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ECONOMIC STATISTICAL DESIGN FOR X-BAR CONTROL CHART

Mr. Nopavut Punyasiri

A Thesis Submitted in Partial Fulfillment of the Requirements

for the Degree of Master of Engineering Program in Engineering Management

The Regional Centre for Manufacturing Systems Engineering

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By	Mr. Nopavut Punyasiri
Field of Study	Engineering Management
Thesis Advisor	Assistant Professor Napassavong Rojanarowan, Ph.D.

Accepted by the Faculty of Engineering, Chulalongkorn University in Partial Fulfillment of the Requirements for the Master's Degree

.... Dean of the Faculty of Engineering

(Associate Professor Boonsom Lerdhirunwong, Dr. Ing.)

THESIS COMMITTEE

.... Chairman

(Professor Sirichan Thongprasert, Ph.D.)

.... Thesis Advisor

(Assistant Professor Napassavong Rojanarowan, Ph.D.)

.... Examiner

(Assistant Professor Prasert Akkharaprathomphong)

.... External Examiner

(Associate Professor James Wallbank, D.Eng.)

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ในปัจจุบันการใช้แบบจำลองทางคณิตศาสตร์เพื่อหาขนาดตัวอย่าง(Sample size) และความถี่ในการสุ่มตัวอย่าง (Sampling frequency) ในแผนภูมิควบคุมไม่เป็นที่นิยมนักเพราะตัวแบบจำลองมักมีความซับซ้อนซึ่งยากต่อการประเมินค่าต่างๆและการหาผลลัพธ์ที่ดีที่สุด จุดประสงค์ของโครงงานนี้คือ พัฒนาแบบจำลองทางคณิตศาสตร์ซึ่งสามารถเข้าใจได้ง่ายและสามารถนำไปใช้ได้จริงโดยพิจารณาต้นทุนคุณภาพซึ่งเกี่ยวกับการใช้แผนภูมิควบคุม แบบจำลองทางคณิตศาสตร์ที่พัฒนาขึ้นนี้สามารถหาขนาดตัวอย่างและความถี่ในการสุ่มตัวอย่างที่ทำให้ต้นทุนคุณภาพโดยรวมต่ำที่สุดโดยระดับคุณภาพของแผนภูมิควบคุมคือ ความผิดพลาดประเภทที่หนึ่ง(α)และความผิดพลาดประเภทที่สอง(β) อยู่ในระดับที่ยอมรับได้ ข้อมูลทั้งทางด้านคุณภาพและค่าใช้จ่ายต่างๆในสภาวะงานจริงของกรณีศึกษาถูกนำมาใช้เพื่อทำให้ตัวแบบจำลองนำไปใช้ได้จริง

ต้นทุนคุณภาพประกอบไปด้วยต้นทุนในการตรวจสอบ (Appraisal costs) และต้นทุนในการแก้ไขข้อบกพร่อง (Failure costs) โดยต้นทุนในการป้องกัน (Prevention costs) ไม่ถูกนำมาคำนวณเพราะเป็นต้นทุนที่ไม่ขึ้นกับขนาดตัวอย่างและความถี่ในการสุ่มตัวอย่าง แบบจำลองทางคณิตศาสตร์ที่พัฒนาจะนำมาใช้เพื่อหาผลลัพธ์ภายใต้สองแผนการ แผนการแรกเพื่อหาแผนการสุ่มที่ดีที่สุดภายใต้จำนวนเครื่องทดสอบที่มีอยู่ ในขณะที่แผนการสองจะยอมให้มีการเพิ่มขึ้นของเครื่องทดสอบ นอกจากนั้นการศึกษาความไว (Sensitivity analysis) ถูกพัฒนาเพื่อแสดงความไวของตัวแปร(Variables)และค่าคงตัว (Parameters)ที่มีต่อค่าใช้จ่าย

ผลที่ได้จากแบบจำลองทางคณิตศาสตร์ที่พัฒนาคือ สามารถลดค่าใช้จ่ายที่เกิดขึ้นและระดับคุณภาพของแผนภูมิควบคุมยังอยู่ในระดับที่ยอมรับได้ โดยขนาดตัวอย่างและความถี่ในการสุ่มตัวอย่างที่เหมาะสม

ภาควิชา ศูนย์ระดับภูมิภาคทางวิศวกรรมระบบการผลิต ลายมือชื่อนิสิต ณพวุฒิ บุญศิริ
สาขาวิชา การจัดการทางวิศวกรรม ลายมือชื่ออ.ที่ปรึกษาวิทยานิพนธ์หลัก นภัส
ปีการศึกษา 2553

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At the present, the economic mathematical models are not widely used to find the optimal sample size and sampling frequency related to the implementation of the control chart because the models are quite complex, and difficult to evaluate and optimize. The objective of this thesis is to develop an understandable economic mathematic model that is easily solved by simple spreadsheet software. The developed model can be used to determine sample size and sampling frequency that minimize total quality cost related to the implementation of control chart while statistical quality constraints which are type 1 error (α) and type 2 error (β) are retained. Both quality and cost criteria under the real situation of the case study company are used in order to make the model realistic.

Quality costs in the model consist of Appraisal costs and Failure costs. Prevention costs are excluded from the model because they are not dependent on sample size and sampling frequency. Costs related to the control chart are collected and analyzed about their relationship to the control chart. Then, the economic mathematical model is optimized under two designed scenarios to find the optimum sample size and sampling frequency that minimize the total cost. The first scenario is to find the optimum sampling plan under the current number of testing machines while the new additional testing machines are allowed in the second scenario. Also, a sensitivity analysis is developed to illustrate how sensitive of each variable and parameter over the costs.

After all, the benefit from the model is that the cost will be reduced while statistical quality constraints is retained by appropriate sample size and sampling frequency.

Department : Regional Centre for Manufacturing System Engineering

Student's Signature

Field of Study : Engineering Management

Advisor's Signature

Academic Year : 2010

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CHAPTER I

INTRODUCTION

1.1 Background of the Thesis

Statistical process control is an effective method for controlling quality and productivity in a firm. The primary tool of statistical process control is statistical control chart. Control chart developed by W.A. Shewhart (1931) is a useful tool to monitor the process to check whether the process is in control or there is a system of assignable causes occurs (Celeno and Fichera, 1999). The objectives of control chart is to help determine either variation in measurements of a product are caused by small, normal variations that cannot be acted upon ("common causes") or by some larger variations ("special cause") that can be acted upon or fixed.

The X-bar chart is one of the variable control chart that is among the most important and useful on-line statistical process monitoring and control techniques. The characteristic of X-bar chart is to control the process average or mean quality level between subgroups.

The design of the X- bar chart involves with the determination of the sample size and the frequency or time between sampling. In practice, the X-bar chart normally has sample size (n) around 4 or 5. The sampling interval is generally based on the production rate and familiarity with the process (Alexander et al., 1995). Sample size and sampling frequency play an important role in the quality of the control chart. Moreover, there are many costs related to the sample size and sampling frequency such as cost of poor product and cost of sampling. Therefore, the quality and costs related to the control chart should be concerned when developing the control chart.

1.2 Statement of Problem

The design of X-bar chart is important in many aspects of different levels in the firm. For example, the parameters needed to control can affect the long run product quality. Managers are interested in achieving quality at the minimum cost.

The main factors that needed to concern when designing X-bar chart are sample size (n) and sampling frequency (f). Sample size and sampling frequency are normally determined based on quality criteria only. Increasing sample size and sampling frequency are necessary when we want to increase the ability of detection. However, increasing sample size and sampling frequency results in an increase in cost of sampling, but cost of scrap and cost of rework can be reduced because we can detect the mean shift more quickly. The causes of variation can be fixed early. On the other hand, decreasing sample size and sampling frequency results in a decrease in probability of detection, causing higher cost of scrap and rework. However, the cost of sampling is reducing.

The problem with the commonly used approach that considering only available aspect when designing control chart is that the cost effectiveness is not obtained. The problem is whether to take large samples at less frequent interval or small samples at more frequent interval (Goel, Jain, and Wu, 1968). Determining sample size and sampling frequency is important for minimizing the costs related. Many researchers have proposed economic models for designing of control chart which give the guideline to answer those questions.

However, Saniga and Shirland (1977) showed that very few economic models for the design of control charts have implemented. The economic models are not widely used because the models are quite complex, and difficult to evaluate and optimize (Alexander et al., 1995). The economic models usually use complex mathematic and statistic, which are hard to understand and applied in real case. Woodal (1986) stated that control chart based on economically optimal design generally have poor statistical properties. Moreover, Montgomery (1980) also stated that the proposed models did not consider all relevant costs and no formal optimization techniques applied to the total cost function.

Hence, this project aims to solve these problems by developing understandable economic mathematical model using both quality and cost criteria. The model is used to determine the control chart parameters which are sample size (n) and sampling frequency (f) that minimizing the total cost of quality while the quality level remains the same. The economic mathematical model will be developed under the real situation of the case study company to make the model realistic.

1.3 Background of the Case Study Company

The case study company is an electronic company that produces specific parts in a motor for hard disk drive (HDD). The production of the product consists of turning process, internal process quality assurance (IPQA), and outgoing quality assurance (OQA). The turning process is a process that turns an internal diameter (ID) of the product. This process is considered to be a long run production process and has a steady standard deviation (SD). The IPQA process is a quality control process which monitors the ID parameter in production line. At the present, the company is using X-bar control chart to monitor the ID as a variable data in IPQA process. The OQA process is the final quality control process before delivering the product to the customer. This thesis will use the ID of the product as a variable data to monitor an X-bar chart.

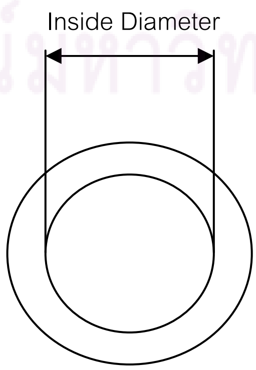

Figure 1.1: Inside Diameter of the Product Studied

1.4 Thesis Objective

1. To develop economic mathematic model by integrating quality costs related to the implementation of control chart.

2. To find the appropriate sample size and sampling frequency for the case study company.

1.5 Scope of the Thesis

1. The scope of the thesis focuses on developing economic mathematical model for the process in the case study company which produces parts in HDD. The thesis uses X-bar chart for monitoring the inside diameter of the product.

2. Constraints about statistical quality are type 1 error (α) and type 2 error (β). Type 1 error and type 2 error can also be illustrated in forms of ARL_0 ($\frac{1}{\alpha}$) and ARL_1 ($\frac{1}{1-\beta}$) respectively.

3. The developed model can be used to determine sample size and sampling frequency that minimize total quality costs with statistical quality constraints.

4. Costs in the model consist of Appraisal costs and Failure costs. Prevention costs are excluded from the model because they are not dependent on sample size and sampling frequency.

5. The production process is considered to be a long run steady process and the measured value is considered to be variable data with normal distribution.

1.6 Methodology

1.6.1 Exploratory Research from Literatures, Books, and Journals

This phase focuses on researching relevant literatures, books, and journals about the design of control chart and costs of quality. Data from the research will be analyzed and identified to set a scope and arrange a schedule for the thesis. The objective of the thesis and its expected benefit are also settled in this phase.

1.6.2 Collect All Related Information from the Case Study Process

The case study process will be used to make the study more realistic and practicable. There are two main data that will be collected from the case study process, which are quality cost and quality characteristic.

1.6.2.1 Quality Cost

There are many costs that related to the control chart and its usage. These related costs will be studied and collected to make the mathematic model precise as much as possible.

1.6.2.2 Quality Characteristic

Quality characteristic measured from the product will be collected and used for designing X-bar control chart.

1.6.3 Develop the Mathematical Model

The economic mathematical model will be developed based on theory, literature, and case study data. The model will include related appraisal and failure costs and statistical quality criteria to determine sample size and sampling frequency. Sample size and sampling frequency are the variables affect both costs and statistical quality of the control chart.

1.6.4 Improve the Mathematical Model

The collected data will be applied in the model for validating and improving the model. The impractical issues and drawbacks in the economic mathematical model will be analyzed and improved to make the effective model. Literature will also be reviewed to ensure that the improvement is in the right direction.

1.6.5 Summarize the Results

Results, advantages, and disadvantages of the economic mathematical model and its application will be illustrated and concluded. Also, limitations and assumptions will be discussed and suggested.

1.7 Research Schedule

Table 1.1: Research Schedule

	Oct 09	Nov 09	Dec 09	Jan 10	Feb 10	Mar 10	Apr 10	May 10	Jun 10	July 10	Aug 10
1. Study research											
2. Collect data											
3. Develop the model											
4. Improve the model											
5. Summarize											

1.8 Expected Benefits

1. Costs of quality will be identified and analyzed about their relationship with the control chart implementation and statistical quality criteria.

2. Developed mathematical model can determine the appropriate sample size and sampling frequency for minimizing total quality costs.

3. After using the new developed model, costs will be reduced while statistical quality is retained.

CHAPTER II

RELATED THEORETICAL STUDIES AND LITERATURE REVIEWS

2.1 Related Theoretical Studies

2.1.1 Statistical Quality Control (Montgomery, 2005)

2.1.1.1 Causes of Quality Variation

A. Chance Causes

In any production process, regardless of how well designed or carefully maintained it is a certain amount of inherent or natural variability that always exists. This natural variability or “background noise” is the cumulative effect of many small, basic unavoidable causes.

In the framework of statistical quality control, this natural variability is often called a “stable system of chance causes”.

A process that is operating with only chance causes of variation present is said to be in statistical control. In other words, the chance causes are an inherent part of the process.

B. Assignable Causes

Another kind of variability may occasionally present in the output of the process. This variability in key quality characteristics usually arises from three sources: improper machines control or adjustment, operator errors, or defective raw material.

Such variability is generally large when compared to the background noise, and it usually represents an unacceptable level of process performance. These sources of variability that are not part of the chance cause pattern are called "assignable causes". A process that is operating in the presence of assignable causes is said to be out of control.

2.1.1.2 The Design of the Control Chart

A. Identify What to Control or the Control Objective

The selected characteristic has to be controlled should be a major characteristic that plays an important role in the product quality. The major characteristic should be controlled rather than a minor characteristic.

B. Specify the Sample Size and the Sampling Frequency.

The sample size and the sampling frequency depend on the following factors:

1. The constancy of production ex. more sampling frequency is required if the production is the new one.
2. The rate of production.
3. Cost of sampling.

C. Collect Sample Data

Gathering the sample data uses different type of data record depending on the type of control chart. After collecting all data, the result is used to formulate control chart from each sample to find sample mean, sample distribution and sample sensitivity.

D. Calculate Control Limits and Develop the Control Chart

Control chart consists of:

1) Upper Control Limit: UCL

$$UCL = \hat{\mu} + L\hat{\sigma}$$

2) Centerline: CL

$$CL = \hat{\mu}$$

3) Lower Control Limit: LCL

$$LCL = \hat{\mu} - L\hat{\sigma}$$

Where $\hat{\mu}$ = the mean of sample

$\hat{\sigma}$ = the standard deviation of sample

L = the distance of control limits from the center line, expressed in standard deviation units

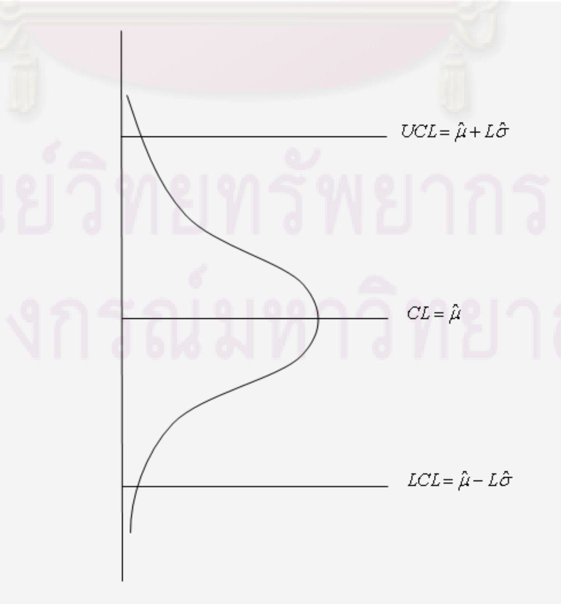

Figure 2.1: UCL, CL, and LCL of the Control Chart

E. Plot and Analyze the Control Chart

Plot the points that have been computed based on measurement on the control charts and then analyze the distribution of the points whether the process is in in-control state or not. The characteristics of in-control state are as follows:

1. Most of the points are near the center line.

2. The points near the upper and the lower limit are in small quantity.

3. Nearly all of the sample points fall between the upper and the lower control limits.

4. The distribution of the points is random.

Using Western Electric Rule to determine the deviation of the process.

F. Improve the Control Chart

If there is a point exceeding the UCL or LCL, the investigation has to be established to fix the causes. After the causes were found, eliminate all the points that plotted outside the control limits and develop new control chart using the points inside the control limits. Use the new control chart to control the process. The important factors are:

1. The center line and the control limits must be computed from in-control state process.

2. The assumption in developing the control chart is that the sample data has to have normal distribution.

G. Use Control Chart to Improve Product Quality

The improvement of product quality is necessary for the competition. Besides, when the process is improved ex. the center line or the sensitivity is change in the better way, the new control limits must be computed to make the control chart conform to the present situation.

2.1.1.3 The X-bar Control Charts

The X-bar charts are among the most important and useful on-line statistical process monitoring and control techniques. Control of the process average or mean quality level is usually done with the control chart of means, called the X-bar chart.

2.1.1.4 Development of X-bar Control Charts

There are 4 important factors in developing X-bar and R charts which are control limit width, sample size, sampling frequency, and method of subgroup selection.

A. Control Limit Width

Control limit width is depended on the value of L since $\hat{\sigma}$ is a value that derived from the process variability which cannot be adjusted. Normally, the L value is set to be 3.

$$UCL = \hat{\mu} + L\hat{\sigma}$$

$$LCL = \hat{\mu} - L\hat{\sigma}$$

The main considerations when developing control chart are lost due to nonconformance, cost of adjusting machine, type 2 error (β), and type 1 error (α).

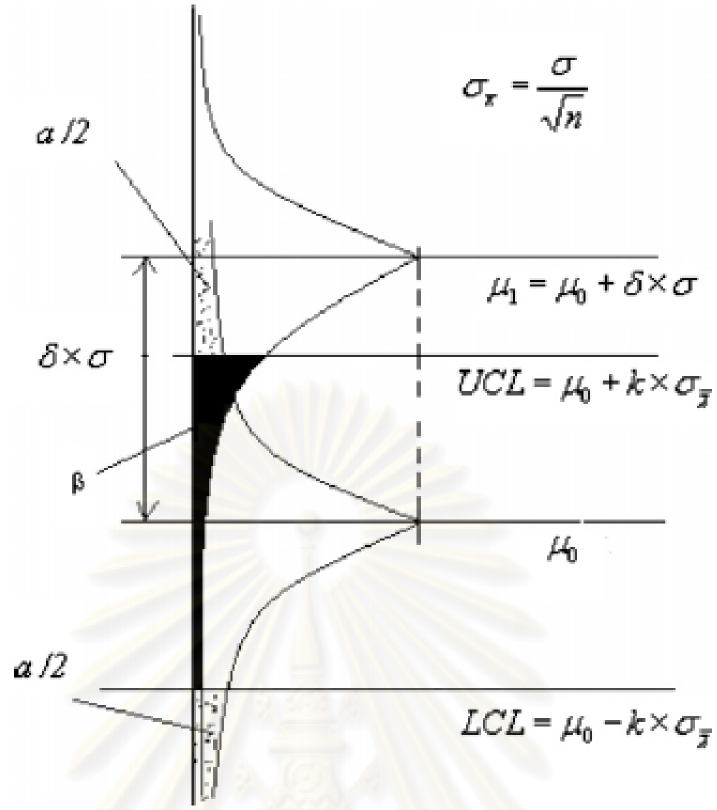

Figure 2.2: X-bar Control Chart Limits and the Shift of the Process Mean from μ_0 to μ_1 (Engin, 2008)

B. Sample Size (n)

Sample size is the number of products which will be formed to be one sample group. There are two aspects of the quantity of sample size as follows.

1. We want a small sample size because we want to save the sampling cost.
2. We want a large sample size because we want the mean sample to distribute as a normal distribution and increase the power of the detection.

Normally, the sample size is around 4-6 pieces.

Sample size and type 2 error (β).

$$\beta = \Phi(L - k\sqrt{n}) - \Phi(-L - k\sqrt{n})$$

Where β is the function of sample size which can detect the magnitude of the shift in various sample sizes (n)

Φ is the standard normal cumulative distribution function

k is the magnitude of the shift from the normal state process expressed in standard deviation units

L is the multiple of standard error in the control limits

ARL (Average Run Length) is the performance measurement of control chart in terms of the expected number of samples taken in order to detect the shift. The large value of ARL means that the large number of samples is required for detection, which resulted in higher cost since a large number of defect products have been produced. ARL is preferred to be small in order to detect the shift early.

$$\text{ARL} = \frac{1}{1-\beta}$$

C. Sampling Frequency (f or 1/h)

The frequency of sampling should be at least equal to the frequency that the special cause variation may arise and also consider about the sampling cost. High sampling frequency is required when the production is new since it can reduce the time required to detect the causes resulting in less defect product produced. However, this can increase the sampling cost due to the fact that it needs more resources to operate. The sampling frequency can be lower when the production is stable.

D. Method of Subgroup Selection

The samples should be selected by an appropriate method so that the common-cause variation can be well-detected and also increase the opportunity to detect the special cause variation. There are two methods of subgroup selection as shown below.

1) Consecutive Sampling

Each sample is randomly tested at the same time (or as closely together as possible) creating higher chance to detect the common-cause variation.

2) Distributed Sampling

Each sample is tested comprehensively from the beginning to the end of the process. This method is popular for stable process since it can detect immediate and short time variance which well-detected in R-chart and also has the ability to detect gradual variance, but faster than Consecutive sampling.

2.1.1.5 Pitfalls in Subgroup Selection

A. Stratification

The samples are from the same process, but different machines resulting in high range of data since the mean from each of the machines are not the same. The control looks like an effective process since the distribution of the mean on X-bar chart is narrow comparing with the width of control limits, but actually it is wider than normal.

B. Mixing

The samples are combined from all machines and then randomly tested. More than one distribution mode might be happened on X-bar chart.

2.1.2 Quality Costs

Since financial costs take an important part in business management, it is essential to integrate financial controls in every department or function including in quality function. Quality costs are emerged into financial control tool so that the company can specify chances to reduce quality costs and consequently reduce overall costs in the company. The main concept of quality costs is to quantify the total cost of quality-related efforts and deficiencies that normally associated with producing, identifying, avoiding, or repairing products that do not meet requirements.

Quality costs can be divided into 3 main categories which are Prevention costs, Appraisal costs, and Failure costs described as follows:

2.1.2.1 Prevention Costs

Prevention costs are costs that a company spends to prevent an error or make it right first time. The company needs to pay attention essentially on the prevention cost because the costs of preventing errors are much cheaper than the costs of fixing errors which might happen later if the prevention is not good enough. The important prevention cost which likely to happen is described as followed:

A. Quality Planning

This cost is related to the creation of the plans involved with the quality such as inspection plan and specialized plans of the quality-assurance function.

B. Investment in Quality-related Information Systems

This cost is involved with acquiring data on product performance and process efficiency and also analyzing these data to identify the problems.

C. Quality Training and Workforce Development

The cost of training consists of developing, preparing, executing, operating, and maintaining quality training program.

D. Process Design

The cost of process design is occurred in order to improve the overall quality of the process and product.

E. Process Control

Costs associated with process control techniques which has the main function to monitor the process in order to reduce variation and improve the product quality.

2.1.2.2 Appraisal Costs

Appraisal costs include all activities associated with measuring, evaluating, and testing products, processes, and services in order to determine whether they conform to the standards or not. This type of quality cost is used to check the products, processes, and services in term of quality to ensure that they can conform to specifications. The major appraisal costs are described below:

A. Test and Inspection of Purchased Materials

Costs occurred from inspecting and testing of all materials to ensure that there is no defect in those incoming materials.

B. Test and Inspection of Products

The cost involved with checking the conformance of the products in each stage of production from the manufacturing through the shipping.

C. Test and Inspection of Equipments

Costs in this section are concerned with maintaining the accuracy and efficiency of equipment in order to keep the equipment in good condition.

D. Quality Audits

Costs associated with a periodic audit of quality-assurance system to ensure that everything is running smoothly.

2.1.2.3 Failure Costs

Failure costs are costs that a company spends to correct products, processes, and services that fail to meet the requirements of the customers. Failure costs can be divided into two types, which are internal failure costs and external failure costs. An internal failure is the failure that happens before the product or service is delivered to the customer, while an external failure is the failure that happens after the product or service is delivered to the customer already. Both internal failure and external failures would disappear if there are no defects in the product.

The major internal failure costs are described as followed:

1. Scrap before Reaching Customers

Scrap cost is loss including labor cost and raw material cost from defective products that cannot be repaired or used.

2. Rework before Reaching Customers

Rework cost is associated with correcting the nonconformance products to make them conform as the specification.

3. Internal Process Failure

This cost deals with internal process failure. It includes the cost of idle production process from machine downtime due to the nonconformance to requirements.

The major external failure costs are described as followed:

1. Cost of Dealing with Customer Complaints

This cost associates with the investigation and adjustment of customers' complaints from the nonconforming products.

2. Product Recall

This type of cost includes costs that involved with returning of nonconforming products including transportation cost and product replacement cost.

3. Indirect Costs

This type of costs involves an indirect cost that arisen from product failure such as customer dissatisfaction and loss of reputation.

Table 2.1: Quality Costs

Type of quality costs		Examples
Prevention costs		Quality planning
		Investment in quality-related information systems
		Quality training and workforce development
		Process design
		Process control
Appraisal costs		Test and inspection of purchased materials
		Test and inspection of products
		Test and inspection of equipments
		Quality audits
Failure costs	Internal failure costs	Scrap before reaching customer
		Rework before reaching customer
		Internal process failure
	External failure costs	Cost of dealing with customer complaints
		Product recall
		Indirect costs

2.2 Literature Review

2.2.1 Development of Economic Statistical Design of Control Chart

2.2.1.1 Statistical Design of Control Chart

Control charts were firstly established in 1924 by Walter A. Shewhart (Shewhart, 1931) as a mean to discriminate between the normal expected random causes and the special assignable causes of the measured values. This Shewhart control chart is called X-bar control chart. In an X- bar control chart, the control limit coefficient (L) is equivalent to 3 based on the normal distribution theory while the sample size (n) is usually around 4 to 5. However, there are no general guidelines for the sampling interval (h).

The design of the control chart will affect directly to the performance and the cost of the control chart. Traditionally, control chart has been designed only by statistical criteria. The type 1 error probability and power are usually specified at the desired level while the sample size (n) and control limits coefficient (L) can be specifically determined. However, the high quality criteria may result in high performance control chart since it can detect an assignable cost early, but it may cause more false alarms and in higher operation cost. Thus, the design of the control chart is very important to its performance in terms of quality and cost criteria.

