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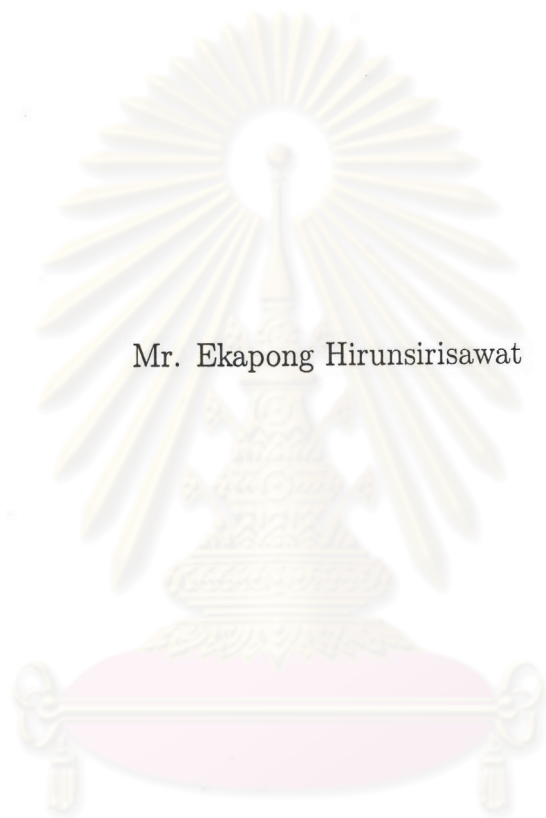
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WAVELET ANALYSIS OF COSMIC MICROWAVE BACKGROUND ANISOTROPY



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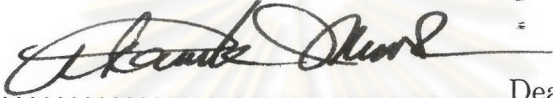
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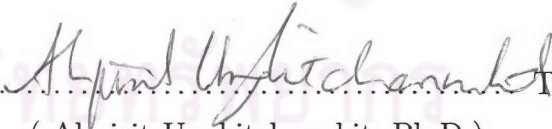
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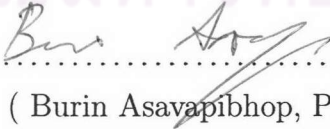
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เอกพงษ์ หิริญญศิริสวัสดิ์: การวิเคราะห์เวฟเลตของแอนไอโซทรอปีของคอสมิกไมโครเวฟแบ็กกราวด์. (WAVELET ANALYSIS OF COSMIC MICROWAVE BACKGROUND ANISOTROPY) อ. ที่ปรึกษา: อ.ดร. อภิสัทธี อึ้งกิจจานุกิจ จำนวนหน้า 69 หน้า. ISBN 974-53-1645-8.

แอนไอโซทรอปีของอุณหภูมิของคอสมิกไมโครเวฟแบ็กกราวด์เป็นสนามสุ่ม (random field) บนท้องฟ้าทรงกลมที่มีคุณสมบัติทางสถิติที่เก็บข้อมูลของเอกภพ โดยเฉพาะอย่างยิ่งเอกภพสมัยเริ่มแรก การค้นหาสัญญาณนอนเกาส์เซียน (non-Gaussian) ในคอสมิกไมโครเวฟแบ็กกราวด์ซึ่งเป็นการศึกษาคุณสมบัติทางสถิติแบบเกาส์เซียน (Gaussian statistics) เป็นวิธีการหนึ่งที่จะช่วยในการศึกษาเอกภพ ในวิทยานิพนธ์นี้ เราทบทวนงานของ Vielva และผู้ร่วมงาน (ถูกตีพิมพ์ใน พ.ศ.2547) และงานของ Mukherjee และ Wang (ถูกตีพิมพ์ใน พ.ศ. 2547) ซึ่งเป็นการค้นหาความเป็นนอนเกาส์เซียน (non-Gaussianity) ในข้อมูลของคอสมิกไมโครเวฟแบ็กกราวด์จากการสังเกตของดาวเทียม WMAP (Wilkinson Microwave Anisotropy Probe) ในปีแรก โดยใช้เวฟเลตหมวกเม็กซิกันแบบทรงกลม (spherical Mexican hat wavelet) งานของพวกเขาค้นพบสัญญาณนอนเกาส์เซียน (non-Gaussian) ในสเกลเวฟเลตประมาณ 4 องศา ซึ่งเทียบเท่ากับขนาดมุมบนท้องฟ้าประมาณ 10 องศาในครึ่งทรงกลมภาคใต้ (southern Galactic hemisphere) นอกจากนี้งานของ Mukherjee และ Wang ยังทำการบังคับค่าของพารามิเตอร์คู่ควบไม่เชิงเส้น (nonlinear coupling parameter)  $f_{NL}$  ซึ่งเป็นตัวบ่งบอกระดับขั้นของความเป็นนอนเกาส์เซียนในรูปแบบของพจน์กำลังสองในเพอร์เทอร์เบชันความโค้งแบบ Bardeen (Bardeen curvature perturbation) โดยใช้ความเบ้ (skewness) ในสเกลเวฟเลตทุกสเกลในการทดสอบทางสถิติ ผลคือค่าของ  $f_{NL}$  ถูกบังคับเป็น  $50 \pm 80$  ที่ช่วงความเชื่อมั่นร้อยละ 68 และน้อยกว่า 280 ที่ช่วงความเชื่อมั่นร้อยละ 99

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The cosmic microwave background (CMB) temperature anisotropy is a random field on the spherical sky whose statistical properties keep the information of the universe, especially the early universe. The study of its Gaussian statistics, namely the search for non-Gaussian signatures, is hopefully a way complementary to others in cosmology. In this thesis, we review the search for non-Gaussianity on the WMAP (Wilkinson Microwave Anisotropy Probe) first-year CMB data using the spherical Mexican hat wavelet of Vielva et al. (2004) and Mukherjee and Wang (2004). Their results are positive; the non-Gaussian signal was found at the wavelet scale around  $4^\circ$  ( $\sim 10^\circ$  in the sky) in the southern Galactic hemisphere. In addition, the nonlinear coupling parameter  $f_{NL}$ , which reflects the degree of non-Gaussianity in the form of the quadratic term in the Bardeen's curvature perturbations, was constrained in Mukherjee and Wang using the skewness at all wavelet scales. Their results constrained  $f_{NL}$  to be  $50 \pm 80$  at 68% confidence, and less than 280 at 99% confidence.

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## List of Symbols

$T$	CMB temperature
$\Delta T/T$	CMB temperature anisotropy
$H$	Hubble expansion rate
$t$	cosmic time
$l$	multipole moment
$P$	pressure
$\rho$	energy density
$G$	Newtonian gravitational constant
$R$	wavelet scale
$S(R)$	skewness spectra
$K(R)$	kurtosis spectra
$f_{NL}$	nonlinear coupling parameter

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