CHAPTER III

FIELD PATTERN ANALYSIS

A. FIELD PATTERN OF THE CYLINDRICAL ANTENNA

It is known that the current distribution and terminal conditions have a larger effect on the input impedance than on the field pattern. If we had assumed sinusoidal current distribution for the computation of input impedance, the result would have had relatively large error especially for the thick cylindrical antenna.

From the thesis authored by Mr. Pakorn Borimasporn(6) there is a very small variation of the field pattern with the antenna thickness, when the current distribution is assumed to be sinusoidal (the value of $^{h}/_{a}$ in his experiment is 12.5) and from the experiment of Dorne (7) the result is nearly the same with some pattern nulls are filled in when the antenna thickness is increased(the lowest value of $^{h}/_{a}$ in his experiment is 8.7). Besides, the current distribution given in eq.(53) is too complicated to be used in the evaluation of electromagnetic field.

In this thesis, the sinusoidal current distribution is assumed and the solutions of the field patterns are based on the thesis titled "The Field Pattern and Gain Analysis of a Hollow Cylindrical Antenna" which is written by Mr. Pakorn Borimasporn. His derival began with

$$A_z = \frac{\mu}{4\pi}\int_0^{2\pi}\int_{-h}^{h}\frac{I(z)}{L}\,dz\ \ d\phi_1 \tag{69}$$

Where $L = \rho - a\ \mathrm{cosec}\ \theta\ \cos(\phi_1-\phi) - z\cos\theta$ (69a)

$$I(z) = I_o\sin\left[\frac{2\pi}{\lambda}(h-z)\right]\ e^{j(wt-\beta L)};\quad \text{for } z > 0 \tag{69b}$$

$$I(z) = I_o\sin\left[\frac{2\pi}{\lambda}(h+z)\right]\ e^{j(wt-\beta L)};\quad \text{for } z < 0 \tag{69c}$$

Eq.(69) and eq.(69c) are the conditions for center-fed cylindrical antenna. In this thesis, the field patterns of the end-fed cylindrical antenna are required.

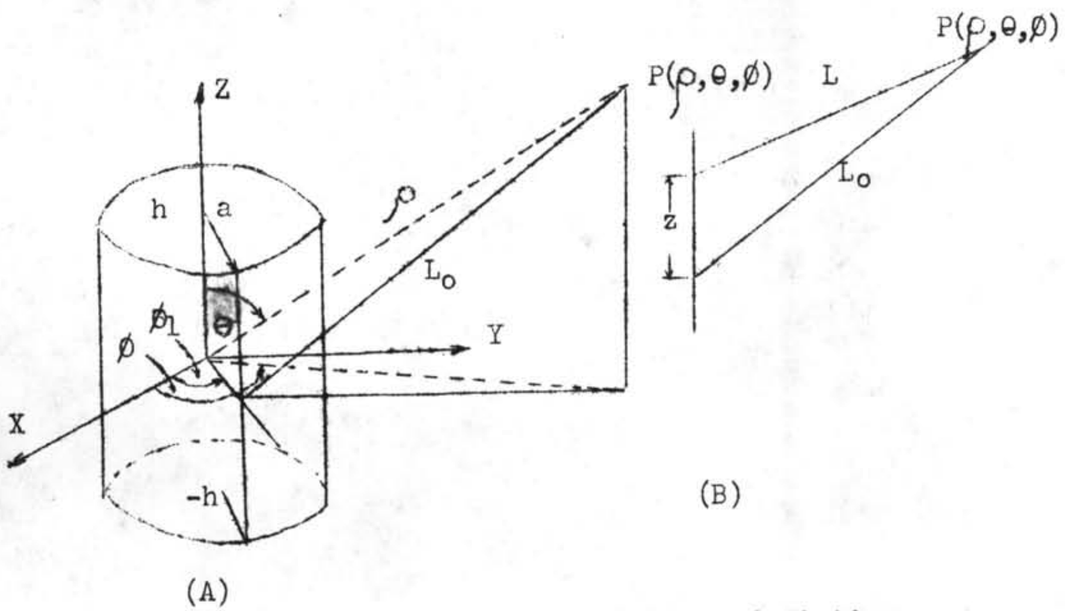

Fig. 2.1 The cylindrical antenna and its orientation.

Hence, for end-fed cylindrical antenna eq.(69) becomes,

$$A_z = \frac{\mu}{4\pi}\int_0^{2\pi}\int_0^h \frac{I(z)}{L}\, dz d\phi_1 \qquad (70)$$

From eq.(11) $$\bar{E} = -j\frac{c^2}{w}\nabla(\nabla.\bar{A}) - jw\bar{A} \qquad (71)$$

In his thesis, it is noted that for $\rho \gg 1, E_\theta \gg E_r$ and $-j\frac{c^2}{w}\nabla(\nabla.\bar{A})$ can be neglected, therefore, when $E_\phi = 0$, eq.(71) becomes,

$$E_\theta = -jwA_\theta \qquad (71a)$$

For cylindrical co-ordinate $A_\theta = - A_z \sin\theta$, hence eq.(71a) becomes,

$$E_\theta = jwA_z \sin\theta \qquad (71b)$$

Substituting eq.(69a), eq.(69b) into eq.(70) and rearranging the terms, we have

$$A_z = \frac{\mu I_o}{4\pi}\int_0^{2\pi} e^{j(wt - \beta\rho + a\beta\operatorname{cosec}\theta\cos(\phi-\phi_1))}.\, d\phi_1 \int_0^h \sin\left[\frac{2\pi}{\lambda}(h-z)\right] e^{j\beta z\cos\theta} dz \qquad (72)$$

The final result of eq.(72) becomes,

$$A_z = \frac{I_o \mu}{8\beta\rho\sin^2\theta} e^{-j(wt-\beta\rho)}\left[4-\beta^2a^2\operatorname{cosec}^2\theta\right]\left[\left\{\cos(\beta h\cos\theta)-\cos\theta)-\cos\beta h\right\} + j\left\{\sin(\beta h\cos\theta) - \cos\theta\sin\beta h\right\}\right] \qquad (73)$$

From eq.(71b) and eq.(73), it is seen that

$$E_\theta = \frac{jw\mu I_o}{8\beta\rho\sin\theta} e^{-j(wt-\beta\rho)}\left[4-\beta^2a^2\operatorname{cosec}^2\theta\right]\left[\left\{\cos(\beta h\cos\theta)-\cos\beta h\right\} + j\left\{\sin(\beta h\cos\theta) - \cos\theta\sin\beta h\right\}\right] \qquad (74)$$

