CHAPTER 2

NETWORK REDUCTION

2.1 Introduction

This chapter describes the methods for the network reduction. The network reduction methods consist of an algorithm for reducing generator buses based-on the coherency and an algorithm for reducing load buses.

The objective of the network reduction algorithm is to reduce the size of the network represention, in terms of both buses and lines, which facilitates studies of the system performance.

2.2 The connection to the network of equivalent generators

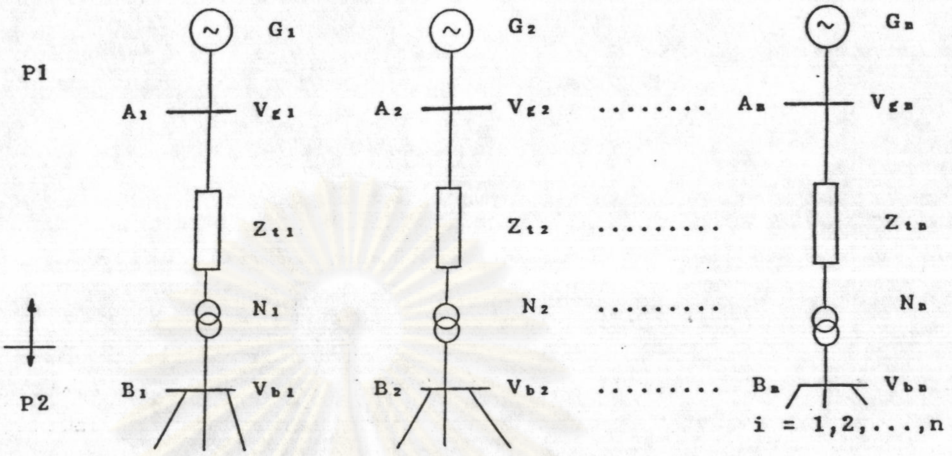
The buses to which a group of coherent generators are connected have to be substituted by a single equivalent generator bus for each one of the coherent group.

2.2.1) The network description

The network is divided into two parts, as shown in Figure 2.1:

- First, the group of generator buses that have been determined to be substituted by a single equivalent generator bus.
- Second, the rest of the network.

The description of the second part of the network is not modified by manipulations within the coherent group and will not be included in the description of the reduction method. One coherent group is considered. The extension to more coherent groups is straightforward.



n = number of generator buses in coherent group

V_{gi} = voltage at generator bus no i

V_{bi} = voltage of the bus which correspondes to the generator bus

Z_{ti} = impedance of transformer no i

N_i = tap of transformer no i

G_i = generator no i

A_i = generator bus no i

B_i = bus which correspondes to the generator bus no i

P1 refered to the first part of the network

P2 refered to the second part of the network

Figure 2.1: The connection to network of the generator buses in actual system

2.2.2 The derivation of the equivalent generator buses

When the generator buses in each coherent group are replaced by the equivalent generator bus, the power flows between the equivalent generator bus and the buses which correspond to the equivalent generator bus should be preserved. The voltage at the generator bus can be selected as mean value of the voltage of the generator buses in each coherent group. The voltage at the equivalent generator bus can be express as follows:

