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OPTIMAL DESIGN OF REINFORCED CONCRETE CANTILEVER RETAINING WALL

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งานวิจัยจำนวนมากเรื่องการหาค่าดีที่สุดของกำแพงกันดินแบบยื่นถูกนำเสนอขึ้นในอดีตที่ผ่านมา ยังไม่มีงานวิจัยใดพิจารณาข้อกำหนดด้านธรณีเทคนิคของเสถียรภาพลาดชันในปัญหาการหาค่าดีที่สุด วัตถุประสงค์ของการวิจัยนี้คือ 1) เพื่อพัฒนาเทคนิคที่มีประสิทธิภาพสำหรับการออกแบบที่ประหยัดที่สุดของกำแพงกันดินคอนกรีตเสริมเหล็กแบบยื่นทั่วไปซึ่งรวมถึงการพิจารณาข้อกำหนดทั้งหมดด้านธรณีเทคนิคคือ การพลิกคว่ำ การเลื่อนไถล กำลังกดและเสถียรภาพของลาดชันรวมทั้งข้อกำหนดด้านโครงสร้าง 2) เพื่อเพิ่มความสามารถของวิธีที่พัฒนาขึ้นในวัตถุประสงค์ข้อ แรกสำหรับการออกแบบที่ประหยัดที่สุดของกำแพงกันดินแบบร่วมกับการรองรับสะพาน ในการศึกษานี้ ตัวแปรการออกแบบของวิธีที่เสนอคือขนาดของกำแพงและเสริมเหล็กในแต่ละส่วนของกำแพง ฟังก์ชันวัตถุประสงค์ ข้อกำหนดพื้นฐานด้านธรณีเทคนิคสามข้อของการวิบัติกำแพง (การพลิกคว่ำ การเลื่อนไถล กำลังกด) รวมทั้งปริมาณเหล็กเสริมเป็นไปตามวิธีเดียวกันกับปัญหาการหาค่าดีที่สุดในอดีตที่ผ่านมา เพื่อพิจารณาเสถียรภาพความลาดชันในปัญหาการหาค่าดีที่สุด ค่าสัดส่วนความปลอดภัยจากวิธีแบ่งชิ้นดินสามัญถูกพิสูจน์เชิงวิเคราะห์ในรูปของตัวแปรไม่ทราบของขนาดกำแพงและจุดศูนย์กลางของพื้นผิววิบัติส่วนโค้งวงกลมสำหรับกำแพงกันดินแบบร่วมกับการรองรับสะพาน ข้อกำหนดเพิ่มของการรับแรงอัดแนวแกนร่วมกับโมเมนต์ดัดถูกนำเข้าไปในระบบสมการไม่เชิงเส้นของการหาค่าดีที่สุดกำแพงกันดิน ความสามารถของสองวิธีที่นำเสนอถูกแสดงผ่านการใช้งานกับปัญหากำแพงกันดินทั่วไปจำนวนมาก

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SOPHEA CHEA: OPTIMAL DESIGN OF REINFORCED CONCRETE CANTILEVER RETAINING WALL. ADVISOR: ASSOC. PROF. BOONCHAI UKRITCHON, Sc.D., 160 pp.

Various researches in optimization of cantilever retaining wall were proposed in the past, none of them considered geotechnical requirement of slope stability in the optimization problem. The objectives of this research are: 1) to develop an efficient technique for optimal design of conventional reinforced concrete cantilever retaining wall including complete geotechnical considerations, namely, overturning, sliding, bearing capacity, and slope stability as well as structural requirements; and 2) to extend capability of the developed method in the first objective for optimal design of integral bridge abutment retaining wall.

In this study, the design variables of the proposed method are dimensions of the wall and steel reinforcements in each wall component. The objective function, three basic geotechnical constraints of wall failures (overturning, sliding, and bearing) as well as required structural reinforcements followed the same method of the optimization problem presented in the past. To take into account of slope stability in the optimization problem, the factor of safety based on the Ordinary Method of Slice is derived analytically in terms of unknown variables of wall dimensions and the center of circular arc failure surface. For integral bridge abutment, additional constraints for combined axial compression load and bending moment are included in the system of nonlinear constraints of the retaining wall optimization. The capabilities of both proposed methods are demonstrated through their applications in varieties of general retaining wall problems.

Department: ....Civil Engineering.......... Student's Signature ....
Field of Study: ....Civil Engineering.......... Advisor's Signature ....
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CHAPTER I
INTRODUCTION

1.1 Introduction

A retaining wall may be referred to a vertical structure that can hold soil movement. It was designed and constructed to resist the lateral soil pressure. For some special cases, a retaining wall is built for supporting vertical loads and for river bank protection system. Generally, retaining walls have been constructed with the plain or reinforced concrete. The study in this research is concerned with reinforced concrete cantilever retaining wall.

Geotechnical stability constraints of wall failures play an important role in controlling wall dimensions in the analysis. Thus, foundations and retaining walls are designed to satisfy adequate safety against failure. The practical problems lead to the minimization problem in finding a set of decision variables that optimize the objective function and satisfy a set of predefined design restrictions. Since the problems are complicated, they require systematic and rational approaches of solution. Due to advantages of numerical method approach, most practical optimization problems are usually solved using computer methods.

1.2 Statement of problem

In design of retaining wall, designers usually assume some dimensions and then check the trial sections against stability conditions. Once, results of stability analysis turn out to be undesirable, these trial sections must be revised until they meet all stability criteria. This conventional design consumes more times and is not efficient. In case that revised structural dimensions lead to safe design, the cost of structures may not be optimal and highly depends on the experience of designers. As a result, designers should analyze the structures to find out the optimal dimensions that satisfy all modes of failures and give the minimum cost of whole structures.

Researches of optimal design of cantilever retaining wall has been studied in the past, namely, Saribas and Erbatur (1996), Ceranic et al. (2001), Castillo et al. (2004), Ypes et al. (2008), Babu and Basha (2008), and Khajehzadeh et al. (2010). These studies have proposed different design constraints and optimization solvers to find the optimal solution of nonlinear programming problem arising from geotechnical and structural constraints. However, none of them have considered geotechnical constraint of slope stability due to difficulties and complexities in mathematically deriving constraint of slope stability.

This is contrast to three basic geotechnical wall failures, namely, overturning, sliding, and bearing capacity, whose constraints are generally and explicitly presented in most foundation textbooks. Similarly, structural constraints of shear and bending moment resistances are easily available in textbooks of reinforced concrete structure. Because those previous researches ignore one of possible wall failures (slope stability) in the analysis, they may not present the most optimal design of cantilever retaining wall.

1.3 Objective of study

The objective of this research is to develop an efficient technique for optimal design of conventional reinforced concrete cantilever retaining wall including geotechnical considerations, namely, overturning, sliding, bearing capacity, and slope stability, as well as structural requirements. The second objective is to extend capability of the developed method in the first objective for optimal design of integral bridge abutment retaining wall.

1.4 Scope of study

Scopes of this study are covered as outlines below:

1. Two-dimensional plane strain analysis
2. Retaining structures with backfill and surcharge loading
3. Neglect effect of water table
4. Soil below base of foundation assumes to be homogenous
5. Conventional retaining wall resist lateral earth pressure without axial load
6. Special retaining wall terms as integral bridge abutment retaining wall resists lateral earth pressure with axial load from bridge

1.5 Research benefits

The benefits of this research are expected as follow:

1. This research makes important contribution to the research of retaining wall, namely, reinforced cantilever wall, and integral bridge abutment wall
2. A new formulation of optimal design of integral bridge abutment is proposed

CHAPTER II
LITERATURE REVIEW

2.1 Introduction

A retaining wall may be referred to a vertical structure that can hold soil movement. It was designed and constructed to resist the lateral soil pressure.

Retaining walls may be classified in many various types as shown in Figure 2.1 such as gravity walls, cantilever retaining walls, counterfort retaining walls, buttressed retaining walls, bridge abutments, box culverts, semi-gravity walls and basement walls.

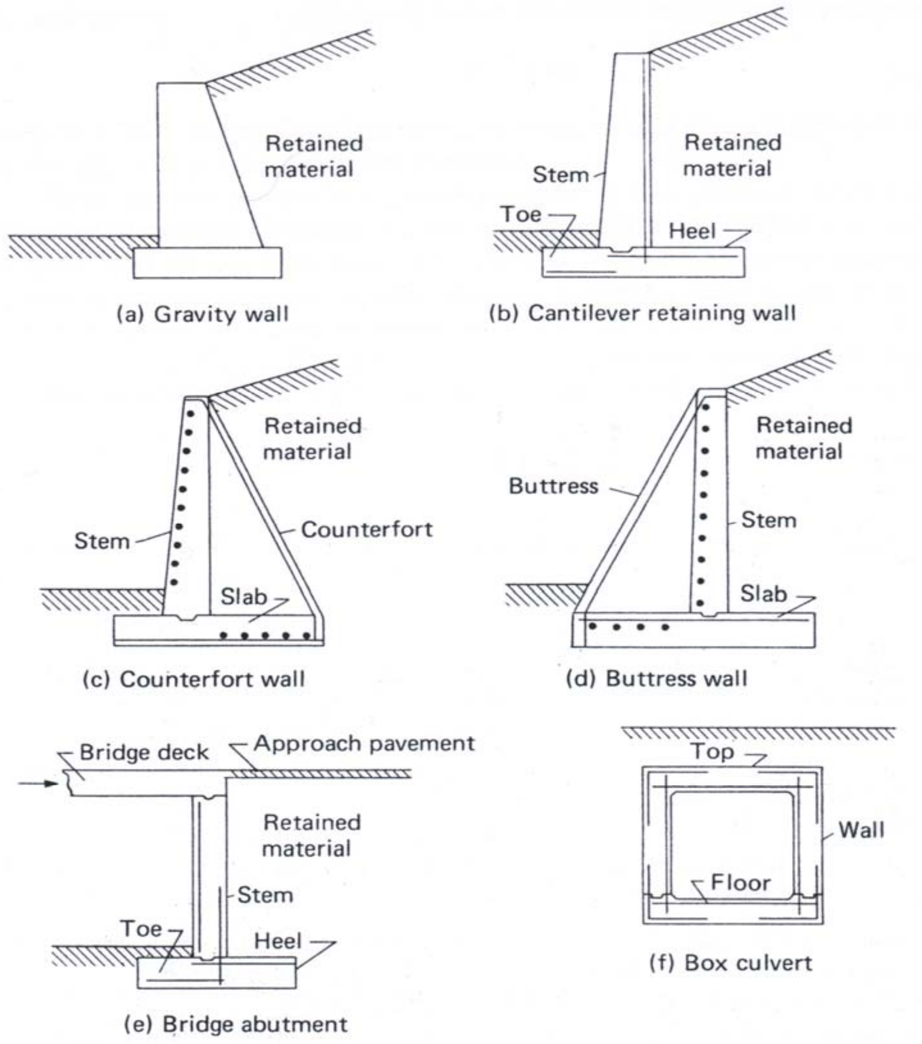

Figure 2.1 Types of retaining structures (Wang, 1992)

A conventional reinforced concrete cantilever retaining walls are constructed from reinforced concrete. Basically, it does not resist to any vertical loads. The typical components of conventional retaining wall composes a thin stem (can be a tapered front face), a toe slab, and a heel slab. Figure 2.2 displays the wall components of reinforced concrete cantilever retaining wall. Another special case of retaining structures is termed as integral bridge abutment. According to Hassoun (2005), bridge abutments are retaining walls that are supported vertical load from bridge deck.

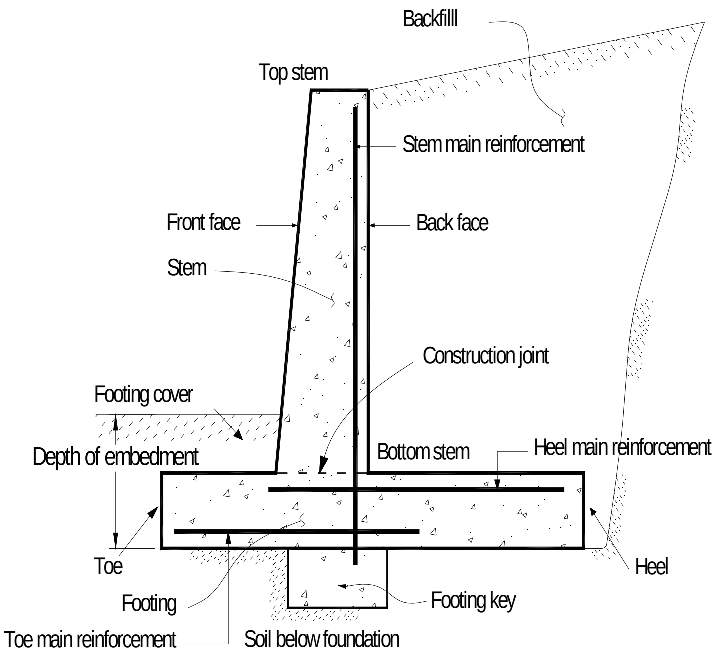

Figure 2.2 Terms used in conventional reinforced concrete cantilever retaining wall

There are two steps in designing of a cantilever retaining wall. First, once calculations of lateral pressures are made, the whole structure is checked for stability. These include check for possible overturning, sliding, eccentricity, and bearing capacity failures. Second, each component of the structure is analyzed for adequate shear and moment strengths, thus steel reinforcement of each component is determined.

Most of practical problems, designers usually select initial dimensions by using approximate proportions of various wall components as shown in Figure 2.3. Once, results of stability analysis turn out to be undesirable, these trial sections must be revised until they meet all stability criteria.

Figure 2.3 presents the approximate proportion in conventional design. Two important parameters are total height of wall (H) and soil embedment (D).

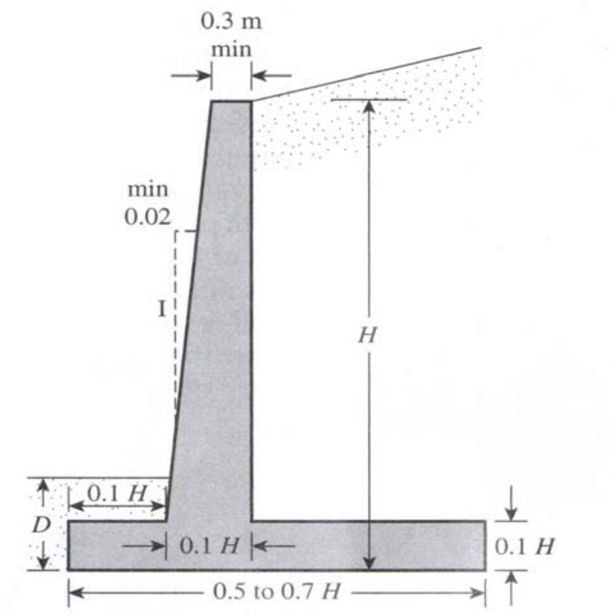

Figure 2.3 Proportions in conventional design of retaining wall (Das, 2007)

2.2 Research literature review

Several researches have been carried out in the past. Thus, some methodologies for analysis and optimal design of retaining structures are developed by many researchers. Saribas and Erbatur (1996) presented a detail study on optimum design of reinforced concrete cantilever retaining walls with seven geometrical and reinforcement design variables. They applied a constrained nonlinear programming to optimum design which was solved by a specially prepared program. Ten modes of wall failure including overturning, sliding, eccentricity, bearing capacity, shear and bending moment of toe slab, heel slab and stem of wall were considered.

Ceranic et al. (2001) studied application of simulated annealing algorithm to a problem with only geometrical design variables. Babu and Basha (2008) presented an approach for reliability-based design optimization of reinforced concrete cantilever retaining wall. Wall failure criteria were considered similarly as Saribas and Erbatur. The analysis was performed by treating input parameters as random variables. Khajehzadeh et al. (2010) presented an effectiveness of particle swarm optimization with passive congregation algorithm to economic design of retaining wall. The problem consisted of eight geometrical and reinforcement design variables. The constraints were the same as Babu and Basha (2008). The optimization algorithm was coded in MATLAB. Table 2.1 summarizes significances and contributions of those researches to the field of optimization of retaining wall.

Table 2.1 Summary of past researches

Researchers	Objective	Result	Remarks
Saribas and Erbatur (1996)	Optimum design (first researchers)	Optimal design and sensitivity studies	No slope constraint
Cernical et al. (2001)	Optimum design using simulated annealing algorithm	Successfully applied	No slope constraint
Castillo et al. (2004)	Add safety factors and probability based optimal design	Optimal design and sensitivity studies	No slope constraint
Ypes et al. (2007)	Economic optimization of retaining wall for road construction	Design a parametric study for different backfill and bearing conditions	No slope constraint
Babu and Basha (2008)	Efficient and economic design	Design charts for wall proportions	No slope constraint
Khajehzadeh et al. (2010)	Optimum design by particle swarm algorithm	Optimal design and more economical	No slope constraint

2.3 Geotechnical considerations

2.3.1 Application of lateral earth pressure theory in retaining wall

Figure 2.4a shows a frictionless retaining wall with a granular backfill (c’ = 0) whose slope makes an angle α with respect to the horizontal. Based on Rankine’s theory, the active earth pressure coefficient may be expressed as:

$$k_a = \cos\alpha \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi'}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi'}} \quad (2.1)$$

where ϕ' = internal friction angle of backfill

α = backfill angle with respect to the horizontal

At any depth z, the Rankine active pressure maybe expressed as:

$$\sigma'_a = \gamma_1 z k_a \quad (2.2)$$

The Rankine active force per unit length of the wall is calculated as:

$$P_a = \frac{1}{2}\gamma_1 H'^2 K_a \tag{2.3}$$

where γ_1 = unit weight of backfill

H' = depth of active soil pressure on wall

K_a = Rankine active earth pressure coefficient

As shown in Figure 2.4a, the resultant force, P_a, is inclined at an angle α to the horizontal and acts at distance $H'/3$ from the base of the wall.

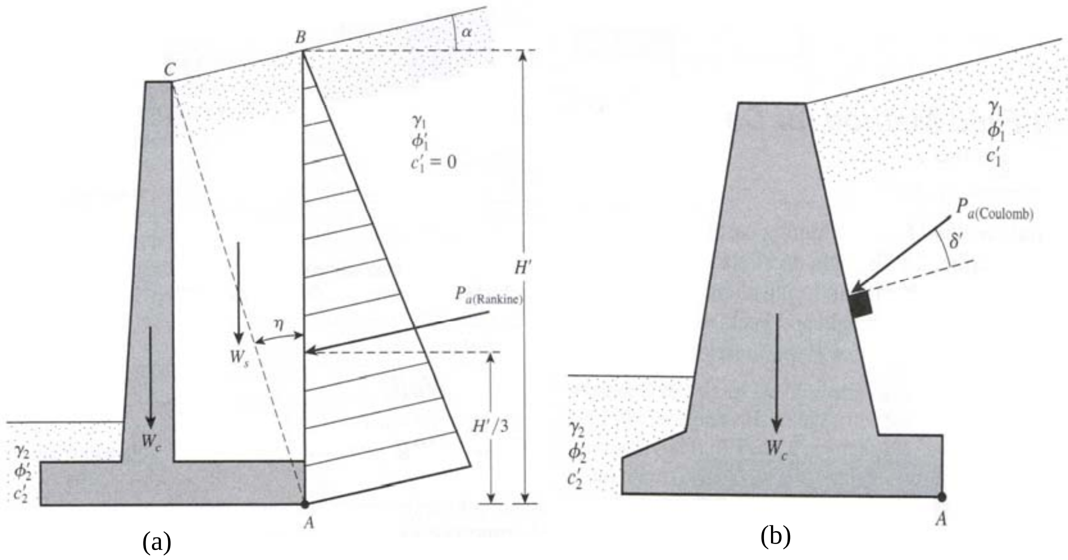

Figure 2.4 Assumption for the determination of lateral earth pressure: (a) cantilever wall (b) gravity wall (Das, 2007)

Figure 2.4b illustrates the application of Coulomb's active earth pressure theory. In Coulomb's theory, Coulomb's active force is function of wall friction angle, δ', which depends on types of backfill material.

According to Das (2007), effects of water table and hydrostatic pressure in the retained soil should not be encountered in retaining wall design. It is important to provide the drainage facilities for retained soils.

2.3.2 Stability of retaining walls

A retaining wall may fail in any of the following geotechnical modes as shown in Figure 2.5.

- It may overturning about its toe (see Figure 2.5a)

- It may sliding along its base (see Figure 2.5b)
- It may fail due to the loss of bearing capacity of the underlying soil below the base (see Figure 2.5c)
- It may fail as slope mechanism of deep-seated shear failure (see Figure 2.5d)
- It may fail due to excessive settlement of the base

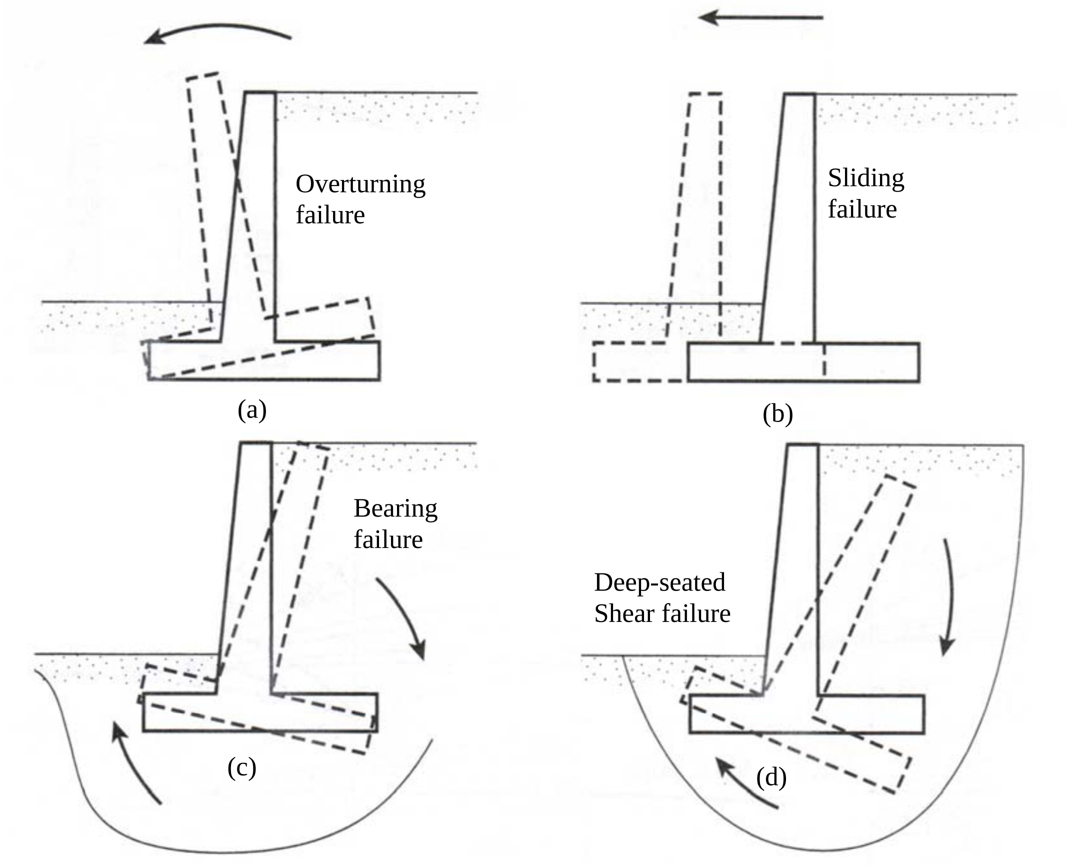

Figure 2.5 Geotechnical failure modes of retaining wall (Das, 2007)

2.3.3 Overturning stability

Figure 2.6 illustrates a cantilever wall with a sloping backfill (α) as well as all forces acting on the wall. Based on the assumption that the Rankine active pressure is valid where:

P_a = active earth pressure acting at a height H'/3 from the base on section AB

$P_h = P_a \cos\alpha$ = horizontal active earth pressure

$P_v = P_a \sin\alpha$ = vertical active earth pressure

P_P = Rankine passive earth pressure at the toe side of the wall

W_1, W_2 = weight of retained soil

W_3, W_4, W_5 = weight of concrete stem and base

$q_{toe} = q_{max}$ = maximum upward soil pressure at left corner of toe slab

$q_{heel} = q_{min}$ = minimum upward soil pressure at right corner of heel slab

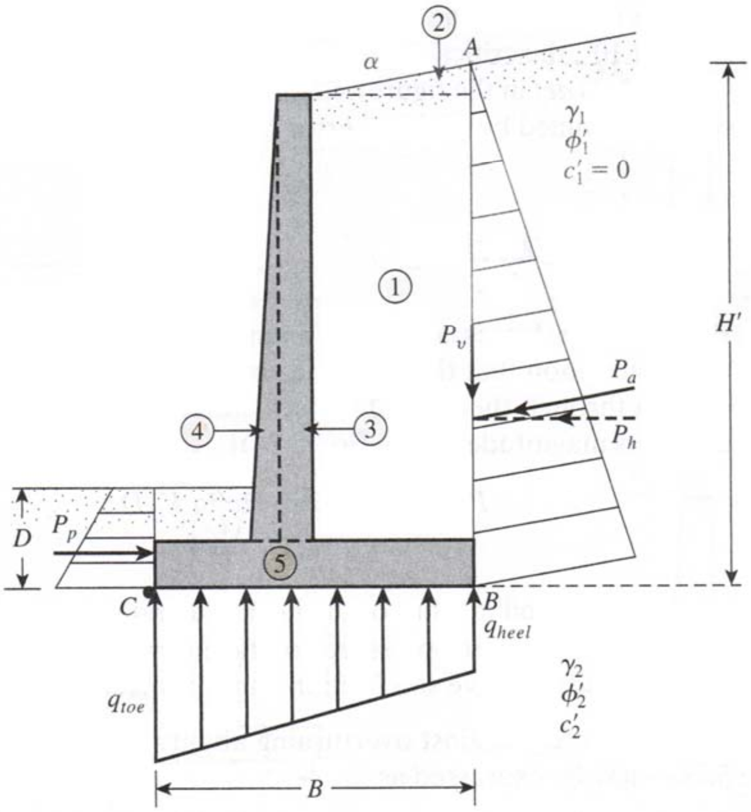

Figure 2.6 Overturning stability assuming that Rankine pressure is valid (Das, 2007)

Rankine passive pressure at the toe side of the wall can be calculated as:

$$P_P = (1/2) K_p \gamma_2 D^2 + 2c'_2 \sqrt{K_p} D \quad (2.4)$$

where γ_2 = unit weight of soil in front of the heel and under the base slab

K_p = Rankine passive earth pressure coefficient

D = depth of soil embedment

c'_2, ϕ'_2 = cohesion and effective angle of internal friction of soil base, respectively

Rankine passive earth pressure coefficient can be expressed as:

$$K_p = \tan^2(45 + \phi_2'/2) \quad (2.5)$$

The factor of safety against overturning about the toe (point C) as shown in Figure 2.6 can be expressed as:

$$FS_{(overturning)} = \frac{\sum M_R}{\sum M_O} \tag{2.6}$$

where $\sum M_O$ = sum of the moment of forces tending to overturn about point C

$\sum M_R$ = sum of the moment of forces tending to resist overturning about C

Das (2007) reported that passive force (P_P) in front of toe may be neglected in calculation of resisting moment (M_R). He also recommended using minimum desirable value of the factor of safety against overturning ranged from 2 to 3.

2.3.4 Sliding stability along the base

Figure 2.7 shows the free body diagram for sliding stability calculation.

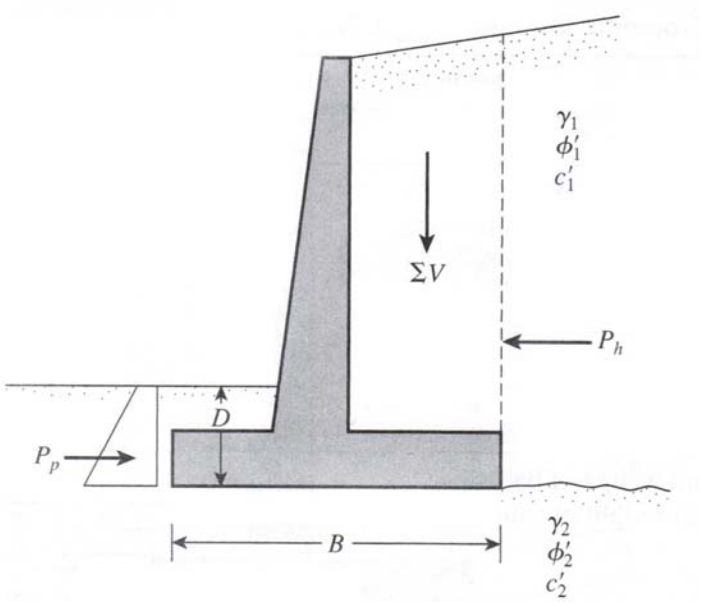

Figure 2.7 Checking for sliding along the base (Das, 2007)

From Figure 2.7, the factor of safety against sliding may be expressed as:

$$FS_{(sliding)} = \frac{\sum F_R}{\sum F_d} \tag{2.7}$$

where $\sum F_R$ = sum of the horizontal resisting forces

$\sum F_d$ = sum of the horizontal driving forces

Sum of the horizontal resistance forces $\sum F_R$ is calculated as below:

$$\sum F_R = \sum(V)\tan\delta' + B\times C'_a + P_P \quad (2.8)$$

where $\sum V$ = sum of the vertical forces

δ' = angle of friction between the soil and the base slab

B = length of the base slab

C'_a = adhesion between the soil and the base slab

P_P = Rankine passive pressure at the back of wall

In many cases, the passive force P_P is ignored in calculating the factor of safety against sliding. In general, the interface shear resistance for friction and base adhesion can be calculated as:

$$\delta' = k_1\phi'_1 \quad (2.9)$$

$$C'_a = k_2 c'_2 \quad (2.10)$$

In most cases, coefficient k_1 and k_2 are in the range from $1/2$ to $2/3$ (Das, 2007).

A minimum factor of safety of 1.5 against sliding is generally required (Das, 2007). NAVFAC (1986) recommended using factor of safety equal to 2.0 and 1.5 for with and without passive force consideration, respectively.

2.3.5 Bearing capacity stability

Figure 2.8 shows the distribution of vertical pressures transmitted from the base slab to the underlying soil. The vertical pressure under the base slab should be checked against the ultimate bearing capacity of the soil.

The maximum and minimum pressures below the base can be calculated based on Figure 2.8 as:

$$q_{max} = q_{toe} = \frac{\sum V}{B}\left(1+\frac{6e}{B}\right) \quad (2.11)$$

$$q_{min} = q_{heel} = \frac{\sum V}{B}\left(1-\frac{6e}{B}\right) \quad (2.12)$$

where $\sum V$ = sum of vertical forces

e = eccentricity of the resultant R

B = length of base slab

When the value of the eccentricity, e, is greater or equal than $B/6$, the sign of q_{min} becomes negative. This means that tensile stress may develop at the end of the heel section. This kind of stress is not desirable because most soils cannot resist tension. If the result of analysis turns out that e is greater than $B/6$, the design should be revised again in order to avoid tensile stress along the wall base.

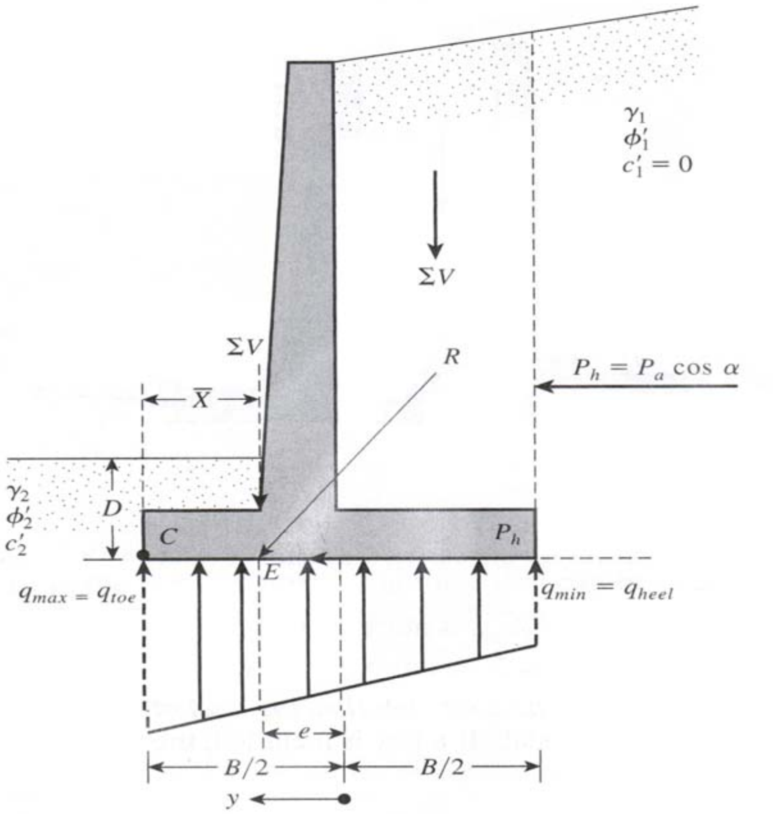

Figure 2.8 Checking bearing capacity and eccentricity under base slab (Das, 2007)

Ultimate bearing capacity of a shallow foundation can be calculated based on the general ultimate bearing capacity equation as follows:

$$q_u = c_2' F_{cs} F_{cd} F_{ci} N_c + q' F_{qs} F_{qd} F_{qi} N_q + \frac{1}{2} \gamma_2 B' F_{\gamma s} F_{\gamma d} F_{\gamma i} N_\gamma \tag{2.13}$$

where c_2' = cohesion of soil below foundation

q’ = effective stress at the level of the bottom of the foundation

γ_2 = unit weight of soil below foundation

B’ = effective width of foundation

$F_{cs}, F_{qs}, F_{\gamma s}$ = shape factors

$F_{cd}, F_{qd}, F_{\gamma d}$ = depth factors

$F_{ci}, F_{qi}, F_{\gamma i}$ = inclination factors

N_c, N_q, N_γ = bearing capacity factors

Effective width of foundation (B') and effective stress at the level of the bottom of the foundation (q') can be calculated based on Figure 2.8 as:

$$q' = \gamma_2 D \tag{2.14}$$

$$B' = B - 2e \tag{2.15}$$

where γ_2 = unit weight of soil below foundation

D = depth of soil embedment

e = eccentricity

B = length of base slab

Bearing Capacity Factors

The value of $N_c, N_q,$ and N_γ for a given soil friction angle can be calculated as:

$$N_q = e^{\pi \tan\phi'} \tan^2\left(45^o + \phi'/2\right) \tag{2.16}$$

$$N_c = \left(N_q - 1\right)\cot\phi' \tag{2.17}$$

$$N_\gamma = 2\left(N_q + 1\right)\tan\phi' \tag{2.18}$$

N_c from Equation (2.17) was derived by Prandtl (1921) and N_q from Equation (2.16) was presented by Reissner (1924). N_γ from Equation (2.18) was given by Caquot and Kerisel (1953) and Vesic (1973).

For total stress analysis where internal friction angle equals to zero, bearing capacity factor values are taken respectively as N_c = 5.14, N_q = 1.0, and $N\gamma$ = 0.0.

Once the ultimate bearing capacity of the soil has been calculated by using Equation (2.13), the factor of safety against bearing capacity failure can be calculated as:

$$FS_{bearing} = \frac{q_u}{q_{max}} \tag{2.19}$$

where q_u = ultimate bearing capacity of a shallow foundation

q_{max} = maximum upward soil ressure below the base

Das (2007) recommends to use factor of safety for bearing capacity equal to 3.

2.3.6 Slope stability

A. Definition of factor of safety in slope stability

Factor of safety in slope stability defined as the ratio of available unit shear stress, τ_f, to required unit shear stress or mobilized shear stress, τ_m. The required shear stress is analyzed by slope calculations. Available shear strength depends on the properties of soil which are measured from laboratory or field test (Chowdhury et al., 2010).

The expression of safety factor can be written as:

$$FS = \frac{\tau_f}{\tau_m} \tag{2.20}$$

where FS = factor of safety

τ_f = available shear stress of soil

τ_m = mobilized shear stress required for equilibrium along the potential failure surface

Based on Mohr-Coulomb equation, shear stress in term of effective stress can be determined as:

$$\tau_f = c' + \sigma'_{nf} \tan\phi' \tag{2.21}$$

$$\tau_m = c'_m + \sigma'_{nf} \tan\phi'_m \tag{2.22}$$

where c', ϕ' = available shear strength parameters

c'_m, ϕ'_m = required or mobilized shear strength parameters

Substituting Equation (2.21) to Equation (2.20), mobilized shear stress τ_m can be written as:

$$\tau_m = \frac{\tau_f}{FS} = \frac{c'}{FS} + \sigma'_{nf} \frac{\tan\phi'}{FS} \tag{2.23}$$

Comparing Equation (2.22) with Equation (2.23), mobilized shear strength parameters can be written as:

$$c'_m = \frac{c'}{FS} \quad (2.24)$$

$$\tan\phi'_m = \frac{\tan\phi'}{FS} \quad (2.25)$$

From Equation (2.24) and (2.25), FS can be defined in the formulae as:

$$FS = c'/c'_m = \tan\phi'/\tan\phi'_m \quad (2.26)$$

The assumption in Equation (2.26) implies that the factor of safety with respect to the cohesion parameter is the same as that with respect to the friction parameter (Chowdhury et al., 2010).

However, several formulas of factor of safety may be defined by various definitions. Figure 2.9 illustrates the definition of factor of safety with different assumption of potential failure surface.

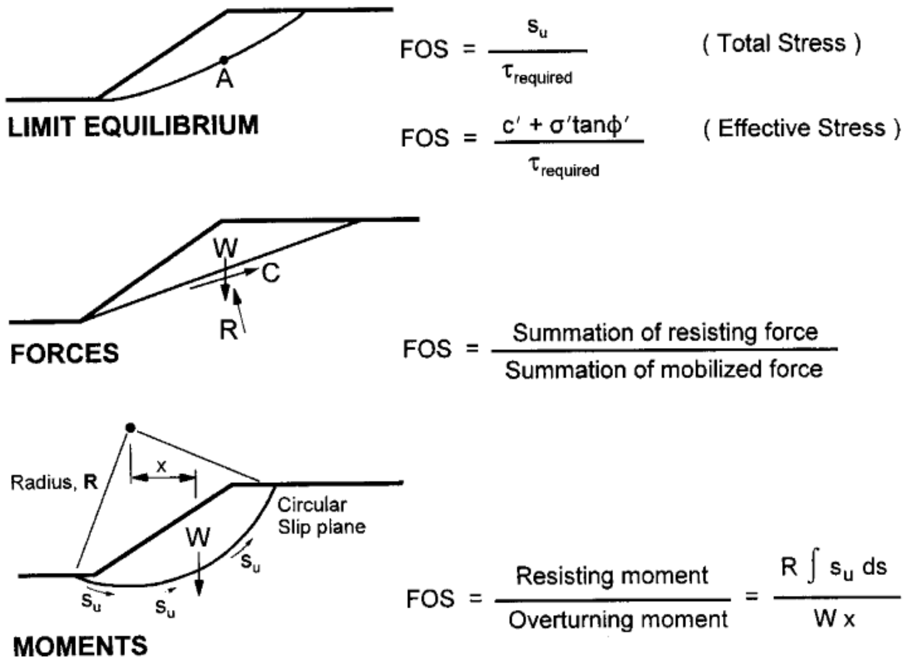

Figure 2.9 Definition of factor of safety with various failure surface (Abramson et al., 2002)

B. Method of Analysis

Slope stability can be analyzed using several methods, namely:

1. Limit equilibrium method (LEM)
2. Limit analysis
3. Finite difference method (FDM)
4. Finite element method (FEM)

C. Limit Equilibrium Method: method of slices

The conventional limit equilibrium methods aim to investigate the equilibrium of the soil mass tending to slide down under the influence of gravity. Limit equilibrium methods for slope stability is analyzed by dividing soil mass of the failure slope into a number of vertical slices and treating each individual slice as a unique sliding block as shown in Figure 2.10. In Figure 2.10, the 14 smaller slices are divided above the slip surface. The groundwater table is presented and surface load acts on inclined slope.