When the phase patterm is disregarded, the field pattern becomes

$$E_\theta = \frac{w\mu I_o}{8\beta\rho\sin\theta}(4-\beta^2a^2\operatorname{cosec}^2\theta).\sqrt{\left[\cos(\beta h\cos\theta)-\cos\beta h\right]^2 + \left[\sin(\beta h\cos\theta) - \cos\theta\sin\beta h\right]^2} \qquad (74a)$$

Let $(\frac{w\mu I_o}{8\beta\rho}) = 1$, therfore, eq.(74a) becomes,

$$E_\theta = \frac{(4-\beta^2a^2\operatorname{cosec}^2\theta}{\sin\theta}\sqrt{\left[\cos(\beta h\cos\theta)-\cos\beta h\right]^2 + \left[\sin(\beta h\cos\theta) - \cos\theta\sin\beta h\right]^2} \qquad (74b)$$

B. FIELD PATTERN OF STACKED CYLINDRICAL ANTENNAS

The consideration is aimed at the inphase, two element stacked cylindrical antennas, with and without ground plane and the distance between elements equal to 0.5λ and 0.6λ as shown in Fig. 3.2

The computation followed the well-known "Law of Multiplication" (8) which states that :

$$E(\theta,\phi) = f(\theta,\phi)\, F(\theta,\phi) \underline{/ f_p(\theta,\phi) + F_p(\theta,\phi)} \qquad (75)$$

Where

$E(\theta,\phi)$ = field pattern of the arrays with distance of array = d

$f(\theta,\phi)$ = field pattern of the individual source

$f_p(\theta,\phi)$ = phase pattern of the individual source

$F(\theta,\phi)$ = field pattern of the array of isotropic sources with distance of array = d

$F_p(\theta,\phi)$ = phase pattern of the array of isotropic sources with distance of array = d

The field pattern of two inphase isotropic point sources A,B in Fig.3.3 can be verified as follows.

$$F_1(\theta,\phi) = E_3\, e^{\psi/2} + E_3\, e^{-\psi/2} \qquad (76)$$

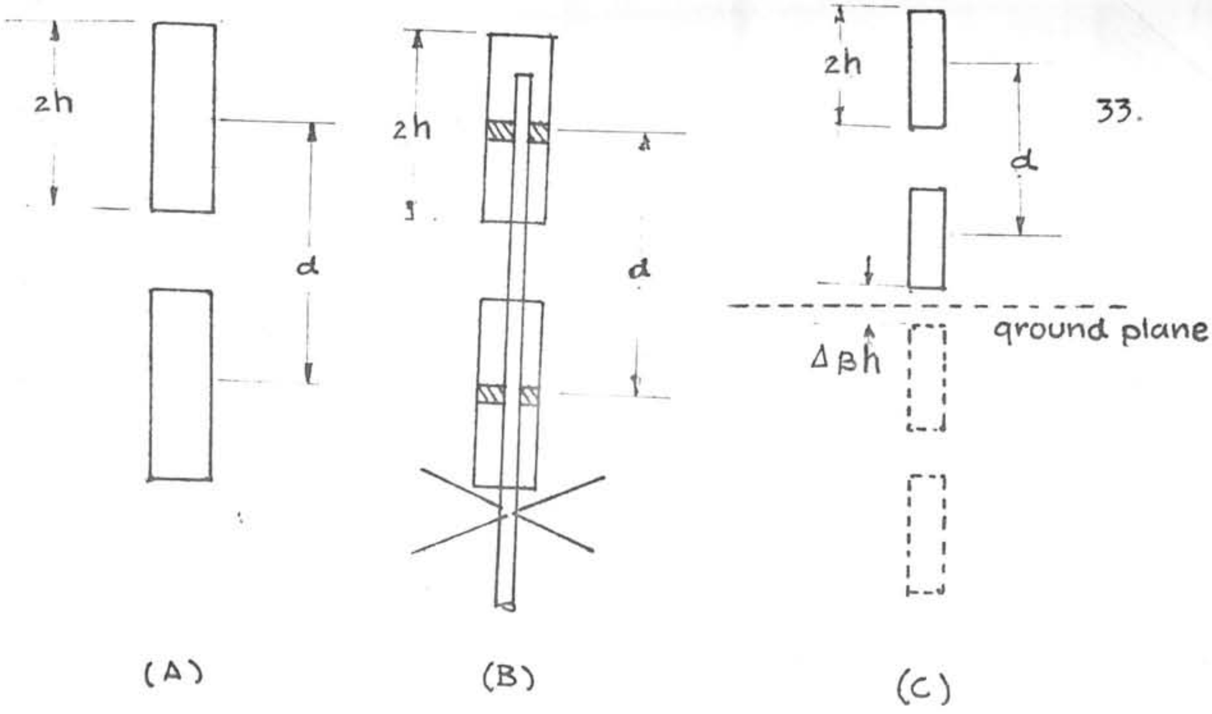

Fig 3.2 Stack cylindrical antenna, (A) without ground plane, (B) with four radial ground rods, (C) equivalent diagram of (B)

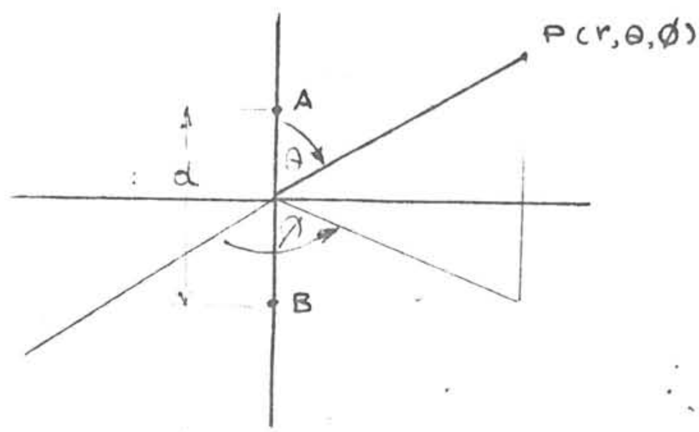

Fig 3.3 Array of two isotropic point sources

E_o = amplitude of the field strength of A,B at distance r, where $r \gg d$

$$= d_r \cos\theta \qquad (77)$$

$$d_r = \frac{2\pi}{\lambda} \cdot d \qquad (78)$$

Hence eq.(76) becomes:

$$F_1(\theta,\phi) = 2E_o \cos\left(\frac{d_r \cos\theta}{2}\right) \qquad (79)$$

Eq.(79) is independent of ϕ, therefore, eq.(79) can be written as

$$F_1(\theta) = 2E_o \cos\left(\frac{d_r \cos\theta}{2}\right) \qquad (80)$$

Let $2E_o = 1$, then the normalized value of eq.(80) becomes,

$$F(\theta) = \cos\left(\frac{d_r \cos\theta}{2}\right) \qquad (80a)$$

Let $E_1(\theta)$ = field pattern of the inphase, two-element stacked cylindrical antennas without ground plane.