$$V_t = (1/n)\sum_{i=1}^{n} V_{gi} \qquad (2.1)$$

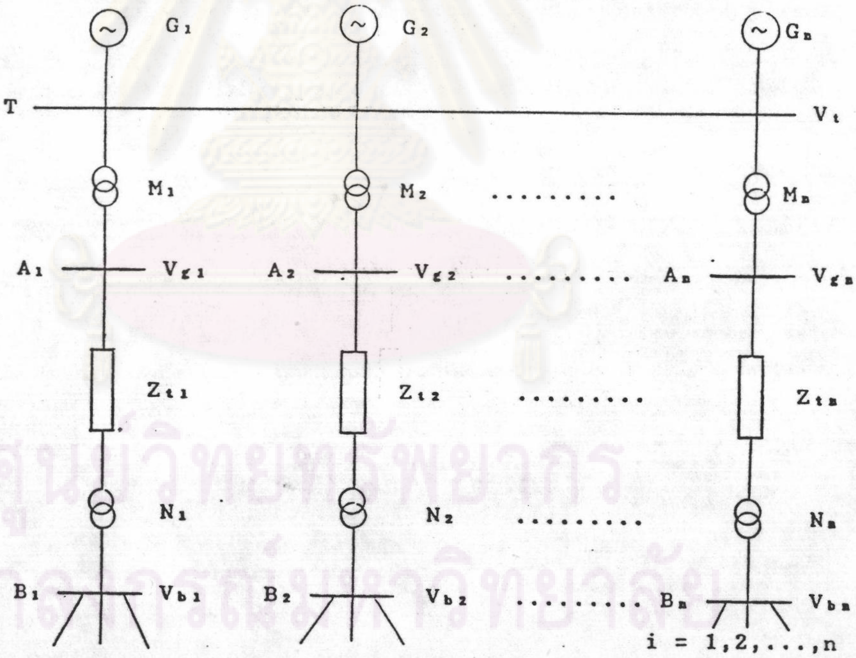
where V_t = voltage at equivalent generator bus

V_{gi} = voltage of generator buses in the coherent group

n = number of generator bus in coherent group

$i = 1,2,\ldots\ldots,n$

The voltage of genertor buses and the equivalent bus are quite different in terms of both the magnitude and the phase angle. They can be linked together by using the phase shifting transformer, as shown in Figure 2.2.



M_i = tap of phase shifting transformer no i

= V_{gi}/V_t

T = equivalent generator bus

v_t = voltage at equivalent generator bus

Figure 2.2 The implementation of phase shifting transformer

After the generator buses and equivalent generator bus have already been linked, they can be eliminated by simplification, a new equivalent generator must be created. The active and reactive power output of equivalent generator must be replaced by the sum of active power output and reactive power output of individual generator in coherent group, and the rated MVA output of the equivalent generators must be replaced by the sum of the rated MVA output of individual generator in coherent group: as shown in Figure 2.3. The active, reactive and rated power output of equivalent generator can be expressed as follows:

$$P_e = \sum_{i=1}^{n} P_i \tag{2.2}$$

$$Q_e = \sum_{i=1}^{n} Q_i \tag{2.3}$$

$$S_{ne} = \sum_{i=1}^{n} S_{ni} \tag{2.4}$$

where P_e = the active power output of equivalent generator

Q_e = the reactive power output of equivalent generator

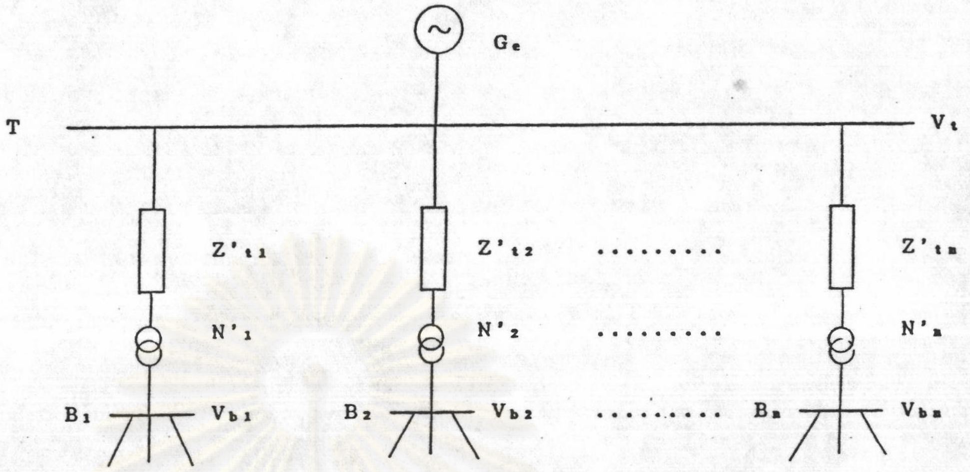
S_{ne} = the rated MVA output of equivalent generator

