Chapter 2

Load-Flow Study

2.1. Introduction

Load-Flow is a solution of the static operating condition of an electric-power transmission system. The results from Load-Flow are needed for planning (1-3), optimization, and also as a part of stability calculation. Load-Flow equations are nonlinear because they express power as a function of voltage. They are algebraic rather than differential, because they describe the steady-state instead of the transient behavior of the network.

Perhaps the electric power-flow problem or Load-Flow is the most studied and documented problem in power engineering. In essence, this problem is the calculation of line loading for given generation and demand levels. The transmission network is nearly linear and demand might superficially expect the power-flow problem to be a linear problem. However, since the power is a product of voltage and current, the problem formulation is non-linear although the transmission network is linear. Additional nonlinearities arise from the specification and use of complex voltages and currents. Also, there are transmission component nonlinearities which may be considered (such as tap-changing transformers, in which the transformer ratio within certain limits is adjusted to keep a given bus voltage magnitude fixed). The sinusoidal steady state is assumed as the result. Sometimes, very slow load or generation variation is taken into consideration, when the variation is slow enough to justify sinusoidal steady state assumption (4).

In general, the power-flow problem has many possible solutions or no solution (5). The case at hand depends on the loading condition. The existence of many possible solutions may give concern since it is desirable to discern which of these solutions is expected in the field. Fortunately, it is usually the case that the solution of bus voltages are close to unit entires. Usually, only one solution will be near the unity bus voltage profile - the other solutions contain one or more bus voltages near zero. A bus voltage soltuion near $1.0 \underline{/ 0.0}$ is referred to as an open circuit solution. The short circuit solutions, while valid mathematically, are rejected on the grounds of violation of usual operating practice.

2.2. Newton-Raphson Method (6-13)

The Load-Flow problem can be solved by the Newton -Raphson method. The method relies on a gradient-like approach. A comparisions between various Load-Flow calculation methods is made in reference 11. It can be concluded that Newton-Raphson Load-Flow (NRLF) is better than many other methods. The table 2.1 shows a comparison between the two popular Load-Flow methods: Newton-Raphson method and Gauss-Seidel techniques. The well-known polar-mismatch NRLF in martix form is:

$$\begin{bmatrix} \Delta P \\ \hline \Delta Q \end{bmatrix} = \left[\begin{array}{c|c} H & N \\ \hline J & L \end{array}\right] \begin{bmatrix} \Delta \delta \\ \hline \Delta V \end{bmatrix} \qquad (2.1)$$

where ΔP is the real power mismatch vector.

ΔQ is the reactive power mismatch vector.

$\left[\begin{array}{c|c} H & N \\ \hline J & L \end{array}\right]$ is the Jacobian matrix.

$\Delta \delta$ is the phase angle correction vector.

ΔV is the voltage correction vector.

The details of eq.(2.1) are summarized in Chapter 5.

Table 2.1 Comparision of Two Load-Flow Methods

Type of Problem	Gauss-Seidel Technique	Newton-Raphson Method
1. Heavily loaded systems.	Usually cannot solve systems with more than 70 degree phase shifts.	Solves systems with phase shifts up to 90 degree.
2. Systems conta-ining negative reactance such as three-wind-ing transform-ers or series line capacitors.	Unable to solve.	Solve with ease.
3. Systems with swing bus at a desired location.	Often requires trial and error to find a swing bus location that will yield a solution.	More tolerant of swing bus locat-ion.
4. Long and short lines terminat-ing on the same	Usually cannot solve if long-to-short ratio	Can solve a system with a long-to-short

bus.	is above 1000:1.	ratio at any bus of 1,000,000:1.
5. Long radial type of system	Difficulty in solving.	Solves a wider range of such problem.
6. Accleration factors.	Number of iterations depends on choice of factor.	None required.

Source: Reference 9.

There are two equations present in eq. (2.1) for every PQ-bus and one equation ΔP, $\Delta\delta$ for PV-bus. No equation for swing bus is required. The Jacobian matrix is not symmetric but similar to [Y-Bus] sparse.

The standard technique for accelerating Newton's method is to re-use the Jacobian matrix from one iteration for several sucessive cycles without recomputation (3). The normal procedure is like this: a) computing mismatch in node power injections, the residuals, b) which are multiplied by the inverse of the Jacobian matrix to obtain the node voltage corrections, c) applying the corrections, d) computing new residuals, etc.(14-16). The process is continued until the problem is solved or the decrease in rate of improvement indicates that the Jacobian matrix should be re-evaluated at a new operating point.

2.3. Approximations to The Jacobian Matrix in NRLF (17)

An inherent characteristic of any practical electric - power transmission system operating in the steady state is the strong interdependence between active powers and bus voltage angles, and between reactive powers and bus voltage magnitudes. Correspondingly, the coupling between these " P-δ " and " Q-V " components of the problem is relatively weak. Applied numerical methods are generally at their most efficient when they are able to take advantage of the physical properties of the system being solved. In the Load-Flow problem, there has been a recent trend towards this objective of " decoupling " (solving separately) the P- δ and Q-V problem.

Because the Jacobian matrix in NRLF is very large and in order to reduce computer storage requirements and arithmetic computations, two approximations are used:

1. The real power flow in an AC system depends mainly on phase-angle differences and,

2. The reactive power flow depends mainly on

voltage magnitude differences.