From eq.(74a) and eq.(75) and eq. (80), the field pattern becomes

$$E_1(\theta) = \left[\frac{w\mu I_o\, 2E_o}{8\beta r}\right]\left[\frac{(4-\beta^2a^2 \operatorname{cosec}^2\theta)}{\sin\theta}\right]$$
$$\left[\sqrt{\left\{\cos(\beta h\cos\theta)-\cos\beta h\right\}^2 + \left\{\sin(\beta h\cos\theta) - \cos\theta \sin\beta h\right\}^2}\right]\left[\cos\left(\frac{d_r\cos\theta}{2}\right)\right] \qquad (81)$$

Let $\left(\frac{w\mu I_o\, 2E_o}{8\beta r}\right) = 1$; hence eq.(81) becomes,

$$E(\theta) = \left[\frac{(4-\beta^2a^2 \operatorname{cosec}^2\theta)}{\sin\theta}\right]\left[\sqrt{\left\{\cos(\beta h\cos\theta)- \cos\beta h\right\}^2 + \left\{\sin(\beta h\cos\theta)-\cos\theta\sin\beta h\right\}^2}\right] \cdot \left[\cos\left(\frac{d_r}{2}\cos\theta\right)\right]$$

Eq.(82) is the field pattern of the in-phase, two-element stacked cylindrical antennas as shown in Fig.3.2(a), with length = h, redius = a, and distance between the elements = d_r

From Fig. 3.2(b), inserting the ground plane causes images on the opposite side of the ground plane [Fig. 3.2(c)]. By assuming that the ground plane is perfect. The computation is performed by following eq.(76) to eq.(82) with the field pattern of individual source $f(\theta,\phi)$ being expressed by eq.(82) and the distance between elements, $d = (d_r + \beta h + \Delta\beta h.)$

As $\Delta\beta h \ll (d_r + \beta h)$, the distance between elements becomes

$$d_1 = (d_r + \beta h)$$

Hence, the field pattern becomes,

$$E(\theta) = \left[\frac{4-\beta^2 a^2 \operatorname{cosec}^2\theta}{\sin\theta}\right]\left[\sqrt{\left\{\cos(\beta h\cos\theta)-\cos\beta h\right\}^2 + \left\{\sin(\beta h\cos\theta)-\cos\theta\sin\beta h\right\}^2}\right]$$
$$\left[\cos\left(\frac{d_r\cos\theta}{2}\right)\right]\left[\cos\frac{(d_r+\beta h)\cos\theta}{2}\right] \qquad (83)$$

Eq.(83) is the expression of the field pattern of the inphase. two-element, stacked cylindrical antennas [as in Fig. 3.2(b)], with length = h, radius = a, and the distance between elements = $(d_r + \beta h)$

Due to the complication of the functions involved in the field pattern analysis, it is advisable to use a computer programmed for these computations.

C. DATA PRECALCULATED FOR NUMERICAL ANALYSIS

The purpose of this analysis is to compute the field pattern of cylindrical stub antenna and stacked cylindrical antennas of which the length to radius ratio = 13.1

Consider eq. (80a)

$$F(\theta) = \frac{\cos(d_r \cos\theta)}{2} \qquad (84)$$

The field pattern of eq.(84) depends upon d_r, Fig. 3.4 shows the field patterns of eq.(84) for d_r = 0.4λ, 0.5λ and 0.6λ respectively.

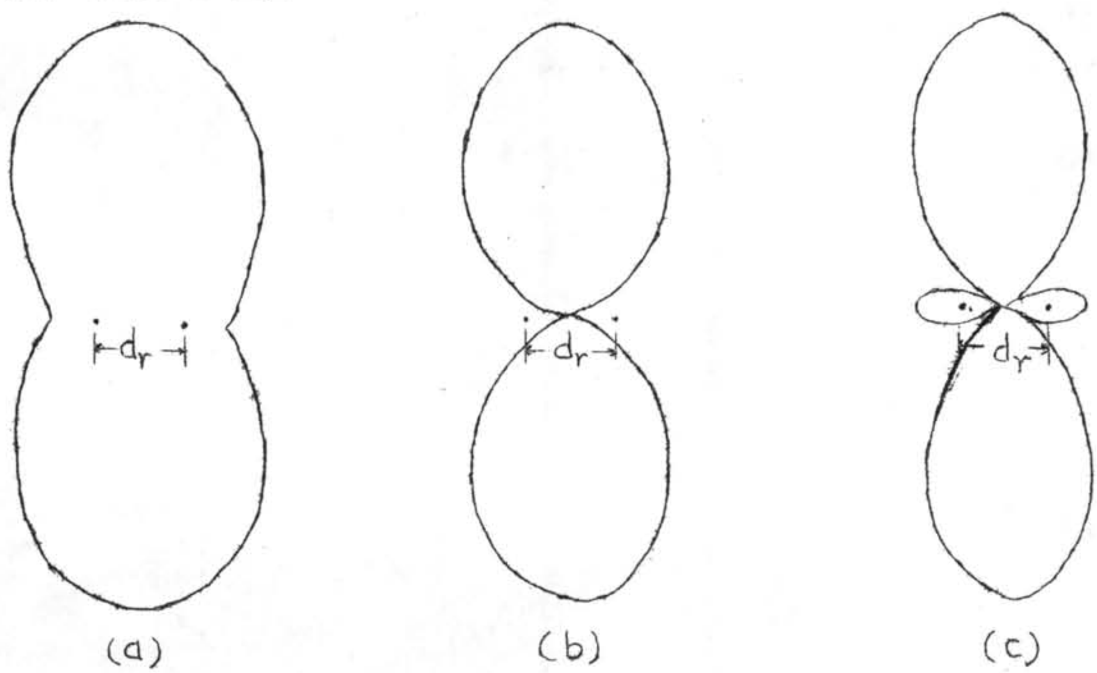

Fig. 3.4 The field pattern of eq.(84) for which (a) d_r = 0.4λ , (b) d_r = 0.5λ and (c) d_r = 0.6λ

C. DATA PRECALCULATED FOR NUMERICAL ANALYSIS

The following analysis is to compute the field pattern of cylindrical stub antenna and stacked cylindrical antennas in which the length to radius ratio = 13.1

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$$F(\theta) = \frac{\cos(d_r \cos\theta)}{2} \qquad (84)$$

The field pattern of eq.(84) depends upon d_r, Fig. 3.4 will show field pattern of eq.(84) for $d_r = 0.4\lambda$, 0.5λ and 0.6λ respectively.

