

การวิเคราะห์หน่วยแรงของชิ้นส่วนเพียโซอิเล็กทริกทรงกระบอกภายใต้แรงกระทำที่ผิวและสนามไฟฟ้า



นายวิชัยรัตน์ แก้วเจือ

ศูนย์วิทยพัทยากร  
จุฬาลงกรณ์มหาวิทยาลัย

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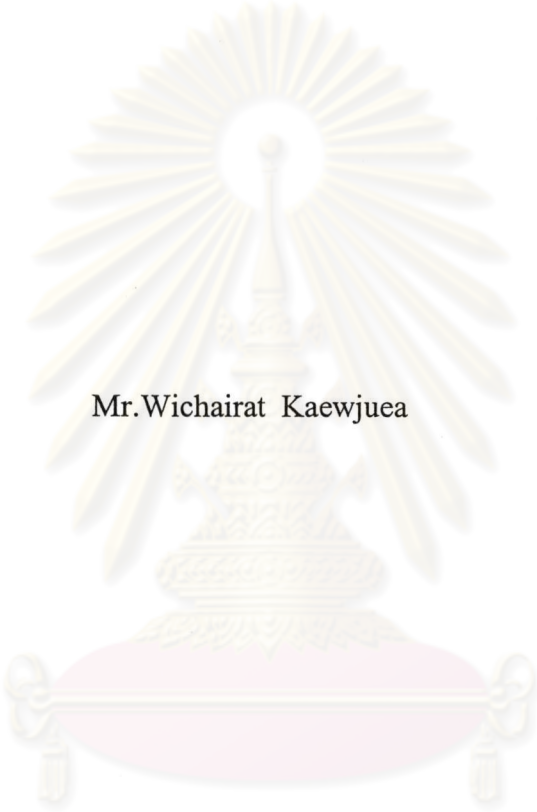
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ลิขสิทธิ์ของจุฬาลงกรณ์มหาวิทยาลัย

STRESS ANALYSIS OF PIEZOELECTRIC CYLINDER UNDER  
SURFACE LOAD AND ELECTRIC FIELD



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งานวิจัยนี้เป็นการศึกษาหน่วยแรงที่เกิดขึ้นกับชิ้นส่วนเพียโซอิเล็กทริกทรงกระบอกที่มี  
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ทรงกระบอกนี้หาได้จากการใช้ฟังก์ชันของการเปลี่ยนแปลงตำแหน่งเพื่อแก้สมการเชิงอนุพันธ์ย่อย  
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ค่าขอบที่สอดคล้องกับแรงในแนวตั้งและสนามไฟฟ้าที่ผิวบนและผิวล่างของทรงกระบอก สำหรับ  
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ชิ้นส่วนเพียโซอิเล็กทริก

ศูนย์วิทยทรัพยากร  
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KEY WORD: SMART MATERIAL / PIEZOELECTRIC MATERIAL / FINITE CYLINDER / AXISYMMETRIC LOADING / SERIES SOLUTION.

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In this paper, the electroelastic fields within a finite transversely isotropic piezoelectric cylinder subjected to axisymmetric loading and electric field at the surface are examined. The general solution for the cylinder is derived by using the displacement potential functions to uncouple the governing differential equations. The general solution is presented in terms of Fourier-Bessel series, which is trigonometric and hyperbolic functions in the  $z$ -direction and Bessel functions in the  $r$ -direction. The boundary value problems corresponding to vertical loading and electric field at the top and bottom surfaces of the cylinder are presented and the arbitrary constants appearing in the general solutions are determined. For applied electrical field, two types of electrodes are considered, i.e., a flexible electrode with the smooth contact surface and a rigid electrode with the rough surface. Selected numerical results for electroelastic fields of a piezoelectric cylinder under mechanical and electrical loading are presented to demonstrate influence of various governing parameters on the cylinder.

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Finally, the author feel proud to dedicate this thesis with due respect to his beloved parents for their wholehearted understanding, encouragement, and patient support throughout his entire study.



ศูนย์วิทยทรัพยากร  
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 คุรุสภา

## LIST OF SYMBOLS

$c$	Vector of elastic stiffness
$d$	Vector of piezoelectric strain constant
$D$	Electric displacement
$e$	Piezoelectric coefficient
$E$	Vector of electric field
$I_n$	Modified Bessel function of the first kind of order $n$
$K_n$	Modified Bessel function of the second kind of order $n$
$J_n$	Bessel function of the first kind of order $n$
$r_0$	Radius of the applied load on the top and bottom surfaces of cylinder
$s$	Vector of compliance coefficient
$u_r$	Mechanical displacement in the $r$ -axis
$u_z$	Mechanical displacement in the $z$ -axis
$Y_n$	Bessel function of the second kind of order $n$
$\phi$	Electric potential
$\tilde{\phi}$	Normalized electric potential
$\epsilon$	Strain vector
$\sigma$	Stress vector
$\varepsilon$	Dielectric constant
$\tilde{\varepsilon}$	Normalized dielectric constant
$\psi$	Potential function
$\lambda_s$	$s^{\text{th}}$ root of $J_1(\lambda_s)$