2.2.1.2 Semi Economic Design of Control Chart

The first work about economic design is done by Girshick and Rubin (Girshick and Rubin, 1952). They proposed in an area of cost modeling of quality control system and also analyze a process model that a machine produces items with a quality characteristic (x). The machine production can be divided in to four stages. Stage 1 and 2 are production stages and the output quality characteristic is described by the probability density function $f_i(x)$ where i = 1, 2. Stage 1 is an in-control stage which has a constant probability of a shift into stage 2 which is an out-of-control stage. The process is not self-correcting so repairing is required to bring the process back to stage 1 or an in-control stage. Stages 3 and 4 are repair stages. They also discussed about 100% inspection and periodic inspection rules. The economic criterion is to maximize expected net income from the process. Since the optimal control rules are hard to derive due to complex integral equations, the usage in practice has been limited. However, they were the first who proposed the expected cost per unit of time which is beneficial to the following researchers which have used this criterion to further development. Bather (1963), Savage (1962), White (1974), and Ross (1971) have investigated generalized formulations of the Girshick and Rubin model, but their results do not lead to practical process control rules. There are also several early researchers proposed their economic design of conventional Shewhart control chart. Weiler (1952) proposed that, for an X-bar chart, the optimum sample size (n) should minimize the total amount of inspection. If there is a shift from an in-control state (μ_0) to an out of control stage $\mu_1 = \mu_0 + \delta\sigma$, Weiler illustrated that the optimal sample size can be calculated from $n = d/\delta^2$ where d depends on the control limits width. However, Weiler did not formally concern about costs but his work aimed for minimizing total inspection resulting in minimizing total costs implicitly. Taylor (1965) showed that control procedures based on fixed sample size (n) and sampling frequency (f) are not optimal. He suggested that these variables should be determined based on posterior probability which the process is in an out-of-control stage. However, in practice, fixed sample size (n) and sampling frequency (f) are widely used because of their administrative simplicity. In conclusion, these economic designs can be classified in the semi-economic design because the proposed models do not consider all related costs or there is no formal optimization techniques applied to the cost function.

2.2.1.3 Economic Models of the X-bar Control Chart

Duncan (1956) proposed the first paper dealing with fully economic model of the Shewhart control chart and incorporating formal optimization methodology to determine the control chart parameters. The design criterion is to maximize the expected net income per unit of time.

There are many assumptions needed to be applied when using the Duncan's model which are as follows:

1. The control chart is used to detect a random single assignable cause.

2. Actions are taken when any point exceeds the control limits since the process is not self-correcting.

3. The process begins from an in-control stage.

4. The distribution of the quality characteristic of the process output is normal.

5. The assignable cause occurs according to a Poisson process and the occurrence time of the assignable cause is an Exponential distribution with parameter $\lambda > 0$, where $1/\lambda$ is the mean time that the process is in the in-control stage.

6. The process is allowed to operate during the search for an assignable cause (continuous process).

7. The rate of production is high so that the occurrence of change in the process during the sample taking is neglected.

8. The process is characterized by an in-control stage μ_0 and the assignable cause occurs at random at magnitude δ which resulted in the mean from μ_0 to either $\mu_0 + \delta$ or $\mu_0 - \delta$. The process is monitored by an X-bar chart with centerline μ_0 and upper and lower control limits $\mu_0 \pm k(\sigma/\sqrt{n})$.

A production cycle consists of four periods which are in-control period, out-of-control period, time to take a sample and interpret the result, and time to find an assignable cause. The expected length of the control period is $1/\lambda$. The number of samples required to produce an out-of-control state is a geometric random variable with mean $1/(1-\beta)$ so the expected length of the out of control period is $h/(1-\beta)-\tau$ where τ is the expected time of occurrence within the interval of adjacent samples. The time required to take a sample and interpret the results is a constant g which is proportional to the sample size (n) while the time required to find the assignable cause is a constant D. Hence, the expected length of the cycle is

$$E(T) = \frac{1}{\lambda} + \frac{h}{1-\beta} - \tau + gn + D \,.$$

The net income per hour when the process is in the in-control state is V_0 while the net income per hour when the process is in the out-of-control state is V_1. The cost of taking sample size (n) is $a_1 + a_2 n$ where a_1 is a fixed cost of sampling and a_2 is a variable cost of sampling. The cost of finding an assignable cost is a_3 and the cost of finding a false alarm is $a_3^{'}$. The expected number of false alarm during a cycle is α times the expected number of sample taken when the process is in the in-control state, or $\alpha e^{-\lambda h}/(1-e^{-\lambda h})$. Thus, the expected net income per cycle is

$$E(c) = \frac{V_0}{\lambda} + V_1[\frac{h}{1-\beta} - \tau + gn + D] - a_3 - \frac{a_3^{'}\ \alpha e^{-\lambda h}}{1-e^{-\lambda h}} - (a_1 + a_2 n)\frac{E(T)}{h}$$

The expected net income per hour $E(A)$ is found by dividing the net income per cycle $E(C)$ by the expected length of the cycle $E(T)$.

$$E(A) = \frac{V_0(1/\lambda) + V_1[h/(1-\beta) - \tau + gn + D] - a_3 - a_3^{'}\alpha e^{-\lambda h}/(1-e^{-\lambda h})}{1/\lambda + h/(1-\beta) - \tau + gn + D} - \frac{a_1 + a_2 n}{h}$$

The objective is to maximize the expected net income per hour [$E(A)$] with respect to sample size (n), interval between sample (h), and multiple of sigma used in control limits (k). There are several numerical approximations are applied in the structure and optimization of the model. An optimization method needs numerical approximation and repetitious procedure to solve for the optimal sample size (n) and multiple of sigma used in control limits (k). However, this is not really a practical optimization method so there are several authors reported optimization methods for the Duncan's model. Goel, Jain, and Wu (1968) devised an iterative procedure that will produce the exact optimum solution which is superior to Duncan's optimization method in some situations.

Although the Duncan's model is the model that integrates fully economic design of the control chart, but there are many unrealistic assumptions and limitations in the model as follows:

1. The Duncan's model does not concern about statistical properties since there is only an economic objective without statistical constraints in the model (Saniga, 1989).

2. The process is allowed to continue in operation during the search for an assignable cause which may be unrealistic in case that the process has to halt to search for an assignable cause (Montgomery, 1980).

3. The cost of eliminating an assignable cause is not charged against the net income for the period (Montgomery, 1980).

4. The cost elements are quite rough since it is hard to value the cost elements in practice (Montgomery, 1980).

5. The optimization method is complicated with no general solving method (Alexander et al., 1995).

These disadvantages and limitations of the Duncan's model persuade many of researchers to develop their models based on Duncan's model for specific usages and assumptions. There are many issues that can be developed from the Duncan's model such as assumptions, actions occurred in the cycle time, cost elements, optimization methods, related control charts, and sampling plans. These issues will be illustrated further.

2.2.1.4 Economic Statistical Design

Both statistical designs and economic designs have strengths and weaknesses (Zhang and Berardi, 1997). The advantage of statistical designs are that they give low error rate both in type 1 and type 2, but may cost more than economic designs due to its strict quality criteria. On the other hand, economic designs focus on cost and ignore statistical properties resulting in low quality criteria level. The economic statistical design was first developed by Saniga (1989) which aimed to integrate advantages from both statistical and economic designs and reduces disadvantages from them. The main objective of economic statistical design is to minimize the expected cost per unit of time as in economic design, but also consider type 1error, type 2 error, and ATS as the constraints (Saniga, 1989). Alternatives and additional constraints can be added depending on the objective of each design. Even though economic statistical design generally cost more than normal economic design, the output qualities or quality statistic properties are better.

2.2.2 Economic and Economic Statistical Design of X-bar Chart with Different Adaptation

2.2.2.1 Weibull Failure Rate Assumption

Most of the previous process models assumed that the process failure mechanism follows a Poisson process and the time that the process is in the in-control state follows an exponential distribution. However, these assumptions may not be suitable for every process. Baker (1971) reported that the optimal economic control chart is sensitive to the process failure mechanism assumption. Hu (1984) discussed an economic design of an X-bar control chart with non-Poisson process. An economic design of an X-bar control chart under a Weibull failure mechanism was proposed by Banerjee and Rahim (1988). Weibull failure mechanism can be generally used in any system with a constant, increasing, or even decreasing failure rate. Thus, the usage of Weibull failure mechanism is wider than Poisson failure mechanism with a constant failure rate. Zhang and Berardi (1997) proposed economic statistical design of X-bar control chart with Weibull in-control time and also added statistical constraint to previous economic design with Weibull in-control time.

2.2.2.2 Multiple Assignable Causes Assumption

The assumption for most of previous models is a single assignable cause assumption. However, many of production processes have several assignable causes. Duncan (1971) extended his single assignable cause model to be a multiple assignable causes model. The occurrence times of the assignable causes assumed to be independently exponentially distributed with mean times $\frac{1}{\lambda_j}$ where j is a number of occurrences. It is assumed that after an initial shift occurred, the second occurrence of an assignable cause is allowed. The shift has been assumed to be at constant magnitude regardless of what the causes are. A new probability (p_j) represents a probability that a point falls outside the control limits after the occurrence of cause A_j. Knappenberger & Grandage (1969) also purposed economic design of the X-bar chart for multiple assignable causes. They assumed that the process can be stopped while action signals are investigated and there are no constraints that limit the number of assignable cause occurrence which is considered to be more realistic assumption and more practical than Duncan's model. A Markov chain model structure is used. In conclusion, multiple assignable cause models are more complex than single assignable cause models and there are more unknown parameters that must be specified in order to determine the optimum control chart design. Duncan (1971) and Knappenberger & Grandage (1969) reported that a single assignable cause model that match the true multiple assignable cause system in certain important ways produces very good results (Montgomery, 1980). Thus, it is reasonable to conclude that a model which contains only a few stages can approximately substitute very complex multistage processes.

2.2.2.3 Joint Economic Design of X-bar and R Control Chart

Saniga (1989) developed an economic statistical model for the joint economic design of X-bar and R control charts. He divided the process into three stages. However, there are statistical constraints on the economic model. In the in-control state, the mean of the process is μ_0 and the standard deviation is σ_0. Two assignable causes can cause an out-of-control-stage. The first assignable cause creates a shift in the process mean to μ_1 while the second assignable cause creates a shift in process standard deviation to σ_1. Since the X-bar and R control charts are statistical independent, the X-bar chart monitors process mean, while the R chart monitors process standard deviation. The design parameters are sample size (n), sampling frequency (f), multiple of sigma used in control limits (K_{x-bar}), and upper control limit factor on the R chart (K_R). This design is a design for two assignable causes so the general modeling is quite similar to Knappenberger and Grandage model. The result from the joint optimized of X-bar and R charts, in general, is that it requires less frequent samples than those in only X-bar chart. This is because the power of the test is increased from using both X-bar and R control chart.

2.2.2.4 Economic Design of Control Chart Using the Taguchi Loss Function

Duncan has roughly defined cost elements in his model but he did not propose how to estimate the cost elements. For example, Duncan integrated a penalty cost for operating out of control in his model but he did not illustrate how this cost element can be obtained and quantified (Alexander et al., 1995). Alexander et al. (1995) proposed the algorithm to estimate costs with Taguchi loss function. The Taguchi loss function gives a mean of considering the loss due to process variability caused by both chance and assignable causes. The Taguchi loss function is defined below.

$$\text{Expected loss/ unit} = \frac{A}{\Delta^2}\nu^2$$

Where A is a cost to society for manufacturing a product out of specification

Δ is a bilateral tolerance of equal value

ν^2 is a mean squared deviation of the process

In this research, the cycle time is similar to Duncan's model and the process is allowed to operate during the investigation and fixing the assignable causes.

2.2.2.5 Minimum-loss Design of X-bar Charts for Correlated Data

In general, the assumption when measuring within a sample is usually assumed to be independent. However, this assumption may not always be realistic for some processes. For example, the process consists of multiple but similar units on a single part. Liu, Chou, and Chen (2002) developed the economic design of X-bar charts for correlated measurements within a sample with Taguchi's loss function. They used correlation model developed by Yang and Hancock (1990) and the loss model from Alexander et al. (1995). The objective function is to minimize the costs in the loss model. Average of the correlation coefficient among the measurements (ρ) was set for the correlation assumption. They implied that positively correlated data resulted in a smaller sample size and a frequent sampling interval. The power of the chart decreases when the correlation coefficient increases. On the contrary, negatively correlated data has a smaller sample size with narrower control limits.

2.2.3 Other Control Charts

2.2.3.1Economic-statistical Design of an Adaptive X-bar Chart

Prabhu, Montgomery, and Runger (1994) proposed an adaptive X-bar chart in year 1994. They called this control procedure a *combined adaptive chart*. An X-bar chart with adaptive design parameter outperforms a traditional fixed sample size (n) and sampling interval (h) X-bar chart (Prabhu, Montgomery, and Runger, 1997). Moreover, Taylor (1965) stated that a constant sample size (n) and a sampling frequency (f) are not optimal. The adaptive X-bar chart can detect the shift faster because of larger sample size (n) and more sampling frequency (f) when the process is running off-target. However, an adaptive chart uses more resources because it uses a frequent sampling rate and a large sample size to improve its performance. Afterward, Prabhu et al. (1997) proposed an economic-statistical design for adaptive X-bar chart with dual sample sizes (n) and dual sampling interval (h). The objective is to minimize the cost function along with a statistical constraint which is Average time to signal (ATS). The adaptive control chart allows the sample size (n) and sampling interval (h) vary over time in order to be consistent with the present situation which is chosen by threshold limits (w and $-w$). The choosing criterion is illustrated below.

$$n(i), t(i) = \begin{cases} (n_2, t_1) \text{ if } w < Z_{i-1} < UCL \\ (n_1, t_2) \text{ if } -w \le Z_{i-1} \le w \\ (n_2, t_1) \text{ if } LCL < Z_{i-1} < -w \end{cases}$$

Where $Z_i = \dfrac{\overline{X_i} - \mu_0}{\sigma / \sqrt{n(i)}}$

n_1, t_2 is a pair of minimum sample size and longest sampling interval

n_2, t_1 is a pair of maximum sample size and shortest sampling interval

Lorenzen and Vance (1986) economic model was extended to an adaptive scheme. A general cycle time contains an in-control state, an out-of-control state, and an investigation & repair period. Moreover, they stated that the average sample size (n) and average sampling interval (h) of the adaptive chart remain identical to the normal chart if the process is running on-target. On the other hand, if the process is running off-target, the adaptive procedure might use more samples and more frequency to make an effective detection of a shift resulting in higher operation costs. However, the adaptive control chart is quite complex and hard to implement in application because of its adaptation procedure.

2.2.3.2 Economic Design of Fraction Defective Control Chart

Most of the previous papers concern about control chart for variable data. However, the control chart can be used for attribute data also. In the economic design of fraction defective control chart, the type 1 and type 2 errors will be computed from a binomial distribution. Further study in the economic design of fraction defective control chart is also interested such as C and U chart. There are several authors who have developed economic design of attribute control chart. Ladany and Alperovitch (1975) proposed an economic model of the fraction defective control chart based on Duncan's model and he assumed that only a single assignable cause can exists. His model consists of cost of sampling, cost of searching for assignable causes, process adjustment, and production in an out-of-control state, but he reported no numerical result. Chiu (1977) purposed his model with a sensitivity analysis. He concluded that the model is insensitive to errors in estimating the cost parameters but requires more precise estimation of the fraction defective in the in-control and out-of-control states. Montgomery, Heikes, and Mance (1975) purposed an economic design of fraction defective control chart with multiple assignable causes. This fraction defective control chart has a fraction defective which calculated from

$$\hat{p} = \frac{D}{n}$$

Where D is the number of defective units found in the sample and n is the sample size

The model consists of cost of sampling, cost of investigating and correcting the process when an out-of-control state occurred, and cost of defective product. The objective is to determine sample size (n), interval between sample (h), and multiple of sigma used in control limits (k). Direct search techniques are used in order to optimize the expected cost function. The sensitivity also presented and the result is that the model is not sensitive to the number of out-of-control state. Montgomery (1980) stated that a proper chosen single assignable cause model would be a suitable approximation for a multiple cause model. Using a single assignable cause will reduce the complexity of the model with an acceptable approximation to multiple assignable causes.

2.2.3.3 An Economic-statistical Design of Double Sampling X-bar Control Chart

Daudin (1992) applied double sampling plans concept to Shewhart's X-bar chart and also adopted two stages Shewhart's X-bar control chart for monitoring process mean. A double sampling X-bar control chart can maintain the advantages of

Shewharts's control chart and improve the abilities to detect mean shift and reducing sample size as well (Torng, Lee, and Liao, 2009) . This is because the double sampling plan X-bar chart is quicker and more sensible to detect a mean shift than Shewhart's X-bar control chart. Torng et al. (2009) developed an economic-statistical design of double sampling X-bar control chart based on Duncan's economic model. Average run length (ARL) is used as a constraint for the economic statistical model.

2.2.3.4 Economic Design of Control Charts with Two Control Limits

Schmidt, Bennett and Case (1980) proposed the control chart with two pairs of control limits for acceptance sampling by variables. Chung (1995) proposed an economic design with two control limits control chart. In the literature review, there are two out-of control states called state 1 and state 2 which assumed to be the independent assignable causes and exponentially distributed with each own means. State 1 represents an out-of-control state caused by a minor assignable cause 1 while state 2 represents an out-of-control state that the process mean is shifted by a major assignable cause. The process standard deviation is assumed to be constant when the process is out-of-control. The correction procedure for operating is designed specifically for this control chart. The authors also proposed a procedure which can be implemented in real time on a personal computer. However, the procedure needs a certain procedure search technique which is hard to implement in practice.

2.2.3.5 Weakness of the Economic Design Control Chart

Saniga and Shirland (1977) reported that only few practitioners have implemented economic model to design their control charts. This is quite strange because most practitioners claim that a major objective in the use of statistical process control procedures is to reduce the costs (Montgomery, 1980). There are major reasons for the lack of practical implementation of economic design. First, the models are complex, difficult to be evaluated, and optimized (Saniga, 1989) which suitable only for research but difficult to implement. Moreover, there is no general solution to optimize the cost function. Some model needs manual computation to solve the model. Second, the cost elements need to be evaluated but most of the presented models do not describe them thoroughly. Although costs do not have to be estimated with high precision (Montgomery, 1980), explicating cost is a very important factor for minimizing cost which is the major objective of the model. The practitioners have to understand deeply about their economic model in order to develop the model effectively.

2.3 Research Plan

This thesis will develop an understandable economic mathematical model for X-bar chart using both quality and cost criteria. The weak points of previous economic designs will be solved, while the strengths will be applied in order to reduce total quality costs. The disadvantages of economic design that it usually ignores statistical properties will be solved by adding more statistical quality constraints to the model to retain quality of the control chart. Also, the advantages of the economic design will be used to help reducing total quality costs. Besides, the mathematical model will be developed in an understandable method which requires non-complex model and optimization method. The assumptions will be established for a practical situation. The costs related to the control chart will be described in details so that the approximation of costs will be more precise. This thesis will decrease the weaknesses of complex specific economic model by providing more understandable and practical model.

The next step of the thesis is to collect related data for the economic mathematical model. Costs related to the control chart will be collected and analyzed about their relationship to the control chart. Appraisal and failure costs are the cost of quality involved in this research. Examples of Appraisal costs are sampling cost, false alarm cost, and true alarm cost. Examples of Failure costs are cost of failure product both occurred in internal and external. These quality costs will be analyzed and applied in the model to create total quality costs function. In order to make the model more effective, the case study data will be collected and studied. Also, the related procedures that affect costs and statistical quality criteria will be studied. The statistical quality constraints are type 1 error and type 2 error. These constraints will be considered in the model to retain the power of the control chart.

In developing the model, cycle time of the control chart will be established. The cycle time will include necessary points of action occurred during the process. The process failure mechanism of the assignable causes will be created as an assumption along with other necessary assumptions. Time of in control period, out of control period, and correcting period will be collected from the case study company. The costs and their causes will be formed in mathematic model. However, the objective of the thesis is to develop economic mathematic model that can be used in general so some procedure and statistical quality criteria will be established in order to acquire a realistic model.

In an improvement step, the collected data will be applied to the model and then the improvement will be executed. The developed model will be validated and the drawback of the model will be improved.

The result will be summarized in the final phase. Advantages, disadvantages, and limitations of the model will be notified. Sensitivity analysis will help to see how the model responds to different values of input parameters.

CHAPTER III

CASE STUDY

Many researchers have proposed economic model under their specific assumptions. However, Saniga and Shirland (1977) showed that very few economic models for the design of control charts have been implemented. The economic models are not widely used because the models are quite complex, and difficult to evaluate and optimize (Alexander et al., 1995).

This thesis aims to solve these problems by developing an understandable economic mathematical model that was generated from realistic assumptions based on case study processes and procedures.

Case study is very crucial to make a realistic economic mathematical model since it can give an important data to formulate quality costs such as sampling plan, probability of defective product, procedure, and penalty cost. Parameters of the developed model can be changed in order to fit various applications.

3.1 Current Problems of the Case Study

The main factors that needed to be concerned when designing X-bar chart are sample size and sampling frequency. Sample size and sampling frequency play an important role in the statistical performance and costs related to the control chart. For example, increasing sample size and sampling frequency is necessary when we want to increase the ability of detection. However, increasing sample size and sampling frequency results in an increase in cost of sampling, but cost of scrap and cost of rework can be reduced because we can detect the mean shift quicker which means that the causes of variation can be fixed early. On the other hand, decreasing sample size and sampling frequency results in a decrease in probability of detection causing higher cost of scrap and rework. However, the cost of sampling is reduced. At the present, the case study company designs the sampling plans of control charts by considering only statistical performance. The costs associated with the implementation of the design sampling plans are not considered.

3.2 Case Study Process Chart

The case study company produces specific parts in a motor for hard disk drive. The production processes consist of turning process, internal process quality assurance (IPQA), and outgoing quality assurance (OQA). The turning process is a process that turns an internal diameter (ID) of the product. This process is considered to be a long run production process which has a steady standard deviation (SD). The IPQA process is a quality control process which monitors the straightness of ID parameter in the production line using x-bar control chart. At the present, the company is using X-bar control chart to monitor the straightness of ID as a variable data in IPQA process. The OQA process is the final quality control process before delivering the product to the customers. The customers are using the variable sampling plan to test the incoming lot. In order to make the model more effective, the case study data will be collected and studied. Also, the related procedures that affect costs and statistical quality criteria will be studied.

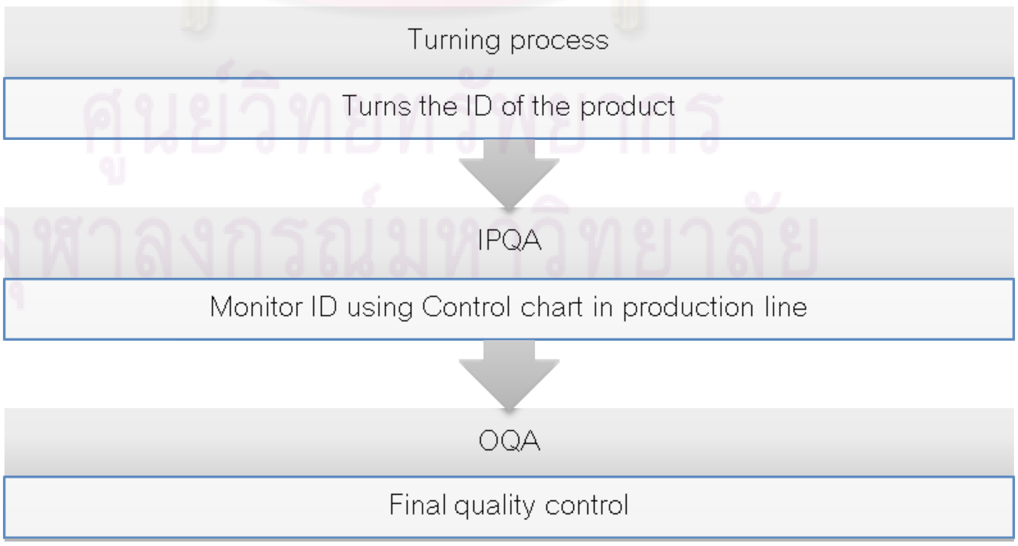

Figure 3.1: Case Study Process Chart

3.3 Turning Process

Turning process is considered to be a steady state production. The quality characteristic of measured value is normally distributed and the standard deviation is known. Details about the turning process will be described by 4M method as followed.

3.3.1 Man

There are the engineer and the technician who responsible for turning machines and also cooperate closely with the quality control staff. When the out-of-control signal is detected in the control chart noticed by the quality control staff, the technician will be called to investigate an assignable cause. Then, if there is an assignable cause, the engineer is required to repair the process.

3.3.2 Machine

There are several turning machines (MC) for each type of product. The machine duty is to turn the ID of the product which has to be controlled the quality in straightness. The turning process is considered to be a steady state with known variance (σ). The probability of defective product produced during in-control and out-of-control periods are γ_1 and γ_2 respectively. The machine needs to be stopped in order to be repaired after the out-of-control situation occurs. Reworking for defective product is also being done by these machines. The probability of defect which occurs in reworking process is assumed to be equal to the in-control period (γ_1).

3.3.3 Material

There are three types of products which required three types of turning machines. The products are M1, M2, and M3.

3.3.4 Method

The machine is automatically run 24 hours with steady production rate (P_r). However, the machine has to be stopped in order to be repaired after an assignable cause had occurred.

3.4 IPQA Process

The IPQA process uses only single upper control limit (UCL) in the control chart to monitor the straightness of the product ID in real time. The product ID straightness has to be controlled within the UCL in the control chart. There are also variable sampling plans to test the product which are IPQA variable sampling plan and IPQA after rework variable sampling plan. IPQA variable sampling plan is the testing process to test the hold product that produced while the process is out-of-control while IPQA after rework variable sampling plan is the testing process to test the product from rework process. Each of the variable sampling plans has each own statistical criteria that can be determined separately. Details about the IPQA process will be described by 4M method as followed.

3.4.1 Man

There are staffs who are responsible for picking up the samples from the production line. These staffs will only pick a sample and then deliver it to the testing staffs. The testing staffs are responsible for all testing activities including control chart testing and variable sampling plan testing.

3.4.2 Machine

There are several testing machines, which measure the straightness of the product ID. Three types of products use the same testing machine, but the machine can test only one piece of product at a time. Hence, these testing machines are considered to be one of the constraints in the model. Investing in the new testing machine will result in more capability of testing. Therefore, this topic will be considered in this thesis.

3.4.3 Material

In this case study, the testing is a non-destructive testing. Therefore, the tested product will be in the same conditions like other products that have not been tested.

3.4.4 Method

Staffs collect the sample with specific amount (n) at specific time (f).If there is an assignable cause detected in the control chart, the technician will be called to search for an assignable cause which takes T1 time. After that, if there is an affirmation of the occurrence of an assignable cause, the production process will be stopped and the engineer will take T2 time for repairing the machine to be back to an in-control state again. The production will be resumed as soon as the assignable cause is fixed. The produced lot during the out-of-control period (Z) will be held for further testing by IPQA variable sampling plan. Reworking process will be applied to the lot that fails to the test. The reworked lot will be tested again by IPQA after rework variable sampling plan to check whether the defect has been corrected or not. If the tested lot still fails, the fail lot will be turned to be a scrap. On the other hand, the lot will be transferred to OQA process if it passed the test.

3.4.5 IPQA Procedure

1. Staffs pick the samples n pieces from each production machine every 1/f hour.

2. Staffs test the ID straightness of those samples with testing machine.

3. If the samples passed the test, the production process is allowed to run continually with γ_1 probability of defect.

4. If the samples fail the test (the measured value is outside the control limits), the production process will be checked by the technician within T_1 time without any stoppage of the production process.

5. If there is no abnormality found by the checking process, it means that there is no assignable cause and the false alarm occurred so the production process is allowed to run continuously.

6. If there is an abnormality found by the checking process, true alarm occurred and there is an assignable cause in the production process. The products produced during an out-of-control period will have γ_2 probability of defect. The process is stopped for repairing by an engineer.

7. The production process is repaired by an engineer within T_2 time.

8. The production process is restarted and allowed to run with γ_1 probability of defect.

9. The defective product that produced while the process is out-of-control with γ_2 probability of defect will be held for further testing by the variable sampling plan.

10. If the testing by IPQA variable sampling plan passed, the hold lot is allowed to pass with 1-Y probability (accept lot). However, the products in this hold lot still have γ_2 probability of defect.

