References

1. D'azzo, John J. . and Houpis, Constantine H. Feedback Control System Analysis & Synthesis. Kogakusha : Mc.Graw-Hill,Inc. 1966.

2. Gelb, Arthur and Velde, Wallace E. Vander Multiple-Input Deseribing Functions and Nonlinear System Design. Mc. Graw-Hill, Inc. 1968.

3. Gibson, John E. Nonlinear Autonomic Control. Tokyo : Mc. Graw-Hill, Inc. 1963.

4. Truxal, John. Automatic Feedback Control System Synthesis: Kogakusha : Mc. Graw-Hill, Inc. 1955.

5. West, John C. Analytical Techniques For Non-Linear Control System. London : The English University Press Ltd. 1960.

APPENDIX A

COMMON TYPES OF DESCRIBING FUNCTION

The describing functions [1,2,3,4,5] of various nonlinearities are summarized here for easy references and the graphs of these nonlinearity characteristic are show in Fig. A.1 to Fig.A.8

A.1 Dead-zone

In this case, we have

$$g(E) = 0 \qquad \text{for } E \leq b \qquad \text{(A1.1)}$$

$$g(E) = \frac{n}{\pi}\left[\pi - 2\sin^{-1}\frac{1}{E/b} - 2\,\frac{1}{E/b}\sqrt{1-\left(\frac{1}{E/b}\right)^2}\,\right]$$

$$\text{for } E > b \qquad \text{(A.1.2)}$$

$$b(E) = 0 \qquad \text{for all } E \qquad \text{(A.1.3)}$$

A.2 Ideal Relay

In this case, we have

$$g(E) = \frac{4M}{\pi E} \qquad \text{for all } E \qquad \text{(A.2.1)}$$

$$b(E) = 0 \qquad \text{for all } E \qquad \text{(A.2.2)}$$

A.3 Relay with deal-zone

In this case, we have

$$g(E) = 0 \qquad \text{for } 0 \leq E \leq a \qquad \text{(A.3,1)}$$

$$g(E) = \frac{4M}{\P E} \cos \qquad \text{for } E > a \qquad (A.3.2)$$

$$b(E) = 0 \qquad \text{for all } E \qquad (A.3.3)$$

where $\Theta = \sin^{-1} \frac{1}{E/a}$

A.4 Relay with hysteresis

Inthis case, we have

$$g(E) = \frac{4M}{\P E} \cos \Theta \qquad \text{for } E > b \qquad (A.4.1)$$

$$b(E) = - \frac{4M}{\P E} \sin \Theta \qquad \text{for } E > b \qquad (A.4.2)$$

where $\Theta = \sin^{-1} \frac{1}{E/b}$

A.5 Relay with hysteresis and dead-zone

In this case, we have

$$g(E) = \frac{2M}{\P E} (\cos\Theta_1 + \cos\Theta_2) \qquad \text{for } E \leq a \text{ and } E \leq d \qquad (A.5.1)$$

$$b(E) = \frac{-2M}{\P E} (\sin\Theta_1 + \cos\Theta_2) \qquad \text{for } E \leq a, \; E \leq d \qquad (A.5.2)$$

where

$$\Theta_1 \quad \sin^{-1} \frac{1}{E/a}$$

$$\Theta_2 \quad \sin^{-1} \frac{1}{E/d}$$

A.6 Backlash

In this case, we have

$$g(E) = \left[\frac{2n}{\P} \quad \frac{\P}{4} + \frac{\Theta}{2} + \frac{1}{4} \sin 2\Theta \right] \quad \text{for } E > d \qquad (A.6.1)$$

$$b(E) = -\frac{2n}{\pi}\cos^2\Theta \qquad \text{for } E > d \qquad \text{(A.6.2)}$$

where $\Theta = \sin^{-1}\frac{1}{E/d}$

A.7 Quintic characteristic

In this case, we have

$$g(E) = \frac{5}{8}E^4 \qquad \text{for all } E \qquad \text{(A.7.1)}$$

$$b(E) = 0 \qquad \text{for all } E \qquad \text{(A.7.2)}$$

A.8 Saturation

In this case, we have

$$g(E) = n \qquad \text{for } E \leq b \qquad \text{(A.8.1)}$$

$$g(E) = \frac{2n}{\pi}\left[\sin^{-1}\left(\frac{1}{E/b}\right) + \frac{1}{E/b}\sqrt{1-\left(\frac{1}{E/b}\right)^2}\,\right] \qquad \text{for } E > b \qquad \text{(A.8.2)}$$

$$b(E) = 0 \qquad \text{for all } E \qquad \text{(A.8.3)}$$

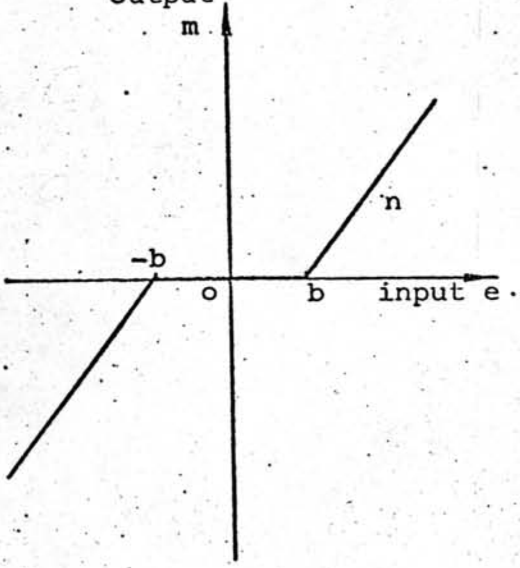

Fig. A.1 A dead - zone

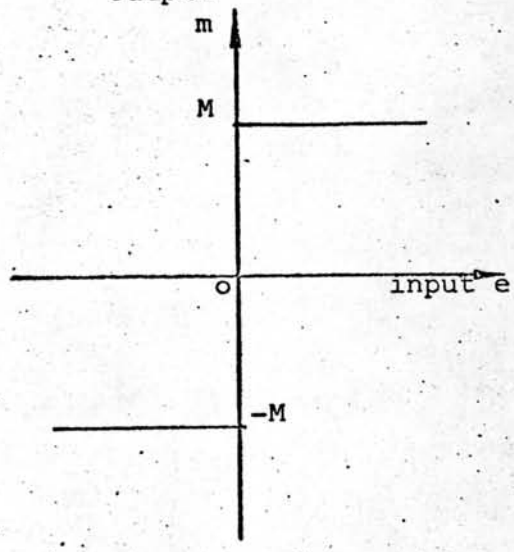

Fig. A.2 An ideal relay

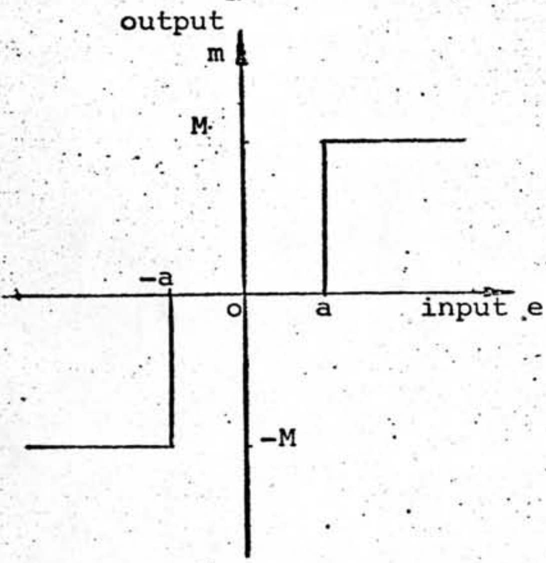

Fig. A.3 A relay with dead - zone

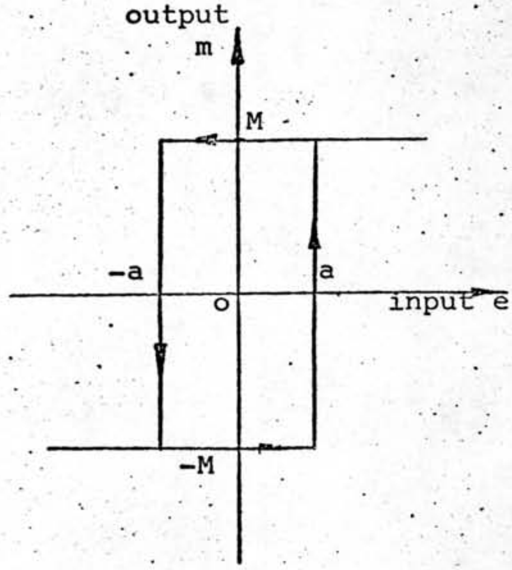

Fig. A.4 A relay with hysteresis

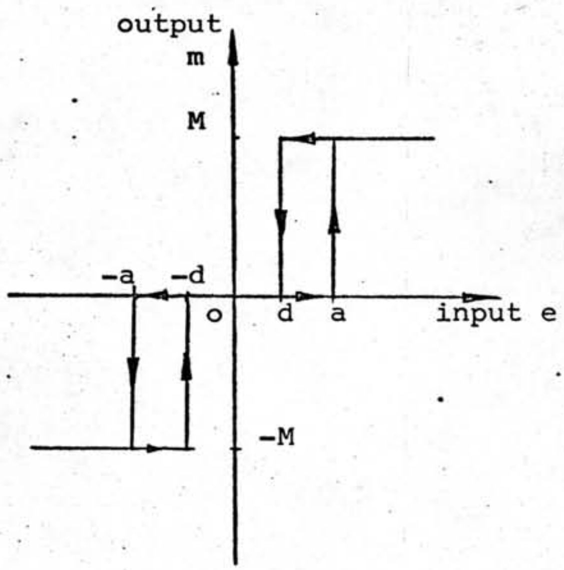

Fig. A.5. A relay with hysteresis and dead - zone

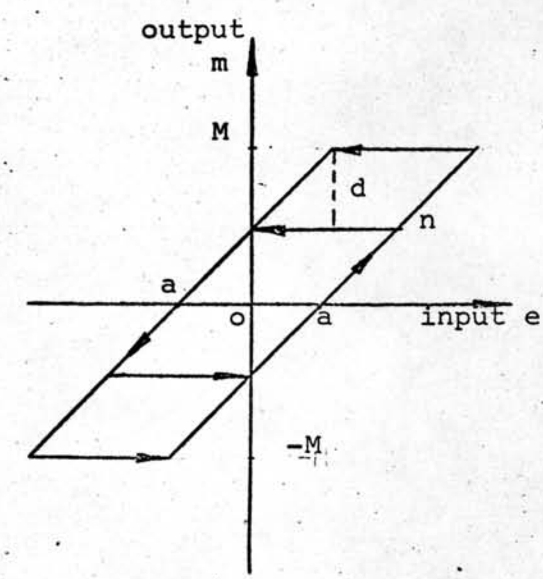

Fig. A.6. A backlash

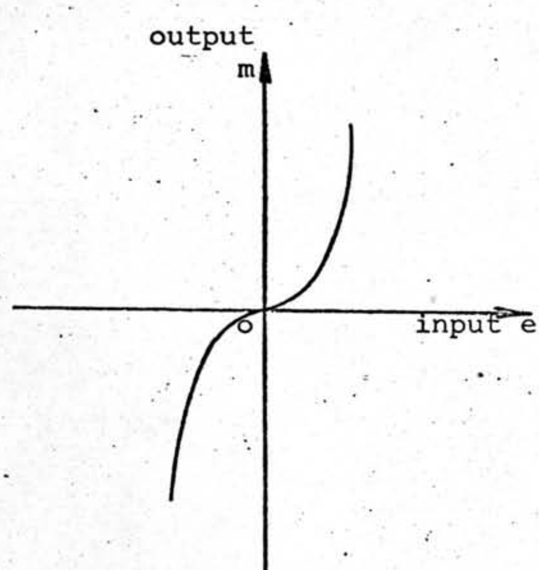

Fig. A.7 A quintic characteristic

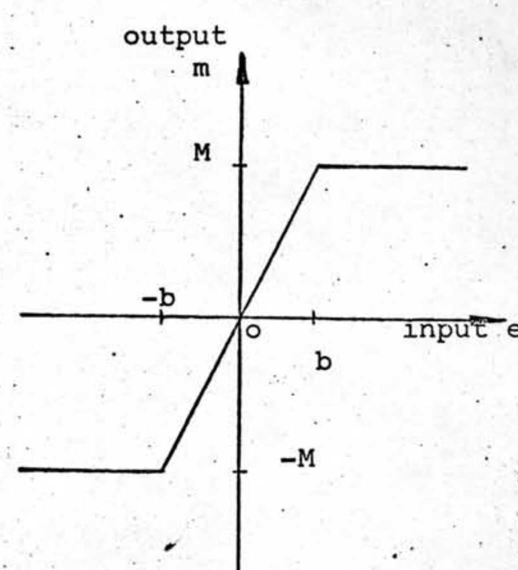

Fig. A.8 A saturation

APPENDIX B

NORMALIZED DESCRIBING FUNCTIONS

The describing functions described in Appendix A can be normalized as follows.

B.1 Dead-zone

From eqns. (A.1.1), (A.1.2) and (A.1.3) it can be easily derived the following equations.

$$g(E) = 0 \qquad \text{for } E/b \leq 1 \qquad \text{(B.1.1)}$$

$$\frac{g(E)}{n} = 1 - \frac{2}{\pi}\left[\sin^{-1}\frac{1}{E/b} + \frac{1}{E/b}\sqrt{1-\left(\frac{1}{E/b}\right)^2}\right] \qquad \text{for } E/b > 1 \qquad \text{(B.1.2)}$$

$$\frac{b(E)}{n} = 0 \qquad \text{for all } E/b \qquad \text{(B.1.3)}$$

In this case the equation of the circle curves is

$$\left[\frac{g_1(w)}{1/n} + \frac{g(E)/n}{\{g(E)/n\}^2 + \{b(E)/n\}^2}\right]^2 + \left[\frac{g_2(w)}{1/n} - \frac{b(E)/n}{\{g(E)/n\}^2+\{b(E)/n\}^2}\right]^2$$

$$= \left[\frac{R/b}{E/b\sqrt{\{g(E)/n\}^2+\{b(E)/n\}^2}}\right]^2 \qquad \text{(B.1.4)}$$

Typical values satisfied the eqn. (B.1.4) are given in Table B.1

Table B.1 Calculated results of eqns. (B.1.1), (B.1.2), (B.1.3) and (B.1.4) for the values of R/b = 1.5, 2.0 and 2.5 respectively.

E/b	$\sin^{-1}\frac{1}{E/b}$	$\frac{g(E)}{n}$	$\frac{g_1(w)}{1/n}$	radius		
				R/b = 1.5	R/b = 2.0	R/b = 2.5
1.1	1.1410	0.0325	-30.76	41.96	55.94	69.93
1.2	0.9851	0.0796	-12.56	15.70	20.94	26.17
1.3	0.8776	0.1284	-7.79	8.99	11.98	14.98
1.4	0.7956	0.1753	-5.70	6.11	8.15	10.19
1.5	0.7297	0.2191	-4.56	4.56	6.08	7.60
1.6	0.6751	0.2596	-3.85	3.61	4.82	6.02
1.7	0.6289	0.2968	-3.37	2.97	3.96	4.95
1.8	0.5890	0.3310	-3.02	2.52	3.36	4.20
1.9	0.5543	0.3622	-2.76	2.18	2.91	3.63
2.0	0.5236	0.3910	-2.56	1.92	2.56	3.20
2.2	0.4719	0.4418	-2.26	1.54	2.06	2.57
2.4	0.4298	0.4852	-2.06	1.29	1.72	2.15
2.6	0.3948	0.5226	-1.91	1.10	1.47	1.84
2.8	0.3652	0.5551	-1.80	0.97	1.29	1.61
3.0	0.3398	0.5836	-1.71	0.86	1.14	1.43
3.1	0.3285	0.5965	-1.68	0.81	1.08	1.35
3.5	0.2898	0.6412	-1.56	0.67	0.89	1.11
4.0	0.2527	0.6850	-1.46	0.55	0.73	0.91
5.0	0.2014	0.7470	-1.34	0.40	0.54	0.67
6.0	0.1674	0.7888	-1.27	0.32	0.42	0.53
8.0	0.1253	0.8413	-1.19	0.22	0.30	0.37
10.0	0.1002	0.8729	-1.15	0.17	0.23	0.29

B.2 Ideal Relay

From eqns. (A.2.1) and (A.2.2) it can be easily derived the following equations.

$$\frac{g(w)}{M} = \frac{4}{\pi E} \qquad \text{for all } E \qquad (B.2.1)$$

$$\frac{b(E)}{M} = 0 \qquad \text{for all } E \qquad (B.2.2)$$

In this case the equation of the circle curves is

$$\left[\frac{g_1(w)}{1/M} + \frac{g(E)/M}{\{g(E)/M\}^2 + \{b(E)/M\}^2}\right]^2 + \left[\frac{g_2(w)}{1/M} - \frac{b(E)/M}{\{g(E)/M\}^2 + \{b(E)/M\}^2}\right]^2$$

$$= \left[\frac{R}{E\sqrt{\{g(E)/M\}^2 + \{b(E)/M\}^2}}\right]^2 \qquad (B.2.3)$$

Typical values satisfied the eqn. (B.2.3) are given in Table B.2.