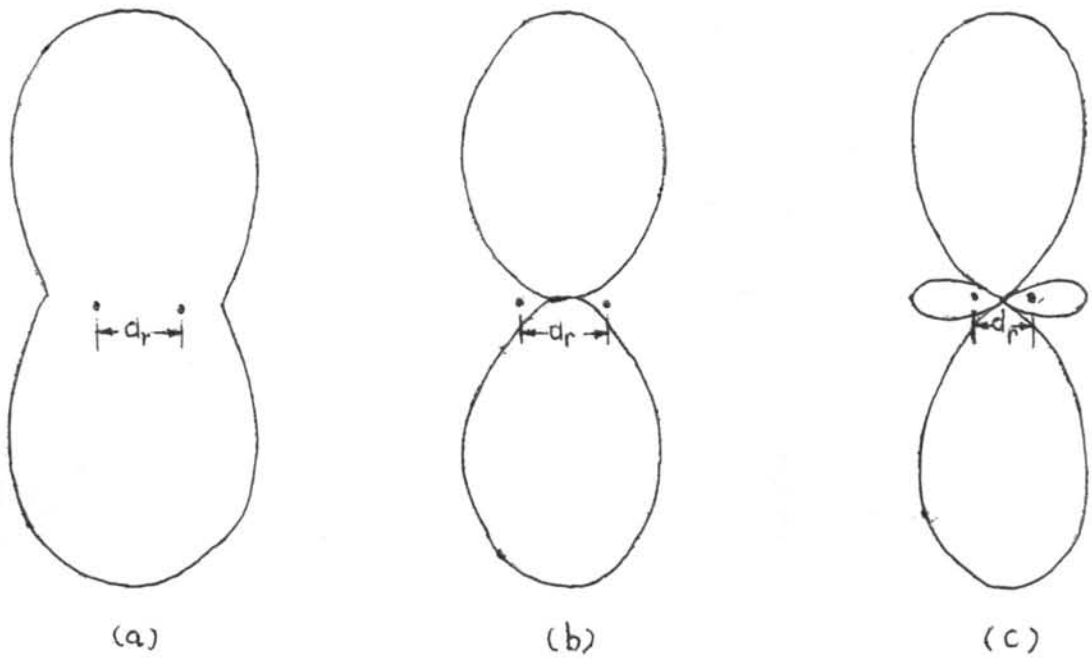

Fig. 3.4 The field pattern of eq.(84) for which (a) $d_r = 0.4\lambda$, (b) $d_r = 0.5\lambda$ and (c) $d_r = 0.6\lambda$

From eq.(84), it is clear that

When $d_r < 0.5\lambda$, the null positions are filled in and increase when d_r is decreased. For stacked antennas the null position is required, so the value of d_r less than 0.5λ is not practically used.

When $d_r = 0.5\lambda$, the null positions are existed and when $d_r > 0.5\lambda$, the directivity is increased, so as the minor loops

In this thesis the value of $d_r = 0.5\lambda$ and 0.6λ are used. The value of h is the near second resonant length, 0.4λ and $h/a = 13.1$.

Let $A = \frac{h}{a}$

$$B = \left[4 - (2\pi \frac{h}{A})^2 \operatorname{cosec}^2\theta\right]$$

$$BR = \sqrt{\left[\cos(\beta h\cos\theta) - \cos\beta h\right]^2 + \left[\sin(\beta h\cos\theta) - \cos\theta \sin\beta h\right]^2}$$

Hence eq.(74b) becomes,

$$E_\theta = \frac{B \cdot BR}{\sin\theta} \qquad (85)$$

Eq. (82) becomes,

$$E_\theta = \frac{B \cdot BR}{\sin\theta} \left\{ \cos(\pi d_r \cos\theta) \right] \qquad (86)$$

And eq.(83) becomes,

$$E_\theta = \frac{B \cdot BR}{\sin\theta} \left[\cos(\pi d_r \cos\theta)\right] \left[\cos\left\{\pi \cdot (d_r + h)\cos\theta\right\}\right] \qquad (87)$$

D. COMPUTER PROGRAM FOR THEORETICAL ANALYSIS

This theoretical analysis is run by the computer NEAC-SERIES 2200 which is installed at the Computer Science Center, Chulalongkorn Univeristy.