Importantly, since computer programs will aid in slope calculation, method of slices can be applied to the complex problem such as:

1. complex slope geometries
2. complex variable soil conditions
3. complex external boundary loads

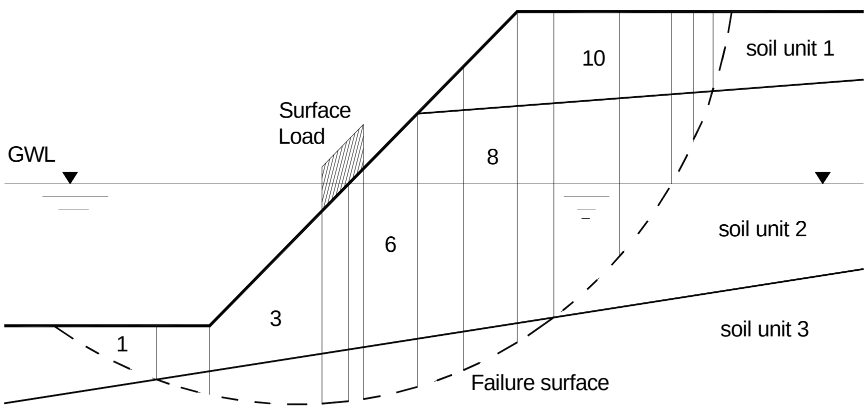

Figure 2.10 Division of soil mass into n vertical slices (Abramson et al., 2002)

Figure 2.11 shows free body diagram of the forces acting on a typical slice taking from Figure 2.10. The represented symbols in Figure 2.11 are summarized in Table 2.2. Table 2.3 shows summary of unknowns and equations associated with method of slices from general cases.

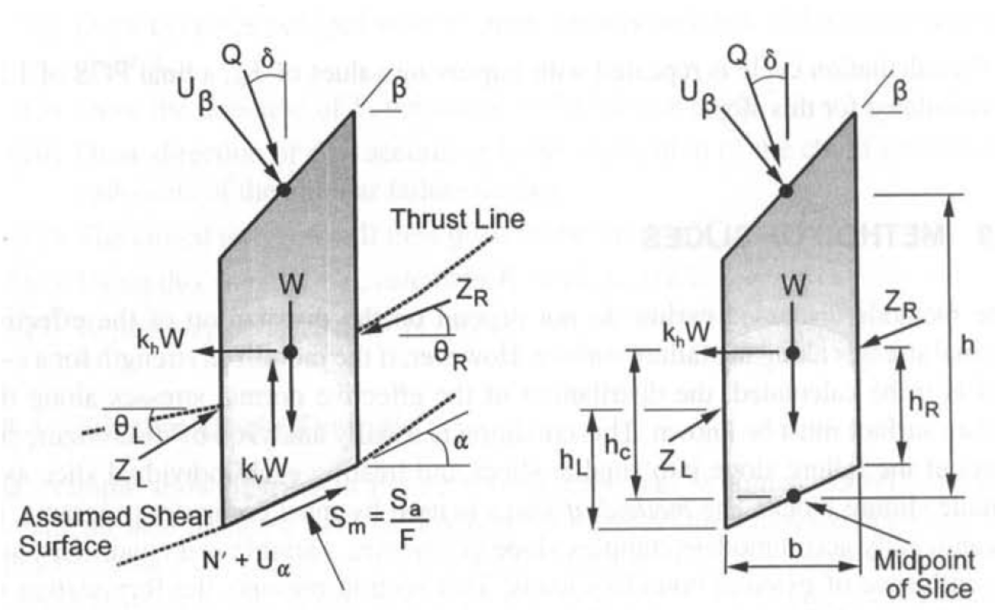

Figure 2.11 Force acting on a typical slice (Abramson et al., 2002)

Table 2.2 Meaning of represented symbols in typical slices

F = factor of safety	Z_L = left interslice force
S_a = available strength = $C + N'\tan\phi$	Z_R = right interslice force
S_m = mobilized strength	θ_L = left interslice force angle
U_α = pore water force	θ_R = right interslice force angle
U_β = surface water force	h_L = height to force Z_L
W = weight of slice	h_R = height to force Z_R
N’ = effective normal force	α = inclination of slice base
Q = external surcharge	β = inclination of slice top
K_v = vertical seismic coefficient	δ = inclination of surcharge
K_h = horizontal seismic coefficient	b = width of slice
h = average height of slice	h_c = height to centroid of slice

***Note**: In Table 2.3, assuming n is number of slices*

Table 2.3 Summary of unknowns and equations associated with method of slices

Number of unknowns	**Total Number**
Factor of safety	1
Normal force at the base of a slice, N’	n
Location of base resultant	n
Mobilized shear force	n
Interslice force resultant, Z	n-1
Interslice force orientation, θ	n-1
Interslice force location (line of thrust)	n-1
Total number of unknowns	6n-2
Number of equations	**Total Number**
Horizontal force equilibrium	n
Vertical force equilibrium	n
Mohr-Coulomb failure criterion	n
Moment equilibrium	n
Total number of equations	4n

Based on Table 2.3, slope problem is statically indeterminate because number of unknowns is greater than the number of available equations. Thus, several assumptions are made in some methods which can be called as non-rigorous solutions such as Ordinary Method of Slices and Bishop’s Simplified. However, in rigorous solutions, all equilibrium equations are completely considered such as Spencer, Morgenstern-Price and Janbu’s rigorous method (Abramson et al., 2002).

D. Application of Ordinary Method of Slices (OMS) in slope stability analysis

Ordinary Method of Slices (OMS) is one of the simplest procedures based on method of slices to estimate the stability of a slope (Abramson et al., 2002). It is considered as the earliest methods that derived factor of safety directly without doing any complicated numerical iterations.

OMS assumes that the interslice force resultants for all slices are parallel to the base of the slice. All interslice forces are neglected in this method. From Figure 2.11

for simple case without seismic coefficient (K_v=K_h=0), the procedure for deriving factor of safety in this method is described as below:

[1] The normal force (N’) is calculated from summation of slice forces perpendicular to the base of the slices (see Figure 2.11).

$$\sum F_{\alpha} = N' + U_{\alpha} - W\cos\alpha - U_{\beta}\cos(\beta-\alpha) - Q\cos(\delta-\alpha) = 0 \qquad (2.27)$$

N’ from Equation (2.27) can be determined as:

$$N' = -U_{\alpha} + W\cos\alpha + U_{\beta}\cos(\beta-\alpha) + Q\cos(\delta-\alpha) \qquad (2.28)$$

[2] The overall moment equilibrium of the forces about center of the circular arc failure surface for each slice is equal to zero.

$$\sum M_0 = \sum_{i=1}^{n}\left[W + U_{\beta}\cos\beta + Q\cos\delta\right]\times R\sin\alpha - \sum_{i=1}^{n}\left[U_{\beta}\sin\beta + Q\sin\delta\right](R\cos\alpha - h) - \sum_{i=1}^{n}[S_m]\times R = 0 \qquad (2.29)$$

where R = radius of the circular failure surface

h = average height of the slice

[3] Mohr-Coulomb mobilized shear strength (S_m) along the base of each slice is computed by:

$$S_m = \frac{C + N'\tan\phi}{FS} \qquad (2.30)$$

where C = cohesion of the soil

$N'\tan\phi$ = frictional shear strength components of the soil

Substituting Equation (2.30) into (2.29), the following equation can be written:

$$\sum_{i=1}^{n}\left[W + U_{\beta}\cos\beta + Q\cos\delta\right]\times R\sin\alpha - \sum_{i=1}^{n}\left[U_{\beta}\sin\beta + Q\sin\delta\right](R\cos\alpha - h) - \sum_{i=1}^{n}\left[\frac{C + N'\tan\phi}{FS}\right]\times R = 0 \qquad (2.31)$$

If FS is assumed to be the same for all slices, thus FS in Equation (2.31) can be computed by:

$$FS = \frac{\sum_{i=1}^{n} \left(C + N' \tan\phi \right) \times R}{\sum_{i=1}^{n} \left[W + U_\beta \cos\beta + Q \cos\delta \right] R \sin\alpha - \sum_{i=1}^{n} \left[U_\beta \sin\beta + Q \sin\delta \right] \left(R \cos\alpha - h \right)} \qquad (2.32)$$

Where $N' = -U_\alpha + W \cos\alpha + U_\beta \cos(\beta - \alpha) + Q \cos(\delta - \alpha)$

The general formulation in Equation (2.32) is usually used to calculate factor of safety according to the assumptions of OMS.

E. Minimum factor of safety associated with a critical slip surface

In summary, many potential slip surfaces of whatever shape can be analyzed by Limit Equilibrium Methods. Each of these slip surfaces corresponds to different factor of safety FS that can be found by trial and error. In addition to the conventional method based on repeated trials, optimization techniques can be used to search for minimum factor of safety corresponding to critical failure surface and location (Chowdhury et al., 2010). Due to complexities of optimization approaches, minimum of factor of safety is usually solved by computer programs.

In the practical design, Teng (1962) recommended to use factor of safety equal to 3.0 for the cohesive resistance and 2.0 for the friction resistance.

2.4 Structural design of reinforced cantilever retaining wall

Structural design of reinforced concrete cantilever retaining wall is concerned with computing reinforcing steel to provide adequate shear and moment strengths for any applied loads on each wall components. Figure 2.12 illustrates the pressure distribution come from external loads and soil pressures which acts on component parts of the wall.

Based on Figure 2.12, the stem will bend as cantilever, so that tensile reinforcing steel will be placed towards the backfill. Heel slab will bend as cantilever which has tensile face upwards due to net downwards pressure from surcharge, soil weight and self-weight of concrete acting on it. Thus, reinforcing steel in placed at upward. Anyway, for toe slab, since the net pressure will act upwards, reinforcing steel must be placed at the bottom face. The thickness of stem, heel, and toe slab must design to provide sufficient compressive stress for resisting any applied shear stress. Generally, shear force and bending moment generated from acting loads should not be allowed to exceed nominal strength from concrete and steel reinforcement. Otherwise, it will develop large crack at critical section of wall as shown in Figure 2.12.

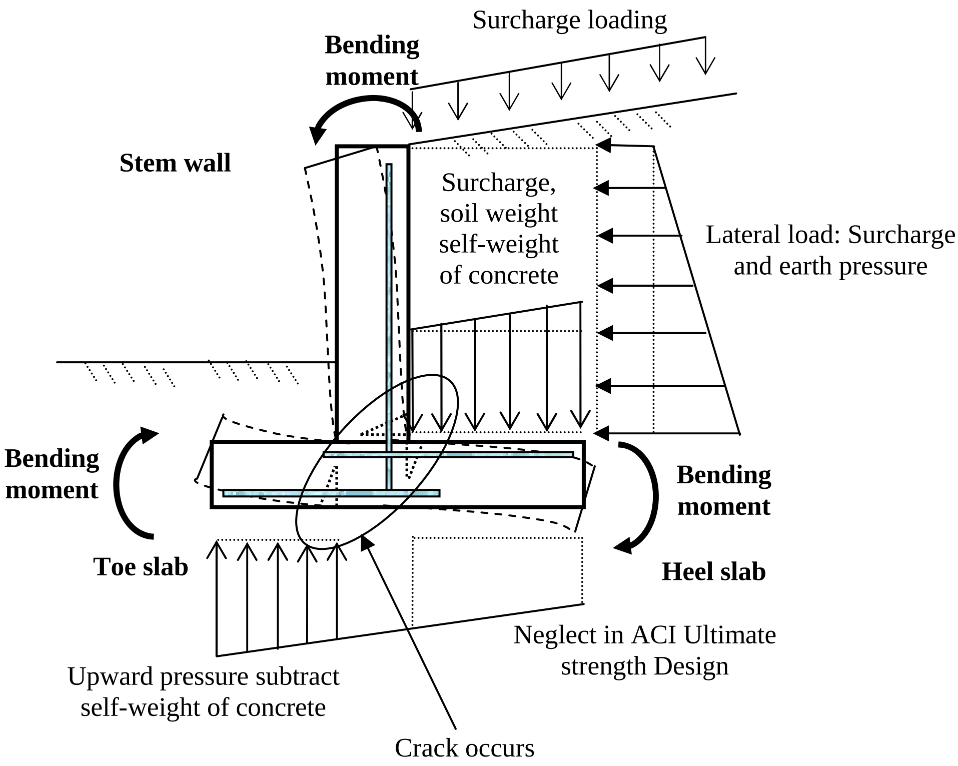

Figure 2.12 Bending and shear failure of reinforced concrete retaining wall

2.4.1 Design load factor based on ACI Code 318-05

When lateral earth pressure H acts with dead load (DL) and live load (LL), ACI Code (2005) specifies that the required strength U be evaluated using the following load factors:

$$U = 1.2D + 1.6L + 1.6H \quad (2.33)$$

From ACI Code (section 9.2.1) in situation where dead load (DL) and live load (LL) reduce the effect of earth pressure (H), the live load is neglect and a factor load of 0.90 is used for the dead load, thus the required strength U can be evaluated as:

$$U = 0.9D + 1.6H \quad (2.34)$$

For any combination of dead load (DL), live load (LL), and earth pressure (H), the required strength U is not to be less than:

$$U = 1.2D + 1.6L \quad (2.35)$$

2.4.2 Design of toe slab

The toe slab is treated as a cantilever beam 1 meter in width fixed at the front face of the wall, with the critical section for the moment at the front face of the wall and the critical section for shear (inclined cracking at a distance d from the front face of the wall (one-way action). In designing of retaining wall, shear force usually controls the adequate thickness of toe. Load factor of 1.60 on soil pressure distribution below base, and 0.90 on the weight of the concrete are used for design load since it reduces the effect of the horizontal earth pressure as recommended by the ACI Code (Section 9.2.1). Weight of soil cover on toe is neglected for conservative design. Detailed expressions of toe slab design are given in appendix A.

2.4.3 Design of heel slab

The applied loads used in computing design moments are the weight of backfill soil, surcharge, and concrete acting downward, along with soil bearing pressure acting upward. The heel slab is treated as a cantilever beam 1 meter in width fixed at the rear face of the wall. Effects of upward pressure under the heel are neglected as recommended by ACI Code (2005). Load factor of 1.20 on the weight of soil and concrete and 1.60 on the weight of any surcharges are used for design loads. Detail expressions of heel slab design are given in appendix A.

2.4.4 Design of stem wall

In stem design, both small axial forces by self-weight of wall and any downward friction (applied by the soil on the back face) are neglected. The stem is assumed to be a cantilever beam 1 meter in width fixed at the top of the footing. Load factor of 1.60 on lateral soil pressures is used for design loads. Normally, the thickness of stem is controlled by the bending moment. Detailed expressions of stem slab design are given in appendix A.

2.5 Design of reinforced concrete beam for flexure (ultimate strength design)

A. Stress distribution and design assumptions

Figure 2.13 illustrates a cross section of a beam with width (b), effective depth (d), and tensile steel (A_s) placed in tension zone at the bottom.

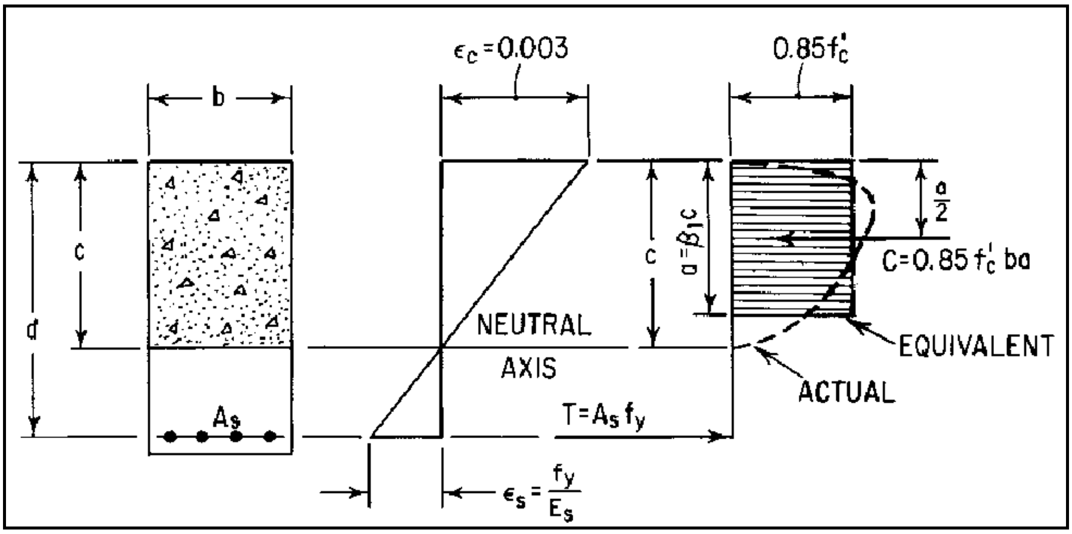

Figure 2.13 Actual and equivalent stress distributions at failure (Ricketts et al. 2003)

Figure 2.13 shows the actual and equivalent stress distributions for tension steel design. In balanced conditions, the concrete reaches its maximum strain of 0.003 while tension steel reaches it yield strength ($f_s = f_y$). The concrete stress of $0.85f'_c$ is assumed to be uniformly distributed over a depth a.

Based on Figure 2.13, the depth of the equivalent rectangular stress block, a, can be calculated by:

$$a = \beta_1 c \tag{2.36}$$

where c = distance between the top of the compressive section and the neutral axis

The factor β_1 can be calculated following the concrete strength as:

$$\beta_1 = \begin{cases} 0.85 & f'_c \le 30 \text{ MPa} \\ 1.09 - 0.008f'_c \quad \text{if} & 30 \text{ MPa} \le f'_c \le 55 \text{ MPa} \\ 0.65 & f'_c \ge 55 \text{ MPa} \end{cases} \tag{2.37}$$

B. Compute tensile steel area for bending moment only

The calculated factored moment, M_u, must not be allowed to exceed nominal flexure strength of a member, ϕM_n, which is stated in Equation (2.38).

$$\phi M_n \ge M_u \tag{2.38}$$

For force equilibrium conditions referred to Figure 2.13, it can be written as:

$$0.85f'_c ab = A_s f_y \tag{2.39}$$

$$a = \frac{A_s f_y}{0.85f'_c\, b} = \frac{\rho f_y d}{0.85f'_c} \tag{2.40}$$

Where $\rho = \frac{A_s}{bd} =$ percentage of tensile steel

Nominal flexure strength provided by reinforcing steel can be computed as:

$$M_n = T\left(d - \frac{a}{2}\right) = A_s f_y\left(d - \frac{a}{2}\right) \tag{2.41}$$

The allowable nominal flexural strength is calculated as:

$$\phi M_n = \phi A_s f_y\left(d - \frac{a}{2}\right) \tag{2.42}$$

$$\phi M_n = \phi A_s f_y d\left(1 - \frac{\rho f_y}{1.7f'_c}\right) \tag{2.43}$$

$$\phi M_n = \phi \rho f_y bd^2\left(1 - \frac{\rho f_y}{1.7f'_c}\right) \tag{2.44}$$

where $\phi = 0.90 =$ for tension controlled section, $\varepsilon_t \geq 0.05$

$\rho = A_s/bd$ = percentage of tensile steel

b,d = width of beam, and effective depth

f_y = yield strength of steel reinforcement

f'_c = compressive strength of concrete

If b and d are known, thus the required reinforcement ratio, ρ, can be determined by equation below:

$$\rho = \frac{0.85f'_c}{f_y}\left(1 - \sqrt{1 - \frac{4M_u}{1.7\phi f'_c\, bd^2}}\right) \tag{2.45}$$

Letting $R_u = \frac{M_u}{\phi bd^2}$ (2.46)

$$\rho = \frac{0.85f'_c}{f_y}\left(1 - \sqrt{1 - \frac{2R_u}{0.85f'_c}}\right) \tag{2.47}$$

A required reinforcing area for design can be calculated as:

$$A_s = \rho \times b \times d \tag{2.48}$$

C. Reinforcing limitations in flexure members

According to ACI Equation 10.3, the minimum amount of tensile reinforcement for preventing a compression failure is computed as:

$$\rho_{min} = 1.4/f_y \tag{2.49}$$

where f_y = yield strength of steel reinforcement (MPa)

The reinforcement ratio, ρ_b, for balanced condition is determined as:

$$\rho_b = \frac{A_{sb}}{bd} = \frac{0.85\beta_1 f'_c}{f_y}\left(\frac{600}{600+f_y}\right) \tag{2.50}$$

where β_1 = rectangular stress block coefficient depends on concrete strength

f'_c = compressive strength of concrete (MPa)

f_y = yield strength of steel reinforcement (MPa)

To ensure concrete beam failing in a ductile manner, ACI Code section 10.3.3 stipulates that the maximum amount of tensile reinforcement, $A_{s,max}$, must not exceed 0.75 of balanced steel area A_{sb}.

Thus, the maximum reinforcement ratio, ρ_{max}, can be computed as:

$$\rho_{max} \le \frac{3}{4}\rho_b \tag{2.51}$$

2.6 Design of reinforced concrete beam for flexure (working stress design)

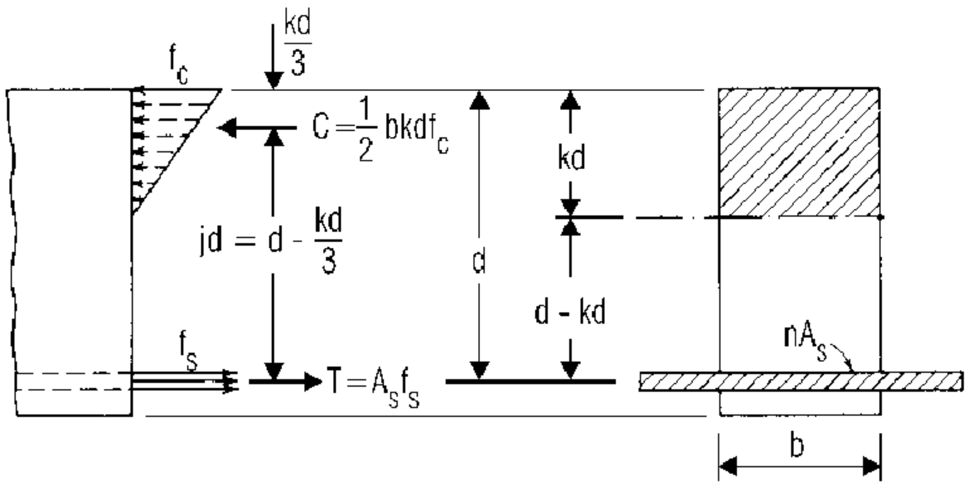

Figure 2.14 Rectangular concrete beam with tension steel only (Ricketts et al. 2003)

Generally, stress distribution in a reinforced concrete beam under service loads is different from that at ultimate strength. The simple procedures of working stress design are presented in the following equations.

Let $n = E_s/E_c$ = nearest integer (2.52)

where $E_s = 200\,000$ MPa = Modulus elasticity of steel

$E_s = 4700\sqrt{f'_c}$ MPa = Modulus elasticity of concrete

For the assumption that stress varies across a beam section with the distance $k \times d$ from neutral axis, value k can be calculated as:

$$k = \frac{1}{1 + f_s/nf_c} \tag{2.53}$$

For Equation (2.53), f_c and f_s can be calculated in the following equations.

$$f_c = 0.45f'_c \tag{2.54}$$

$$f_s = 0.50f_y \tag{2.55}$$

where f'_c = compressive strength of the concrete (MPa)

f_c = allowable compressive stress in extreme surface of concrete (MPa)

f_s = allowable stress in steel (MPa)

f_y = yield strength of steel reinforcement (MPa)

f_y = 140 MPa = allowable stress in steel reinforcement for grade 40, 50

f_y = 170 MPa = allowable stress in steel reinforcement for grade 60

Distance $j \times d$ between the centroid of compression and the centroid of tension can be computed by Equation (2.56) as:

$$j = 1 - k/3 \tag{2.56}$$

The allowable moment resistance of the steel reinforcement in working stress design can be expressed as:

$$M_s = T \times j \times d = A_s \times f_s \times j \times d \tag{2.57}$$

where A_s = cross section area of tensile steel

f_s = allowable stress in steel

$j \times d$ = distance between the centroid of compression and tension

2.7 Design of beam for shear (ultimate strength design)

In safety criterion based on ACI Code, the calculated factored shear force, V_u, must be less than or equal to nominal shear strength of a member, ϕV_n.

$$\phi V_n \geq V_u \tag{2.58}$$

where V_u = shear force due to the factored loads

ϕ = strength reduction factor, taken equal to 0.75 (ACI 318-05 section 9.2)

V_n = nominal shear resistance of a member

From Equation (2.58), nominal shear strength at the considered section is computed by:

$$V_n = V_c + V_s \tag{2.59}$$

where V_c = nominal shear strength provide by concrete

V_s = nominal shear strength provided by stirrups

In most structural design books, stirrups are not used in either the wall or the footing. Whether the shear capacity carried by concrete is adequate or not, it is necessary to increases the thickness members rather than using stirrups (Leet, 1989).

- *Application of shear design in RC cantilever retaining walls*

In previous literature reviews on cantilever retaining wall design, both stem and footing members (toe, heel) are designed as a cantilever beam with strip 1 meter as width. Since stirrups are not usually provided in shear design, shear force generated from factored loads should not be allowed to exceed nominal shear strength of concrete. Equation (2.60) shows the basic safety design criterion as:

$$\phi V_c \geq V_u \quad \text{(in term of shear force)} \tag{2.60}$$

Equation (2.60) can be expressed in term of shear stresses, ν, as:

$$\phi \nu_c \geq \nu_u \tag{2.61}$$

where ϕ = strength reduction factor, taken equal to 0.75 (ACI 318-05)

$\nu_u = V_u / b_w d$ = ultimate shear stress on the cross section (kN/m^2)

$\nu_c = V_c / b_w d$ = nominal shear stress provided by concrete (kN/m^2)

b_w = web width

d = effective depth for shear calculations

Shear strength carried by concrete is generally calculated as:

$$V_c = (1/6)\sqrt{f'_c}\,b_w d \tag{2.62}$$

where V_c = shear strength of concrete (kN)

f'_c = compressive strength of concrete (kN/m^2)

b_w = web width

d = effective depth for shear calculations

2.8 Design of beam for shear (working stress design)

According to ACI (1963), the shear stress applied in reinforced concrete members can be calculated by Equation (2.63) while allowable shear stress of concrete can be calculated by Equation (2.64).

$$\nu_s = V_s / bd \tag{2.63}$$

$$\nu_c = 0.09\sqrt{f'_c} \tag{2.64}$$

where ν_s = shear stress produced by service loads (kN/m^2)

ν_c = allowable shear stress of concrete (kN/m^2)

V_s = shear force produced by service loads (kN)

b = web width

d = effective depth for shear calculations

f'_c = compressive strength of concrete (kN/m^2)

- *Application of shear design in RC cantilever retaining walls*

In previous literature reviews on cantilever retaining wall design, both stem and footing members (toe, heel) are designed as a cantilever beam with strip 1 meter as width. Stirrups are not usually provided for shear design. Thus, shear stress produced by service loads must not less than or equal to the nominal shear stress of concrete. Equation (2.65) shows the basic safety design criterion as:

$$\nu_c \geq \nu_s \tag{2.65}$$

where ν_s = shear stress produced by service loads (kN/m^2)

ν_c = allowable shear stress of concrete (kN/m^2)

2.9 Design of reinforced concrete column by ACI Code

2.9.1 Introduction

A column is the reinforced concrete member which subjected to axial compression with or without bending (McGregor, 2002). These forces are produced by external loads such as dead loads, live loads and wind loads.

Figure 2.15 shows a cross section of column which subjected to combined bending and axial load in column.

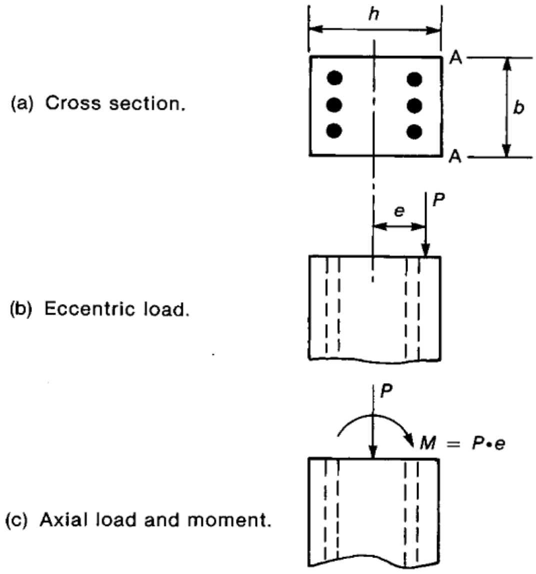

Figure 2.15 Combined bending and axial load in column (McGregor, 2002)

Figure 2.15a shows a cross section of reinforced column in which subjected to axial compression force P and moment M. When axial compression P applies on the cross section, there maybe have a misalignment of this load on the column as shown in Figure 2.15b. Similarly, when moment applies on the cross section, there may be produced a portion of the unbalanced moment at the end of the beams where column acts as support. The ratio of the moment to the axial force is usually referred as the eccentricity of the load. In Figure 2.15b, the eccentric load P can be replaced by a center load P acting in the centroidal axis plus a moment $M = P \times e$ about the centroid. The applied load P and moment M are obtained from structural analysis and they are normally referred to the acting load in the centroidal axis.

According to Nilson (2004), the typical failures modes of a column depend on the value of eccentricity e.

The principle analysis and design of reinforced concrete column are based on the stress equilibrium, compatibility condition, and uniaxial constitutive laws of materials (Hsu, 2010).

Interaction diagram for concrete column

A strength interaction diagram is a better approach for practical design of reinforced concrete column (Nilson, 2004). In the interaction diagram, nominal bending moment, M_n, and nominal axial compression, P_n, are plotted as a unique pair for any given eccentricity. The failure axial loads and failure moments for a column is defined by eccentricity range from zero to infinity. The unique pair of values M-P is plotted on a graph as shown in Figure 2.16.

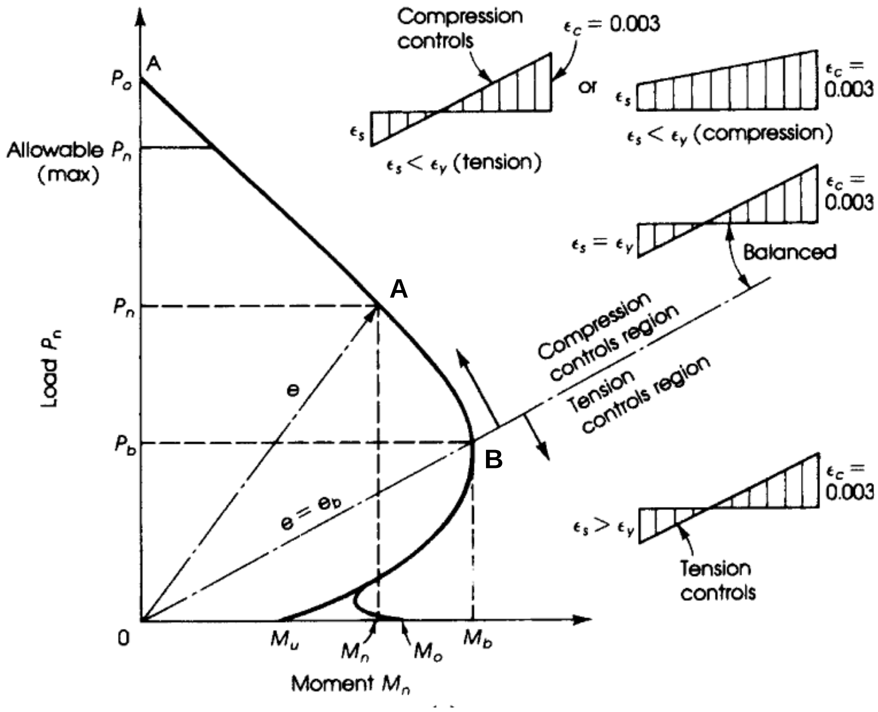

Figure 2.16 Load-moment strength interaction diagram (Hassoun, 2005)

This interaction curve is divided into compression failure region and a tension failure region.

Point A-B represents the compression failure mode which corresponds to small eccentricities. In this region, the concrete will reach its limit strain (ε_u) before the tension steel starts yielding. Compression steel may be yielding.

Point B represents the balanced condition. The balanced failure mode corresponds to balanced eccentricity (e_b) with the balanced nominal load (P_b) and bending moment (M_b). The concrete strain reaches its limitation (ε_u). At the same time, tensile steel of the column reaches yield strain (ε_{st}).

Point B-M_0 represents the tension failure mode which corresponds to large eccentricity. In this region, the concrete reaches its ultimate strain (ε_u). The tensile steel of the column reaches yield strain (ε_{st}) while the compression steel may or may not have yielded.

The theoretical case assuming that a large axial load P_0 is acting at the plastic centroid where e = 0 and M_n = 0. However, ACI Sections 10.3.6.1 and 10.3.6.2 permit to use maximum axial load capacity of a column equaled to 0.85 times of that from centroid P_0 for tied-columns.

$$\phi P_{n(max)} = 0.80\phi\left[0.85f'_c\left(A_g - A_{st}\right) + f_y\left(A_{st}\right)\right] \tag{2.66}$$

where ϕ = strength reduction factor

$P_{n(max)}$ = the maximum nominal strength of the column cross section

A_g = gross area of column

A_{st} = cross section area of longitudinal steel reinforcement

f'_c = compressive strength of concrete

2.9.2 Strength design method for columns

For column design according to the ACI Code, the nominal strengths multiplying with reduction factors must be greater than or equal to the design strength calculated by load factors. Equation (2.67) and (2.68) shows the basic safety design criterion as:

$$\phi P_n \geq P_u \tag{2.67}$$

$$\phi M_n \geq M_u \tag{2.68}$$

where P_u, M_u = factored load and moment applied to the column, computed from a structural analysis

P_n, M_n = nominal strength of the column cross section

ϕ = strength reduction factor

2.9.3 Investigation of column strength in compression-controlled region

In compression controlled region, compression strength P_n exceeds the balanced strength P_b or when the eccentricity e is less than the balanced value e_b.

Figure 2.17 shows compression failure in a column section. In this region, the concrete strain (ε_u) reaches its maximum value of 0.003, while the strain in tensile steel (ε_s) is less than maximum yield strain (ε_y). Since the strain in compressive steel (ε'_s) reaches the maximum yield strain, yield strength of compressive can be taken the same as the maximum yield strength ($f'_s = f_y$).

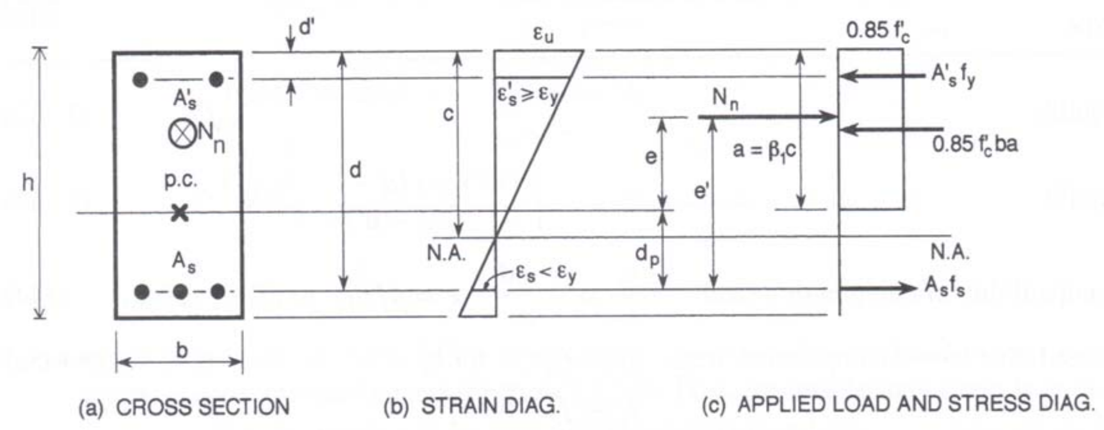

Figure 2.17 Compression controlled failure in column section (Hsu, 2010)

Based on Figure 2.17, three types of forces are normally taken into account, namely, compressive force carried out by concrete (C_c), compressive steel (C_s) (top), and tensile steel (T_s) (bottom). The expressions of these forces can be written as:

1. Compressive force carried by concrete

$$C_c = (0.85f'c)(\beta_1 c)(ba) \tag{2.69}$$

2. The force carried by the top steel

$$C_s = (A'_s)(f's - 0.85f'c) \tag{2.70}$$

3. The force carried by the bottom steel

$$T_s = (f_s)(A_s) \tag{2.71}$$

Use equilibrium condition, P_n can be calculated as:

$$P_n = C_c + C_s - T_s \tag{2.72}$$

By taking moment on neutral axis based on free body diagram in Figure 2.17

$$P_n \times e = C_c\left(\frac{h}{2}-\frac{a}{2}\right)+C_s\left(\frac{h}{2}-d'\right)+T_s\left(d-\frac{h}{2}\right) \quad (2.73)$$

Or $$M_n = P_n \times e = C_c\left(\frac{h}{2}-\frac{a}{2}\right)+C_s\left(\frac{h}{2}-d'\right)+T_s\left(d-\frac{h}{2}\right) \quad (2.74)$$

Thus, nominal axial compression P_n and bending moment M_n can be calculated based on Equation (2.73) and (2.74), respectively.

However, yielding strength of compression steel and tensile steel can be computed by strain compatibility equation as their values is varied from compression controlled region to tensile controlled region.

Table 2.4 List of strain limit table in column design

Section Condition	**Concrete Strain**	**Steel Strain**	**If f_y = 420 MPa**
Compression controlled	0.003	$\varepsilon_t \le fy/E_s$	$\varepsilon_t \le 0.002$
Tension controlled	0.003	$\varepsilon_t \ge 0.005$	$\varepsilon_t \ge 0.005$
Transition controlled	0.003	$fy/E_s \le \varepsilon_t \le 0.005$	$0.002 \le \varepsilon_t \le 0.005$
Balanced strain	0.003	$\varepsilon_s = fy/E_s$	$\varepsilon_s = 0.002$
Transition region (flexure)	0.003	$0.004 \le \varepsilon_t \le 0.005$	$0.004 \le \varepsilon_t \le 0.005$

Table 2.4 presents the strain limitation in column design if yielding strength 420 MPa is used. Based on this table, yielding strength of compression steel and tensile steel can be determined for each failure condition.

Nilson (2004) stated that it is sometimes more convenient to compute the nominal moment capacity in function of ratio c/d rather than the net tensile strain.

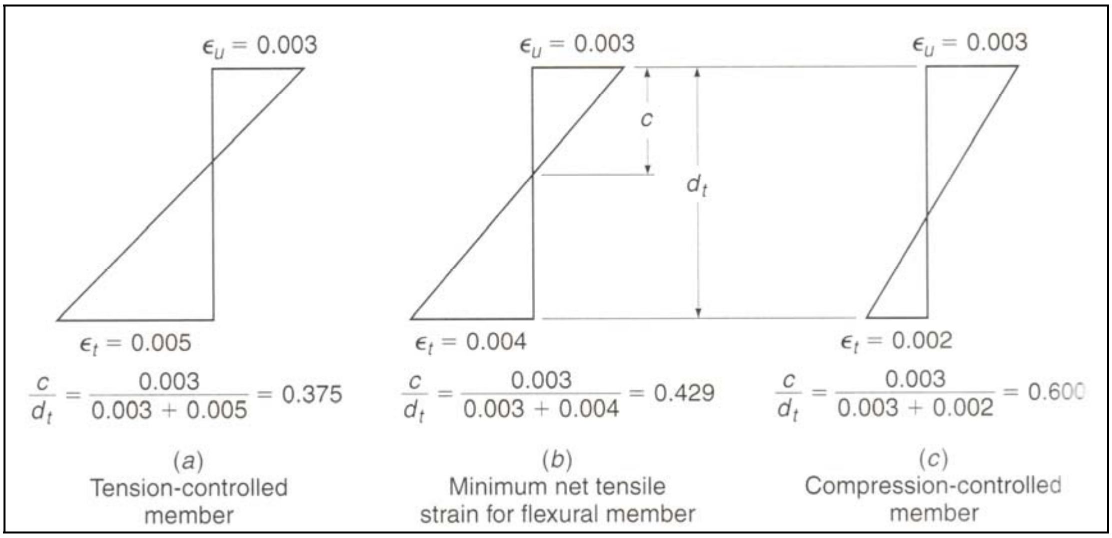

Figure 2.18 Net tensile strain in column section (Nilson, 2004)

Figure 2.18 shows the ratio of c/d_t which corresponds to the three regions of reinforced concrete column members.

More details of iterative procedure for analysis are given in appendix B.

2.9.4 Strength reduction factors

The ACI Code (2005) provides basic reduction factors for members subjected to axial compression as:

- *0.65* for compression controlled region. A value of *0.70* may be used if members are spiral section
- *0.90* for tension controlled region (both spiral and other section)
- Varies linearly between *0.65* (or *0.70*) and *0.90* for transition region using Equation (2.75) and (2.76).

$$\phi = 0.567 + 66.7\varepsilon_t \quad \text{(spiral section)} \tag{2.75}$$

$$\phi = 0.483 + 83.3\varepsilon_t \quad \text{(other section)} \tag{2.76}$$

Figure 2.19 presents the variation of strength reduction factor in compression, transition, and tension controlled regions.