Table B.2 Calculated results of eqns. (B.2.1), (B.2.2) and (B.2.3) for the values of R = 1.5, 2.0 and 2.5 respectively.

E	g(E)/M	$\frac{g_1(w)}{1/M}$	radius		
			R = 1.5	R = 2.0	R = 2.5
0.0	∞	-0.00	1.18	1.57	1.96
0.5	2.5465	-0.39	1.18	1.57	1.96
1.0	1.2732	-0.79	1.18	1.57	1.96
1.5	0.8488	-1.18	1.18	1.57	1.96
2.0	0.6366	-1.60	1.18	1.57	1.96
2.5	0.5093	-1.96	1.18	1.57	1.96
3.0	0.4244	-2.36	1.18	1.57	1.96
3.5	0.3638	-2.75	1.18	1.57	1.96
4.0	0.3183	-3.14	1.18	1.57	1.96
4.5	0.2829	-3.53	1.18	1.57	1.96
5.0	0.2546	-3.93	1.18	1.57	1.96
5.5	0.2315	-4.32	1.18	1.57	1.96
6.0	0.2122	-4.71	1.18	1.57	1.96
6.5	0.1959	-5.11	1.18	1.57	1.96
7.0	0.1819	-5.50	1.18	1.57	1.96
7.5	0.1698	-5.89	1.18	1.57	1.96
8.0	0.1592	-6.28	1.18	1.57	1.96

B.3 Relay with Dead-zone

From eqns. (A.3.1), (A.3.2) and (A.3.3) it can be easily derived the following equations.

$$\frac{g(E)}{M/a} = 0 \qquad \text{for } E/a \leq 1 \qquad \text{(B.3.1)}$$

$$\frac{g(E)}{M/a} = \frac{4}{\pi (E/a)}\sqrt{1 - \left(\frac{1}{E/a}\right)^2} \qquad \text{for } E/a > 1 \qquad \text{(B.3.2)}$$

$$\frac{b(E)}{M/a} = 0 \qquad \text{for all } E/a \qquad \text{(B.3.3)}$$

In this case the equation of the circle curves is

$$\left[\frac{g_1(w)}{a/M} + \frac{\frac{g(E)}{M/a}}{\left(\frac{g(E)}{M/a}\right)^2 + \left(\frac{b(E)}{M/a}\right)^2}\right]^2 + \left[\frac{g_2(w)}{a/M} - \frac{\frac{b(E)}{M/a}}{\left(\frac{g(E)}{M/a}\right)^2 + \left(\frac{b(E)}{M/a}\right)^2}\right]^2$$

$$= \left[\frac{R/a}{E/a\sqrt{\left(\frac{g(E)}{M/a}\right)^2 + \left(\frac{b(E)}{M/a}\right)^2}}\right]^2 \qquad \text{(B.3.4)}$$

Typical values satisfied the eqn. (B.3.4) are given in Table 3.3.

Table B.3 Calculated results of eqns. (B.3.1), (B.3.2), (B.3.3) and (B.3.4) for the values of R/a = 1.5, 2.0 and 2.5 respectively.

E/a	$\frac{g(E)}{M/a}$	$\frac{g_1(w)}{a/M}$	radius		
			R/a = 1.5	R/a = 2.0	R/a = 2.5
1.1	0.4822	-2.07	2.83	3.77	4.71
1.2	0.5865	-1.71	2.13	2.84	3.55
1.3	0.6258	-1.60	1.84	2.46	3.07
1.4	0.6365	-1.57	1.68	2.24	2.80
1.6	0.6212	-1.61	1.51	2.01	2.52
1.8	0.5882	-1.70	1.42	1.89	2.27
2.0	0.5513	-1.81	1.36	1.81	2.27
2.5	0.4668	-2.14	1.29	1.71	2.14
3.0	0.4001	-2,50	1.25	1.67	2.08
3.5	0.3486	-2.87	1.23	1.64	2.05
4.0	0.3082	-3.24	1.22	1.62	2.03
4.5	0.2759	-3.62	1.21	1.61	2.01
5.0	0.2495	-4.01	1.20	1.60	2.00
5.5	0.2276	-4.39	1.20	1.60	2.00
6.0	0.2092	-4.78	1.21	1.59	2.01
6.5	0.1936	-5.17	1.20	1.59	2.00
7.0	0.1800	-5.56	1.19	1.59	1.98
7.5	0.1682	-5.95	1.19	1.58	1.98

B.4 Relay With Hysteresis

From eqns. (A.4.1) and (A.4.2) it can be easily derived the following equations.

$$\frac{g(E)}{M/b} = \frac{4}{\pi(E/b)} \sqrt{1 - \left(\frac{1}{E/b}\right)^2} \qquad \text{for } E/b > 1 \qquad \text{(B.4.1)}$$

$$\frac{b(E)}{M/b} = \frac{-4}{\pi(E/b)^2} \qquad \text{for } E/b > 1 \qquad \text{(B.4.2)}$$

In this case the equation of the circle curves is

$$\left[\frac{g_1(w)}{b/M} + \frac{\frac{g(E)}{M/b}}{\left(\frac{g(E)}{M/b}\right)^2 + \left(\frac{b(E)}{M/b}\right)^2}\right]^2 + \left[\frac{g_2(w)}{b/M} - \frac{\frac{b(E)}{M/b}}{\left(\frac{g(E)}{M/b}\right)^2 + \left(\frac{b(E)}{M/b}\right)^2}\right]^2$$

$$= \left[\frac{R/b}{E/b\sqrt{\left(\frac{g(E)}{M/b}\right)^2 + \left(\frac{b(E)}{M/b}\right)^2}}\right]^2 \qquad \text{(B.4.3)}$$

Typical values satisfied the eqn. (B.4.3) are given in Table B.4.

Table B.4 Calculated results of eqns. (B.4.1), (B.4.2) and (B.4.3) for the values of R/b = 1.5, 2.0 and 2.5 respectively.

E/b	$\frac{g(E)}{M/b}$	$\frac{b(E)}{M/b}$	$\frac{g_1(w)}{b/M}$	$\frac{g_2(w)}{b/M}$	radius		
					R/b=1.5	R/b=2.0	R/b=2.5
1.0	0.0000	-1.2732	-0.00	-0.79	1.18	1.57	1.96
1.2	0.5965	-0.8842	-0.52	-0.79	1.18	1.57	1.96
1.4	0.6365	-0.6496	-0.77	-0.79	1.18	1.57	1.96
1.6	0.6212	-0.4974	-0.98	-0.79	1.18	1.57	1.96
1.8	0.5882	-0.3930	-1.18	-0.79	1.18	1.57	1.96
2.0	0.5513	-0.3183	-1.36	-0.79	1.18	1.57	1.96
2.5	0.4668	-0.2037	-1.80	-0.79	1.18	1.57	1.96
3.0	0.4001	-0.1415	-2.22	-0.79	1.18	1.57	1.96
3.5	0.3486	-0.1039	-2.63	-0.79	1.18	1.57	1.96
4.0	0.3082	-0.0800	-3.04	-0.79	1.18	1.57	1.96
4.5	0.2759	-0.0628	-3.45	-0.79	1.18	1.57	1.96
5.0	0.2495	-0.0509	-3.55	-0.79	1.18	1.57	1.96
5.5	0.2276	-0.0421	-4.25	-0.79	1.18	1.57	1.96
6.0	0.2092	-0.0354	-4.65	-0.79	1.18	1.57	1.96
6.5	0.1936	-0.0301	-5.04	-0.79	1.18	1.57	1.96
7.0	0.1800	-0.0260	-5.44	-0.79	1.18	1.57	1.96
7.5	0.1682	-0.0226	-5.84	-0.79	1.18	1.57	1.96

B.5 Relay with Hysteresis and dead-zone

From eqns. (A.5.1) and (A.5.2) it can be easily derived the following equations.

$$\frac{g(E)}{M/a} = \frac{2}{\pi(E/a)}\sqrt{1-\left(\frac{1}{E/a}\right)^2} + \sqrt{1-\left(\frac{d/a}{E/a}\right)^2} \quad \text{for } E/a > 1 \text{ and } E/d > 1 \tag{B.5.1}$$

$$\frac{b(E)}{M/a} = \frac{-2}{\pi(E/a)}\left[\frac{1}{E/a} - \frac{d/a}{E/d}\right] \quad \text{for } E/a > 1 \text{ and } E/d > 1 \tag{B.5.2}$$

In this case the equation of the circle curves is

$$\left[\frac{g_1(w)}{a/M} + \frac{\frac{g(E)}{M/a}}{\left(\frac{g(E)}{M/a}\right)^2 + \left(\frac{b(E)}{M/a}\right)^2}\right]^2 + \left[\frac{g_2(w)}{a/M} - \frac{\frac{b(E)}{M/a}}{\left(\frac{g(E)}{M/a}\right)^2 + \left(\frac{b(E)}{M/a}\right)^2}\right]^2$$

$$= \left[\frac{R/a}{E/a\sqrt{\left(\frac{g(E)}{M/a}\right)^2 + \left(\frac{b(E)}{M/a}\right)^2}}\right]^2 \tag{B.5.3}$$

Typical values satisfied the eqn. (B.5.3) are given in Table B.5.

Table B.5 Calculated results of eqns. (B.5.1), (B.5.2) and (B.5.3) for the values of R/a = 1.5, 2.0 and 2.5 respectively.

E/a	$\frac{b(E)}{M/a}$	$\frac{g(E)}{M/a}$	$\frac{g_1(w)}{a/M}$	$\frac{g_2(w)}{a/M}$	radius		
					R/a=1.5	R/a=2.0	R/a=2.5
1.0	-0.3183	0.5513	-1.36	-0.79	2.36	3.14	3.93
1.1	-0.2631	0.7566	-1.18	-0.41	1.70	2.27	2.84
1.2	-0.2210	0.7755	-1.19	-0.34	1.55	2.07	2.58
1.4	-0.1624	0.7430	-1.28	-0.28	1.41	1.88	2.35
1.6	-0.1243	0.6886	-1.41	-0.25	1.34	1.79	2.23
2.0	-0.0796	0.5839	-1.68	-0.23	1.27	1.70	2.12
2.5	-0.0509	0.4829	-2.05	-0.22	1.24	1.65	2.06
3.0	-0.0354	0.4093	-2.43	-0.21	1.22	1.62	2.03
3.5	-0.0260	0.3543	-2.81	-0.21	1.21	1.61	2.01
4.0	-0.0199	0.3120	-3.19	-0.20	1.20	1.60	2.00
4.5	-0.0157	0.2785	-3.58	-0.20	1.19	1.59	1.99
5.0	-0.0127	0.2514	-3.97	-0.20	1.19	1.59	1.99
5.5	-0.0105	0.2291	-4.35	-0.20	1.19	1.59	1.98
6.0	-0.0088	0.2104	-4.74	-0.20	1.19	1.58	1.98
6.5	-0.0075	0.1944	-5.14	-0.20	1.19	1.58	1.98
7.0	-0.0065	0.1807	-5.53	-0.20	1.19	1.58	1.98
7.5	-0.0057	0.1688	-5.92	-0.20	1.18	1.58	1.97

B.6 Backlash

From eqns. (A.6.1) and (A.6.2) it can be easily derived the following equations.

$$\frac{g(E)}{n} = 0.5 + \frac{1}{\pi}\left[\sin^{-1}\frac{1}{E/d} + \frac{1}{E/d}\sqrt{1-\left(\frac{1}{E/d}\right)^2}\right] \quad \text{for } E/d > 1 \qquad (B.6.1)$$

$$\frac{b(E)}{n} = \frac{-2}{\pi} \times \left[1-\left(\frac{1}{E/d}\right)^2\right] \quad \text{for } E/d > 1 \qquad (B.6.2)$$

In this case the equation of the circle curves is

$$\left[\frac{g_1(w)}{1/n} + \frac{g(E)/n}{\left(g(E)/n\right)^2 + \left(b(E)/n\right)^2}\right]^2 + \left[\frac{g_2(w)}{1/n} - \frac{b(E)/n}{\left(g(E)/n\right)^2 + \left(b(E)/n\right)^2}\right]^2 = \left[\frac{R/d}{E/d\sqrt{\left(g(E)/n\right)^2 + \left(b(E)/n\right)^2}}\right]^2 \qquad (B.6.3)$$

Typical values satisfied the eqn. (B.6.3) are given in Table B.6.

Table B.6 Calculated results of eqns. (B.6.1), (B.6.2) and (B.6.3) for the values of R/d = 1.5, 2.0 and 2.5 respectively.

E/d	$\sin^{-1}\frac{1}{E/d}$	b(E)/n	g(E)/n	$\frac{g_1(w)}{1/n}$	$\frac{g_2(w)}{1/n}$	radius		
						R/d=1.5	R/d=2.0	R/d=2.5
1.0	1.5708	0.0000	1.0000	-1.00	0.00	1.50	2.00	2.50
1.2	0.9851	-0.0990	0.9609	-1.03	-0.11	1.29	1.72	2.15
1.4	0.7956	-0.1578	0.9142	-1.06	-0.18	1.16	1.54	1.93
1.6	0.6751	-0.1960	0.8697	-1.09	-0.25	1.05	1.40	1.75
1.8	0.5890	-0.2185	0.8352	-1.14	-0.30	0.97	1.30	1.62
2.0	0.5236	-0.2387	0.8045	-1.14	-0.34	0.84	1.19	1.49
2.5	0.4115	-0.2674	0.7477	-1.19	-0.42	0.76	1.01	1.26
3.0	0.3398	-0.2836	0.7073	-1.22	-0.49	0.65	0.87	1.09
3.5	0.2898	-0.2915	0.6806	-1.24	-0.53	0.58	0.77	0.96
4.0	0.2527	-0.2984	0.6575	-1.26	-0.57	0.52	0.69	0.87
4.5	0.2241	-0.3029	0.6396	-1.28	-0.60	0.47	0.63	0.79
5.0	0.2014	-0.3056	0.6265	-1.29	-0.63	0.43	0.57	0.72
5.5	0.1828	-0.3080	0.6145	-1.30	-0.65	0.40	0.53	0.66
6.0	0.1674	-0.3091	0.6066	-1.31	-0.67	0.36	0.48	0.60
6.5	0.1545	-0.3112	0.5964	-1.32	-0.69	0.34	0.46	0.57

B.7 Quintic Characteristic

The normalized describing function of quintic characteristic are also the same of eqns. (A.7.1) and (A.7.2) rewritten below.

$$g(E) = \frac{5}{8} E^4 \qquad \text{for all } E \qquad (B.7.1)$$

$$b(E) = 0 \qquad \text{for all } E \qquad (B.7.2)$$

In this case the equation of the circle curves is

$$\left[g_1(w) + \frac{g(E)}{g^2(E) + b^2(E)} \right]^2 + \left[g_2(w) - \frac{g(E)}{g^2(E) + b^2(E)} \right]^2$$

$$= \left[\frac{R}{E\sqrt{g^2(E) + b^2(E)}} \right]^2 \qquad (B.7.3)$$

Typical values satisfied the eqn. (B.7.3) are given in Table B.7.

Table B.7 Calculated results of eqns. (B.7.1), (B.7.2) and (B.7.3) for the values of R = 1.5, 2.0 and 2.5 respectively.