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C     PROGRAM SOLVING THE FIELD STRENGTH OF STACK CYLINDRICAL RADIATO
C     E(I) = FIELD STRENGTH OF THE ANTENNA
      DIMENSION THETA (38),E (37), EN (37)
      PI = 3.14159
      DO 50 II= 1,2
      READ(2,5)A,AK,AKK
    5 FORMAT(3F10.3)
      DO 100JJ=1,3
    7 WRITE(3,9)
    9 FORMAT(1H1,10X,42HTHE FIELD STRENGTH OF CYLINDRICAL RADIATOR,/)
      WRITE(3,10) A
   10 FORMAT(10X,28HRATIO OF LENGTH TO RADIOUS = ,F10.3)
      WRITE(3,17)AKK
   17 FORMAT(10X,17HLENGTH OF ARRAY =,F10.3)
      WRITE(3,20)AK
   20 FORMAT(10X,20HANTENNA LENGTH      = ,F5.2//)
   25 WRITE(3,30)
   30 FORMAT(10X,6HDEGREE,10X,5HVALUE,10X,10HNORMALIZE,/)
      THETA(1) = 0
      DO 35 I=2,38
      THETA(I) =THETA(I-1)+5.
         THE   = THETA(I-1)*PI/180.
      S = SIN(THE)
      C =COS(THE)
      CL = COS(THE)*PI
      AL=2.*PI*AK
      SL=SIN(AL)
      CLL=COS(AL)
      B=(4.-((AL/A)**2)/S**2)
      BR=SQRT(1.+CLL**2+(C**2)*(SL**2)-2.*(CLL*COS(AL*C)+C*SL*SIN(AL*C
     -)
      GO TO (41,42,43),JJ
   41 E(I-1)=B*BR/S
      GO TO 35
   42 E(I-1)=B*BR*(COS(AKK*CL))/S
      GO TO 35
   43 E(I-1)=B*BR*(COS(AKK*CL))*(COS((AKK+AK)*CL))/S
      GOTO 35
   35 CONTINUE
      DO 40 I=1,37
   40 EN(I)=E(I)/E(19)
      WRITE(3,45) (THETA(I), E(I), EN(I), I=1,37 )
   45 FORMAT (10X,F6.1,7X,F8.4,10X,F8.5)
  100 CONTINUE
   50 CONTINUE
      STOP
      END
```

THE FIELD STRENGTH OF CYLINDRICAL RADIATOR

RATIO OF LENGTH TO RADIOUS = 13.100
LENGTH OF ARRAY = .500
ANTENNA LENGTH = .40

DEGREE	VALUE	NORMALIZE,
0.0	* .9E+99	* .1E+99
5.0	-.1110	-.01549
10.0	.7301	.10184
15.0	1.3606	.18977