2.9.5 Reinforcement ratio permitted in column design

ACI Code recommended that longitudinal reinforcement area, $A_{st,}$ in tied column should not less than 0.01 times and not more than 0.08 times the gross area A_g. The expression of reinforcement ratio is given as:

$$\rho_t = A_{st}/A_g \tag{2.77}$$

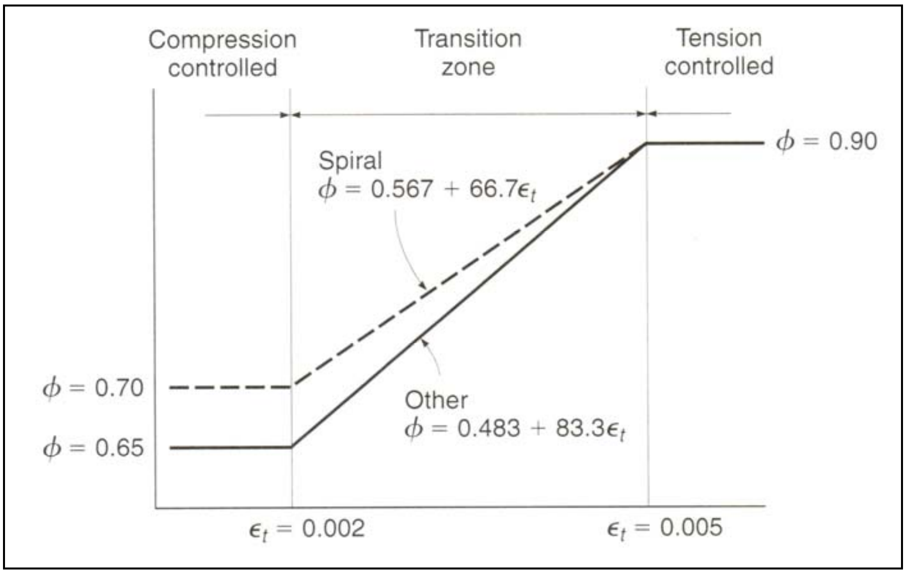

Figure 2.19 Variation of strength reduction factor with net tensile strain (Nilson, 2004)

McGregor (2002) stated that reinforcement ratio of most economical tied-column section generally ranged from 1 to 2 percent.

In a rectangular column, minimum number of bars is limited to four according to the ACI Section 10.9.2. In simple practice, it is generally to use an even number of bars. Particularly, all bars have the same size (McGregor, 2002).

2.9.6 Slenderness ratio

A slender column deflects laterally under any applied load. This load will increases the moments in the column and hence weakens the column (MacGregor, 2002). In that case, column is designed as slender column which takes into account of slenderness ratio.

According to ACI Section 10.12.2, a slenderness ratio in braced frames is neglected if the expression in Equation (2.78) is satisfied.

$$\frac{k\ell_u}{r} \leq 34 - 12\left(M1/M2\right) \tag{2.78}$$

where k = effective length factor

ℓ_u = unsupported height of column from top of floor to the bottom of the beams or slab in the floor above r = the radius of gyration, equal to 0.3 and 0.25 times the overall depth of rectangular and circular columns, respectively

$M1/M2$ = the ratio of the moments at the two ends of the column

The range of effective length factor, k, for different columns and frames is illustrated in Figure 2.20.

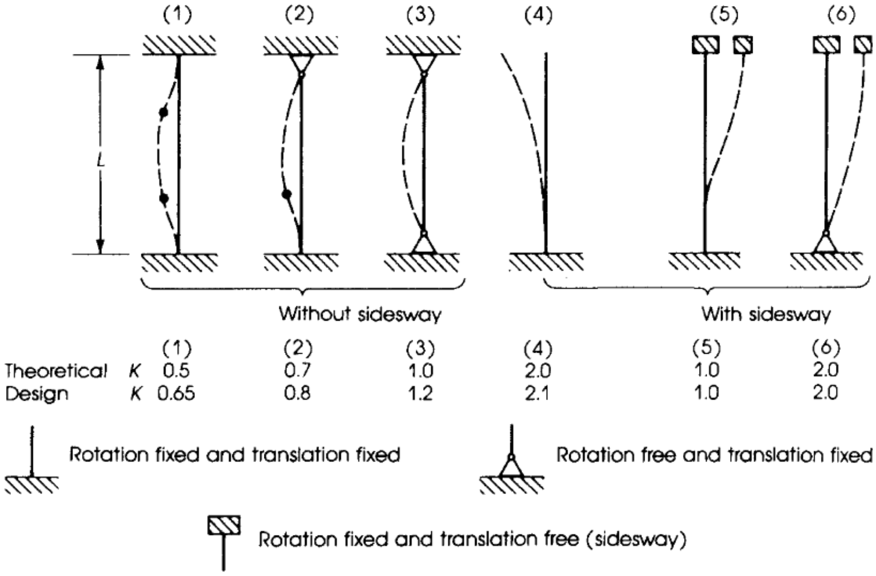

Figure 2.20 Effective lengths of columns and length factor k (Hassoun, 2005)

2.9.7 Shear force in columns

According to ACI Code section 11.3.1.2, the shear force carried by the concrete for a member subjected to axial compression is expressed as:

$$V_c = \left(1 + \frac{N_u}{14A_g}\right)\left(\frac{\sqrt{f'_c}}{6}\right)b_w d \qquad (2.79)$$

where V_c = shear forced carried by concrete (kN)

N_u = the factored axial force (kN)

A_g = gross area of column

f'_c = compressive strength of concrete (kN/m^2)

b_w = web width

d = effective depth of column

ACI Sections 7.10.5.1, 7.10.5.2, and 7.10.5.3 specify that the maximum tie size is No. 10M bar for longitudinal bars up to No. 32 and a No. 13M bar for larger longitudinal bars or for bundled bars. The vertical spacing of ties shall less than or equal to 16 times of longitudinal bar diameters, and shall not exceed 48 tie diameters. The maximum spacing is also limited to the least dimension of the column. In seismic regions, much closer spacings are required as mention in ACI section 21.4.4.

- For ties design without providing additional shear reinforcement, ACI Code recommended that applied factored shearing forces, V_u, should not greater than $\phi V_c/2$.
- If the shear V_u exceeds $\phi V_c/2$, shear reinforcement are additionally provided.
- If the shear V_u is greater than $0.5\phi V_c$ and smaller than ϕV_c ($0.5\phi V_c < V_u < \phi V_c$), it would be necessary to satisfy ACI Sections 7.10.5, 11.5.4.1, and 11.5.5.3.

2.10 Optimization theory

2.10.1 Introduction

In general, optimization is the process of finding something that is as effective as possible or is the act of creating the best result under a prescribed set of conditions (Rao, 2009). From a mathematical perspective, optimization deals with finding the maxima or minima of a function with one or more variables. The minima and maxima in mathematical definition are shown in Figure 2.21.

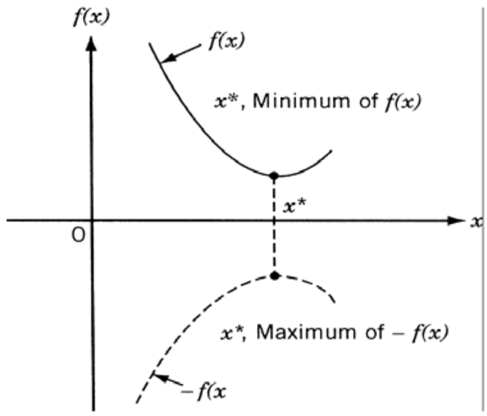

Figure 2.21 Minimum of f(x) and maximum of –f(x) at point x*

In application of optimization into engineering problems, optimization can be applied to solve civil engineering structures such as frames, foundations, bridges, towers, chimneys, and dams for minimum cost (Rao, 2009).

2.10.2 Problem statement of an optimization

An optimization or a mathematical programming problem can be stated as:

$$\text{Find } X = \begin{Bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{Bmatrix} \quad \text{which minimizes } f(X) \tag{2.80}$$

$$\text{Subject to the constraints} \quad g_i(X) \leq 0, \quad i=1,2,\ldots m \tag{2.81}$$

$$L_j(X) = 0, \quad j=1,2,\ldots p \tag{2.82}$$

where X = an n-dimensional vector called the design variable vector

f(X) = term of objective function

$g_i(X)$, $L_j(X)$ = inequality and equality constraints, respectively

The problem in Equation (2.80) can be called as a constrained optimization problem because there are equality constraints L(X) and inequality constraints g(X). Some optimizations that do not have any constraint as in Equation (2.81) and (2.82) can be called as unconstrained optimization problems.

Three major components of an optimization problem are design variables, design constraints, and objective function.

A. Design variables

In any engineering system, a set of quantities which are usually fixed in design process are called as *preassigned parameters* while other certain quantities which treat as variables are called as *design or decision variables*. Thus, the design variables can be represented as one set of design vectors referred to programming problem definition in Equation (2.80).

B. Design constraints

In many practical problems, the design variables are chosen if they satisfy certain specified functional and other requirements. Thus, those requirements that make the design variables satisfy in order to produce an acceptable result are called as *design constraints*. In this sense, constraints that impose physical limitations on design variables such as availability, fabricability, and transportability can be defined as *geometric or side constraints*.

C. Objective function

An *objective function* is the main criterion which was taken to minimize or maximize. Its expression is function of the design variables.

Rao (2009) reported that in civil engineering structural design, the main aim of optimization is to minimize the cost.

D. Constraint surface

This section will present the boundary of constraint surface in two dimensional design spaces and its shape.

Consider an optimization problem with only inequality constraints which represent as $g_j(X) \leq 0$. The design variables X that satisfy the equality constraints $[g_j(X)=0]$ forms a boundary surface in the design space called as *constraint surface*. The constraint surface divides the design space into two regions. The feasible region where constraints are satisfied is referred to $g_j(X) < 0$, and the infeasible region where constraints are violated is referred to $g_j(X) > 0$.

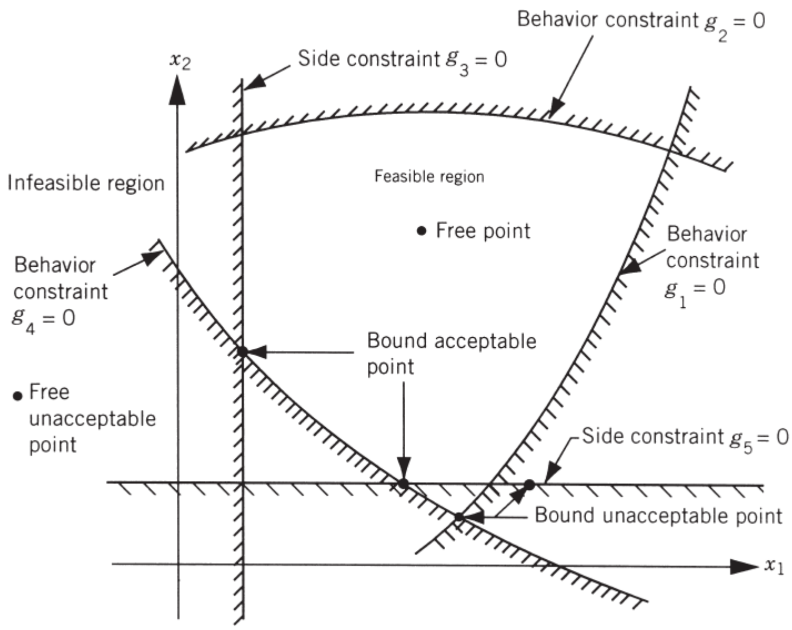

Figure 2.22 Boundary points of optimization (Rao, 2010)

Figure 2.22 shows a two-dimensional design space where the infeasible region is sketched by hatched lines. In case that the constraints are linear such as g_3 and g_5, they are plotted as straight lines as shown in figure. However, when constraints are nonlinear such as g_1, g_2, and g_4, they are constructed as curve following that case. A design point that lies on one or more than one constraint surface is called as *bound points*, and the associated constraint is called as an *active constraint. Free point*s are defined as those that lie in feasible region and do not lie on any constraint surface.

2.10.3 Classic optimization approaches

The classical methods of optimization becomes useful in calculating the optimum solution if the function to be optimized are continuous or differentiable. These methods are based on technique of differential calculus in finding the optimum point.

A. Theorem 1: necessary condition

In calculus, the first derivative test uses the first derivative of a function to determine whether a given critical point of a function is a local maximum, a local minimum, or neither.

Consider a function f(X) is continuous or differentiable with n-variables. The necessary condition stated that if the first partial derivatives of $f(X^*)$ equal to zero, then f(X) has a either relative minimum or maximum at $X=X^*$.

The theorem can be expressed as:

$$\frac{\partial f}{\partial x_1}\left(X^*\right)=\frac{\partial f}{\partial x_2}\left(X^*\right)=\ldots=\frac{\partial f}{\partial x_n}\left(X^*\right)=0 \qquad (2.83)$$

In general, a point X^* satisfies Equation (2.83) is called *stationary point*. This point can be a minimum or maximum.

B. Theorem 2: sufficient condition

The sufficient condition for the minimum or maximum value of the function f(X) at $X=X^*$ can be stated by evaluating the matrix of the second partial derivatives (Hessian matrix) at $X=X^*$.

The Hessian matrix of the second partial derivatives can be written in quadratic form as:

$$H = \sum_{i=1}^{n}\sum_{j=1}^{n} h_i h_j \frac{\partial^2 f}{\partial X_i \partial X_j} \tag{2.84}$$

(i) If the Hessian is positive definite, then X^* is a relative minimum point

(ii) If the Hessian is negative definite, then X^* is a relative maximum point

(iii) If the Hessian have both positive and negative, then X^* is a saddle point

(iv) If the Hessian is equal to zero, then X^* is inclusive

2.11 Cost optimization of concrete structures

Adeli (2006) reported that optimization of concrete structures usually deal with cost minimization since different materials are used. Concrete structures could include reinforced concrete, prestressed concrete, and fiber-reinforced concrete structures. In concrete structure at least three different cost items, namely, cost of concrete, steel, and formwork should be considered in optimization.

The general cost function for beam structures (including reinforced, fiber, or prestressed beam) can be written as:

$$Cost_m = Cost_{cb} + Cost_{sb} + Cost_{pb} + Cost_{fb} + Cost_{sbv} + Cost_{fib} \tag{2.85}$$

where $Cost_m$ = the total cost of material in beam

$Cost_{cb}$ = the total cost of concrete

$Cost_{sb}$ = the total cost of steel reinforcement

$Cost_{pb}$ = the total cost of prestressing steel

$Cost_{fb}$ = the total cost of the formwork

$Cost_{sbv}$ = the total cost of shear steel

$Cost_{fib}$ = the total cost of fiber in concrete

In simplified way, Equation (2.85) can be expressed as:

$$\begin{aligned} Cost_m &= \omega_{con} L_{beam}\left(A_{conb} - A_{stb} - A'_{shb} - A_{preb}\right)U_{con} + \omega_{st} L_{beam}\left(A_{conb} + A'_{stb}\right)U_{st} \\ &\quad + \omega_{pre} L_{beam} A_{preb} U_{pre} + L_{beam} p_{forb} U_{for} + Cost_{sbv} + Cost_{fib} \end{aligned} \tag{2.86}$$

where $Cost_m$ = the total cost of material in beam

L_{beam} = length of the beam

$\omega_{con}, \omega_{st}, \omega_{pre}$ = unit weights of concrete, reinforcing steel, and prestressing steel respectively

$A_{conb}, A_{stb}, A'_{stb}, A_{preb}$ = cross sectional areas of concrete, reinforcing steel, compression reinforcing steel, and prestressing steel, in beam structures respectively

U_{con}, U_{st}, U_{pre}, U_{for} = unit cost of concrete, reinforcing steel, prestressing steel, and formwork, respectively

p_{forb} = cross sectional perimeter of the beam form

The general cost function for column structures (including reinforced or prestressed column) can be expressed as:

$$Cost_{total} = Cost_{cc} + Cost_{sc} + Cost_{pc} + Cost_{fc} + Cost_{tc} \tag{2.87}$$

where $Cost_m$ = the total cost of material in column

$Cost_{cc}$ = the total cost of concrete

$Cost_{sc}$ = the total cost of reinforcing steel

$Cost_{pc}$ = the total cost of prestressing steel

$Cost_{fc}$ = the total cost of formwork

$Cost_{tc}$ = the total cost of lateral stirrups

Equation (2.87) can be written in simplified way as:

$$\begin{aligned} Cost_m &= \omega_{con} H_{column} \left(A_{conc} - A_{stc} - A_{prec}\right) U_{con} + \omega_{st} H_{column} A_{stc} U_{st} \\ &+ \omega_{pre} H_{column} A_{prec} U_{pre} + H_{column} p_{forc} U_{for} + V_{stirrupc} U_{st} \end{aligned} \tag{2.88}$$

where $Cost_m$ = the total cost of material in column

H_{column} = height of the column

A_{conc}, A_{stc}, A_{prec} = cross sectional areas of concrete, reinforcing steel, and prestressing steel, in column structures respectively

U_{con}, U_{st}, U_{pre}, U_{for} = unit cost of concrete, reinforcing steel, prestressing steel, and formwork, respectively

p_{forb} = cross sectional perimeter of the column form

$V_{stirrupc}$ = volume of the lateral stirrups

2.12 Optimization problem solving techniques

As optimization techniques choose the best solution from a set of many acceptable design variables, the best solution of design variables must satisfy only the functional and other constrained restrictions.

Although optimization techniques can sometimes be calculated analytically, most practical optimization problems require computer methods as primarily solving tools (Rao, 2000). In this sense, some solver algorithms in optimization problems have been developed in commercial software packages. In this thesis, build-in function solvers, namely, MAPLE, MATLAB, ISML FORTRAN, and KNITRO will be used to find minimum of an objective function.

2.12.1 Mathwork's MATLAB

MATLAB's build-in **"***fmincon***"** command solves a minimum of constrained nonlinear multivariable function. It uses sequential quadratic programming (SQP) optimization algorithm as the optimization searching technique (Mathwork, 2010).

The characteristics of *fmincon* command are defined by Equation (2.89) as below:

Find a minimum of a constrained nonlinear multivariable function, *f(x)*

$$\min_{x} f\left(x\right) \text{ subject to } \begin{cases} c(x) \leq 0 \\ ceq(x) = 0 \\ A.x \leq b \\ Aeq.x = beq \\ lb \leq x \leq ub \end{cases} \tag{2.89}$$

Where x, b, beq, lb, ub = vectors,

A, Aeq = matrices,

c(x), ceq(x) = functions that return vectors

f(x) = function that returns a scalar

f(x), c(x), ceq(x) = nonlinear functions.

Syntax

x = fmincon(fun,x0,A,b,Aeq,beq,lb,ub,nonlcon,options)

where fun= the function to be minimized

x0 = initial point for x

A = matrix for linear inequality constraints

b = vector for linear inequality constraints

Aeq = matrix for linear equality constraints

beq = vector for linear equality constraints

lb,ub = vector of lower bounds and upper bounds

nonlcon = nonlinear constraint function

options = options structure

2.12.2 Maplesoft MAPLE

The *NLPSolve* command solves a nonlinear constrained optimization, which computes a real-value objective function. Constrained optimization problems can be solvable in case that the objective function and the constraints is twice continuously differentiation. Even though these conditions are not met, *NLPSolve* sometimes still finding to solutions.

The characteristic of *NLPSolve* command is detailed as:

Optimization [NLPSolve] = solve a nonlinear program

Calling sequence

NLPSolve (obj, constr, bd, opts)

NLPSolve (opfobj, ineqcon, eqcon, opfbd, opts)

Parameters

obj	algebraic; objective function
constr	(optional) set(relation) or list(relation); constraints
bd	(optional) sequence of name = range; bounds for one or more variables
opfobj	procedure; objective function
ineqcon	(optional) set(procedure) or list(procedure); inequality constraints
eqcon	(optional) set(procedure) or list(procedure); equality constraints
opfbd	(optional) sequence of ranges; bounds for all variables
opts	(optional) equation(s) of the form option = value where option is one of assume, feasibilitytolerance, infinitebound, initialpoint, iterationlimit, maximize, method, optimalitytolerance, or output; specify options for the NLPSolve command.

2.12.3 IMSL FORTRAN

The routine DNCONF solves a nonlinear programming problem using the successive quadratic programming algorithm (SQP). This routine was developed by Schittkowski (1986). The following description presents the usage and arguments of this routine in FORTRAN which can be found in IMSL Math/Library (1997).

The problem is defined as follow:

$$\min_{x \in R^n} f(x)$$

$$\text{subject to} \quad \begin{cases} g_j(x) = 0 & \text{for } j = 1, \ldots., m \\ g_j(x) \geq 0 & \text{for } j = m_e + 1, \ldots., m \\ x_l \leq x \leq x_u \end{cases} \quad (2.90)$$

where all problem functions are assumed to be continuously differentiable

Usage

CALL DNCONF (FCN, M, ME, N, XGUESS, IBTYPE, XLB, XUB, XSCALE, IPRINT, MAXITN, X, FVALE)

Arguments

FCN User-supplied SUBROUTINE to evaluate the functions at a given point

The usage is CALL FCN (M, ME, N, X, ACTIVE, F, G)

where M = Total number of constraints (Input)

ME = Number of equality constraints (Input)

N = Number of variables (Input)

X = The point at which the functions are evaluated (Input)

X should not be changed by FCN

ACTIVE = Logical vector of length MMAX indicating the active constraints (input)

MMAX = MAX (1, M)

F = The computed function value at the point X (Output)

G = Vector of length MMAX containing the values of constraints at point X (Output)

FCN must be declared EXTERNAL in the calling program.

M Total number of constraints (Input)

ME Number of equality constraints (Input)

N Number of variables (Input)

XGUESS Vector of length N containing an initial guess of the computed solution (Input)

IBTYPE Scalar indicating the types of bounds on variables (Input)

IBTYPE	Action
0	User will supply all the bounds
1	All variables are nonnegative
2	All variables are nonpositive
3	User supplies only the bounds on 1st variable; all other variables will have the same bounds

XLB Vector of length N containing the lower bounds on variables (Input, if IBTYPE = 0; output, if IBTYPE = 1 or 2; input/output, if IBTYPE = 3)

If there is no lower bound for a variable, then the corresponding XLB value should be set to -1.0E6.

XUB Vector of length N containing the upper bounds on variables (Input, if IBTYPE = 0; output, if IBTYPE = 1 or 2; input/output, if IBTYPE = 3)

If there is no upper bound for a variable, then the corresponding XLB value should be set to 1.0E6.

XSCALE Vector of length N containing the diagonal scaling matrix for the variables (Input)

All values of XSCALE must be greater than zero. In the absence of other information, set all entries to 1.0.

IPRINT Parameter indicating the desired output level (Input)

IPRINT	Action
0	No output printed
1	Only a final convergence analysis is given
2	One line of intermediate results are printed in each iteration
3	Detailed information is printed in each iteration

MAXITN Maximum number of iterations allowed (Input)

X Vector of length N containing the computed solution (Output)

FVALUE Scalar containing the value of the objective function at the computed solution (Output)

2.12.4 Ziena's Optimization KNITRO

KNITRO is a software package for solving large scale mathematical optimization problems (from http://www.ziena.com). It is specialized in solving the nonlinear problems powerfully. *KNITRO* optimization solvers can be written in *C, C++, FORTRAN, or Java* as a software routine to solve the problem.

KNITRO stands for "*N*onlinear *I*nterior point *T*rust *R*egion *O*ptimization" while the "*K*" is silent.

The characteristic of *KNITRO* command is defined as:

$$\min_{x \in R^n} f(x)$$

$$\text{subject to} \quad \begin{cases} g_i(x) \geq 0 & i \in I \\ h_i(x) = 0 & i \in E \\ l_i \leq x_i \leq u_i & i = 1, 2, \ldots.n \end{cases} \tag{2.91}$$

where I, E = finite non negative integers subsets

f, g_i, h_i = objective function assumed to be twice differentiable

CHAPTER III
RESEARCH METHODOLOGY

3.1 Design variables

Figure 3.1 shows problem definition and design variables related to geometry of wall dimensions and cross section of main bars area.

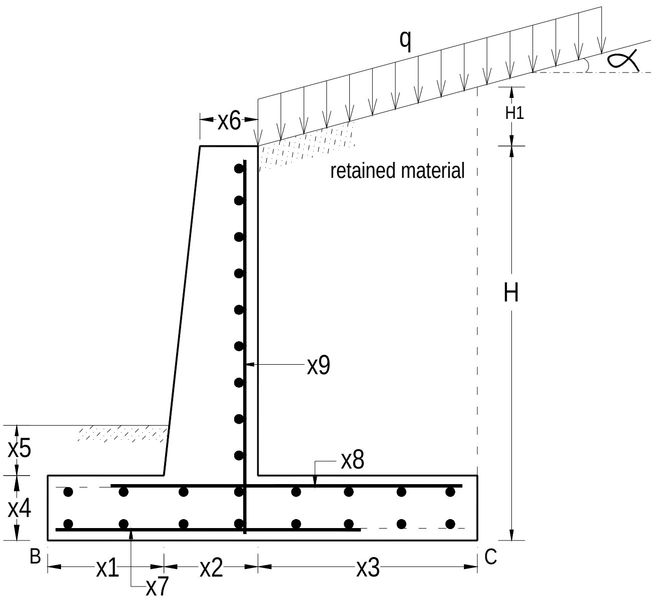

Figure 3.1 Cross section of the RC cantilever retaining wall used for optimum design

Nine design variables are taken into consideration. These include the following:

1) X_1 total width of toe (m)
2) X_2 stem thickness at bottom (m)
3) X_3 total width of heel (m)
4) X_4 thickness of base slab (m)
5) X_5 soil cover (m)
6) X_6 stem thickness at top (m)
7) X_7 horizontal reinforcing area of the toe per unit length of wall (mm^2/m)
8) X_8 horizontal reinforcing area of the heel per unit length of wall (mm^2/m)
9) X_9 vertical reinforcing area of the stem per unit length of wall (mm^2/m)

3.2 Objective function of reinforced retaining wall

Objective function for the analysis is the total cost of material in retaining wall which includes total cost of concrete, steel reinforcement and formwork. The total cost of retaining wall is defined as:

$$f(X) = C_{con} \times V_{con} + C_{st} \times W_{st} + C_{fw} \times S_{fw} \quad (3.1)$$

where C_{con} = cost of concrete per 1 cubic meter (฿/m^3)

V_{con} = volume of concrete (m^3)

C_{st} = cost of steel per 1 kilogram (฿/kg)

W_{st} = weight of steel (kg)

C_{fw} = cost of formwork per 1 square meter (฿/m^2)

S_{fw} = area of formwork (m^2)

3.2.1 Concrete volume and steel weight calculation

Figure 3.2a shows a cross section of reinforced concrete beam with length, L, width, b, and height, h. A_s denotes tensile steel area placing along the beam length as shown in Figure 3.2b.

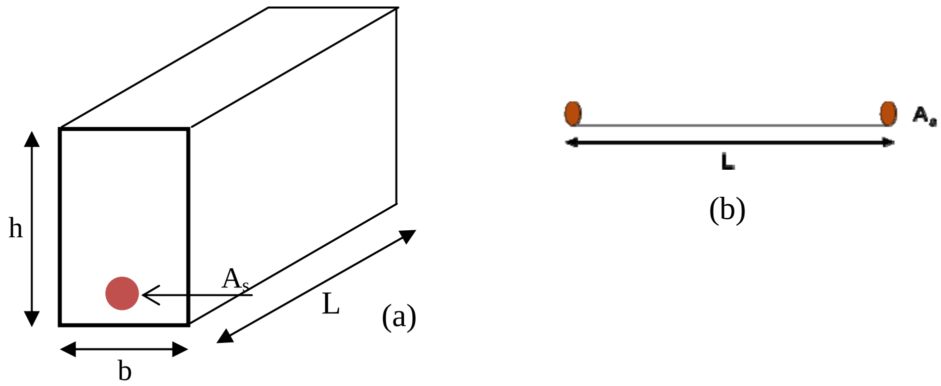

Figure 3.2 (a) Shape of a reinforced concrete beam (b) steel bar cross section

Volume of concrete can be calculated as:

$$V_{con} = b \times h \times L \quad (3.2)$$

Where b = width of reinforced concrete beam

h = height of concrete beam

L = length of beam

Weight of steel is calculated by:

$$V_{st} = A_{st} \times L$$
$$W_{st} = V_{st} \times \gamma_{st} \qquad (3.3)$$

where V_{st} = volume of steel

A_{st} = tensile steel area

γ_{st} = unit weight of steel = 7850 kg/m^3

3.2.2 Formwork area calculation

Area of formwork can be calculated per 1 meter of wall length based on Figure 3.3 as follow:

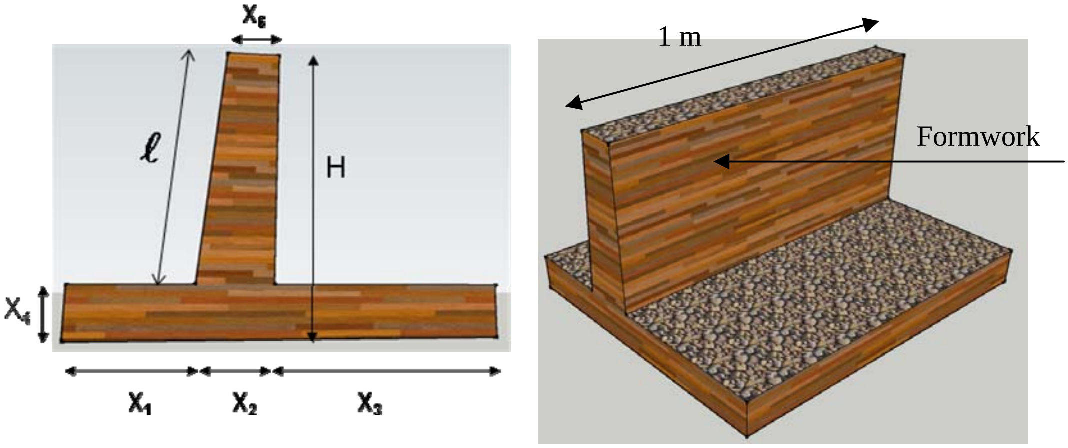

Figure 3.3 Formwork calculation

$$S_f = H + X_4 + \ell = H + X_4 + \sqrt{(X_2 - X_6)^2 + (H - X_4)^2} \qquad (3.4)$$

where H = height of retaining wall

ℓ = length of inclined front face of stem

X_1 = length of toe slab

X_2 = bottom stem thickness

X_3 = length of heel slab

X_4 = thickness of footing

X_6 = top stem thickness

It should be noted that the cost of formwork should be calculated at two vertical area of the base and at inclined and vertical surface of the stem. The bottom area of the base, the top area of the base, and the top area of the stem are not included in calculation of the total cost since formworks in those areas are not required during construction.

By applying expressions from Equation (3.2) until Equation (3.4) for retaining wall, the general expression of objective function (with main bars only) can be written as:

$$\begin{aligned} F = {} & \frac{1}{2}(H - x_4)(x_2 + x_6) + (x_1 + x_2 + x_3)(x_4) + \left(x_7 * 10^{-6}\right)(x_1 + x_2 + x_3)(\gamma_{st}) \\ & + \left(x_8 * 10^{-6}\right)(x_1 + x_2 + x_3)(\gamma_{st}) + \left(x_9 * 10^{-6}\right)(H)(\gamma_{st}) \\ & + H + X_4 + \sqrt{(X_2 - X_6)^2 + (H - X_4)^2} \end{aligned} \tag{3.5}$$

where X_7, X_8, X_9 = reinforcing area in square millimeter (mm^2)

3.3 Cost of material in Thailand

Table 3.1 and 3.2 list the unit price of concrete, steel reinforcement of type SD40 in Thailand according to Bureau of Trade and Economics indices Ministry of Commerce Thailand (http://www.price.moc.go.th). The cost of formwork is taken as 150 B per square meter based on the average cost from contractors in Thailand while it is not reported by the Bureau of Trade and Economics.

Table 3.1 Unit price index of concrete

Strength of concrete	**Unit cost (฿/m^3)**						
	2005	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**
17 MPa	2337.5	2470	2470	2470	2470	2470	2470
21 MPa	2377.5	2510	2510	2510	2510	2510	2510
25 MPa	2417.5	2550	2550	2550	2550	2550	2550
28 MPa	2497.5	2630	2630	2630	2630	2630	2630
31 MPa	2547.5	2680	2680	2680	2680	2680	2680
32 MPa	2607	2740	2740	2740	2740	2740	2740
35 MPa	2677	2810	2810	2810	2810	2810	2810

The unit price of concrete depends on its strength ranging from 19 MPa to 35 MPa.

Table 3.2 Unit price index of steel reinforcement SD40

Diameter of reinforcement	Unit cost (฿/kg)						
	2005	2006	2007	2008	2009	2010	2011
12 mm	17.62	19.58	23.28	18.92	19.10	20.56	22.56
16 mm	17.42	19.38	23.08	18.70	18.90	20.37	22.40
20 mm	17.42	19.38	23.08	18.70	18.90	20.37	22.40
25 mm	17.42	19.38	23.08	18.70	18.90	20.37	22.40
28 mm	17.42	19.38	23.08	18.70	18.90	20.37	22.40

The unit price of steel reinforcement also depends on its diameters ranging from 12 mm to 28 mm.

3.4 Formulation of design constraints for retaining wall

3.4.1 Overturning stability constraint

Factor of safety against overturning can be written as the ratio of the sum of the resisting moment (M_R) about point B to that of the driving moment (M_D) about point B in Figure 3.1.

$$FS_{ov} = \sum M_R \Big/ \sum M_{ov} \tag{3.6}$$

3.4.2 Sliding stability constraints

Factor of safety against sliding failure can be expressed as the ratio of the sum of horizontal resisting forces (F_R) to that of the horizontal driving forces (F_D). There are two options of horizontal resisting forces. The first option considers passive force (P_P) at toe side of the wall and the other neglects this passive force.

$$FS_{sd1} = \sum F_R \Big/ \sum F_d \quad (F_R \text{ with } P_P) \tag{3.7}$$

$$FS_{sd2} = \sum F_R \Big/ \sum F_d \quad (F_R \text{ without } P_P) \tag{3.8}$$

3.4.3 Bearing stability constraints

Bearing capacity failure gives two constraints, namely eccentricity failure and factor of safety against bearing failure of underlying soils as:

$$\bar{x} = \frac{\sum M_{net}}{\sum V} = \frac{\sum M_R - \sum M_{OV}}{\sum V} \tag{3.9}$$

$$(1/3)B \leq \bar{x} \leq (2/3)B \quad (3.10)$$

where $\bar{x}$ = distance from left corner of toe slab to resultant force R

$B = x_1 + x_2 + x_3$ = base width of the wall

M_R = sum of the moment of forces tending to resist overturning

M_{OV} = sum of the moment of forces tending overturn

Bearing failure is defined as the maximum contact pressure at the interface between the wall structures to the ultimate bearing capacity of the foundation soil:

$$FS_{bearing} = q_{max}/q_u \quad (3.11)$$

In general bearing capacity equation, it is to enforce the condition of Hansen's depth factor, where the ratio of depth soil cover to the base length is smaller or equal to 1.

$$\left(\frac{x_4 + x_5}{B - 2e}\right) \leq 1 \quad (3.12)$$

3.4.4 Slope stability constraints

A. Shape of circular arc failure surfaces

If the base soil consists of medium to soft clay, a circular slip surface failure may develop as shown in Figure 3.4a. In the proposed optimization problem, the circular arc failure surface is assumed to pass right at the corner of the wall base and does not intersect the concrete base. This critical mechanism is also obtained by extensive studies using AutoSLOPE (2004) for different geometries of cantilever retaining wall.

The studies showed that the circular arc failure surfaces which do not pass right at the corner of the wall base and penetrate more deeper are always not critical and their factor of safety is not the least as shown in Figure 3.4b. This important result is logical and valid provided that the soil underlying the wall base has the constant shear strength properties and there is no presence of the weak soil layer below the base of the wall.

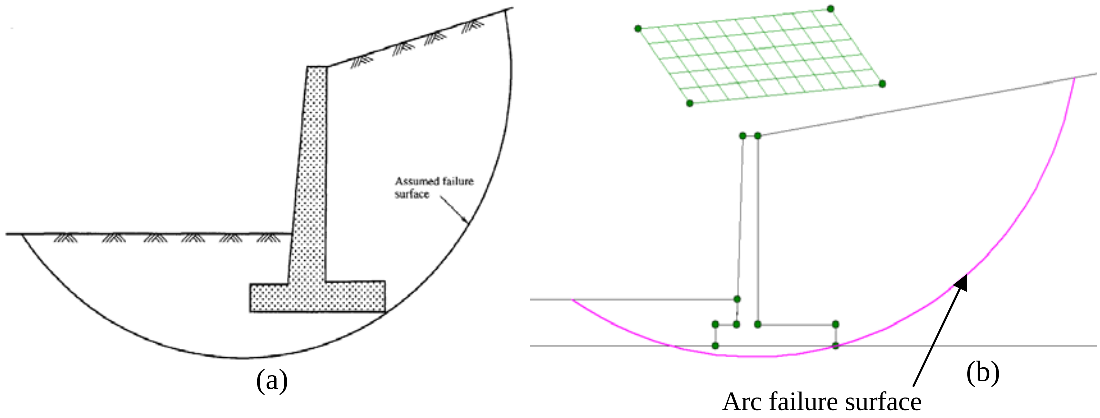

Figure 3.4 (a) Conventional circular arc failure surface (Murthy, 2002) and (b) critical failure mechanism using AutoSLOPE (Ukritchon et al., 2004)

Because slope failure shape is arc circular, factor of safety can be written as the ratio of the sum of the resisting moment to that of driving moment. Thus, factor of safety based on Ordinary Method of Slices is derived analytically in terms of unknown variables of wall dimensions and center of circular arc failure surface as illustrated in Figure 3.5.

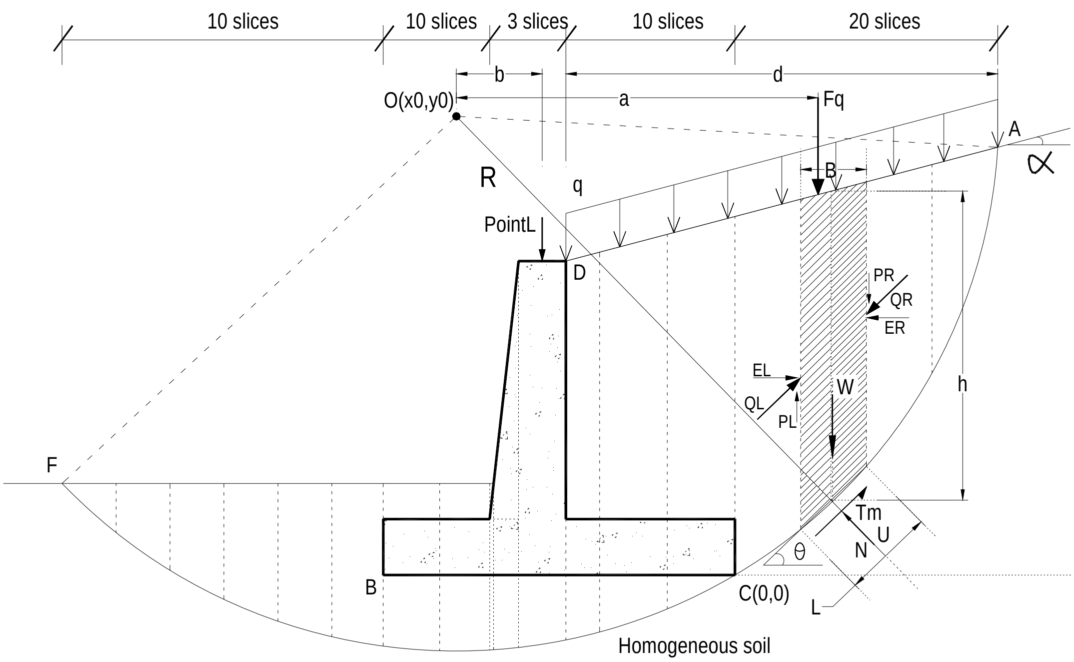

Figure 3.5 Ordinary Method of Slices in RC cantilever retaining wall

Expression of safety factor of slope stability for reinforced concrete cantilever retaining wall by Ordinary Method of Slices can be written as following:

$$FS = \frac{\sum_{i=1}^{n}\left[c'L_i + \left(N_i - U_i\right)\tan\phi'\right]}{\sum_{i=1}^{n} W_i \sin\theta_i R + q\left(B/\cos\alpha\right)\times a + P\times b} \tag{3.13}$$

where $N_i = \left[W_i + q_i\left(B_i/\cos\alpha\right) + P_i\right]\cos\theta_i$ = total normal force

W_i = total weight of slices

q_i = distribution line load

B_i = width of each slices

α = angle of slope backfill

P_i = point load

θ_i = angle of interslice force orientation (degree)

U_i = pore water force at base

c’, ϕ' = cohesion and internal friction angle of soil

L = base length

R = radius of circular arc

a, b = distance from center of arc to point, and line load, respectively

Finally, the factor of safety is derived analytically as function below:

$$FS = f\left(x_0, y_0, x_1, x_2, x_3, x_4, x_5, x_6, H, \gamma_1, c_1, \phi_1, \gamma_2, c_2, \phi_2, q, \alpha\right) \tag{3.14}$$

where x_0,y_0 = center of critical failure

x_1, x_2, x_3, x_4, x_5, x_6 = wall parameters

H = total height of wall

$\gamma_1, c_1, \phi_1, \gamma_2, c_2, \phi_2$ = soil properties (unit weight, cohesion, and friction angle respectively)

q = surcharge loading

α = angle of slope backfill

B. Procedure to derive the additional constraints for slope stability

In literature review, minimum factor of safety corresponded to critical failure surface are calculated by optimization methods.