E	g(E)	$g_1'(w)$	radius		
			R = 1.5	R = 2.0	R = 2.5
1.00	0.6250	-1.60	2.40	3.20	4.00
1.02	0.6765	-1.48	2.17	2.90	3.62
1.03	0.7034	-1.42	2.07	2.76	3.45
1.04	0.7312	-1.37	1.97	2.63	3.29
1.06	0.7890	-1.27	1.79	2.39	2.99
1.10	0.9151	-1.09	1.49	1.99	2.48
1.15	1.0931	-0.91	1.19	1.59	1.99
1.17	1.1712	-0.85	1.09	1.46	1.82
1.19	1.2533	-0.80	1.01	1.34	1.68
1.21	1.340	-0.75	0.93	1.23	1.54
1.25	1.5259	-0.66	0.79	1.05	1.31
1.30	1.7851	-0.56	0.65	0.86	1.08
1.35	2.0759	-0.48	0.54	0.71	0.89
1.40	2.4010	-0.42	0.45	0.59	0.74
1.50	3.1641	-0.32	0.32	0.42	0.53
1.60	4.0960	-0.24	0.23	0.31	0.38
1.80	6.5610	-0.15	0.13	0.17	0.21
2.00	10.0000	-0.10	0.08	0.10	0.13

B.8 Saturation

From eqns. (A.8.1), (A.8.2) and (A.8.3) it can be easily derived the following equations.

$$\frac{g(E)}{n} = 1 \qquad \text{for } E/b \leq 1 \qquad \text{(B.8.1)}$$

$$\frac{g(E)}{n} = \frac{2}{\P} \left[\sin^{-1}\left(\frac{1}{E/d}\right) + \frac{1}{E/b}\sqrt{1 - \left(\frac{1}{E/b}\right)^2} \right]$$

$$\text{for } E/b > 1 \qquad \text{(B.8.2)}$$

$$\frac{b(E)}{n} = 0 \qquad \text{for all } E/b \qquad \text{(B.8.3)}$$

In this case the equation of the circle curves is

$$\left[\frac{g_1(w)}{1/n} + \frac{g(E)/n}{\left(g(E)/n\right)^2 + \left(b(E)/n\right)^2}\right]^2 + \left[\frac{g_2(w)}{1/n} - \frac{b(E)/n}{\left(g(E)/n\right)^2 + \left(b(E)/n\right)^2}\right]^2$$

$$= \left[\frac{R/b}{E/b\sqrt{\left(g(E)/n\right)^2 + \left(b(E)/n\right)^2}}\right]^2 \qquad \text{(B.8.4)}$$

Typical values satisfied the eqn. (B.8.4) are given in Table B.8.

Table B.8 Calculated results of eqns. (B.8.1), (B.8.2) and (B.8.3) for the values of R/b = 1.5, 2.0 and 2.5 respectively.

E/b	$\sin^{-1}\frac{1}{E/b}$	g(E)/n	$\frac{g_1(w)}{1/n}$	radius		
				R/b = 1.5	R/b = 2.0	R/b = 2.5
0.4	-	1.000	-1.00	3.75	5.00	6.25
0.6	-	1.000	-1.00	2.50	3.33	4.17
0.8	-	1.000	-1.00	1.88	2.50	3.13
1.0	-	1.000	-1.00	1.50	2.00	2.50
1.2	0.985	0.920	-1.09	1.36	1.81	2.26
1.4	0.796	0.825	-1.21	1.30	1.73	2.17
1.8	0.589	0.669	-1.49	1.25	1.66	2.08
2.0	0.524	0.609	-1.64	1.23	1.64	2.05
2.5	0.412	0.495	-2.02	1.21	1.61	2.02
3.0	0.340	0.416	-2.40	1.20	1.60	2.00
3.5	0.290	0.359	-2.79	1.19	1.59	1.99
4.0	0.253	0.315	-3.17	1.19	1.59	1.98
4.5	0.224	0.281	-3.56	1.19	1.58	1.98
5.0	0.201	0.253	-3.95	1.19	1.58	1.98
5.5	0.183	0.203	-4.34	1.18	1.58	1.97
6.0	0.167	0.211	-4.73	1.18	1.58	1.97
6.5	0.154	0.195	-5.13	1.18	1.58	1.97
7.0	0.143	0.181	-5.52	1.18	1.58	1.97
7.5	0.134	0.169	-5.91	1.18	1.58	1.97
8.0	0.125	0.159	-6.30	1.18	1.57	1.97

APPENDIX C

THE STANDARD NORMALIZED GRAPHS OF CIRCLE CURVES

For the future practical applications, the standard normalized graphs of circle curves in different common types of nonlinearities are constructed in seperate graphs.