20.0	1.9386	.27040
25.0	2.4947	.34797
30.0	3.0377	.42370
35.0	3.5689	.49779
40.0	4.0863	.56996
45.0	4.5860	.63966
50.0	5.0622	.70608
55.0	5.5082	.76829
60.0	5.9165	.82523
65.0	6.2792	.87582
70.0	6.5886	.91899
75.0	6.8377	.95373
80.0	7.0204	.97921
85.0	7.1320	.99477
90.0	7.1695	1.00000
95.0	7.1320	.99477
100.0	7.0204	.97921
105.0	6.8378	.95373
110.0	6.5887	.91899
115.0	6.2792	.87582
120.0	5.9165	.82523
125.0	5.5082	.76829
130.0	5.0622	.70608
135.0	4.5860	.63966
140.0	4.0864	.56997
145.0	3.5689	.49779
150.0	3.0377	.42370
155.0	2.4948	.34797
160.0	1.9386	.27040
165.0	1.3607	.18979
170.0	.7303	.10186
175.0	-.1114	-.01554
180.0	*-.7E+12	*-.1E+12

THE FIELD STRENGTH OF CYLINDRICAL RADIATOR

RATIO OF LENGTH TO RADIOUS = 13.100
LENGTH OF ARRAY = .500
ANTENNA LENGTH = .40

DEGREE	VALUE	NORMALIZE,
0.0	* .9E+99	* .1E+99
5.0	-.0007	-.00009
10.0	.0174	.00243
15.0	.0728	.01015
20.0	.1834	.02558
25.0	.3658	.05103
30.0	.6346	.08851
35.0	1.0003	.13952
40.0	1.4681	.20478
45.0	2.0363	.28402
50.0	2.6937	.37572
55.0	3.4198	.47699
60.0	4.1836	.58353
65.0	4.9456	.68982
70.0	5.6604	.78952
75.0	6.2804	.87599
80.0	6.7609	.94301
85.0	7.0652	.98546
90.0	7.1695	1.00000
95.0	7.0652	.98546
100.0	6.7609	.94301
105.0	6.2804	.87599
110.0	5.6605	.78952
115.0	4.9457	.68983
120.0	4.1836	.58353
125.0	3.4198	.47699
130.0	2.6938	.37573
135.0	2.0363	.28402
140.0	1.4682	.20478
145.0	1.0003	.13952
150.0	.6346	.08851
155.0	.3659	.05103
160.0	.1834	.02558
165.0	.0728	.01015
170.0	.0174	.00243
175.0	-.0007	-.00009
180.0	*-.3E+07	*-.4E+06

THE FIELD STRENGTH OF CYLINDRICAL RADIATOR

RATIO OF LENGTH TO RADIOUS = 13.100
LENGTH OF ARRAY = .500
ANTENNA LENGTH = .40