Thus, in order to obtain the critical center position of the circular failure surface corresponding to the least of the factor of safety against slope failure, two additional equalities are constrained using the optimal condition of a function. This condition states that the first derivative of the function - the factor of safety - with respect to the unknown variable (x_0,y_0) must be zero:

$$\partial FS / \partial X_0 = 0 \tag{3.15}$$

$$\partial FS / \partial Y_0 = 0 \tag{3.16}$$

Based on theory of optimization, the first derivative in the Equation (3.15) and (3.16) can give only the stationary point of coordinate of critical center position (X_0,Y_0). The second partial derivative test is a method to determine if a critical stationary point (X_0,Y_0) of a function FS(X_0,Y_0) is a minimum, maximum or saddle point.

Thus, it should be mentioned that there is no need to apply such zero equality constraints to other unknown variables (X_1-X_6) since the analysis searches the critical position of the circular arc failure surface and treat those unknown variables as the constant terms.

Equation (3.17) and (3.18) present second partial derivative test and Hessian Matrix to stipulate the maxima and minima function.

$$H\Big|_{(X_0,Y_0)} = \begin{bmatrix} \dfrac{\partial^2 FS}{\partial X_0^2} & \dfrac{\partial^2 FS}{\partial X_0 \partial Y_0} \\ \dfrac{\partial^2 FS}{\partial X_0 \partial Y_0} & \dfrac{\partial^2 FS}{\partial Y_0^2} \end{bmatrix}_{(X_0,Y_0)} \tag{3.17}$$

The determinants of the square submatrices of H are:

$$H_1 = \left.\frac{\partial^2 FS}{\partial X_0^2}\right|_{(X_0,Y_0)} \tag{3.18}$$

$$H_2 = \left.\left[\frac{\partial^2 FS}{\partial X_0^2}\frac{\partial^2 FS}{\partial Y_0^2} - \left(\frac{\partial^2 FS}{\partial X_0 \partial Y_0}\right)^2\right]\right|_{(X_0,Y_0)} \tag{3.19}$$

Thus, the sufficient condition for a minimum FS(X_0,Y_0) at point (X_0,Y_0) exists if the Hessian Matrix H_1 and H_2 evaluated at (X_0,Y_0) is positive definite. As a result, all expressions in Equation (3.18) and Equation (3.19) must be positive or higher than zero.

C. Additional constraints for critical center position

Figure 3.6 shows the circular arc failure surface in reinforced concrete cantilever retaining wall.

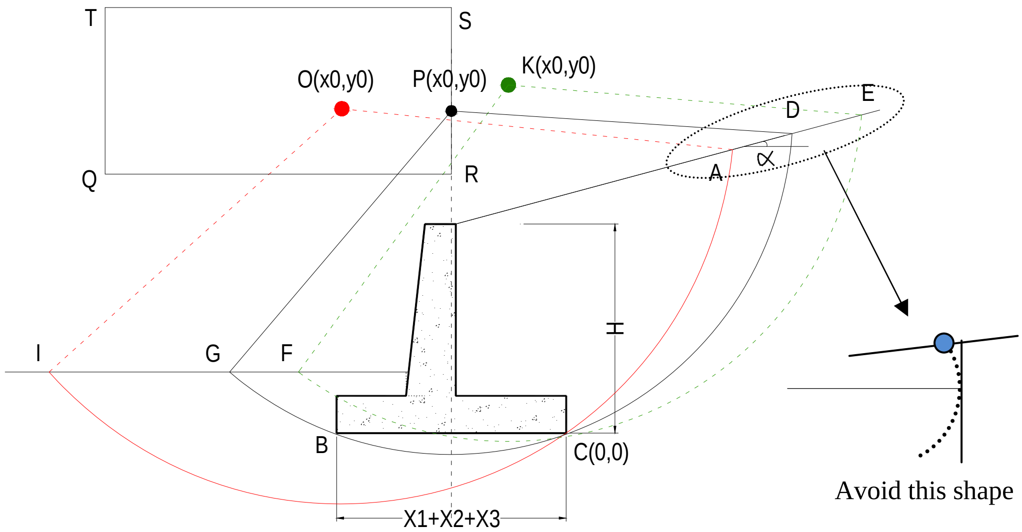

Figure 3.6 Arc failure surfaces correspond to their center positions

In this figure, arc OIA and PGBCD do not intersect the wall components since its axis locates in search region QRST. In contrast, arc KFCE intersects the wall component. The feasible center position of circular arc should stay at the left-hand side from middle of base length.

Ordinates of center positions are constrained to lie vertically assuming between lines (QR) and (TS) in order to avoid the intersection shape as shown in small figure on the right. When this shape appears, using procedures for calculating safety factor of slope stability are generally impractical.

Table 3.3 presents the lower side and upper side constraints which center of circular arc failure surface does not intersect the wall components as shown in Figure 3.6.

Table 3.3 Side constraints on critical center of circular arc failure surface

Description	Lower side constraints	Upper side constraints
Axis of critical center	$X_0 \geq (1/2)(x_1 + x_2 + x_3)$	$X_0 \leq (3/2)(x_1 + x_2 + x_3)$
Ordinate of critical center	$Y_0 \geq 1.2 * H$	$Y_0 \leq 2 * H$

D. Verification of validation of FS slope expression in MAPLE

In order to verify validation of FS_{SLOPE} expression derived from Ordinary Method of Slice in MAPLE, a comparison of critical safety factor between MAPLE and other slope programs have been made. Table 3.4 shows the results of minimum factor of safety, FS_{min}, by AutoSLOPE and *NLPSolve* (MAPLE). The example for comparison is taken from geotechnical textbook by Das (2007). In that example, surcharge loading (q) is added as shown in Figure 3.7.

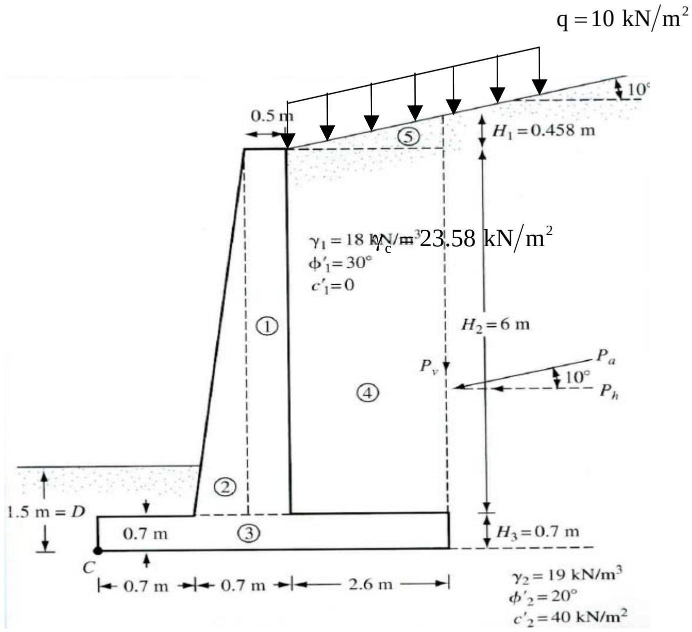

Figure 3.7 Input of example for comparison of critical safety factor of slope

Table 3.4 FS_{min} and coordinate of critical center position (x_0, y_0)

Description	MAPLE (NLPSolve)	Maharak (2007)	AutoSLOPE (2004)
FSmin	1.9957	1.9940	2.0140
X_0	1.200	1.390	1.171
Y_0	9.322	9.456	9.863

Based on Table 3.4, minimum safety factor, FS_{min}, by *NLPSolve* is slightly smaller than that by Maharak (2007). Thus, the expression of safety factor of slope derived by MAPLE is valid and suitable to use in optimization problem of reinforced concrete cantilever retaining wall.

3.4.5 Additional side constraints

Bowles (1996) suggested using thickness of cantilever wall equal to 250 mm. The practical minimum and maximum values of steel ratio in ACI code (2005) are considered. Table 3.5 summarized additional constraints arising from certain minimum criteria of thickness of stem and reinforcement ratio in each component of retaining wall.

Table 3.5 Additional lower and upper side constraints

Description	Lower side constraints	Upper side constraints
Stem thickness at top (m)	$X_6 \geq 0.25$ m	----
Horizontal steel area of the toe X_7 (mm^2)	$X_7 \geq \rho_{min}\left(X_4 - c - \Phi/2\right)$	$X_7 \leq \rho_{max}\left(X_4 - c - \Phi/2\right)$
Horizontal steel area of the heel X_8 (mm^2)	$X_8 \geq \rho_{min}\left(X_4 - c - \Phi/2\right)$	$X_8 \leq \rho_{max}\left(X_4 - c - \Phi/2\right)$
Vertical steel area of the stem X_9 (mm^2)	$X_9 \geq \rho_{min}\left(X_2 - c - \Phi/2\right)$	$X_9 \leq \rho_{max}\left(X_2 - c - \Phi/2\right)$

3.5 Proposed method for optimizing integral bridge abutment wall

3.5.1 Load considered into design

External loads apply in abutment support can be come from:

A. vertical loads from self weight of bridge slab
B. vertical load from live loading (Truck)
C. vertical load from self weight of abutment wall
D. horizontal loads from temperature, creep movements and wind
E. horizontal loads from braking and skidding effects of vehicles
F. horizontal pressure from exerted by the retained materials
G. vertical loading from the weight of the fill acts on the footing

H. surcharge loading on the rear of the wall

Appendix C gives detail on simply loads counted in bridge design and the combination of strength design based on AASHTO specification (2007).

However, the structural design on abutment wall is respected to ACI Code 318-05 which defined quite different from AASHTO.

3.5.2 Design variables

Figure 3.8 shows problem definition and design variables of integral bridge abutment walls related to geometry of wall dimensions and cross section of main bars area. In that figure, vertical line load (P) is assumed to be acting on the center line of abutment stem.

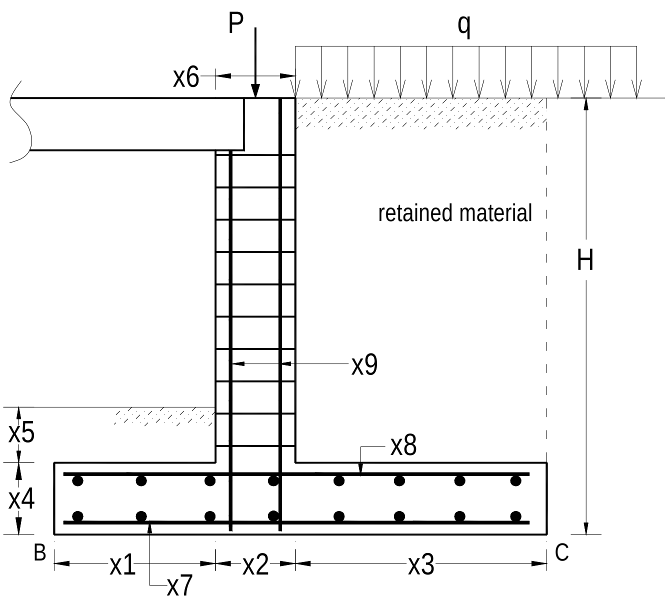

Figure 3.8 Design variables of integral bridge abutment wall

Nine design variables will be calculated. Assuming that top stem is equal bottom stem (X_2=X_6), the problem reduce to eight design variables. These include the following:

1. X_1 = total width of toe (m)
2. X_3 = total width of heel (m)
3. X_4 = thickness of base slab (m)
4. X_5 = soil cover (m)

5. X_6 = stem thickness at top (m)
6. X_7 = horizontal reinforcing area of the toe per unit length of wall (mm^2/m)
7. X_8 = horizontal reinforcing area of the heel per unit length of wall(mm^2/m)
8. X_9 = vertical reinforcing area of the stem per unit length of wall (mm^2/m)

3.5.3 Objective function of integral bridge abutment wall

Objective function in integral bridge abutment wall is the total cost of concrete, steel reinforcement, tied stirrups and formwork. The method to calculate total cost of bridge abutment wall is reported the same to conventional retaining wall. In integral bridge abutment wall, stirrups are necessary to place on stem wall in order to resist shear force. The procedure to calculate total cost of stirrups is summarized in the following equation.

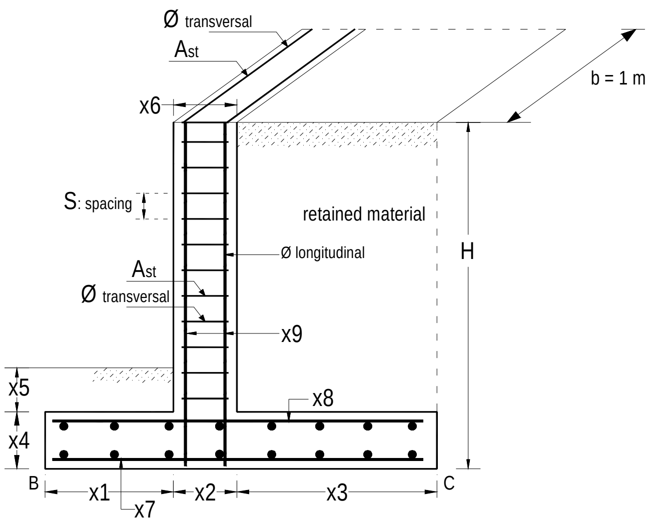

Figure 3.9 Reinforcing steel used in bridge abutment wall

The diameter of transversal bar is 10 mm with cross section denoted as A_{st}. Based on ACI Code (2005), vertical spacing (S) of stirrup is calculated as:

$$s = \min\left(16\phi_{longitudinal}, 48\phi_{transversal}\right) \tag{3.20}$$

where $\phi_{longitudinal}$ = diameters of longitudinal bars (mm)

$\phi_{transversal}$ = diameters of transversal bars (mm)

Numbers of stirrup (n_{st}) can be calculated as:

$$n_{st} = \left(H - x_4\right)/s \tag{3.21}$$

where H = total height of abutment wall

X_4= thickness of footing

s = spacing from one tie to one tie

Volume of one stirrup can be calculated as:

$$V_{st} = A_{st} \times 2(b + X_6) \tag{3.22}$$

where A_{st} = area of one stirrups

b = 1 meters (strip width)

X_6 = width of stem wall

Total cost of tie ($tcost_{stirrup}$) can be calculated as:

$$W_{st} = V_{st} \times \gamma_s$$
$$tcost_{stirrup} = W_{st} \times n_{st} \times C_S = A_{st} \times 2(b + X_6) \times \gamma_s \times \frac{(H - x_4)}{s} \times C_S \tag{3.23}$$

where W_{st} = weight of one tie

V_{st} = volume of one tie

γ_s = unit weight of reinforcing steel

n_{st} = numbers of tie

C_S = unit cost of reinforcing steel (B/Kg)

3.5.4 Geotechnical design considerations

The concept of geotechnical designs of bridge abutment wall is the same to conventional cantilever retaining wall. Three typical failure modes simply taken to consider are namely, sliding failure, overturning failure, and bearing failure. Those failure modes must be checked to satisfy required factor of safety. According to Chen (2000), the required factor of safety against sliding should not be less than 1.50; required factor of safety against overturning should not be less than 2.0, and factor of safety against bearing failure should not be less than 3.0. These values are applied in the abutment with spread footing under service load.

In this study, slope stability constraints are included in optimal design.

3.5.5 Slope stability analysis in bridge abutment wall

The procedure for calculating factor of safety against slope failure in bridge abutment wall is the same to conventional retaining wall. However, in this case, safety factor of slope is also derived in function of line load, P, which acts vertically on stem wall.

- *Verification of validation of FS slope expression in MAPLE*

In order to verify validation of safety factor of slope expression (FS_{SLOPE}) derived from Ordinary Method of Slice in MAPLE, a comparison of critical safety factor between MAPLE and other slope programs have been made. Table 3.6 shows the results of minimum factor of safety, FS_{min}, by AutoSLOPE, and *NLPSolve* (MAPLE).

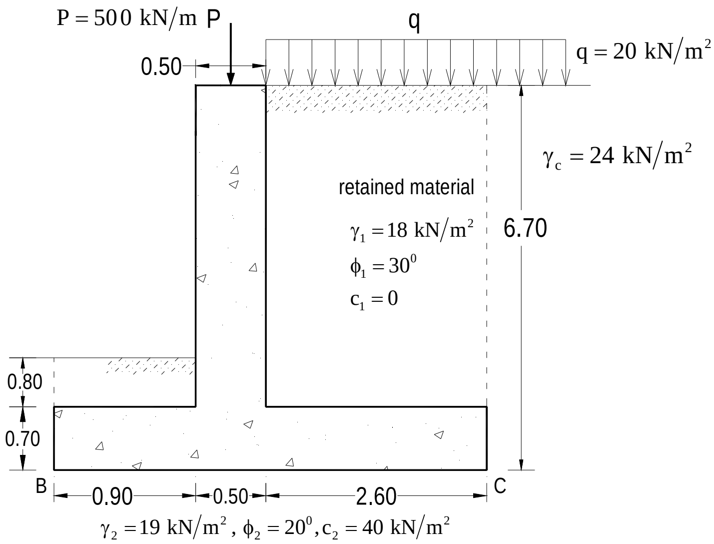

Figure 3.10 Input of example for comparison of critical safety factor of slope

Table 3.6 FS_{min} and coordinate of critical center position (x_0,y_0)

Description	MAPLE (NLPSolve)	Maharak (2007)	AutoSLOPE (2004)
FSmin	2.078	2.060	2.060
X_0	-1.534	-1.590	-1.519
Y_0	6.710	6.840	6.799

Based on Table 3.6, FS_{min} by *NLPSolve* is slightly smaller than that by Maharak with 1%. Thus, the expression of safety factor of slope derived by MAPLE is valid and suitable to use in optimization problem of integral bridge abutment retaining wall.

3.5.6 Structural design requirements

A. Design assumptions

Due to the fact that the bridge abutment walls are subjected to axial loads and moments from lateral earth pressure causing weak-axis bending, it can be designed as bearing walls. McGregor (2005) defined bearing walls are the walls that are laterally supported and braced by the rest of the structure, that resist primarily in-plane vertical loads acting downward on the top of the wall. The vertical load may develop an eccentricity on the wall, causing weak axis bending. Moreover, according to ACI Section 14.4, it stated that the design of bearing wall is carried out by following procedures as:

1. by using the one-way column design and slenderness requirements in ACI Section 10.11, 10.12 and 10.13 or
2. by the empirical design method in ACI Section 14.5.

McCormac (2005) stated that reinforced concrete bearing wall can be design either as columns or slender walls using an alternative procedure specified in ACI Section 14.8.

In order to avoid complicated design when the wall is often classified as slender column, constraints in enforcing design as short column should be related to slenderness ratio computation.

B. Support modeling

Base of bridge abutment can be acted as fixed end support since there is no movement and sliding on bottom wall. The bridge slab will be acted as roller support as shown in Figure 3.11.

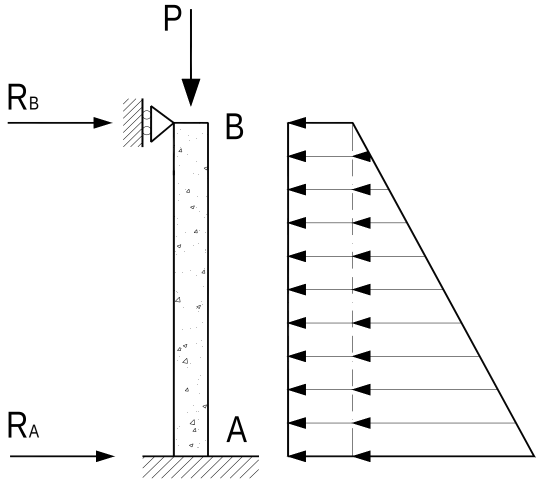

Figure 3.11 Support modeling of abutment wall

From modeling of acting force in Figure 3.11, maximum bending moment and shear force can be calculated based on structural analysis. The reaction and moment at end of support and zero shears are presented in Table 3.7.

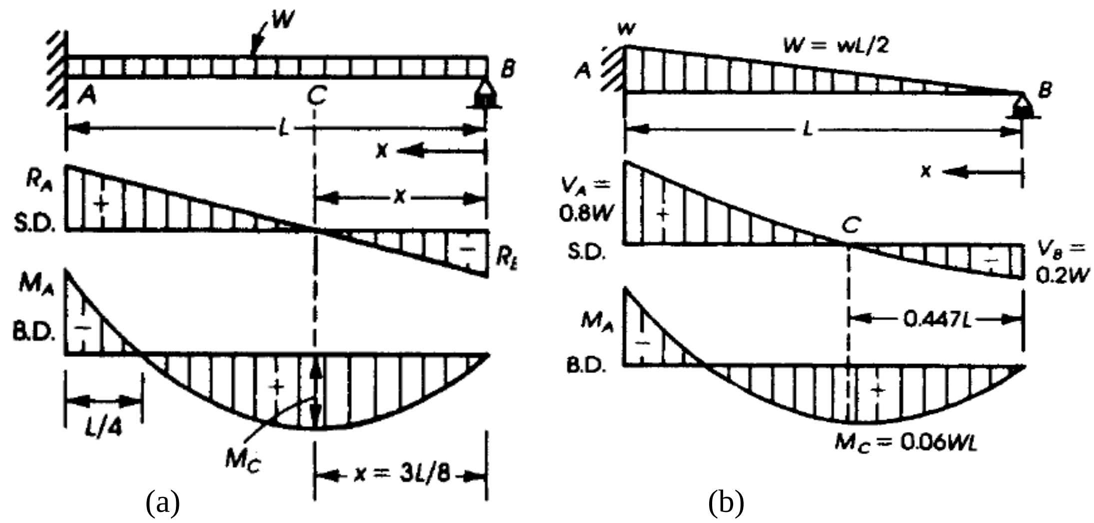

Figure 3.12 Moment and shear diagram (a) due to uniform load (b) due to triangular load (Hassoun, 2005)

Table 3.7 Reaction and moment at end of supports and zero shears

	Uniform Load	Triangular Load
$R_A = V_A$	$5W/8$	$4W/5$
$R_B = V_B$	$3W/8$	$W/5$
M_A (-)	$WL/8$	$2WL/15$
M_B (+)	0	0
M_C (+)	$9WL/128$	$3WL/50$

C. *Important constraints in optimal design of bridge abutment wall*

i. *Constraints on stem shape*

Stem shape constraints are used to limit design length of stem which must be greater or equal to 0.25 meter and smaller than 1.0 meter as presented in Table 3.8.

ii. *Constraints on reinforcement ratio in column*

Based on ACI Code, reinforcement ratio in column should not less than 0.01 and not more than 0.08. Thus, these constraints are summarized in Table 3.8.

Table 3.8 Summary of side constraints in abutment wall

Description	Lower bound	Upper bound
Shape X_6 (m)	$X_6 \geq 0.25$	$X_6 \leq 1.00$
Steel ratio in column	$\rho_{column} \geq 0.01$	$\rho_{column} \leq 0.08$

iii. *Constraints enforcing design as short column according to ACI Code*

The slenderness effects in column design are neglected if the following equation is satisfied.

$$\frac{k\ell_u}{r} \leq 34 - \frac{M_1}{M_2} \tag{3.24}$$

where k = 0.70 (as in Figure 3.11: fixed support at bottom with rotation free and translation fixed support at top)

$\ell_u = H - X_4$ = unsupported length

$r = 0.3 \times X_6$ = radius of gyration (rectangular section)

$M1/M2 = 0$ (moment at translation roller support at top is equal to zero based on Figure 3.11)

iv. *Constraints enforcing design in compression-controlled region*

To enforce design in compression-controlled regions, proportion of value c/d should be constrained in one interval whose values limit compression failures mode.

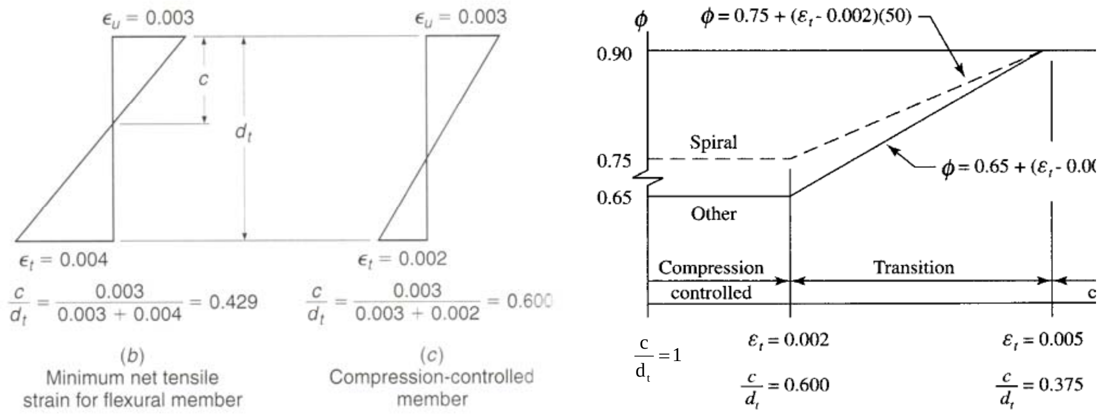

Figure 3.13 Net tensile strains and proportion of c/d (Nilson, 2004)

The strain compatibility gives rise to the following relationships:

$$\frac{c}{d-c} = \frac{\varepsilon_u}{\varepsilon_t} = \frac{\varepsilon_u}{f_y/E_s} \tag{3.25}$$

$$\frac{c}{d} = \frac{\varepsilon_u}{f_y/E_s + \varepsilon_u} \tag{3.26}$$

The conditions on compression controlled design can be expressed as:

$$c \geq 0.003E_s d / \left(f_y + 0.003E_s\right) \tag{3.27}$$

$$\frac{c}{d} \leq 1.00 \tag{3.28}$$

v. *Constraints on strength design*

According to ACI Code, the nominal strengths multiplied with reduction factors must be greater than or equal to the design strength calculated by load factors.

The resulting constraints are given by:

$$\phi P_n = \phi\left(0.85f'_c\left(\beta_1 c\right)b + A'_s\left(f_y - 0.85f'_c\right) - A_s\left(E_s\right)\left[\left(0.003\right)\left(\frac{d}{c} - 1\right)\right]\right) \geq P_{applied} \quad (3.29)$$

$$\phi M_n = \phi\left[\begin{array}{l} 0.85f'_c\left(\beta_1 c\right)b\left(\frac{h}{2} - \frac{\left(\beta_1 c\right)}{2}\right) + A'_s\left(f_y - 0.85f'_c\right)\left(\frac{h}{2} - d'\right) \\ +A_s\left(E_s\right)\left[\left(0.003\right)\left(\frac{d}{c} - 1\right)\right]\left(d - \frac{h}{2}\right) \end{array}\right] \geq M_{applied} \quad (3.30)$$

The maximum axial load capacity of a column multiplied with strength reduction factor must exceed the applied axial factored compression as:

$$\phi P_{n(max)} = \phi * 0.80\left[0.85f'_c\left(A_g - A_{st}\right) + f_y\left(A_{st}\right)\right] \geq P_{applied} \quad (3.31)$$

vi. *Constraints on shear force*

The applied factored shearing forces (V_u) should not greater than one and half of shearing force provided by concrete (V_c). This condition ensures that stirrups designed according ACI Section 7.10.5.1, 7.10.5.2, and 7.10.5.3 can resist the external applied shear force and there is no requirement of additional shear reinforcing steel.

$$\phi V_c / 2 \geq V_u \quad (3.32)$$

3.6 Solvers for optimization problem

The optimization of retaining wall is formulated as a constrained nonlinear programming (Saribas and Erbatur, 1996). Thus, various optimization algorithms can be used depending on mathematical structure of the problem. Some available build-in function of optimization in MAPLE, MATLAB, IMSL FORTRAN, and KNITRO can be used to compute minimum or maximum of a real-value nonlinear objective function and nonlinear constraints.

3.6.1 MAPLE and MATLAB

MAPLE is used mainly for deriving expressions of all constraints in reinforced concrete cantilever retaining wall. Code generation command in MAPLE is useful for generating analytical expressions from MAPLE to MATLAB. For additional constraints in slope stability, both MAPLE and MATLAB can compute the first and second partial derivative of expressions by command *diff*. MAPLE and

MATLAB can generate equality constraints ($g_j(X) = 0$) without causing computation errors. Although expression of safety factor of slope and additional constraints are too long (more than 1000 lines), MAPLE can evaluate their analytical expressions and store analytical results in internal memory.

3.6.2 IMSL FORTRAN and KNITRO

Because FORTRAN and KNITRO only evaluate the first and second derivative expressions numerically, it is impossible to use build differentiation command for enforcing additional slope constraints. To resolve those problems, Finite Difference Method (FDM) is applied to perform the approximation of derivatives by central, forward or backward. Figure 3.14 presents the finite difference mesh in one and two dimensions where u is function of x and y [u(x,y)].

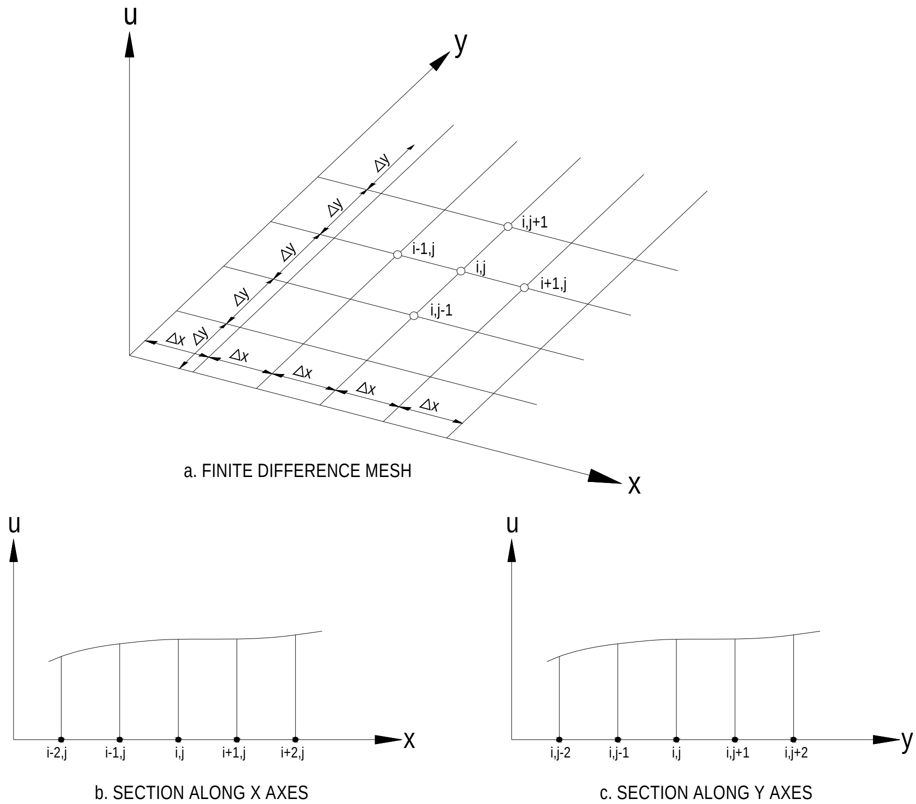

Figure 3.14 Finite difference approximations in one and two dimensions

- *First derivative*

The simple formulas of central-difference approximation of u respected to x and y can be written as:

$$\frac{\partial u}{\partial x}=\frac{u_{i+1,j}-u_{i-1,j}}{2\Delta x}=\frac{u(x+\Delta x,y)-u(x-\Delta x,y)}{2\Delta x} \tag{3.33}$$

$$\frac{\partial u}{\partial y}=\frac{u_{i,j+1}-u_{i,j-1}}{2\Delta y}=\frac{u(x,y+\Delta y)-u(x,y-\Delta y)}{2\Delta y} \tag{3.34}$$

- *Second derivative*

The simple formulas of central-difference approximation of u respected to x and y can be written as:

$$\frac{\partial^2 u}{\partial x^2}=\frac{u_{i-1,j}-2u_{i,j}+u_{i+1,j}}{(\Delta x)^2}=\frac{u(x-\Delta x,y)-2u(x,y)+u(x+\Delta x,y)}{(\Delta x)^2} \tag{3.35}$$

$$\frac{\partial^2 u}{\partial y^2}=\frac{u_{i,j-1}-2u_{i,j}+u_{i,j+1}}{(\Delta y)^2}=\frac{u(x,y-\Delta y)-2u(x,y)+u(x,y+\Delta y)}{(\Delta y)^2} \tag{3.36}$$

$$\begin{aligned}\frac{\partial^2 u}{\partial x\partial y}&=\frac{u_{i-1,j-1}-u_{i-1,j+1}-u_{i+1,j-1}+u_{i+1,j+1}}{4(\Delta x)(\Delta y)}\\&=\frac{u(x-\Delta x,y-\Delta y)-u(x-\Delta x,y+\Delta y)-u(x+\Delta x,y-\Delta y)+u(x+\Delta x,y+\Delta y)}{4(\Delta x)(\Delta y)}\end{aligned} \tag{3.37}$$

For numerical calculation of safety factor of slope, the first and second derivative of safety factor of slope respected to X_0 and Y_0 from Equation (3.35) to Equation (3.37) can be expressed as:

$$\frac{\partial FS}{\partial X_0}=\frac{FS(X_0+\Delta X,Y_0)-FS(X_0-\Delta X,Y_0)}{2\Delta X} \tag{3.38}$$

$$\frac{\partial FS}{\partial Y_0}=\frac{FS(X_0,Y_0+\Delta Y)-FS(X_0,Y_0-\Delta Y)}{2\Delta Y} \tag{3.39}$$

$$\frac{\partial^2 FS}{\partial X_0^2}=\frac{FS(X_0-\Delta X,Y_0)-2FS(X_0,Y_0)+FS(X_0+\Delta X,Y_0)}{(\Delta X)^2} \tag{3.40}$$

$$\frac{\partial^2 FS}{\partial Y_0^2}=\frac{FS(X_0,Y_0-\Delta Y)-2FS(X_0,Y_0)+FS(X_0,Y_0+\Delta Y)}{(\Delta Y)^2} \tag{3.41}$$

$$\begin{aligned}\frac{\partial^2 FS}{\partial X_0 Y_0}&=\frac{FS(X_0-\Delta X,Y_0-\Delta Y)-FS(X_0-\Delta X,Y_0+\Delta Y)}{4(\Delta X)(\Delta Y)}\\&+\frac{-FS(X_0+\Delta X,Y_0-\Delta Y)+FS(X_0+\Delta X,Y_0+\Delta Y)}{4(\Delta X)(\Delta Y)}\end{aligned} \tag{3.42}$$

For FORTRAN coding, the research uses tolerance of ΔX and ΔY equal to 10^{-5}.

To make central-difference approximation more accuracy, it is possible to use the finite difference coefficient in central which are listed in Table 3.9.

Table 3.9 Central finite difference coefficient

Derivative	Order	-3	-2	-1	0	1	2	3
1	1			-1/2	0	1/2		
	2		1/12	-2/3	0	2/3	1/12	
	3	-1/60	3/20	-3/4	0	3/4	-3/20	1/60
2	1			1	-2	1		
	2		-1/12	4/3	-5/2	4/3	-1/12	
	3	1/90	-3/20	3/2	-49/18	3/2	-3/20	1/90

For example, the second derivative with second-order of 5 coefficients of accuracy is:

$$\frac{\partial^2 f}{\partial x^2} = \frac{-\frac{1}{12} f\left(x-2\Delta X\right)+\frac{4}{3} f\left(x-\Delta X\right)-\frac{5}{2} f\left(x\right)+\frac{4}{3} f\left(x+\Delta X\right)-\frac{1}{12} f\left(x+2\Delta X\right)}{\Delta X^2} \quad (3.43)$$

This research uses second-order of 5 coefficients of accuracy.

3.7 Summary of all constraints and objective function

This section summarizes an objective function of total cost of construction materials and all constraints in the optimal design of conventional and bridge abutment retaining wall. The subscript, *d* and *t*, refers to *design or requirements*, and *total stress*, respectively.

- *Optimization problem of conventional retaining wall*

Minimize total cost F

$$\begin{aligned} F = {} & \frac{1}{2}\left(H-x_4\right)\left(x_2+x_6\right)+\left(x_1+x_2+x_3\right)\left(x_4\right)+\left(x_7 * 10^{-6}\right)\left(x_1+x_2+x_3\right)\left(\gamma_{st}\right) \\ & +\left(x_8 * 10^{-6}\right)\left(x_1+x_2+x_3\right)\left(\gamma_{st}\right)+\left(x_9 * 10^{-6}\right)\left(H\right)\left(\gamma_{st}\right) \\ & +H+X_4+\sqrt{\left(X_2-X_6\right)^2+\left(H-X_4\right)^2} \end{aligned} \quad (3.44)$$

Subjected to

$$g_{j1} = FS_{ovd} - FS_{ov} \le 0 \tag{3.45}$$

$$g_{j2} = FS_{sddp} - FS_{sdp} \le 0 \tag{3.46}$$

$$g_{j3} = FS_{sdd0} - FS_{sd0} \le 0 \tag{3.47}$$

$$g_{j4} = (1/2)B - \left(\sum M_R - \sum M_{ov} / \sum V\right) \le 0 \tag{3.48}$$

$$g_{j5} = \left(\sum M_R - \sum M_{ov} / \sum V\right) - (2/3)B \le 0 \tag{3.49}$$

$$g_{j6} = FS_{bearingd} - FS_{bearing} \le 0 \tag{3.50}$$

$$g_{j7} = \tau_{utoe} - \phi\tau_c \le 0 \tag{3.51}$$

$$g_{j8} = \tau_{uheel} - \phi\tau_c \le 0 \tag{3.52}$$

$$g_{j9} = \tau_{ustem} - \phi\tau_c \le 0 \tag{3.53}$$

$$g_{j10} = M_{utoe} - MR_{toe} \le 0 \tag{3.54}$$

$$g_{j11} = M_{uheel} - MR_{heel} \le 0 \tag{3.55}$$

$$g_{j12} = M_{ustem} - MR_{stem} \le 0 \tag{3.56}$$

$$g_{j13} = \rho_{min}\left(X_4 - c - \Phi/2\right) - X_7 \le 0 \tag{3.57}$$

$$g_{j14} = \rho_{min}\left(X_4 - c - \Phi/2\right) - X_8 \le 0 \tag{3.58}$$

$$g_{j15} = \rho_{min}\left(X_2 - c - \Phi/2\right) - X_9 \le 0 \tag{3.59}$$

$$g_{j16} = X_7 - \rho_{max}\left(X_4 - c - \Phi/2\right) \le 0 \tag{3.60}$$

$$g_{j17} = \rho_{max}\left(X_4 - c - \Phi/2\right) - X_8 \le 0 \tag{3.61}$$

$$g_{j18} = \rho_{max}\left(X_2 - c - \Phi/2\right) - X_9 \le 0 \tag{3.62}$$

$$g_{j19} = 0.25 - x_6 \le 0 \tag{3.63}$$

$$g_{j20} = \left(\frac{x_4 + x_5}{B - 2 \times e}\right) - 1 \le 0 \tag{3.64}$$

$$g_{j21} = FS_{sloped} - FS_{slope} \le 0 \tag{3.65}$$

$$g_{j22} = (1/2)\left(X_1 + X_2 + X_3\right) - X_0 \le 0 \tag{3.66}$$

$$g_{j23} = X_0 - (3/2)\left(X_1 + X_2 + X_3\right) \le 0 \tag{3.67}$$

$$g_{j24} = 1.2 \times H - Y_0 \le 0 \tag{3.68}$$

$$g_{j25} = Y_0 - 2 \times H \leq 0 \tag{3.69}$$

$$g_{j26} = -\left(\partial^2 FS_{slope} \big/ \partial^2 X_0\right) < 0 \tag{3.70}$$

$$g_{j27} = \left(\partial^2 FS_{slope} \big/ \partial X_0 Y_0\right)^2 - \left(\partial^2 FS_{slope} \big/ \partial X_0^2\right)\left(\partial^2 FS_{slope} \big/ \partial Y_0^2\right) \leq 0 \tag{3.71}$$

$$L_{j28} = \partial FS_{slope} \big/ \partial X_0 = 0 \tag{3.72}$$

$$L_{j29} = \partial FS_{slope} \big/ \partial Y_0 = 0 \tag{3.73}$$

$$L_{j30} = x_4 + x_5 = D \tag{3.74}$$

$$g_{j31} = FS_{sdd0t} - FS_{sd0t} \leq 0 \tag{3.75}$$

$$g_{j32} = FS_{sddpt} - FS_{sdpt} \leq 0 \tag{3.76}$$

$$g_{j33} = FS_{sdd0t} - FS_{sd0t} \leq 0 \tag{3.77}$$

$$g_{j34} = FS_{bearingdt} - FS_{bearingt} \leq 0 \tag{3.78}$$

$$g_{j35} = \tau_{stoe} - \tau_c \leq 0 \tag{3.79}$$

$$g_{j36} = M_{stoe} - MR_{stoe} \leq 0 \tag{3.80}$$

$$g_{j37} = \tau_{sheel} - \tau_c \leq 0 \tag{3.81}$$

$$g_{j38} = M_{sheel} - MR_{sheel} \leq 0 \tag{3.82}$$

$$g_{j39} = \tau_{sstem} - \tau_c \leq 0 \tag{3.83}$$

$$g_{j40} = M_{sstem} - MR_{sstem} \leq 0 \tag{3.84}$$

Total design constraints in the optimal design of conventional and bridge abutment retaining wall are summarized in Table 3.10, and Table 3.11, respectively.