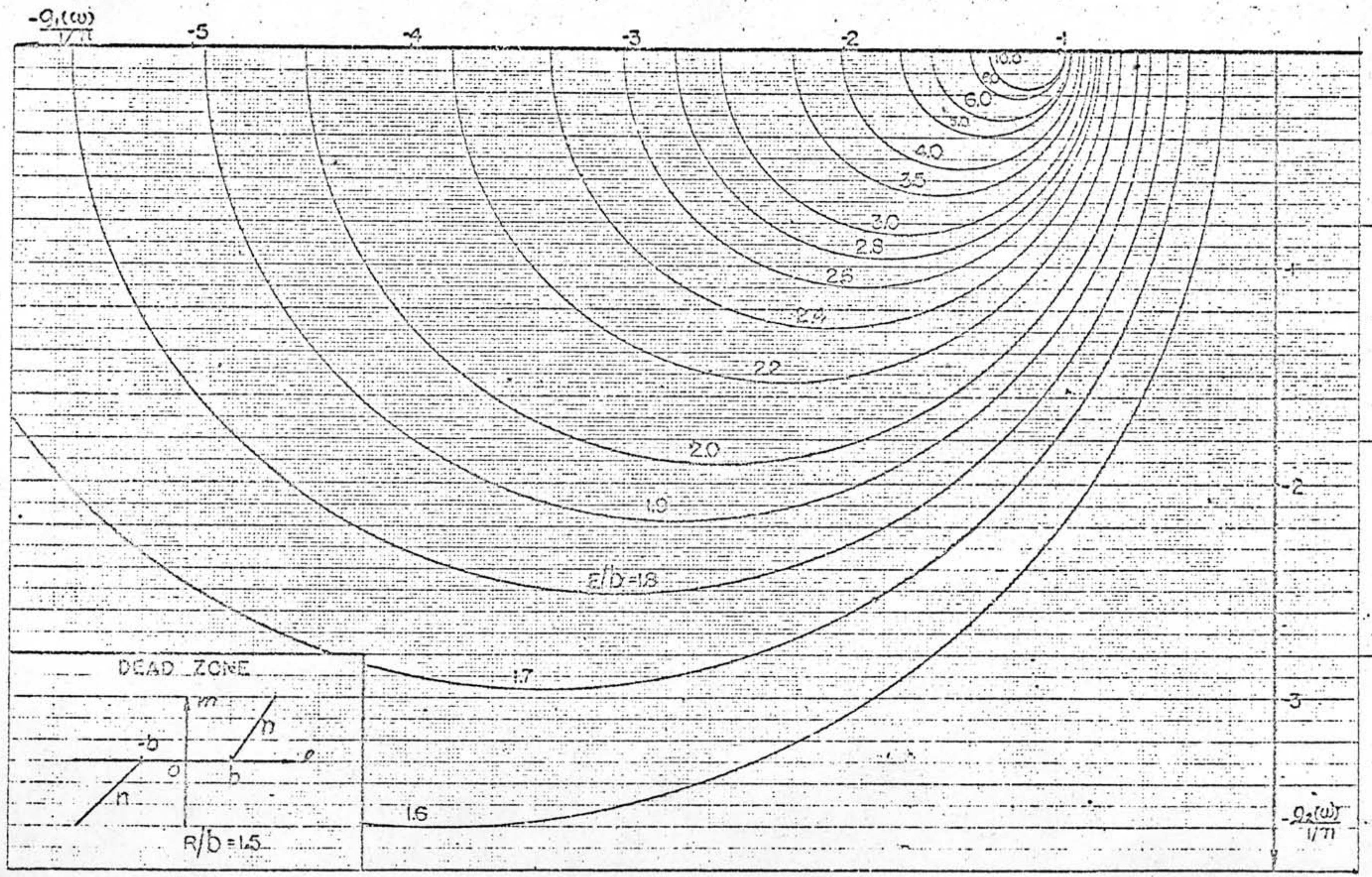

FIG. C.1.1. Normalized circle curves for dead zone with $R/b = 1.5$

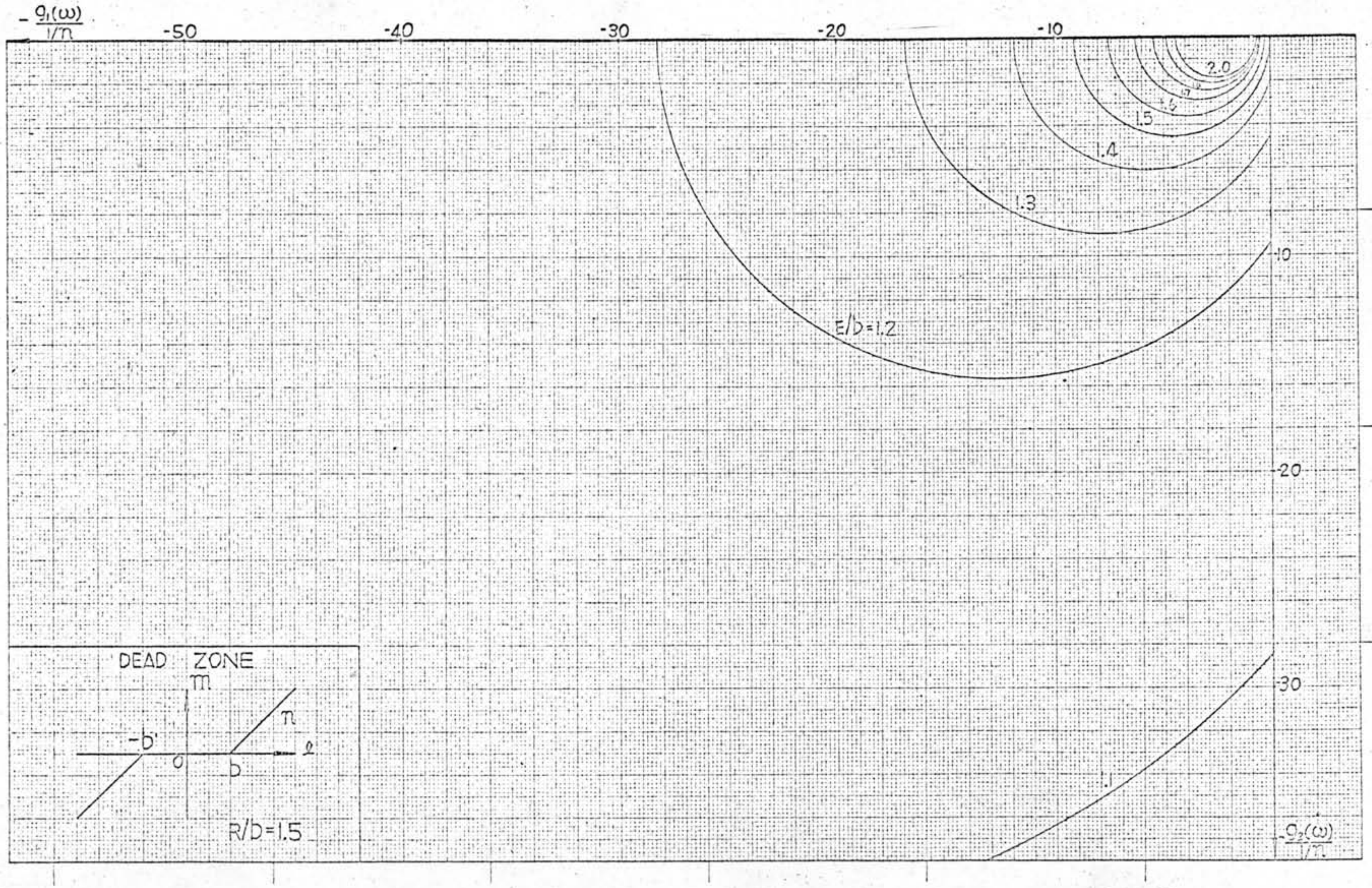

Fig. C. 1.1a Additional normalized circle curves for dead zone with R/b = 1.5

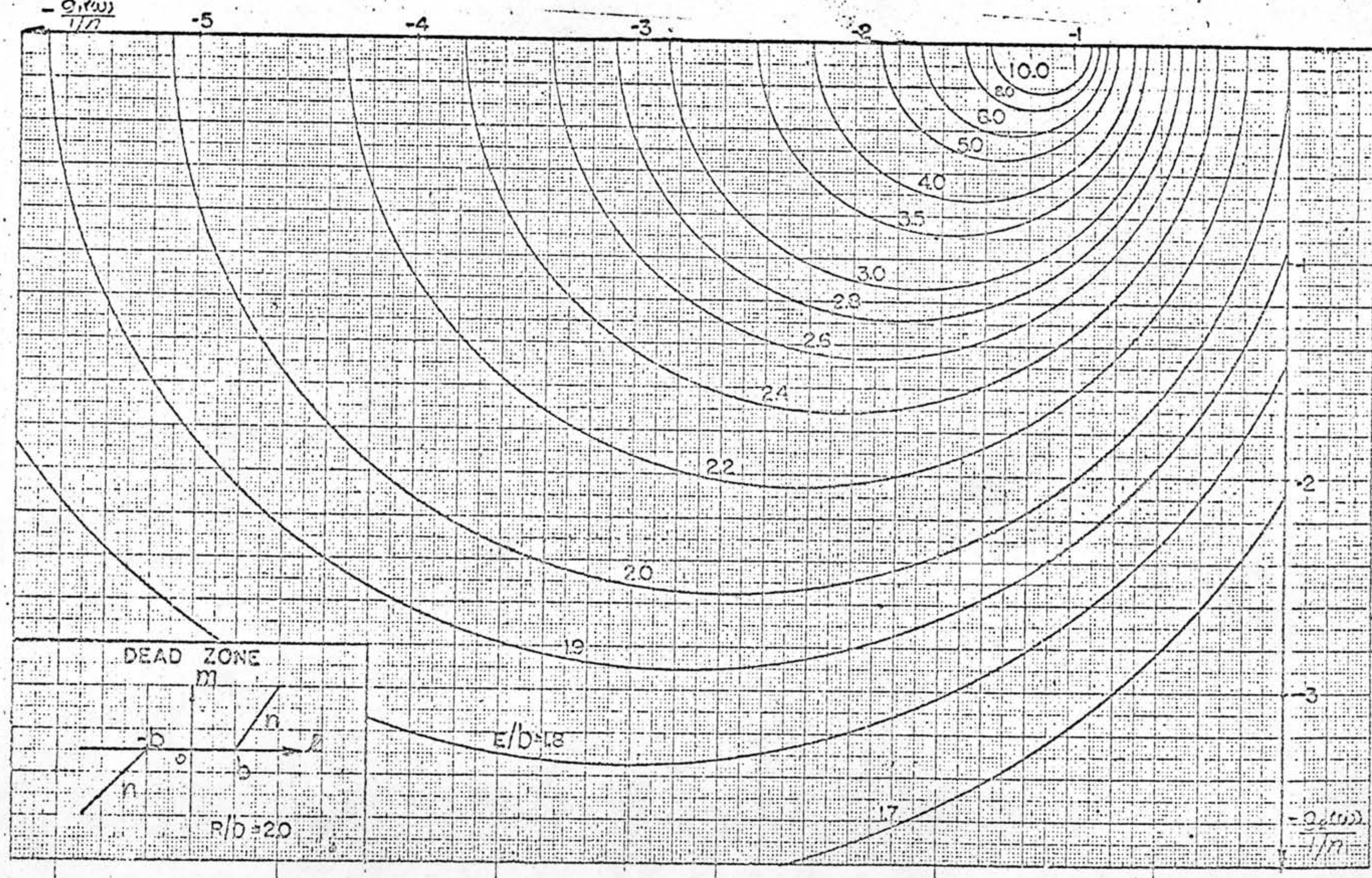

FIG. C.1.2 Normalized circle curves for dead zone with R/b =2.0

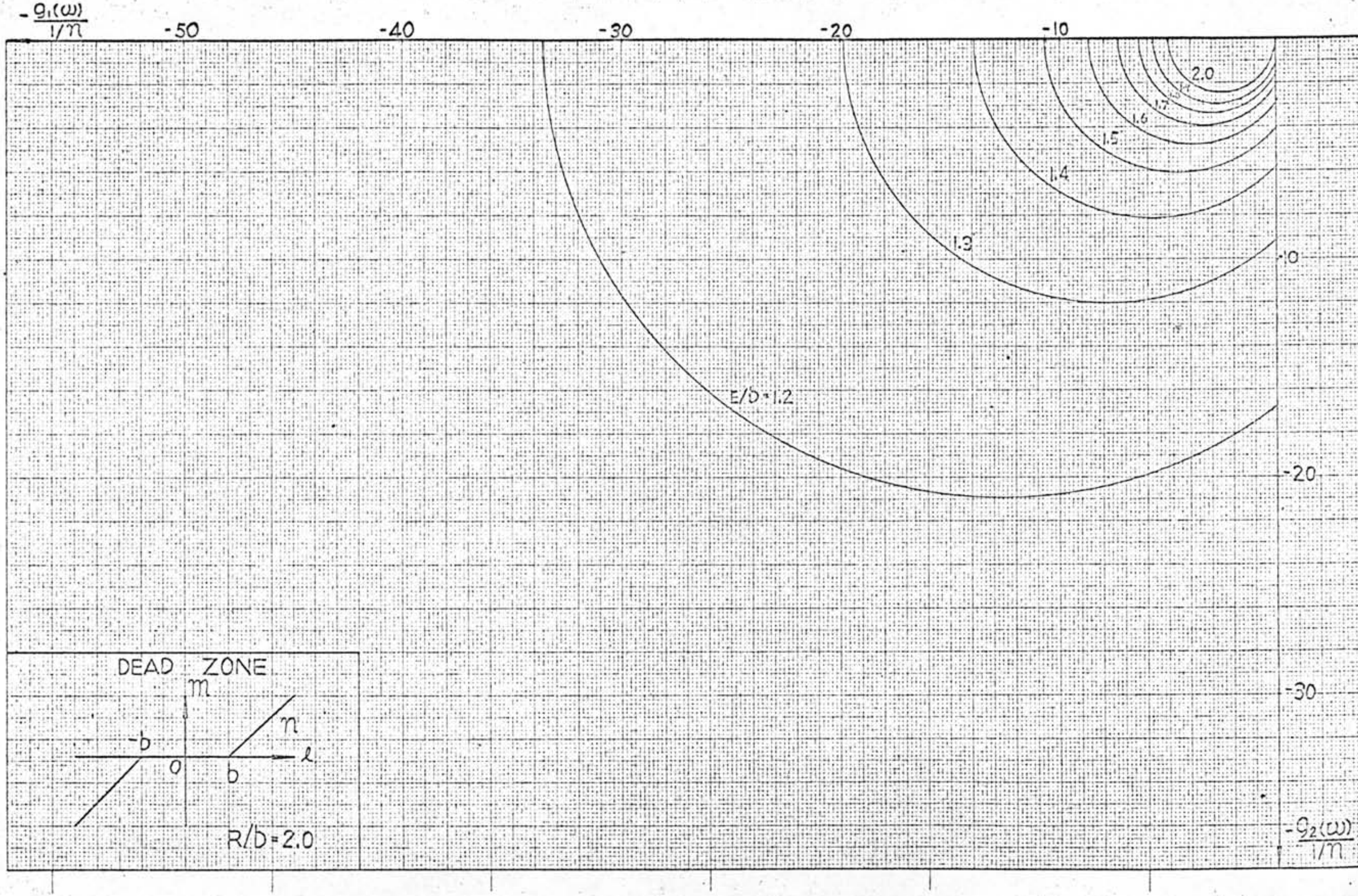

Fig. C. 1.2a Additional normalized circle curves for dead zone with R/b = 2.0

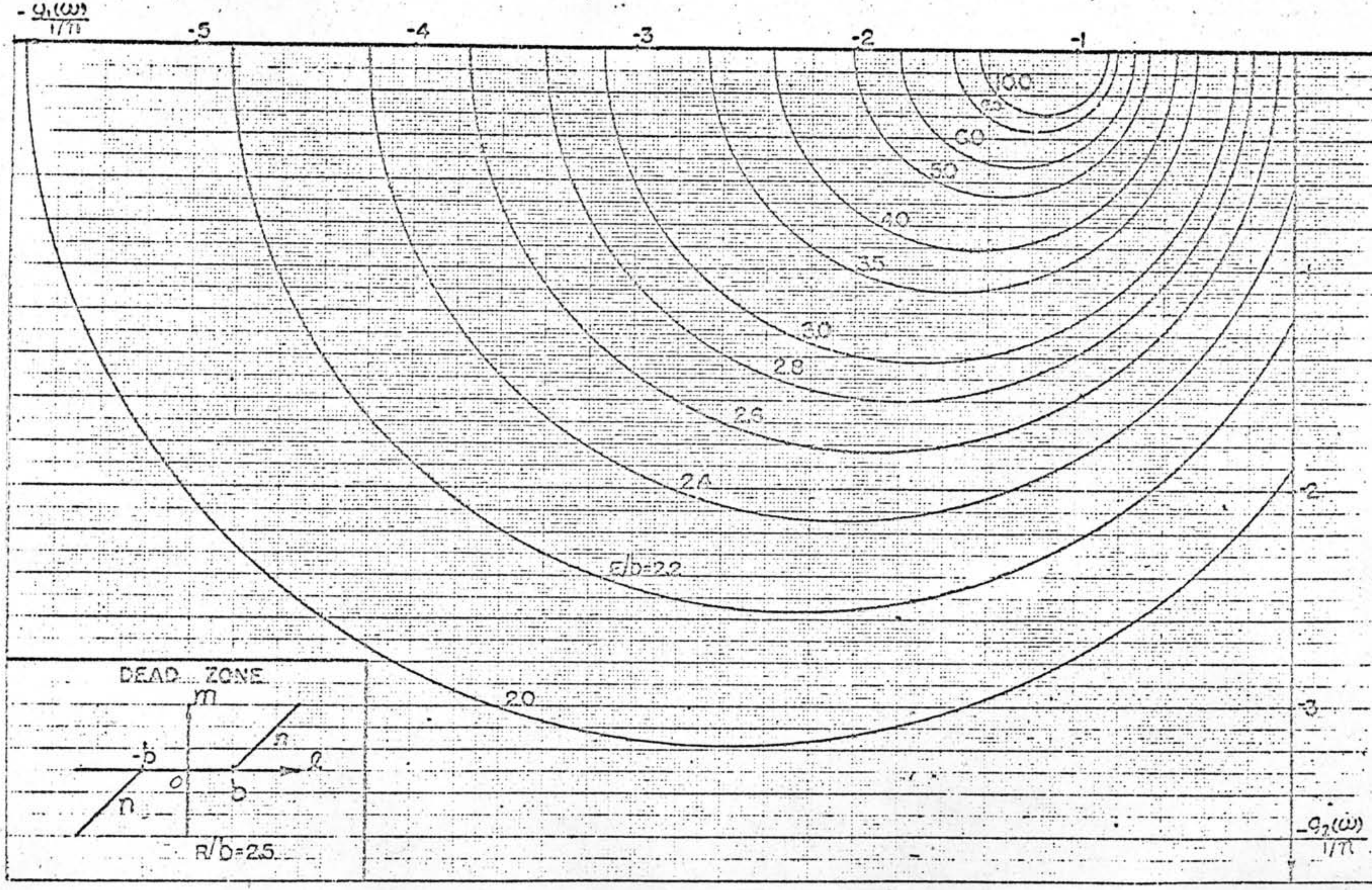

FIG. C.1.3 Normalized circle curves for dead zone with R/b =2.5

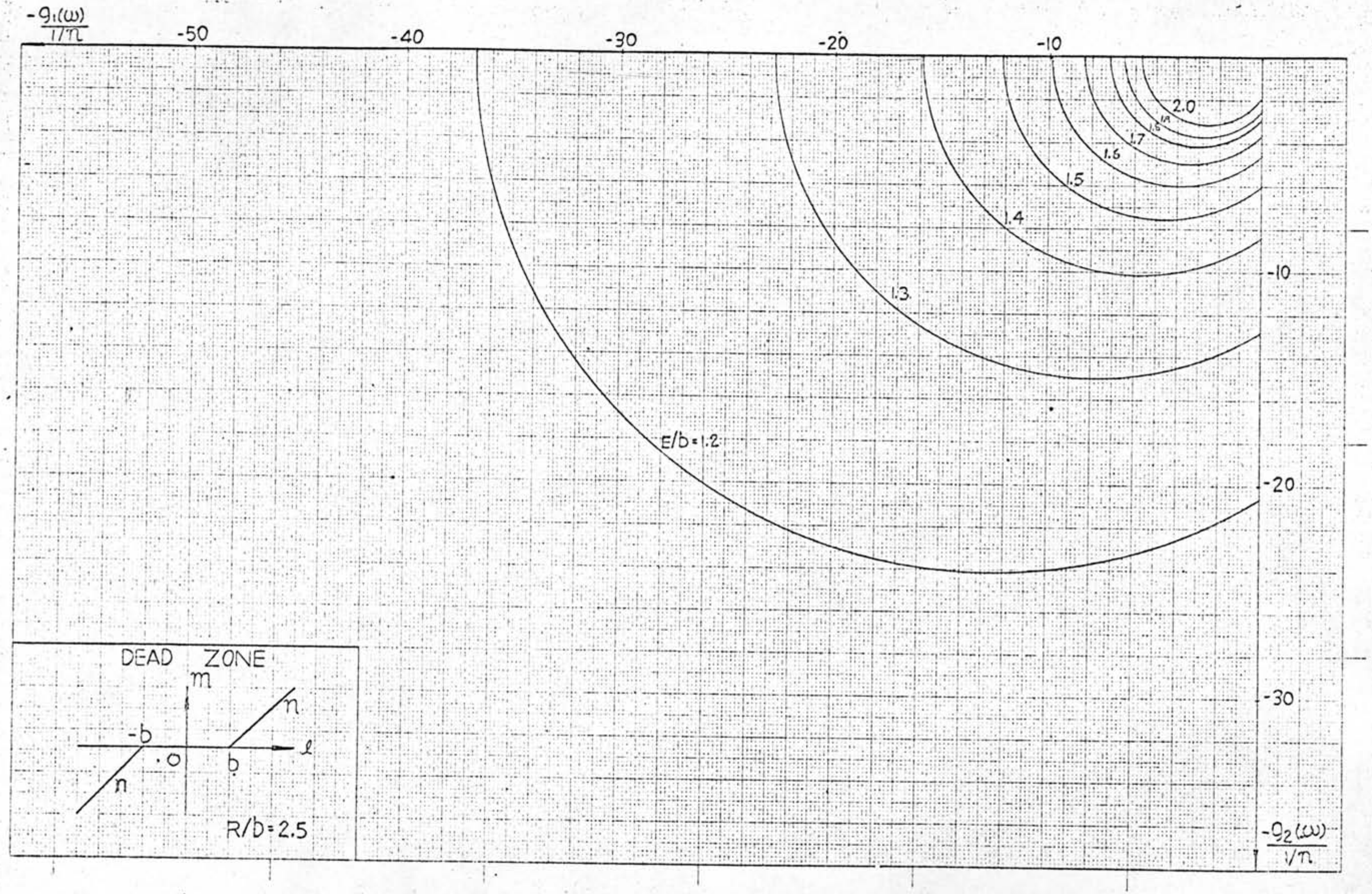

