CHAPTER IV

NUMERICAL EXAMPLES AND RESULTS

The programme FEAP (20) was modified and employed to analyze elastic-plastic plane frames under static and dynamic loadings with and without substructuring. To demonstrate the accuracy of the proposed method, four problems were solved and the results of substructure and full system analyses were compared. In the first example, static analysis to predict the collapse load of a vierendeel girder was performed. Dynamic responses of two planar frames subjected to sinusoidal base excitations were obtained in the second and the third examples. For the last example a tall building frame subjected to the 1940 EL Centro Earthquake was examined.

In all examples the structures were assumed to be made of an elastic-plastic material. In the analyses the material was approximated by an elastic-viscoplastic model characterized by the parameters $n = 51$ and $\tau = 0.1$ sec. Lumped masses were assumed and rotational masses ignored. Computation was performed on an IBM 370/138 machine using single precision arithmetics unless otherwise noted.

4.1 Static Collapse Load of a Vierendeel Girder

A vierendeel girder was analyzed for the static collapse load using the proposed method. The geometry, member properties of this frame together with the loading are shown in Fig. 2 (a). In addition to the material model mentioned earlier, another one with $n = 51$ and $\tau = 3$ sec. was employed in a separate analysis to study the sensitivity of the solutions to variation of τ for large n. The structure was discretized into 39 elements and a three-point Gauss quadrature scheme was employed for each element. In the numerical integration of the constitutive equations the time step was set equal to τ and $\tau/30$ for the two material models, respectively. The loading was applied incrementally at proportional load factor, λ, of 0.4, 0.8, 1.2, 1.6, 1.7, 1.8 and at increments of 0.05 thereafter. The maximum number of iterations was limited to twenty-five for each time step. Analyses were performed both for the full system and the substructured model. The latter consisted of two substructures connected along the center line of the frame.

The numerical results are shown in Figs. 2 (b) to 2 (d). The solution from the substructure analysis was practically identical with the full system solution. Figure 2 (b) shows the load factor-center displacement curves. When the proportional load factor was greater than 1.8 the curve deviated from the straight line and convergence was not achieved within the maximum number of iterations allowed for each time step. Difficulty arises in predicting the collapse load from the load-displacement curve since it does not exhibit a flat plateau. The method of predicting the collapse load based on equilibrium consideration (4) was employed. Figure 2 (c) shows the error in vertical support reactions as a function of the load factor, λ. We may observe that the error increases very rapidly beyond $\lambda = 1.95$ for $\tau = 0.1$ sec. and beyond $\lambda = 2.05$ for $\tau = 3$ sec. If we consider 5% error in vertical reactions as an indication of failure to maintain equilibrium of the structure the collapse load factor is then found to be 2.1 for $\tau = 0.1$ sec. and 2.25 for $\tau = 3$ sec. These values are 2.4% and 9.7% higher than the actual collapse load factor predicted by the rigid-plastic theory (14), respectively. As noted earlier by Chang (4), the collapse load is not sensitive to the value of τ used when the parameter n is large.

Figure 2 (d) shows the collapse mechanism together with the plastic moments at the plastic hinges. This collapse mechanism is the same as the one given by Massonnet (14), except for some discrepancies in the plastic moments. Higher plastic moments are obtained when $\tau = 3$ sec. while lower values are obtained for $\tau = 0.1$ sec. The discrepancy is due to the fact that the dynamic yield stress for constant strain rate loading is proportional to $|\tau\dot{\varepsilon}|^{\frac{1}{n}}$ as depicted by Eq. (5). If the value of $|\tau\dot{\varepsilon}|$ is higher than unity the right hand side of Eq. (5) will be greater than unity, resulting in a higher dynamic yield stress than the static yield value. The opposite is true when $|\tau\dot{\varepsilon}|$ is smaller than unity.

4.2 Five-Story Frame Subjected to a Sinusoidal Ground Acceleration

The second example is a five story frame subjected to a sinusoidal ground acceleration of 10 sin 6t ft/sec^2. as shown in Fig. 3 (a). Table 1 lists the member properties and the floor masses. Advantage was taken of symmetry to model half of the structure as shown in Fig. 3 (b). Three Gauss points for each beam element and four for each column element were employed in the numerical integration with time steps of 0.1 sec. Five modes were included for the full system and three for the substructure analyses. In the latter two substructures with reduced d.o.f. as shown in Fig. 3 (c) were considered. The natural periods of vibration for the substructured system are shown and compared with the values obtained from the full system analysis in Table 2. Results of dynamic response are shown in Table 3 and Fig. 3 (d).

It is seen from Table 2 that the natural periods of vibration are predicted with satisfactory accuracy only for the first two modes in the substructure analysis. The third mode is much in error. Obviously the fourth and fifth modes of the full system which are translational modes cannot be reproduced by the substructured model.