DEGREE	VALUE	NORMALIZE.
0.0	* .9E+99	* .1E+99
5.0	.0006	.00009
10.0	-.0163	-.00228
15.0	-.0667	-.00931
20.0	-.1623	-.02263
25.0	-.3062	-.04271
30.0	-.4882	-.06809
35.0	-.6784	-.09462
40.0	-.8231	-.11480
45.0	-.8461	-.11801
50.0	-.6577	-.09173
55.0	-.1742	-.02429
60.0	.6545	.09128
65.0	1.8155	.25322
70.0	3.2136	.44824
75.0	4.6724	.65171
80.0	5.9622	.83161
85.0	6.8518	.95569
90.0	7.1695	1.00000
95.0	6.8518	.95569
100.0	5.9622	.83162
105.0	4.6725	.65172
110.0	3.2137	.44825
115.0	1.8155	.25323
120.0	.6545	.09129
125.0	-.1741	-.02429
130.0	-.6577	-.09173
135.0	-.8461	-.11801
140.0	-.8231	-.11480
145.0	-.6784	-.09462
150.0	-.4882	-.06810
155.0	-.3062	-.04271
160.0	-.1623	-.02263
165.0	-.0668	-.00931
170.0	-.0163	-.00228
175.0	.0006	.00009
180.0	* .2E+07	* .3E+06

THE FIELD STRENGTH OF CYLINDRICAL RADIATOR

RATIO OF LENGTH TO RADIOUS = 13.100
LENGTH OF ARRAY = .600
ANTENNA LENGTH = .40

DEGREE	VALUE	NORMALIZE,
0.0	* .9E+99	* .1E+99
5.0	.0336	.00468
10.0	-.2056	-.02868
15.0	-.3365	-.04694
20.0	-.3861	-.05385
25.0	-.3421	-.04771
30.0	-.1871	-.02609
35.0	.0954	.01331
40.0	.5169	.07210
45.0	1.0809	.15076
50.0	1.7794	.24819
55.0	2.5905	.36133
60.0	3.4776	.48506
65.0	4.3899	.61231
70.0	5.2662	.73453
75.0	6.0400	.84246
80.0	6.6477	.92722
85.0	7.0359	.98137
90.0	7.1695	1.00000
95.0	7.0359	.98137
100.0	6.6477	.92722
105.0	6.0401	.84247
110.0	5.2662	.73454
115.0	4.3900	.61232
120.0	3.4777	.48506
125.0	2.5905	.36133
130.0	1.7794	.24819
135.0	1.0809	.15077
140.0	.5169	.07210
145.0	.0954	.01331
150.0	-.1871	-.02609
155.0	-.3421	-.04771
160.0	-.3861	-.05385
165.0	-.3365	-.04694
170.0	-.2057	-.02869
175.0	.0337	.00470