Fig. C. 1.3a Additional normalized circle curves for dead zone with R/b = 2.5

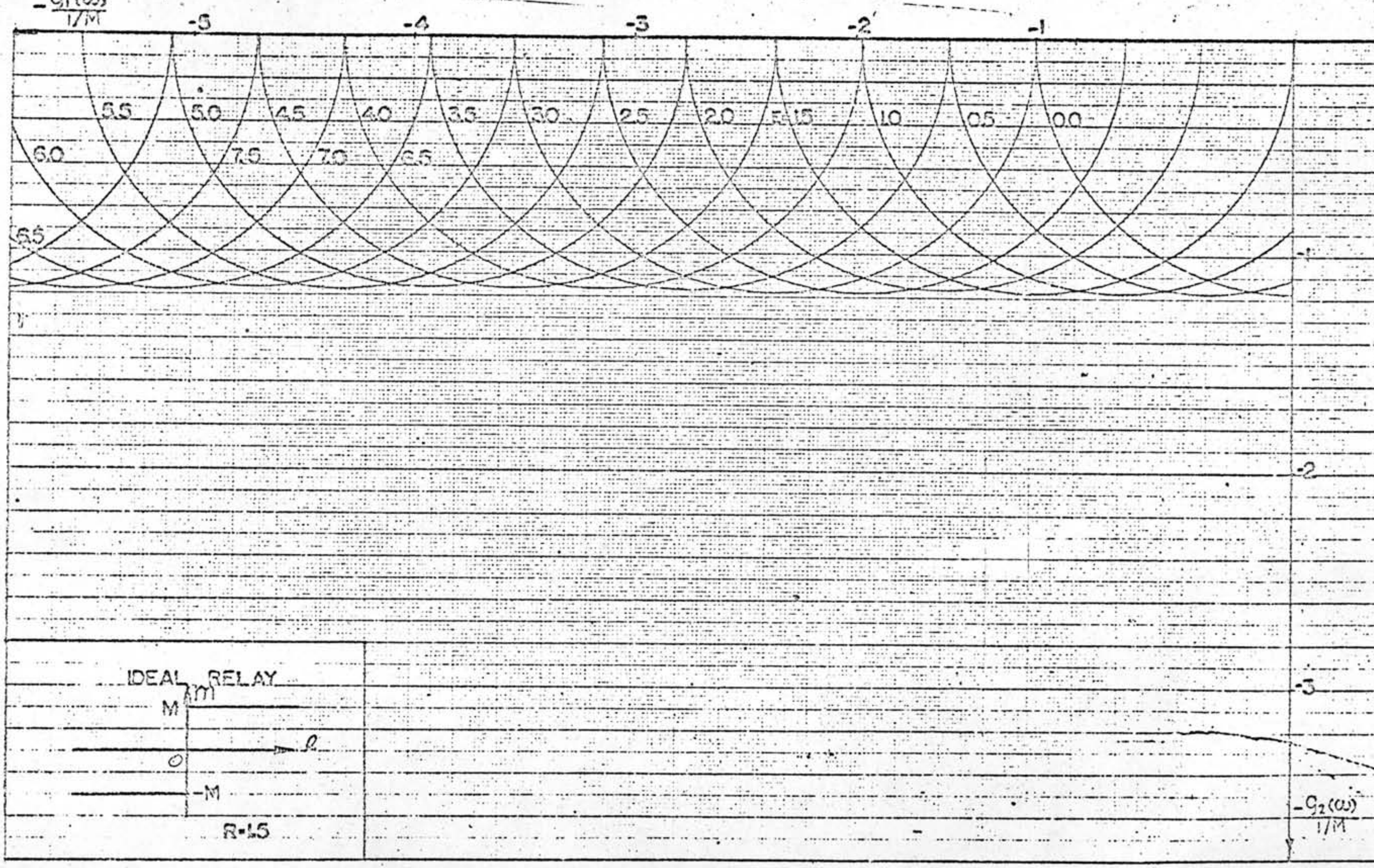

FIG. C.2.1 Normalized circle curves for ideal relay with R =1.5

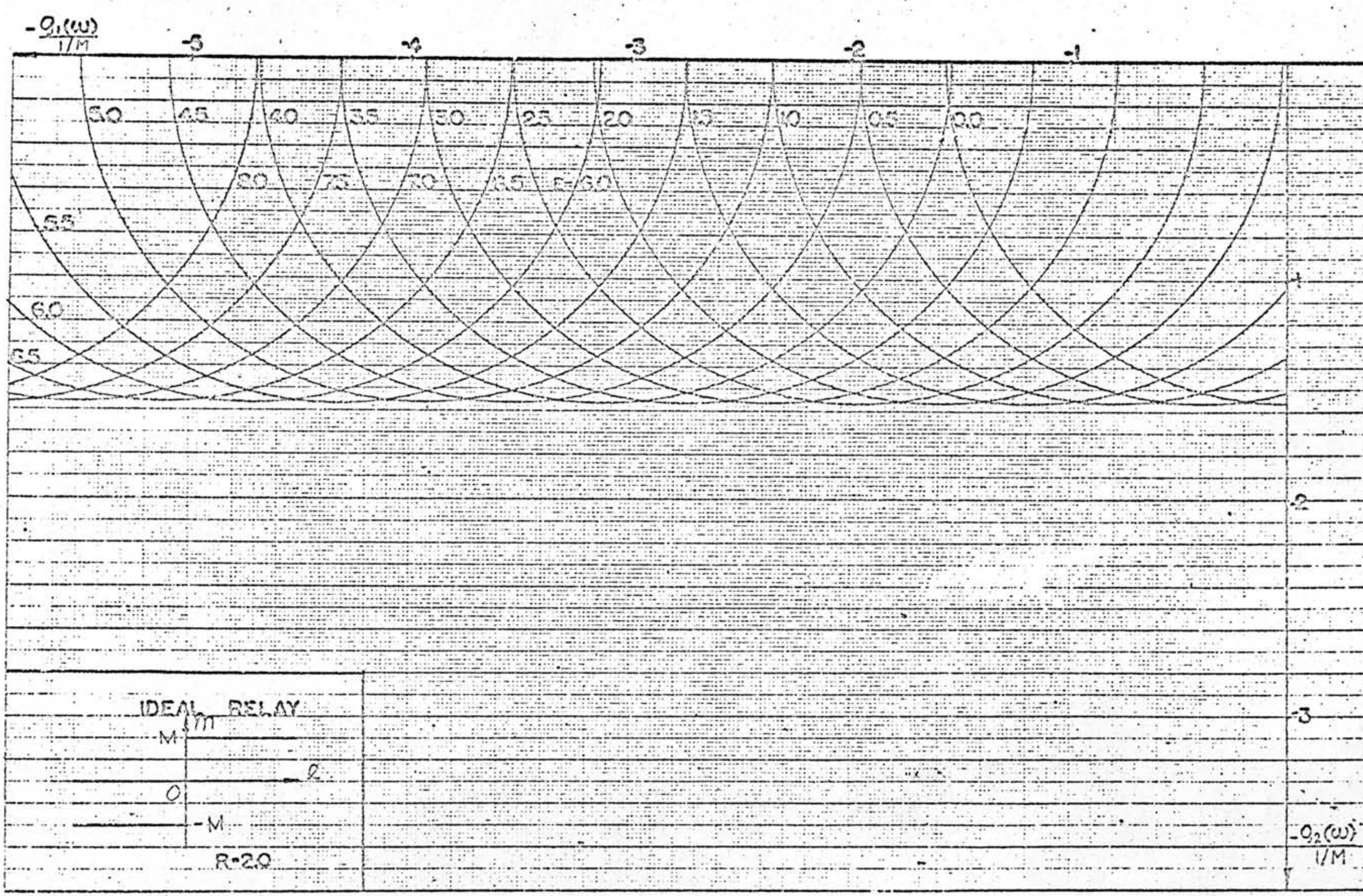

FIG. C.2.2 Normalized circle curves for ideal relay with R =2.0

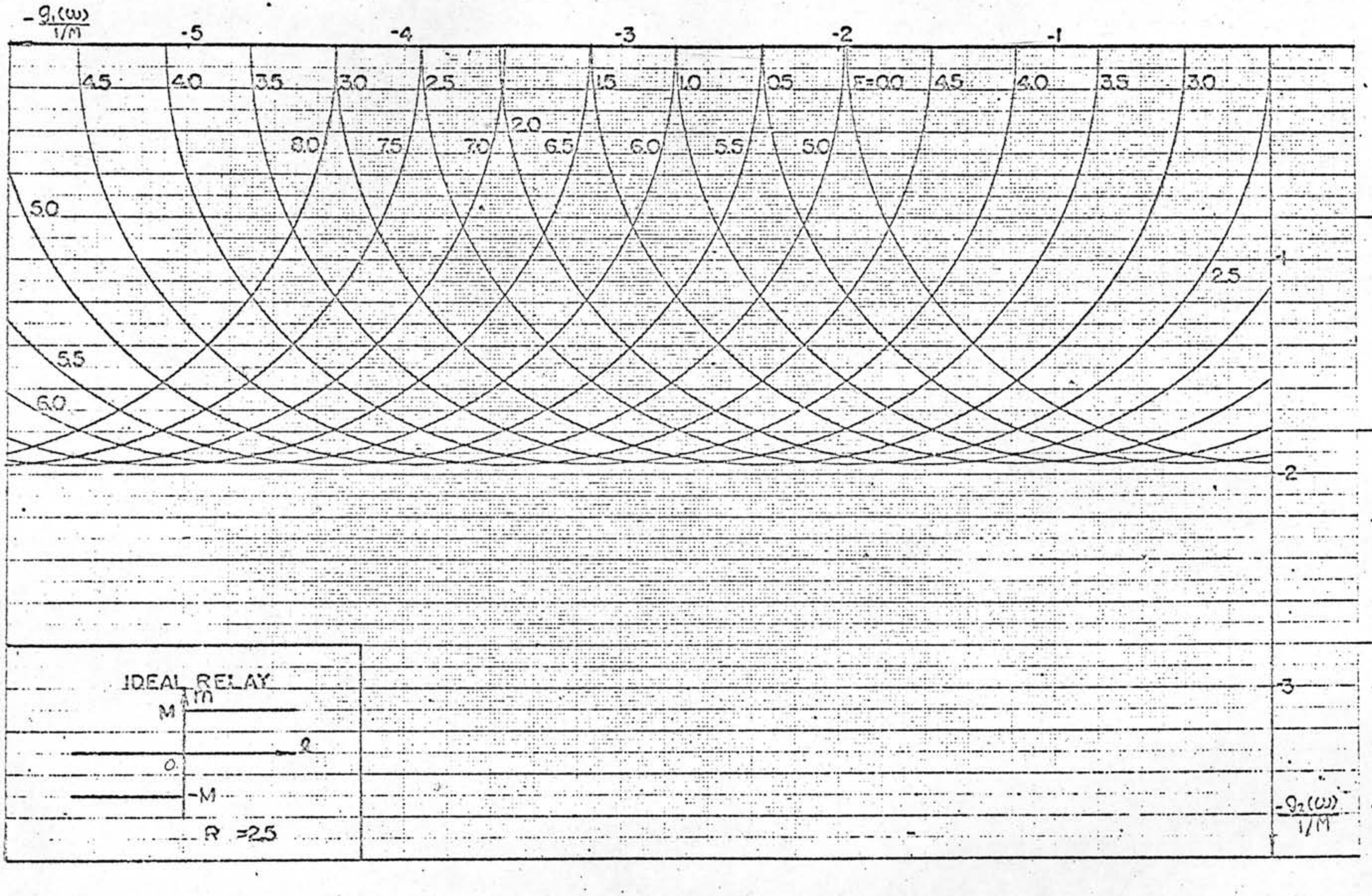

FIG. C.2.3 Normalized circle curves for ideal relay with R =2.5

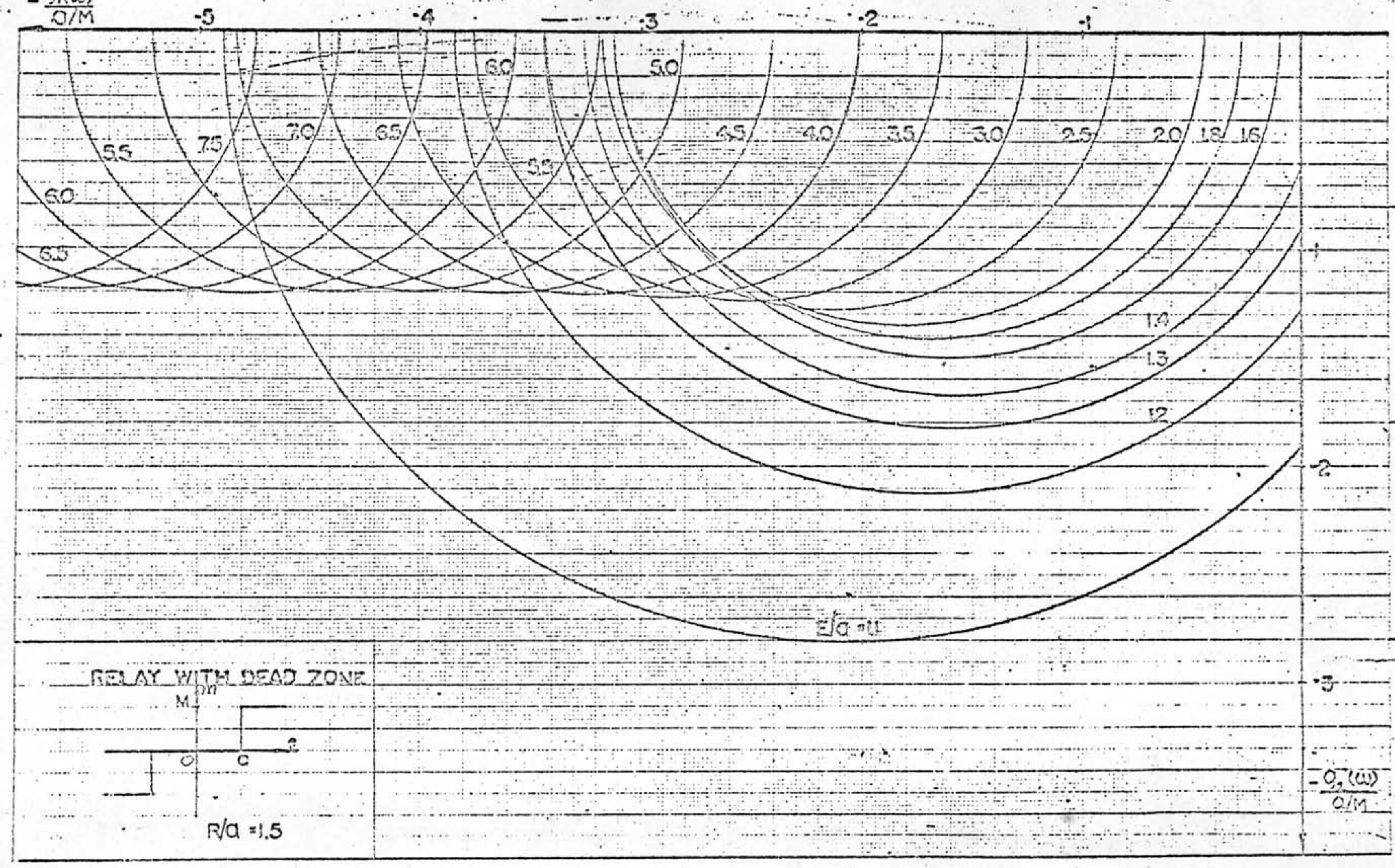

FIG. C.3.1 Normalized circle curves for relay with dead zone with R/a =1.5

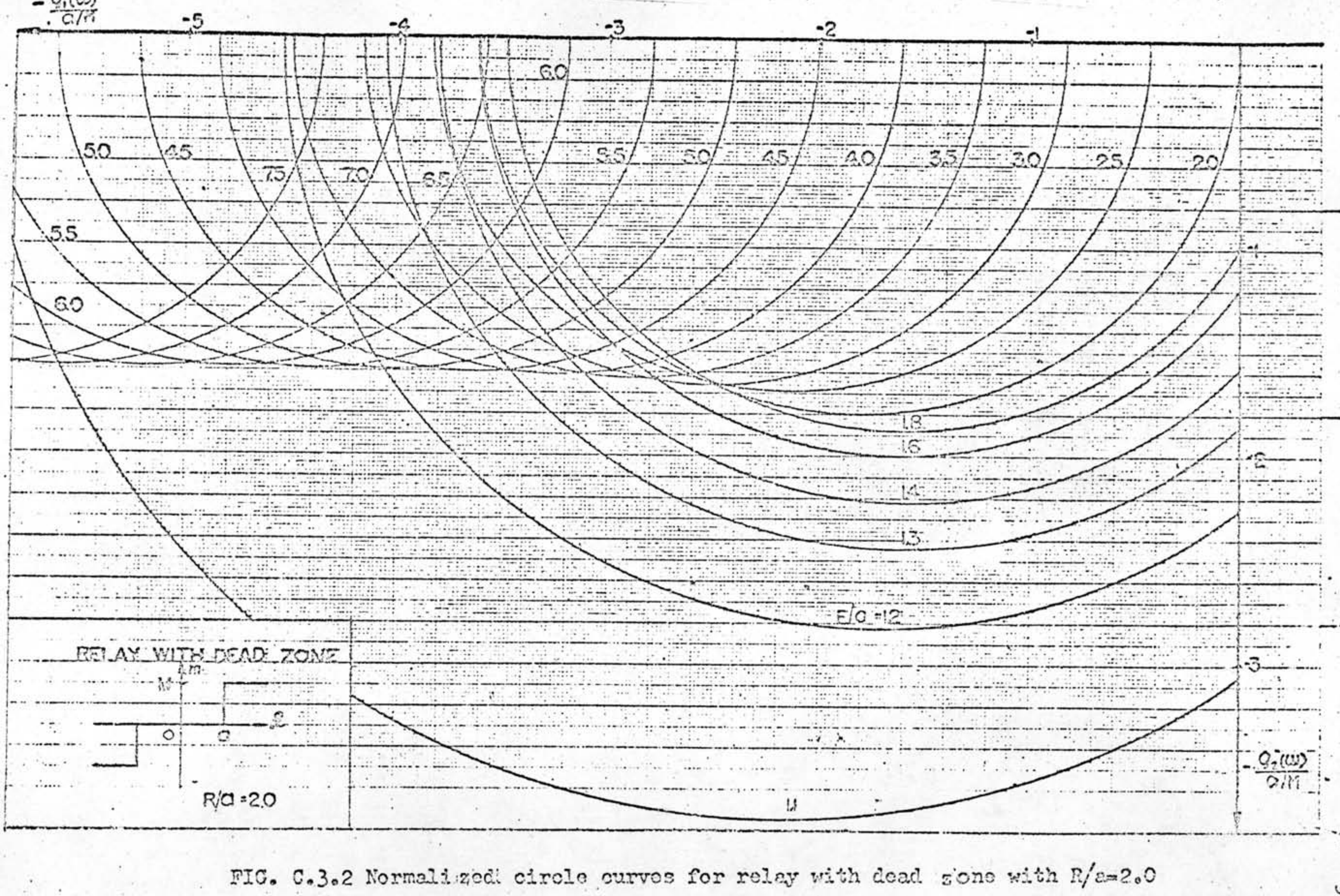

FIG. C.3.2 Normalized circle curves for relay with dead zone with R/a=2.0