11. If the testing by IPQA variable sampling plan failed, the hold lot will be reworked with Y probability (reject lot).

12. Reworking on the hold lot. The reworked products will have γ_1 probability of defect.

13. The reworked lot will be tested again by IPQA after rework variable sampling plan.

14. If the testing by IPQA after rework variable sampling plan passed, the reworked lot is allowed to pass with (1-H) probability (accept lot). These products will have γ_1 probability of defect.

15. If the testing by IPQA after rework variable sampling plan fails, the reworked lot will be turned to be a scrap with H probability (reject lot).

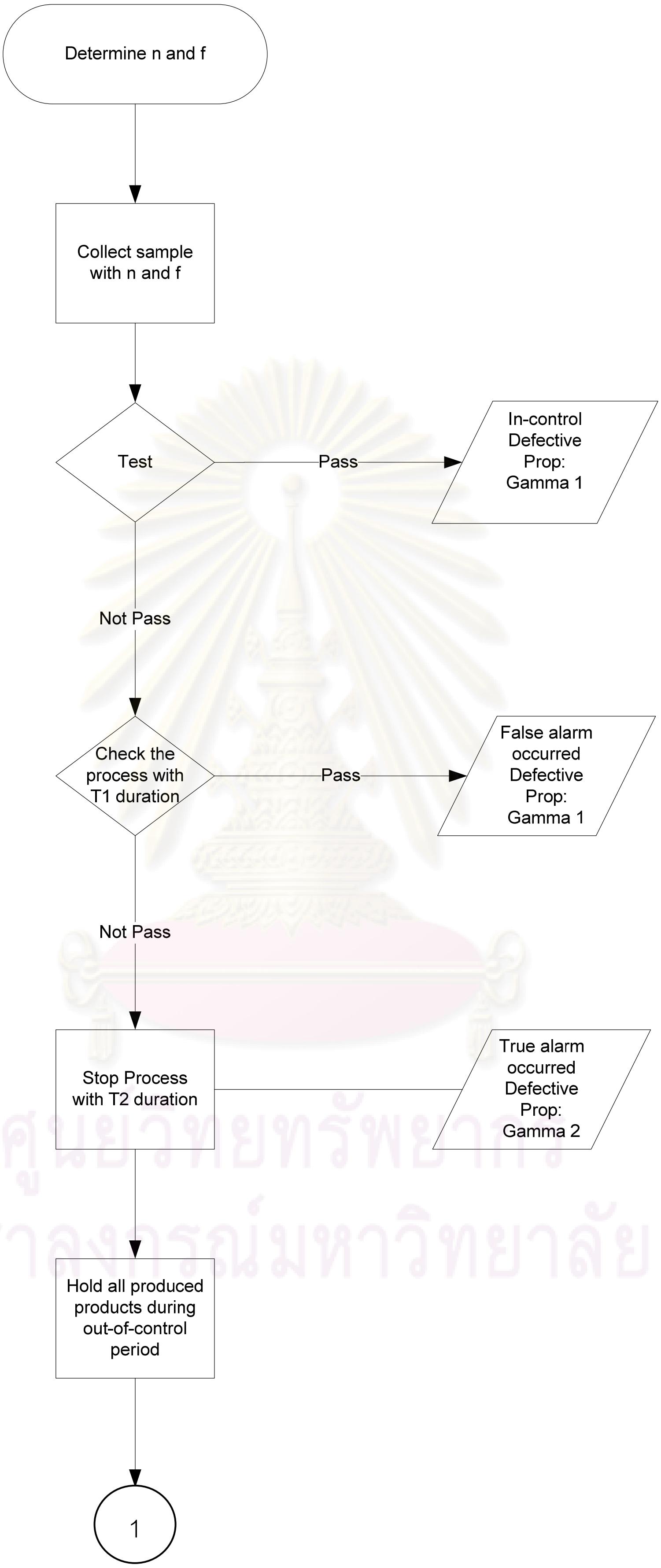

Figure 3.2: IPQA Process Chart

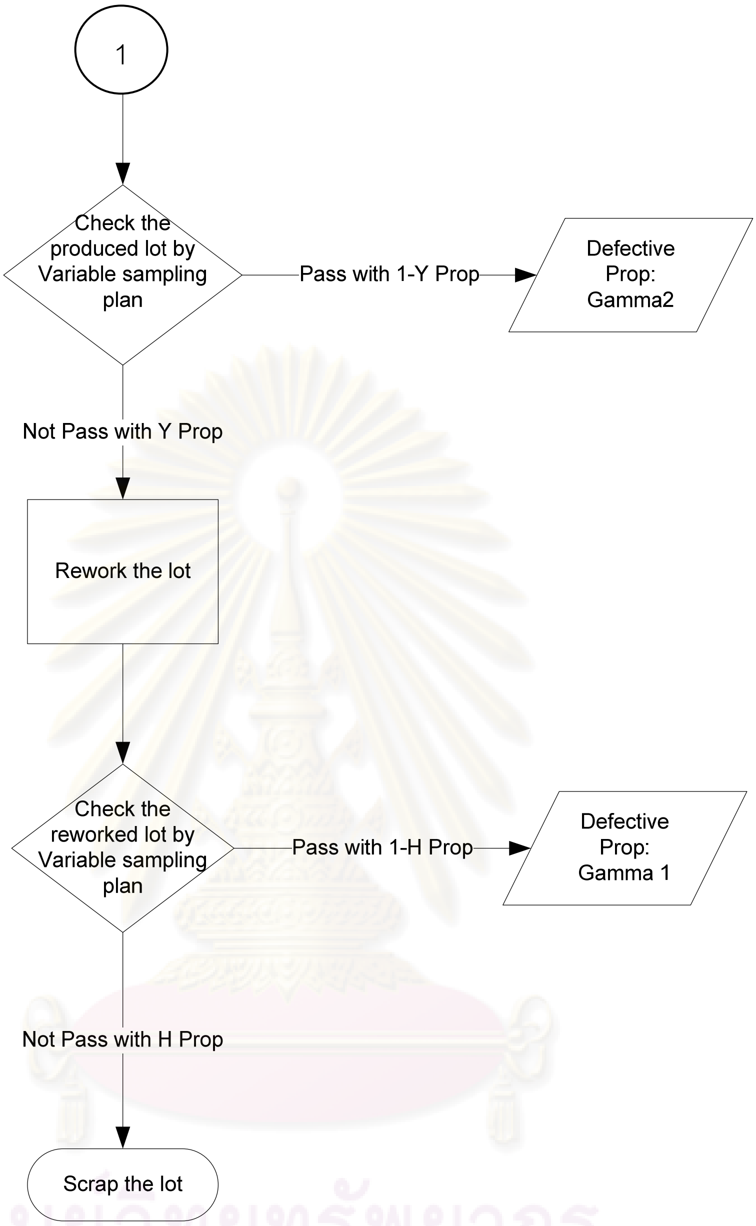

Figure 3.2: IPQA Process Chart (Cont.)

3.5 OQA Process

OQA is the final quality control process which uses variable sampling plan to test the lot before delivering to the customer. The first variable sampling plan in OQA process is OQA variable sampling plan. The second variable sampling plan in OQA process is the variable sampling plan for after rework process called OQA after rework variable sampling plan. Each of the OQA variable sampling plans has its own statistical criteria. Details about the OQA process will be described by 4M method as followed.

3.5.1 Man

Staffs will have responsibility to test the samples using variable sampling plan.

3.5.2 Machine

The testing machines used in OQA process are the same machines that used in IPQA process. The capacity of the testing machines has to be shared for both OQA and IPQA processes. There are no dedicated testing machines for each process.

3.5.3 Material

The testing is a non-destructive testing. The passed samples will be loaded with the other good products waiting for delivering to the customer.

3.5.4 Method

The product will be divided in lot amount O_{Size} waiting for testing with OQA variable sampling plan before delivering to the customer. If the tested lot fails, the fail lot will be reworked 100%. Then, the reworked lot will be tested again by OQA after rework variable sampling plan. If the lot fails again, the lot will be turned to be a scrap. Contrarily, the pass lot will be delivered to the customer as a lot.

3.5.5 OQA Procedure

1. Finished products from IPQA process with P probability of defect are divided into lots.

2. Each lot will be tested by OQA variable sampling plan before delivering to the customers.

3. If the testing by OQA variable sampling plan passed, the lot is allowed to pass with 1-P_0 probability (accept lot) while the products have P probability of defect.

4. If the testing by OQA variable sampling plan fails, the lot will be reworked with P0 probability (reject lot).

5. Reworking the whole lot. The reworked products will have γ_1 probability of defect.

6. The reworked lot will be tested again by OQA after rework variable sampling plan.

7. If the testing by OQA after rework variable sampling plan passed, the reworked lot is allowed to pass with 1-P_d probability (accept lot) while the products have γ_1 probability of defect.

8. If the testing by OQA after rework variable sampling plan fails, the lot will be turned to be a scrap with P_d probability (reject lot).

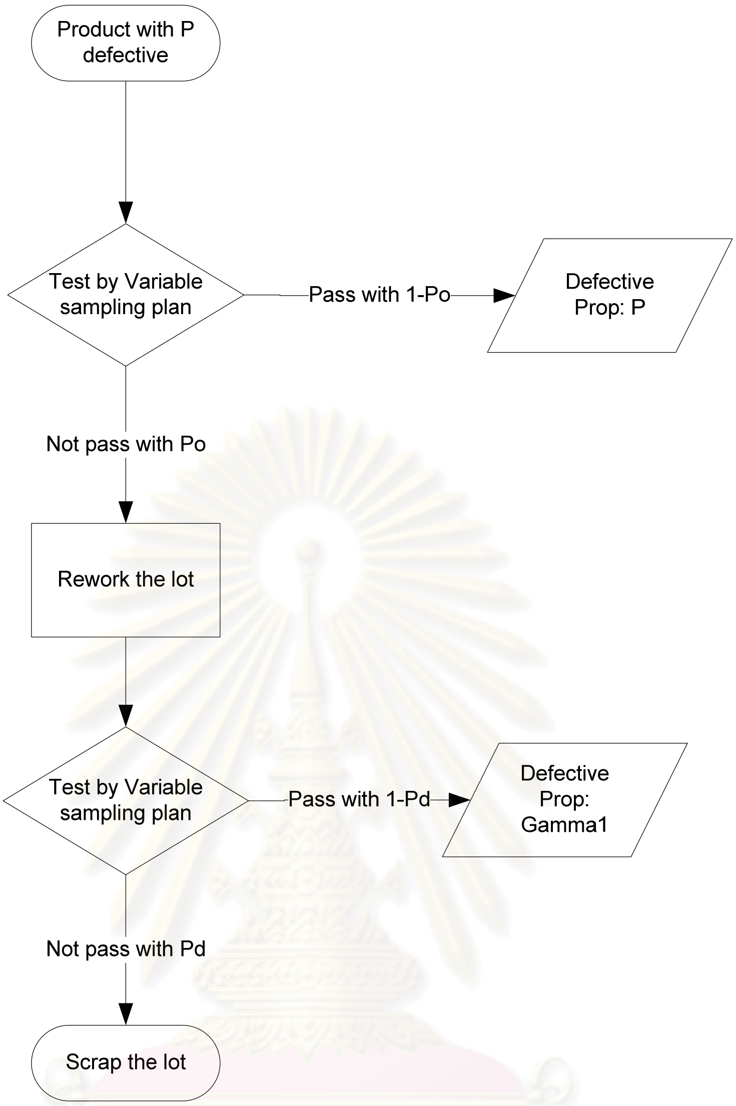

Figure 3.3: OQA Process Chart

3.6 Customer Process

Customer is one of the main factors that influences to the cost related to the control chart. This cost will be described further in term of external failure cost. The company has to deliver the lot of finished product to the customer as a routine process. The customer will check the product lot by using customer variable sampling plan. If the lot is rejected by the customer, the customer will claim the company and request for replacement. If the testing by customer variable sampling plan passes, the product will be allowed to enter to the customer manufacturing process. In the customer manufacturing process, if the product fails to assembly, the customer will also claim for the new product from the company. In addition, the customer will charge for a penalty from the company for the product that fails to assembly with agreed rate. The company will deliver the product for replacement every specific interval. The entire products that are delivered and rejected by the customer are considered to be worthless. The customer will not accept the old rejected products again.

3.6.1 Customer Procedure

1. The product lots are delivered to the customer with P’ probability of defective from OQA process.

2. Each of the product lot will be tested by customer variable sampling plan before delivering to the customer manufacturing process.

3. If the testing by customer variable sampling plan fails, the lot will be turned to be a scrap with P_{rej} probability (reject lot)

4. If the testing by customer variable sampling plan passed, the lot is allowed to pass with $1\text{-}P_{rej}$ probability (accept lot). The product will have P’ probability of defect.

5. The pass lot will be delivered to the customer manufacturing process to assemble the product with other components in their manufacturing process.

6. If the assembly fails with probability P_m, the product and other components will be turned to be scraps. The penalty will be charged from the case study company with determined rate per piece (P_{en}).

7. If the assembly passed with probability 1-P_m, the product is definitely accepted.

8. The case study company has to deliver the new product for replacement to the customer every specific interval (I_T).

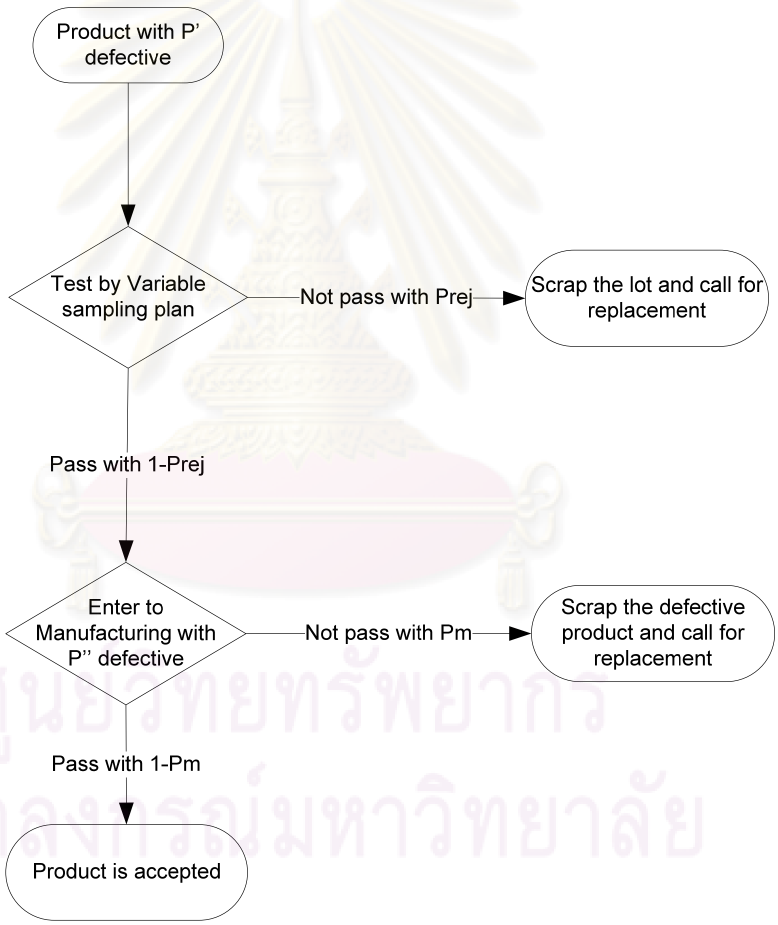

Figure 3.4: Customer Process Chart

CHAPTER IV

ECONOMIC MATHEMATIC MODEL

Due to the importance of knowing assumptions and theories involved with the model, the derivation of the proposed model starting from previous models that had been carried out is illustrated in this chapter. The scopes, assumptions, cycle time, and variables and parameters used in this thesis are also described.

4.1 Previous Models

The first work about economic design is done by Girshick and Rubin (Girshick and Rubin, 1952). They proposed economic design in an area of cost modeling of quality control system and also analyzed the process model that a machine produces items with a quality characteristic (x). The machine production can be divided into four stages. Stage 1 and 2 are production stages and the output quality characteristic is described by the probability density function f(x) where i = 1, 2. Stage 1 is an in-control stage which has a constant probability of a shift into stage 2 which is an out-of-control stage. Stages 3 and 4 are repair stages.

Duncan (1956) proposed the first paper dealing with fully economic model of the Shewhart control chart and incorporating formal optimization methodology to determine the control chart parameters. A production cycle consists of four periods which are in-control period, out-of-control period, time to take a sample and interpret the result, and time to find an assignable cause.

Although the Duncan's model is the model that integrates fully economic design of the control chart, but there are many unrealistic assumptions and limitations in the model as follows:

1. The Duncan's model does not concern about statistical properties since there is only an economic objective without statistical constraints in the model (Saniga, 1989).

2. The process is allowed to continue in operation during the search for an assignable cause which may be unrealistic in case that the process has to halt to search for an assignable cause (Montgomery, 1980).

3. The cost of eliminating an assignable cause is not charged against the net income for the period (Montgomery, 1980).

4. The cost elements are quite rough since it is hard to value the cost elements in practice (Montgomery, 1980).

5. The optimization method is complicated with no general solving method (Alexander et al., 1995).

These disadvantages and limitations of the Duncan's model persuade many of researchers to develop their models based on Duncan's model for specific usages and assumptions.

Knappenberger & Grandage (1969) also proposed economic design of the X-bar chart for multiple assignable causes. Duncan (1971) and Knappenberger & Grandage (1969) reported that a single assignable cause model that match the true multiple assignable cause system in certain important ways produces very good results (Montgomery, 1980).

Duncan integrated a penalty cost for operating out of control in his model but he did not illustrate how this cost element can be obtained and quantified (Alexander et al., 1995). Alexander et al. (1995) proposed the algorithm to estimate costs with Taguchi loss function. The Taguchi loss function gives a mean of considering the loss due to process variability caused by both chance and assignable causes.

Chung (1995) proposed an economic design with two control limits control chart. In the literature review, there are two out-of control states called state 1 and state 2 which assumed to be the independent assignable causes and exponentially distributed with each own means. State 1 represents an out-of-control state caused by a minor assignable cause while state 2 represents an out-of-control state that the process mean is shifted by a major assignable cause.

Although there are many model proposed, only few practitioners have implemented economic model to design their control charts. This may be because the models are complex, difficult to be evaluated, and optimized (Saniga, 1989) which suitable only for research but difficult to implement. For example, the cost elements are quite rough since it is hard to value the cost elements in practice (Montgomery, 1980) and the optimization method is complicated with no general solving method (Alexander et al., 1995). Moreover, there is no general solution to optimize the cost function. Some model needs manual computation to solve the model. Second, there are many cost elements occurred and need to be evaluated on order to minimize them but most of the presented models do not describe them thoroughly. Although costs do not have to be estimated with high precision (Montgomery, 1980), explicating cost is a very important factor for minimizing cost which is the major objective of the model.

4.2 The Proposed Model

The main objective of economic statistical design is not only minimize the expected cost per unit of time as in economic design, but also consider type 1error, type 2 error, and ATS as the statistical quality constraints (Saniga, 1989). This thesis will develop an understandable economic mathematical model for X-bar chart using both quality and cost criteria. Besides, the developed mathematical model is non-complex and easily solved by Solver function in Microsoft Excel and doesn’t require complex optimization method to solve for solution.

4.3 Model Assumptions

To develop the model, specific assumptions are necessary to be specified. For example, certain assumptions about the behavior of the production process are required to formulate an economic model for the design of a control chart (Montgomery, 1980). The model in this thesis will be developed based on the assumptions which gained from both previous research and the case study company.

4.3.1 Control Chart Assumptions

1. The model parameters are deterministic.

2. Assignable cause will occur seemingly at random

3. The X-bar control chart is used to monitor a process mean which is subject to an assignable cause (Chiu and Huang, 1996).

4. The distribution of quality characteristic of measured value is normal. Applying the central limit theorem, this result is still approximately correct for the X- bar chart even if the underlying distribution is nonnormal (Chiu and Huang, 1996).

5. Taking action only when a point exceeds the control limits (Duncan, 1956).

6. Rate of production is sufficiently high which we can neglect the possibility of a change in the process occurring during the taking of a sample (Duncan, 1956).

7. There will be no repeatable assignable cause occurring again before the current assignable cause is identified and removed.

8. The process is not self-correcting.

9. Measurements within a sample are independent.

10. The standard deviation is assumed to be stable even though the process mean may change. (Gibra, 1971)

11. The process is subject to the occurrence of a single assignable cause of variation which takes the form of a shift of known magnitude $\pm\delta\sigma$ in the process mean. (Gibra, 1971)

12. The production process is allowed to stop while repairing an assignable cause.

4.3.2 General Assumptions

1. Production rate equals export rate and customer production rate.

2. Probability of defective product after rework assumed to be equal to the probability of the product that is in-control (γ_1).

4.4 The Cycle Time

In order to develop the model, the cycle time and actions that occur in each stage have to be generated. Duncan (1956) proposed a production cycle which consists of four periods which are in-control period, out-of-control period, time to take a sample and interpret the result, and time to find an assignable cause. He also assumed that at the start, the control chart is maintained to detect a single assignable cause that occurs at random and results in a change in the process of known proportions. However, Duncan's model does not allow the process to be shut down when a search for the assignable cause is being carried out and it does not include the time and cost of repairing the process if it is found to be out-of-control (Chiu, 1975).

In this thesis, the cycle time is divided into four stages illustrated below.

1. The in-control stage in T_0 period. Also as Duncan's model, it will be presumed that the process begins in a state of control at the level indicated by the standard values. In this stage, the process is allowed to operate continuously with γ_1 probability of defect. The samples are collected by specified sample size and sampling frequency. False alarm can occur in this stage with α probability.

2. The delay detection stage in ATS period. The ATS is the mean time the control scheme takes to detect an out-of-signal condition from the time of occurrence of the assignable cause (Prabhu, Montgomery, and Runger, 1997). After the production process goes from an in-control stage to an out-of-control stage at the mean shift point, it takes ATS time to detect an assignable cause in the control chart. In this period, the defective products are produced with γ_2 probability.

3. The finding an assignable cause stage in T_1 period. The process is searched for an assignable cause in order to stop and repair if there is an assignable cause occurred. At this stage, the process is still allowed to run and the defective products are produced with γ_2 probability.

4. The repairing stage in T_2 period. The process is allowed to stop for repairing in T_2 period. After the repairing stage, the process is then resumed to run in an in-control stage again and a new cycle begins. However, T_2 value can be equal to zero to represent the continuous process.

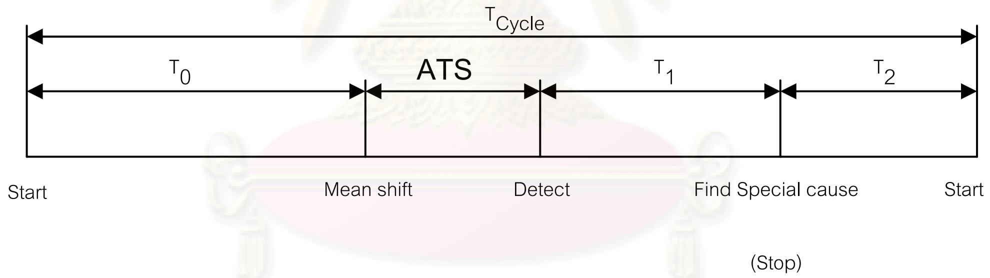

Figure 4.1: The Cycle Time

T_0 : Time period that the process is in an in-control stage

ATS : Average Time to Signal. ATS will represent the average time between the shift and its detection (Montgomery, 2005).

T_1 : Time period for finding an assignable cause

T_2 : Time period that the process is stopped for repairing

T_{Cycle} : The cycle time

$T_{Cycle} = T_0+ATS+T_1+T_2$

4.5 Variables and Parameters

J : Product set = {M1, M2, M3}

$\forall i \in J$

4.5.1 Variables

n_i : Sample size Piece/Time

f_i : Sampling frequency Times/Hour

4.5.2 Primary Parameters

4.5.2.1 Control Chart Related Parameters

α_i : Probability of type I error

β_i : Probability of type II error

$$\beta_i = \Phi(L - k\sqrt{n_i}) - \Phi(-L - k\sqrt{n_i}),$$

where L = 3 and k = 2 (Montgomery, 2005)

γ_{1i} : Probability of defect occurs when the process is in-control

γ_{2i} : Probability of defect occurs when the process is out-of-control

T_{0i} : Time period that the process is in an in-control stage Hour/Time

ATS_i : Average Time to Signal Hour/Time

$$ATS_i = \left(\frac{1}{1-\beta_i} \times \frac{1}{f_i}\right)$$

T_{1i} : Time period for finding an assignable cause Hour/Time

T_{2i}	: Time period that the process is stopped for repairing	Hour/Time
T_{Cyclei}	: The cycle time	Hour/Time
	$T_{Cycle} = T_0 + ATS + T_1 + T_2$	
$P_{r\,i}$	: Production rate for each product	Piece/hour
M_i	: Testing machine operating rate	Baht/Hour/Machine
R_i	: Rework rate	Baht/Piece
P_{ni}	: Product profit	Baht/Piece
C_{Ti}	: Cost of transportation to the customer	Baht/Time
O_{Ti}	: Transportation lot size	Piece/Time
I_{Ti}	: Transportation interval	Hour
$C_{P\,IPQAi}$	: Product cost in IPQA process	Baht/Piece
$C_{P\,OQAi}$	: Product cost in OQA process	Baht/Piece
$C_{P\,Cusi}$	: Product cost when delivered to the customer	Baht/Piece
P_{eni}	: Customer penalty rate	Bath/Piece
L_M	: Staff labor rate	Baht/Hour/Person
T_{Pick}	: Time for collecting samples from each machine	Hour person/Time
T_{Testi}	: Time for testing a sample per piece	Hour person/Piece
K_i	: Testing machine capacity	Piece/Machine/Hour

$$K_i = \frac{1}{T_{testi}}$$

D_i : Cost of destructed product Baht/Piece

S_1 : Engineer labor rate Baht/Hour

S_2 : Technician labor rate Baht/Hour

MC_i : Number of production machine Machine

O_{Sizei} : OQA variable sampling plan lot size Piece/Time

O_{Custi} : Customer variable sampling plan lot size Piece/Time

F_i : Number of testing machine in the present Machine

G_i : Number of testing machine allowed Machine

W_i : Working hour per year Hour/Year

4.5.2.2 New Testing Machine Investment

PV : Testing machine in present value Baht

a : Testing machine operating life Year

r : Interest rate in the business Percent/Year

AV : Testing machine in annual value Baht/Year

4.5.2.3 Variable Sampling Plan

USL_i : Upper specification limit for every variable sampling plan

σ_i : Standard deviation for each product

4.5.2.4 IPQA Variable Sampling Plan

AQL_{IPQAi} : Acceptable quality level in IPQA variable sampling plan

$LTPD_{IPQAi}$: Lot tolerance percent defective in IPQA variable sampling plan

$Alpha_{IPQAi}$: Type I error in IPQA variable sampling plan

$Beta_{IPQAi}$: Type II error in IPQA variable sampling plan

$n_{IPQA\ i}$: Sample size in IPQA variable sampling plan

K_{IPQAi} : Critical distance in IPQA variable sampling plan

Y_i : Probability of rejecting the lot in IPQA variable sampling plan

$1\text{-}Y_i$: Probability of accepting the lot in IPQA variable sampling plan

4.5.2.5 IPQA After Rework (AR) Variable Sampling Plan

$AQL_{IPQA\ ARi}$: Acceptable quality level in IPQA AR variable sampling plan

$LTPD_{IPQA\ ARi}$: Lot tolerance percent defective in IPQA AR variable sampling plan