Table 3.10 Summary of all design constraints in conventional retaining wall

Design criteria	**Constraints in TSA ESA (WSD)**		**Constraints in TSA ESA (USD)**	
	No Slope	**With Slope**	**No Slope**	**With Slope**
overturning	1	1	1	1
Two eccentricity	2	2	2	2
Two Sliding with and without P_p **(total and effective separately)**	2	2	2	2
Bearing **(total and effective separately)**	1	1	1	1
Shear and moment **(WSD)** Toe, Heel, and Stem	6	6	-	-
Shear and moment **(USD)** Toe, Heel, and Stem	-	-	6	6
Steel reinforcement ratio in beam **(USD)**	-	-	6	6
Top stem and Hansen's depth factor	2	2	2	2
FS Slope	-	1	-	1
Two lower bound and upper bond for critical center constraints in X_0 and Y_0	-	4	-	4
Hessian Matrix elements	-	2	-	2
Partial derivative FS **(equality con-)**	-	2	-	2
Depth of embedment **(equality con-)**	1	1	1	1
Total Number of constraints	**15**	**24**	**21**	**30**

Table 3.11 Summary of all design constraints in integral bridge abutment wall

Design criteria	**Constraints in TSA ESA (USD)**	
	No Slope	**With Slope**
overturning	1	1
Two eccentricity	2	2
Two Sliding with and without P_p **(total and effective separately)**	2	2
Bearing **(total and effective separately)**	1	1
Shear and moment **(USD)** [Toe, Heel]	4	4
Steel reinforcement ratio in beam **(USD)** [Toe, H	4	4
Top stem and Hansen's depth factor	2	2
Shear in column wall	1	1
Reinforcement steel ratio in column	2	2
Slenderness ratio	1	1
Strength constraints on P_n, M_n, P_{max}	3	3
Compression controlled by neutral axis	2	2
FS Slope	-	1
Two lower bound and upper bond for critical center constraints in X_0 and Y_0	-	4
Hessian Matrix elements	-	2
Partial derivative FS **(equality con-)**	-	2
Depth of embedment **(equality con-)**	1	1
Total Number of constraints	**25**	**34**

CHAPTER IV
RESULTS AND DISCUSSIONS

4.1 General

This section presents two numerical examples of conventional retaining wall and bridge abutment wall in order to demonstrate significant effect of slope stability constraints to optimal design of cantilever retaining wall.

Table 4.1 lists input parameters for optimal design of two different examples. The initial input must consist of:

- Total height of wall
- Angle of backfill and surcharge loading
- Soil properties of backfill (unit weight, cohesion, internal friction angle)
- Soil properties of base foundation (unit weight, cohesion, internal friction angle)
- Interface shear resistance for friction angle and base adhesion
- Factor of safety against overturning, sliding, bearing, and slope failure
- Factor of safety against shear and moment failures
- Concrete and reinforcing steel properties (strength, unit weight)
- Structural design (initial diameter, concrete diameter)
- Cost of materials (concrete, steel, formwork)

Unit price of concrete and reinforcing steel depend on its strength and diameters, respectively as reported in Table 3.1 and Table 3.2. In this study, the unit cost of concrete, steel, and formwork are given by 2,550.0 B/m^3, 22.0 B/kg, and 150.0 B/m^2, respectively.

Table 4.1 Initial input used in optimal design of conventional retaining wall (Effective stress analysis)

Input parameters	Symbol	Unit	Ex1	Ex2
Total Height	H	m	6.0	5.2
Depth of embedment	D	m	1.8	1.0
Yield strength of steel reinforcement	f_y	MPa	400.0	400.0
Compressive strength of concrete	f_c'	MPa	25.0	25.0
Unit weight of steel reinforcement	γ_{st}	Kg/m^3	7850.0	7850.0
Unit weight of concrete	γ_c	kN/m^3	24.0	24.0
Concrete cover	c	mm	70.0	70.0
Diameters of bar	Φ	Mm	20.0	16.0
Surcharge loading	q	kN/m^2	10.0	10.0
Backfill of slope	α	Degree	5.0	0.0
Unit weight of backfill soil	γ_1	kN/m^3	18.0	16.8
Cohesion of backfill soil	c_1	kN/m^2	0.0	0.0
Internal friction angle of backfill soil	ϕ_1	degree	30.0	30.0
Unit weight of soil below foundation	γ_2	kN/m^3	19.0	17.6
Cohesion of soil below foundation	c_2	kN/m^2	40.0	30.0
Undrained shear strength of base soil	S_u	kN/m^2	100.0	100.0
Internal friction angle of base soil	ϕ_2	degree	20.0	28.0
Interface shear resistance for friction angle and base adhesion	k_1, k_2	-	0.6667	0.6667
FS for overturning stability	FS_{ov}	-	2.0	2.0
FS for sliding stability (include P_P)	FS_{sdp}	-	2.0	2.0
FS for sliding stability (exclude P_P)	FS_{sd0}	-	1.5	1.5
FS for bearing stability	FS_{be}	-	3.0	3.0
FS for slope stability	FS_{slope}	-	2.5	2.5
Factor of safety against shear and moment failures	FS_s,FS_m	-	1.0	1.0
Unit cost of concrete	C_C	฿/m^3	2,550	2,550
Unit cost of steel	C_{ST}	฿/kg	22.0	22.0
Unit cost of formwork	C_F	฿/m^2	150.0	150.0

Table 4.2 Initial input used in optimal design of conventional retaining wall (Total stress analysis)

Input parameters	Symbol	Unit	Ex1	Ex2
Total Height	H	m	6.0	5.5
Depth of embedment	D	m	1.5	1.0
Yield strength of steel reinforcement	f_y	MPa	400.0	400.0
Compressive strength of concrete	f_c'	MPa	25.0	25.0
Unit weight of steel reinforcement	γ_{st}	Kg/m^3	7850.0	7850.0
Unit weight of concrete	γ_c	kN/m^3	24.0	24.0
Concrete cover	c	mm	70.0	70.0
Diameters of bar	Φ	Mm	20.0	16.0
Surcharge loading	q	kN/m^2	10.0	10.0
Backfill of slope	α	Degree	5.0	0.0
Unit weight of backfill soil	γ_1	kN/m^3	18.0	17.0
Cohesion of backfill soil	c_1	kN/m^2	0.0	0.0
Internal friction angle of backfill soil	ϕ_1	degree	30.0	30.0
Unit weight of soil below foundation	γ_2	kN/m^3	19.0	18.0
Cohesion of soil below foundation	c_2	kN/m^2	110.0	110.0
Undrained shear strength of base soil	S_u	kN/m^2	100.0	110.0
Internal friction angle of base soil	ϕ_2	degree	0.0	0.0
Interface shear resistance for friction angle and base adhesion	k_1, k_2	-	0.6667	0.6667
FS for overturning stability	FS_{ov}	-	2.0	2.0
FS for sliding stability (include P_P)	FS_{sdp}	-	2.0	2.0
FS for sliding stability (exclude P_P)	FS_{sd0}	-	1.5	1.5
FS for bearing stability	FS_{be}	-	3.0	3.0
FS for slope stability	FS_{slope}	-	2.5	2.5
Factor of safety against shear and moment failures	FS_s,FS_m	-	1.0	1.0
Unit cost of concrete	C_C	฿/m^3	2,550	2,550
Unit cost of steel	C_{ST}	฿/kg	22.0	22.0
Unit cost of formwork	C_F	฿/m^2	150.0	150.0

4.2 Trial solutions

This section will study the effect of variation of one wall dimension on factor of safety of slope stability and total cost of materials. The study is carried out on the second example (Ex2) in Table 4.1 before applying the proposed optimization technique.

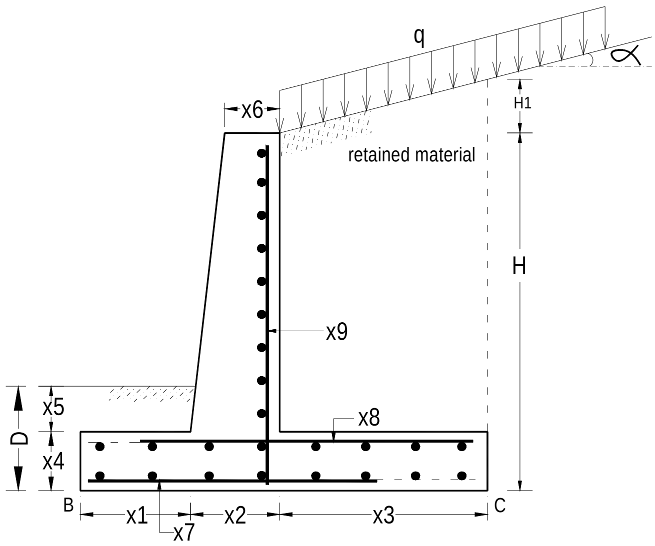

Figure 4.1 Design variables $X_{[i]}$

Figure 4.1 shows the design parameters of retaining wall. The study on trial solutions is focused on changing X_1, X_2, and X_3 since these three design variables act as controlled variables in design. It can be noticed that unknown X_4 is designed by shear. Design of unknown X_5 depends on a given embedment depth (D) constant. Design of unknown X_6 is related to the minimum value recommended by Bowles (1996). Reinforcing areas such as X_7, X_8, and X_9 are mainly depended on X_1, X_2, and X_3.

A. *Trial solutions by changing X_1*

The same input parameters in example 2 of Table 4.1 are considered into trial solutions using ultimate strength design. The study maintains X_2 and X_3 as constant values while the value of X_1 is changed. X_4 is given as 0.60 meter since it was calculated by conventional proportion and this thickness is sufficient for shear design. The result in summarized in Table 4.3.

Table 4.3 Trial solutions by changing X_1

X	Trial1	Trail2	Trial3	Trial4	Trial5	Trial6	Trial7	Trial8
x1 (m)	*0.30*	*0.40*	*1.00*	*1.80*	*2.50*	*2.80*	*3.00*	*4.00*
x2 (m)	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40
x3 (m)	2.20	2.20	2.20	2.20	2.20	2.20	2.20	2.20
x4 (m)	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60
x5 (m)	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40
x6 (m)	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
FSov	2.415	2.569	3.352	4.864	6.100	6.648	7.020	8.950
FSsdp	2.945	2.975	3.150	3.384	3.589	3.677	3.736	4.029
FSsd0	1.610	1.639	1.815	2.049	2.254	2.342	2.401	2.693
FSbe	4.088	4.424	7.100	12.845	20.719	25.157	28.553	51.410
Ecentricity	not ok	ok	ok	ok	ok	ok	ok	ok
FSstoe	3.820	3.170	2.140	1.93	1.840	1.800	1.770	1.600
FSsheel	1.180	1.180	1.180	1.180	1.180	1.180	1.180	1.180
FSsstem	1.920	1.920	1.920	1.920	1.920	1.920	1.920	1.920
FSslope (min)	2.455	2.456	2.466	2.483	2.500	2.510	2.520	2.550
Tcost (B)*10000	1.3291	1.3547	1.4804	1.6533	1.8046	1.8694	1.9126	2.1287

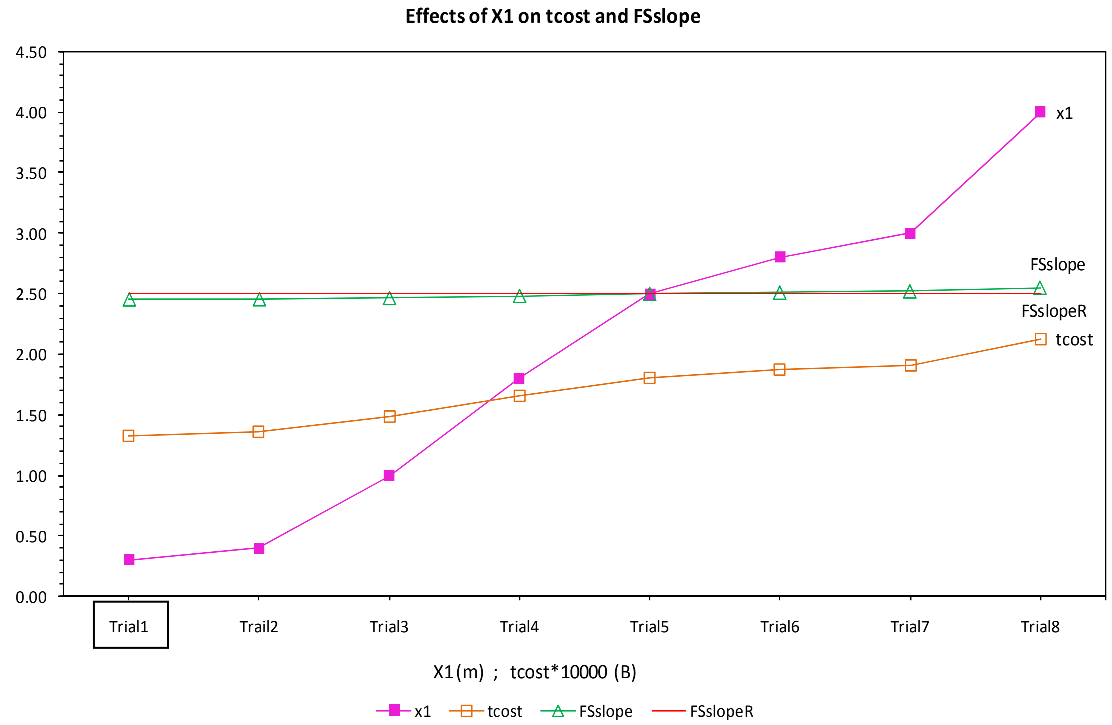

Figure 4.2 Effect of X_1 on safety factor of slope and total cost

From Figure 4.2, when X_1 increases, total cost increases accordingly. In trial 1, total base length does not satisfy eccentricity failures although this trial gives the lowest total cost. All factors of safety against geotechnical and structural failures are greater than those of requirements from trial 5, which correspond to X_1=2.50 meters. This can be concluded that the required safety factor of slope, FSslope, can be achieved by mainly increasing value of X_1. As a result, an optimum solution does exist, where the value of FSslope can satisfy the required value of 2.50.

B. Trial solutions by changing X3

Based on Ex 2 in Table 4.1, the study maintains X_1 and X_2 as constant value while the value X_3 is changed. X_4 is given as 0.60 meter since it was calculated by conventional proportion and this thickness is sufficient for shear design. The result is summarized in Table 4.4.

Table 4.4 Trial solutions by changing X_3

X	Trial1	Trial2	Trial3	Trial4	Trial5	Trial6
x1 (m)	0.30	0.30	0.30	0.30	0.30	0.30
x2 (m)	0.55	0.55	0.55	0.55	0.55	0.55
x3 (m)	*1.90*	*2.20*	*2.40*	*2.60*	*2.80*	*3.00*
x4 (m)	0.60	0.60	0.60	0.60	0.60	0.60
x5 (m)	0.40	0.40	0.40	0.40	0.40	0.40
x6 (m)	0.25	0.25	0.25	0.25	0.25	0.25
FSov	2.164	2.666	3.029	3.416	3.825	4.258
FSsdp	2.840	3.015	3.132	3.249	3.366	3.483
FSsd0	1.505	1.680	1.797	1.914	2.031	2.148
FSbe	3.840	4.495	4.956	5.427	5.903	6.382
ecentricity	not ok	ok	ok	ok	ok	ok
FSstoe	3.710	4.150	4.41	4.660	4.890	5.100
FSsheel	1.360	1.180	1.080	1.000	0.920	0.860
FSsstem	3.000	3.000	3.000	3.000	3.000	3.000
Fsslope(min)	2.400	2.470	2.530	2.570	2.620	2.670
tcost (B)*10000	1.3680	1.4328	1.4859	1.5514	1.6217	1.6971

From Table 4.4, total cost increases as X_3 increases. However, increasing X_3 does not have any effects in trial 5 and 6 because factor of safety against shear failure is less than that of requirements. In trial 3 and trail 4, factor of safety against slope failure is satisfied in case that lengths of heel slab X_3 ranges from 2.40 meters to 2.60 meters. This can be concluded that X_3 has significant influence on critical safety factor of slope.

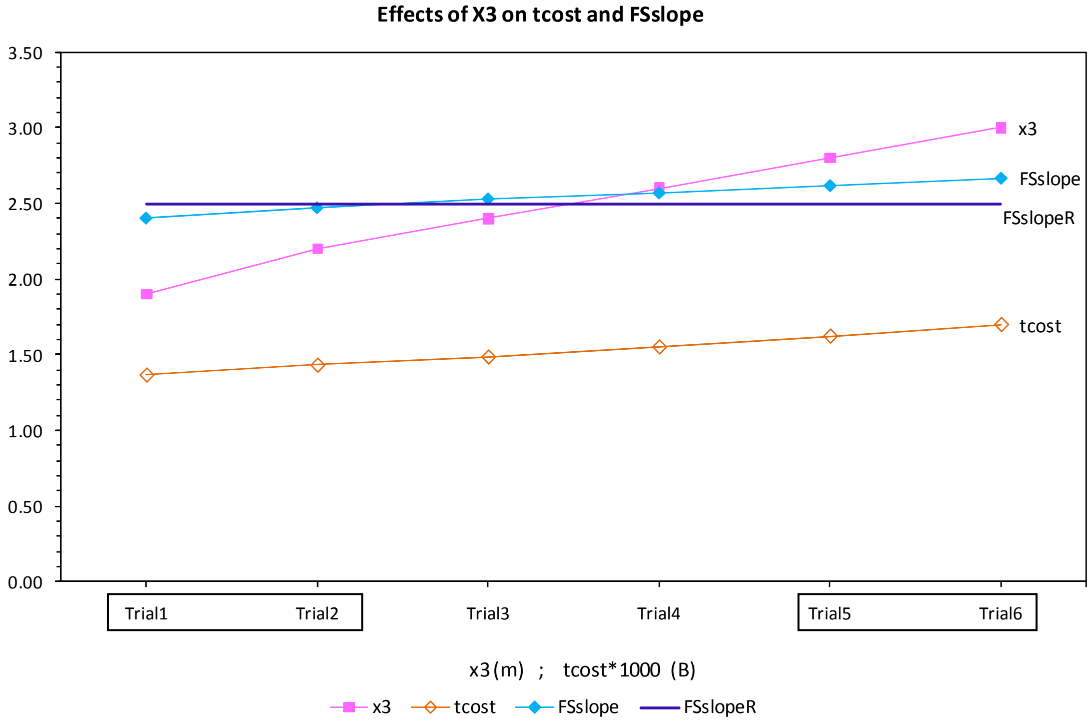

Figure 4.3 Effects of X_3 on safety factor of slope and total cost

C. Trial solutions by changing X2

The effects of X_2 are presented in Table 4.5.

Table 4.5 Trial solutions by changing X_2

X	**Trial1**	**Trial2**	**Trial3**	**Trial4**	**Trial5**	**Trial6**
x1 (m)	0.30	0.30	0.30	0.30	0.30	0.30
x2 (m)	*0.60*	*0.80*	*1.00*	*1.20*	*1.40*	*1.60*
x3 (m)	2.00	2.00	2.00	2.00	2.00	2.00
x4 (m)	0.60	0.60	0.60	0.60	0.60	0.60
x5 (m)	0.40	0.40	0.40	0.40	0.40	0.40
x6 (m)	0.25	0.25	0.25	0.25	0.25	0.25
FSov	2.406	2.741	3.096	3.470	3.865	4.279
FSsdp	2.922	3.016	3.109	3.203	3.296	3.390
FSsd0	1.587	1.680	1.774	1.867	1.961	2.054
FSbe	4.184	4.757	5.394	6.092	6.849	7.667
ecentricity	not ok	ok	ok	ok	ok	ok
FSstoe	3.980	4.440	4.930	5.420	5.940	6.470
FSsheel	1.290	1.290	1.290	1.290	12.590	1.290
FSsstem	3.390	5.120	7.180	9.960	12.590	16.160
Fsslope(min)	2.437	2.469	2.501	2.534	2.570	2.600
tcost (B)*10000	1.4455	1.6691	1.8930	2.1169	2.3409	2.5651

The solutions presented in Table 4.5 are calculated based on Ex 2 in Table 4.1. The study maintains X_1 and X_3 as constant value while X_2 is changed. X_4 is given as 0.60 meter since it was calculated by conventional proportion and this thickness is sufficient for shear design.

From Table 4.5, total cost increases as X_2 increases. However, increasing X_2 can make safety factor of slope, FSslope, satisfying with which of requirement, FSslopereq. However, the case of X_2 greater than 1 meter can satisfy required safety factor of slope of 2.50. This can be concluded that increasing X_2 is slightly significant.

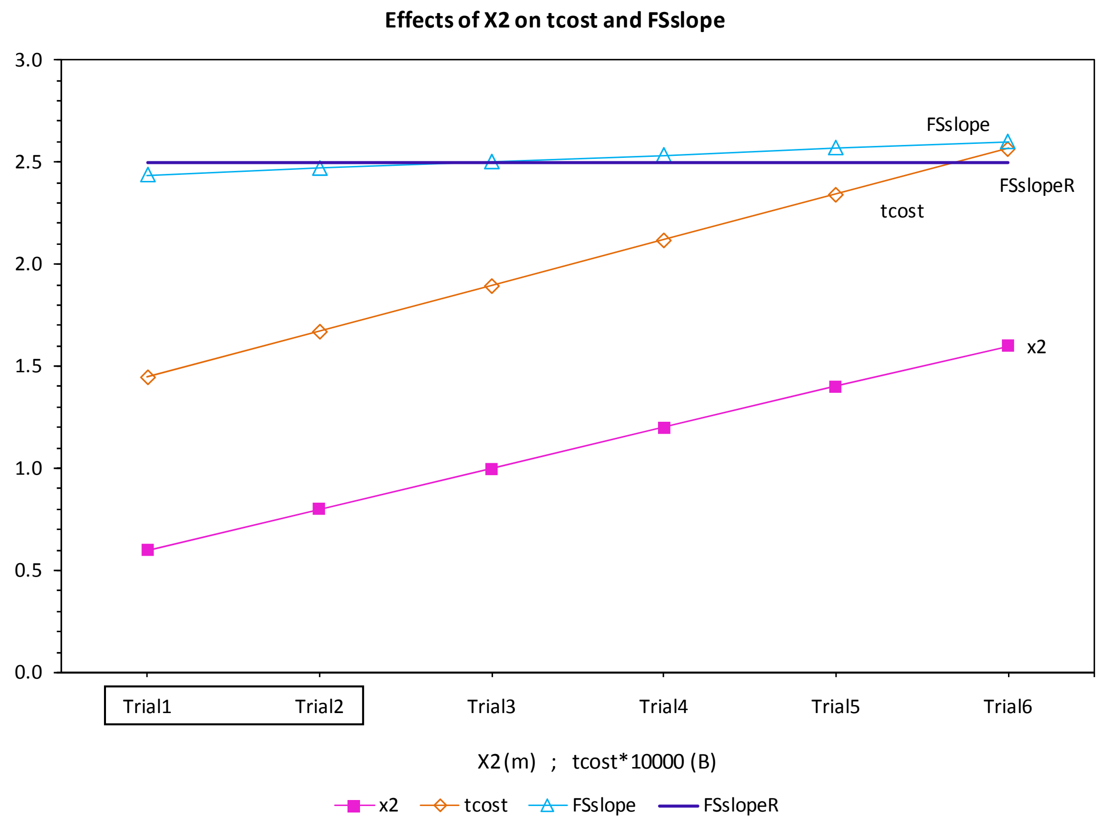

Figure 4.4 Effects of X_2 on safety factor of slope and total cost

D. Trial solutions by changing X1 and X3

The same input parameters in example 2 of Table 4.1 are taking into trial design using ultimate strength design. Both X_1 and X_2 are changed while X_2 are fixed. X_4 is given by 0.60 m since it is calculated by conventional proportion and its thickness is controlled by shear design. The result is summarized in Table 4.6.

Table 4.6 Trial solutions by changing X1 and X3

X	Trial1	Trial2	Trial3	Trial4	Trial5
x1 (m)	0.60	0.70	0.80	0.80	1.00
x2 (m)	*0.50*	*0.50*	*0.50*	*0.50*	*0.50*
x3 (m)	1.80	2.00	2.20	2.40	2.60
x4 (m)	0.60	0.60	0.60	0.60	0.60
x5 (m)	0.40	0.40	0.40	0.40	0.40
x6 (m)	0.25	0.25	0.25	0.25	0.25
FSov	2.346	2.842	3.387	3.802	4.617
FSsdp	2.846	2.992	3.138	3.255	3.431
FSsd0	1.511	1.657	1.803	1.920	2.095
FSbe	4.445	5.389	6.469	7.108	8.997
ecentricity	ok	ok	ok	ok	ok
FSstoe	2.380	2.380	2.380	2.480	2.360
FSsheel	1.440	1.290	1.180	1.080	1.000
FSsstem	2.620	2.620	2.620	2.620	2.620
Fsslope(min)	2.378	2.428	2.478	2.530	2.580
tcost (B)*10000	1.3553	1.4201	1.4849	1.5394	1.6529

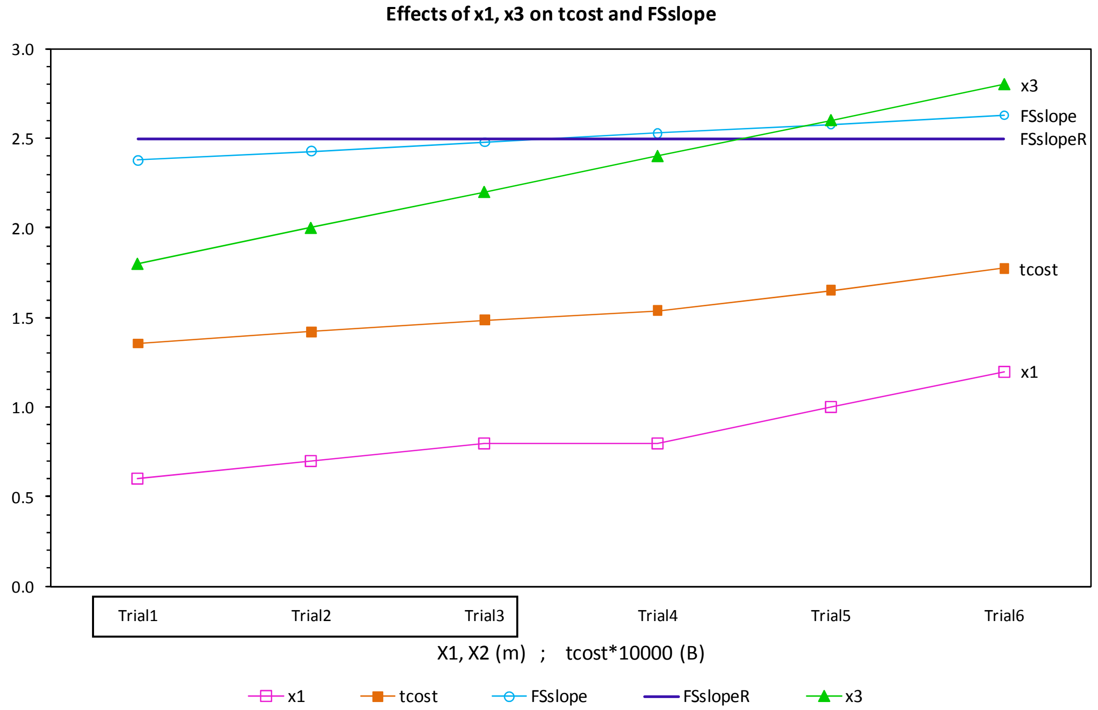

Figure 4.5 Effect of changing X_1 and X_3 on factor of slope and total cost

From Figure 4.5, total cost increases as X_1 and X_3 increase. In trial 1, even though factor of safety against sliding is slightly greater than that of requirement, the design is unsafe because critical safety factor of slope stability is not satisfied. It can be concluded that the total cost in this trial is not optimal. For trial 3 and 4 where toe length (X_1) is set constant, slightly increase of heel slab (X_3) can increase critical safety factor of slope ($FS_{SLOPE\ REQ}$). This is true as mentioned in trial solutions by varying X_3 section 4.2.2.

4.3 Optimization solutions from computer methods

This section presents optimization solutions from computer methods. Four optimization solver algorithms are used in the optimal design, namely, *NLPSolve* (MAPLE), *fimincon* (MATLAB), *DNCONF* (IMSL FORTRAN), and *KNITRO*.

A. Optimization solutions without including slope constraints

Example 2 in Table 4.1 is solved with all algorithms presented above. Table 4.7 shows results from computer calculations without including any slope constraints. Ultimate strength design (USD) is used in structural design.

Table 4.7 Optimal solutions with different optimization solvers

Optimal solutions	**NLPSolve**	**fmincon**	**DNCONF**	**KNITRO**
X_1 (m)	1.208	1.208	1.208	1.208
X_2 (m)	0.443	0.443	0.443	0.443
X_3 (m)	1.549	1.549	1.550	1.550
X_4 (m)	0.386	0.386	0.386	0.386
X_5 (m)	0.614	0.614	0.614	0.614
X_6 (m)	0.250	0.250	0.250	0.250
X_7 (mm^2)	1077.51	1077.51	1077.50	1077.50
X_8 (mm^2)	1405.57	1405.57	1405.57	1405.57
X_9 (mm^2)	1822.26	1822.26	1822.26	1822.26
Total cost (฿)	11975.83	11975.83	11975.83	11975.83

It can be noticed that all optimal solutions are the same in all algorithms. *NLPSolve* produces the same solutions in comparing with *fimincon*, *DNCONF* and *KNITRO* although those used different initial search points. It can be concluded that the optimal solution by computer methods is the global minimum. These results are valid and reasonable.

B. Optimization solutions with slope constraints

This section presents the optimization solutions from example 2 in Table 4.1 which is solved by the same algorithms presented above. Ultimate strength design (USD) as well as its constraints are included in computation. The solutions are summarized in Table 4.8.

Table 4.8 Optimal solutions with different optimization solvers in case of including slope constraints

Optimal solutions	NLPSolve	fmincon	DNCONF	KNITRO
X_1 (m)	0.246	N/A	N/A	0.246
X_2 (m)	0.435	N/A	N/A	0.435
X_3 (m)	2.367	N/A	N/A	2.367
X_4 (m)	0.554	N/A	N/A	0.554
X_5 (m)	0.446	N/A	N/A	0.446
X_6 (m)	0.250	N/A	N/A	0.250
X_7 (mm^2)	1665.495	N/A	N/A	1665.495
X_8 (mm^2)	2146.310	N/A	N/A	2146.310
X_9 (mm^2)	1689.484	N/A	N/A	1689.484
FS_{SLOPE}^{MIN}	2.500	N/A	N/A	2.500
Total cost (฿)	13447.091	N/A	N/A	13447.091
Running time (s)	180.34 s	N/A	N/A	18.68 s

Note: N/A stands for “Not Available”.

According to Table 4.8, *NLPSolve* and *KNITRO* can determine optimal design with including slope constraints. These two methods give the same results. The others such as *fmincon*, and *IMSL DNCONF* cannot determine the optimal solution.

However, it is not always that *MAPLE* and *KNITRO* can find the optimal solution due to several reasons such as:

1. algorithm may not rigorous
2. input values does not satisfy requirement constraints in slope stability with safety factor equal to 2.50
3. Square root terms are appeared in FS slope expression

It can be concluded that slope constraints is very nonlinear and complex, namely, first and second derivative of safety factor of slope with respect to center of critical circular arc failure surface ($\partial FS/\partial X_0$, $\partial FS/\partial Y_0$) and all elements of Hessian matrix ($\frac{\partial^2 FS}{\partial X^2_0}$, $\frac{\partial^2 FS}{\partial Y^2_0}$, $\frac{\partial^2 FS}{\partial X_0 \partial Y_0}$) for sufficiency of the optimal condition.

4.4 Comparison between conventional and optimal design of RC cantilever retaining wall

The conventional method for designing reinforced concrete cantilever retaining wall has been presented already in literature review by using approximate proportions of wall components. A comparison between the conventional and optimal design is made in order to demonstrate the efficiency of optimization technique.

Table 4.9 Conventional design of RC cantilever retaining wall

Design variables	Trial dimension and required safety factor	Con design 1	Con design 2	Con design 3	Con design 4
X_1 (m)	$0.1\times H$	0.55	0.55	0.55	0.55
X_2 (m)	$0.1\times H$	0.55	0.55	0.55	0.55
X_3 (m)	$0.3\times H$ to $0.5\times H$	2.00	2.20	2.30	2.40
X_4 (m)	$0.1\times H$	0.60	0.60	0.60	0.60
X_5 (m)	$D - 0.1\times H$	0.40	0.40	0.40	0.40
X_6 (m)	0.250	0.25	0.25	0.25	0.25
X_7 (mm^2)	-	1827.00	1827.00	1827.00	1827.00
X_8 (mm^2)	-	1827.00	1827.00	1834.36	1834.36
X_9 (mm^2)	-	1652.00	1652.00	1652.00	1652.00
FS_{OV}	2.00	2.701	3.070	3.263	3.462
FS_{SDP}	2.00	2.972	3.089	3.147	3.205
FS_{SD0}	1.50	1.637	1.753	1.812	1.870
FS_{BE}	3.00	4.942	5.482	5.757	6.032
$FS_{SLOPE\ REQ}$	2.50	2.433	2.481	2.505	2.529
Total cost (฿)	-	14436.27	14868.48	15088.91	15407

Table 4.9 shows total cost and factor of safety of slope stability from conventional design. USD is used in structural design of stem, heel, and toe slab. The input of the conventional design is reported the same as example 2 in Table 4.1. Two important parameters taken into trial selection are total height (H=5.20 m) and soil embedment (D=1.0 m).

The different percentage with respect to total cost of conventional method can be calculated as:

$$\text{Different percentage (\%)} = \frac{\text{tcost(con)} - \text{tcost(opt)}}{\text{tcost(con)}} \times 100\% \qquad (4.1)$$

Since critical safety factor of slope from *Con design 3* is equal to required, value, the solutions in that column will be used to compare with optimal design. Thus from Equation (4.1), it can be concluded that the total cost of whole structure decreases about 10.88% when optimization technique is applied.

4.5 Results and discussion on two examples

Two examples are presented in this section. In each example, the optimal design is focused on effective stress analysis (ESA) with structural design based on USD and WSD. Table 4.10 presents both USD and WSD without and with slope constraints. Optimal solutions are obtained by running KNITRO solver algorithm.

All symbols of wall design variables are defined as:

X_1, X_2, X_3 (m) = total width of toe, stem thickness at bottom, and total width of heel, respectively

X_4 (m) = thickness of footing slab

X_5 (m) = depth of soil cover above slab footing

X_6 (m) = stem thickness at top

X_7 (mm^2) = horizontal reinforcing area of toe per unit length of wall

X_8 (mm^2) = horizontal reinforcing area of heel per unit length of wall

X_9 (mm^2) = horizontal reinforcing area of stem per unit length of wall

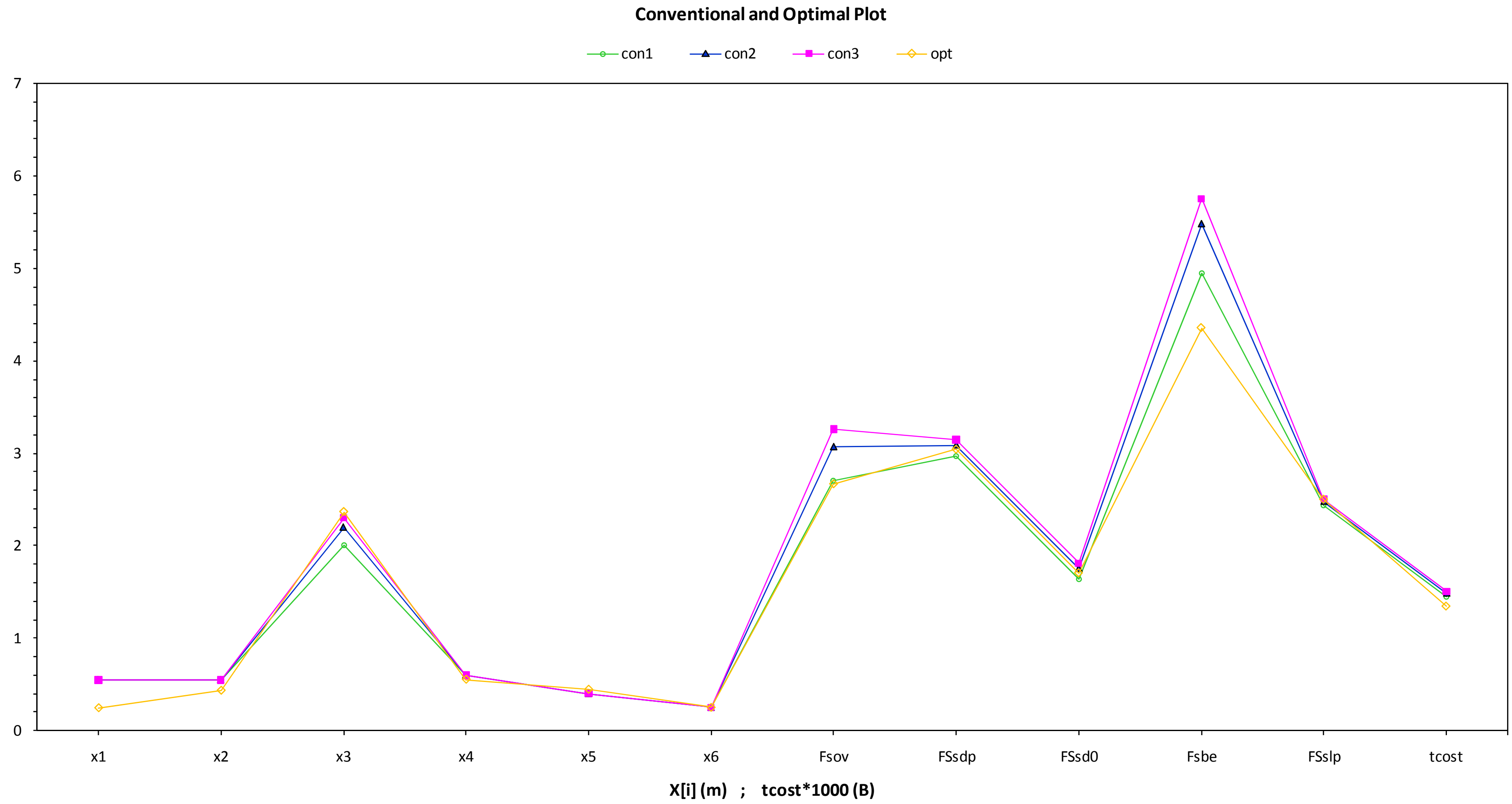

Figure 4.6 Plot of conventional design and optimal design

4.5.1 Optimization with effective stress analysis (ESA)

A. Optimal solutions of example 1 (ESA)

Table 4.10 presents optimal solutions of example 1 analyzed by effective stress condition. The optimal solution by working stress and ultimate strength design including slope constraints are also reported in that table.

Table 4.10 Optimization solutions of example 1 (ESA)

Design Variables	Exercise 1 (WSD)		Exercise 1 (USD)	
	Without slope	With slope	Without slope	With slope
	Optimum values	Optimum values	Optimum values	Optimum values
X_1 (m)	1.518	0.879	1.838	0.896
X_2 (m)	0.626	0.619	0.549	0.553
X_3 (m)	2.077	2.522	1.917	2.551
X_4 (m)	0.487	0.536	0.544	0.705
X_5 (m)	1.313	1.264	1.255	1.095
X_6 (m)	0.250	0.250	0.250	0.250
X_7 (mm^2)	1494.744	652.462	1625.079	2188.110
X_8 (mm^2)	2922.433	3871.956	1731.317	2306.400
X_9 (mm^2)	2702.530	2672.498	2112.667	1922.707
$MinFS_{SLOPE}$	2.453	2.50	2.432	2.500
Optimal Cost (฿)	19229.907	19263.569	18020.115	19509.764

In example 1, depth of soil embedment is considered as an important input. If designers use shallow depth, solvers cannot find optimal solutions which give minimum safety factor of slope equal to 2.50. Generally, the width of heel (X_3) and thickness of base slab (X_4) of the optimal solution with slope stability is higher than that without slope constraints. The larger values will satisfy required safety factor of slope failure, wall sliding failure along the base without passive force, and shear heel failure. In addition, main reinforcement areas in each section are higher.

