Chapter 3

Diakoptics or The Piecewise Solution of Large-Scale System

3.1. Introduction

As pointed out by Branin (20), it is an interesting historical coincidence that just prior to the advent of the high-speed digital computer, the pioneering work of Gabriel Kron, whose first paper on tensor analysis laid the foundations of the method of "tearing" more than sixty years ago, demonstrated convincingly the superior organizational powers of the matrix-tensor notation in Power System theory. Kron emphasized not only the conceptual elegance of this notation but also its virtually automatic way of handling the tedious bookeeping chores of Power System analysis. He thereby laid some of the logical and procedual foundations necessary for programing a digital computer to analyze Power System-that is, both to compile and to solve the Power System equations automatically.

The matrix formulation of the Power System problem (21-24), is the direct outgrowth of Kron's work and that of Roth (25) who first provided an algebraic topological explanation of the network problem as an abstraction. This formulation has been used at least in part by such program as TAP (26,27), NET-1 (28), and ECAP (29). Kron's work has also been used as the basic for much of the computer analysis of networks encountered in the electrical industry (4,6).

The historical background of his work may be of interest. The closed-path or circuit have been familiar to electrical engineers from the time of Maxwell, but not open-path. In the early 1930's, the originator of Diakoptics (method of tearing), G. Kron, searched for the dual of the Maxwell's mesh method, based upon closed-path, and discovered the junction pair concept. By means of this concept, the differences of potential appearing between any two junctions of an electrical network are assume to be variables. His procedure differed from the conventional methods in that it allowed more general reference frames than that of node-to-datum. Kron found that the open-path which he then called the open-mesh, was a more correct dual to the closed-mesh than the junction-pair concept, but his original work was limited to only the definition of the open-mesh. In his classical work, Tensor Analysis of Network, Kron introduced the concept of orthogonal networks possessing nonsingular connection tensor C and A (where A is the inverse of C). This new concept assumes that every electrical network is the direct sum of a closed-path network and an open-path network, with coupling (mutual impedance or admittance) terms existing between the two types of networks. Subsequenly, in the 1950's, he conceived of Diakoptics and then more clearly realized the importance of the open-path concept; but the theoretical foundation of Diakoptics still remained to be developed (30).

The word "Diakoptics" comes from the Greek word "kopto", which means to break or to tear apart, and "dia", which reinforces "kopto". Using this method, large-scale Power System is torn into a number of pieces that are solved separately; the results are then combined to yield the solution of the total system. The concept of Diakoptics for Power System or related problem is that, in the first step, the problem is solved as if all individual subsystems are isolated from each other. Then, in subsequent steps, these solutions are modified to take the interconnections into consideration. The separation or tearing can be based upon strictly computation considerations, on individual ownerships as in interconnected individual utilities, on economic considerations, on control considerations, or on any other set of considerations. The tearing of the Power System is particularly convenient in such multiprocessing environments where installed computers, although at the same, location but equiped with multi-processors which can operate and solve problems in parallel, or when the computers are distributed to various location but tied together by communication links (30-36).

3.2. Diakoptics in Electric Power System

The basic idea of Diakoptics is as just said to solve a large system by breaking or tearing it apart into smaller subsystems or areas, solving the individual parts, combine and modify the solutions of the torn parts to yield the solution of the original untorn problem. The result of the procedure is identical to one that would have been obtained if the whole system had been solved as unity.

Considering Fig. 3.1, for example, which may represent a large Power System before tearing. Subsystem A and subsystem B are tied together by elements called tie-lines. The original Power System is divided to two areas, A and B, as shown in Fig. 3.2. The solution, in Diakoptics, is obtained by tearing the original system apart, to solve each part separately with no contribution from the neighboring parts considered. The contribution to the total solution from the interconnections is then considered separately.

These torn parts or subsystems often occur quite naturally and thus, also, the corresponding lines of tear as, for example, the boundaries between power companies. The uses of Diakoptics are at least twofold: in the first application larger systems can be solved efficiently by the use of Diakoptics on a given computer by processing the solution through the computer serially. The second application employs a multiplicity of computers that essentially operate in parallel and thus provide more speed of execution than that obtained by using a single computer. The computers can be physically next to each other thus forming a cluster of computers, or they can be kilometers apart. Each computer can work on the solution of a given part. In Fig. 3.1 two computers would be required. Extra computational capability is required to calculate the contribution to the total solution due to the interconnections. This capability can be provided by the computers working on the torn parts or by extra computers provided for that purpose.

Both applications above involve conventional computers that operate more or less serially. Future computers may well have parallell computing capability, similar to the second application. In such a case we can expect larger problems to be solved with greater speed by use of Diakoptics than by solving the problem by conventional methods.