$Alpha_{IPQA\ ARi}$: Type I error in IPQA AR variable sampling plan

$Beta_{IPQA\ ARi}$: Type II error in IPQA AR variable sampling plan

$n_{IPQA\ ARi}$: Sample size in IPQA AR variable sampling plan

$K_{IPQA\ ARi}$: Critical distance in IPQA AR variable sampling plan

H_i : Probability of rejecting the lot in IPQA AR variable sampling plan

$1\text{-}H_i$: Probability of accepting the lot in IPQA AR variable sampling plan

4.5.2.6 OQA Variable Sampling Plan

AQL_{OQAi} : Acceptable quality level in OQA variable sampling plan

$LTPD_{OQAi}$: Lot tolerance percent defective in OQA variable sampling plan

$Alpha_{OQAi}$: Type I error in OQA variable sampling plan

$Beta_{OQAi}$: Type II error in OQA variable sampling plan

n_{OQAi} : Sample size in OQA variable sampling plan

K_{OQAi} : Critical distance in OQA variable sampling plan

P_{0i} : Probability of rejecting the lot in OQA variable sampling plan

$1-P_{0i}$: Probability of accepting the lot in OQA variable sampling plan

4.5.2.7 OQA After Rework (AR) Variable Sampling Plan

$AQL_{OQA\ ARi}$: Acceptable quality level in OQA AR variable sampling plan

$LTPD_{OQA\ ARi}$: Lot tolerance percent defective in OQA AR variable sampling plan

$Alpha_{OQA\ ARi}$: Type I error in OQA AR variable sampling plan

$Beta_{OQA\ ARi}$: Type II error in OQA AR variable sampling plan

$n_{OQA\ ARi}$: Sample size in OQA AR variable sampling plan

$K_{OQA\ ARi}$: Critical distance in OQA AR variable sampling plan

P_{di} : Probability of rejecting the lot in OQA AR variable sampling plan

$1-P_{di}$: Probability of accepting the lot in OQA AR variable sampling plan

4.5.2.8 Customer Variable Sampling Plan

AQL_{CUSi} : Acceptable quality level in customer variable sampling plan

$LTPD_{CUSii}$: Lot tolerance percent defective in customer variable sampling plan

$Alpha_{CUSi}$: Type I error in customer variable sampling plan

$Beta_{CUSi}$: Type II error in customer variable sampling plan

n_{CUSi} : Sample size in customer variable sampling plan

K_{CUSi} : Critical distance in customer variable sampling plan

P_{reji} : Probability of rejecting the lot in customer variable sampling plan

$1\text{-}P_{reji}$: Probability of accepting the lot in customer variable sampling plan

4.5.2.9 Customer Manufacturing

P_{mi} : Probability of defective found in the customer manufacturing process

$1\text{-}P_{mi}$: Probability of defective not found in the customer manufacturing process

4.5.3 Secondary Variables

P_i : Percentage of defective product produced from IPQA process

$$P_i = \frac{[(ATS_i + T_{1i}) \times (\gamma_{2i} \times (1 - Y_i) + Y_i \times \gamma_{1i} \times (1 - H_i))] + [T_{0i} \times \gamma_{1i}]}{T_{Cycle\,i} - T_{2i}}$$

The percentage of defective product produced from IPQA process can be obtained from three sources which are in-control stage, out-of-control stage, and after rework process. The defective product will occur with γ_1 probability during T_1 time in an in-control period and γ_{2i} probability during ATS $+T_1$ time in an out-of-control period. The defective product produced during ATS+T_1 period will pass from IPQA when it passes the test by variable sampling plan with 1-Y probability. The product after rework will have γ_1 probability of defect as in an in-control stage. The defective product occurs during reworking process will pass IPQA after rework sampling plan with 1-H probability. (See Figure 4.2: IPQA Process Chart)

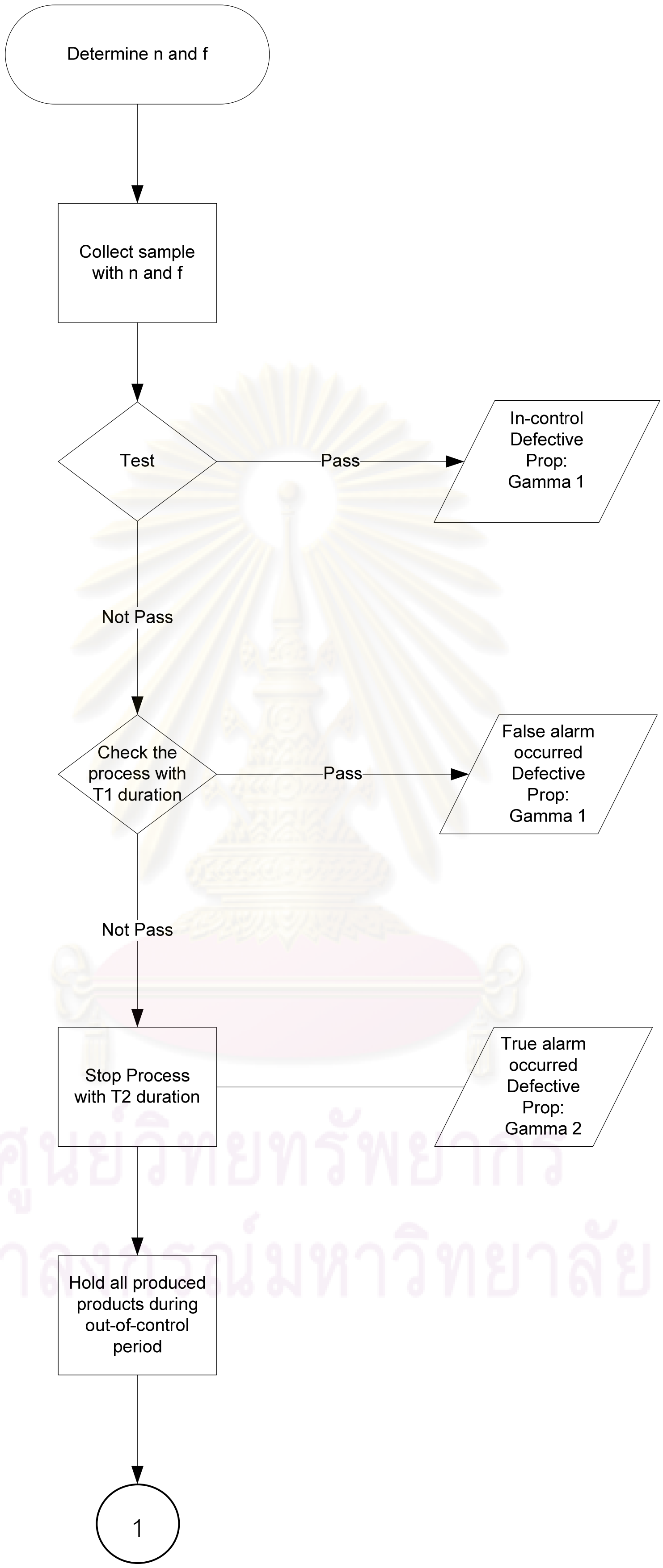

Figure 4.2: IPQA Process Chart

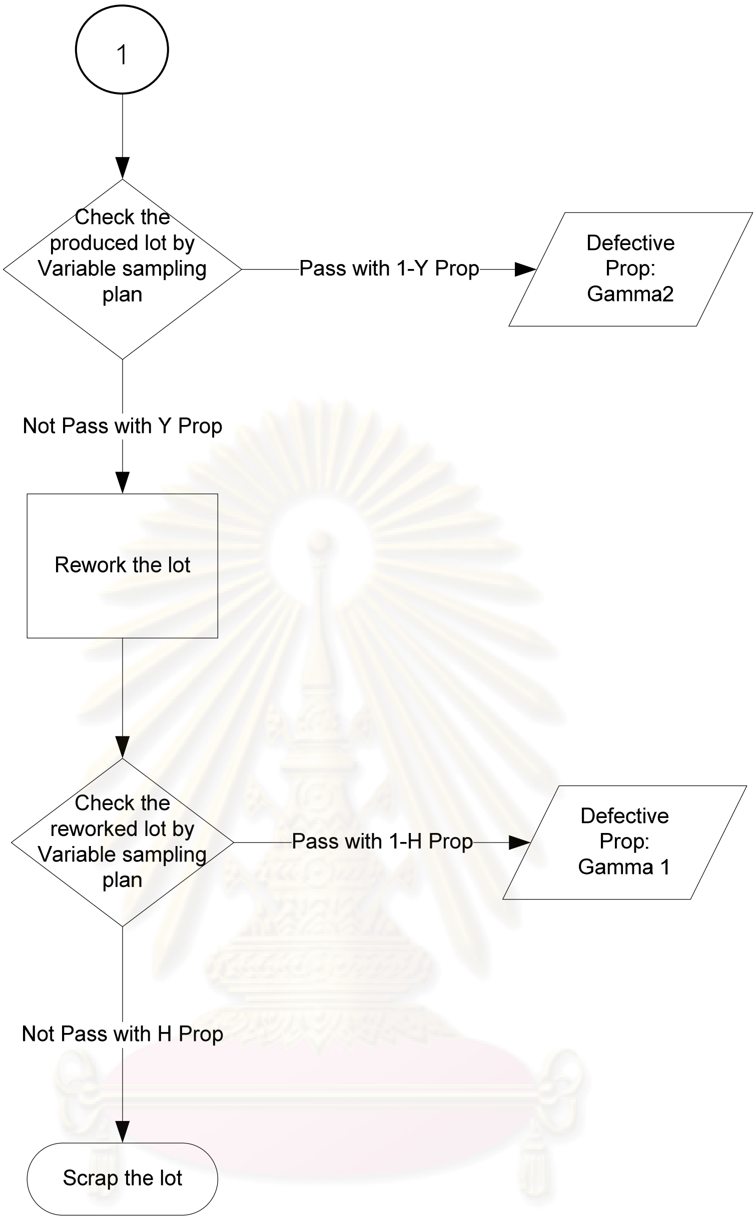

Figure 4.2: IPQA Process Chart (Cont.)

Z_i : Amount of product that has to be held for checking Piece/Time

$$Z_i = \Pr_i \times (ATS_i + T_{1i})$$

The amount of product that will be held is the product produced during the process is out-of-control during ATS+T_1 time.

P'_i : Percentage of defective product produced from OQA process

$$P'_i = P_i \times (1 - P_{0i}) + P_0 \times \gamma_{1i} \times (1 - P_{di})$$

The percentage of defective product produced from OQA process can be obtained from two sources which IPQA process and after rework process. The defective product that passes OQA variable sampling plan with 1-P_0 probability will have P defective as it passed from IPQA process. The product after rework will have γ_1 probability of defect. The defective product occurs during reworking will pass OQA after rework sampling plan with 1-P_d probability. (See figure 4.3: OQA Process Chart)

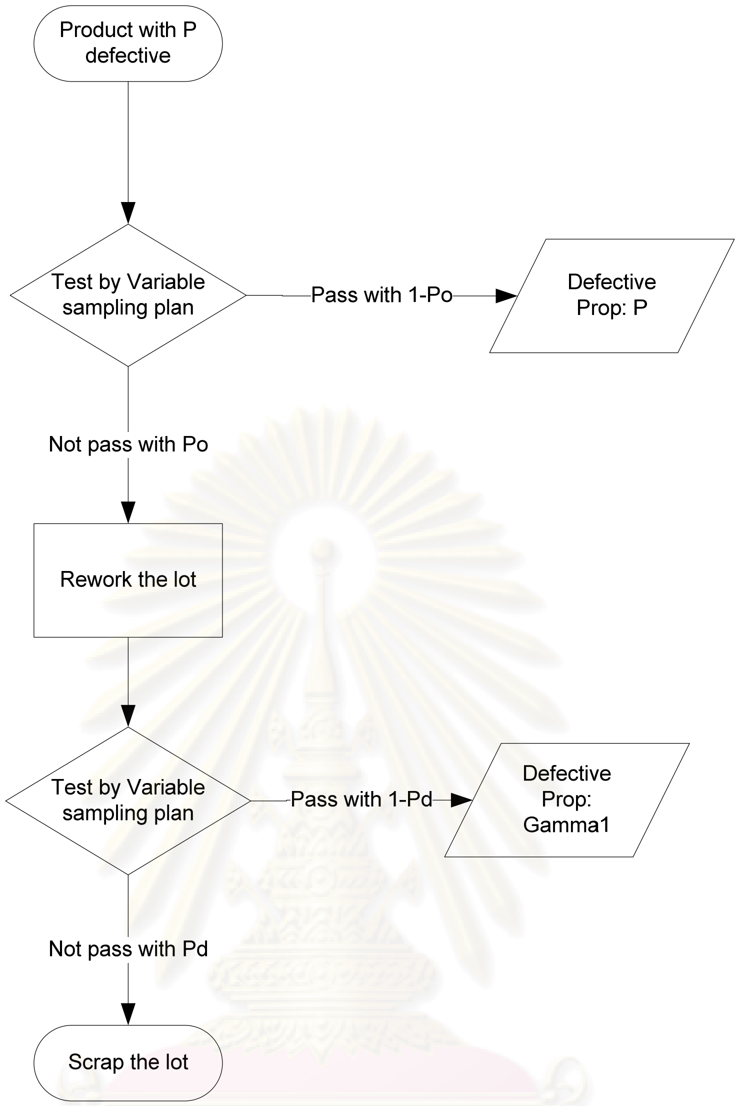

Figure 4.3: OQA Process Chart

ATU : All testing machine used in the process Machine

$$\text{ATU} = \sum_{i=1}^{m}\left(\frac{n_i \times f_i}{K_i}\right) + \sum_{i=1}^{m}\left(\frac{n_{OQAi} \times \Pr_i}{K_i \times O_{Sizei}}\right) + \sum_{i=1}^{m}\left(\frac{n_{IPQAi}}{K_i \times T_{Cyclei}}\right) + \sum_{i=1}^{m}\left(\frac{Y_i \times n_{IPQA\,ARi}}{K_i \times T_{Cyclei}}\right) + \sum_{i=1}^{m}\frac{(P_{oi} \times n_{OQA\,ARi} \times \Pr_i)}{O_{Sizei} \times K_i}$$

ATU is a number of testing machine used in all processes which consist of control chart sampling, OQA variable sampling plan, IPQA variable sampling plan, IPQA after rework variable sampling plan, and OQA after rework variable sampling plan. This parameter will be used as a constraint due to the fact that ATU has to be less than or equal to the number of machines allowed (G).

4.6 Economic Model Criteria

Girshick and Rubin (Girshick and Rubin, 1952) set the economic criterion to maximize expected net income from the process. Duncan (1956) also proposed design criterion which is to maximize the expected net income per unit of time. Nevertheless, since net income considered being independent of the variables, the criteria of optimum design will therefore be minimum cost (Duncan, 1956).Thus, maximum net income will be equal to the minimum cost since the net income is a constant value. Economic model are generally formulated using a total cost per unit time function (Montgomery, 1980).

This thesis will develop the economic mathematical model that consists of costs related to the implementation of control chart. Then, the model will be optimized by finding the appropriate sample size and sampling frequency that minimize total costs.

4.7 Cost Occurred from Each Stage in the Cycle Time

In order to make a clear picture of costs related to the control chart, it is necessary to describe costs that occur from each stage defined earlier. The cost elements will be generated from each stage both directly and indirectly.

1. Stage 1

In this stage, costs occur from sampling activity and defective product with γ_1 probability of defect.

2. Stage 2

Costs occur from sampling activity and defective product with γ_2 probability of defect.

3. Stage 3

Costs occur from sampling activity, finding assignable cost, and defective product with γ_2 probability of defect.

4. Stage 4

Costs occur from opportunity lost which arising from ceasing the production process and repairing an assignable cost.

CHAPTER V

ECONOMIC MATHEMATICAL MODEL'S COST ELEMENTS & OPTIMIZATION

In this chapter, the cost elements in the economic mathematical model are described about their relationship with the control chart. The cost formula, objective function, constraints, optimization technique, input parameters, and scenarios studied are also illustrated.

5.1 Economic Mathematical Model's Cost Elements

Duncan's cost model includes the cost of sampling and inspection, the cost of defective products, the cost of false alarm, the cost of searching an assignable cause, and the cost of process correction (Liu, Chou, and Chen, 2002). However, the cost elements of Duncan's are quite rough since it is hard to value the cost elements in practice (Montgomery, 1980).

In this thesis, all cost elements will be classified by quality cost concept to make the model more understandable. The model will use cost per hour unit which is calculated from the cost per cycle time divided by the cycle time.

Costs in the model consist of appraisal costs and failure costs. Prevention costs are excluded from the model because they are not depended on sample size and sampling frequency. Improvement from prevention investment will lead to the next scenario of the model by changing parameters such as in-control time period. For example, if the turning machine is improved by investing the new high precision turning machine, the in-control time period will be changed in the model and the sampling plan will also be changed because of the new value of parameter (T_0).

Appraisal costs are costs occurred from routine sampling activities. The appraisal costs consist of control chart sampling cost, false alarm cost, and OQA sampling cost. Control chart sampling cost is a cost that occurs during the usage of control chart in

IPQA process. False alarm cost is a cost of investigating process when there is an alarm in the control chart even though the process is still in-control. OQA sampling cost occurs from sampling activities before delivering finished products to the customer.

Failure costs consist of internal and external failure costs. Internal failure costs occur from defective products and their sequences of creating loss in the company process. Internal failure costs consist of retest cost, defect cost, cease cost, and true alarm cost. Retest cost occurs from retesting activities. The retest cost is considered to be one of the failure costs since retesting can only occur from the defective products that have to be reworked. Defect cost is a cost that occurs from defective products and activities needed for repairing them. Cease cost is a cost that occurs from an opportunity lost due to the stoppage of production process needed for repairing the process. True alarm is a cost that occurs from activities needed for repairing an assignable cause in the production process.

External failure costs are costs that occur after the products are delivered to the customer already. The external failure costs consist of transportation cost and replacement cost. Replacement cost is a cost of replacing the defective products with the new products and also the penalty cost to the customer. Transportation cost is a cost of transporting new products to replace the rejected defective products.

These costs elements and their formula will be described in details in the Figure 5.1: Total Cost Elements.

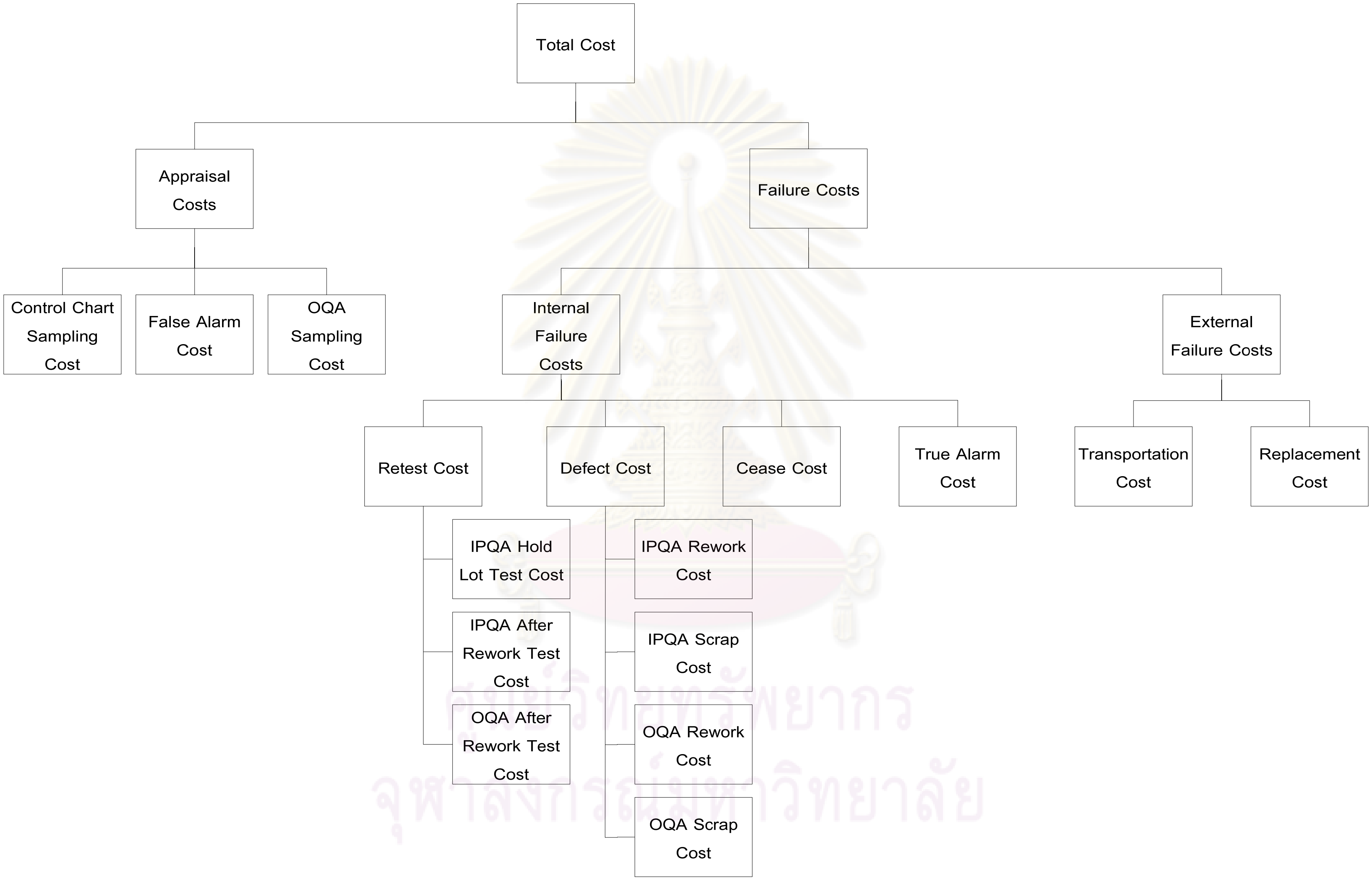

Figure 5.1: Total Cost Elements

5.2 Appraisal Costs

Appraisal costs are costs that occur from testing activities in order to determine whether the products conform to the specification or not. These costs occur by routine testing procedure which do not caused from defect products or processes. The company uses the control chart to monitor the straightness of ID of the product as a process control tool in IPQA process while variable sampling plan testing is used to test the product lot in OQA process. Therefore, there will be control chart sampling cost and false alarm cost occur in control chart activity and OQA sampling cost occurs in OQA variable sampling activity.

5.2.1 Control Chart Sampling Cost

Control chart sampling cost is arisen from sampling and testing activities in IPQA process which directly related to the use of the control chart. It is considered to be an appraisal cost because it is a routine testing to monitor the process. Duncan (1956) assumed that the time to take and inspect a sample and to compute is proportional to the sample size. In this model, the sampling cost is proportional to both sample size and sampling frequency. This sampling cost occurs only when the sampling activity occurred with f frequency.

Control chart sampling cost consists of labor cost, testing machine cost, and material cost.

5.2.1.1 Labor Cost

Labor cost is a cost of labor that used to pick the samples from the production line and test those samples. There are two activities in the control chart sampling which are picking and testing the samples. In picking sample activity, it is assumed that the staff uses the same amount of time (T_{Pick}) to pick the sample for all sample size. However, testing time (T_{Test}) is varied with the size of sample because the staff has to test the sample one by one. The more sample size has to be tested, the more testing time spent.

Labor cost = Labor rate x Labor used

$$Labor\cos t = L_m \times \sum_{i=1}^{m} (T_{Pick} \times f_i \times + T_{Testi} \times n_i \times f_i)$$

5.2.1.2 Machine Cost

Machine cost is a machine operating cost such as electricity cost and maintenance cost. This cost is depending on the sample size and sampling frequency.

Machine cost = Machine operating rate x Machine used

$$Machine\ \cos t = M \times \sum_{i=1}^{m} (\frac{n_i \times f_i}{K_i})$$

Since K_i is a testing machine capacity and n_i x f_i is a number of pieces needed to be tested in one hour so $\sum_{i=1}^{m} (\frac{n_i \times f_i}{K_i})$ will represent the number of machine used in this testing activity.

5.2.1.3 Material Cost

Material cost is a cost of product destroyed in testing activity. However, the testing in this case study is a non-destructive testing so the material cost is none. This cost is also depending on the sample size and sampling frequency.

Material cost = Cost of product destroyed x Number of product destroyed

$$Material\cos t = \sum_{i=1}^{m} (D_i \times n_i \times f_i)$$

5.2.2 False Alarm Cost

False alarm cost is a cost of investigating an assignable cause which does not really exist. The false alarm will occur with α probability only during an in-control period of the process. When the in-control ARL (ARL0) increases, it indicates there is a smaller probability for the occurrence of false alarms (Torng, Lee, and Liao, 2009). However, this cost cannot be denied since there is a probability of false alarm occurrence. The probability of false alarm is depended on the control limit coefficient (L) which set to be 3 in order to remain the standard statistical property. The company uses the technician to investigate an assignable cause. Thus, the cost of investigation will come from technician labor rate (S_2) multiply by time period used to find an assignable cause (T_1). The in-control time is T_0, thus the fraction of time that this cost can occur in one cycle is $\frac{T_{oi}}{T_{Cyclei}}$.

False alarm cost = Cost of investigation per hour x investigation time x probability of false alarm x time that false alarm can occur (in-control time)

$$Falsealarm\mathrm{cost} = \sum_{i=1}^{m}(S_2 \times T_{1i} \times \alpha_i \times f_i \times \frac{T_{oi}}{T_{Cyclei}})$$

5.2.3 OQA Sampling Cost

OQA sampling cost is a cost that occurs in a routine OQA variable sampling plan process which is the first variable sampling plan in OQA process. This cost is considered to be an appraisal cost because it occurs from a routine testing activity that used to test the product lot before delivering to the customer. Since the production rate is P_r and the OQA sampling cost occurs in every lot amount (O_{Size}), this cost will occur with $\frac{\mathrm{Pr}}{O_{Size}}$ frequency in order to test every produced lot.

OQA sampling cost consists of labor cost, testing machine cost, and material cost.

5.2.3.1 Labor Cost

Labor cost is a cost of labor that used to test the sample in OQA process with variable sampling plan. Staff will collect the sample with designed amount (n_{OQA}) from every product lot amount O_{size} before delivering to the customer.

Labor cost = Labor rate x Labor used

$$Labor\,\mathrm{cost} = L_m \times \sum_{i=1}^{m} \frac{(T_{Testi} \times n_{OQAi} \times \mathrm{Pr}_i)}{O_{Sizei}}$$

5.2.3.2 Machine Cost

Machine cost is a machine operating cost such as electricity cost and maintenance cost.

Machine cost = Machine operation rate x Machine used

$$Machine\,\mathrm{cost} = M \times \sum_{i=1}^{m} \left(\frac{n_{OQAi} \times \mathrm{Pr}_i}{K_i \times O_{Sizei}}\right)$$

Since K_i is a testing machine capacity and (n_i x Pr_i)/ O_{size} is a number of sample needed to be tested in one hour, thus $\sum_{i=1}^{m} \left(\frac{n_{OQAi} \times \mathrm{Pr}_i}{K_i \times O_{Sizei}}\right)$ will represent the number of machine used in this testing activity.

5.2.3.3 Material Cost

Material cost is a cost of product destroyed in testing activity. However, the testing in this case study is a non-destructive testing, thus the material cost is none.

Material cost = Cost of product destroyed x Number of product destroyed

$$Material\,\mathrm{cost} = D_i \times \sum_{i=1}^{m} \frac{(n_{OQAi} \times \mathrm{Pr}_i)}{O_{Sizei}}$$

5.3 Failure Costs

Failure costs are costs that occur from defective products and processes both directly and indirectly. In control chart implication, there are many costs occurred as a sequence of defective products and processes such as retesting and repairing cost. Whereas Duncan (1956) applied a penalty cost for operating out of control, he did not show how this cost can be obtained or quantified (Alexander et al., 1995). This thesis will illustrate the failure costs and their relation to the control chart using realistic case study.