The results also show that even though the optimal solution without slope stability constraints gives lower cost of material, such design is not adequate and unsafe because the minimum safety factor of slope failure is violated with the required FS_{SLOPE}. On the other hand, the optimal solution with including slope stability constraints give higher total cost of material, but the design is safe and valid because the minimum FS_{SLOPE} is equal to the required value.

This result means that slope constraint is active and its safety factor is the controlled value for the optimal solution. Other active constraints are wall sliding failure along the base without passive force and shear heel failure.

B. Optimal solutions of example 2 (ESA)

Table 4.11 presents optimal solutions of example 2 analyzed by effective stress condition. The similar discussion can be applied to example 1.

Table 4.11 Optimization solutions of example 2 (ESA)

Design Variables	**Example 2 (WSD)**		**Example 2 (USD)**	
	Without slope	**With slope**	**Without slope**	**With slope**
	Optimum values	**Optimum values**	**Optimum values**	**Optimum values**
X_1 (m)	0.606	0.228	1.208	0.246
X_2 (m)	0.496	0.491	0.443	0.435
X_3 (m)	1.809	2.337	1.550	2.367
X_4 (m)	0.376	0.422	0.386	0.554
X_5 (m)	0.624	0.578	0.614	0.446
X_6 (m)	0.250	0.250	0.250	0.250
X_7 (mm^2)	554.627	82.869	1077.506	1665.495
X_8 (mm^2)	2459.913	3569.290	1405.567	2146.310
X_9 (mm^2)	2307.069	2272.199	1822.260	1689.484
minFS	2.375	2.500	2.3111	2.500
Optimal Cost (฿)	12528.285	13338.788	11975.826	13447.091

The result indicates that slope constraint is active as well as shear and eccentricity. Safety factor of shear in heel is the controlled value for the optimal solution since heel slab resists directly to surcharge on surface and soil weight of backfill.

From results in both two examples, it can be concluded that constraints of slope failure mechanism, shear in heel, wall sliding failure along the base without passive force, and eccentricity failure have significant effects on the optimal dimensions and reinforcements of cantilever retaining wall.

4.5.2 Optimization with total stress analysis (TSA)

A. Optimal solution of example 1 (TSA)

Table 4.12 presents optimal solutions of example 1 analyzed by total stress condition. The optimal solution by working stress and ultimate strength design including slope constraints can be determined by solvers.

Table 4.12 Optimization solutions of example 1 (TSA)

Design Variables	**Example 1 (WSD)**		**Example 1 (USD)**	
	Without slope	**With slope**	**Without slope**	**With slope**
	Optimum values	**Optimum values**	**Optimum values**	**Optimum values**
X_1 (m)	2.144	0.623	1.705	0.707
X_2 (m)	0.612	0.621	0.541	0.544
X_3 (m)	1.15	3.809	1.715	3.743
X_4 (m)	0.431	0.780	0.506	1.015
X_5 (m)	1.069	1.119	1.994	0.485
X_6 (m)	0.250	0.250	0.250	0.250
X_7 (mm^2)	2860.766	205.625	1616.320	3280.5838
X_8 (mm^2)	1029.775	5861.907	1496.815	3388.981
X_9 (mm^2)	2842.349	2341.086	2189.600	1643.685
minFS	3.679	3.423	3.689	3.407
Optimal Cost (฿)	17801.903	25371.675	16847.842	27231.442

Generally, the width of heel (X_3) thickness of base slab (X_4) of the optimal solution with slope stability is higher than that without slope constraints. The higher values can satisfy required safety factor of slope failure, and bearing capacity failures. In addition, main reinforcement areas in each section are higher. This result shows that bearing capacity constraint and shear in heel constraint are active and its safety factor is the controlled value for the optimal solution.

B. Optimal solutions of example 2 (TSA)

Table 4.13 presents optimal solutions of exercise 2 analyzed by total stress condition. The optimal solutions by working stress and ultimate strength design including slope constraints can be determined by solvers

Table 4.13 Optimization solutions of example 2 (TSA)

Design Variables	Example 2 (WSD)		Example 2 (USD)	
	Without slope	With slope	Without slope	With slope
	Optimum values	Optimum values	Optimum values	Optimum values
X_1 (m)	1.586	0.457	1.403	0.513
X_2 (m)	0.532	0.534	0.470	0.469
X_3 (m)	1.179	2.885	1.431	2.867
X_4 (m)	0.380	0.532	0.411	0.693
X_5 (m)	0.620	0.468	0.589	0.307
X_6 (m)	0.250	0.250	0.250	0.250
X_7 (mm^2)	2151.073	191.574	1363.522	2151.777
X_8 (mm^2)	1096.259	4405.265	1164.444	2591.615
X_9 (mm^2)	2521.805	2311.379	1996.451	1698.917
minFS	4.014	3.679	3.996	3.686
Optimal Cost (฿)	14194.153	17147.983	13123.023	17627.117

Generally, the width of heel (X_3) thickness of base slab (X_4) of the optimal solution with slope stability is higher than that without slope constraints. The higher values can satisfy required safety factor of slope failure, and bearing capacity failures. In addition, main reinforcement areas in each section are higher.

This result shows that bearing capacity constraint and shear in heel constraint are active and its safety factor is the controlled value for the optimal solution.

4.6 Results on integral bridge abutment

The first part of input parameters to find maximum shear in abutment support is described in Table 4.14 (for more details on analysis formulation, see appendix C).

Table 4.14 Input parameters for finding maximum shear force

Name	Symbol	Value	Unit
Length of bridge span	L	6.50	m
Width of barrier	wb	0.20	m
Width of bitumen laying	wl	7.40	m
Total width	wt	7.80	m
Thickness of bitumen	Tb	0.05	m
Thickness of bridge slab	T	0.40	m
Thickness of barrier	Tb	0.60	m
Width of sidewalk	wsw	0.00	m
Thickness of sidewalk	tsw	0.00	m
Pedestrian Load	PL	3.60	kN/m^2
Thickness of stem beam	Tbe	0.60	m
Width of stem beam	wbeam	0.40	m
Width from stem beam to end slab side	wbe	0.90	m
Number of stem beam	nbeam	0	-
Unit weight of concrete	γ_c	24.00	kN/m^3
Unit weight of bitumen	γ_{bi}	22.50	kN/m^3
Design Truck (AASHTO)	LL	HS20*	-
Width design lane	wlane	3.60	m
Design lane load	Lane	9.30	kN/m
Dynamic allowance factor	IM	33	%

The line load distributed per 1 meter can be expressed as:

$$P_1 = V_{max}/wt \tag{4.2}$$

where P_1 = axial compression force from external load per unit length of wall

V_{max} = maximum reaction of applied load on bridge (kN)

wt = total width of bridge (m)

* HS20 = Truck 20 tons according to AASHTO (2007)

Table 4.15 Input parameters for optimal design of bridge abutment wall

Input parameters	Symbol	Unit	Ex1	Ex2
Point Load	P_1	kN/m	180.0	180.0
Total Height	H	m	6.0	5.8
Depth of embedment	D	m	1.0	1.0
Yield strength of reinforcing steel	f_y	MPa	400.0	400.0
Compressive strength of concrete	f_c'	MPa	25.0	25.0
Unit weight of steel	γ_{st}	Kg/m^3	7850.0	7850.0
Unit weight of concrete	γ_c	kN/m^3	24.0	24.0
Concrete cover	cover	mm	70.0	70.0
Diameters of bar	Φ_{long}	mm	25	25
Diameter of stirrups	Φ_{st}	mm	10	10
Surcharge loading	q	kN/m^2	20.0	20.0
Backfill of slope	α	Degree	0.0	0.0
Unit weight of backfill soil	γ_1	kN/m^3	18.0	16.8
Cohesion of backfill soil	c_1	kN/m^2	0.0	0.0
Internal friction angle of backfill soil	ϕ_1	degree	30.0	30.0
Unit weight of base soil	γ_2	kN/m^3	19.0	18.0
Cohesion of base soil	c_2	kN/m^2	40.0	20.0
Undrained shear strength of base soil	S_u	kN/m^2	80.0	80.0
Internal friction angle of base soil	ϕ_2	degree	20.0	30.0
FS for overturning stability	FS_{ov}	-	2.0	2.0
FS for sliding stability (include P_P)	FS_{sdp}	-	2.0	2.0
FS for sliding stability (exclude P_P)	FS_{sd0}	-	1.5	1.5
FS for bearing stability	FS_{be}	-	3.0	3.0
FS for slope stability	FS_{slope}	-	2.2	2.2
Factor of safety against shear and moment failures	FS_s,FS_m	-	1.0	1.0
Unit cost of concrete	C_C	฿/m^3	2,550	2,550
Unit cost of steel	C_{ST}	฿/kg	22.0	22.0
Unit cost of formwork	C_F	฿/m^2	150.0	150.0

From structural analysis, maximum shear force (V_{max}) equals to 1403 kN. From Equation 4.2, the axial compression force per unit length of wall (P_1) equal to 180 kN/m.

The total axial load (P) including self-weight of stem wall is calculated as:

$$P = P_1 + \gamma_c x_6 \left(H - x_4\right) \qquad (4.3)$$

where P_1 = axial compression force from external load per unit length of wall

γ_c = unit weight of concrete

x_6 = width of stem wall

H, x_4 = total height, and slab footing thickness, respectively

The optimization problem of bridge abutment wall is solved in *MAPLE* program with Ultimate Strength Design in ACI Code 318-05. The results of these two examples by Total Stress Analysis and Effective Stress Analysis are presented in Table 4.16 and Table 4.17.

Table 4.16 Optimal dimension and reinforcement of example 1

Design Variables	Example 1 (USD)(TSA)		Example 1 (USD)(ESA)	
	Without slope	With slope	Without slope	With slope
	Optimum values	Optimum values	Optimum values	Optimum values
X_1 (m)	2.916	2.474	1.84	1.456
X_2 (m)	0.526	0.524	0.520	0.511
X_3 (m)	0.720	1.944	1.260	2.922
X_4 (m)	0.765	0.780	0.808	0.867
X_5 (m)	0.235	0.220	0.192	0.133
X_6 (m)	0.526	0.524	0.520	0.511
X_7 (mm^2)	2387.78	2439.03	2541.22	2746.74
X_8 (mm^2)	2387.78	2439.03	2541.22	2746.74
X_9 (mm^2)	5261.48	5239.94	5197.14	5111.68
minFS	2.10	2.296	2.15	2.200
Optimal Cost (฿)	26309.23	28673.30	25183.23	29709.59

From Table 4.16, although analysis without slope case give lower cost than analysis with slope case, the design is not safe because factor of safety of slope in the former case does not satisfy slope requirement.

Generally, for both total stress and effective stress analysis, the width of toe (X_1) is smaller while the width of heel is higher.

Table 4.17 Optimal dimension and reinforcement of example 2

Design Variables	Example 2 (USD)(TSA)		Example 2 (USD)(ESA)	
	Without slope	With slope	Without slope	With slope
	Optimum values	Optimum values	Optimum values	Optimum values
X_1 (m)	2.661	2.348	1.386	0.662
X_2 (m)	0.482	0.480	0.471	0.471
X_3 (m)	0.593	1.785	0.913	2.917
X_4 (m)	0.726	0.740	0.810	0.810
X_5 (m)	0.273	0.260	0.190	0.190
X_6 (m)	0.482	0.480	0.471	0.471
X_7 (mm^2)	2253.95	2299.23	2525.27	2427.20
X_8 (mm^2)	2253.95	2299.23	2525.27	2621.00
X_9 (mm^2)	4818.35	4800.88	4706.62	4705.88
minFS	2.201	2.425	1.783	2.20
Optimal Cost (฿)	23085.96	25561.35	21039.91	24866.58
Times (s)	13.572	13.572	45.693	45.693

From Table 4.17, the total cost of without slope case is always lower than that of with slope case. The design in slope case is safe because factor of slope is satisfied with requirement.

Generally, for both total stress and effective stress, the width of toe (X_1) is smaller while the width of heel is higher.

CHAPTER V
PARAMETRIC STUDY

5.1 Introduction

The parametric study considers the effect of input (design) parameters on the final design values. In this study, two previous examples are used and each input parameter is changed following the actual field problems, then its effect is investigated using optimization technique.

5.2 Analysis process

Example 2 of Table 4.1 is used in this section. Table 5.1 lists the input parameters and investigated values on sensitivity study. In order to investigate the effect of each design parameter on the design variables, one parameter is varied while other parameters remain constant. This numerical analysis is solved using computer code written in *KNITRO* and *MAPLE*.

Table 5.1 Input parameters used in the analysis

N^0	Input Parameter	Symbol	Unit	Reference Value	Investigated Value
1	Compressive strength of concrete	f_c'	MPa	25.0	17, 21, *25*, 28, 31, 32
2	Yield strength of reinforcing steel	f_y	MPa	400.0	300, *400*
3	Internal friction angle of backfill soil	ϕ_1	degree	30.0	22, 24, 26, 28, *30*, 32, 34, 36
4	Cohesion of base soil	c_2	kN/m^2	30.0	*30*, 40, 50, 60, 70, 80
5	Internal friction angle of base soil	ϕ_2	degree	28.0	26, *28*, 30, 32, 34, 36, 38, 40

5.3 Sensitivity study on concrete strength

Concrete strength ranging from 17 MPa to 35 MPa is usually used in practical design. In this sensitivity study, the unit price of concrete per one cubic meter in

Thailand is mentioned already in methodology. Table 5.2 shows the optimal solution with different value of concrete strength.

Table 5.2 Effect of concrete strength on optimal solution (ESA-USD)

f'$_c$ (MPa)	**17**	**21**	**25**	**28**	**31**	**33**
X_1 (m)	0.233	0.241	0.246	0.250	0.254	0.256
X_2 (m)	0.452	0.442	0.435	0.427	0.422	0.418
X_3 (m)	2.354	2.362	2.367	2.372	2.376	2.378
X_4 (m)	0.656	0.590	0.554	0.527	0.505	0.499
X_5 (m)	0.344	0.402	0.446	0.472	0.494	0.501
X_6 (m)	0.250	0.250	0.250	0.250	0.250	0.250
X_7 (mm^2)	2022.740	1818.720	1665.490	1574.340	1496.120	1473.540
X_8 (mm^2)	2022.740	1962.570	2146.300	2276.560	2399.130	2440.280
X_9 (mm^2)	1535.190	1624.900	1689.480	1748.030	1791.690	1820.300
Optimal Cost (฿)	13931.930	13571.370	13447.090	13555.000	13587.110	13735.860

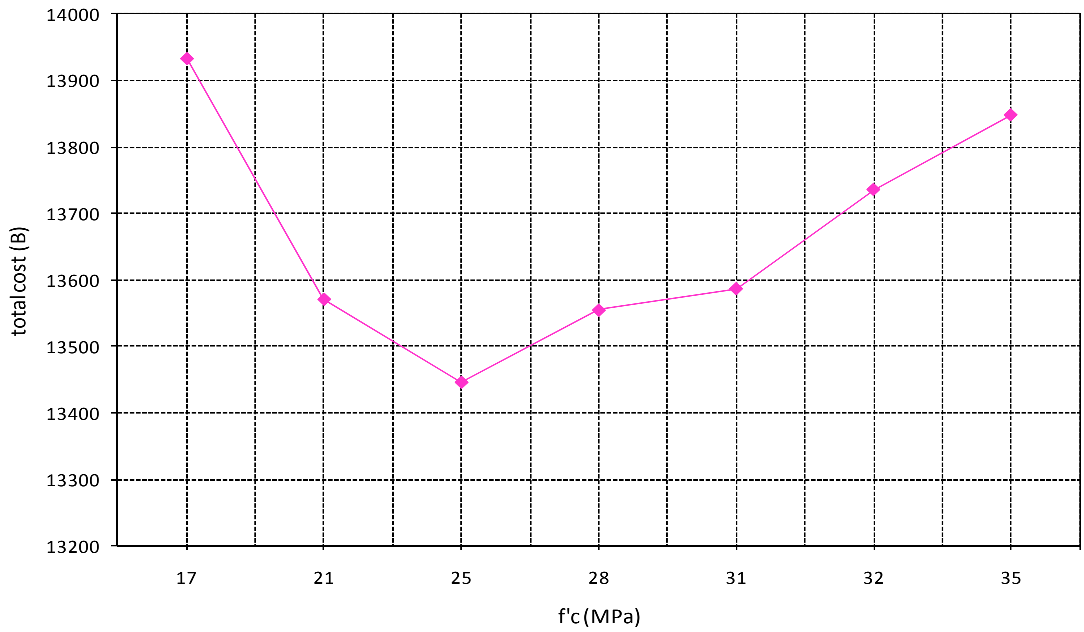

Figure 5.1 Effect of concrete strength on total cost

According to Figure 5.1, compressive strength of concrete equal to 25 MPa gives minimum total cost since its unit price depends on its strength.

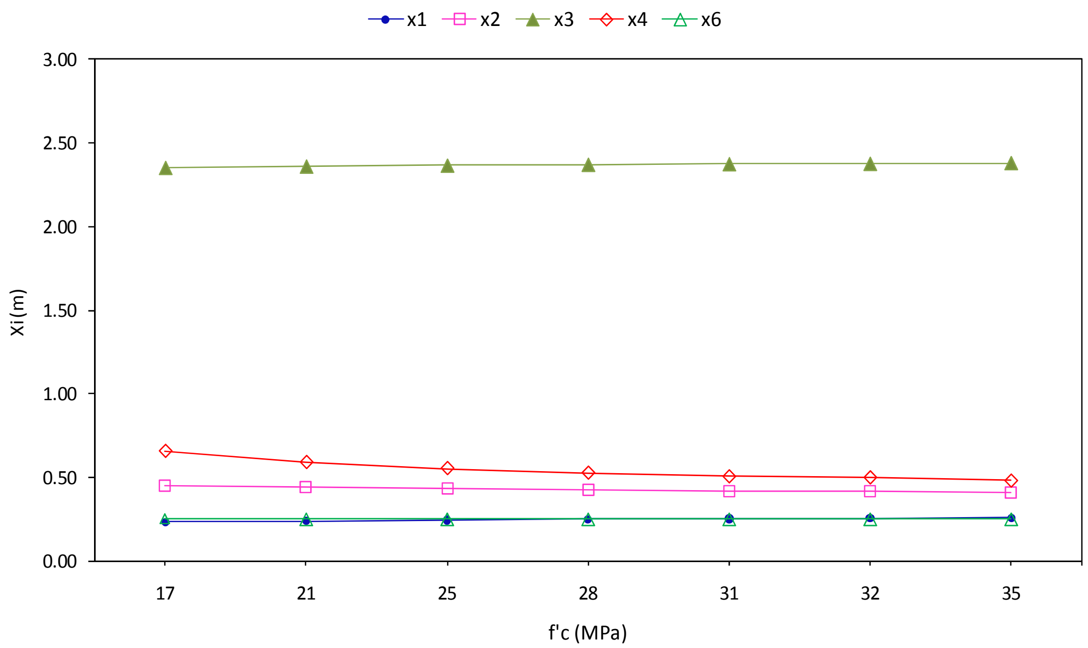

Figure 5.2 Effect of concrete strength on optimal dimensions

In Figure 5.2, when various strength of concrete is used, several design dimensions slightly changes. The most significant change is thickness of footing slab which it depends on concrete strength since its thickness is controlled by shear design. Higher concrete strength can reduce footing thickness.

For economical saving, it is highly recommended to use compressive strength of concrete equal to 25 MPa

5.4 Sensitivity study on yield strength of reinforcing steel

Two typical yield strength of reinforcing steel used in practical design are SD30 (300 MPa) and SD40 (400 MPa). In this sensitivity study, unit price of reinforcing steel per one kilogram in Thailand is mentioned already in the methodology section. Table 5.3 shows the optimal solutions with different value of yielding strength.

Table 5.3 Effect of yielding strength on optimal solution (ESA-USD)

f_y **(MPa)**	**300**	**400**
X_1 (m)	0.233	0.246
X_2 (m)	0.490	0.434
X_3 (m)	2.335	2.367
X_4 (m)	0.547	0.554
X_5 (m)	0.453	0.446
X_6 (m)	0.250	0.250
X_7 (mm^2)	2188.780	1665.490
X_8 (mm^2)	2822.050	2146.310
X_9 (mm^2)	1934.760	1689.480
Optimal Cost (฿)	14578.980	13447.090

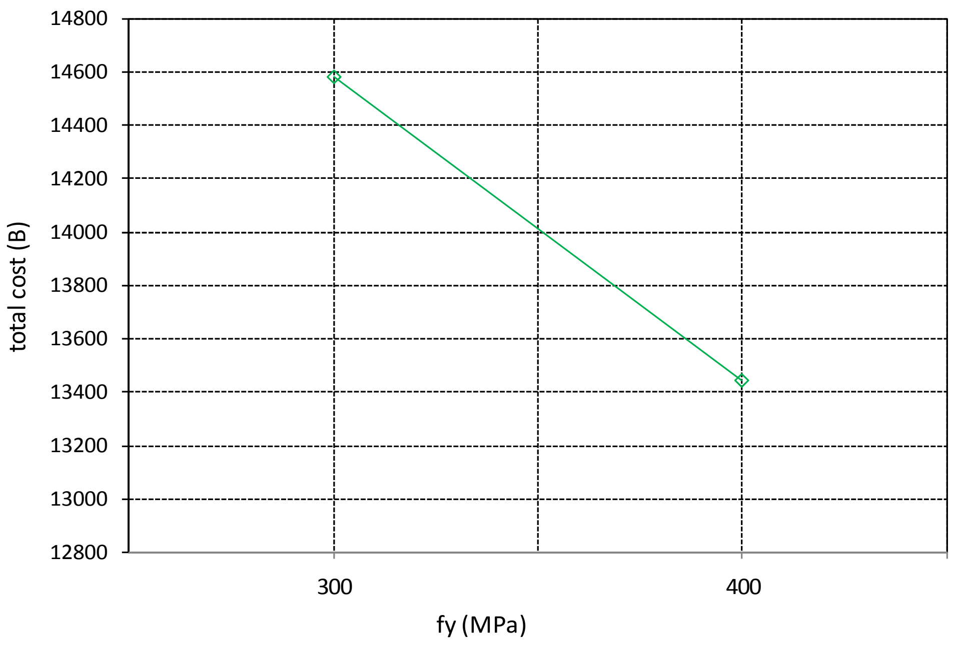

Figure 5.3 Effect of steel yield strength on total cost

According to Figure 5.3, yield strength of reinforcing steel equal to 400 MPa gives minimum total cost due to its high strength.

In Figure 5.4, when various yield strengths of steel are used, several design dimensions slightly increase except X_2. Optimal solution of type SD30 or SD40 are quite the same.

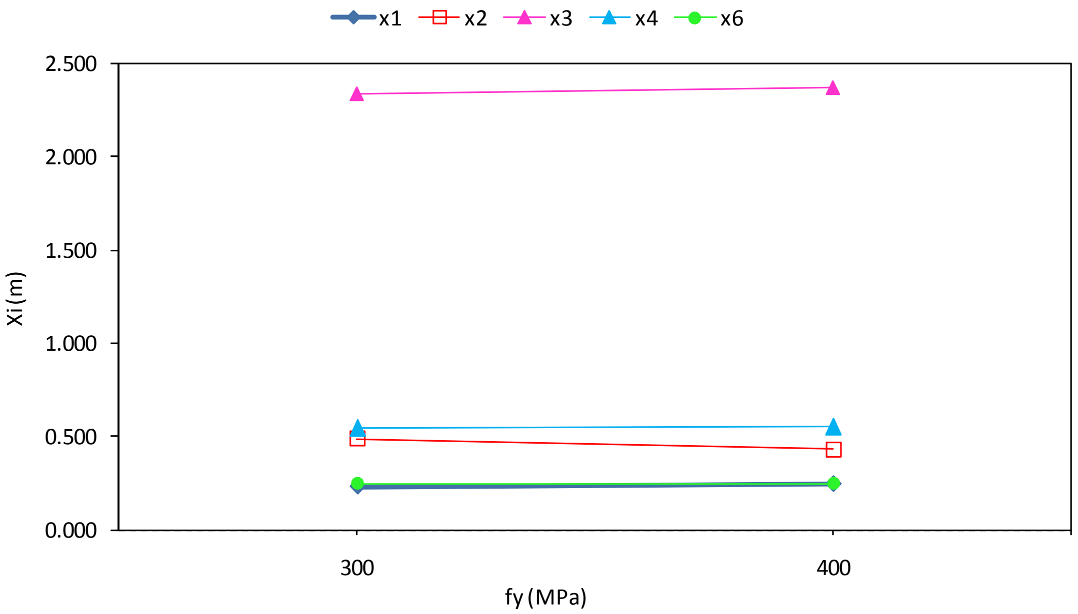

Figure 5.4 Effect of concrete strength on optimal dimensions

Thus for economical saving, it is highly recommended to use yield strength of reinforcing steel equal to 400 MPa.

5.5 Sensitivity study on internal friction angle of backfill soil

Internal friction angle of backfill is varied following the actual field problems. Its effect on optimal dimensions and cost are summarized in Table 5.4.

Table 5.4 Effect of internal friction angle on optimal solution (ESA-USD)

ϕ_1 **(degree)**	**22**	**24**	**26**	**28**	**30**	**32**	**34**	**36**
X_1 (m)	0.149	0.165	0.185	0.212	0.246	0.291	0.351	0.434
X_2 (m)	0.498	0.481	0.465	0.450	0.435	0.420	0.406	0.393
X_3 (m)	2.970	2.829	2.683	2.530	2.367	2.191	1.998	1.779
X_4 (m)	0.680	0.650	0.620	0.587	0.554	0.517	0.477	0.432
X_5 (m)	0.319	0.349	0.380	0.412	0.446	0.482	0.522	0.567
X_6 (m)	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
X_7 (mm^2)	2107.410	2003.620	1896.360	1784.210	1665.490	1537.900	1398.110	1241.02
X_8 (mm^2)	2691.340	2564.220	2432.270	2293.700	2146.300	1987.090	1811.690	1613.37
X_9 (mm^2)	1810.920	1780.510	1719.780	1748.030	1689.480	1659.780	1631.140	1604.31
Optimal Cost (฿)	16768.260	15908.900	15071.640	14252.390	13447.090	12651.420	11860.700	11069.51

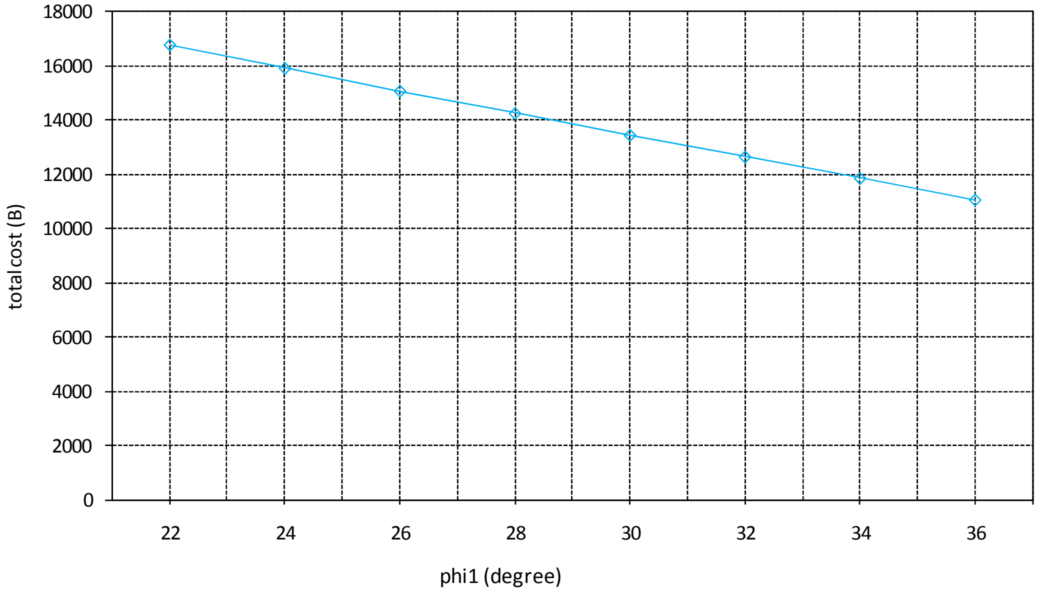

Figure 5.5 Effect of internal friction angle of backfill soil on total cost

According to Figure 5.5, internal friction angle have significant effects on total cost and optimal dimensions. Total cost decreases when angle of internal friction is quite large.

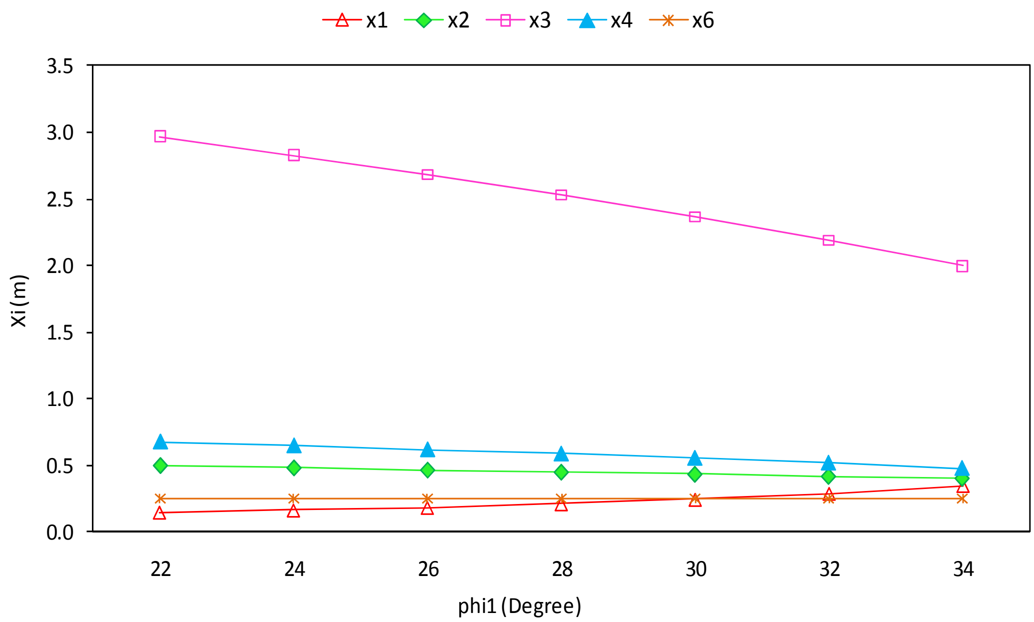

Figure 5.6 Effect of internal friction angle of backfill soil on optimal dimensions

In Figure 5.6, the higher angle of internal friction decreases bottom stem thickness, heel length, and thickness of footing. However, toe length increases as heel length decreases. It is highly recommended to use higher angle of internal friction angle for economizing total cost of construction.

5.6 Sensitivity study on cohesion of base soil

Internal friction angle of backfill is varied following the actual field problems. Its effect on optimal dimensions and cost are reported in table 5.5.

Table 5.5 Effect of cohesion of base soil on optimal solution (ESA-USD)

c_2 (kN/m^2)	30	35	40	50	60	70	80
X_1 (m)	0.246	0.748	1.085	0.966	0.769	0.606	0.468
X_2 (m)	0.435	0.439	0.438	0.442	0.439	0.437	0.436
X_3 (m)	2.367	1.699	1.355	1.466	1.675	1.868	2.047
X_4 (m)	0.554	0.416	0.379	0.369	0.411	0.451	0.487
X_5 (m)	0.446	0.584	0.620	0.631	0.589	0.549	0.512
X_6 (m)	0.250	0.250	0.250	0.250	0.250	0.250	0.250
X_7 (mm^2)	1665.49	1184.06	1053.53	1018.57	1166.50	1304.74	1434.01
X_8 (mm^2)	2146.30	1541.15	1089.56	1330.33	1518.84	1693.96	1856.84
X_9 (mm^2)	1689.48	1812.04	1856.20	1846.63	1814.15	1782.78	1752.85
Optimal Cost (฿)	13447.09	11814.21	11309.60	11355.13	11763.47	12181.40	12605.02

According to Figure 5.7, cohesion of base soil has more effects on total cost and optimal dimensions. Total cost decreases when angle of internal friction increases. However, the total cost is the most economical when cohesion of base soil equal 40 kN/m^2 is used.

In Figure 5.8 both X_1 and X_3 are increased and decreased at the same time. Thickness of footing becomes the smallest when cohesion of base soil cohesion equal 40 kN/m^2 is used. It is important to investigate the cohesion of soil base since it affects on both total cost and dimensions.

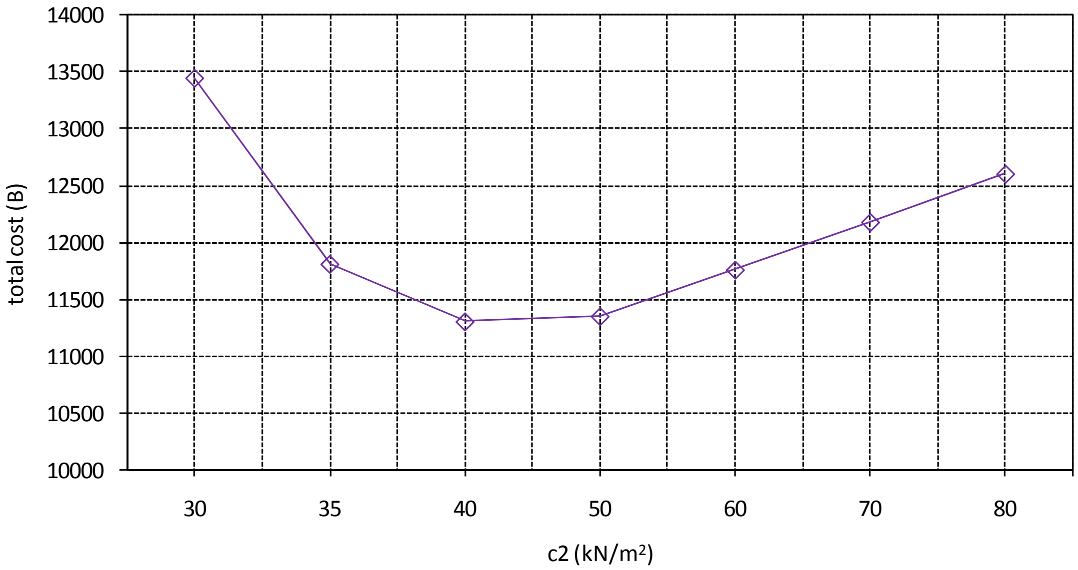

Figure 5.7 Effect of cohesion of base soil on total cost

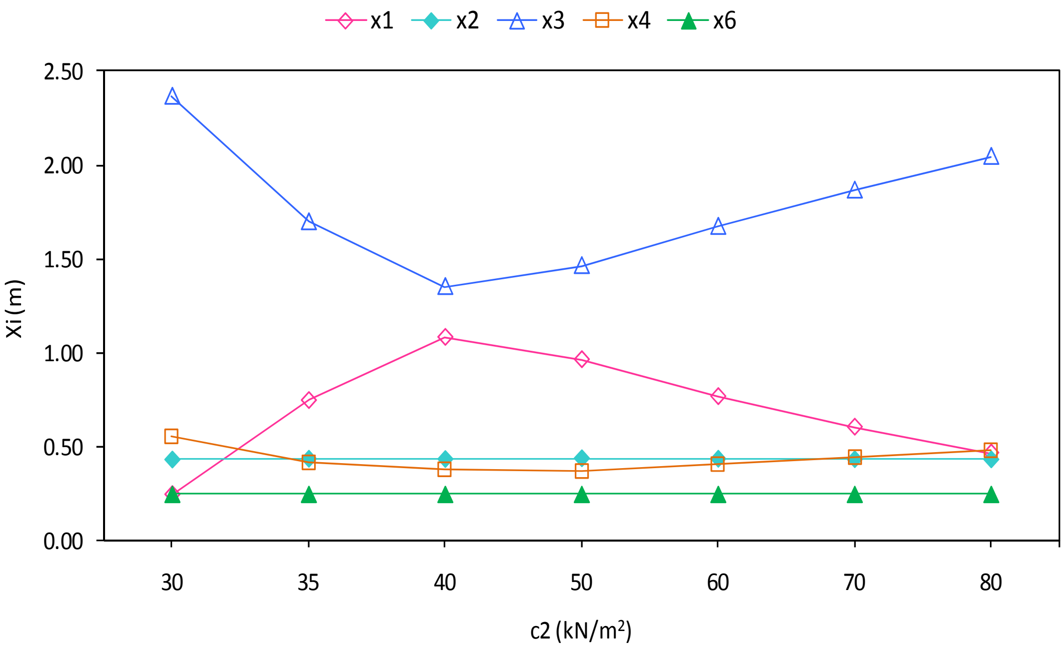

Figure 5.8 Effect of cohesion of base soil on optimal dimensions

5.7 Sensitivity study on internal friction angle of base soil

Internal friction angle of backfill is varied following the actual field problems. Its effect on optimal dimensions and cost are reported in Table 5.6.

Figure 5.9 shows the effects of internal friction of base soil on total cost.

Table 5.6 Effect of internal friction angle of base soil on optimization solutions (ESA-USD)

ϕ_2 **(degree)**	**26**	**28**	**30**	**32**	**34**	**36**	**38**	**40**
X_1 (m)	0.100	0.246	0.460	0.672	0.883	1.086	1.086	1.086
X_2 (m)	0.434	0.435	0.437	0.438	0.441	0.438	0.438	0.438
X_3 (m)	2.730	2.367	2.056	1.787	1.552	1.355	1.355	1.355
X_4 (m)	0.630	0.554	0.490	0.434	0.386	0.379	0.379	0.379
X_5 (m)	0.370	0.446	0.510	0.565	0.614	0.620	0.620	0.620
X_6 (m)	0.250	0.250	0.250	0.250	0.250	0.250	0.250	0.250
X_7 (mm^2)	1930.13	1665.490	1440.43	1247.26	1078.89	1053.53	1053.53	1053.53
X_8 (mm^2)	2473.88	2146.30	1864.90	1621.27	1407.34	1089.56	1089.56	1089.56
X_9 (mm^2)	1616.82	1689.48	1744.84	1796.80	1836.98	1856.20	1856.20	1856.20
Optimal Cost (฿)	14539.06	13447.09	12628.63	12003.08	11516.22	11309.60	11309.60	11309.60

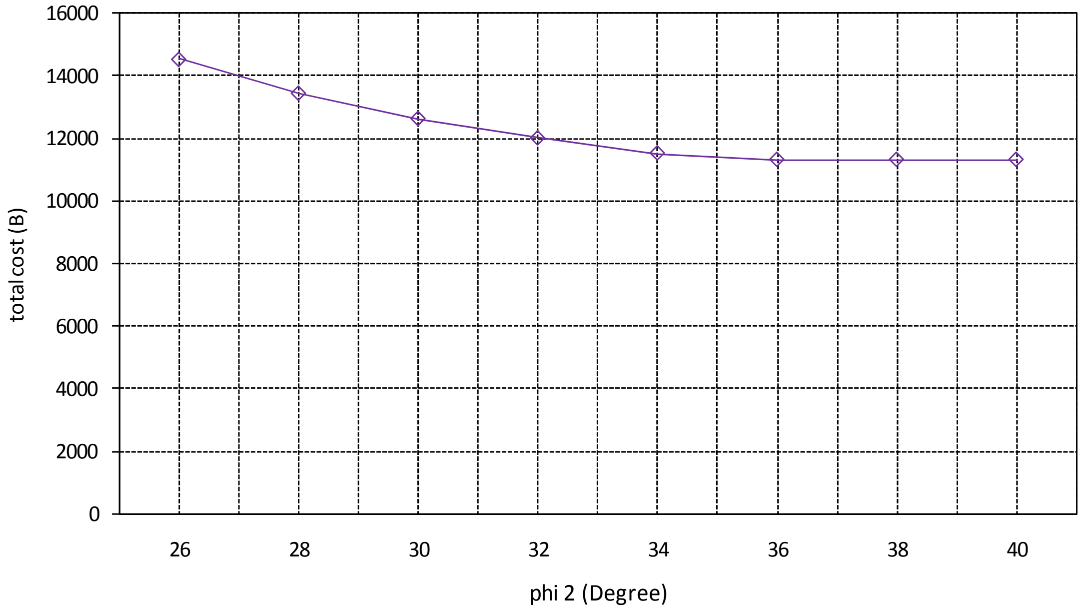

Figure 5.9 Effect of angle of internal friction of base soil on total cost

According to Figure 5.9, cohesion of base soil has more effects on total cost and optimal dimensions. Total cost decreases when angle of internal friction increases.