180.0	* .2E+12	* .3E+11

THE FIELD STRENGTH OF CYLINDRICAL RADIATOR

RATIO OF LENGTH TO RADIOUS = 13.100
LENGTH OF ARRAY = .600
ANTENNA LENGTH = .40

DEGREE	VALUE	NORMALIZE.
0.0	* .9E+99	* .1E+99
5.0	-.0336	-.00468
10.0	.2054	.02865
15.0	.3346	.04667
20.0	.3791	.05288
25.0	.3274	.04566
30.0	.1707	.02381
35.0	-.0804	-.01122
40.0	-.3835	-.05349
45.0	-.6547	-.09132
50.0	-.7717	-.10763
55.0	-.5935	-.08278
60.0	-.0000	-.00000
65.0	1.0567	.14739
70.0	2.5077	.34977
75.0	4.1510	.57898
80.0	5.6828	.79263
85.0	6.7738	.94481
90.0	7.1695	1.00000
95.0	6.7738	.94482
100.0	5.6828	.79264
105.0	4.1511	.57899
110.0	2.5077	.34978
115.0	1.0568	.14740
120.0	.0000	.00000
125.0	-.5935	-.08277
130.0	-.7717	-.10763
135.0	-.6547	-.09132
140.0	-.3835	-.05349
145.0	-.0804	-.01122
150.0	.1707	.02381
155.0	.3274	.04566
160.0	.3791	.05288
165.0	.3346	.04667
170.0	.2055	.02866
175.0	-.0337	-.00469
180.0	*-.2E+12	*-.3E+11

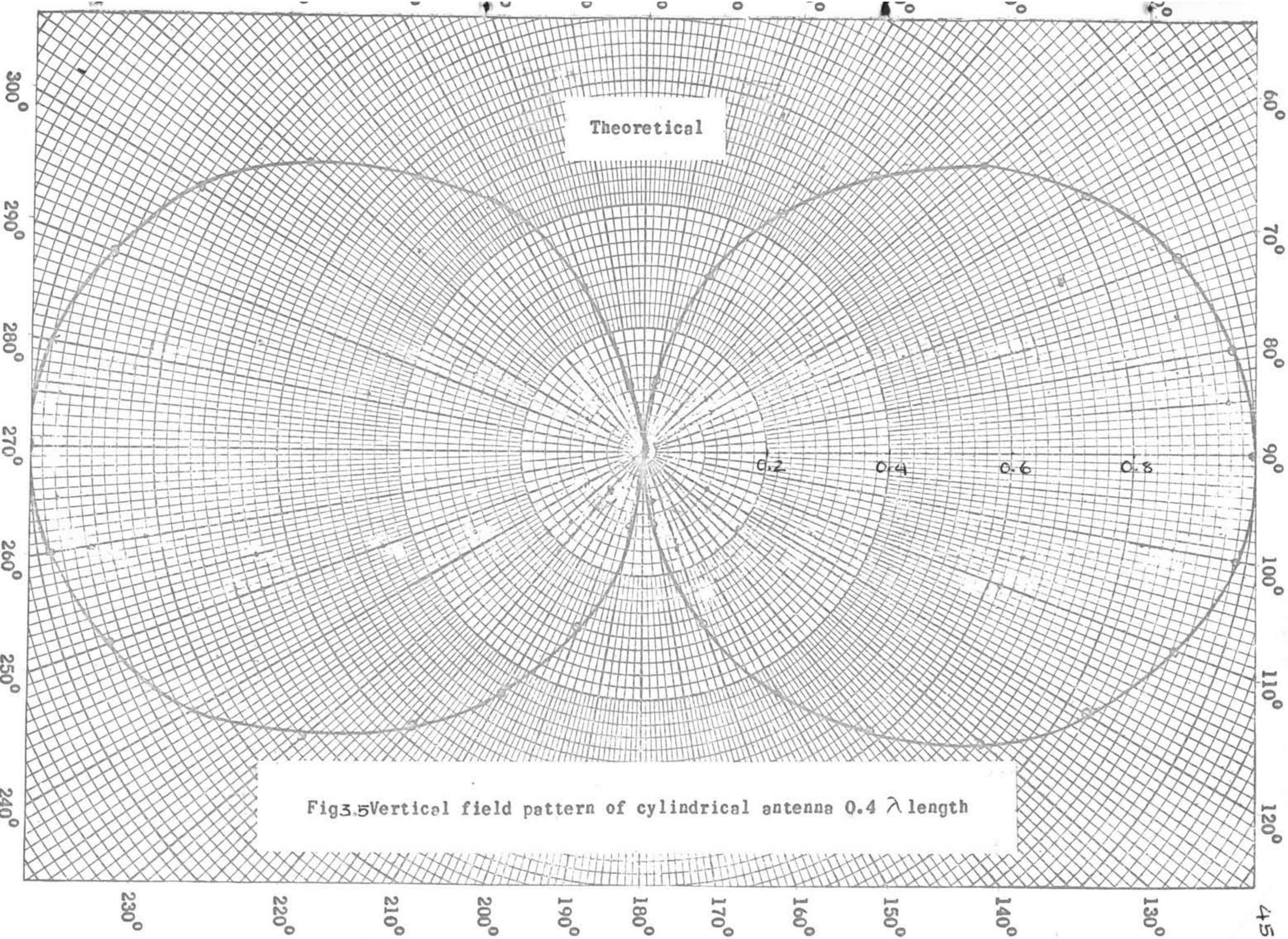

Fig3.5Vertical field pattern of cylindrical antenna 0.4 λ length

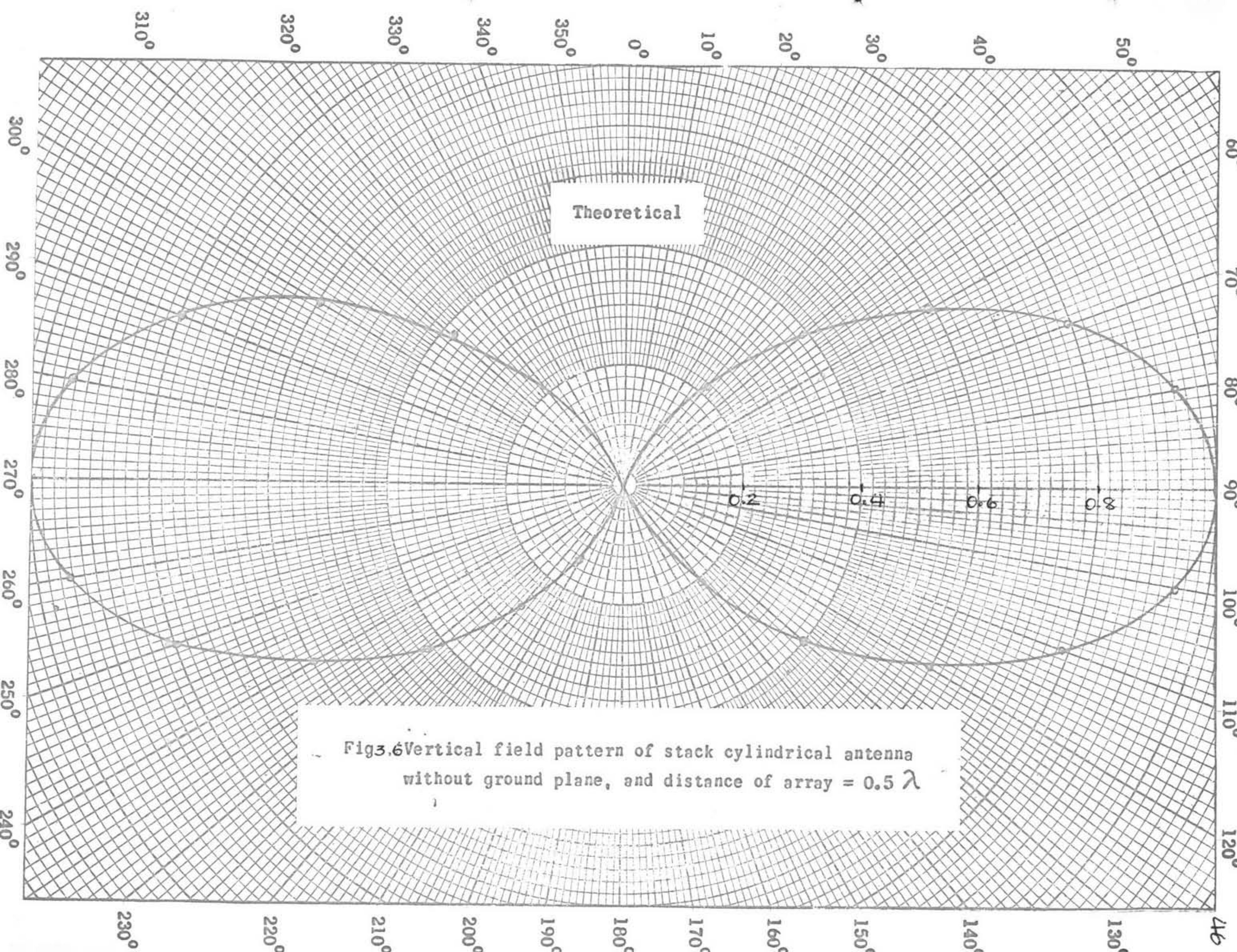

Fig3.6Vertical field pattern of stack cylindrical antenna without ground plane, and distance of array = 0.5 λ

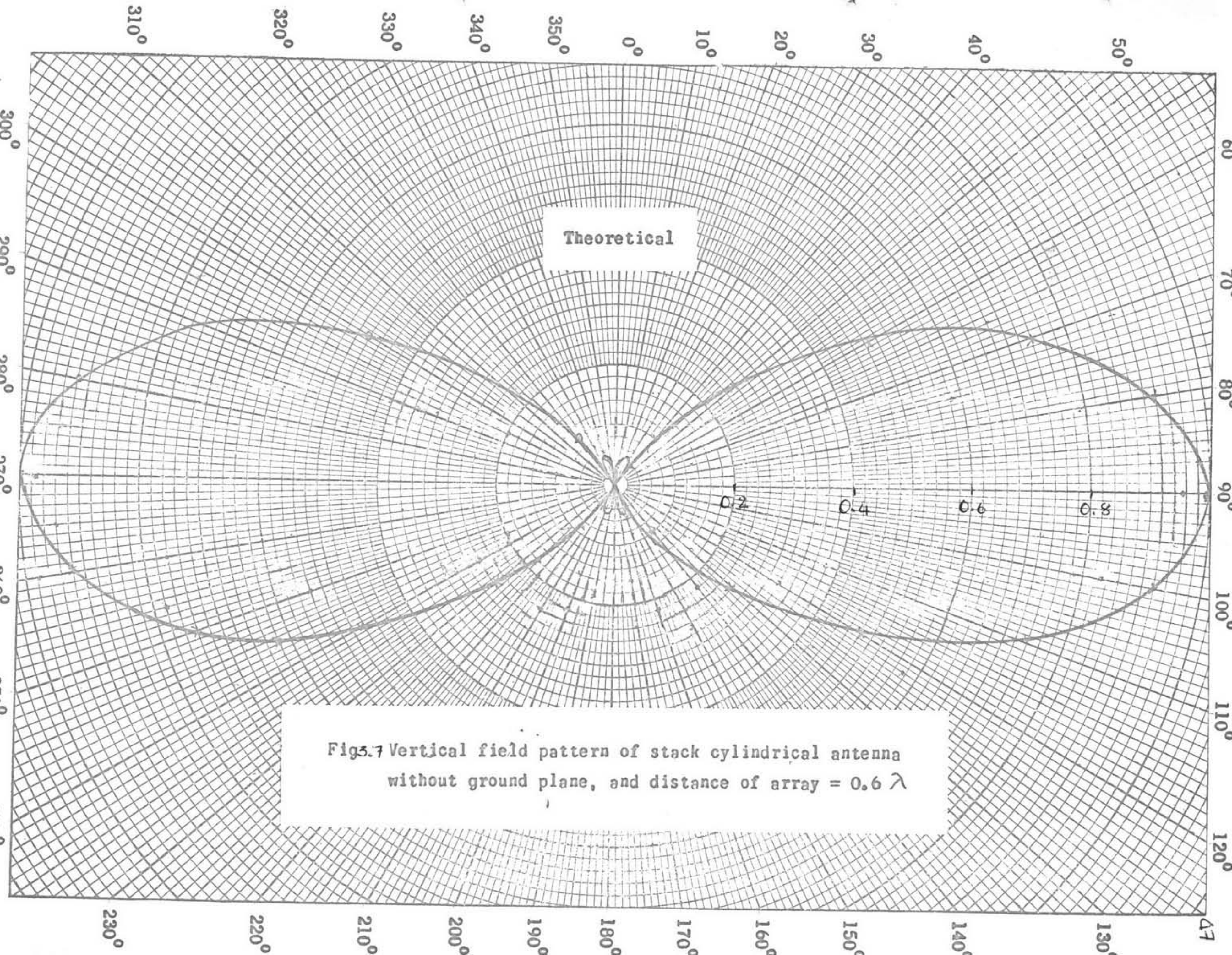

Fig5.7 Vertical field pattern of stack cylindrical antenna without ground plane, and distance of array = 0.6 λ

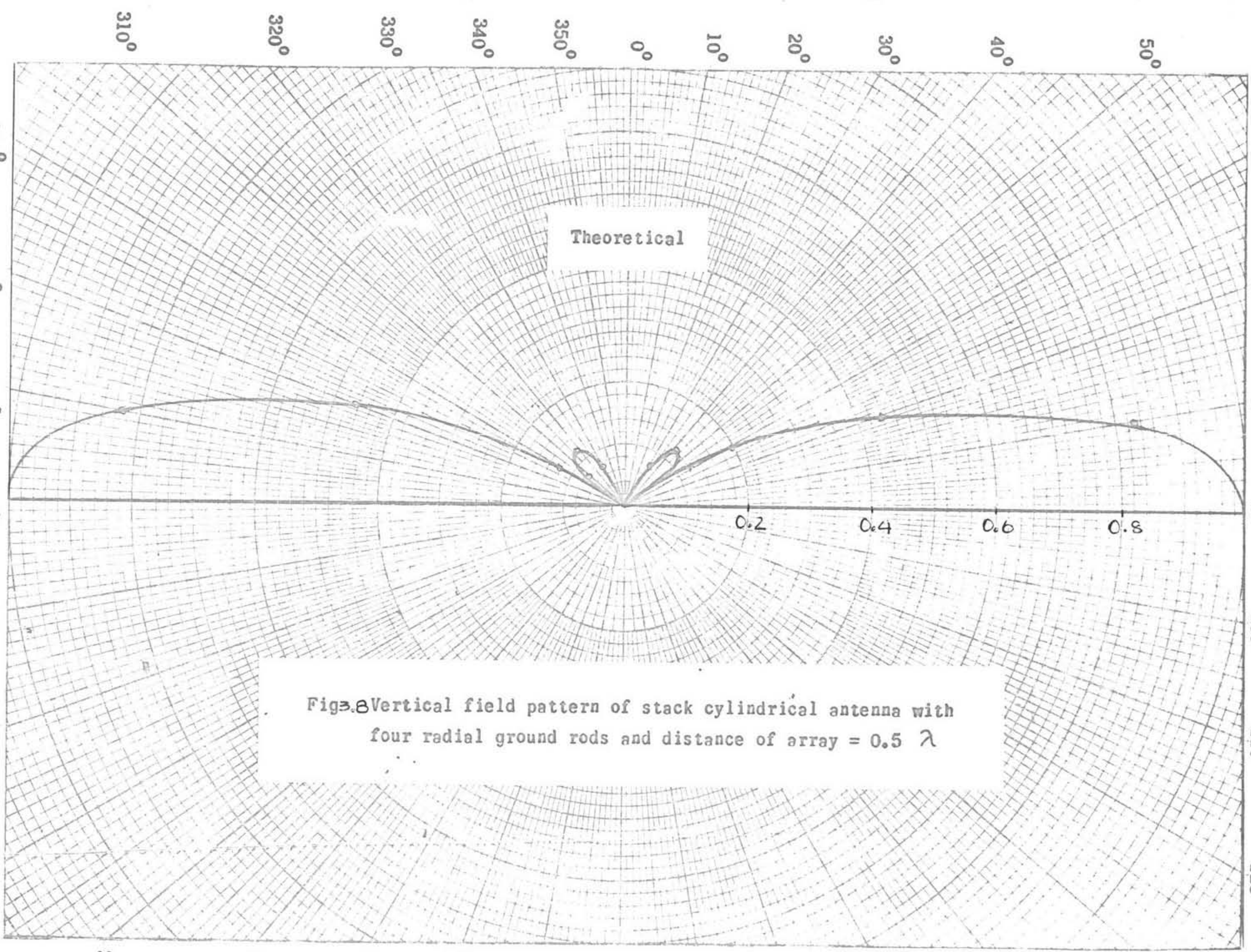

Fig3.8 Vertical field pattern of stack cylindrical antenna with four radial ground rods and distance of array = 0.5 λ

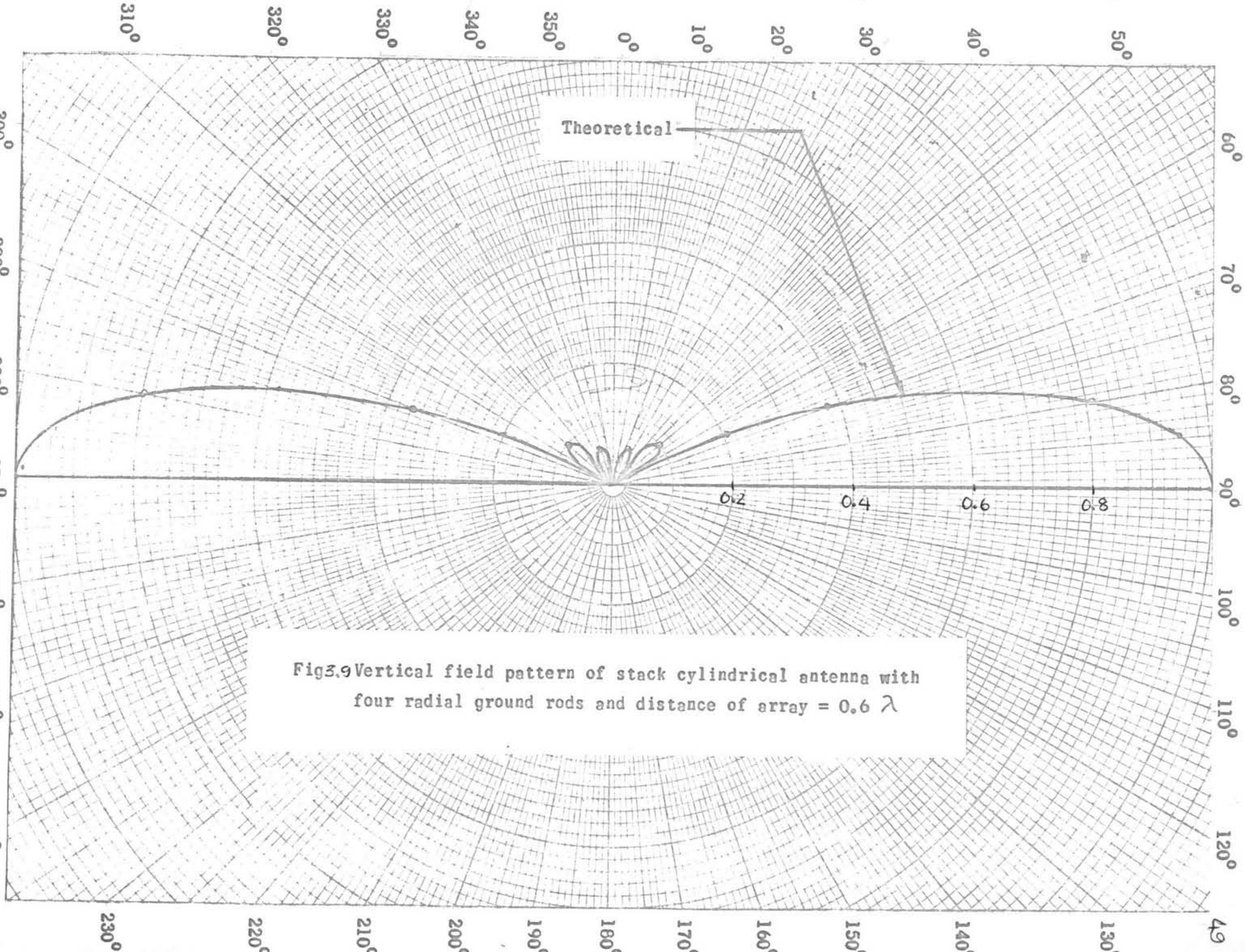

Fig3.9 Vertical field pattern of stack cylindrical antenna with four radial ground rods and distance of array = 0.6 λ