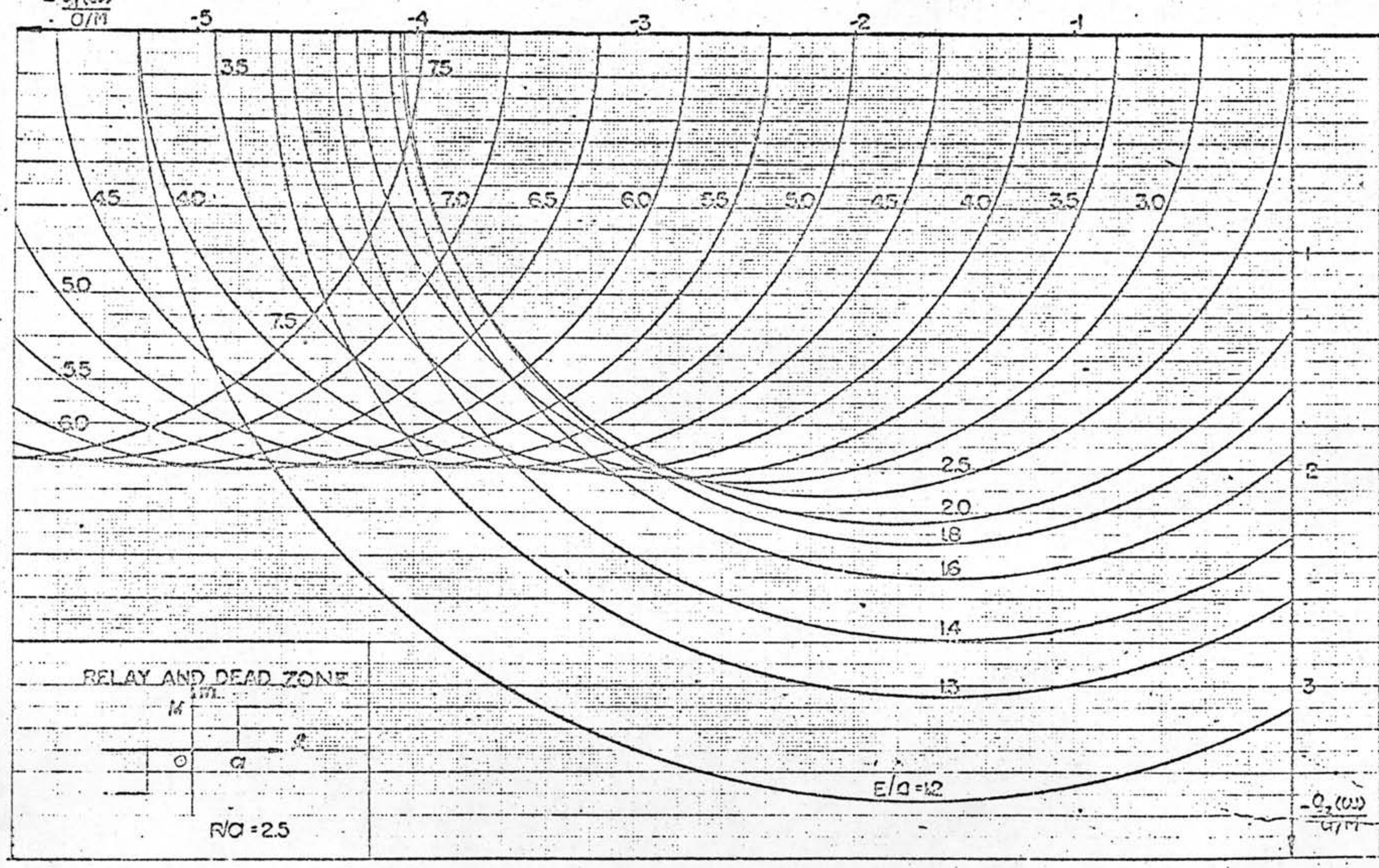

FIG. C.3.3 Normalized circle curves for relay and dead zone with R/a =2.5

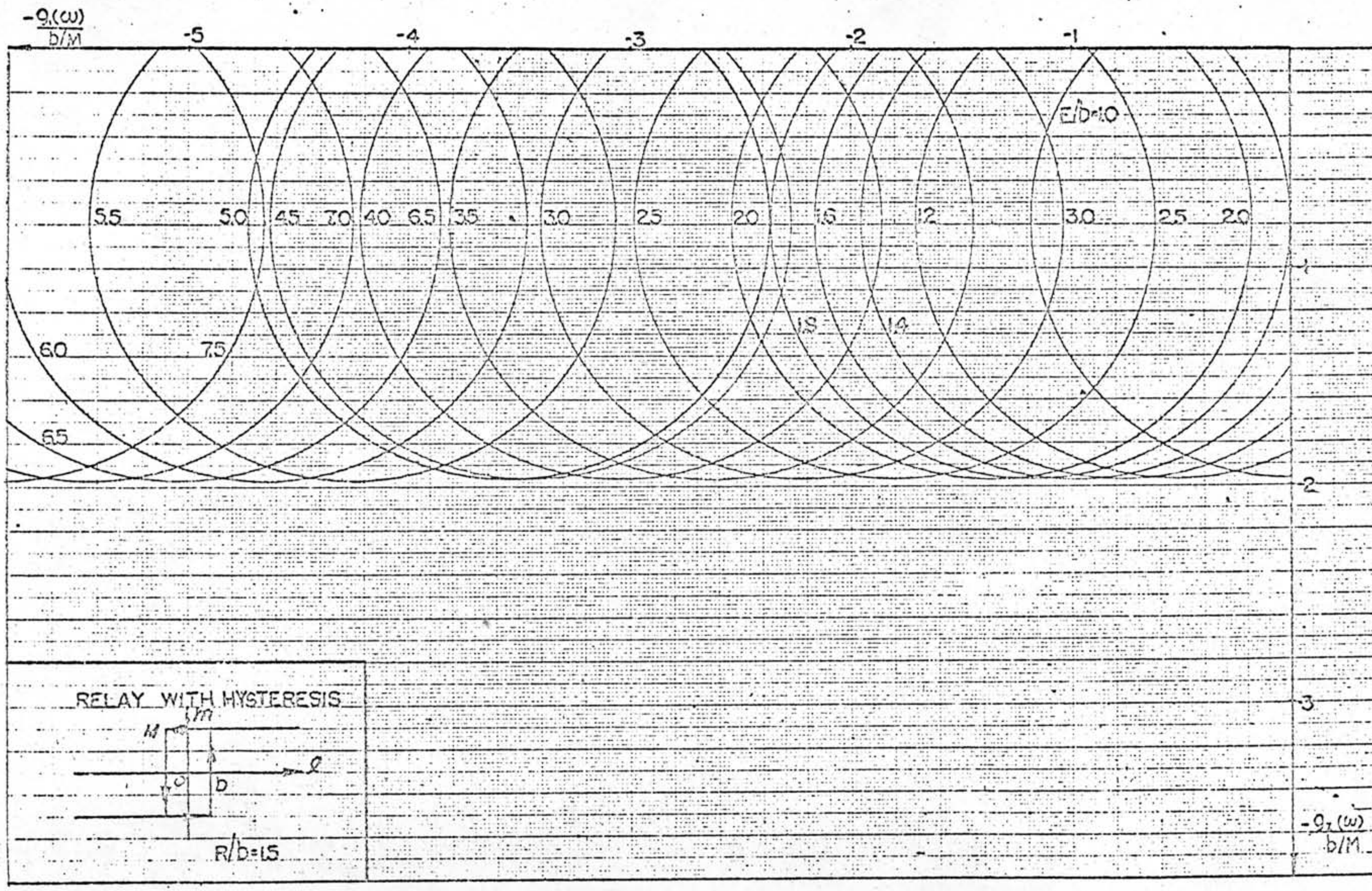

FIG. C.4.1 Normalized curves for relay with hyteresis with R/b =1.5

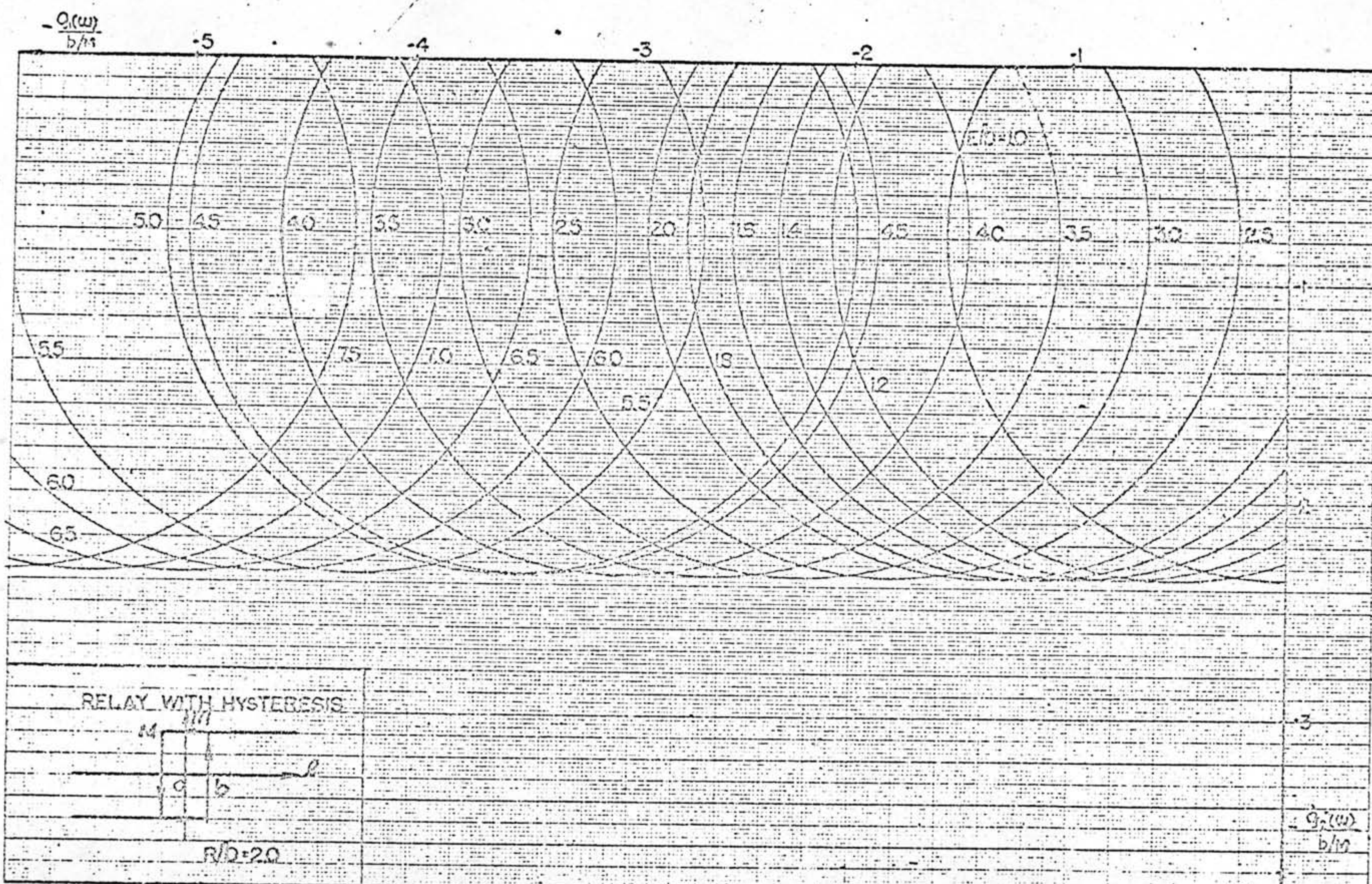

FIG. C.4.2 Normalized circle curves for relay with hyteresis with R/b =2.0

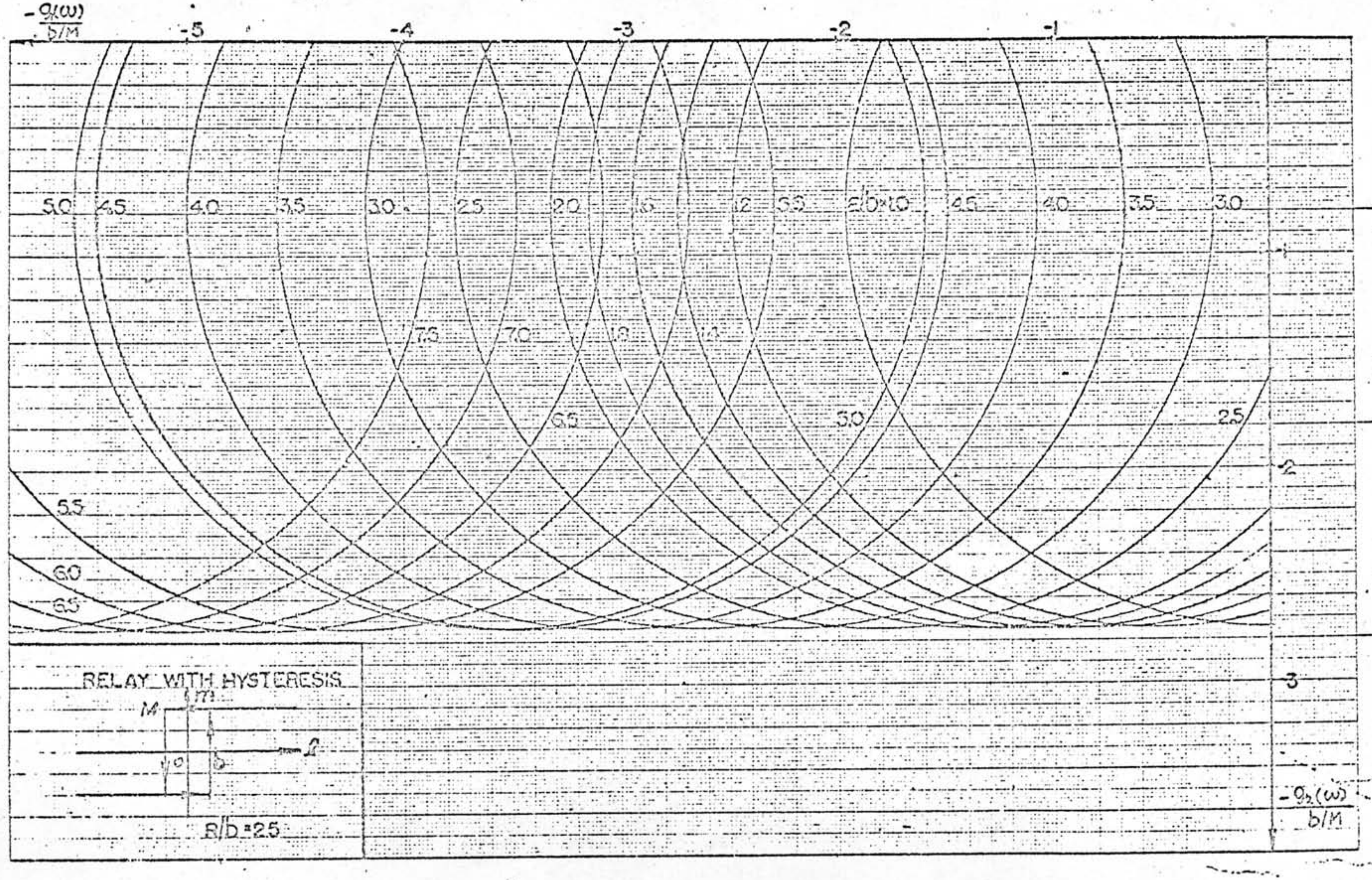

FIG. C.4.3 Normalized circle curves for relay with hyteresis with R/b =2.5

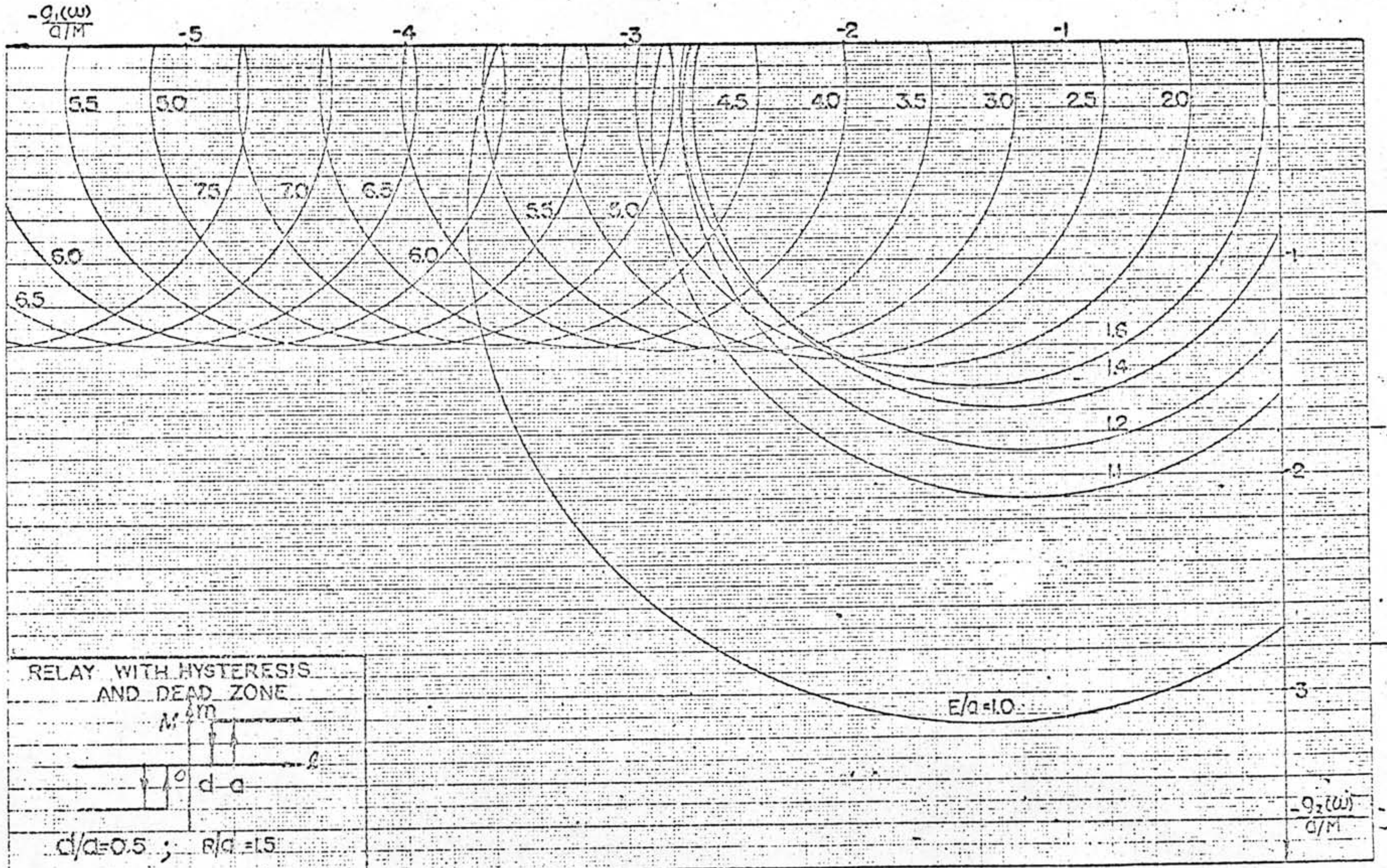

FIG.C.5.1 Normalized circle curves for relay with hyteresis and dead zone with d/a =0.5,R/a =1.5