Failure cost consists of internal failure costs and external failure costs. Internal failure costs occur within the company before delivering products to the customer while external failure costs occur after delivering products to the customer.

5.4 Internal Failure Costs

Internal failure costs occur from defective products and their consequences which consist of true alarm cost, cease cost, retest cost, and defect cost.

5.4.1 True Alarm Cost

Duncan (1956) assumed that the cost of repair and the cost of bringing the process back to a state of control subsequent to the discovery of the assignable cause will not be charged. However, this assumption of Duncan is not realistic for the real case because there must be an activity needed to bring the process back to the normal stage and its cost is undeniable.

True alarm cost is the cost of investigating and repairing the out-of-control stage process to be back into in-control stage again. The investigation will be done by the technician while repairing will be done by the engineer. In one cycle time, the true alarm can occur only one time because an assignable cause is allowed to occur once for each cycle.

True alarm cost = Technician labor rate x Technician time used + Engineer labor rate x Engineer time used

$$True\,alarm\,\mathrm{cost} = \sum_{i=1}^{m} \frac{(S_2 \times T_{1i} + S_1 \times T_{2i})}{T_{Cyclei}}$$

5.4.2 Cease Cost

Duncan (1956) assumed that the process is not shut down while the search for the assignable cause is in progress. However, there are many production processes that needed to be shut down in order to repair the assignable cause. Knappenberger & Grandage (1969) made an assumption where the process is stopped while out-of-control signals are being investigated. In this thesis, the model allows the process to be shut down for T_2 period to repair the assignable cause. Cease cost is a cost occurred from an opportunity loss since the production process has to be stopped for repairing an assignable cause. There will be cease cost once in a cycle time because the process is allowed to stop once in a cycle time.

Cost of cease = Production rate x Profit per product x Time that process stopped

$$Cease\,\mathrm{cost} = \sum_{i=1}^{m} \left(\frac{T_{2i} \times \mathrm{Pr}_i \times Pn_i}{T_{Cyclei}}\right)$$

5.4.3 Retest Cost

Retest cost is a cost that occurs by retesting suspicious product lots that might have defective product so it is considered to be a failure cost. The testing activity in this process is not a routine testing, but it occurs because the previous testing indicated that there is a defective product.

5.4.3.1 IPQA Hold Lot Test Cost

After products produced during the out-of-control process are held, the testing is needed to determine whether the lot should be reworked or not. IPQA hold lot test cost is a cost that occurs from testing samples from the hold lot (Z) that produced during an out-of-control period. There will be IPQA hold lot test cost once in a cycle time.

IPQA hold lot test cost consists of labor cost, testing machine cost, and material cost.

A. Labor Cost

Labor cost is a cost of labor that used to test the sample in IPQA hold lot with variable sampling plan. Staff will collect the sample with designed amount (n_{IPQA}) from the hold lot.

Labor cost = Labor rate x Labor used

$$Labor\ \mathrm{cost} = L_m \times \sum_{i=1}^{m} \left(\frac{T_{Testi} \times n_{IPQAi}}{T_{Cyclei}}\right)$$

B. Machine Cost

Machine cost is a machine operating cost such as electricity cost and maintenance cost.

Machine cost = Machine operation rate x Machine used

$$Machine\ \mathrm{cost} = M \times \sum_{i=1}^{m} \left(\frac{n_{IPQAi}}{K_i \times T_{Cyclei}}\right)$$

Since K_i is a testing machine capacity and n_i / T_{Cycle} is a number of sample needed to be tested in one hour so $\sum_{i=1}^{m} \left(\frac{n_{IPQAi}}{K_i \times T_{Cyclei}}\right)$ will represent the number of machine used in this testing activity.

C. Material Cost

Material cost is a cost of products destroyed in testing activity. However, the testing in this case study is a non-destructive testing so the material cost is none.

Material cost = Cost of product destroyed x Number of product destroyed

$$Material\,\text{cost} = \sum_{i=1}^{m} \left(\frac{D_i \times n_{IPQAi}}{T_{Cyclei}}\right)$$

5.4.3.2 IPQA after Rework Test Cost

After the hold lot fails to pass variable sampling plan testing, the hold lot will be reworked with Y probability. Then, the reworked lot will be tested by IPQA after rework variable sampling plan to check the rework process quality. IPQA after rework test cost will occur with Y probability once in a cycle time.

IPQA after rework test cost consists of labor cost, testing machine cost, and material cost.

A. Labor Cost

Labor cost is a cost of labor that used to test the sample in IPQA after rework hold lot with variable sampling plan. Staff will collect the sample with designed amount ($n_{IPQA\ AR}$) from the hold lot.

Labor cost = Labor rate x Labor used

$$Labor\,\text{cost} = L_m \times \sum_{i=1}^{m} \left(\frac{Y_i \times T_{Testi} \times n_{IPQAARi}}{T_{Cyclei}}\right)$$

B. Machine Cost

Machine cost is a machine operating cost such as electricity cost and maintenance cost.

Machine cost = Machine operation rate x Machine used

$$Machine\ \cos t = M \times \sum_{i=1}^{m} \left(\frac{Y_i \times n_{IPQA\ ARi}}{K_i \times T_{Cyclei}}\right)$$

Since K_i is a testing machine capacity and $(Y_i \times n_i) / T_{Cycle}$ is a number of sample needed to be tested in one hour so $\sum_{i=1}^{m} \left(\frac{Y_i \times n_{IPQA\ ARi}}{K_i \times T_{Cyclei}}\right)$ will represent the number of machine used in this testing activity.

C. Material Cost

Material cost is a cost of products that destroyed in testing activity. However, the testing in this case study is a non-destructive testing so the material cost is none.

Material cost = Cost of product destroyed x Number of product destroyed

$$Material\ \cos t = \sum_{i=1}^{m} \left(\frac{Y_i \times D_i \times n_{IPQA\ ARi}}{T_{Cyclei}}\right)$$

5.4.3.3 OQA after Rework Test Cost

After the product lot fails to pass OQA variable sampling testing, the lot will be reworked with P_0 probability. Then, the reworked lot will be tested by OQA after rework variable sampling plan to check the rework process quality.

OQA after rework test cost is a cost that occurs when there is a testing of the sample from the reworked lot in OQA process. OQA after rework test cost will occur with P_0 probability.

OQA after rework sampling cost consists of labor cost, testing machine cost, and material cost.

A. Labor Cost

Labor cost is a cost of labor that used to test the sample in OQA after rework process with variable sampling plan. Staff will collect the sample with designed amount ($n_{OQA\ AR}$) from the product lot amount (O_{size}).

Labor cost = Labor rate x Labor used

$$Labor\,\mathrm{cost} = L_m \times \sum_{i=1}^{m} \frac{(P_{0i} \times T_{Test} \times n_{OQAARi} \times \Pr_i)}{O_{Sizei}}$$

B. Machine Cost

Machine cost is a machine operating cost such as electricity cost and maintenance cost.

Machine cost = Machine operation rate x Machine used

$$Machine\,\mathrm{cost} = M \times \sum_{i=1}^{m} \frac{(P_{oi} \times n_{OQAARi} \times \Pr_i)}{O_{Sizei} \times K_i}$$

Since K_i is a testing machine capacity and $(P_{0i} \times Pr_i)/ O_{sizei}$ is a number of samples needed to be tested in one hour so $\sum_{i=1}^{m} \frac{(P_{oi} \times n_{OQAARi} \times \Pr_i)}{O_{Sizei} \times K_i}$ will represent the number of machine used in this testing activity.

C. Material Cost

Material cost is a cost of product that destroyed in testing activity. However, the testing in this case study is a non-destructive testing so the material cost is none.

Material cost = Cost of product destroyed x Number of product destroyed

$$Material\text{cost} = \sum_{i=1}^{m} \frac{(P_{oi} \times D_i \times n_{OQAARi} \times \text{Pr}_i)}{O_{Sizei}}$$

5.4.4 Defect Cost

The defect cost in the model consists of rework cost and scrap cost. These costs occur because the production has produced defective products especially when the process is out-of-control (with γ_{2i} probability of defect). In general, the sooner we find that the process is out-of-control, the lower the cost of repairing it will be. On the other hand, the longer the process remains in the out-of-control state, the more defective items will be produced (Chiu and Huang, 1996). The defect cost can be decreased by decreasing delay detection period (ATS + T1). If the out-of-control ARL (ARL1) decreases, it means that the process variation will be detected faster (Torng, Lee, and Liao, 2009).

5.4.4.1 IPQA Rework Cost

IPQA rework cost is a cost of reworking the hold defective product lot in IPQA process. The product produced while the production process is out-of-control (Z) will be held for the test. Then, if the test fails with probability Y_i, the hold lot will be reworked. The IPQA rework cost is depended on the amount of product produced from an out-of-control period. . IPQA after rework cost will occur with Y probability once in a cycle time.

Rework cost = Rework rate x Number of product reworked

$$IPQA rework \cos t = \sum_{i=1}^{m} \frac{Y_i \times R_i \times \Pr_i \times (ATS_i + T_{1i})}{T_{Cyclei}}$$

5.4.4.2 IPQA Scrap Cost

IPQA scrap cost is a cost of scrap product that occurs from an IPQA process. IPQA scrap cost occurs when the defective product lot failed to pass the IPQA after rework variable sampling plan testing. IPQA scrap cost will occur with H_i probability after the rework process occurs.

$$IPQA scrap \cos t = \sum_{i=1}^{m} \frac{Y_i \times H_i \times C_{P\,IPQAi} \times \Pr_i \times (ATS_i + T_{1i})}{T_{Cyclei}}$$

5.4.4.3 OQA Rework Cost

OQA rework cost is a cost of reworking defective product lot in OQA process. Product lot amount (O_{size}) will be tested before delivering to the customer. Then, if the test fails with probability $P_{0,}$ the lot will be reworked.

Rework cost = Rework rate x Number of product reworked

$$OQA rework \cos t = R \times \sum_{i=1}^{m} (P_{oi} \times \Pr_i)$$

5.4.4.4 OQA Scrap Cost

OQA scrap cost is a cost of scrap product that occurs from an OQA process. IPQA scrap cost occurs when the reworked lot failed to pass the OQA after rework variable sampling plan testing. OQA scrap cost will occur with P_d probability after OQA rework process occurs.

$$OQA scrap \cos t = \sum_{i=1}^{m} (P_{oi} \times P_{di} \times C_{P\,OQAi} \times \Pr_i)$$

5.5 External Failure Costs

5.5.1 Transportation Cost

Transportation cost occurs when the defective products are rejected by the customer and the company has to transport the new products to replace those amounts of rejected products. There are two causes that the product can be rejected. First, the products are rejected with P_{Reji} probability after the customer tested the product lot by customer variable sampling plan. Second, the products are rejected after they fail to assembly with P_{mi} probability in customer manufacturing process. The company will transport the new product for replacement every I_T interval (in this case study is a week or 168 hours). The number of defective products found in each interval will be summed and then replaced by the new products. The transportation cost is C_T per transportation lot (O_T). The $\frac{I_T}{O_T}$ will represent the number of lot required to transport the entire products found in the period.

Transportation cost = Cost of transport per lot x Number of product lot transported

$$Transportation\cos t = \sum_{i=1}^{m} C_{Ti} \times \left(\frac{roundup(\Pr_i \times (P_{reji} + (1 - P_{reji})(P_{mi})(P'_i)) \times \frac{I_{Ti}}{O_{Ti}})}{I_{Ti}} \right)$$

5.5.2 Replacement Cost

Woodall noted that economic models assign a cost to customer dissatisfaction and liability claims among other costs (Saniga, 1989). In this model, replacement cost consists of two costs which are a cost of product that rejected by the customer and a penalty cost. The company will be charged for the penalty amount (P_{en}) for each product if the product fails to assembly in customer manufacturing process with P_m probability.

Replacement cost = Cost of product x Number of product rejected + Penalty rate x Number of product fail to assembly

$$Replacement\cos t = \sum_{i=1}^{m} (\Pr_i \times (P_{\mathrm{Re}\,ji} \times C_{Pcusi} + P_{mi} \times P'_i \times (1 - P_{reji}) \times (C_{Pcusi} + P_{eni})))$$

5.6 Machine Investment Cost

This cost is a cost of machine that the company has to invest in order to increase the number of the testing machine. This cost is not included in the model because it does not relate to the control chart directly. It is a cost that the company has to pay in order to increase the number of testing machine which is one of the constraints in the model. However, this investment option and its cost will be considered in order to find the best solution that minimizes the total cost and the machine investment cost.

The machine present value is PV and the machine operating life is a year. The interest rate in the business is r per year. Therefore, the annual value (AV) for the testing machine will gain from the PV, a, and r parameter. The AV can be considered to be the machine depreciation per year because at the end of year a, the machine value is assumed to be zero. The AV is the machine depreciation for a year so the cost of investing machine per hour can be obtained by dividing the AV by working hour per year (W). The number of current machine is F and the maximum number of machine allowed in the model is G so G-F will represent the number of the new invested machine.

The machine has certain operating life time which is about a year no matter how much the machine has been used. Therefore, the machine investment cost is considered to be a fixed cost which does not depend on the sample size and sampling frequency, but it depends on the number of machines invested. On the other hand, the machine operating rate (M) is the cost that depends on the usage of the machine. The more the sample size and sampling frequency are, the more the machine operating rate is.

Machine investment cost = Machine cost per hour x Number of new machine invested

$$Machine investment \cos t = \frac{AV}{W} \times (G - F)$$

5.7 Objective Function

The objective function in this model is to minimize the total cost function by finding optimum sample size and sampling frequency. The main objective of this economic statistical design is to minimize the expected cost per unit of time by considering type 1error, type 2 error, and ATS as the constraints (Saniga, 1989).

Objective function = min Total cost (n, f)

Where Total cost = Appraisal cost + Failure cost

= Control chart sampling cost + False alarm cost + OQA Sampling cost + True alarm cost + Cease cost + IPQA hold lot test cost + IPQA after rework test cost + OQA after rework test cost + IPQA rework cost + IPQA scrap cost + OQA rework cost + OQA scrap cost + Transportation cost + Replacement cost

$$= L_m \times \sum_{i=1}^{m} (T_{Pick} \times f_i \times + T_{Testi} \times n_i \times f_i) + M \times \sum_{i=1}^{m} (\frac{n_i \times f_i}{K_i}) +$$

$$\sum_{i=1}^{m} (D_i \times n_i \times f_i) + \sum_{i=1}^{m} (S_2 \times T_{1i} \times \alpha_i \times f_i \times \frac{T_{oi}}{T_{Cyclei}}) +$$

$$L_m \times \sum_{i=1}^{m} \frac{(T_{Testi} \times n_{OQAi} \times \Pr_i)}{O_{Sizei}} + M \times \sum_{i=1}^{m} (\frac{n_{OQAi} \times \Pr_i}{K_i \times O_{Sizei}}) +$$

$$D_i \times \sum_{i=1}^{m} \frac{(n_{OQAi} \times \Pr_i)}{O_{Sizei}} + \sum_{i=1}^{m} \frac{(S_2 \times T_{1i} + S_1 \times T_{2i})}{T_{Cyclei}} +$$

$$\sum_{i=1}^{m} (\frac{T_{2i} \times \Pr_i \times Pn_i}{T_{Cyclei}}) + L_m \times \sum_{i=1}^{m} (\frac{T_{Testi} \times n_{IPQAi}}{T_{Cyclei}}) +$$

$$M \times \sum_{i=1}^{m} (\frac{n_{IPQAi}}{K_i \times T_{Cyclei}}) + \sum_{i=1}^{m} (\frac{D_i \times n_{IPQAi}}{T_{Cyclei}}) +$$

$$L_m \times \sum_{i=1}^{m} (\frac{Y_i \times T_{Testi} \times n_{IPQA\,ARi}}{T_{Cyclei}}) + M \times \sum_{i=1}^{m} (\frac{Y_i \times n_{IPQA\,ARi}}{K_i \times T_{Cyclei}}) +$$

$$
\begin{aligned}
&\sum_{i=1}^{m}\left(\frac{Y_i \times D_i \times n_{IPQAARi}}{T_{Cyclei}}\right) + L_m \times \sum_{i=1}^{m}\frac{(P_{0i} \times T_{Test} \times n_{OQAARi} \times \Pr_i)}{O_{Sizei}} + \\
&M \times \sum_{i=1}^{m}\frac{(P_{oi} \times n_{OQAARi} \times \Pr_i)}{O_{Sizei} \times K_i} + \sum_{i=1}^{m}\frac{(P_{oi} \times D_i \times n_{OQAARi} \times \Pr_i)}{O_{Sizei}} + \\
&\sum_{i=1}^{m}\frac{Y_i \times R_i \times \Pr_i \times (ATS_i + T_{1i})}{T_{Cyclei}} + \\
&\sum_{i=1}^{m}\frac{Y_i \times H_i \times C_{P\,IPQAi} \times \Pr_i \times (ATS_i + T_{1i})}{T_{Cyclei}} + R \times \sum_{i=1}^{m}(P_{oi} \times \Pr_i) + \\
&\sum_{i=1}^{m}(P_{oi} \times P_{di} \times C_{P\,OQAi} \times \Pr_i) + \\
&\sum_{i=1}^{m} C_{Ti} \times \left(\frac{roundup(\Pr_i \times (P_{reji} + (1-P_{reji})(P_{mi})(P'_i)) \times \frac{I_{Ti}}{O_{Ti}})}{I_{Ti}}\right) + \\
&\sum_{i=1}^{m}(\Pr_i \times (P_{\mathrm{Re}\,ji} \times C_{Pcusi} + P_{mi} \times P'_i \times (1-P_{reji}) \times (C_{Pcusi} + P_{eni})))
\end{aligned}
$$

5.8 Constraints

Traditionally, control chart has been designed only by statistical criteria. The type 1 error probability and power are usually specified at the desired levels while the sample size (n) and control limits coefficient (L) usually be specifically determined.

In economical design, Woodal (1986) stated that control chart based on economically optimal design generally have poor statistical properties. The Duncan's model does not concern about statistical properties since there is only an economic objective without statistical constraints in the model (Saniga, 1989).

In order to create the economic statistical design, statistical constraints are added to the economic model. The metric used to evaluate the statistical performance of a control chart is generally either the average run length (ARL) or the average time to signal (ATS). As Celeno and Fichera (1999) said that the objective function must be minimized in order to pursue the economic goal, whereas the statistical objectives are reached by minimizing the α value and maximizing the 1-β value.

In this thesis, the control limit coefficient (L) is set to be 3 which is the standard use as Montgomery (2005) said that the multiple usually chosen is three; hence, three-sigma limits are customarily employed on control chart, regardless of the type of chart employed. Therefore, with 3 control limit coefficient, the α value will be 0.0027 and ARL_0 will be 370 to retain the standard statistical property.

Another crucial statistical constraint in this thesis is β value. The β can be expressed as ARL $\left(\frac{1}{1-\beta_i}\right)$ or ATS $\left(\frac{1}{1-\beta_i}\times\frac{1}{f_i}\right)$. The defective products produced during an out-of-control period can be reduced significantly by reducing the ATS. The significant variables that affect the ATS in the model are β and f while the significant variable that affects β is n since β is a function of sample size.

The constraints are

β_i	$\leq$	0.2	$\forall i \in J$	(i)
β_i	$\geq$	0	$\forall i \in J$	(ii)
n_i	$\geq$	1	$\forall i \in J$	(iv)
f_i	$\leq$	$\frac{1}{T_{picki}}$	$\forall i \in J$	(v)
f_i	$>$	0	$\forall i \in J$	(vi)
ATU_i	$\leq$	G_i	$\forall i \in J$	(vi)

Constraint (i): β value has to be less than or equal to 0.2 or ARL_1 is less than 1.54. This constraint will significantly control the statistical property of the control chart to ensure that the solution of the model will meet the statistical criteria. However, this constraint value can be changed in order to satisfy the company policy.

Constraint (ii): β value has to be more than or equal to 0. This constraint is added due to the fact that the β value cannot be negative.

Constraint (iii): n value has to be more than or equal to 1 because the least sample size allowed is 1.

Constraint (iv): f value has to be less than or equal to the capability of picking samples. This is because staff has to use T_{Pick} time to pick the sample so there will be limited frequency of picking the samples.

Constraint (vi): f value has to be more than 0. This is because the sampling activity has to be done at least one time.

Constraint (vii): All testing machine used has to be less than or equal to testing machine allowed. The machine allowed (G) can be increased if the company buys the new testing machine. The ATU value can be increased also.

5.9 Optimization Technique

Montgomery (1980) states that the mathematical models and their associated optimization schemes are relatively complex and are often presented in a manner that is difficult for the practitioner to understand and use. The main difficulties in the use of economic design are the computations involved (Chung, 1995).The availability of computer programs for these models and the development of simplified approximate optimization procedures suitable for manual computation would help alleviate this problem.

The economic mathematical model in this thesis consists of mathematical formulas that can be solved by Solver function in Microsoft Excel. This can help the practitioner solve their problems without the problem of using complex program.

5.10 Input Parameters

The input parameters illustrated in Table. 5.1 are gained from the case study company as an example. The mathematical model allows parameter value to be changed so that the model can be applicable to many cases. Constraints, statistical criteria, probability, and cost can be modified to match each case. For example, if the production is a continuous process where there is no need to be stopped while repairing the process, the T_2 value can be filled with zero to represent the continuous model.

Table 5.1: Input Parameters Value

Input value				Source
	M1	M2	M3	
n	-	-	-	Variable
f	-	-	-	Variable
α	0.0027	0.0027	0.0027	Determined by control limit coefficient = 3
β	-	-	-	f(n)
γ_1	0.0006	0.0023	0.0017	Control chart data
γ_2	0.0208	0.0554	0.0911	Control chart data
T_0	7.8853	19.0230	1222.2650	Process in-control time
T_1	0.0375	0.0375	0.05	Time for investigating process
T_2	0.4750	0.4750	0.4750	Time for repairing process
ATS	-	-	-	f(β, f)
T_{Cycle}	-	-	-	f(ATS,T_1, T_2)
Pr	1205	3893	929	Production rate
M	2.84	2.84	2.84	Machine operation rate
R	2.18	1.32	1.04	Rework rate
P_n	1.46	2.96	7.37	Product profit
C_T	1028	1028	1028	Transportation cost
O_T	10000	10000	10000	Transportation lot size

Table 5.1: Input Parameters Value (Cont.)

Input value				Source
	M1	M2	M3	
I_T	168	168	168	Transportation interval
$C_{P\ IPQA}$	10.83	9.43	6.9	Product cost in IPQA
$C_{P\ OQA}$	11.83	10.43	7.9	Product cost in OQA
$C_{P\ Cus}$	12.31	12.43	14.31	Product cost when delivering to the customer
P_{en}	123.10	124.30	143.10	Penalty rate for customer
L_M	25.18	25.18	25.18	Staff labor rate
T_{Pick}	0.083	0.083	0.083	Time to pick samples
T_{Test}	0.0208	0.0208	0.0333	Time to test sample
K	45.6007	45.6007	28.5000	$f(T_{Test})$
D	0	0	0	Cost of destructed product
S_1	64.10	64.10	64.10	Engineer labor rate
S_2	35.26	35.26	35.26	Technician labor rate
MC	16	57	19	Number of production machine
O_{Size}	1000	1000	1000	OQA variable sampling plan lot size
O_{Cust}	1000	1000	1000	Customer variable sampling plan lot size
F	5			Number of current testing machine
G	-			Determined
W	7488	7488	7488	Working hour per year
PV	1,300,000	1,300,000	1,300,000	Testing machine in present value
A	10	10	10	Testing machine operating life
R	0.12	0.12	0.12	Interest rate in the business
AV	-	-	-	f(PV,r,a)
USL	0.4	0.4	0.4	Determined

Table 5.1: Input Parameters Value (Cont.)

Input value				Source
	M1	M2	M3	
σ	0.0421	0.0573	0.0524	Measured value standard deviation
AQL_{IPQA}	0.02	0.02	0.02	Determined
$LTPD_{IPQA}$	0.08	0.08	0.08	Determined
$Alpha_{IPQA}$	0.05	0.05	0.05	Determined
$Beta_{IPQA}$	0.2	0.2	0.2	Determined
n_{IPQA}	-	-	-	$f(AQL_{IPQAi,}\ LTPD_{IPQAi},\ Alpha_{IPQAi},\ Beta_{IPQAi})$
K_{IPQA}	-	-	-	$f(AQL_{IPQAi,}\ LTPD_{IPQAi},\ Alpha_{IPQAi},\ Beta_{IPQAi},\ n_{IPQA})$
Y	-	-	-	$f(USL,\ K_{IPQA},\ \sigma\ ,\ \gamma_{2i},\ n_{IPQA}\ )$
1-Y	-	-	-	f(Y)
$AQL_{IPQA\ AR}$	0.02	0.02	0.02	Determined
$LTPD_{IPQA\ AR}$	0.08	0.08	0.08	Determined
$Alpha_{IPQA\ AR}$	0.05	0.05	0.05	Determined
$Beta_{IPQA\ AR}$	0.2	0.2	0.2	Determined
$n_{IPQA\ AR}$	-	-	-	$f(AQL_{IPQA\ ARi,}\ LTPD_{IPQA\ ARi},\ Alpha_{IPQA\ ARi},\ Beta_{IPQA\ ARi})$
$K_{IPQA\ AR}$	-	-	-	$f(AQL_{IPQA\ AR,}\ LTPD_{IPQA\ AR},\ Alpha_{IPQA\ AR},\ Beta_{IPQA\ AR},\ n\ _{IPQA\ AR})$
H	-	-	-	$(USL,\ K_{IPQA\ AR},\ \sigma\ ,\ \gamma_{1i},\ n_{IPQA\ AR}\ )$
1-H	-	-	-	f(H)
AQL_{OQA}	0.02	0.02	0.02	Determined
$LTPD_{OQA}$	0.08	0.08	0.08	Determined
$Alpha_{OQA}$	0.05	0.05	0.05	Determined
$Beta_{OQA}$	0.2	0.2	0.2	Determined

Table 5.1: Input Parameters Value (Cont.)

Input value				Source
	M1	M2	M3	
n_{OQA}	-	-	-	f($AQL_{OQAi,}$ $LTPD_{OQAi}$, $Alpha_{OQAi}$, $Beta_{OQAi}$)
K_{OQA}	-	-	-	f($AQL_{OQAi,}$ $LTPD_{OQAi}$, $Alpha_{OQAi}$, $Beta_{OQAi}$, n_{OQA})
P_{0i}	-	-	-	(USL, K_{OQA}, σ ,P_i, n_{OQA})
1-P0i	-	-	-	f(P0)
$AQL_{OQA\ AR}$	0.02	0.02	0.02	Determined
$LTPD_{OQA\ AR}$	0.08	0.08	0.08	Determined
$Alpha_{OQA\ AR}$	0.05	0.05	0.05	Determined
$Beta_{OQA\ AR}$	0.2	0.2	0.2	Determined
$n_{OQA\ AR}$	-	-	-	f($AQL_{OQA\ ARi,}$ $LTPD_{OQA\ ARi}$, $Alpha_{OQA\ ARi}$, $Beta_{OQA\ ARi}$)
$K_{OQA\ AR}$	-	-	-	f($AQL_{OQA\ ARi,}$ $LTPD_{OQA\ ARi}$, $Alpha_{OQA\ ARi}$, $Beta_{IPQA\ ARi}$, $n_{IPQA\ AR}$)
P_d	-	-	-	(USL, $K_{OQA\ AR}$, σ , γ_{1i}, $n_{OQA\ AR}$)
1-P_d	-	-	-	f(P_d)
AQL_{CUS}	0.02	0.02	0.02	Determined
$LTPD_{CUS}$	0.08	0.08	0.08	Determined
$Alpha_{CUS}$	0.05	0.05	0.05	Determined
$Beta_{CUS}$	0.2	0.2	0.2	Determined
n_{CUS}	-	-	-	f($AQL_{CUS,}$ $LTPD_{CUSi}$, $Alpha_{CUSi}$, $Beta_{CUSi}$)
K_{CUS}	-	-	-	f($AQL_{CUSi,}$ $LTPD_{CUSi}$, $Alpha_{CUSi}$, $Beta_{CUS}$, n_{CUS})
P_{rej}	-	-	-	(USL, K_{CUS}, σ , P', n_{CUS})
1-P_{rej}	-	-	-	f(P_{reji})

Table 5.1: Input Parameters Value (Cont.)