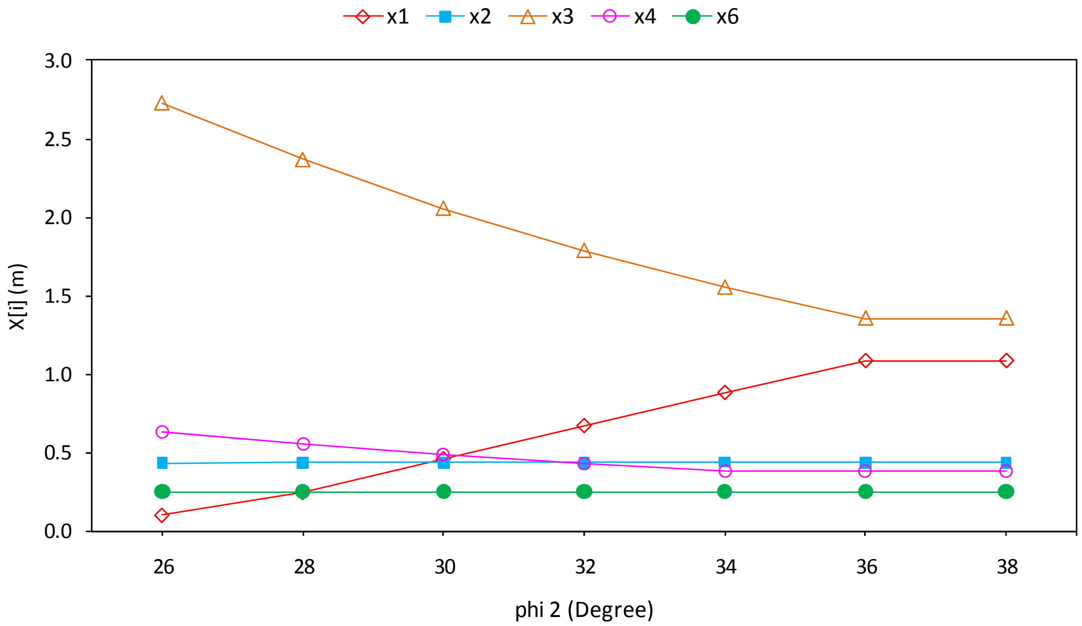

Figure 5.10 Effect of angle of internal friction of base soil on design dimensions

In Figure 5.10, angle of internal friction of base soil has significant effects on decreases heel length and thickness of footing. However, as toe length increases, heel length decreases until it becomes constant for higher internal friction angle. It is highly recommended to use higher angle of internal friction angle for economizing total cost of construction. On the other hand, higher angle can lead to a constant total cost and dimensions.

CHAPTER VI
CONCLUSION AND RECOMMENDATIONS

6.1 Conclusion

This thesis presented the application of the optimization techniques to optimal design of reinforced concrete cantilever retaining wall. Two numerical examples were mainly solved by optimization solvers in *MAPLE* and *KNITRO*. Comparisons between conventional and optimal design have been reported. A parametric study for optimization technique has also been investigated.

We can conclude that this study provides a complete optimal design method of cantilever retaining wall satisfying all geotechnical constraints where past researches in this field were unable to achieve. The result obtained in this study is the most optimal, and thus there is no need to further analyze slope stability, because it is already include in the analysis. On the other hand, optimal results obtained from previous researches were still required to check sufficiency of factor of safety against slope stability because it lacks of in the analysis. Furthermore, their results may not be the most optimal.

This research is successful in developing the optimal design that satisfies all imposed restrictions. The capabilities of both proposed methods are demonstrated through their applications in varieties of general retaining wall problems.

According to parametric study, compressive strength of concrete has the effect on thickness of footing since the shear design does not require shear reinforcing. The internal friction angle of backfill has significantly effects on total cost, bottom of stem, footing thickness, and heel length. The internal friction angle of soil base also has the same effect similar to that of backfill.

6.2 Recommendations for future work

After having completed this research, several recommendations for future works can be summarized below:

- Further study on slope stability constraints in case that slope failure is non-circular shape (wedge failure surface)

- Apply other methods of slices to derive expression of safety factor and make a comparison on these methods in optimal solutions
- Construct design charts for reinforced concrete cantilever retaining wall
- Optimization reinforced concrete cantilever retaining wall subjected to seismic loading. This will be more advantageous since some regions in Thailand are in earthquake zone.

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APPENDICES

APPENDIX A

Formulation for Designing Reinforced Concrete Cantilever Retaining Wall Ultimate Strength and Working Stress Design

Reinforced Concrete Cantilever Retaining Wall

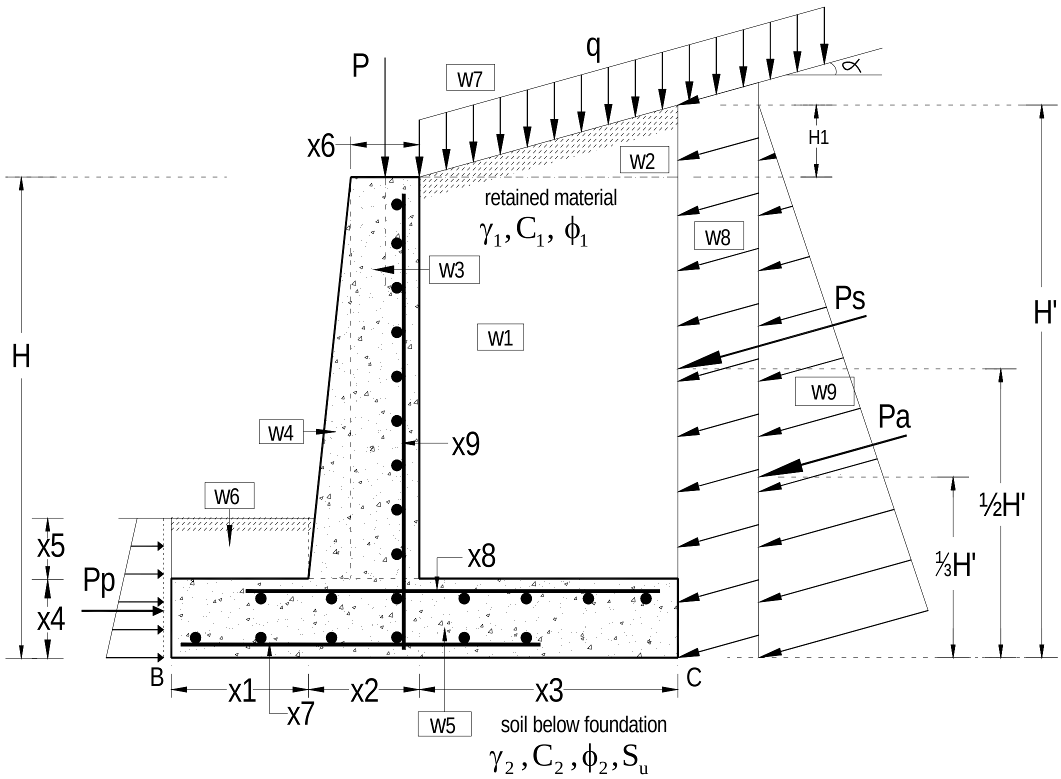


Figure A.1 Typical section of RC cantilever retaining wall

1. Calculate H'

$$H' = H + H_1$$

$$H_1 = x_3 \tan\alpha$$

$$H' = H + x_3 \tan\alpha$$

2. Rankine's active earth pressure and passive earth pressure

$$K_a = \cos\alpha \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi_1}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi_1}}$$

$$K_p = \tan^2\left(45 + \frac{\phi_2}{2}\right)$$

$K_{pt} = 1$ (Total stress analysis)

Table A.1 Important input parameters

N^0	Input parameters (1)	Unit (2)	Symbol (3)
1	Total height of wall	m	H
2	Depth embedment	m	D_{cover}
3	Point Load P	kN/m	P
4	Yield strength of reinforcing steel	MPa	f_y
5	Compressive strength of concrete	MPa	$f_c^{'}$
6	Unit weight of concrete	kN/m^3	γ_c
7	Unit weight of steel	Kg/m^3	γ_s
8	Concrete cover	mm	c
9	Diameters of bars	mm	Φ
10	Surcharge load	kN	q
11	Backfill slope	Degree	α
12	Unit weight of backfill soil	kN/m^3	γ_1
13	Cohesion of backfill soil	KPa	C_1
14	Internal friction angle of backfill soil	Degree	ϕ_1
15	Unit weight of base soil	kN/m^3	γ_2
16	Cohesion of base soil	KPa	C_2
17	Internal friction angle of base soil	Degree	ϕ_2
18	Undrained shear strength of base soil	kN/m^2	S_u
19	Cost of steel	฿/kg	C_s
20	Cost of concrete	฿/m^3	C_c
21	Cost of formwork	฿/m^2	C_f
22	Dead Load factor	-	DL
23	Live Load factor	-	LL

3. Rankine’s active force and passive force per unit length of wall

- *Effective stress analysis (ESA)*

$$P_a = (1/2) K_a \gamma_1 H'^2$$

$$P_{av} = P_a \sin\alpha = (1/2) K_a \gamma_1 H'^2 \sin\alpha = (1/2) K_a \gamma_1 (H + x_3 \tan\alpha)^2 \sin\alpha$$

$$P_{ah} = P_a \cos\alpha = (1/2) K_a \gamma_1 H'^2 \cos\alpha = (1/2) K_a \gamma_1 (H + x_3 \tan\alpha)^2 \cos\alpha$$

$$P_p = (1/2) K_p \gamma_2 (x_4 + x_5)^2 + 2c_2 \sqrt{K_p} (x_4 + x_5)$$

- *Total stress analysis*

$$P_{pt} = (1/2)\gamma_2 (x_4 + x_5)^2 + 2S_u (x_4 + x_5)$$

4. Surcharge force per unit length of wall

$$P_s = K_a qH' = K_a q(H + x_3 \tan\alpha)$$

$$P_{sv} = K_a qH'\sin\alpha = K_a q(H + x_3 \tan\alpha)\sin\alpha$$

$$P_{sh} = K_a qH'\cos\alpha = K_a q(H + x_3 \tan\alpha)\cos\alpha$$

5. Surcharge force acting downward

- [In case surcharge q is present only**]**

$$P_q = q(x_3/\cos\alpha)$$

- [In case surcharge q = 0**]**

$$P_q = 0$$

1. Factor of safety against overturning failure

Table A.2 Summation of resisting moment acting on retaining wall

N⁰	**Area**	**Weight/unit length of wall**	**Moment arm from C**	**Moment about B**
	(m^2)	(kN/m)	(m)	(kN.m/m)
1	$A1 = x_3(H - x_4)$	$W_1 = \gamma_1 A_1$	$a_1 = x_1 + x_2 + (1/2)x_3$	$M1 = W_1 \text{ x } a_1$
2	$A2 = (1/2)x_3(x_3 \tan\alpha)$	$W_2 = \gamma_1 A_2$	$a_2 = x_1 + x_2 + (2/3)x_3$	$M2 = W_2 \text{ x } a_2$
3	$A3 = x_6(H - x_4)$	$W_3 = \gamma_c A_3$	$a_3 = x_1 + x_2 - (1/2)x_6$	$M3 = W_3 \text{ x } a_3$
4	$A4 = \frac{1}{2}(x_2 - x_6)(H - x_4)$	$W_4 = \gamma_c A_4$	$a_4 = x_1 + \frac{2}{3}(x_2 - x_6)$	$M4 = W_4 \text{ x } a_4$
5	$A5 = (x_1 + x_2 + x_3)x_4$	$W_5 = \gamma_c A_5$	$a_5 = \frac{1}{2}(x_1 + x_2 + x_3)$	$M5 = W_5 \text{ x } a_5$
6	$A6 = x_1 x_5$	$W_6 = \gamma_2 A_6$	$a_6 = (1/2)x_1$	$M6 = W_6 \text{ x } a_6$
7	Surcharge Loading	$P_q = q\frac{x_3}{\cos\alpha}$	$a_7 = x_1 + x_2 + x_3/2$	$P_q \text{ x } a_7$
8	Inclined surcharge	$P_{sv} = P_s \sin\alpha$	$a_8 = x_1 + x_2 + x_3$	$P_s \sin\alpha \times a_8$
9	Inclined earth pressure	$P_{av} = P_a \sin\alpha$	$a_9 = x_1 + x_2 + x_3$	$P_a \sin\alpha \times a_9$
10	Point Load P	P	$a_P = x_1 + x_2 - x_6/2$	$P \times a_P$
		$\sum V = \sum W_i + P_q + P_{sv} + P_{av} + P$		$\sum M_R = \sum_i^6 W_i a_i + (P_s + P_a)\sin\alpha \times a_8 + P_q \times a_7 + P \times a_P$

Note: 1) Section 6 is usually ignored by designers because of erosion condition (Das, 2007)

2) In working stress design (WSD), section 6 is included

3) P_P is neglected in calculating moment for overturning stability

Table A.3 Summation of driving moment acting on retaining wall

description	Force	Moment arm from C	Moment about C
	(kN/m)	(m)	(kN.m/m)
Surcharge	$P_{sh} = P_s \cos\alpha$	$a_s = \frac{1}{2}(H + x_3 \tan\alpha)$	$M_s = P_{sh} \times a_s$
Earth-pressure	$P_{ah} = P_a \cos\alpha$	$a_a = \frac{1}{3}(H + x_3 \tan\alpha)$	$M_a = P_{ah} \times a_a$
	$\sum H = P_s \cos\alpha + P_a \cos\alpha$		$\sum M_{ov} = P_s \cos\alpha \times a_s + P_a \cos\alpha \times a_a$

Factor of safety against overturning:

A. *Total overturning*

$$FS^1_{ov} = \frac{\sum M_R}{\sum M_{ov}} = \frac{\sum_{i=1}^{6} W_i a_i + P_q \times a_7 + (P_s + P_a)\sin\alpha \times a_8 + P \times a_P}{P_s \cos\alpha\left(H'/2\right) + P_a \cos\alpha\left(H'/3\right)}$$

B. *Partial overturning*

$$FS^2_{ov} = \frac{\sum M_R}{\sum M_{ov} - \sum M_{earth}} = \frac{\sum_{i=1}^{6} W_i a_i + P_q \times a_7 + P \times a_P}{\left[P_s \cos\alpha\left(H'/2\right) + P_a \cos\alpha\left(H'/3\right)\right] - \left[(P_s + P_a)\sin\alpha \times a_8\right]}$$

2. Factor of safety against sliding failure

- *Effective stress analysis*

Maximum resisting force derived from the soil per unit length of the wall along the bottom the base slab

$$R' = \left(\sum V\right)\tan\delta' + BC'_a$$

where V = summation of vertical forces (kN)

δ' = angle of friction between soil and the base slab (degree)

C'_a = cohesion between soil and the base slab (kN/m^2)

B = length of base slab (m)

Summation of horizontal resisting force F_R

$$\sum F_R = \sum V \times \tan\delta' + B \times C'_a + P_p$$

If assuming $P_P = P_{p(mobi)}$ = mobilized passive forces by designer's preferences.

$$P_{P(mobi)} = \frac{1}{n} P_P$$

where n = input variable factor for passive mobilized n = 1,2,3…

Interface between concrete base and soil base can be calculated as:

$$\delta' = k_1 \phi_2$$

$$C_a' = k_2 C_2$$

where k_1 = 1/2 to 2/3 (Das, 2007)

k_2 = 1/3 to 2/3 (Das, 2007)

k = input parameters

$$\sum F_R = \sum V \times \tan(k_1\phi_2) + B \times (k_2 C_2) + P_p$$

Factor of safety against sliding:

A. Consider P_P

$$FS^1_{(sliding)} = \frac{\sum F_R}{\sum F_h} = \frac{\sum V \times \tan\delta' + BC'_a + P_p}{\left[(1/2) K_a \gamma_1 H'^2 + K_a q H'\right]\cos\alpha}$$
$$= \frac{\sum V \times \tan(k_1\phi_2) + B \times (k_2 C_2) + P_p}{\left[(1/2) K_a \gamma_1 H'^2 + K_a q H'\right]\cos\alpha}$$

B. If $P_{P(mobilized)}$ is used:

$$FS^2_{sliding} = \frac{\sum F_R}{\sum F_h} = \frac{\sum V \times \tan(k_1\phi_2) + B(k_2 C_2) + (1/n) P_p}{\left[(1/2) K_a \gamma_1 H'^2 + K_a q H'\right]\cos\alpha}$$

C. If $P_P = 0$

$$FS^3_{sliding} = \frac{\sum F_R}{\sum F_h} = \frac{\sum V \times \tan(k_1\phi_2) + B(k_2 C_2)}{\left[(1/2) K_a \gamma_1 H'^2 + K_a q H'\right]\cos\alpha}$$

- *Total stress analysis*

In this case, $C_2 = S_u$ and $\phi_2 = 0$

Factor of safety against sliding in total stress analysis:

A. Consider P_P

$$FS^{1(t)}_{(sliding)} = \frac{\sum F_R}{\sum F_h} = \frac{B \times (k_2 S_u) + P_p}{\left[(1/2) K_a \gamma_1 H'^2 + K_a q H'\right] \cos\alpha}$$

B. If $P_{P(mobilized)}$ is used:

$$FS^{2(t)}_{sliding} = \frac{\sum F_R}{\sum F_h} = \frac{B(k_2 S_u) + (1/n) P_p}{\left[(1/2) K_a \gamma_1 H'^2 + K_a q H'\right] \cos\alpha}$$

C. If $P_P = 0$

$$FS^{3(t)}_{sliding} = \frac{\sum F_R}{\sum F_h} = \frac{B(k_2 S_u)}{\left[(1/2) K_a \gamma_1 H'^2 + K_a q H'\right] \cos\alpha}$$

3. Factor of safety against bearing failure

- *Eccentricity below foundation*

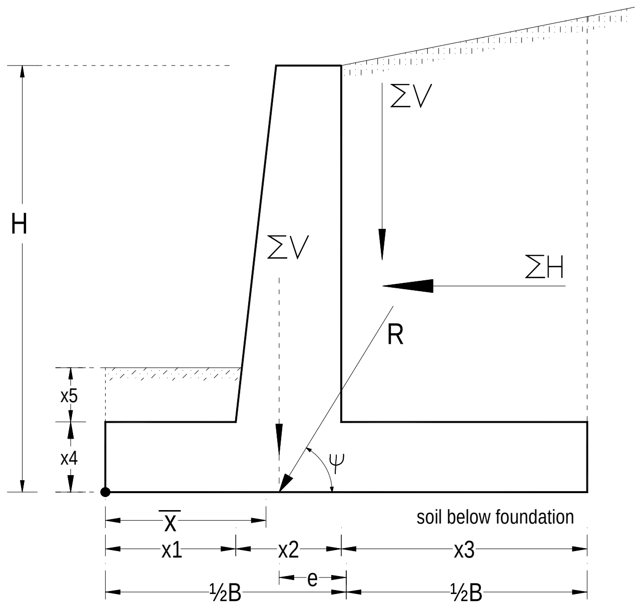


Figure A.2 Checking for eccentricity and bearing capacity failure

Taking moment about the toe of the base at B, the resultant vertical force at the base is located at $\bar{x}$, where $B/3 \le \bar{x} \le 2B/3$ (Budhu, 2008).

$$\bar{x} = \frac{\sum M_{net}}{\sum V} = \frac{\sum M_R - \sum M_{ov}}{\sum V}$$

$$\frac{1}{3}(x_1+x_2+x_3) \le \frac{\sum M_R - \sum M_{ov}}{\sum V} \le \frac{2}{3}(x_1+x_2+x_3)$$

e = eccentricity of the resistance force R (m)

$$e = \frac{B}{2} - \frac{\sum M_R - \sum M_{ov}}{\sum V}$$

$$q_{max} = q_{toe} = \frac{\sum V}{B}\left(1+\frac{6e}{B}\right)$$

$$q_{min} = q_{heel} = \frac{\sum V}{B}\left(1-\frac{6e}{B}\right)$$

if $e \le (B/6)$, then $q_{min} \ge 0$, pressure distribution is trapezoidal

if $e \ge (B/6)$, then $q_{min} < 0$, avoid this case !!!!

- *General ultimate bearing capacity equation*

  General bearing capacity equation:

  - *Effective stress analysis*

$$q_u = c_2 N_{cs} N_{cd} N_{ci} N_c + q N_{qs} N_{qd} N_{qi} N_q + (1/2)\gamma_2 B' N_{\gamma s} N_{\gamma d} N_{\gamma i} N_\gamma$$

  - *Total stress analysis*

$$q_u^t = S_u N_{cs} N_{cd} N_{ci} N_c + q N_{qs} N_{qd} N_{qi} N_q + (1/2)\gamma_2 B' N_{\gamma s} N_{\gamma d} N_{\gamma i} N_\gamma$$

- *Bearing capacity factors*

  - *Effective stress analysis*

$$N_q = e^{\pi \tan\phi_2} \tan^2\left(45^o + \frac{\phi_2}{2}\right)$$

$$N_c = (N_q - 1)\cot\phi_2$$

$$N_\gamma = 2(N_q + 1)\tan\phi_2$$

$$q = \gamma_2 D = \gamma_2 (x_4 + x_5)$$

$$B' = B - 2 \times e$$

- *Total stress analysis*

$N_c = 5.14$
$N_q = 1$
$N_\gamma = 0$

- *Shape factor (DeBeer)*

$N_{cs} = 1 + \left(N_q / N_c\right)\left(B'/L'\right)$
$N_{qs} = 1 + \left(B'/L'\right)\tan\phi_2$
$N_{\gamma s} = 1 - 0.4\left(B'/L'\right)$

Wall footing is infinite length (L=∞), thus $N_{cs} = N_{qs} = N_{\gamma s} = 1$

- *Depth factor (Hansen)*

- *Effective stress analysis*

In case $\left(D_f / B'\right) \le 1$

$N_{cd} = 1 + 0.4\left(D_f / B'\right)$
$N_{qd} = 1 + 2\tan\phi_2\left(1 - \sin\phi_2\right)^2\left(D_f / B'\right)$
$N_{\gamma d} = 1$

- *Total stress analysis ($\phi_2 = 0$)*

$N_{cd} = 1 + 0.4\left(D_f / B'\right)$
$N_{qd} = 1$
$N_{\gamma d} = 1$

- *Inclination factor (Meyerhof)*

- *Effective stress analysis*

ψ : inclined angle

$\psi^o = \tan^{-1}\left(\sum F_h / \sum V\right)$

$N_{ci} = N_{qi} = \left(1 - \psi^o / 90^o\right)^2$
$N_{\gamma i} = \left(1 - \psi^o / \phi_2^o\right)^2$

- *Total stress analysis ($\phi_2 = 0$)*

$N_{ci} = N_{qi} = \left(1 - \psi^o / 90^o\right)^2$
$N_{\gamma i} = 0$

Factor of safety of bearing capacity can be calculated as:

- *Effective stress analysis*

$FS_{be} = q_u / q_{max}$

- *Total stress analysis*

$FS^t_{be} = q_u / q_{max}$

STRUCTURAL DESIGN (ACI CODE)

Typical formulas used in *Working Stress Design (WSD)* for designing beams with tension reinforcement only

1. Compressive strength of concrete $f_c = 0.45f'_c$ (MPa)
2. Strength of steel in WSD $f_{sw} = 0.50f_y$ (MPa)
3. Design strength of steel $f_s = \min(f_{sw}, 170)$ (MPa)
4. Modulus elasticity of steel $E_s = 200\,000$ (MPa)
5. Modulus elasticity of concrete $E_s = 4700\sqrt{f'_c}$ (MPa)

   $n = E_s / E_c =$ the nearest integer

   $k = \dfrac{1}{1 + f_s / nf_c}$

   $j = 1 - k/3$
6. Required steel area $A_s = \dfrac{M}{f_s jd}$
7. Shear force of concrete $V_c = 0.09\sqrt{f'_c} \times bd$
8. Shear stress of concrete $\nu_c = 0.09\sqrt{f'_c} \times 1000$

Typical formulas used in Ultimate Strength Design for designing beams with tension reinforcement only

1. Equivalent rectangular stress block $a = \beta_1 c$
2. The factor β_1 : $\beta_1 = \begin{cases} 0.85 & f'_c \le 30 \text{ MPa} \\ 1.09 - 0.008f'_c & \text{if } 30 \text{ MPa} \le f'_c \le 55 \text{ MPa} \\ 0.65 & f'_c \ge 55 \text{ MPa} \end{cases}$

3. Minimum steel ratio $\rho_{min} = 1.4/f_y$

4. Steel ratio in balanced condition $\rho_b = \frac{A_{sb}}{bd} = \frac{0.85\beta_1 f'_c}{f_y}\left(\frac{600}{600+f_y}\right)$

5. Maximum steel ratio $\rho_{max} \le \frac{3}{4}\rho_b$

6. Reinforcing steel ratio design $\rho = \frac{0.85f'_c}{f_y}\left(1-\sqrt{1-\frac{4M_u}{1.7\phi f'_c bd^2}}\right)$

If letting $R_u = \frac{M_u}{\phi bd^2}$ $\rho = \frac{0.85f'_c}{f_y}\left(1-\sqrt{1-\frac{2R_u}{0.85f'_c}}\right)$

7. The usable flexural strength $\phi M_n = \phi A_s f_y d\left(1-\frac{\rho f_y}{1.7f'_c}\right)$

$$\phi M_n = \phi\rho f_y bd^2\left(1-\frac{\rho f_y}{1.7f'_c}\right)$$

8. Required steel area $A_s = \rho bd$

9. Shear force of concrete $V_c = \phi(1/6)\sqrt{f'_c} \times bd$

10. Shear stress of concrete $\nu_c = \phi(1/6)\sqrt{f'_c}$

Code	**DL**	**LL**	**Shear ϕ**	**Moment ϕ**
ACI 318-99	1.4	1.7	0.85	0.9
ACI 318-05	1.2	1.6	0.75	0.9

4. Factor of safety against toe shear and moment failure mode

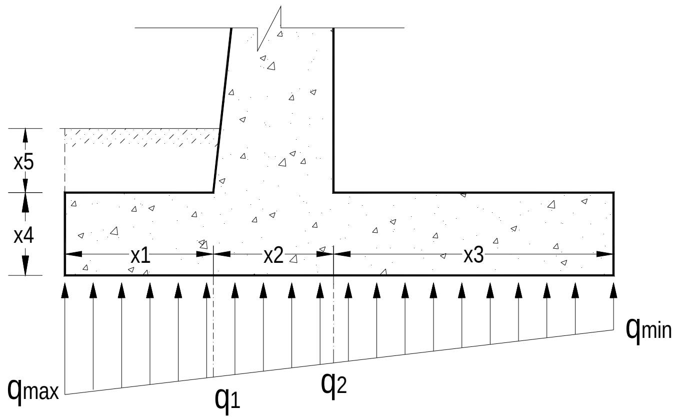


Figure A.3 Pressure distribution under footing

$$q_1 = (q_{max} - q_{min})\frac{x_2 + x_3}{x_1 + x_2 + x_3} + q_{min}$$

$$q_2 = (q_{max} - q_{min})\frac{x_3}{x_1 + x_2 + x_3} + q_{min}$$

$$q_0 = (q_{max} - q_1)\left(\frac{d}{x_1}\right) + q_1 \qquad \text{(USD)}$$

$$q_3 = (q_{max} - q_1)\left(\frac{d}{2x_1}\right) + q_1 \qquad \text{(WSD)}$$

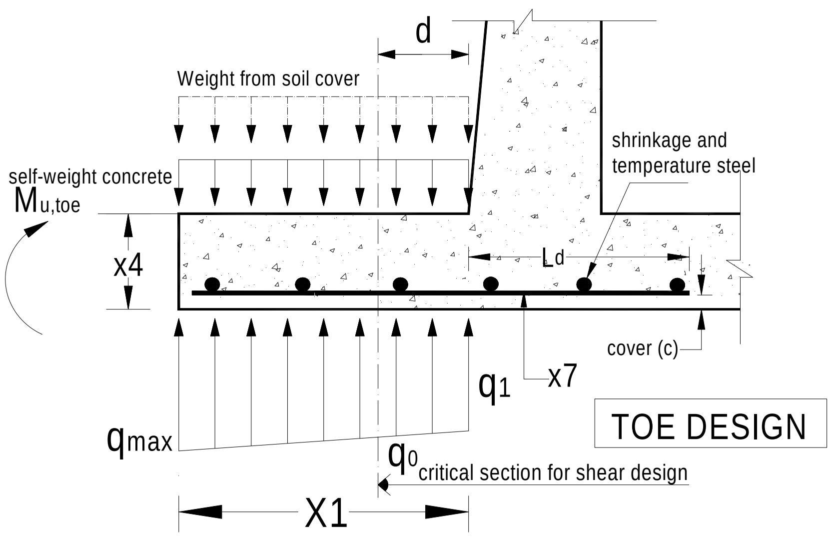


Figure A.4 Pressure distribution for toe design

Force accounting in toe design:

1. Upward soil pressure for shear (WSD) $W_{sup} = q_3 + (1/2)(q_{max} - q_3)$
2. Upward soil pressure for shear (USD) $W_{uup} = q_0 + (1/2)(q_{max} - q_0)$
3. Upward soil pressure for moment $W_{up} = q_1 + (1/2)(q_{max} - q_1)$
4. Self-weight of concrete $W_{sc} = x_4 \gamma_c$
5. Weight of soil covers $W_{ssc} = x_5 \gamma_2$

- *WSD (Working Stress Design)*

Effective depth for design $d = x_4 - c - \Phi/2$

Critical section for shear in working stress design $d/2 = (1/2)(x_4 - c - \Phi/2)$

- *Toe shear force*

$$W_{s,toe} = W_{sup} - W_{sc} - W_{ssc}$$

$$V_{s,toe} = \left[q_3 + (1/2)(q_{max} - q_3)\right](x_1 - d/2) - x_4\gamma_c(x_1 - d/2) - x_5\gamma_2(x_1 - d/2)$$

Service shear stress $\nu_{s,toe} = V_{s,toe} / (b \times d)$ (kN/m^2)

Resisting shear stress $\nu_{sr,toe} = (1/6)\sqrt{f'_c} \times 1000$ (kN/m^2)

- *Toe bending moment*

$$M_{s,toe} = \left[q_1 x_1\left(\frac{x_1}{2}\right) + \frac{1}{2}(q_{max} - q_1)x_1\left(\frac{2}{3}x_1\right)\right] - \left[\gamma_c x_4 x_1\left(\frac{x_1}{2}\right) + \gamma_2 x_5 x_1\left(\frac{x_1}{2}\right)\right]$$

Compute reinforcing area X_7^s

$$x_7^s = M_{s,toe} / (f_s jd)$$

Resisting moment of toe in WSD

$$M_{sr,toe} = x_7^s f_s jd$$

- Factor of safety against toe shear failure mode

$$FS_{ss,toe} = \nu_{s,toe} / \nu_{sr,toe}$$

- Factor of safety against bending moment failure mode

$$FS_{sm,toe} = M_{sr,toe} / M_{s,toe}$$

- *USD (Ultimate Strength Design)*

+ ACI-99 Load factor DL=1.4, LL=1.7, DLw=0.9
+ ACI-02 Load factor DL=1.2, LL=1.6, DLw=0.9

Combination load factor for shear design $U = LL \times W_{upward} - DLw \times W_c$

- The critical section for shear design is located at a distance d from the front face of the stem which $d = x_4 - c - \Phi/2$
- Limbrunner and Aghayere (2007) do not consider critical section for shear. !!!!!!

- *Toe shear force (when consider critical section d)*

$$V_{u,toe} = LL \times \left[q_0 + (1/2)(q_{max} - q_0)\right](x_1 - d) - DLw \times \gamma_c x_4 (x_1 - d)$$

- *Toe shear force (when do not consider critical section)*

$$V_{u,toe} = LL \times \left[q_1 + (1/2)(q_{max} - q_1)\right](x_1) - DLw \times \gamma_c x_4 (x_1)$$

Ultimate shear stress $\tau_{u,toe} = V_{u,toe} / (b \times d)$ (kN/m^2)

Resisting shear stress $\tau_{ur,toe} = \phi(1/6)\sqrt{f'_c} \times 1000$ (kN/m^2)

+ ACI-99 $\phi = 0.85$
+ ACI-02 $\phi = 0.75$

- *Toe bending moment*

$$M_{u,toe} = LL \times \left[q_1 x_1 \left(\frac{x_1}{2}\right) + \frac{1}{2}(q_{max} - q_1) x_1 \left(\frac{2}{3} x_1\right)\right] - DLw \times \gamma_c x_4 x_1 \left(\frac{x_1}{2}\right)$$

Compute reinforcing area X_7

where $\rho = \frac{0.85 f'_c}{f_y}\left[1 - \sqrt{1 - \frac{4M_u}{1.7\phi f'_c bd^2}}\right]$

$\phi = 0.90$

b = 1 m (strip 1m for analysis)

If let $R_u = \frac{M_u}{\phi bd}$ $\rho = \frac{0.85 f'_c}{f_y}\left[1 - \sqrt{1 - \frac{2R_u}{0.85 f'_c}}\right]$

Required reinforcing steel ratio $\rho_{req} \geq \rho_{min}$
$\rho_{req} \leq \rho_{max}$

Thus, $X_7 = \rho \times b \times d$ (m^2)

Resisting moment of toe slab

$$M_{ur,toe} = \phi \rho f_y b d^2 \left(1 - \frac{\rho f_y}{1.7 f_c^{'}}\right)$$

$$\rho = \frac{A_s}{bd} = \frac{x_7}{bd}$$ = reinforcement ratio in toe slab

$$M_{ur,toe} = \phi x_7 f_y d \left(1 - \frac{x_7}{1.7bd}\frac{f_y}{f_c^{'}}\right)$$

- Factor of safety against toe shear failure mode

$$FS_{us,toe} = \tau_{ur,toe} / \tau_{u,toe}$$

- Factor of safety against bending moment failure mode

$$FS_{um,toe} = M_{ur,toe} / M_{u,toe}$$

5. Factor of safety against heel shear and moment failure mode

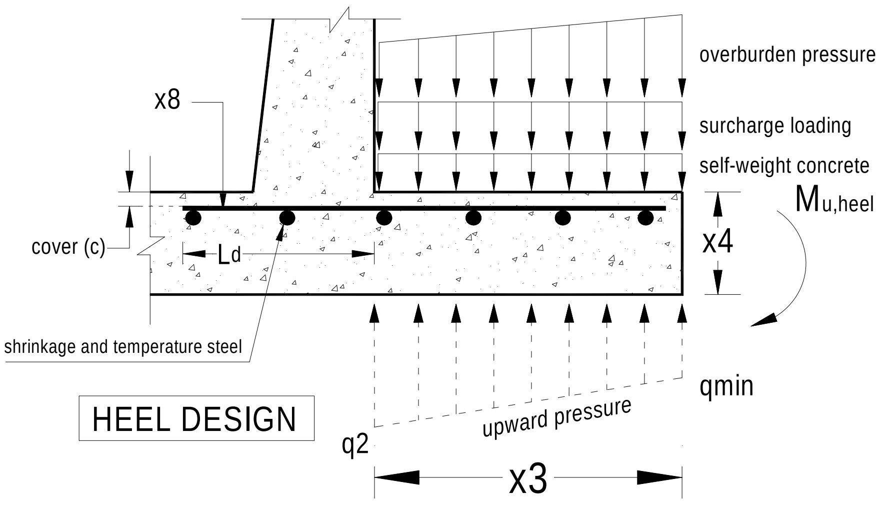


Figure A.5 Pressure distribution for heel design

Force accounting in heel design

1. Overburden pressures $W_{over} = \gamma_1 \left[(H - x_4) + (1/2) x_3 \tan\alpha \right]$

2. Surcharge loading $W_{sur} = q/\cos\alpha$

3. Self-weight of concrete $W_c = x_4\gamma_c$

4. Upward soil pressure $W_{up} = q_{min} + (1/2)(q_2 - q_{min})$

➢ *WSD (Working Stress Design)*

Cernica (1966) reported that moment and shear for the case of zero foundation pressure shall multiply the by 2/3 for conservative design.