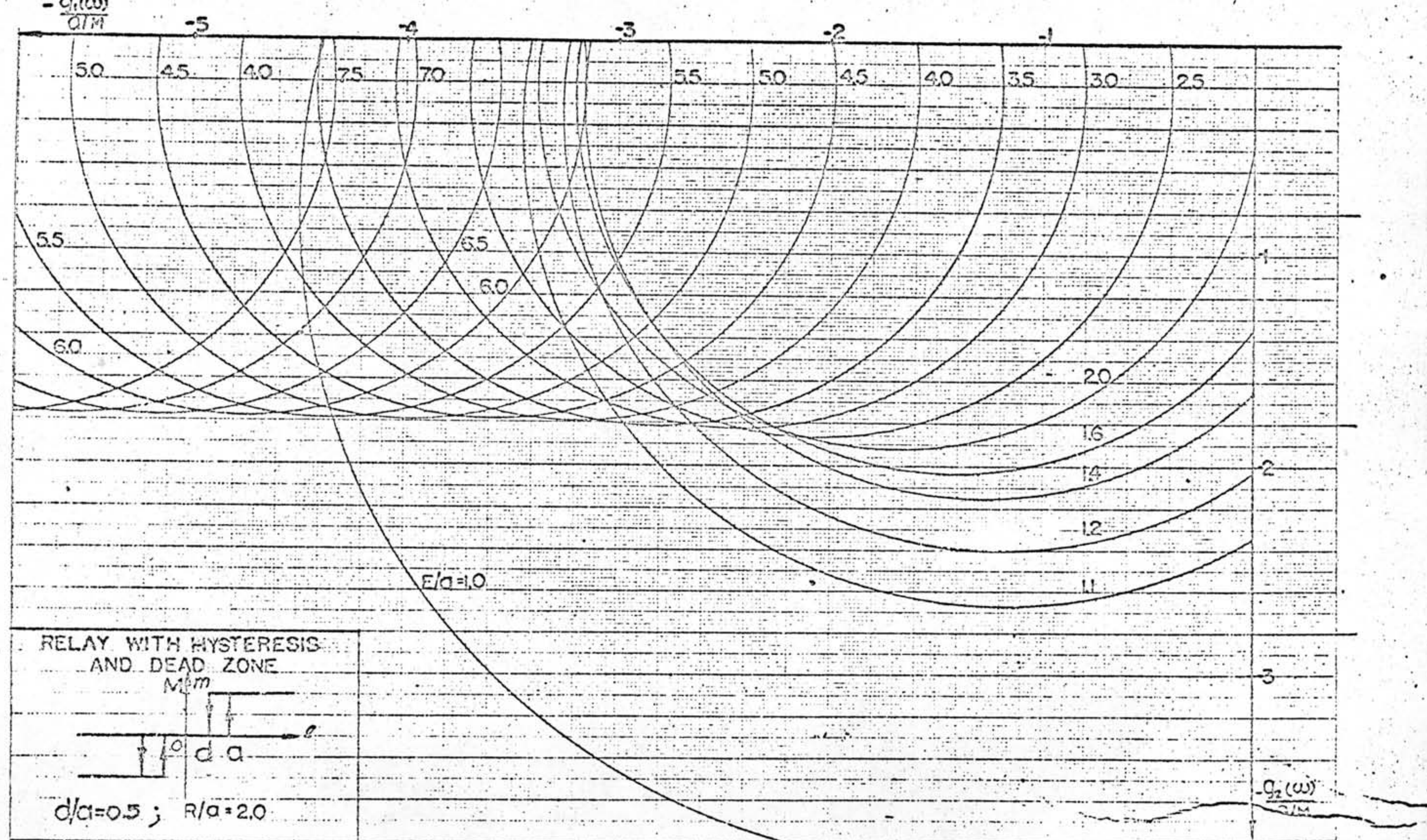

FIG. C.5.2 Normalized circle curves for relay with hyterosis and dead zone with d/a =0.5,R/a =2.0

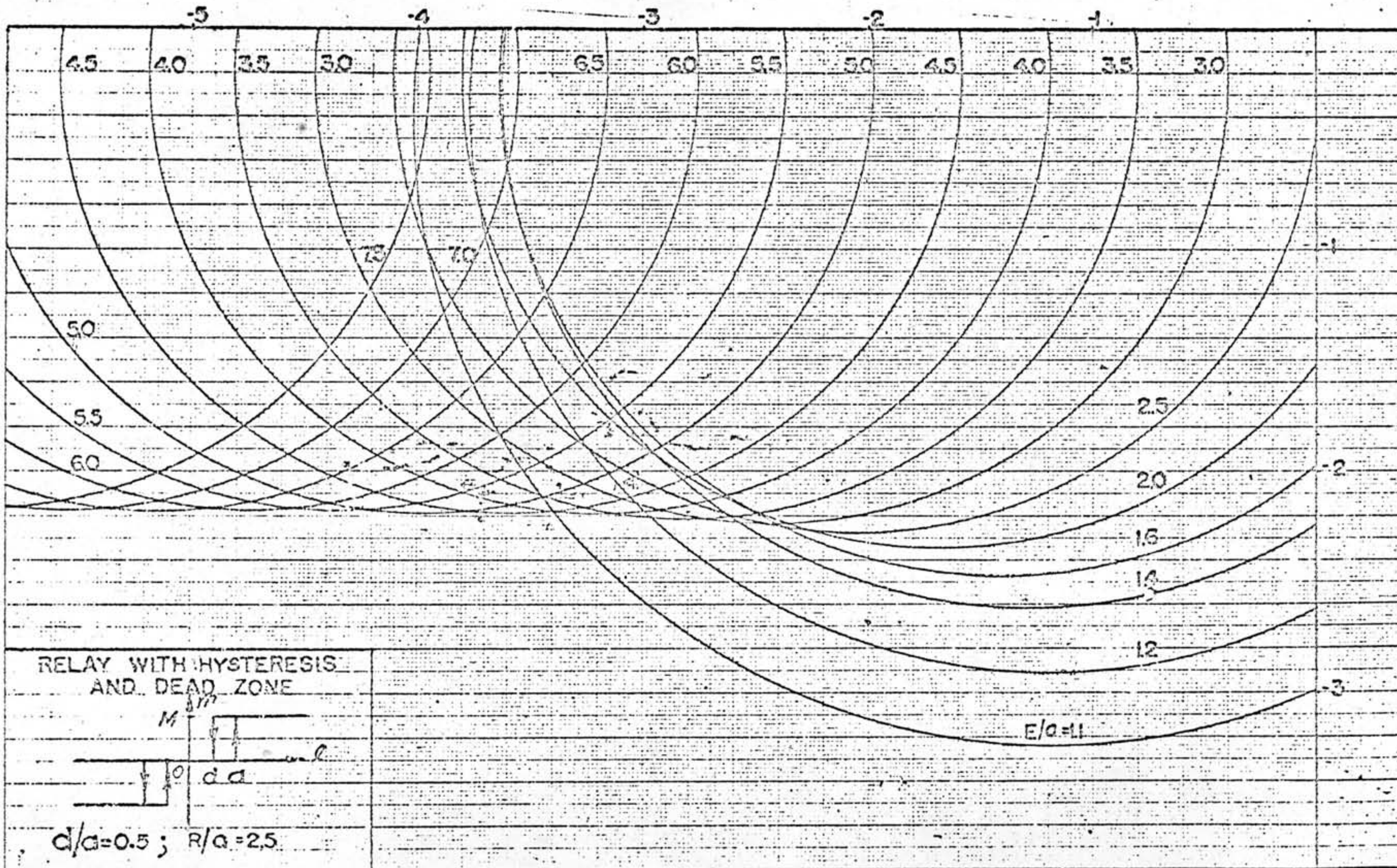

FIG. C.5.3 Normalized circle curves for relay with hyteresis and dead zone with d/a =0.5;R/a =2.5