P_i = the active power output of generator no i

Q_i = the reactive power output of generator no i

S_{ni} = the rated MVA output of generator no i

$i = 1, 2, \ldots\ldots\ldots, n$



N'_i = modified tap of transformer no i
= $N_i.M_i$
Z'_{ti} = modified impedance of transformer no i
= $Z_t/(M_i.M^*_i)$
G_e = equivalent generator

Figure 2.3 The connection to network of the equivalent generator

2.2.3 How to preserve the power flows.

In Figure 2.1, the power flows from bus Bj to bus Aj and power flows from bus Aj to bus Bj can be expressed as follows[4];

$$P_{BjAj} - jQ_{BjAj} = V^*_{bj}(V_{bj} - N_j.V_{gj})/(Z_{tj}.N_j.N^*_j) \tag{2.5}$$

$$P_{AjBj} - jQ_{AjBj} = V^*_{gj}(N_j.V_{gj} - V_{bj})/(Z_{tj}.N_j) \tag{2.6}$$

In Figure 2.3, the power flows from bus B_j to bus T and power flows from bus T to bus B_j can be expressed as follows[4];

$$P_{BjT} - jQ_{BjT} = V^*_{bj}(V_{bj} - N'_j.V_t)/(Z'_{tj}.N'_j.N'^*_j) \quad (2.7)$$

$$P_{TBj} - jQ_{TBj} = V^*_t(N'_j.V_t - V_{bj})/(Z'_{tj}.N'_j) \quad (2.8)$$

Since $N'_j = N_j.V_{gj}/V_t$

$= N_j.M_j$

$Z'_{tj} = Z_{tj}/(M_j.M^*_j)$

Substituting for N'_j and Z'_{tj} into Eq(2.7) and Eq(2.8)

$$P_{BjT} - jQ_{BjT} = V^*_{bj}(V_{bj} - N_j.V_{gj})/(Z_{tj}.N_j.N^*_j) \quad (2.9)$$

$$P_{TBj} - jQ_{TBj} = V^*_{gj}(N_j.V_{gj} - V_{bj})/(Z_{tj}.N_j) \quad (2.10)$$

therefore Eq(2.2) = Eq(2.6) and Eq(2.3) = Eq(2.7)

$$P_{BjAj} - jQ_{BjAj} = P_{BjT} - jQ_{BjT} \quad (2.11)$$

$$P_{AjBj} - jQ_{AjBj} = P_{TBj} - jQ_{TBj} \quad (2.12)$$

Eq(2.11) and Eq(2.12), show that when the generator buses in coherent group are substituted by using the method described here, the power flows in the network can be miantained.

2.2.4 Summary of the equivalent

Suppose the coherent group contains n generator buses which have to be substituted by the equivalent generator bus, t. The voltage of generator bus no i is V_{gi}.

Table 2.1 Summary of the equivalent

- Voltage of the equivalent generator bus t.

$$V_t = (1/n).\sum_{i=1}^{n} V_{gi} \qquad ; \quad i = 1,2,\ldots\ldots,n$$

- The modified impedance of transformer.

$$Z'_{ti} = Z_t/(M_i.M^*_i) \qquad ; \text{ see Figure 2.3}$$

- The modified tap of transformer.

$$N'_i = N_i.M_i \qquad ; \text{ see Figure 2.3}$$

- The tap of implementation phase shifting transformer.

$$M_i = V_{gi}/V_t \qquad ; \text{ see Figure 2.2}$$

- The generation of the equivalent bus is the sum of the generation at each bus eliminated.

- The load at the equivalent bus is the sum of the load at each bus eliminated.