- *Heel shear force*

$$V_{s,heel} = (2/3)(W_{ov} + W_{sur} + W_c)$$

$$V_{s,heel} = (2/3)\left[\gamma_1\left[(H - x_4) + (1/2)x_3\tan\alpha\right]x_3 + (q/\cos\alpha)x_3 + (\gamma_c x_4)x_3\right]$$

Service shear stress $\nu_{s,heel} = V_{s,heel}/(b \times d)$ (kN/m^2)

Resisting shear stress $\nu_{sr,heel} = 0.09\sqrt{f'_c} \times 1000$ (kN/m^2)

- *Heel bending moment*

$$M_{s,heel} = (2/3)(M_{ov} + M_{sur} + M_c)$$

$$M_{s,heel} = \left(\frac{2}{3}\right)\gamma_1\left[(H - x_4)x_3\frac{1}{2}x_3 + \left(\frac{1}{2}\right)x_3\tan\alpha \times x_3\frac{2}{3}x_3\right] + \left(\frac{2}{3}\right)\left[\left(\frac{q}{\cos\alpha}\right)x_3\frac{1}{2}x_3 + (\gamma_c x_4)x_3\frac{1}{2}x_3\right]$$

Compute reinforcing area X_8^s

$$x_8^s = M_{s,heel}/(f_s jd)$$

Resisting moment of heel in WSD

$$M_{sr,heel} = x_8^s f_s jd$$

- Factor of safety against toe heel failure mode

$$FS_{ss,heel} = \nu_{sr,heel}/\nu_{s,heel}$$

- Factor of safety against moment

$$FS_{sm,heel} = M_{sr,heel}/M_{s,heel}$$

- *USD (Ultimate Strength Design)*

+ ACI-99 Load factor DL=1.4, LL=1.7
+ ACI-02 Load factor DL=1.2, LL=1.6
+ Combination load factor $U=DL\times W_c + DL\times W_{over} + LL\times W_{sur}$

d = effective height given by $d = x_4 - c - \Phi/2$

- *Heel shear force*

$$V_{u,heel} = DL\times\gamma_1\left[(H-x_4)+(1/2)x_3\tan\alpha\right]x_3 + LL\times(q/\cos\alpha)x_3 + DL\times(\gamma_c x_4)x_3$$

Ultimate shear stress $\tau_{u,heel} = V_{u,heel}/(b\times d)$ (kN/m^2)

Resisting shear stress $\tau_{ur,heel} = \phi(1/6)\sqrt{f'_c}\times 1000$ (kN/m^2)

+ ACI-99 $\phi = 0.85$
+ ACI-02 $\phi = 0.75$

- *Heel bending moment (conservative design)*

Conservative design in ultimate strength neglects effect of upward pressure below heel (ACI Code). !!!!!

$$M_{u,heel} = DL\times\gamma_1\left[(H-x_4)x_3\frac{1}{2}x_3 + (1/2)x_3\tan\alpha\times x_3\frac{2}{3}x_3\right] + LL\times\left(\frac{q}{\cos\alpha}\right)x_3\frac{1}{2}x_3 + DL\times(\gamma_c x_4)x_3\frac{1}{2}x_3$$

Compute reinforcing area X_8

Method to calculate reinforcement ratio in heel slab is reported the same as in design of toe slab. Thus, requirement top steel area in heel slab is expressed as:

$X_8 = \rho\times b\times d$ (m^2)

Resisting moment of heel slab

$$M_{ur,heel} = \phi\rho f_y bd^2\left(1-\frac{\rho f_y}{1.7f'_c}\right)$$

$\rho = \frac{A_s}{bd} = \frac{x_8}{bd}$ Reinforcement ratio in heel slab

$$M_{ur,heel} = \phi x_8 f_y d\left(1-\frac{x_8}{1.7bd}\frac{f_y}{f'_c}\right)\times 10^{-6}$$

$\phi = 0.90$

- Factor of safety against toe heel failure mode

$FS_{us,heel} = \tau_{ur,heel} / \tau_{u,heel}$

- Factor of safety against moment

$FS_{um,heel} = M_{ur,heel} / M_{u,heel}$

6. Factor of safety against stem shear failure mode

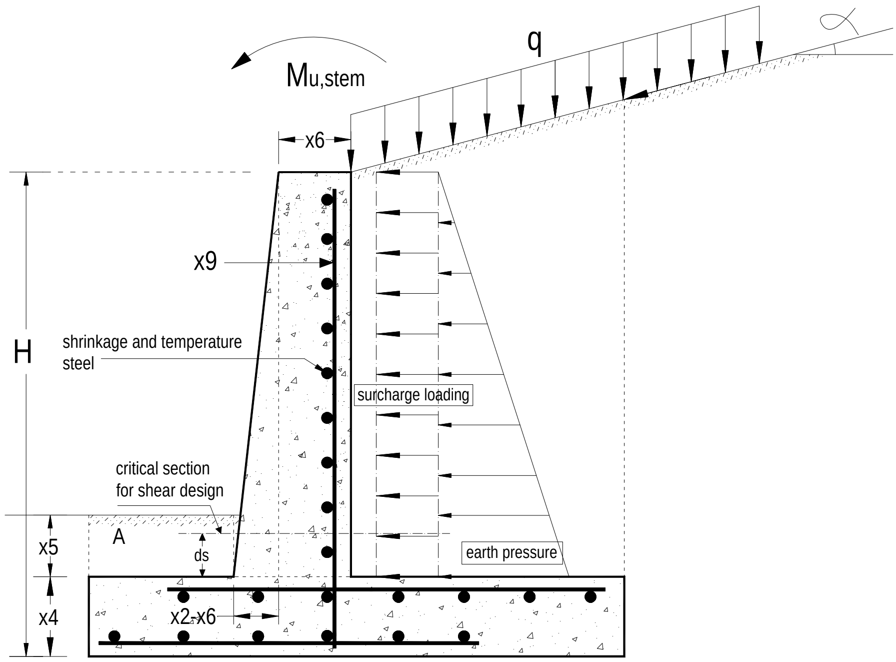


Figure A.6 Force distribution on stem

1. Lateral force applied to the stem

$P_a = (1/2) K_a \gamma_1 (H - x_4)^2$ Inclination alpha α

$P_a = (1/2) K_a \gamma_1 (H - x_4)^2 \cos\alpha$ Horizontal direction

\+ Arm $A_a = (1/3)(H - x_4)$

2. Surcharge force applied to the stem

$P_s = K_a q (H - x_4)$ Inclination alpha α

$P_s = K_a q (H - x_4) \cos\alpha$ Horizontal direction

+ Arm $A_s = (1/2)(H - x_4)$

- *WSD (Working stress design)*
  - *Stem shear force*

$$V_{s,stem} = (1/2)K_a\gamma_1(H - x_4)^2\cos\alpha + K_a q(H - x_4)\cos\alpha$$

Service shear stress $\nu_{s,stem} = V_{s,stem}/(b \times d)$ (kN/m^2)

Resisting shear stress $\nu_{sr,heel} = 0.09\sqrt{f'_c} \times 1000$ (kN/m^2)

- *Stem bending moment*

$$M_{s,stem} = (1/2)K_a\gamma_1(H - x_4)^2\cos\alpha \times \frac{1}{3}(H - x_4) + K_a q(H - x_4)\cos\alpha \times \frac{1}{2}(H - x_4)$$

Compute reinforcing area X_9^s

$$x_9^s = M_{s,stem}/(f_s jd)$$

Resisting moment of stem in WSD

$$M_{sr,stem} = x_9^s f_s jd$$

- Factor of safety against stem failure mode

$$FS_{ss,stem} = \nu_{sr,stem}/\nu_{s,stem}$$

- Factor of safety against moment

$$FS_{sm,stem} = M_{sr,stem}/M_{s,stem}$$

- *USD (Ultimate strength design)*

+ ACI-99 Load factor DL=1.4, LL=1.7
+ ACI-02 Load factor DL=1.2, LL=1.6
+ Combination load factor U=LL×P_{active} + LL×$P_{surcharge}$

The critical section for shear is located at a distance d_s out of the stem height

$$d_s = x_2 - c - \Phi/2$$

- *Stem shear force*

$$V_{u,stem} = LL \times \left[(1/2)K_a\gamma_1(H - x_4 - d_s)^2\cos\alpha + K_a q(H - x_4 - d_s)\cos\alpha\right]$$

Ultimate shear stress $\tau_{u,stem} = V_{u,stem} / (b \times d_s)$ (kN/m^2)

Resisting shear stress $\tau_{ur,stem} = \phi 0.17 \sqrt{f'_c} \times 1000$ (kN/m^2)

+ ACI-99 $\phi = 0.85$

+ ACI-02 $\phi = 0.75$

- *Stem bending moment*

$$M_{u,stem} = LL \times (P_a \times A_a + P_s \times A_s)$$

$$M_{u,stem} = LL \times \left[\left(\frac{1}{2}\right) K_a \gamma_1 (H - x_4)^2 \cos\alpha \times \frac{1}{3}(H - x_4) + K_a q (H - x_4) \cos\alpha \times \frac{1}{2}(H - x_4) \right]$$

$$M_{u,stem} = LL \times \left[(1/6) K_a \gamma_1 (H - x_4)^3 \cos\alpha + (1/2) K_a q (H - x_4)^2 \cos\alpha \right]$$

Compute reinforcing area X_9

Method to calculate reinforcement ratio in heel slab is reported the same as in design of toe slab. Thus, requirement top steel area in heel slab is expressed as:

$x_9 = \rho \times b \times d$ (m^2)

Resisting moment of stem

$$M_{ur,stem} = \phi \rho f_y b d^2 \left(1 - \frac{\rho f_y}{1.7 f_c'} \right)$$

$\rho = \frac{A_s}{bd} = \frac{x_9}{bd}$ reinforcement ratio in stem

$$M_{ur,stem} = \phi x_9 f_y d \left(1 - \frac{x_9}{1.7bd} \frac{f_y}{f_c'} \right) \times 10^{-6}$$

$\phi = 0.90$

- Factor of safety against stem failure mode

$$FS_{ss,stem} = \tau_{ur,stem} / \tau_{u,stem}$$

- Factor of safety against moment

$$FS_{um,stem} = M_{ur,stem} / M_{u,stem}$$

Stem face steel

$\rho_h = 0.0025$ = horizontal reinforcement ratio

$\rho_v = 0.0015$ = vertical reinforcement ratio

A_{st} = shrinkage and temperature $A_{st} = \rho_h \times b \times h_{av}$

h_{av} = average depth

A_s = Horizontal front face $A_{sh,front} = (2/3) \times A_{st}$

A_s = Horizontal back face $A_{sh,back} = (1/3) \times A_{st}$

A_s = Vertical front face $A_{sv,front} = \rho_v \times b \times h_{av}$

APPENDIX B

Typical Formulas in Analyses of Reinforced Concrete Column Section

Design Assumptions in the ACI Code

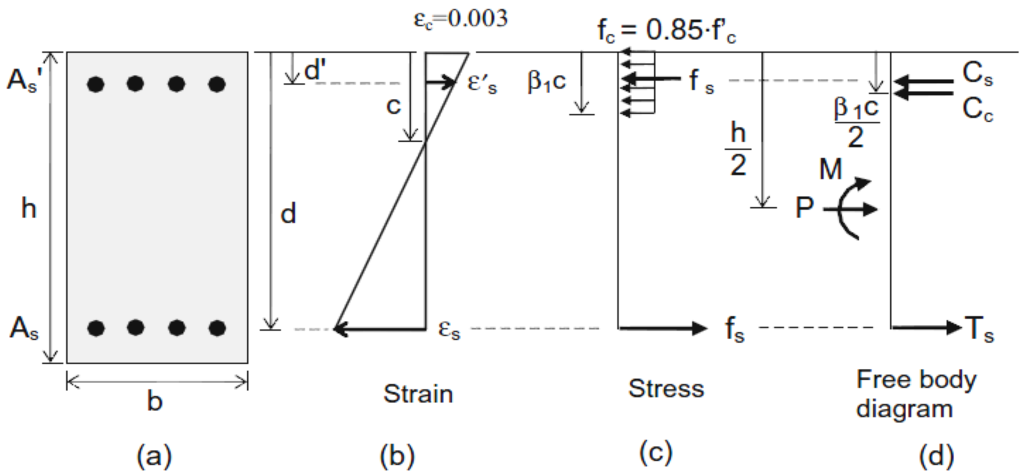


Figure B.1 Strain and stress diagrams for ultimate strength design (Lee, 2009)

Summary of acting force based on Figure B.1.

1. Compressive force carried by concrete

$$C_c = (0.85f'c)(\beta_1 c)(b)$$

2. The force carried by the top steel

$$C_s = (A'_s)(f's - 0.85f'c)$$

Based on strain compatibility

$$f's = (E_s)(\varepsilon'_s) = (E_s)\left[(0.003)\left(1 - \frac{d'}{c}\right)\right] \qquad \text{if } 0 \le \varepsilon'_s \le fy/E_s$$

$$f's = (E_s)(\varepsilon'_s) = f_y \qquad \text{if } \varepsilon'_s \ge fy/E_s$$

3. The force carried by the bottom steel

$$T_s = (f_s)(A_s)$$

Based on strain compatibility

$$fs = (E_s)(\varepsilon_s) = (E_s)\left[(0.003)\left(\frac{d}{c} - 1\right)\right] \qquad \text{if } 0 \le \varepsilon_s \le fy/E_s$$

$$fs = (E_s)(\varepsilon_s) = f_y \qquad \text{if } \varepsilon_s \ge fy/E_s$$

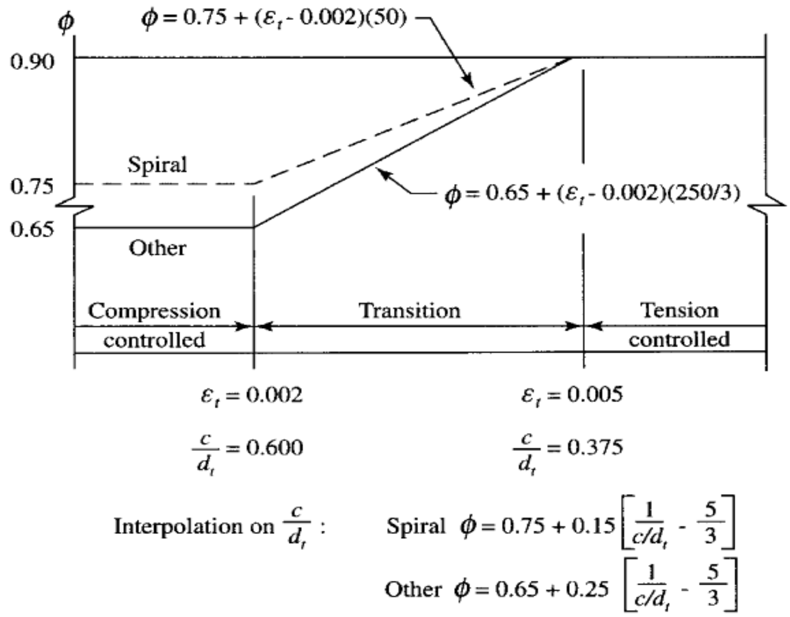


Figure B.2 Variation of ϕ with net tensile strain ε_t and c/d for Grade 60 reinforcement steel (Hassoun, 2005)

Since variation of term c/d establishes the compression-controlled, transition zone, and tension-controlled based on figure B.2, four domains of column analysis are represented explicitly in term of c/d (Lee, 2009).

Four domains are considered:

i. *Domain 1 : compression-controlled region*

$$0.003E_s d/\left(f_y + 0.003E_s\right) \le c \le d$$

In this domain $\phi = 0.65$ and top reinforcement is yielding

ii. *Domain 2 : Transition zone*

$$0.003E_s d'/\left(0.003E_s - f_y\right) \le c \le 0.003E_s d/\left(f_y + 0.003E_s\right)$$

In this domain $\phi = 0.233 + 0.25\,d/c$ and both the top and bottom reinforcement is yielding

iii. *Domain 3 : Transition zone*

$$3d/8 \le c \le 0.003E_s d'/\left(0.003E_s - f_y\right)$$

In this domain $\phi = 0.233 + 0.25\,d/c$ and bottom reinforcement is yielding

iv. *Domain 4 : Tension-controlled region*

$d'/\beta_1 \le c \le 3d/8$

In this domain $\phi = 0.90$ and bottom reinforcement is yielding

Note: The domain with $c \le d'/\beta_1$ and the domain with $c > d$ are excluded in the study.

Compression-controlled region

Force equilibrium based on free body diagram in Figure B.1

$$P_n = C_c + C_s - T_s$$

$$P_n = 0.85f'_c\,ab + A'_s\left(f'_s - 0.85f'_c\right) - A_s f_s$$

$$P_n = 0.85f'_c\left(\beta_1 c\right)b + A'_s\left(f_y - 0.85f'_c\right) - A_s\left(E_s\right)\left[\left(0.003\right)\left(\frac{d}{c} - 1\right)\right]$$

Taking moment on neutral axis based on free body diagram in Figure B.1.

$$P_n \times e = C_c\left(\frac{h}{2} - \frac{a}{2}\right) + C_s\left(\frac{h}{2} - d'\right) + T_s\left(d - \frac{h}{2}\right)$$

$$M_n = P_n \times e = 0.85f'_c\,ab\left(\frac{h}{2} - \frac{a}{2}\right) + A'_s\left(f'_s - 0.85f'_c\right)\left(\frac{h}{2} - d'\right) + A_s f_s\left(d - \frac{h}{2}\right)$$

$$M_n = 0.85f'_c\left(\beta_1 c\right)b\left(\frac{h}{2} - \frac{\left(\beta_1 c\right)}{2}\right) + A'_s\left(f_y - 0.85f'_c\right)\left(\frac{h}{2} - d'\right)$$
$$+ A_s\left(E_s\right)\left[\left(0.003\right)\left(\frac{d}{c} - 1\right)\right]\left(d - \frac{h}{2}\right)$$

APPENDIX C
Typical Formulas in Analyses of External Design Load for the Bridge

Determine Maximum Bending Moment and Maximum Shear Force

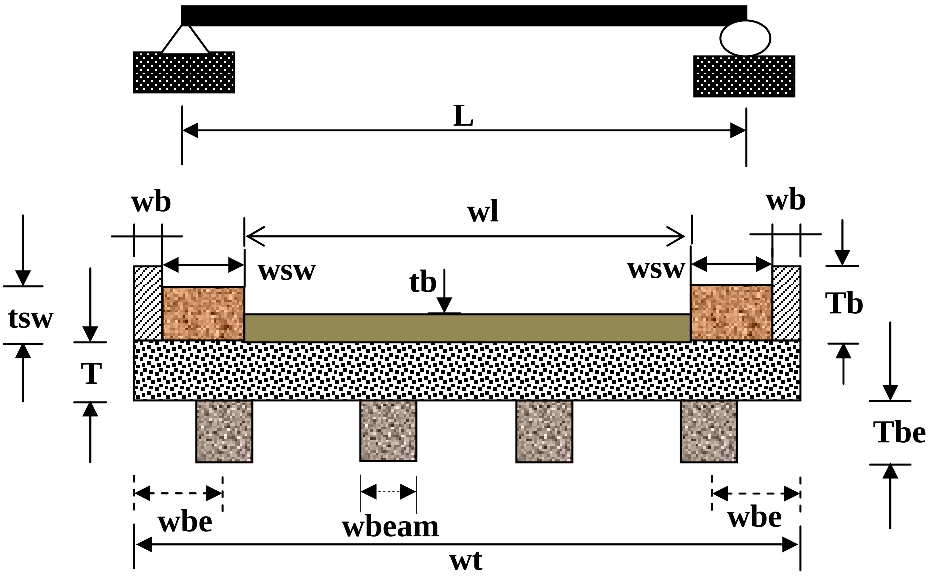


Figure C.1 Assuming cross section of bridge slab on simply supported span L

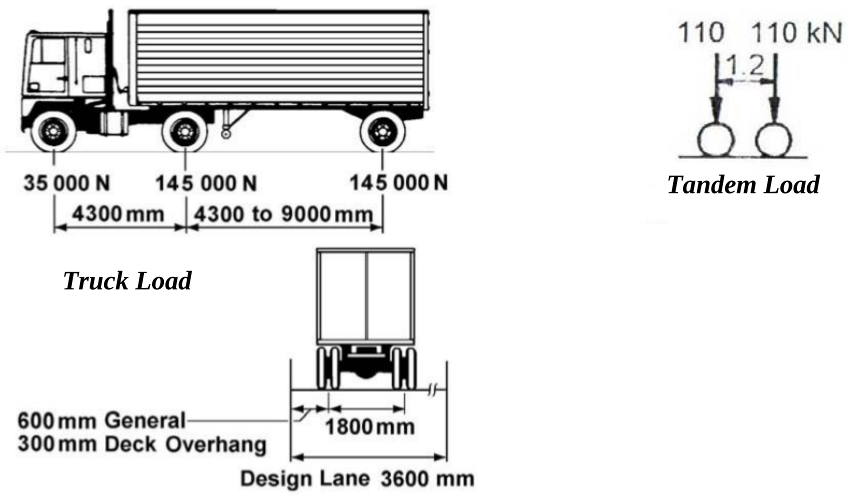
Table C.1 Input parameters

Name	Symbol
Length of bridge span	L
Width of barrier	wb
Width of bitumen laying	wl
Total width	wt
Thickness of bitumen	tb
Thickness of bridge slab	T
Thickness of barrier	Tb
Width of sidewalk	wsw
Thickness of sidewalk	tsw
Pedestrian Load	PL
Thickness of stem beam	Tbe
Width of stem beam	wbeam
Width from stem beam to end slab side	wbe
Number of stem beam	nbeam

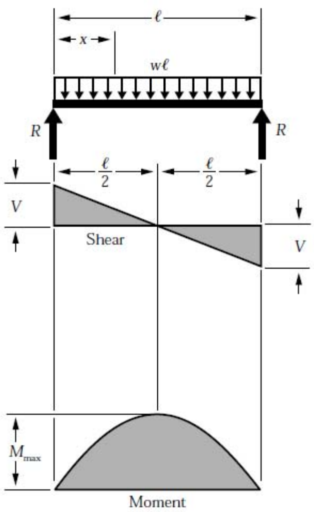
Table C.1 Input parameters

Unit weight of concrete	γ_c
Unit weight of bitumen	γ_{bi}
Design Truck (AASHTO)	HS20*
Width design lane	wlane
Design lane load	Lane

(*) Design truck HS20 characteristics



For simply supported beam as shown in Figure C.1, maximum shear and bending moment can be calculated as:



$$R = V \quad \dots\dots\dots\dots \quad = \frac{w\ell}{2}$$

$$V_x \quad \dots\dots\dots\dots \quad = w\left(\frac{\ell}{2} - x\right)$$

$$M_{max} \text{ (at center)} \quad \dots\dots\dots \quad = \frac{w\ell^2}{8}$$

$$M_x \quad \dots\dots\dots\dots \quad = \frac{wx}{2}(\ell - x)$$

$$\Delta_{max} \text{ (at center)} \quad \dots\dots\dots \quad = \frac{5w\ell^4}{384\,EI}$$

$$\Delta_x \quad \dots\dots\dots\dots \quad = \frac{wx}{24\,EI}(\ell^3 - 2\ell x^2 + x^3)$$

Figure C.2 Shear and bending moment due to uniformly distribution load

1) Shear and moment due to future wearing surface (DW)

Laying width $wl = wt - 2\times wb - 2\times wsw$

Total weight $DW = \gamma_b \times tb \times wl = \gamma_b \times tb \times (wt - 2\times wb - 2\times wsw)$

$$M_{DW,max} = DW \times L^2/8$$
$$V_{DW,max} = DW \times L/2$$

2) Shear and moment due to structure component and attachment (DC)

- Weight of barrier

$$Wb = wb \times Tb \times \gamma_c \times 2$$

- Weight of sidewalk concrete

$$Wcsw = (wsw)\times(tsw)\times\gamma_c \times 2$$

- Weight of concrete slab

$$Wc = wt \times T \times \gamma_c$$

- Weight of stem beam (if presents)

$$Wbe = wbeam \times Tbe \times \gamma_c \times nbeam$$

Total weight of component and attachment

$$DC = Wb + Wc + Wcsw + Wbe = wb \times Tb \times \gamma_c \times 2 + (wsw)\times(tsw)\times\gamma_c \times 2 + wt \times T \times \gamma_c + wbeam \times Tbe \times \gamma_c \times nbeam$$

$$M_{DC,max} = DC \times L^2/8$$
$$V_{DC,max} = DC \times L/2$$

3) Shear and moment due to Pedestrian load

If pedestrian load is present on sidewalk,

PL=3.60 kN/m^2 (AASHTO s3.6.1.6)

$$DPL = PL \times wsw$$

$$M_{DPL,max} = DPL \times L^2/8$$
$$V_{DPL,max} = DPL \times L/2$$

4) Shear and moment due to lane load

Clear road way width $cw = wt - 2\times wb - 2\times wsw$

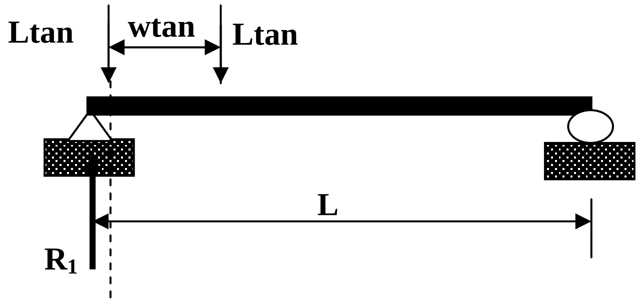
Number of design lane $dlane = cw/wlane$ (nearest int eger)

Llane = 9.30 kN/m (AASTHO s3.6.1.2.4)

$$M_{1Lane,max} = Lane \times L^2/8$$
$$V_{1Lane,max} = Lane \times L/2$$

5) Shear and moment due to tandem load

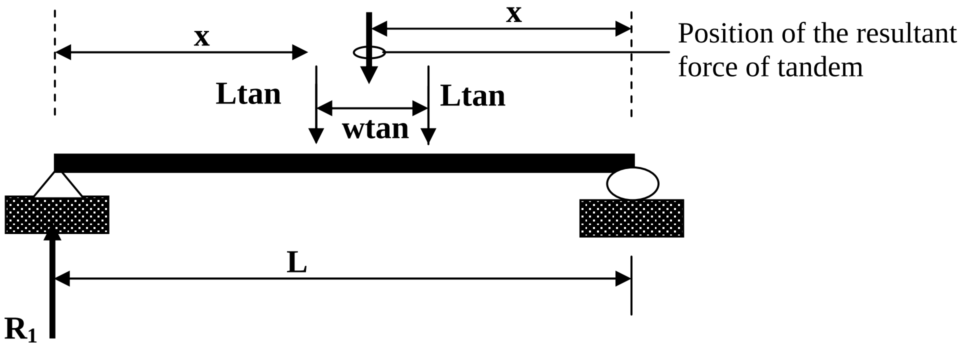
- Maximum shear force



$$R_1 L = (L\tan) \times L + (L\tan) \times (L - w\tan)$$

$$V_{1Tandem,max} = \frac{(L\tan) \times L + (L\tan) \times (L - w\tan)}{L}$$

- Maximum bending moment for span



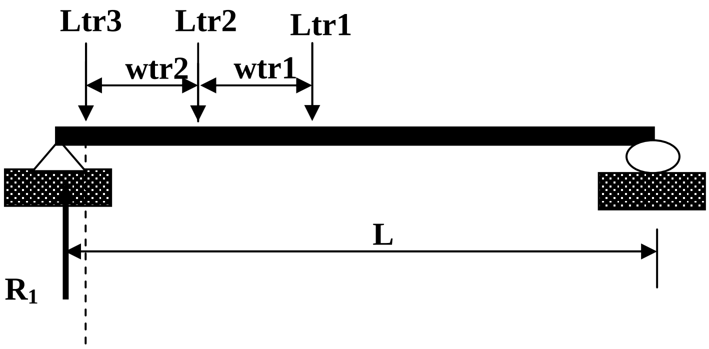
$$x + (w\tan/2) + x = L$$
$$x = (1/2)\left[L - (w\tan/2)\right]$$

$$R_1 L = (L\tan) \times (L - x) + (L\tan) \times (L - x - w\tan)$$
$$R_1 = \frac{(L\tan) \times (L - x) + (L\tan) \times (L - x - w\tan)}{L}$$

$$M_{1Tandem,max} = R_1 x = x\,\frac{(L\tan) \times (L - x) + (L\tan) \times (L - x - w\tan)}{L}$$

6) Shear and moment due to truck load

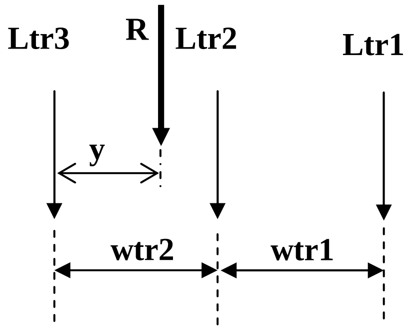
- Maximum shear force



$$R_1 L = (Ltr3)\times L + (Ltr2)\times(L - wtr2) + (Ltr1)\times(L - wtr1 - wtr2)$$

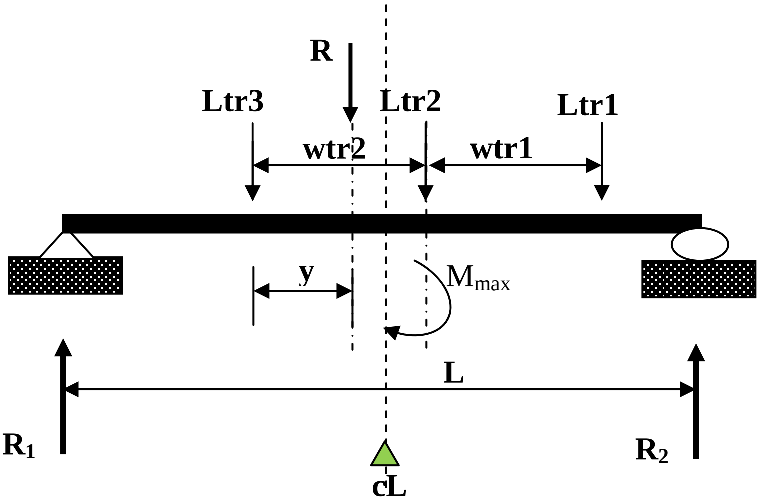
$$V_{1Truck,max} = \frac{(Ltr3)\times L + (Ltr2)\times(L - wtr2) + (Ltr1)\times(L - wtr1 - wtr2)}{L}$$

- Maximum bending moment for span



$$R(y) = (Ltr2)\times wtr2 + (Ltr1)\times(wtr1 + wtr2)$$

$$y = \frac{(Ltr2)\times wtr2 + (Ltr1)\times(wtr1 + wtr2)}{Ltr1 + Ltr2 + Ltr3}$$



$$R_1 + R_2 = Ltr1 + Ltr2 + Ltr3$$

$$R_2 L = (Ltr3)\left(\frac{L}{2} - y - \frac{1}{2}(wtr2 - y)\right) + (Ltr2)\left(\frac{L}{2} + \frac{1}{2}(wtr2 - y)\right)$$
$$+ (Ltr1) \times \left(\frac{L}{2} + \frac{1}{2}(wtr2 - y) + wtr1\right)$$

$$R_2 = (1/L)\left[\begin{array}{l}(Ltr3)\left(\frac{L}{2} - y - \frac{1}{2}(wtr2 - y)\right) + (Ltr2)\left(\frac{L}{2} + \frac{1}{2}(wtr2 - y)\right) \\ + (Ltr1) \times \left(\frac{L}{2} + \frac{1}{2}(wtr2 - y) + wtr1\right)\end{array}\right]$$

$$M_{1Truck,max} = (Ltr1 + Ltr2 + Ltr3 - R_2)\left(\frac{L}{2} + \frac{1}{2}(wtr2 - y)\right) - (Ltr3)(wtr2)$$

Vehicle design load (ASSHTO) include IM

Dynamic Load Allowance (IM = 33% : all other limit state), thus

$$M_L = (1 + IM/100)\max(M_{Truck}, M_{Tandem}) + M_{Lane}$$
$$V_L = (1 + IM/100)\max(V_{Truck}, V_{Tandem}) + V_{Lane}$$

M_L and V_L : are include Dynamic Load Allowance !!!!!

Load combinations in design strength

- Strength I

$$M_I = \left[\eta_1 \begin{Bmatrix}1.25 \\ 0.90\end{Bmatrix} DC + \eta_2 \begin{Bmatrix}1.50 \\ 0.65\end{Bmatrix} DW + \eta_3 \begin{Bmatrix}1.75 \\ 0.00\end{Bmatrix} (LL + IM)\right]$$

- Strength II

$$M_{II} = \left[\eta_1 \begin{Bmatrix}1.25 \\ 0.90\end{Bmatrix} DC + \eta_2 \begin{Bmatrix}1.50 \\ 0.65\end{Bmatrix} DW + \eta_3 \begin{Bmatrix}1.35 \\ 0.00\end{Bmatrix} (LL + IM)\right]$$

In load combinations taken into account of *multiple present of Live Load factor m, the combination becomes:*

i. Case 1 Lane Load and pedestrian Load: *m1=1.20* for vehicular live load
m2=1.0 for pedestrian load

ii. Case 2 Lane Load : *m1=1.20, m2=0.0*

iii. Case 2 Lane Load and pedestrian Load : *m1=m2=0.85*

iv. Case greater Lane Load and pedestrian Load: *m1=m2=1.0*

v. Case 3 Lane Load : *m1=0.85, m2=0.0*

vi. Case greater Lane Load : *m1=0.65, m2=0.0*

➢ *Bending Moment*

Case 1 Lane Load: [dlane = 1]

$$M_{STR[1]} = 1.25M_{DC} + 1.5M_{DW} + 1.75\left[\left(M_L \times m1\right) \times 1 + M_{PL} \times m2\right]$$

Case 2 Lane Loads: [dlane = 2]

$$M_{STR[2]} = 1.25M_{DC} + 1.5M_{DW} + 1.75\left[\left(M_L \times m1\right) \times 2 + M_{PL} \times m2\right]$$

➢ *Shear*

Case 1 Lane Load: [dlane = 1]

$$M_{STR[1]} = 1.25M_{DC} + 1.5M_{DW} + 1.75\left[\left(M_L \times m1\right) \times 1 + M_{PL} \times m2\right]$$

Case 2 Lane Loads: [dlane = 2]

$$M_{STR[2]} = 1.25M_{DC} + 1.5M_{DW} + 1.75\left[\left(M_L \times m1\right) \times 2 + M_{PL} \times m2\right]$$

APPENDIX D
Detail of optimal calculations

Detail of Optimal Calculation

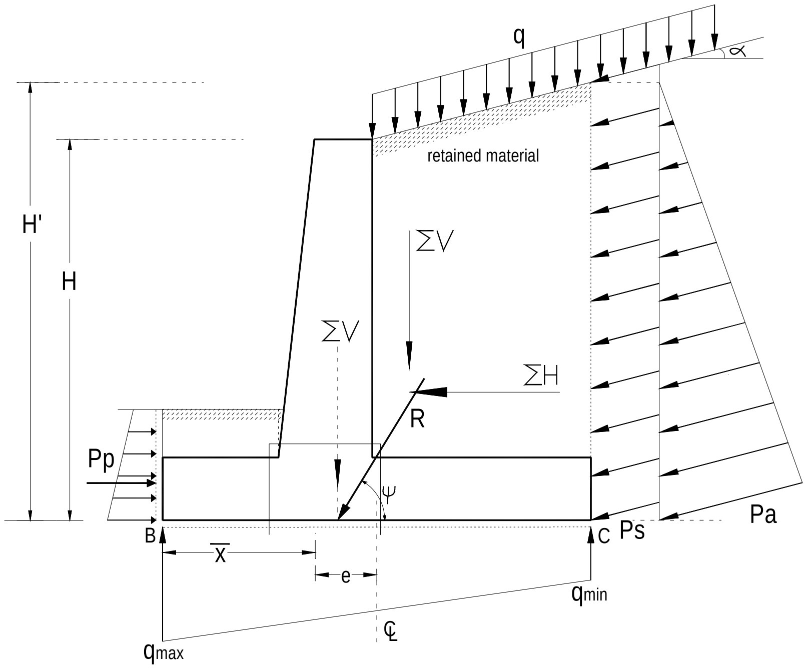


Figure D.1 Pressure distribution on conventional retaining wall

Detail of optimal calculations on conventional retaining wall *Ex1 (EAS-USD)*

Term	**Value**	**Unit**
H’	6.223	m
K_a	0.3372	-
K_p	2.0396	-
K_{pt}	1.000	-
Pa	117.537	$kN/m^2/m$
Ps	20.985	$kN/m^2/m$

Overturning Criterion	Unit	M_R	M_D	Optimal FS	Required FS
	kN.m/m	1003.504	307.447	3.258	2.000

Sliding criterion	Unit	F_R	P_p	F_D	Optimal FS	Required FS
With P_P	kN/m	206.992	268.432	137.995	3.445	2.000
without P_P	kN/m	206.992	-	137.995	1.500	1.500

Bearing criterion	Unit	q_{max}	q_{min}	q_u	Optimal FS	Required FS
	$kN/m^2/m$	162.48	49.18	627.85	3.86	3.000

Eccentricity criterion $\overline{X}$	Unit	$(1/3)B$	$(2/3)B$	$(1/6)B$	e	Result $\overline{X}$	Result e
	m	1.333	2.666	0.667	0.357	Ok	Ok

Slope criterion	Unit	M_R	M_D	M_{Dq}	R (m)	Optimal FS	Required FS
	kN.m/m	6070.61	2107.82	320.42	8.04	2.5000	2.5000

Shear criteria	Unit	V_U	V_R	Optimal FS	Required FS
Toe	kN/m	201.168	390.944	1.943	1.000
Heel	kN/m	390.944	390.944	1.000	1.000
Stem	kN/m	138.475	295.512	2.134	1.000
Moment criteria	Unit	M_U	M_R	Optimal FS	Required FS
Toe	kN.m/m	92.862	476.753	5.134	1.000
Heel	kN.m/m	501.637	501.637	1.000	1.000
Stem	kN.m/m	314.957	314.957	1.000	1.000

Detail of optimal calculations on conventional retaining wall *Ex1 (ESA-WSD)*

Term	**Value**	**Unit**
H’	6.221	m
K_a	0.3372	-
K_p	2.0396	-
K_{pt}	1.000	-
Pa	117.537	$kN/m^2/m$
Ps	20.976	$kN/m^2/m$

Overturning Criterion	Unit	M_R	M_D	Optimal FS	Required FS
	kN.m/m	1004.40	307.596	3.265	2.000

Sliding criterion	Unit	F_R	P_p	F_D	Optimal FS	Required FS
With P_P	kN/m	206.836	268.432	137.890	3.447	2.000
without P_P	kN/m	206.836	-	137.890	1.500	1.500

Bearing criterion	Unit	q_{max}	q_{min}	q_u	Optimal FS	Required FS
	$kN/m^2/m$	159.53	49.54	624.87	3.92	3.000

Eccentricity criterion $\overline{X}$	Unit	$(1/3)B$	$(2/3)B$	$(1/6)B$	e	Result $\overline{X}$	Result e
	m	1.340	2.680	0.670	0.353	Ok	Ok

Slope criterion	Unit	M_R	M_D	M_{Dq}	R (m)	Optimal FS	Required FS
	kN.m/m	6054.50	2101.40	320.40	8.04	2.5000	2.5000

Shear criteria	Unit	V_U	V_R	Optimal FS	Required FS
Toe	kN/m	74.093	207.343	2.798	1.000
Heel	kN/m	207.343	207.343	1.000	1.000
Stem	kN/m	108.681	245.217	2.256	1.000
Moment criteria	Unit	M_U	M_R	Optimal FS	Required FS
Toe	kN.m/m	44.332	44.332	1.000	1.000
Heel	kN.m/m	263.056	263.056	1.000	1.000
Stem	kN.m/m	214.737	214.737	1.000	1.000

Detail of optimal calculations on conventional retaining wall *Ex2 (ESA-USD)*

Term	Value	Unit
H’	5.20	m
K_a	0.3333	-
K_p	2.7698	-
K_{pt}	1.000	-
Pa	75.7120	$kN/m^2/m$
Ps	17.3333	$kN/m^2/m$

Overturning Criterion	Unit	M_R	M_D	Optimal FS	Required FS
	kN.m/m	470.109	176.300	2.666	2.000

Sliding criterion	Unit	F_R	P_p	F_D	Optimal FS	Required FS
With P_P	kN/m	158.646	124.231	93.045	3.040	2.000
without P_P	kN/m	158.646	-	93.045	1.705	1.500

Bearing criterion	Unit	q_{max}	q_{min}	q_u	Optimal FS	Required FS
	kN/m^2/m	189.667	0.000	826.014	4.355	3.000

Eccentricity criterion $\overline{X}$	Unit	$(1/3)B$	$(2/3)B$	$(1/6)B$	e	Result $\overline{X}$	Result e
	m	1.016	2.032	0.508	0.508	Ok	Ok

Slope criterion	Unit	M_R	M_D	M_{Dq}	R (m)	Optimal FS	Required FS
	kN.m/m	3376.44	1145.87	204.70	6.439	2.5000	2.5000

Shear criteria	Unit	V_U	V_R	Optimal FS	Required FS
Toe	kN/m	68.777	297.410	4.324	1.000
Heel	kN/m	297.410	297.410	1.000	1.000
Stem	kN/m	105.302	222.998	2.117	1.000
Moment criteria	Unit	M_U	M_R	Optimal FS	Required FS
Toe	kN.m/m	8.593	275.914	32.106	1.000
Heel	kN.m/m	352.072	352.072	1.000	1.000
Stem	kN.m/m	207.337	207.337	1.000	1.000

Detail of optimal calculations on conventional retaining wall *Ex1 (TSA-USD)*

Term	Value	Unit
H'	6.327	m
K_a	0.3372	-
K_p	1.000	-
K_{pt}	1.000	-
Pa	121.503	kN/m^2/m
Ps	21.336	kN/m^2/m

Overturning Criterion	Unit	M_R	M_D	Optimal FS	Required FS
	kN.m/m	1625.198	322.539	5.038	2.000

Sliding criterion	Unit	F_R	P_p	F_D	Optimal FS	Required FS
With P_P	kN/m	332.933	321.375	142.295	4.598	2.000
without P_P	kN/m	332.933	-	142.295	2.339	1.500

Bearing criterion	Unit	q_{max}	q_{min}	q_u	Optimal FS	Required FS
	kN/m^2/m	145.22	84.08	435.68	3.000	3.000

Eccentricity criterion $\overline{X}$	Unit	$(1/3)B$	$(2/3)B$	$(1/6)B$	e	Result $\overline{X}$	Result e
	m	1.664	3.329	0.832	0.222	Ok	Ok

Slope criterion	Unit	M_R	M_D	M_{Dq}	R (m)	Optimal FS	Required FS
	kN.m/m	10537.57	2682.62	410.50	9.374	3.406	2.5000

Shear criteria	Unit	V_U	V_R	Optimal FS	Required FS
Toe	kN/m	143.909	585.818	4.070	1.000
Heel	kN/m	585.818	585.818	1.000	1.000
Stem	kN/m	123.063	291.127	2.365	1.000
Moment criteria	Unit	M_U	M_R	Optimal FS	Required FS
Toe	kN.m/m	51.459	1070.507	20.802	1.000
Heel	kN.m/m	1104.634	1104.634	1.000	1.000
Stem	kN.m/m	266.475	266.475	1.000	1.000

Detail of optimal calculations on conventional retaining wall *Ex2 (TSA-USD)*

Term	Value	Unit
H’	5.50	m
K_a	0.3333	-
K_p	1.000	-
K_{pt}	1.000	-
Pa	85.708	$kN/m^2/m$
Ps	18.333	$kN/m^2/m$

Overturning Criterion	Unit	M_R	M_D	Optimal FS	Required FS
	kN.m/m	792.381	207.548	3.817	2.000

Sliding criterion	Unit	F_R	P_p	F_D	Optimal FS	Required FS
With P_P	kN/m	282.306	229.00	104.042	4.914	2.000
without P_P	kN/m	282.306	-	104.042	2.713	1.500

Bearing criterion	Unit	q_{max}	q_{min}	q_u	Optimal FS	Required FS
	$kN/m^2/m$	149.036	43.871	447.109	3.000	3.000

Eccentricity criterion $\overline{X}$	Unit	$(1/3)B$	$(2/3)B$	$(1/6)B$	e	Result $\overline{X}$	Result e
	m	1.283	2.566	0.642	0.349	Ok	Ok

Slope criterion	Unit	M_R	M_D	M_{Dq}	R (m)	Optimal FS	Required FS
	kN.m/m	6293.27	1327.71	247.31	7.250	3.995	2.5000

Shear criteria	Unit	V_U	V_R	Optimal FS	Required FS
Toe	kN/m	108.965	384.246	3.526	1.000
Heel	kN/m	384.246	384.246	1.000	1.000
Stem	kN/m	111.956	244.513	2.184	1.000
Moment criteria	Unit	M_U	M_R	Optimal FS	Required FS
Toe	kN.m/m	28.459	460.555	16.182	1.000
Heel	kN.m/m	550.834	550.834	1.000	1.000
Stem	kN.m/m	229.495	229.495	1.000	1.000

BIBIOGRAPHY

Sophea CHEA was born on 18 March 1986 in Battambang province, kingdom of Cambodia. He finished high school in 2004 at Preash Monivong high school in above province. He received bachelor of Engineering from Department of Rural Engineering, Institute of Technology of Cambodia (I.T.C.) in 2009. After his graduation, he was awarded the AUN/SEED-NET scholarship (JICA) to study the graduate level for master's degree in the field of Geotechnical Engineering, Chulalongkorn University, Thailand in academic year 2009-2011. In the future, he plans to continue his study for doctoral degree in the same field in order to expand his knowledge horizon and to serve as the staff of institution upon his graduation.