If tearing has no effect to the original network, then the current through tie-lines (i_1, i_2, i_3 and i_4) is not changed. Procedure according to principals of Diakoptics are (30,32,33):

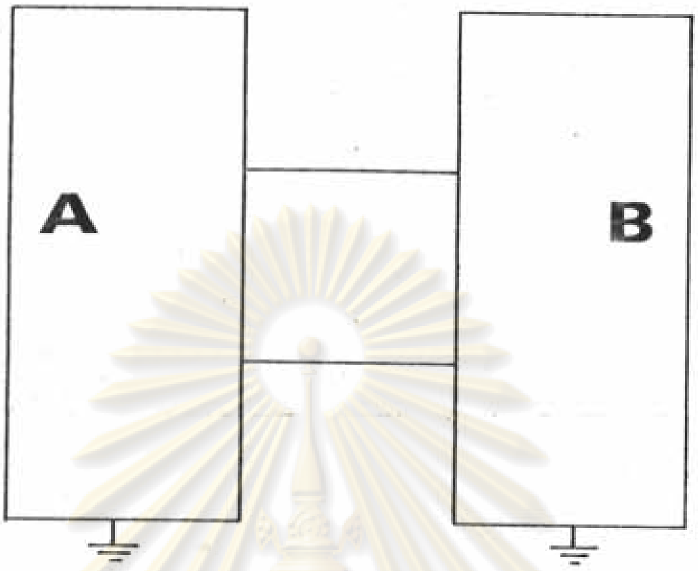

Fig. 3.1 An Electric Power System.

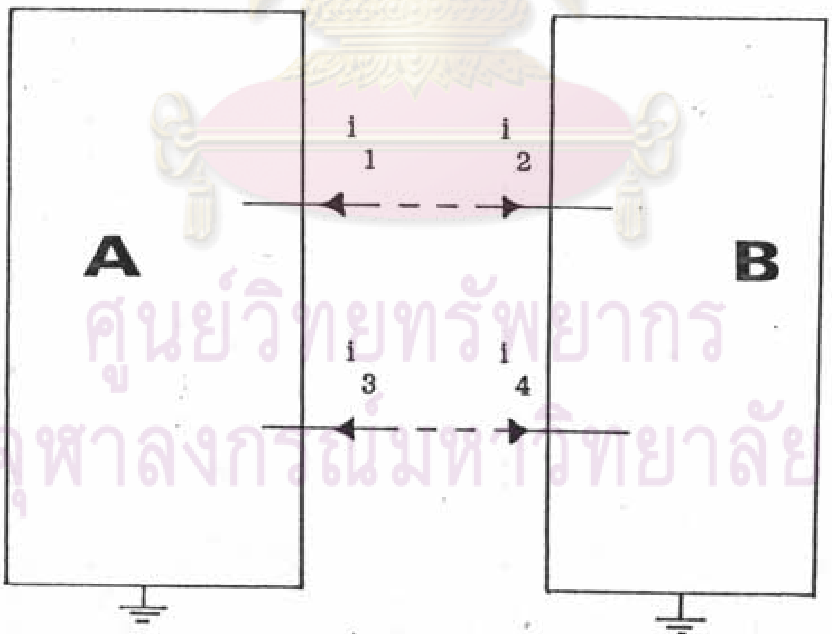

Fig. 3.2 Tearing of The Original Power System to Subsystems or Areas.

1. Tear the whole original system into subsystems.

2. Solve each subsystems's equations.

3. Find current through the tie-lines.

4. Combine subsystems's solution. The combination of subsystems's solution comprises the solution of the whole system.

The tearing divides the impedance matrix of the system as in eq. (3.1):

$$[Z] = \left[\begin{array}{cc|c} Z_{aa} & & \\ \cline{1-2} & Z_{bb} & Z_2 \\ \hline Z_3 & & Z_4 \end{array}\right] \qquad (3.1).$$

In compact form $[Z_1]$ is defined as a block-diagonal matrix:

$$[Z_1] = \left[\begin{array}{c|c} Z_{aa} & \\ \hline & Z_{bb} \end{array}\right] \qquad (3.2).$$

The matrices $[Z_{aa}]$ and $[Z_{bb}]$ are Bus Impedance Matrix of area A and B respectively. References 10,18 and 29 show the caculation techniques for matrices $[Z_2]$, $[Z_3]$ and $[Z_4]$ from $[Z_1]$.

Let E be the vector of bus voltages of the original system,

I be the vector of bus currents of the original system and

i be the vector of tie-line's currents.

Then, the equation in matrix form is:

$$\left[\begin{array}{c} E \\ \hline 0 \end{array}\right] = \left[\begin{array}{c|c} Z_1 & Z_2 \\ \hline Z_3 & Z_4 \end{array}\right] \left[\begin{array}{c} I \\ \hline i \end{array}\right] \qquad (3.3).$$

By Kron's reduction formula (30,32,33), eq. (3.3) can be reduced to:

$$[E] = \{ [Z_1] - [Z_2][Z_4]^{-1}[Z_3] \} [I] \quad (3.4).$$

If [I] is known, the solution for [E] can be solved by six-step as below:

1. Find solutions of torn subsystems:

$$[e'] = [Z_1][I] \quad (3.5).$$

2. Compute voltage across torn subsystems by any given branch sign conversion:

$$[Ec] = [c][e'] \quad (3.6).$$

where [c] is connection matrix, as defined by given direction of tie-line (30,33).

3. Compute closed-path or tie-line's current:

$$[i] = [Z_4]^{-1}[Ec] \quad (3.7).$$

4. Convert closed-path current through injected tie-line's current by given sign of the current.

5. Find voltage contributions in subsystems due to tie-line's current:

$$[e''] = [Z_1][i] \quad (3.8).$$

6. The voltage solution is the sum of voltages in step 1 and 5:

$$[\,E\,] = [\,e'\,] + [\,e''\,] \qquad (3.9).$$

Note that, during step 1 through 6, there is no approximation in any step. By this method the accurate solution of the whole network is obtained with less calculation time and computer storage requirement than if the entire network was treated as one unity.