2.3 Reduction of passive network

After the coherent generator buses in each coherent group have already been substituted by the equivalent generator buses, the system has created a complicated network structure. The passive network between the equivalent generator buses must be reduced so that only the generator buses and a few, selected, buses in the system are retained. The Gaussian matrix reduction formular is used for reduction of the passive network. The bus admittannce equation of the system constructed with the load treated as follows;

- The constant impedance portion of load is included within the admittance matrix
- The constant current portion of load on a bus to be eliminated (bus j) is converted to $[\,I_{Ij}\,]$.
- The constant power portion of load on a bus to be eliminated (bus j) is converted to $[\,I_{Sj}\,]$.

The bus admittance matrix equation of the system can be expressed in the following portioned form:

$$\begin{bmatrix} I_1 \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} 0 \\ I_{I2} \end{bmatrix} + \begin{bmatrix} 0 \\ I_{S2} \end{bmatrix} \tag{2.13}$$

where Subscript 1 refers to retained buses
Subscript 2 refers to buses to be eliminated
$[\,I_1\,]$ = injected current for the buses to be retained
$[\,V_1\,]$ = voltage for the buses to be retained
$[\,V_2\,]$ = voltage for the buses to be eliminated

In the reduced system of equation, only $[\,I_1\,]$ and $[\,V_1\,]$ are to be included in explicit terms. $[\,I_{I2}]$, $[\,I_{S2}]$ and $[\,V_2\,]$ are assumed to be linearly dependent of $[\,I_1\,]$ and $[\,V_1\,]$ are included implicitly in the system of equation.

The upper and lower lines of Eq(2.13) can be expressed as follows;

$$[\,I_1\,] = [\,Y_{11}\,].[\,V_1\,] + [\,Y_{12}\,].[\,V_2\,] \tag{2.14}$$

$$[\,V_2\,] = -[\,Y^{-1}{}_{22}\,].([\,Y_{21}\,].[\,V_1\,] + [\,I_{I2}] + [\,I_{S2}]) \tag{2.15}$$

Substituting for Eq(2.15) into Eq(2.14)

$$[\,I_1\,] = [\,Y'_{11}\,].[\,V_1\,] + [\,I'_{I1}] + [\,I'_{S1}] \tag{2.16}$$

wher $[\,Y'_{11}\,]$ = equivalent admittance matrix

$= [\,Y_{11}\,] - [\,Y_{12}\,].[\,Y^{-1}{}_{22}\,].[\,Y_{21}\,]$

$[\,I'_{I1}] = -[\,Y_{12}\,].[\,Y^{-1}{}_{22}\,].[\,I_{I2}]$

$[\,I'_{S1}] = -[\,Y_{12}\,].[\,Y^{-1}{}_{22}\,].[\,I_{S2}]$

The current $[\,I'_{I1}]$ and $[\,I'_{S1}]$ are the equivalent current on the retained buses. These currents, $[\,I'_{I1}]$ and $[\,I'_{S1}]$, are reconverted to constant current and constant power load respectively by using these equations;

$$[\,S_{Ij}\,] = [\,V_j\,].[\,I'^{*}{}_{Ij}\,] \tag{2.17}$$

$$[\,S_{Sj}\,] = [\,V_j\,].[\,I'^{*}{}_{Sj}\,] \tag{2.18}$$

where $[S_{Ij}]$ = constant current load portion on retained bus j

$[S_{Sj}]$ = constant power load portion on retained bus j

The constant impedance portion of the equivalent load is obtained by calculating the shunt admittance $[Y_{jj}]$ from the j^{th} row of equivalent admittance matrix, $[Y'_{11}]$

The constant impedance type component of complex power is then given by

$$[S_{Zj}] = [V^2_j].[Y^*_{jj}] \qquad (2.19)$$

2.4 The nonlinearities of parameters in the network

The method for passive network reduction described here taken into account only the nonlinearities of the loads. The system in which is dependent of the load characteristics can only be reduced partly by the method described. Some parts must be unreduced so the correct response is obtained.