Input value				Source
	M1	M2	M3	
P_m	0.8	0.8	0.8	Probability of defect found in customer manufacturing process
$1\text{-}P_m$	0.2	0.2	0.2	$f(P_m)$
P	-	-	-	$f(ATS, T_1, \gamma_{2i}, Y, \gamma_{1i}, H, T_0, T_2, T_{Cycle})$
Z	-	-	-	$f(Pr, ATS, T_1)$
P'	-	-	-	$f(P, P_0, \gamma_1, P_d)$
ATU	-	-	-	$f(n, f, K, n_{OQA}, P_r, O_{Size}, n_{IPQA}, T_{Cycle}, Y, n_{IPQA\ AR}, P_0, n_{OQA\ AR})$

5.10.1 Primary Parameter

α : This parameter is set to be 0.0027 because the control limits coefficient is set to be 3.

β : This parameter is a function of control chart sample size. However, this value cannot exceed 0.2 to maintain the statistical property of the control chart.

γ_1, γ_2 : These parameters can be gained from the historical data of the control chart.

T_0 : This parameter can be gained from an average in-control time period.

T_1 : This parameter can be obtained from an average time that the technician used to find an assignable cause for each product.

T_2 : This parameter can be obtained from an average time that the engineer used to repair the process.

P_r : This parameter can be obtained from the production policy. It is assumed that the company has a steady production rate all day. Changing in production rate will result in the new sampling plan.

M : This parameter can be obtained from a testing machine operating rate. Changing type of testing machine will change this value.

R : This parameter can be obtained from a rework cost per piece.

P_n : This parameter can be obtained from a profit that the company gained from each product.

P_{ce} : This parameter can be obtained from a product price that the company charges the customer per piece.

C_T : This parameter can be obtained from a cost of transportation per lot size (O_T).

O_T : This parameter can be obtained from a transportation lot size.

I_T : This parameter can be obtained from a transportation interval that the company and the customer agreed.

$C_{P\ IPQA}$: This parameter can be obtained from a product cost in IPQA process.

$C_{P\ OQA}$: This parameter can be obtained from a product cost in OQA process. The product cost in OQA process is higher than the product cost in IPQA process because the company has to put resources in OQA process more than that in IPQA process.

$C_{P\ CUS}$: This parameter can be obtained from a product cost when delivered to the customer. This product cost can also considered as product price that the customer has to pay to the company.

P_{en} : This parameter can be obtained from a penalty rate per piece that the customer charges the company. The penalty rate will depend on the cost of assembly parts in the customer manufacturing process. If the assembled parts are expensive, the penalty rate will be high.

L_M : This parameter can be obtained from an average salary of staff in one month divided by working hour in a month.

T_{Pick} : This parameter can be obtained from a time that staff picks the samples from turning machine.

T_{Test} : This parameter can be obtained from a time that staff tests the sample with testing machine.

K : This parameter is a function of T_{test}.

D : This parameter can be obtained from a cost of product that destroyed in destructive testing.

S_1 : This parameter can be obtained from an engineer labor cost in turning process.

S_2 : This parameter can be obtained from a technician labor cost in turning process.

MC : This parameter can be obtained from a number of turning machine for each product.

O_{Size} : This parameter can be obtained from a lot size in OQA variable sampling plan.

O_{Cust} : This parameter can be obtained from a lot size in customer variable sampling plan.

F : This parameter can be obtained from a number of current testing machines.

G : This parameter can be obtained from a number of machine allowed included the new invested machine. This value can be changed for each design experiment. If the companies use the current number of machine, the G value will equal to F value. If the company invest more in testing machine, the G value will be greater than F value.

W : This parameter can be obtained from the company working hour per year.

PV : This parameter can be obtained from a new machine price in the present.

a : This parameter can be obtained from a machine operating life.

r : This parameter can be obtained from an interest rate in the business.

5.11 Scenarios Studied

There will be three scenarios for the model. Each of the scenarios will be illustrated about its sample size, sampling frequency, and costs.

First, the present sampling plan will be used in the model to let the model illustrates the present cost and present quality statistical performance. After that, the alternatives will be illustrated.

Second, the model will be solved under the statistical quality constraints and present resources to find the optimum sampling plan that minimizes total cost. The number of testing machine will be limited to the current number of machine that the company already has which is 5.

Third, the additional new testing machine will be allowed in the model. The Number of testing machine allowed (G) in constraints will be allowed to increase by the new invested machine (G-F). There will be machine investment cost added since the company has to invest in new machines. Investing in the new testing machine will increase the testing capacity which may result in a decreasing of a failure cost. This is because an assignable cause can be detected earlier by the sufficient testing capacity. This scenario will find the optimum sampling plan for each number of machines. After all, the sampling plan and the number of machine invested that minimize total cost will be illustrated.

CHAPTER VI

RESULTS

In this chapter, results for each scenario are shown including the statistical properties and costs. Also, discussion and conclusion about the results are illustrated.

6.1 Present Sampling Plan

At present the case study company has no formal methodology to design the sampling plan for its process. The company has designed the sample size and sampling frequency by the traditional routine. However, the company wants to increase the performance in its process by developing a methodology to design the sampling plan which can lower the costs involved and also satisfy the statistical constraints.

The result of the present sampling plan is shown in order to demonstrate the present cost and the statistical performance before conducting the improvement. Since there is no further invested machine in the present sampling plan, the number of testing machine allowed (G) in the present sampling plan is 5 machines.

The present sample size, sampling frequency, and other crucial parameters are illustrated in the table below.

Table 6.1: Present Sampling Plan Result

	M1	M2	M3
n	2	2	1
f	0.25	0.25	0.25
1/f	4	4	4
β	0.57	0.57	0.84
ATS	9.26	9.26	25.21
P	0.011	0.010	0.002
Z	11206	36202	23468
P'	0.011	0.010	0.002
ATU	4.48		

The sample size for product M1, M2, and M3 are 2, 2, and 1piece per time respectively. This causes the β value to exceed the constraint limit in this model. The β value considered to be high which means that there is a high probability that the defective products will be produced while the process is thought to be in-control despite the fact that it is already out-of-control. The sampling frequency is set to be the same for all products based on the operation is 0.25 time per hour. However, each product has its own details so the sampling frequency should be designed individually to create efficiency of using the control chart. The All testing machine used in the process (ATU) is 4.48 machines while the allowed number of machine is 5 machines. This means that there is an abandoned testing capacity which can be used if necessary.

Table 6.2: Present Sampling Plan Costs

Appraisal Costs		
- Control chart sampling cost	31.91	1.73%
- False alarm cost	0.06	0.00%
- OQA Sampling cost	57.97	3.14%
Sum appraisal costs	89.94	4.88%
Internal failure costs		
- True alarm cost	92.16	5.00%
- Cease cost	240.00	13.02%
- IPQA hold lot test cost	25.62	1.39%
- IPQA after rework test cost	10.18	0.55%
- OQA after rework test cost	0.18	0.01%
- All retest cost	35.97	1.95%
- IPQA rework cost	1,003.59	54.43%
- IPQA scrap cost	0.01	0.00%
- OQA rework cost	31.57	1.71%
- OQA scrap cost	0.00	0.00%
- All defect costs	1,035.17	56.15%
External failure costs		
-Cost of replacement	332.13	18.01%
-Cost of transport	18.36	1.00%
Sum failure costs	1,753.79	95.12%
Total cost	1,843.74	100.00%
Machine investment cost	-	
Total cost + machine investment cost	1,843.74	

The failure cost is the major cost in the present sampling plan accounted for 95.12 % while the appraisal cost is only 4.88 %. This means that the company pays large amount and also uses most of the resources to fix the defects that already occurred. The major defect cost is from IPQA rework cost which used for reworking the defective products produced while the process is out-of-control. Cost of replacement is also considered to be the major cost which occurs when the defective products delivered to the customer. These failures costs can be reduced by improving the detection performance to detect the error earlier before the defective products are produced in a large amount.

6.2 Optimum Sampling Plan under the Current Number of Testing Machines

This experiment is done to find the optimum sample size and sampling frequency that can minimize the total cost in the model while the constraints are satisfied under the existing testing machines.

Table 6.3: Optimum Sampling Plan under the Current Number of Testing Machines Result

	M1	M2	M3
n	4	4	4
f	0.13	0.22	0.03
1/f	7.54	4.62	38.99
β	0.16	0.16	0.16
ATS	8.96	5.49	46.34
P	0.011	0.008	0.002
Z	10843	21531	43096
P'	0.011	0.008	0.002
ATU		5	

The sample size is increased to be 4 pieces per time in this solution. This sample size causes the β value to be 0.16 which can satisfy the constraint. The sampling frequency is decreased independently based on the characteristic of each product. The sampling frequency for product M3 is decreased significantly from 0.25 to 0.03 time per hour because the M3 has a high steady production resulted in longer in-control period

(T_0). Thus, there is no need to collect the sample frequently. The ATU value is 5 machines which mean that the testing machines are used in a full capacity.

Table 6.4: Optimum Sampling Plan under the Current Number of Testing Machines

Costs

Appraisal Costs		
- Control chart sampling cost	39.68	2.71%
- False alarm cost	0.04	0.00%
- OQA Sampling cost	57.97	3.96%
Sum appraisal costs	97.69	6.68%
Internal failure costs		
- True alarm cost	102.12	6.98%
- Cease cost	269.39	18.42%
- IPQA hold lot test cost	28.37	1.94%
- IPQA after rework test cost	11.62	0.79%
- OQA after rework test cost	0.08	0.01%
- All retest cost	40.08	2.74%
- IPQA rework cost	728.95	49.84%
- IPQA scrap cost	0.01	0.00%
- OQA rework cost	17.41	1.19%
- OQA scrap cost	0.00	0.00%
- All defect costs	746.37	51.03%
External failure costs		
-Cost of replacement	188.69	12.90%
-Cost of transport	18.36	1.26%
Sum failure costs	1,365.02	93.32%
Total cost	1,462.70	100.00%
Machine investment cost	-	
Total cost + machine investment cost	1,462.70	

The total cost is reduced from 1,834.74 to 1,462.70 Baht per hour which arisen from a reduction of failure costs and an increasing of appraisal costs. IPQA rework cost is decreased from 1003.59 to 724.79 Baht per hour while the cost of replacement is decreased from 1,003.59 to 728.95 Baht per hour. The failure costs are reduced since an assignable cause can be detected faster. The appraisal costs are increased from 89.94 to 97.69 Baht per hour due to the increasing activities in testing process. From the figure, it can be seen that increasing in appraisal costs has a highly effect to a reduction of the failure costs in this case.

6.3 Optimum Sampling Plan under the Additional New Testing Machines

This experiment allows the model to increase the number of testing machine in order to increase the testing capacity. As a consequence, the machine investment cost is added to the model as a machine investment cost separately. The optimum sample size and sampling frequency for each number of machines allowed (G) will be illustrated case by case.

6.3.1 Optimum Sampling Plan under 6 Testing Machines

Table 6.5: Optimum Sampling Plan under 6 Testing Machines Result

	M1	M2	M3
n	4	4	4
f	0.20	0.35	0.04
1/f	4.92	2.83	25.03
β	0.16	0.16	0.16
ATS	5.85	3.36	29.75
P	0.009	0.006	0.002
Z	7098	13225	27682
P'	0.009	0.006	0.002
ATU	6		

The sample size is remained the same as in the optimum sampling plan under 5 machines, but the sampling frequency is increased thoroughly. This means that the additional testing capacity is needed to lower the total cost which obviously seen from the increasing of all testing machine used in the process (ATU) value to be 6 instead of 5 machines. This is because if the additional testing capacity is not necessary, the model will not use the additional capacity and have the same optimum sample size and sampling frequency. The average time to signal (ATS) value which is a function of β and f is reduced because the sampling frequency is increased. The amount of product that has to be held for checking (Z) value for every product is decreased since an assignable cause can be detected earlier resulting in a reduction amount of the hold lot.

Table 6.6: Optimum Sampling Plan under 6 Testing Machines Costs

Appraisal Costs		
- Control chart sampling cost	64.16	5.33%
- False alarm cost	0.07	0.01%
- OQA Sampling cost	57.97	4.82%
Sum appraisal costs	122.19	10.15%
Internal failure costs		
- True alarm cost	115.26	9.57%
- Cease cost	300.31	24.95%
- IPQA hold lot test cost	32.01	2.66%
- IPQA after rework test cost	12.75	1.06%
- OQA after rework test cost	0.03	0.00%
- All retest cost	44.79	3.72%
- IPQA rework cost	498.38	41.40%
- IPQA scrap cost	0.00	0.00%
- OQA rework cost	6.34	0.53%
- OQA scrap cost	0.00	0.00%
- All defect costs	504.73	41.93%
External failure costs		
-Cost of replacement	98.13	8.15%
-Cost of transport	18.36	1.52%
Sum failure costs	1,081.57	89.85%
Total cost	1,203.77	100.00%
Machine investment cost	30.73	
Total cost + machine investment cost	1,234.49	

Comparing the results between using 5 testing machines and 6 testing machines, the total cost is reduced from 1,462.70 to 1,203.77 Baht per hour which arisen from a reduction in failure costs and an increasing in appraisal costs. The failure costs are reduced from 1,365.02 to 1,081.57 Baht per hour while the appraisal costs are increased from 97.69 to 122.19 Baht per hour. The new added cost is a machine investment cost which is 30.73 Baht per hour. However, the sum of total cost and machine cost which is 1,234.49 Baht per hour is still lower than the total cost in the optimum sampling plan using 5 machines which is 1,462.70 Baht per hour. Thus, this can be concluded that the new additional machine is needed to lower the total cost. The new additional machine can help lower the total cost by increasing the testing capacity which resulted in lower failure costs.

6.3.2 Optimum Sampling Plan under 7 Testing Machines

Table 6.7: Optimum Sampling Plan under 7 Testing Machines Result

	M1	M2	M3
n	4	4	4
f	0.26	0.51	0.06
1/f	3.80	1.97	17.93
β	0.16	0.16	0.16
ATS	4.52	2.34	21.31
P	0.008	0.005	0.002
Z	5492	9241	19848
P'	0.008	0.005	0.002
ATU	7		

The sample size is remained the same as in the optimum sampling plan under 6 machines, but the sampling frequency is increased in every product. This is because the further testing capacity is still needed in order to test the sample more frequently which leads to a reduction of total cost. The out-of control state can be detected earlier because of an enhancement of the testing capacity resulting in lower average time to signal (ATS) value in every product. Amount of product that has to be held for checking (Z) is decreased which leads to lower cost of rework. The percent of defective product produced from IPQA process (P) is also reduced because the production process can be repaired quicker.

Table 6.8: Optimum Sampling Plan under 7 Testing Machines Costs

Appraisal Costs		
- Control chart sampling cost	90.92	8.28%
- False alarm cost	0.10	0.01%
- OQA Sampling cost	57.97	5.28%
Sum appraisal costs	148.99	13.57%
Internal failure costs		
- True alarm cost	122.64	11.17%
- Cease cost	317.56	28.92%
- IPQA hold lot test cost	34.06	3.10%
- IPQA after rework test cost	13.37	1.22%
- OQA after rework test cost	0.01	0.00%
- All retest cost	47.45	4.32%
- IPQA rework cost	371.73	33.86%
- IPQA scrap cost	0.00	0.00%
- OQA rework cost	3.08	0.28%
- OQA scrap cost	0.00	0.00%
- All defect costs	374.82	34.14%
External failure costs		
-Cost of replacement	68.14	6.21%
-Cost of transport	18.36	1.67%
Sum failure costs	948.96	86.43%
Total cost	1,097.94	100.00%
Machine investment cost	61.45	
Total cost + machine investment cost	1,159.39	

Comparing the results between using 6 testing machines and 7 testing machines, the total cost is reduced from 1,203.77 to 1,097.94 Baht per hour. The failure costs which include IPQA rework cost and cost of replacement are reduced. The control chart sampling cost is increased from 64.16 to 90.92 Baht per hour which leads to an increasing in appraisal costs due to the further testing activity. The machine investment cost is increased because the model is allowed to invest in the new machine up to 2 machines. Besides, the sum of total cost and machine investment cost which is 1,159.39 Baht per hour is lower than the sum of total cost and machine investment cost in the optimum sampling plan using 6 machines (1,234.49 Baht per hour).

6.3.3 Optimum Sampling Plan under 8 Testing Machines

Table 6.9: Optimum Sampling Plan under 8 Testing Machines Result

	M1	M2	M3
n	4	4	4
f	0.32	0.67	0.07
1/f	3.17	1.49	13.80
β	0.16	0.16	0.16
ATS	3.77	1.77	16.40
P	0.007	0.004	0.002
Z	4584	7025	15283
P'	0.007	0.004	0.002
ATU	8		

The sample size is remained the same at 4 pieces per time while the sampling frequency is increased compared to the sampling frequency in the optimum sampling plan under 7 machines. The machine testing capacity is still needed in order to increase the sampling frequency to detect the out-of-control state earlier resulting in lower value of average time to signal (ATS) and amount of product that has to be held for checking (Z). The additional machine is used in full capacity which can be monitored in ATU. In addition, the percent of defective both P and P' are decreased which lead to a reduction in defective products.

Table 6.10: Optimum Sampling Plan under 8 Testing Machines costs

Appraisal Costs		
- Control chart sampling cost	118.78	11.29%
- False alarm cost	0.14	0.01%
- OQA Sampling cost	57.97	5.51%
Sum appraisal costs	176.88	16.82%
Internal failure costs		
- True alarm cost	127.29	12.10%
- Cease cost	328.27	31.21%
- IPQA hold lot test cost	35.35	3.36%
- IPQA after rework test cost	13.74	1.31%
- OQA after rework test cost	0.01	0.00%
- All retest cost	49.10	4.67%
- IPQA rework cost	295.47	28.09%
- IPQA scrap cost	0.00	0.00%
- OQA rework cost	1.78	0.17%
- OQA scrap cost	0.00	0.00%
- All defect costs	297.26	28.26%
External failure costs		
-Cost of replacement	54.56	5.19%
-Cost of transport	18.36	1.75%
Sum failure costs	874.84	83.18%
Total cost	1,051.72	100.00%
Machine investment cost	92.18	
Total cost + machine investment cost	1,143.90	

Comparing the results between using 7 testing machines and 8 testing machines, the total cost is reduced from 1,097.94 to 1,051.72 Baht per hour. The failure costs are reduced from 948.96 to 874.84 Baht per hour while the appraisal costs are increased from 148.99 to 176.88 Baht per hour. There is a tradeoff between failure costs and appraisal costs. The testing capacity is needed to detect an assignable cause at the early stage which required the cost of machine investment due to the new additional testing machine. Nevertheless, the sum of total cost and machine investment cost which is 1,143.90 Baht per hour is still lower than those of using 7 machines which is 1,159.39 Baht per hour.

6.3.4 Optimum Sampling Plan under 9 Testing Machines

Table 6.11: Optimum Sampling Plan under 9 Testing Machines Result

	M1	M2	M3
n	4	4	4
f	0.36	0.84	0.09
1/f	2.75	1.19	11.13
β	0.16	0.16	0.16
ATS	3.27	1.41	13.23
P	0.006	0.004	0.002
Z	3988	5645	12333
P'	0.006	0.004	0.002
ATU	9		

The sample size is still remained the same at 4 pieces per time while the sampling frequency is increased compared to the sampling frequency in the optimum sampling plan under 8 machines which enhances the ability to detect an assignable cause faster. The amount of product that has to be held for checking (Z) is also reduced continuously. The additional machines are used in full capacity which can be monitored in ATU.

Table 6.12: Optimum Sampling Plan under 9 Testing Machines Costs

Appraisal Costs		
- Control chart sampling cost	147.22	14.24%
- False alarm cost	0.18	0.02%
- OQA Sampling cost	57.97	5.61%
Sum appraisal costs	205.36	19.87%
Internal failure costs		
- True alarm cost	130.50	12.63%
- Cease cost	335.53	32.46%
- IPQA hold lot test cost	36.24	3.51%
- IPQA after rework test cost	13.99	1.35%
- OQA after rework test cost	0.01	0.00%
- All retest cost	50.24	4.86%
- IPQA rework cost	245.44	23.75%
- IPQA scrap cost	0.00	0.00%
- OQA rework cost	1.15	0.11%
- OQA scrap cost	0.00	0.00%
- All defect costs	246.59	23.86%
External failure costs		
-Cost of replacement	47.04	4.55%
-Cost of transport	18.36	1.78%
Sum failure costs	828.26	80.13%
Total cost	1,033.62	100.00%
Machine investment cost	122.91	
Total cost + machine investment cost	1,156.53	

The solution in this model has an interesting point. Comparing the results between using 8 testing machines and 9 testing machines, the failure costs are reduced from 847.84 to 828.26 Baht per hour while the appraisal costs are increased from 176.88 to 205.36 Baht per hour. The total cost is still reduced from 1,051.72 to 1,033.62 Baht per hour which means that the additional testing capacity helps to decrease the failure costs. However, the sum of total cost and machine investment cost which is 1,156.53 Baht per hour is higher than those of using 8 machines which is 1,143.90 Baht per hour. Therefore, the optimum sampling plan under 8 testing machine can give the minimum of summed of total cost and machine investment cost and it is also the best solution in this experiment. This is because investing in the new machines costs more than the saving from a decreasing in failure costs.

6.3.5 Optimum Sampling Plan under 10 Testing Machines

Table 6.13: Optimum Sampling Plan under 10 Testing Machines Result

	M1	M2	M3
n	4	4	4
f	0.40	0.97	0.10
1/f	2.52	1.03	9.68
β	0.16	0.16	0.16
ATS	3.00	1.22	11.51
P	0.006	0.004	0.002
Z	3654	4915	10736
P'	0.006	0.004	0.002
ATU	9.75		

The sample size is still remained the same at 4 pieces per time while the sampling frequency is increased compared to the sampling frequency in the optimum sampling plan under 9 machines. The average time to signal (ATS) is reduced from the increasing in sampling frequency. The percent of defective product is the same of those using 9 machines. The all testing machine used in the process (ATU) is 9.75 machines while the number of machine allowed (G) is 10 machines. This means that the testing capacity is not used in full capacity.

Table 6.14: Optimum Sampling Plan under 10 Testing Machines Costs

Appraisal Costs		
- Control chart sampling cost	168.80	16.38%
- False alarm cost	0.21	0.02%
- OQA Sampling cost	57.97	5.63%
Sum appraisal costs	226.97	22.03%
Internal failure costs		
- True alarm cost	132.34	12.84%
- Cease cost	339.64	32.96%
- IPQA hold lot test cost	36.75	3.57%
- IPQA after rework test cost	14.13	1.37%
- OQA after rework test cost	0.00	0.00%
- All retest cost	50.88	4.94%
- IPQA rework cost	218.06	21.16%
- IPQA scrap cost	0.00	0.00%
- OQA rework cost	0.86	0.08%
- OQA scrap cost	0.00	0.00%
- All defect costs	218.93	21.25%
External failure costs		
-Cost of replacement	43.30	4.20%
-Cost of transport	18.36	1.78%
Sum failure costs	803.45	77.97%
Total cost	1,030.42	100.00%
Machine investment cost	153.63	
Total cost + machine investment cost	1,184.06	

Comparing the results between using 9 testing machines and 10 testing machines, the total cost is slightly reduced from 1,033.62 to 1,030.42 Baht per hour due to the fact that the amount of failure costs reduced almost equals to the amount of appraisal costs increased. The sum of total cost and machine investment cost which is 1,184.06 Baht per hour is higher than those of using 9 machines which is 1,156.53 Baht per hour. This is because the new machines can save about 3 Baht per hour, but it costs more about 30 Baht per hour. Thus, investing in the new machine is not a good solution anymore.

6.3.6 Optimum Sampling Plan under 11 Testing Machines

Table 6.15: Optimum Sampling Plan under 11 Testing Machines Result

	M1	M2	M3
n	4	4	4
f	0.40	0.97	0.10
1/f	2.52	1.03	9.68
β	0.16	0.16	0.16
ATS	3.00	1.22	11.51
P	0.006	0.004	0.002
Z	3654	4915	10736
P'	0.006	0.004	0.002
ATU	9.75		

From the table, there is no difference in optimum sampling plan between using 10 machines and 11 machines. The sample size and sampling frequency is also the same. The all testing machine used in the process (ATU) value is 9.75 while the machine allowed (G) is 11 machines. This is because the number of testing machine beyond 9.75 is not significant to the reduction of the total cost anymore.

Table 6.16: Optimum Sampling Plan under 11 Testing Machines Costs

Appraisal Costs		
- Control chart sampling cost	168.80	16.38%
- False alarm cost	0.21	0.02%
- OQA Sampling cost	57.97	5.63%
Sum appraisal costs	226.97	22.03%
Internal failure costs		
- True alarm cost	132.34	12.84%
- Cease cost	339.64	32.96%
- IPQA hold lot test cost	36.75	3.57%
- IPQA after rework test cost	14.13	1.37%
- OQA after rework test cost	0.00	0.00%
- All retest cost	50.88	4.94%
- IPQA rework cost	218.06	21.16%
- IPQA scrap cost	0.00	0.00%
- OQA rework cost	0.86	0.08%
- OQA scrap cost	0.00	0.00%
- All defect costs	218.93	21.25%
External failure costs		
-Cost of replacement	43.30	4.20%
-Cost of transport	18.36	1.78%
Sum failure costs	803.45	77.97%
Total cost	1,030.42	100.00%
Machine investment cost	184.36	
Total cost + machine investment cost	1,214.78	

The total cost is the same as in the optimum sampling plan under 10 machines. This is because increasing in appraisal costs cannot reduce the failure costs in significant level due to the fact that increasing in appraisal costs may cost more than those savings from failure costs. Therefore, there is no need to invest in the testing capacity anymore since it does not help to reduce the total cost. Moreover, the sum of total cost and machine investment cost is much higher than those of using 10 machines. This is because the total cost is the same, but there is an investment cost which is about 30 Baht per hour added.

6.3.7 Optimum Sampling Plan under 12 Testing Machines

Table 6.17: Optimum Sampling Plan under 12 Testing Machines Result

	M1	M2	M3
n	4.00	4.00	4.00
f	0.40	0.97	0.10
1/f	2.52	1.03	9.68
β	0.16	0.16	0.16
ATS	3.00	1.22	11.51
P	0.006	0.004	0.002
Z	3654	4915	10735
P'	0.006	0.004	0.002
ATU	9.75		

Again, the result is the same as using 10 and 11 machines. There is no need to invest in the testing capacity anymore since it does not help to reduce the total cost.