In Table 3 are tabulated the maximum horizontal displacement of the top story and the maximum values of normalized moment in the second floor beam. The normalized moment is defined as the ratio of the moment to the static yield moment of the same section. The full system and the substructure analyses yielded almost identical numerical results with 1.34% and 1.44% discrepancies in the maximum displacement and the maximum moment response, respectively. Almost identical results were obtained for the displacement responses at the top story of both models so that they could not be plotted to show the differences. The plot of the response of normalized moment at the end of the second floor beam (Fig. 3 (d)) shows good accuracy in the substructure analysis in which three modes were included in the computation. However when five modes were used convergence could not be achieved within the number of iterations allowed for each time step and large errors in the displacement and normalized moment response occurred. This was attributed to the large errors in the higher modes obtained from the substructured model. Thus, the number of modes to be included in computation for the reduced system should be chosen properly by considering the accuracy of the eigen modes predicted.

4.3 Ten-Story Frame Subjected to a Sinusoidal Ground Acceleration

Next we considered a ten-story frame subjected to the same ground acceleration as in the five-story frame. The member properties and the geometry are shown in Fig. 4 (a) and the finite element models shown in Figs. 4 (b-d). Four point Gauss quadrature for each element was employed in the analyses. The effect of using single and double precision arithmetics on the numerical results was studied in this example owing to the reasonable size of this problem which is big enough for significant comparison and yet not exceeds the incore storage of the computer avaliable for double precision arithmetics. Also the accuracy of the results with different numbers of modes included in the computation was investigated.

In the full system analyses the results obtained using single and double precision arithmatics were almost identical. Therefore analyses of all examples were performed with single precision arithmatics to save computer storage and C.P.U. time. Displacement responses obtained from the five and eight mode solutions for the full system are shown in Fig. 4 (e). The latter is slightly greater than the former and shifting of the peaks in the negative direction is evident. When ten modes were included in the computation, instability occurred. In the substructure analyses, two substructure models were considered. Model 10 A (Fig. 4 (c)) and Model 10 B (Fig. 4 (d)) consisted of three and five substructures, respectively. The rotational d.o.f. of these models were condensed in the final reduced system. The integrating time step was taken to be 0.1 sec. Table 4 tabulates the natural periods for the two models. The results of the displacement and the normalized moment responses are shown in Table 5, Figs. 4 (f) through 4(h).

Table 4 reveals that the natural periods obtained from model 10 B are in reasonable agreement with the solution for the full system up to the fifth mode. Results from model 10 A deteriorate quickly for modes higher than the second one.

Table 5 shows that when five modes were included in model 10 A large errors occured in the maximum displacement and normalized moment responses due to the same reason as in the five story frame example. Inclusion of four modes in model 10 B yielded a slightly better result in the prediction of the maximum displacement than the case when five modes were included. However, the five mode solution was slightly more accurate in predicting the normalized moment.

Figures 4(f) through 4 (h) show the displacement-time curve and the normalized moment-time curves for model 10 A and model 10 B compared with the full system results. Again, better agreements were obtained with model 10 B which has more generalized co-ordinates.

4.4 Twenty-Story Frame Subjected to the 1940 EL Centro Earthquake

The last example considered was a 20-story frame subjected to the

North-South component of the 1940 EL Centro Earthquake. The configuration of the structure and the analysis models are shown in Figs. 5 (a) and 5 (b), respectively. Table 6 lists the relative member stiffness of the columns and girders. Four point Gauss quadrature for each element was employed in the analyses. The base of the structure was subjected to the 1940 EL Centro Earthquake for four seconds during which the most severe motion occurred (5). Each time step of integration was limited to 0.1 sec. or less. Two substructured models (Figs. 5 (c) and 5 (d)) with four and five substructures, respectively, were analyzed. The numbers of generalized co-ordinates are 30 for model 20 A and 36 for model 20 B, which are 18.75% and 22.50% of the full system, respectively. The results are shown in Tables 7, 8 and Figs. 5 (e) through 5 (g).

From Table 7 it is seen that the natural periods obtained from model 20 B were very good for the first two modes, and practically acceptable for modes up to the fifth one. Model 20A gave less accurate results especially for the higher modes. Therefore, five or seven modes were included in forced vibration analyses for model 20 A while seven modes were included in model 20 B.

Table 8 discloses that inclusion of seven modes in model 20 A gave practically the same degree of accuracy as using five modes in the computation.

Figures 5 (e) to 5 (g) show the displacement-time curve and the normalized moment-time curve for the two models as well as the full system model. Satisfactory prediction of peak values of both the horizontal displacement and the normalized moment response were obtained for the two reduced models, slightly better results being obtained from model 20 B. Some shifting of the response occurred after yielding in the last second.

In this example the required incore stotage and the C.P.U. time consumed in the substructure analysis were evaluated to study the advantages and disadvantages of the proposed method. When the number of degrees of freedom is reduced to 25% of the full system in the substructure analysis the required incore storage would be about 50% of that for the full system. But Table 9 discloses that the C.P.U. time required was 24.85% to 33.86% more due to transferring of substructure data into and out of core. When this transfer of data was eliminated by performing condensation of the full system, slightly less C.P.U. time was required in model 20 A than in the full system analysis.