilities of the hypothetical multiquark star. Assuming that the star is spherical symmetric with the content of the multiquark matter, we solve the Tolman-Oppenheimer-Volkoff equation with the equations of state derived from the thermodynamic relations presented in this thesis. The difference between two power-law relations of pressure versus baryon number density has been found to be apparent in separate part of the star, namely the power-law relation $P \sim d^2$ for small density is responsible for the crust region of the star, while the power-law relation $P \sim d^{7/5}$ for large density is responsible for the region of the core of the star. The gravitational stabilities are explored as shown in the results of [98].

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# Appendix A

## Force condition at the D8-branes

There are three forces acting on a D4-brane locating inside the D8-branes, one from the D8, another from the radial strings pulling down towards horizon and lastly the force from its own "weight" in the background. The equilibrium can be sustained only when these three forces are balanced. As is shown in [96], variation of the total action with respect to $u_c$ and the constant of motion with respect to $x_4(u)$ lead to

$$x_4'(u_c) = \left(\tilde{L}(u_c) - \frac{\partial S_{source}}{\partial u_c}\right) \Bigg/ \frac{\partial \tilde{S}_{D8}}{\partial x_4'}\Bigg|_{u_c}, \tag{A.1}$$

$$= \frac{1}{d}\sqrt{\frac{9u_c^2(1+\frac{d^2}{u_c^5})}{1+\frac{1}{2}(\frac{u_T}{u_c})^3 + 3n_s\sqrt{f(u_c)}} - \frac{d^2 u_c^{-3}}{f(u_c)}} \tag{A.2}$$

where the Legendre transformed action is

$$\tilde{S}_{D8} = \int_{u_c}^{\infty} \tilde{L}(x_4'(u), d)\, du, \tag{A.3}$$

$$= \mathcal{N}\int_{u_c}^{\infty} du\ u^4\sqrt{f(u)(x_4'(u))^2 + u^{-3}}\sqrt{1+\frac{d^2}{u^5}}, \tag{A.4}$$

and the source term is given by

$$S_{source} = \mathcal{N}d\Big[\frac{1}{3}u_c\sqrt{f(u_c)} + n_s(u_c - u_T)\Big]. \tag{A.5}$$

There are two contributions from the D-branes and strings as the sources for the baryon chemical potential. Additional strings increase the baryonic chemical potential of the exotic multiquark states. Since the number of total charge on each D4 is $N_c$ which is absorbed into $\mathcal{N}$, the number of radial strings stretched down to the horizon, $n_s$, is thus given in unit of $N_c$.

# VITAE

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## Publications

Author names are in Alphabetical order. For the up-to-date publication list and citation numbers, type-in `FIND A HIRUNSIRISAWAT` at the SPIRES homepage `http://inspirehep.net/`

1. P. Burikham, E. Hirunsirisawat "Holographic Multiquarks in the Quark-gluon Plasma: A Review," Advances in High Energy Physics **2011** (2011): Article ID 123184.
2. P. Burikham, E. Hirunsirisawat and S. Pinkanjanarod, "Thermodynamic Properties of Holographic Multiquark and the Multiquark Star," JHEP **1006** (2010) 040 [arXiv:1003.5470 [hep-ph]].
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