Table 6.18: Optimum Sampling Plan under 12 Testing Machines Costs

Appraisal Costs		
- Control chart sampling cost	168.80	16.38%
- False alarm cost	0.21	0.02%
- OQA Sampling cost	57.97	5.63%
Sum appraisal costs	226.97	22.03%
Internal failure costs		
- True alarm cost	132.34	12.84%
- Cease cost	339.64	32.96%
- IPQA hold lot test cost	36.75	3.57%
- IPQA after rework test cost	14.13	1.37%
- OQA after rework test cost	0.00	0.00%
- All retest cost	50.88	4.94%
- IPQA rework cost	218.06	21.16%
- IPQA scrap cost	0.00	0.00%
- OQA rework cost	0.86	0.08%
- OQA scrap cost	0.00	0.00%
- All defect costs	218.93	21.25%
External failure costs		
-Cost of replacement	43.30	4.20%
-Cost of transport	18.36	1.78%
Sum failure costs	803.45	77.97%
Total cost	1,030.42	100.00%
Machine investment cost	215.08	
Total cost + machine investment cost	1,245.51	

All costs including total cost are the same as the optimum sampling plan under 10 and 11 machines. This is because the sampling plan is the same as those of using 10 and 11 machines. The total cost cannot be reduced by increasing in the activity of testing. Moreover, there is an increasing in machine investment cost due to the additional number of testing machines. Therefore, the sum of total cost and machine investment cost is higher than others.

6.4 Discussion

In this session, all results including the present sampling plan, the optimum sampling plan under the current number of testing machines, and the optimum sampling plan under the additional new testing machines will be presented in graphs and tables. The Figure 6.1 shows the cost trend under numbers of machine. Also, the Table 6.19 shows the details of results under numbers of machine.

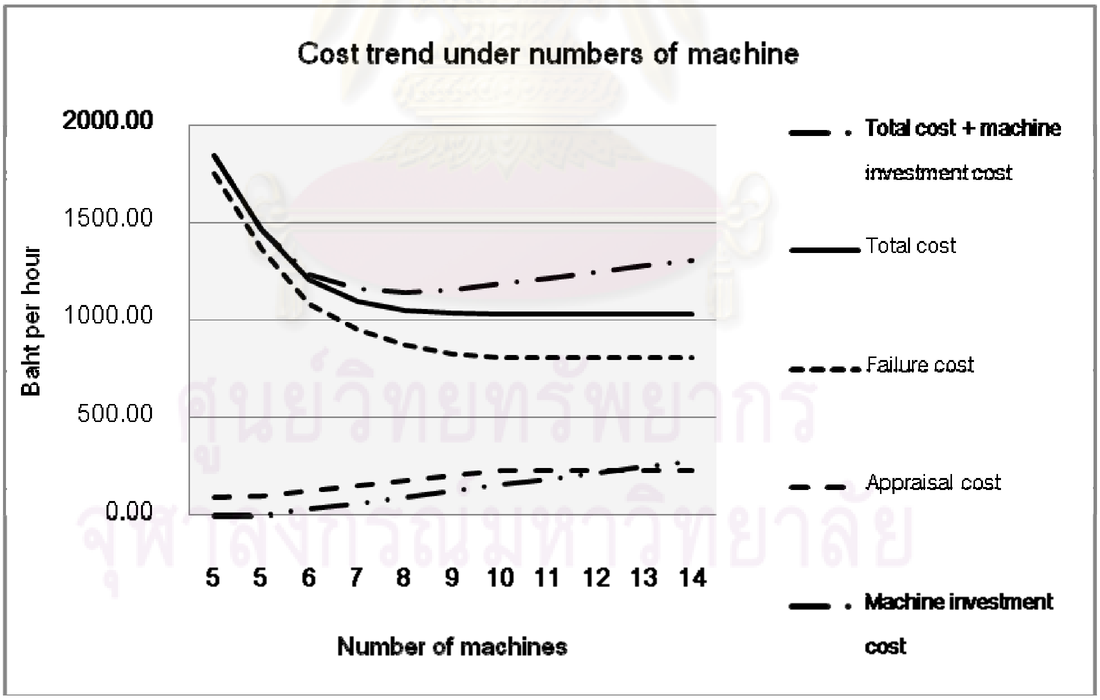

Figure 6.1: Cost Trend under Numbers of Machine

The total cost in the optimum sampling plan under 5 machines is lower than the present sampling plan in significant level even though the number of machines used in both sampling plan are the same. This is because the failure costs are reduced abundantly while the appraisal costs are increased slightly. These effects are caused by an increasing in sample size which affects directly to the β value. The testing capacity is utilized in full capacity to screen the defective product before delivering to the customer resulting in lower failure costs. There is no additional new testing machine so the machine investment cost is none.

Table 6.19: Optimum Sampling Plan under the Additional New Testing Machines Result Conclusion

No. of Machines	n			f			β			P			P'		
	M1	M2	M3	M1	M2	M3	M1	M2	M3	M1	M2	M3	M1	M2	M3
5	2	2	1	0.25	0.25	0.25	0.57	0.57	0.84	0.011	0.010	0.002	0.011	0.010	0.002
5	4	4	4	0.13	0.22	0.03	0.16	0.16	0.16	0.011	0.008	0.002	0.011	0.008	0.002
6	4	4	4	0.20	0.35	0.04	0.16	0.16	0.16	0.009	0.006	0.002	0.009	0.006	0.002
7	4	4	4	0.26	0.51	0.06	0.16	0.16	0.16	0.008	0.005	0.002	0.008	0.005	0.002
8	4	4	4	0.32	0.67	0.07	0.16	0.16	0.16	0.007	0.004	0.002	0.007	0.004	0.002
9	4	4	4	0.36	0.84	0.09	0.16	0.16	0.16	0.006	0.004	0.002	0.006	0.004	0.002
10	4	4	4	0.40	0.97	0.10	0.16	0.16	0.16	0.006	0.004	0.002	0.006	0.004	0.002
11	4	4	4	0.40	0.97	0.10	0.16	0.16	0.16	0.006	0.004	0.002	0.006	0.004	0.002
12	4	4	4	0.40	0.97	0.10	0.16	0.16	0.16	0.006	0.004	0.002	0.006	0.004	0.002
13	4	4	4	0.40	0.97	0.10	0.16	0.16	0.16	0.006	0.004	0.002	0.006	0.004	0.002
14	4	4	4	0.40	0.97	0.10	0.16	0.16	0.16	0.006	0.004	0.002	0.006	0.004	0.002

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Table 6.20: Optimum Sampling Plan under the Additional New Testing Machines Costs Conclusion

No. of Machines	Appraisal cost	Failure cost	Total cost	Machine investment cost	Total cost + machine investment cost
5	89.94	1,753.79	1,843.74	-	1,843.74
5	97.69	1,365.02	1,462.70	-	1,462.70
6	122.19	1,081.57	1,203.77	30.73	1,234.49
7	148.99	948.96	1,097.94	61.45	1,159.39
8	176.88	874.84	1,051.72	92.18	1,143.90
9	205.36	828.26	1,033.62	122.91	1,156.53
10	226.97	803.45	1,030.42	153.63	1,184.06
11	226.97	803.45	1,030.42	184.36	1,214.78
12	226.97	803.45	1,030.42	215.08	1,245.51
13	226.97	803.45	1,030.42	245.81	1,276.24
14	226.97	803.45	1,030.42	276.54	1,306.96

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The failure costs are reduced significantly. This is because the testing capacity is more sufficient for testing the product before the failure costs occurred. The trend of the failure costs are shown in Figure 6.1: Cost Trend under Numbers of Machines .The failure costs are reduced continuously until the number of machines is 10. From that point, the failure costs are steady without any declining trend. The more decreasing the failure costs is, the more effort the testing activity had to be put. However, higher level of testing activity requires higher appraisal costs also. As a consequence, a lot amount of appraisal costs will result in higher optimum total cost so the model does not allow that consequence to be happened.

The appraisal costs are higher due to the improvement of testing activity by an increasing in testing machine investment. The rising amount of appraisal costs is considered to be small compared to the saving from failure costs. This is why the total cost can be reduced. The appraisal costs are increased continuously until the number of testing machine is 10 machines. From that point, an increasing in testing activity cannot reduce the total cost anymore. This is because the increasing amount of appraisal costs is more than the saving from failure costs reduction.

Without considering the machine investment cost, the total cost is decreased continuously until it reaches its optimum solution in using 10 machines. The reason why there is the point which the total cost will not be decreased anymore is that the testing activity also costs money even it can help reduce some failure costs. Nevertheless, the model already found its optimum sample size and sampling frequency that can minimize the total cost so the sampling plan is not changed afterwards.

The lowest total cost is done by the optimum sampling plan using 10 machines. However, the additional new testing machines also cost money. This issue is turned to be the main concern that has to be considered further.

Truly, the model can be optimized by using the optimum sample size and sampling frequency under the 10 machines. However, the company does not already own 10 machines at the present. The company has to invest for those new machines. Thus, the sum of total cost and machine investment cost has the lowest value in 8 machines solution. After that, the sum value of total cost and machine investment cost is continuously increased due to the fact that the amount of total cost saved is not enough to cover the cost of machine investment. Therefore, the 8 machine solution can give the optimum summed of total cost and machine investment cost which is the best solution for the company.

In statistical quality perspective, the β value is decreased significantly from 0.57and 0.84 to 0.16 when using the new optimum sampling plan. This is because increasing in sample size will reduce the β value directly. This means that defective products will be produced in smaller amount resulting in lower failure cost. Percentage of defective product produced from IPQA process (P) and Percentage of defective product produced from OQA process (P') are also decreased along with the failure cost.

6.5 Conclusion

6.5.1 Present Number of Testing Machine

The optimized model can find the sample size and sampling frequency that reduce the total cost while there is no need to invest in an additional testing machine. This can be done by reducing the failure costs which arisen from an increasing in the testing activity. However, increasing in the testing activity will affects the appraisal costs also. The sample size should be increased to satisfy the statistical constraint because the present β values are considered to be high at present (0.57 and 0.84). The high value of type two error (β) resulted in large amounts of defective products produced before the control chart can detect. The defective products require many processes to repair them such as retesting and reworking process. The sampling frequency is adjusted individually in the optimum sampling plan model according to the characteristic of each product. The sampling frequency is decreased because increasing in sample size can bring about more benefits than increasing in sampling frequency in this case. However, the sampling frequency will be increased if the testing capacity is escalated by investing in the additional new testing machine. The appraisal costs are increased because of an increasing in testing activities. Nevertheless, the increasing amount of the appraisal costs is much less than the saving amount from the failure costs.

Table 6.21: Optimum Sampling Plan under the Current Number of Testing Machines Conclusion

	M1	M2	M3
n	4	4	4
f	0.13	0.22	0.03
1/f	7.54	4.62	38.99
β	0.16	0.16	0.16
ATS	8.96	5.49	46.34
P	0.011	0.008	0.002
Z	10843	21531	43096
P'	0.011	0.008	0.002
ATU	5.00		
Appraisal costs	97.69		
Failure costs	1,365.02		
Total cost	1,462.70		

6.5.2 Investing Machine Solution

Investing in the additional new testing machine can reduce the total cost because the new testing machine can increase the current testing capacity. The failure costs can be reduced significantly by trading off with the slightly additional appraisal costs which caused by the testing activity.

From the optimized models under the additional new testing machines, the question is what amount of the new testing machine will deliver the best benefits to the company. After developing all of the optimum sampling plans, the best result is from 10 machines solution which can minimize the total cost. However, there is a machine investment cost occur from every number of the new invested machine. This is because the company has owned only 5 machines so it has to invest for the extra machine. The sum of total cost and machine investment cost has the lowest value in 8 machines solution. Since the 2 more added machines from 8 machines can reduce the total cost in fewer amounts than their machine investment cost, the sum of total cost and machine investment cost for 10 machines is not the best solution. Therefore, the 8 machines solution is the best solution that can provide minimum sum of total cost and machine investment cost. From the model, the company should investing in 3 more machines to increase the number of machine to be 8 machine and apply the purpose sampling plan to minimize the summed of total cost and machine investment cost.

Table 6.22: Optimum Sampling Plan under 8 Testing Machines Conclusion

	M1	M2	M3
n	4	4	4
f	0.32	0.67	0.07
1/f	3.17	1.49	13.80
β	0.16	0.16	0.16
ATS	3.77	1.77	16.40
P	0.007	0.004	0.002
Z	4584	7025	15283
P'	0.007	0.004	0.002
ATU	8.00		
Appraisal cost	176.88		
Failure cost	874.84		
Total cost	1,051.72		
Machine investment cost	92.18		
Total cost + machine investment cost	1,143.90		

CHAPTER VII

SENSITIVITY ANALYSIS

This chapter represents the model sensitivity analysis. This chapter consists of objective of the sensitivity analysis and all of the sensitivity analysis designed.

7.1 Objective of the Sensitivity Analysis

In previous research, a sensitivity analysis is performed to examine the cost effect of changing the controlled properties such as type I error rate, power, average time to signal (ATS) (Zhang and Berardi, 1997). The new optimum sampling plan for changed parameters is developed along with the new total cost.

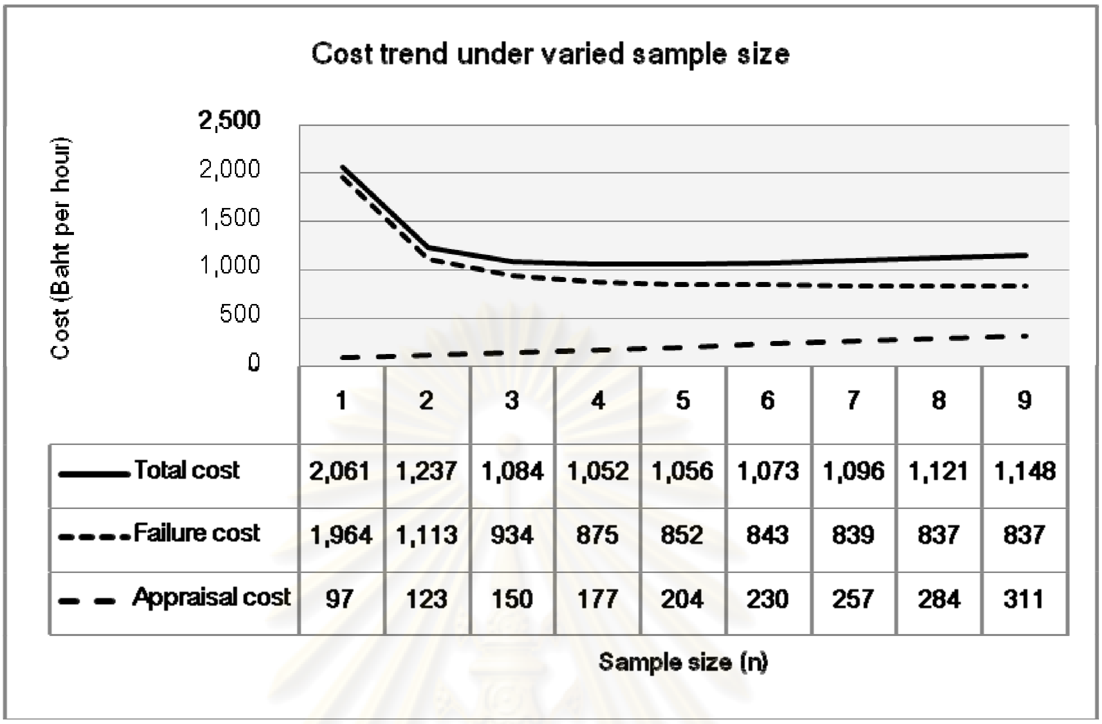
In this thesis, there will be three designed sensitivity analysis to illustrate how sensitive of each variable and parameter over the total cost is. The sensitivity analysis will be applied on the optimum sampling plan under eight machines which can deliver the best solution to minimize the sum of total cost and machine investment cost. The sensitivity analysis is applied on the optimum sampling plan under eight machines because it is the best solution that the company should apply it in real case.

The first designed sensitivity analysis is to vary the sample size and sampling frequency to see the change of the total cost. The objective of this sensitivity analysis is to analyze the effect of the changed value of sample size and sampling frequency over the total cost. Knowing the effect in changing the value of sample size and sampling frequency beforehand is beneficial for the company since the unexpected circumstance such as testing machine error and human error can occur all the time. In addition, the company can decide whether to reduce the sample size or the sampling frequency if the testing capacity is limited. However, the machine investment cost will not be considered in this sensitivity analysis since the machine investment cost is not a function of sample size and sampling frequency and it is a fixed cost that the company already invested. Changing in sample size and sampling frequency will not affect the machine investment cost. Therefore, the cost considered in this sensitivity analysis is the total cost, not the sum of total cost and machine investment cost.

The second designed sensitivity analysis is to vary some selected parameter values up to 50% change to see the change of the total cost by supposing that the user incorrectly estimates any of the values by up to 50 % and the values of the parameters given in the example are assumed to be the basic case (Yu and Chen, 2005). The objective of this sensitivity analysis is to analyze the effect of the changed value of parameters over the total cost. The new sample size and sampling frequency for changed parameter values will be illustrated along with the new total cost. For example, the total cost and the new sampling plan will be calculated in case that the labor rate is changed up to 50%.

The objective of the third designed sensitivity analysis is to classify the parameters that have a crucial effect to the total cost in order to control them. The methodology in analyzing which parameter has a crucial effect to the total cost is to change the value of parameter one by one until the total cost is changed 10% higher than the optimum total cost. The optimum total cost is 1,052 Baht per hour so the maximum limit of cost is 1,157 Baht per hour which is higher than the optimum total cost 10%. For example, the labor rate (L_m) has to increase up to 52% from the normal rate to reach the cost limit which means that the labor rate can increase up to 52% before it exceeds the cost limit.

7.2 Sample Size Sensitivity Analysis

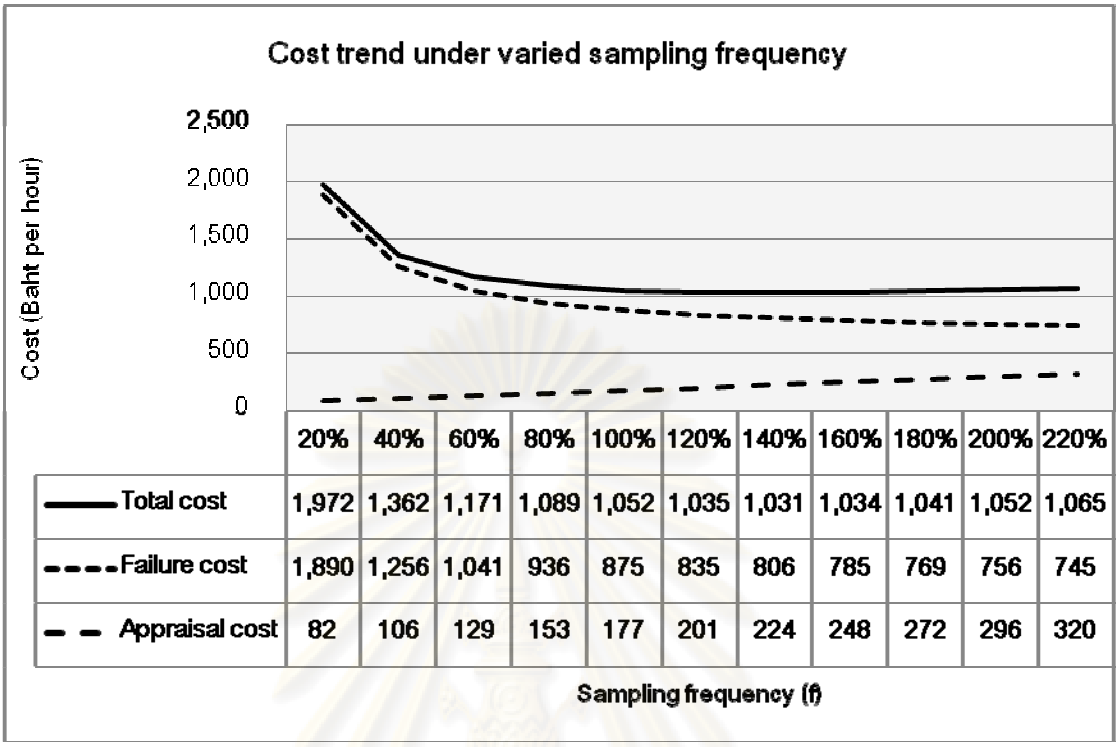


	1	2	3	4	5	6	7	8	9
Total cost	2,061	1,237	1,084	1,052	1,056	1,073	1,096	1,121	1,148
Failure cost	1,964	1,113	934	875	852	843	839	837	837
Appraisal cost	97	123	150	177	204	230	257	284	311

Figure 7.1: Cost Trend under Varied Sample Size

The total cost is reduced dramatically when the sample size is increased from 1 to 4 pieces per time especially from 1 to 2 pieces per time. This is because the failure costs are reduced rapidly while the appraisal costs are increased slightly. The total cost starts to rise from its lowest value where the sample size is 4 pieces per time due to the fact that the appraisal costs are higher along with the sample taken while the failure costs are reduced in a smaller value compared to those increased in appraisal costs. Further increasing in sample size from 4 pieces per time will give no satisfied solution in term of cost.

7.3 Sampling Frequency Sensitivity Analysis



	20%	40%	60%	80%	100%	120%	140%	160%	180%	200%	220%
Total cost	1,972	1,362	1,171	1,089	1,052	1,035	1,031	1,034	1,041	1,052	1,065
Failure cost	1,890	1,256	1,041	936	875	835	806	785	769	756	745
Appraisal cost	82	106	129	153	177	201	224	248	272	296	320

Figure 7.2: Cost Trend under Varied Sampling Frequency

The total cost is reduced rapidly when there is an increasing in sampling frequency during 20% to 80% of the optimum sampling frequency. This is because more frequency of sampling activity can reduce the failure costs by detecting an assignable cause quicker. However, the more frequency the sampling activity is, the more the appraisal costs increased. Increasing in sampling frequency more than 140% of the optimum sampling frequency will make the total cost higher.

7.4 Parameter Sensitivity Analysis

The parameters will be classified into 4 categories in this parameter sensitivity analysis which are time, production, labor and machine, and product cost category. Time category consists of time period that the process is in an in-control stage (T_0), time period for finding an assignable cause (T_1), time period that the process is stopped for repairing (T_2), time for collecting samples from each machine (T_{Pick}), and time for testing a sample per piece (T_{Test}). Production category consists of probability of defect occurs when the process is in-control (γ_1), probability of defect occurs when the process is out-of-control (γ_2), and production rate (P_r). Labor and machine category consists of engineer labor rate (S_1), technician labor rate (S_2), staff labor rate (L_m), testing machine operating rate (M), and rework rate (R). Product cost category consists of product profit (P_n), product cost in IPQA process ($C_{P\ IPQA}$), product cost in OQA process ($C_{P\ OQA}$), product cost when delivered to the customer ($C_{P\ Cus}$), cost of transportation to the customer (C_T), and customer penalty rate (P_{en}).

The parameters value will be varied up and down 50% to illustrate the changed sampling plan and the total cost.

7.5 Time Category Sensitivity Analysis

7.5.1 Time Period that the Process Is in an In-control Stage (T_0)

Table 7.1: Time Period that the Process Is in an In-control Stage (T_0) Sensitivity

	T_0			Optimized n			Optimized f			Total cost
	M1	M2	M3	M1	M2	M3	M1	M2	M3	
Case1 50%	3.94	9.51	611.13	4	4	4	0.27	0.47	0.06	1,852
Case2 100%	7.89	19.02	1,222.26	4	4	4	0.32	0.67	0.07	1,052
Case3 150%	11.83	28.53	1,833.40	4	4	4	0.35	0.76	0.08	794

The total cost can be reduced significantly when the time period that the process is in an in-control stage (T_0) is increased. This is because the defective products produced while the process is in-control are much lower than those produced when the process is out-of-control. Longer T_0 means that the process is in an in-control stage longer before the failure occurs. On the other hand, the lower the T_0 value, the higher the total cost. The sample size is remained the same but the sampling frequency is decreased in case 1 and increased in case 3.

7.5.2 Time Period for Finding an Assignable Cause (T_1)

Table 7.2: Time Period for Finding an Assignable Cause (T_1) sensitivity

	T1			Optimized n			Optimized f			Total cost
	M1	M2	M3	M1	M2	M3	M1	M2	M3	
Case1 50%	0.02	0.02	0.02	4	4	4	0.32	0.67	0.07	1,047
Case2 100%	0.04	0.04	0.05	4	4	4	0.32	0.67	0.07	1,052
Case3 150%	0.06	0.06	0.07	4	4	4	0.32	0.67	0.07	1,057

Changing in time period for finding an assignable cause (T_1) value has a slight effect to the total cost and has no effect to the optimum sampling plan. The lower in T_1 value can reduce the total cost because the T_1 parameter has direct effect to the amount of hold product (Z). If the technician can find an assignable cause quicker, the amount of hold product will be lower and the amount of defective product that may be reworked further will be lower also.

7.5.3 Time Period that the Process Is Stopped for Repairing (T_2)

Table 7.3: Time Period that the Process Is Stopped for Repairing (T_2) Sensitivity

	T2			Optimized n			Optimized f			Total cost
	M1	M2	M3	M1	M2	M3	M1	M2	M3	
Case1 50%	0.24	0.24	0.24	4	4	4	0.35	0.66	0.07	834
Case2 100%	0.48	0.48	0.48	4	4	4	0.32	0.67	0.07	1,052
Case3 150%	0.71	0.71	0.71	4	4	4	0.28	0.69	0.08	1,263

The total cost can be reduced significantly by the reduction in time period that the process is stopped for repairing (T_2) value. This is because T_2 parameter directly affects to the cease cost which is considered to be a high portion in failure costs. On the other hand, increasing in T_2 value can increase the total cost apparently. Therefore, the sooner the engineer can repair the process, the lower the cease cost caused from an opportunity lost occurs. Also, the sampling frequency is changed by changing in T_2 value.

7.5.4 Time for Collecting Samples from Each Machine (T_{Pick})

Table 7.4: Time for Collecting Samples from Each Machine (T_{Pick}) Sensitivity

	T_{Pick}			Optimized n			Optimized f			Total cost
	M1	M2	M3	M1	M2	M3	M1	M2	M3	
Case1 50%	0.042	0.042	0.042	4	4	4	0.32	0.67	0.07	1,046
Case2 100%	0.083	0.083	0.083	4	4	4	0.32	0.67	0.07	1,052
Case3 150%	0.125	0.125	0.125	4	4	4	0.32	0.67	0.07	1,058

Changing in time for collecting samples from each machine value has a slight effect to the total cost and has no effect to the optimum sampling plan. Reduction in time for collecting samples from each machine (T_{Pick}) can reduce the total cost because staff can use less time for sampling activity resulting in lower cost of labor.

7.5.5 Time for Testing a Sample per Piece (T_{Test})

Table 7.5: Time for Testing a Sample per Piece (T_{Test}) Sensitivity

	T_{Test}			Optimized n			Optimized f			Total cost
	M1	M2	M3	M1	M2	M3	M1	M2	M3	
Case1 50%	0.010	0.010	0.017	4	4	4	0.48	1.33	0.14	888
Case2 100%	0.021	0.021	0.033	4	4	4	0.32	0.67	0.07	1,052
Case3 150%	0.157	0.259	0.030	4	4	4	0.32	0.67	0.07	1,419

Changing in time for testing a sample per piece (T_{Test}) has a high effect to the total cost and the sampling plan. The total cost can be reduced significantly if there is a reduction in time for testing a sample per piece (T_{Test}). This is because the testing machine capacity (K) which is a function of T_{Test} ($K = \frac{1}{T_{testi}}$) can be increased in high level due to the reduction in time for testing a sample per piece (T_{Test}) without any investment in the new testing machine. Also, the sampling frequency can be increased significantly because of an increasing in the testing machine capacity.

7.6 Production Category Sensitivity Analysis

7.6.1 Probability of Defect Occurs when the Process Is In-control (γ_1)

Table 7.6: Probability of Defect Occurs when the Process Is In-control (γ_1) Sensitivity

	γ_1			Optimized n			Optimized f			Total cost
	M1	M2	M3	M1	M2	M3	M1	M2	M3	
Case1 50%	0.0003	0.0012	0.0009	4	4	4	0.31	0.67	0.07	1,040
Case2 100%	0.0006	0.0023	0.0017	4	4	4	0.32	0.67	0.07	1,052
Case3 150%	0.0009	0.0035	0.0026	4	4	4	0.32	0.67	0.07	1,068

The lower the defective products produced, the lower the failure costs occur. The total cost can be reduced by the reduction in the probability of defect occurs when the process is in-control (γ_1). Turning machine quality has direct effect to the γ_1 value. However, changing in γ_1 value has a slight effect to the total cost and the sampling frequency because the γ_1 value is considered to be small compared with the γ_2 value.