The most significant effect of these approximations is the decoupling of real power part of Jacobian matrix from the reactive power part. Eq. (2.1) is reduced to:

$$[\Delta P] = [H][\Delta\delta] \qquad (2.2)$$

and

$$[\Delta Q] = [L][\Delta V] \qquad (2.3).$$

Eq. (2.2) is called P - δ problem and eq. (2.3) is called Q-V problem. The method is called Decoupled Load-Flow (DLF). The order of [H] and [L] are approximately half of the entire Jacobian matrix.

Equations (2.2) and (2.3) can be constructed and solved simultaneously at each iteration. A much better approach however is to conduct each iteration cycle by first solving eq. (2.2) for $\Delta\delta$, and use the updated δ in constructing and then solving eq. (2.3) for ΔV. These step wise solutions can be performed alternately in the same computer storage area.

In this form, eq.(2.2) and (2.3), the decoupled method convergence is as reliable as that of the formal Newton version as eq. (2.1) from which it was derived. The convergence to practical accuracies however usually takes more iterations, because over all quadratic convergence is lost (see Appendix A).

2.4. Fast Decoupled Load-Flow (18)

In eq. (2.2) and (2.3), there are still coupling whereas [H] depends on V and [L] depends on δ . Further approximations applied are as follow:

1. Electrical angles at buses are close to each other.

2. All bus voltages are close to 1.0 P.U..

3. The ratio of reactance to resistance (X/R) is very high, and line charging is omitted.

Thus, [H] and [L] become constant and this approximation turn eq. (2.2) and (2.3) into eq. (2.4) and (2.5),

$$[\Delta P] = [B'][\Delta\delta] \quad (2.4)$$

and

$$[\Delta Q] = [B''][\Delta V] \quad (2.5).$$

These are called Fast Decoupled Load-Flow (FDLF) equations.

Finally, the FDLF assumed for the Jacobian that shunt reactance from buses to ground are negligible and that all transformers have unity for the off-nominal tap ratio, without any phase shift. [B'] and [B"] are observed to have a nonzero pattern for elements corresponding to the Bus Admittance Matrix. The advantage of the FDLF is that neither [B'] nor [B"] vary in any iterations, so they are constant matrices which are inverted only once and repeat solutions are calculated. The number of iterations required by the FDLF's convergence is usually higher than the NRLF's (11,18), however the net benefit is that the FDLF method works on smaller matrices which require less memory storage and half the number of multiplications by the inverse to reach solutions. But the convergence of FDLF is not as strong as the convergence of the NRLF. In cases of high line loading, high reactive power line dissipation, low X/R ratio, low bus voltages, and low load power factor, [N] and [J] this may be significant. However, as stated in reference 19, in case of series and parallel compensation techniques, the paper has shown, without proving, that

$$B'_{ij} = -B_{ij} - \frac{G_{ij}^2}{B_{ij}} \qquad (2.6)$$

$$B'_{ii} = -\sum_{j=1}^{n} B_{ij} \qquad (2.7)$$

where n is total number of bus is in the system.

and

$$B''_{ij} = G_{ij} - B_{ij} \qquad (2.8)$$

$$B''_{ii} = G_{ii} - B_{ii} \qquad (2.9).$$

where $Y_{ij} = G_{ij} + j\,B_{ij}$

Y_{ij} is an element of [Y-Bus].

The technique is less efficient than the FDLF for system with high X/R ratio.

The FDLF method offers a uniquely attrative combination of advantage over the established methods, including Newton's, in terms of speed, reliability, simplicity, and storage, for conventional Load-Flow solutions. The basic algorithm remains unchanged for a variety of different applications. Given a set of good ordered elimination routines, the basic program can be coded efficiently, and its speed and storage requirements are roughly proportional to the system size. The table 2.2 shows the advantages and disadvantages among NRLF, DLF and FDLF which can be a guidance to select the appropriated method.

2.5. <u>Modified Fast Decoupled Load-Flow</u> (13)

The reduction time of forming [B'] and [B"] can be implemented through the three approximations of the FDLF directly to [H] and [L ']. From this point, the matrices are become (13):

$$[B'] = -Im [Y\text{-}Bus] \quad (2.10)$$

also,

$$[B''] = -Im [Y\text{-}Bus] \quad (2.11)$$

where Im is an imaginary part of complex number. From eq. (2.10) and (2.11), the [B'] and [B"] can be formulated by [Y-Bus] which is completely construction and also, the solution of this technique, the modified FDLF, is as good as the FDLF. This is the reason why the modified FDLF should be applied in Load-Flow software.

Table 2.2 Load-Flow Solution Activities Selection Guide for NRLF,DLF and FDLF.

Method	Advantages	Disadvantages
NRLF	Rapid convergence on well-conditioned cases. Very small bus mismatches can be achieved.	Intolerant of data errors. Cannot start from poor voltage estimates. No indicataion of

		cause of problem when failing to convergence. Can give problems converging cases where reactive power limits are restrictive.
DLF	Rapid convergence on well-conditioned cases. Small bus mismatches can be achieved.	As for NRLF
FDLF	As for DLF	As mismatches are reduced, rate of improvement be allowed. Cannot handle network with low X/R ratio branches (e.g. equivalents). As for NRLF.

Source: References 46 and 61.