7.6.2 Probability of Defect Occurs when the Process Is Out-of-control (γ_2)

Table 7.7: Probability of Defect Occurs when the Process Is Out-of-control (γ_2) Sensitivity

	γ_2			Optimized n			Optimized f			Total cost
	M1	M2	M3	M1	M2	M3	M1	M2	M3	
Case1 50%	0.0104	0.0277	0.0456	4	4	4	0.03	0.43	0.07	723
Case2 100%	0.0208	0.0554	0.0911	4	4	4	0.32	0.67	0.07	1,052
Case3 150%	0.0312	0.0831	0.1367	4	4	4	0.48	0.57	0.05	1,298

The probability of defect occurs when the process is out-of-control (γ_2) has more influence to the total cost than γ_1. This is because the value of γ_2 is more than the value of γ_1 which resulted in more defective products produced. Therefore, increasing in γ_2 will increase the total cost and the amount of defective product significantly. On the other hand, decreasing in γ_2 will lower the total cost significantly. The sampling frequency is affected by changing in γ_2 value. In product M1, the sampling frequency is increased significantly because the increasing in γ_2 value.

7.6.3 Production Rate (P_r)

Table 7.8: Production Rate (P_r) Sensitivity

	P_r			Optimized n			Optimized f			Total cost
	M1	M2	M3	M1	M2	M3	M1	M2	M3	
Case1 50%	603	1,947	465	4	4	4	0.28	0.66	0.07	682
Case2 100%	1,205	3,893	929	4	4	4	0.32	0.67	0.07	1,052
Case3 150%	1,808	5,840	1,394	4	4	4	0.26	0.49	0.05	1,517

The production rate (P_r) is one of the major parameters that influence the total cost in the high degree. Increasing in P_r will increase the total cost significantly. This is because the total cost is the cost that paid for checking and repairing the product and process so if the amount of product is high, the total cost is high also. The sampling frequency is changed if the P_r changed due to the fact that the sampling frequency has to be high in order to check the process sufficiently. As Alexander et al. (1995) stated that the sampling interval is generally based on the production rate and familiarity with the process. However, even P_r will increase the total cost, the profits from selling more product will be increased also.

7.7 Labor and Machine Category Sensitivity Analysis

7.7.1 Engineer Labor Rate (S_1)

Table 7.9: Engineer Labor Rate (S_1) Sensitivity

	S1			Optimized n			Optimized f			Total cost
	M1	M2	M3	M1	M2	M3	M1	M2	M3	
Case1 50%	32.05	32.05	32.05	4	4	4	0.33	0.67	0.07	991
Case2 100%	64.10	64.10	64.10	4	4	4	0.32	0.67	0.07	1,052
Case3 150%	96.15	96.15	96.15	4	4	4	0.30	0.68	0.07	1,113

Changing in engineer labor rate (S_1) has a direct effect to the total cost. The total cost can be reduced when the engineer labor rate is reduced. This is because the engineer has to be used to repair an assignable cause which resulted in an occurrence of true alarm cost. The sampling frequency is changed also by changing in S_1 value.

7.7.2 Technician Labor Rate (S_2)

Table 7.10 Technician Labor Rate (S_2) Sensitivity

	S2			Optimized n			Optimized f			Total cost
	M1	M2	M3	M1	M2	M3	M1	M2	M3	
Case1 50%	17.63	17.63	17.63	4	4	4	0.32	0.67	0.07	1,049
Case2 100%	35.26	35.26	35.26	4	4	4	0.32	0.67	0.07	1,052
Case3 150%	52.88	52.88	52.88	4	4	4	0.31	0.67	0.07	1,054

Changing in technician labor rate (S_2) has a slight effect to the total cost. The total cost can be reduced when the technician labor rate is reduced. However, changing in the total cost from the technician labor cost is considered to be less compared to the engineer labor cost. The sampling frequency is slightly changed in case 3.

7.7.3 Staff Labor Rate (L_m)

Table 7.11 Staff Labor Rate (L_m) Sensitivity

	L_m			Optimized n			Optimized f			Total cost
	M1	M2	M3	M1	M2	M3	M1	M2	M3	
Case1 50%	12.59	12.59	12.59	4	4	4	0.32	0.67	0.07	950
Case2 100%	25.18	25.18	25.18	4	4	4	0.32	0.67	0.07	1,052
Case3 150%	37.77	37.77	37.77	4	4	4	0.32	0.67	0.07	1,153

The total cost is increased significantly when the staff labor rate (L_m) increased. This is because staff plays an important role in picking and testing the products. An increasing in L_m can affect the total cost since the company has to pay for its labors in a higher rate. However, the sampling frequency and sample size are considered to be the same even though the L_m value is changed.

7.7.4 Testing Machine Operating Rate (M)

Table 7.12 Testing Machine Operating Rate (M) Sensitivity

	M			Optimized n			Optimized f			Total cost
	M1	M2	M3	M1	M2	M3	M1	M2	M3	
Case1 50%	1.42	1.42	1.42	4	4	4	0.32	0.67	0.07	1,040
Case2 100%	2.84	2.84	2.84	4	4	4	0.32	0.67	0.07	1,052
Case3 150%	4.26	4.26	4.26	4	4	4	0.32	0.67	0.07	1,063

Testing machine operating rate (M) has a slight influence to the total cost. Decreasing in testing machine operating rate can reduce the total cost. However, the sampling frequency is not changed by changing in M value.

7.7.5 Rework Rate (R)

Table 7.13: Rework Rate (R) Sensitivity

	R			Optimized n			Optimized f			Total cost
	M1	M2	M3	M1	M2	M3	M1	M2	M3	
Case1 50%	1.09	0.66	0.52	4	4	4	0.31	0.65	0.07	903
Case2 100%	2.18	1.32	1.04	4	4	4	0.32	0.67	0.07	1,052
Case3 150%	3.27	1.98	1.56	4	4	4	0.32	0.67	0.07	1,200

The rework rate (R) has a significant influence to the total cost. The total cost can be reduced by the reduction in rework rate (R). This is because every defective product needed to be reworked so the lower the rework rate, the lower the failure costs and total cost. The sampling frequency is decreased in case 1 and remained the same in case 3.

7.8 Product Cost Sensitivity Analysis

7.8.1 Product Profit (P_n)

Table 7.14: Product Profit (P_n) Sensitivity

	P_n			Optimized n			Optimized f			Total cost
	M1	M2	M3	M1	M2	M3	M1	M2	M3	
Case1 50%	0.73	1.48	3.69	4	4	4	0.34	0.66	0.07	887
Case2 100%	1.46	2.96	7.37	4	4	4	0.32	0.67	0.07	1,052
Case3 150%	2.19	4.44	11.06	4	4	4	0.29	0.68	0.08	1,215

Changing in product profit (P_n) has a significant effect to the total cost. The total cost has direct variation to the product profit. This is because P_n parameter directly affects to the cease cost which is considered to be a high portion in failure costs. However, the company may not want to reduce the value of this parameter since everyone usually prefers having high profit. The sampling frequency also changed by changing the value of P_n.

7.8.2 Product Cost in IPQA Process ($C_{P\ IPQA}$)

Table 7.15: Product Cost in IPQA Process ($C_{P\ IPQA}$) Sensitivity

	$C_{P\ IPQA}$			Optimized n			Optimized f			Total cost
	M1	M2	M3	M1	M2	M3	M1	M2	M3	
Case1 50%	5.42	4.72	3.45	4	4	4	0.32	0.67	0.07	1,052
Case2 100%	10.83	9.43	6.90	4	4	4	0.32	0.67	0.07	1,052
Case3 150%	16.25	14.15	10.35	4	4	4	0.32	0.67	0.07	1,052

Changing in product cost in IPQA process ($C_{P\ IPQA}$) has an insignificant effect to the total cost. This is because the amount of the product that turned to be a scrap is considered to be low. The reworking process is efficient enough to repair the defective product. The sampling frequency and sample size are the same also.

7.8.3 Product Cost in OQA Process ($C_{P\ OQA}$)

Table 7.16: Product Cost in OQA Process ($C_{P\ OQA}$) Sensitivity

	$C_{P\ OQA}$			Optimized n			Optimized f			Total cost
	M1	M2	M3	M1	M2	M3	M1	M2	M3	
Case1 50%	5.92	5.22	3.95	4	4	4	0.32	0.67	0.07	1,052
Case2 100%	11.83	10.43	7.90	4	4	4	0.32	0.67	0.07	1,052
Case3 150%	17.75	15.65	11.85	4	4	4	0.32	0.67	0.07	1,052

Also, changing in product cost in OQA process has an insignificant effect to the total cost as changing in $C_{P\ IPQA}$. The amount of the defective product that turned to be a scrap is low. Hence, the sampling plan and the total cost still equal to the original value.

7.8.4 Product Cost when Delivered to the Customer ($C_{P\ Cus}$)

Table 7.17: Product Cost when Delivered to the Customer ($C_{P\ Cus}$) Sensitivity

	$C_{P\ Cus}$			Optimized n			Optimized f			Total cost
	M1	M2	M3	M1	M2	M3	M1	M2	M3	
Case1 50%	6.16	6.22	7.16	4	4	4	0.27	0.69	0.07	1,045
Case2 100%	12.31	12.43	14.31	4	4	4	0.32	0.67	0.07	1,052
Case3 150%	18.47	18.65	21.47	4	4	4	0.34	0.66	0.07	1,057

Product cost when delivered to the customer ($C_{P\ Cus}$) has a slight effect to the total cost and the optimum sampling plan. The total cost is higher if the $C_{P\ Cus}$ is increased. This is because the cost of replacement is the function of $C_{P\ Cus}$. If $C_{P\ Cus}$ is higher, the company has to pay for the replacement cost in a higher amount in case that there is an external failure occurred. However, the $C_{P\ Cus}$ is one of the values that cannot be improved or changed to reduce the total cost. The sampling frequency is changed by changing in $C_{P\ Cus}$ value.

7.8.5 Customer Penalty Rate (P_{en})

Table 7.18: Customer Penalty Rate (P_{en}) Sensitivity

	P_{en}			Optimized n			Optimized f			Total cost
	M1	M2	M3	M1	M2	M3	M1	M2	M3	
Case1 50%	61.55	62.15	71.55	4	4	4	0.31	0.68	0.07	1,030
Case2 100%	123.10	124.30	143.10	4	4	4	0.32	0.67	0.07	1,052
Case3 150%	184.65	186.45	214.65	4	4	4	0.32	0.67	0.07	1,073

Changing in customer penalty rate (P_{en}) has an effect to the total cost in moderate degree. The total cost will be increased if the customer penalty rate is increased. This is because the company has to pay the higher penalty charge to the customer for each defective product. However, the effect is not very high due to the fact that the number of the product needed to be replaced is limited due to the low rate of defective products delivered. This can be implied that the company is now having high quality control procedure so that it can screen the defective product quite well resulting in a low amount of the defective product rejected. The sampling frequency is changed when the P_{en} is reduced in case1.

7.8.6 Cost of Transportation to the Customer (C_T)

Table 7.19: Cost of Transportation to the Customer (C_T)

	C_T			Optimized n			Optimized f			Total cost
	M1	M2	M3	M1	M2	M3	M1	M2	M3	
Case1 50%	514.00	514.00	514.00	4	4	4	0.32	0.67	0.07	1,043
Case2 100%	1,028.00	1,028.00	1,028.00	4	4	4	0.32	0.67	0.07	1,052
Case3 150%	1,542.00	1,542.00	1,542.00	4	4	4	0.32	0.67	0.07	1,061

The change in cost of transportation to the customer (C_T) has a slight effect to the total cost. This is because the number of the product that needed to be replaced is limited due to the low rate of defective product delivered. An increasing in C_T can increase the total cost because the company has to pay more in transportation cost which is considered to be an external failure cost. The sampling frequency and the sample size are remained the same because the number of defective products delivered to the customer is consider to be small so changing in cost of transportation to the customer (C_T) has no significant effect to the sampling plan solved.

7.9 Parameter Sensitivity Analysis Conclusion

Table 7.20 illustrated the changed value of each parameter that leads the total cost to be in a higher direction. Time period that the process is in an in-control stage (T_0) is the only one parameter that has opposite direction with the total cost. Furthermore, it can be seen that some of the parameter can change the total cost significantly while some cannot change the total cost even the percentage of the changed value is the same. The optimal total cost is 1,052 Baht per hour. Therefore, the parameters which have a crucial effect to the total cost will be studied further in the next section in order to help the company control the factors that influence the total cost crucially.

Table 7.20: Parameter Sensitivity Conclusion

Categories	Parameters	Changed value	Total cost	% change from optimum total cost
Time category	T_0	50%	1,852	76%
	T_1	150%	1,057	0%
	T_2	150%	1,263	20%
	T_{Pick}	150%	1,058	1%
	T_{Test}	150%	1,419	35%
Production category	γ_1	150%	1,068	2%
	γ_2	150%	1,298	23%
	P_r	150%	1,517	44%
Labor and machine category	S_1	150%	1,113	6%
	S_2	150%	1,054	0%
	L_m	150%	1,153	10%
	M	150%	1,063	1%
	R	150%	1,200	14%
Product cost category	P_n	150%	1,215	15%
	C_{PIPQA}	150%	1,052	0%
	C_{POQA}	150%	1,052	0%
	C_{PCUS}	150%	1,057	0%
	P_{en}	150%	1,073	2%
	C_T	150%	1,061	1%

7.10 Parameter which Has a Crucial Effect to the Total Cost

The objective of this sensitivity analysis is to classify the parameters that have a crucial effect to the total in order to control them. The way to justify whether the parameter has a crucial effect to the total cost or not is to vary each parameter with the same optimum plan until it reach the cost limited of the total cost which is 10% change. The table below shows the percentage of each parameter that changed from the optimum total cost to the cost limited. Therefore, the lower the percentage, the higher effect to the total cost.

Table 7.21: Parameter which Has a Crucial Effect to the Total Cost

Parameters	Percentage changed
T_0	13%
P_r	14%
γ_2	20%
T_2	25%
P_n	32%
R	36%
T_{Test}	49%
L_m	52%
S_1	86%
γ_1	178%
P_{en}	240%
M	460%
C_T	570%
T_{Pick}	900%
C_{PCUS}	940%
T_1	1060%
S_2	1930%
C_{PIPQA}	-
C_{POQA}	-

From the table above, it can be seen that the time period that the process is in an in-control stage (T_0) has an effect to the total cost the most. Its value can be changed up to 13% before the cost limited is reached. The production rate (P_r) has a high effect to the total cost. However, reducing the production rate (P_r) may not considered being a good choice for the company since a high production rate can create more money due to higher amount of products produced resulting in higher profits given to the company. Probability of defect occurs when the process is out-of-control (γ_2) also has a high effect to the total cost since it makes defective products produced in a high amount while the process is out-of-control. Time period that the process is stopped for repairing (T_2), product profit (P_n), rework rate (R), time for testing a sample per piece (T_{Test}), staff labor rate (L_M), and engineer labor rate (S_1) also have a significant effect to the total cost. Changing in these parameters has to be monitored since it can make the total cost over the cost limited easily. Product cost in IPQA process ($C_{P\ IPQA}$) and product cost in OQA process $C_{P\ OQA}$ have very few effects to the total cost so that its effect is unable to see. This is because the product that needed turned to be a scrap is very low.

This sensitivity analysis is used to identify which parameter has a high effect to the total cost in percentage. However, the possibility of changing each parameter is also considered to be the main consideration. For example, time for testing a sample per piece (T_{Test}) can be changed up to 49% before it reaches the cost limit. Nevertheless, the possibility for this parameter to be changed up to 49% is considered to be low since the testing machine is an automatic operation. Therefore, analyzing historical data and also considering about the possibility of changing along with the sensitivity analysis result is important in order to create the feasible solution that can control the total cost below the cost limit.

CHAPTER VIII

DISCUSSION AND CONCLUSION

Every mathematical model has to be generated based on specific assumptions so the model can be applied with specific circumstances also. In the same way as every mathematical model, the economic mathematical model in this thesis was developed based on many assumptions. For example, the production process has to be a steady state production so that the X-bar control chart can be applied. The standard deviation (SD) has to be known and stable even the process mean changed. Rate of production is sufficiently high which we can neglect the possibility of a change in the process occurred during the period of taking samples. Truly, these assumptions are used widely in this field of research, but some might not be compatible when using the model in the realistic way. For example, the production process may not be considered to be a steady state production and the process standard deviation may change. The testing may have an error which resulted in an error in interpreting the results. Therefore, it should be concerned that the assumptions should be satisfied before using the model.

Although the model already included many costs related to the implementation of the control chart, some costs and loss are still missing in the real case due to the fact that the missing costs and loss are considered to be difficult to estimate in the model. This model has considered an opportunity loss from the ceasing in production and generated it as a cease cost. Cease cost is calculated from the profits that should be gained during the production stoppage. However, opportunity loss may be generated from many reasons and the effect of the opportunity loss may cause more than what we called cease cost considered in this model. Opportunity loss may occur from losing its market due to the failure in delivery and customer dissatisfaction. These loss and effects are difficult to estimate and apply in a model. However, the user can put the opportunity loss in the model in another way by changing P_n parameter which is a cost per product from an opportunity loss. The user has to evaluate the effect and then applied in P_n parameter by themselves. The accuracy of the model depends on the accuracy in estimating this loss. Moreover, the developed model can be extended to cover other external costs, which are costs of investigating causes of defects, cost of complaints in warranty, and loss of reputation.

The probability of defect occurs when the process is in an in-control state (γ_1) and an out-of-control state (γ_2) can be obtained from the control chart data which calculated from USL, LSL, μ and σ. They can also represent the process capability since the process capability ratio can be calculated from

$$C_{pk} = \min(C_{pu} = \frac{USL - \mu}{3\sigma}, C_{pl} = \frac{\mu - LSL}{3\sigma})$$

The probability of defect occurs when the process in control and out-of-control plays a significant role in both quality and cost criteria. High probability of defect means that the process has low process capability resulting in higher amount defective products produced and cost of quality. The user should improve the process quality until the process capability is satisfied and stable before using the control chart to monitor the process.

The economic mathematical model in this thesis considered to be a deterministic model. Its solution is calculated based on the historical data which contain no forecasting method. Therefore, accuracy in collecting parameters in the past is very important to the solution of the model. The real process and cost parameters can change with time so the parameters should be updated regularly. The user should keep collecting parameters and monitoring the process closely. Once the parameter changed, the model should be optimized with the latest value of the changed parameters.

The mathematical model in this thesis is developed to help the user to determine the sample size and sampling frequency that can minimize total cost with statistical quality constraints. However, the user should concern about the assumptions that needed to be satisfied before using the model. There can be some inaccuracy when the model applied in realistic situation mentioned above. Therefore, the user should continuously analyze the real historical data along with the usage of the model.

Economic statistical designs are economic designs which included statistical constraints. Economic statistical designs are generally more costly than economic designs due to the added constraints (Zhang and Berardi, 1997). There are many researchers have purposed their economic model. However, Saniga and Shirland (1977) showed that very few economic models for the design of control charts have implemented. The economic models are not widely used because the models are quite complex, and difficult to evaluate and optimize (Alexander et al., 1995). Woodal (1986) stated that control chart based on economically optimal design generally have poor statistical properties. Moreover, Montgomery (1980) also stated that the proposed models did not consider all relevant costs and no formal optimization techniques applied to the total cost function. This thesis proposes an economic statistical design of the x-bar control chart by integrating quality costs related to the implementation of the control chart. The proposed model is used to determine the control chart parameters which are sample size (n) and sampling frequency (f) that minimize the total cost of quality while the quality level remains satisfactory. The economic mathematical model is developed under the real situation of the case study company.

Costs in the model consist of appraisal costs and failure costs. Prevention costs are excluded from the model because they are not depended on sample size and sampling frequency. The cost elements are classified by quality cost concept to make the model more understandable. Total cost of quality is optimized, where the cost per hour unit is calculated from the cost per cycle time divided by the cycle time.

The cycle time in this thesis consists of four stages which are in-control stage, delay detection stage, finding an assignable cause stage, and repairing stage. The production process is allowed to operate continuously and the sample are collected with specific sample size and sampling frequency in the in-control stage. The delay detection stage is a time that the control scheme takes to detect an out-of-signal condition from the time of occurrence of the assignable cause. The process is searched for an assignable cause in finding-an-assignable-cause stage. Then, the process is allowed to stop when the process is repaired in repairing stage.

Appraisal costs are costs occurred from routine sampling activities. The appraisal costs consist of control chart sampling cost, false alarm cost, and OQA sampling cost. Control chart sampling cost is a cost that occurs during the usage of control chart in IPQA process. False alarm cost is a cost of investigating process when there is an alarm in the control chart even though the process is still in-control. OQA sampling cost occurs from sampling activities before delivering finished products to the customer.

Failure costs consist of internal and external failure costs. Internal failure costs occur from defective products and their sequences of creating loss in the company process. Internal failure costs consist of retest cost, defect cost, cease cost, and true alarm cost. Retest cost occurs from retesting activities. Defect cost is a cost that occurs from defective products and activities needed for repairing them. Cease cost is a cost that occurs from an opportunity loss due to the stoppage of production process needed for repairing the process. True alarm is a cost that occurs from activities needed for repairing an assignable cause in the production process.

External failure costs are costs that occur after the products are delivered to the customer already. The external failure costs consist of transportation cost and replacement cost. Replacement cost is a cost of replacing the defective products with the new products and also the penalty cost from the customer. Transportation cost is a cost of transporting new products to replace the defective products rejected.

In this thesis, the control limit coefficient (L) is set to be 3 which is the standard use as Montgomery (2005) said that the multiple usually chosen is three; hence, three-sigma limits are customarily employed on control chart, regardless of the type of chart employed. Another crucial statistical constraint in this thesis is β value. The β can be expressed as ARL $\left(\frac{1}{1-\beta_i}\right)$ or ATS $\left(\frac{1}{1-\beta_i} \times \frac{1}{f_i}\right)$.

There are three scenarios for the model. First, the present sampling plan is used in the model to let the model illustrates the present cost and present statistical quality performance. Second, the model is solved to find the optimum sampling plan that minimizes total cost under the statistical quality constraints and present resources. The number of testing machine is limited to the current number of machine that the company already own which is 5 machines. Third, the additional new testing machine is allowed in the model. Investing in the new testing machine will increase the testing capacity which may result in a decreasing of failure costs. This scenario will provide the number of machines and its sampling plan that deliver the minimum cost.

At the present number of testing machine, the result shows that the optimized model can find the sample size and sampling frequency that can minimize the total cost while the number of testing machine is the same. The sample size is increased to satisfy the statistical constraints since the β values are considered to be high at present. The high value of type two error (β) resulted in large amount of defective products produced before the control chart can detect the out-of-control pattern. Instead of wasting the money in repairing the defective products, the company should focus on an increasing of the testing activity to help reduce the number of defective products. An appraisal costs are increased due to an increasing of testing activities. However, the increasing amount of appraisal costs is much less than the saving amount from the reduction of failure costs.

Investing in the additional new testing machine can reduce the total cost because it can enhance the testing capacity. The failure costs can be reduced significantly while the appraisal costs are increased slightly due to an increasing of the testing activity. From the model, the best solution that minimizes the total cost is 10 machines solution. However, there is a machine investment cost that has to be added since the company has only 5 machines currently. The best solution that minimizes the summed of total cost and machine investment cost is 8 machines solution. Therefore, the optimum solution under 8 machines solution is the best solution that can minimize the cost that the company has to pay. The 2 more machines which are added from 8 machines can reduce the total cost in fewer amounts than their investment costs. Further investment in the testing machine cannot reduce the total cost anymore. This is because the appraisal costs will be increased in a higher amount than the saving from the failure costs. Therefore, an appraisal action has the limit on reducing the total cost since the appraisal itself also costs money. Preventive action may be the solution to reduce the failure costs.

However, the preventive action would change the model parameters such as time period that the process is in an in-control stage and probability of defect occurs when the process is in-control. Changing these parameters will reduce the total cost apparently as shown in the sensitivity analysis. Nevertheless, the cost of preventive action should be concerned about its advantages and disadvantages.

A sensitivity analysis is performed to show the effects of variables and parameters on the total cost of quality in the economic mathematical model of the control chart. There will be 3 designed sensitivity analysis in this thesis to illustrate how sensitive of each variable and parameter over the total cost. The parameter sensitivity analysis will classify the parameters into 4 categories which are time, production, labor and machine, and product cost category. The sensitivity analysis will be applied on the optimum sampling plan under 8 machines which is the one that delivers the best solution to minimize the sum of total cost and machine investment cost.

The first designed sensitivity analysis is to vary the sample size and sampling frequency to see the change of the total cost. The objective of the first designed sensitivity analysis is to analyze the effect of the changed value of sample size and sampling frequency over the total cost while the objective of the second design sensitivity analysis is to analyze the effect of the changed values of parameters over the total cost and sampling plan. For the third design sensitivity analysis, the objective is to classify the parameters that have a crucial effect to the total cost in order to control them. The way to justify whether the parameter has a crucial effect to the total cost or not is to vary each parameter with the same optimum sampling plan until it reach the cost limit of the total cost which is 10% change. Therefore, the maximum changed value for every parameter that makes the total cost reach the cost limited will be calculated.

From the sensitivity analysis, we can find that changing variables which are sample size and sampling frequency can affect the total cost directly. Insufficient sample size and sampling frequency can cost a lot of money due to an increasing amount of defective products occurred. Increasing in sample size and sampling frequency can reduce the total cost significantly. However, the reduction of the total cost by an increasing in sample size and sampling frequency is not continuously decreased because there is an optimum point. After reaching the optimum point, the total cost started to rise due to the significant portion of appraisal costs.

Parameters also have significant effect to the total cost and sampling plan. From the sensitivity analysis, we can find that the total cost is sensitive to many parameters such as time period that the process is in an in-control stage (T_0), time period that the process is stopped for repairing (T_2), probability of defect occurs when the process is out-of-control (γ_2), rework rate (R), time for testing a sample per piece (T_{Test}), and staff labor rate (L_m). Product costs in each process has a slight effect to the total cost since the amount of products that turned to be a scrap is considered to be low and the amount of defective products delivered to the customer is low also. The sampling plan can be affected by changing in many parameters as well.

From the third designed sensitivity analysis, there are many parameters that needed to be controlled otherwise the total cost will be increased rapidly. From the calculation, the first top 5 parameters that needed to be controlled are time period that the process is in an in-control stage (T_0), production rate (P_r), probability of defect occurs when the process is out-of-control (γ_2), time period that the process is stopped for repairing (T_2), and product profit (P_n) respectively. This sensitivity analysis identified which parameter has a high effect to the total cost in percentage. However, the possibility of changing each parameter is also considered to be the main consideration. Therefore, analyzing historical data and also considering about the possibility of changing each parameter along with the sensitivity analysis result is important in order to create the feasible solution that can control the total cost below the cost limit.

The sensitivity analysis can also indicate the way to reduce the total cost further. For example, the further reduction can be done by improving the turning process to increase the time period that the process is in an in-control stage (T_0) resulting in lower total cost. This preventive action can be applied in the model as well by changing related parameters which is considered to be the new scenario for the model. Therefore, the model can find the new optimum sampling plan for the new parameters and resources.

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BIOGRAPHY

Mr. Nopavut Punyasiri is the author of this thesis. He was born in May 14, 1986. He graduated from Chulalongkorn University in 2008 with a Bachelor degree in Industrial Engineering in faculty of Engineering.