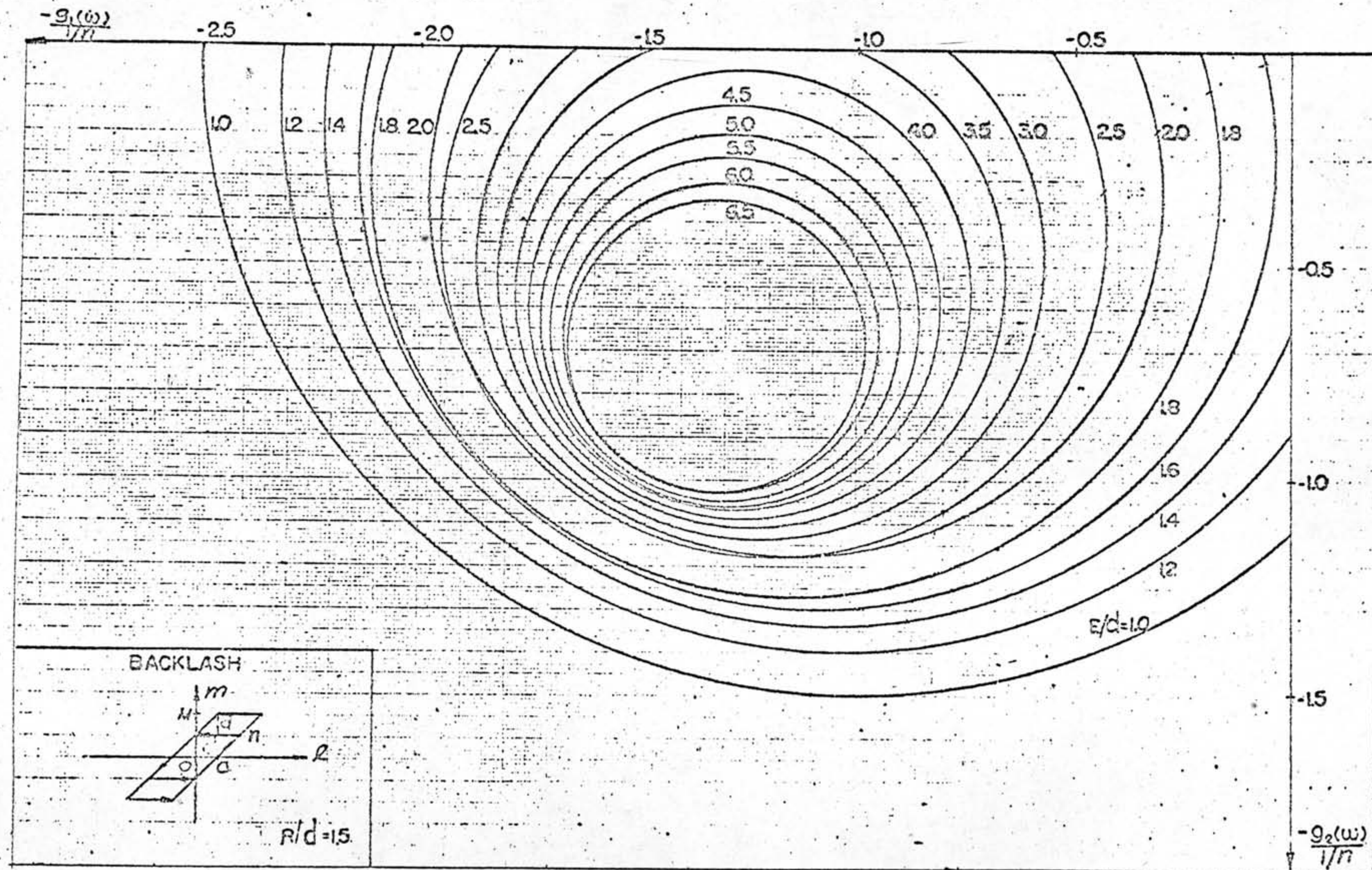

FIG. C.6.1 Normalized circle curves for backlash with R/d =1.5

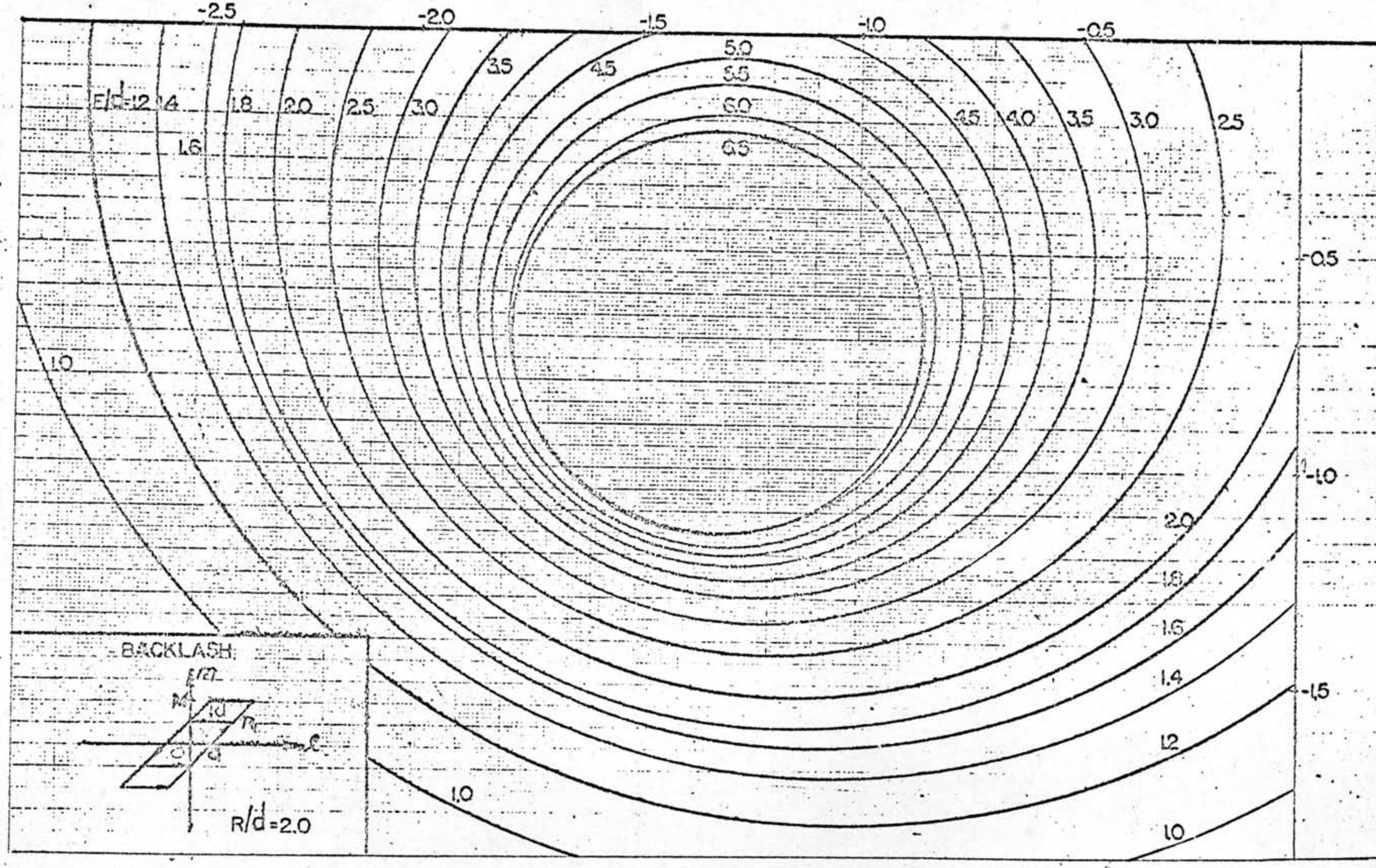

FIG. C.6.2 Normalized circle curves for backlash with R/d = 2.0

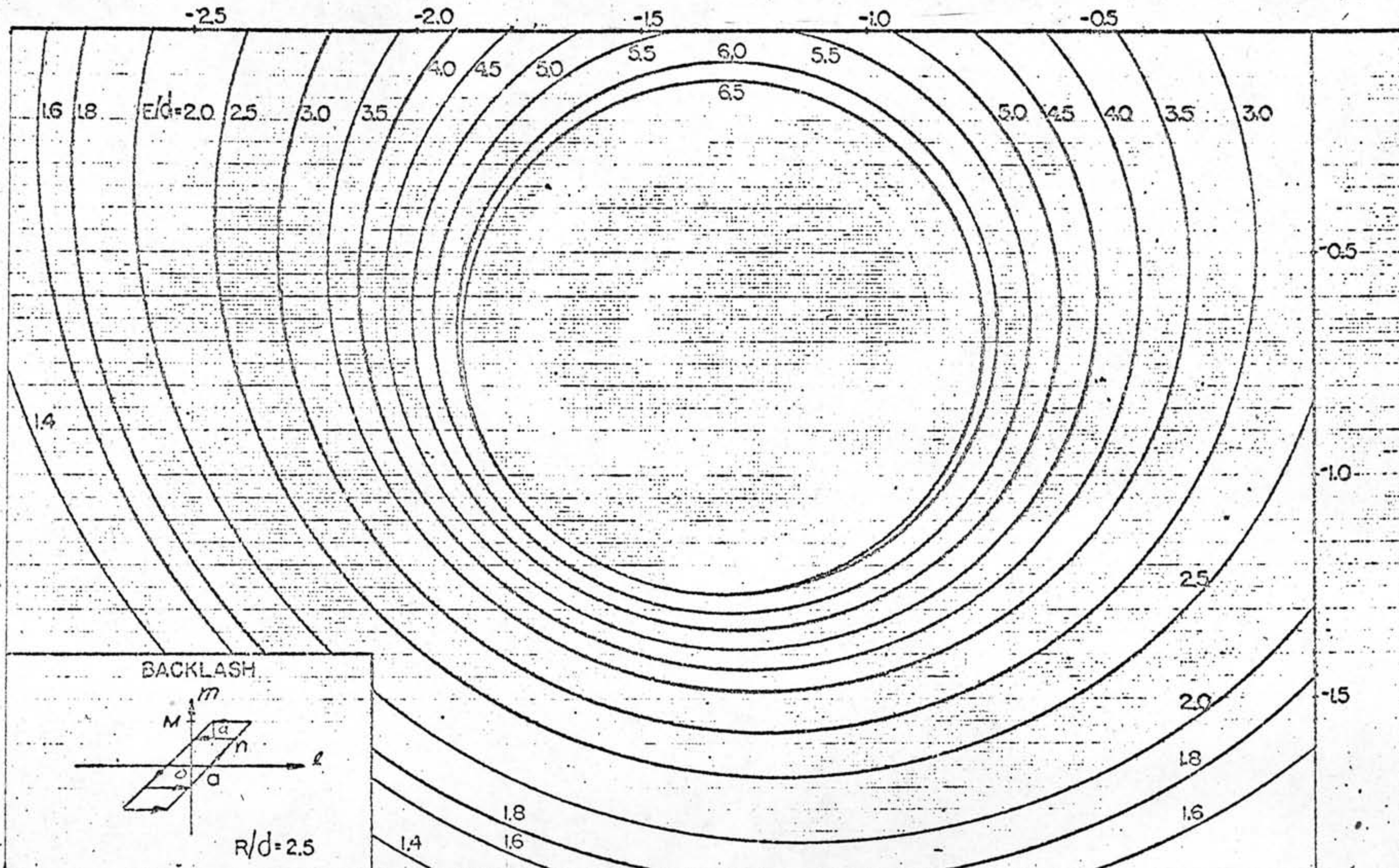

FIG. C.6.3 Normalized circle curves for backlash with R/d = 2.5

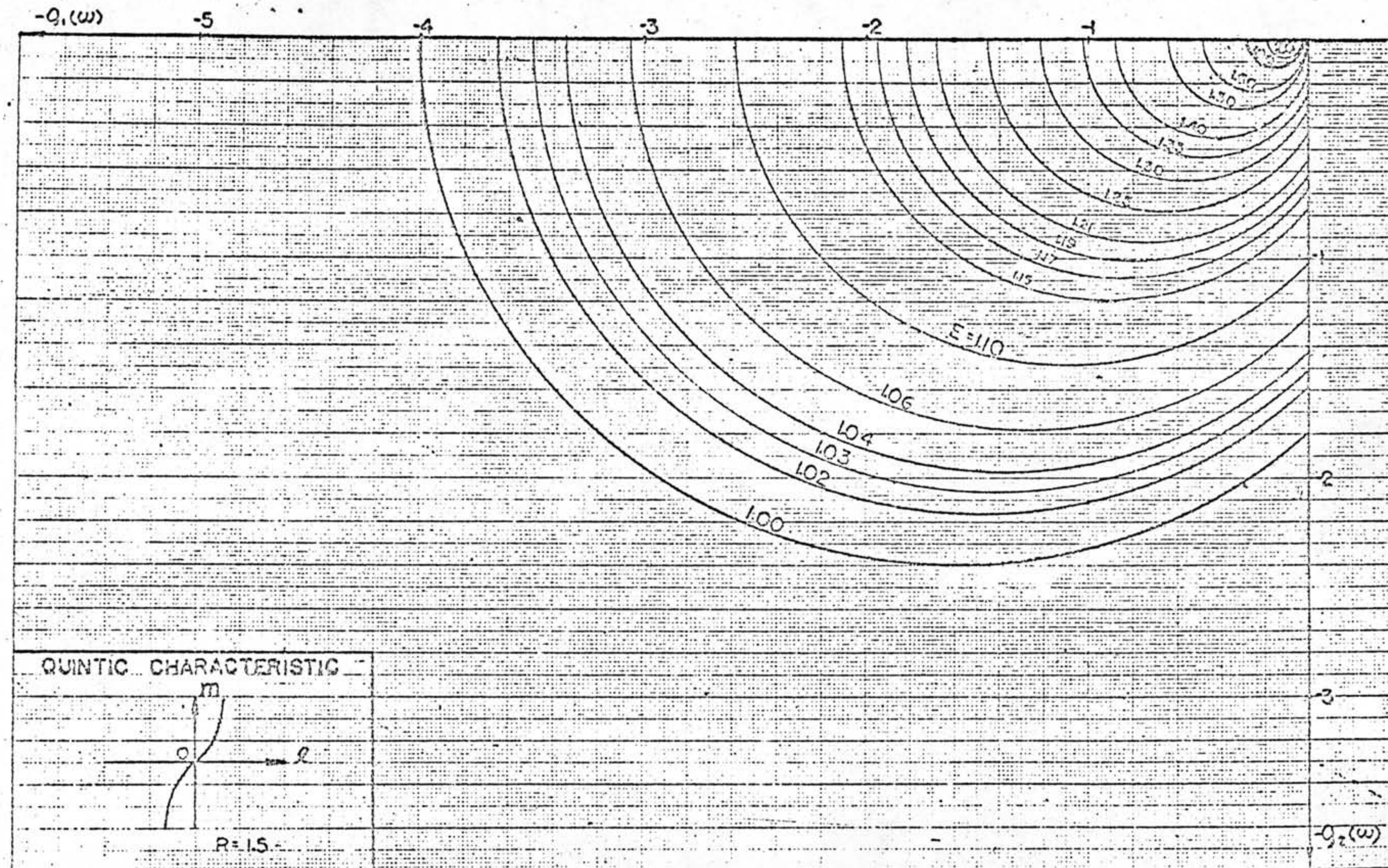

FIG. C.7.1 Normalized circle curves for quintic characteristic with R = 1.5

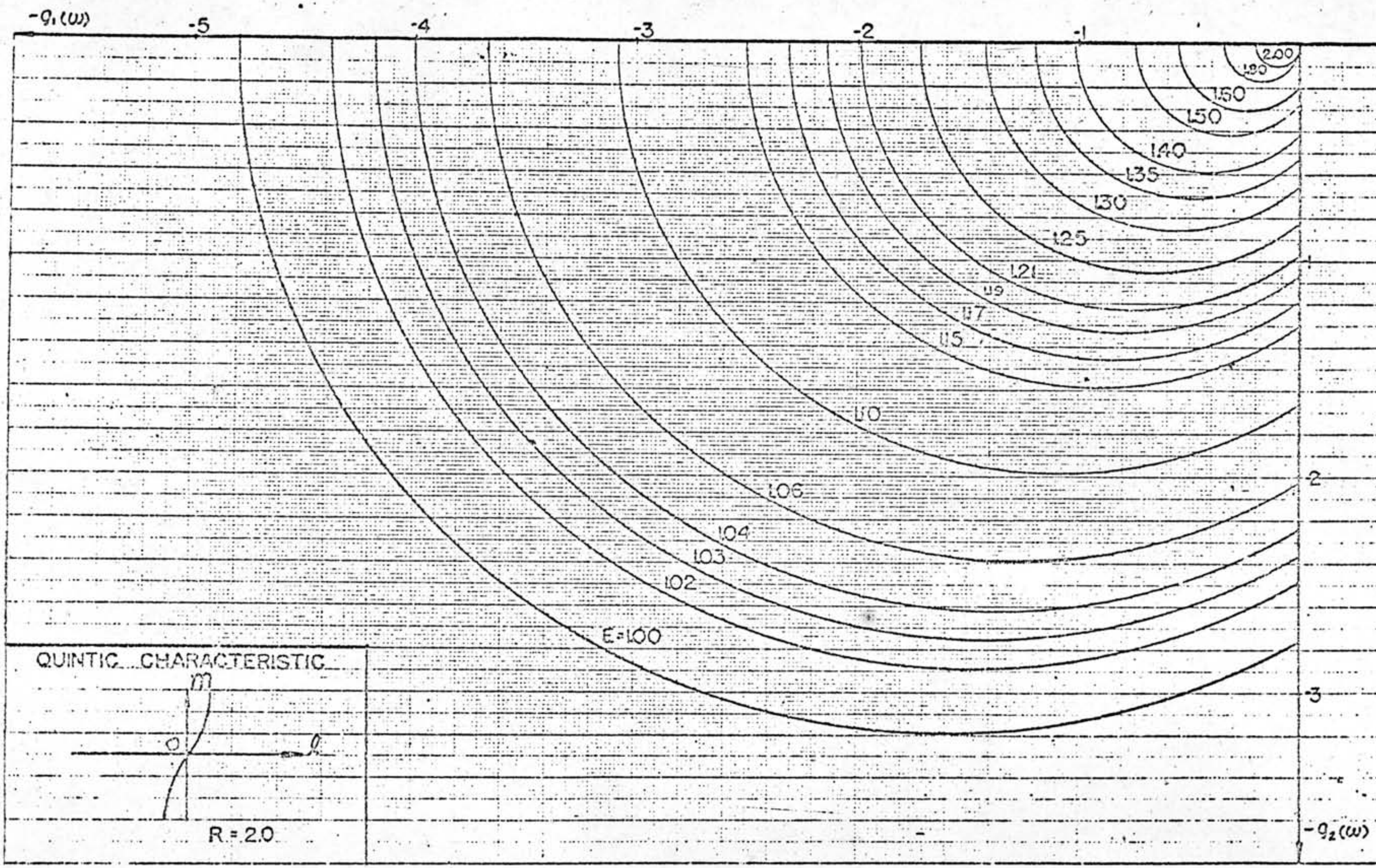

FIG. C.7.2 Normalized circle curves for quintic characteristic with R =2.0

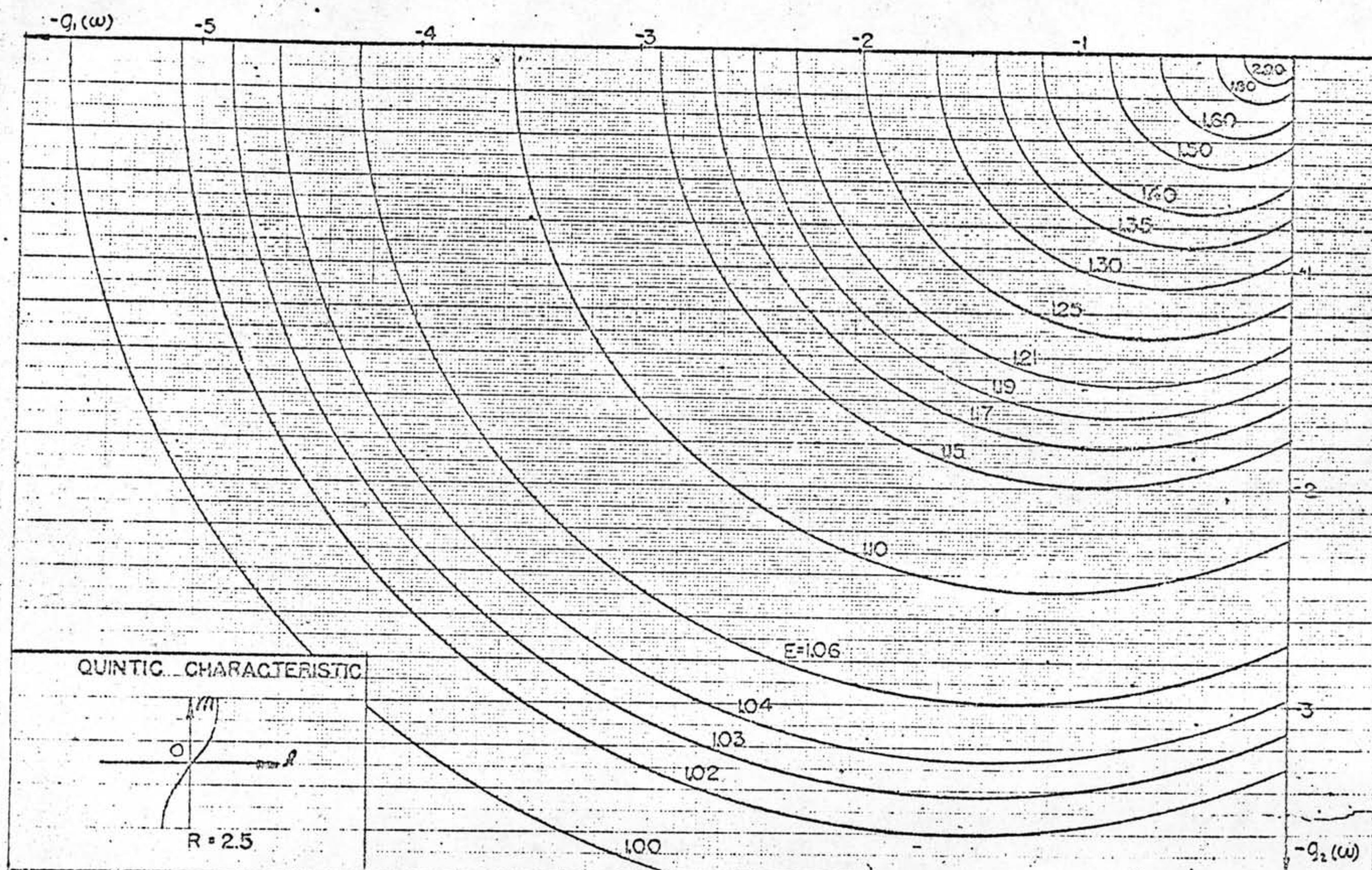

FIG. C.7.3 Normalized circle curves for quintic characteristic with R = 2.5

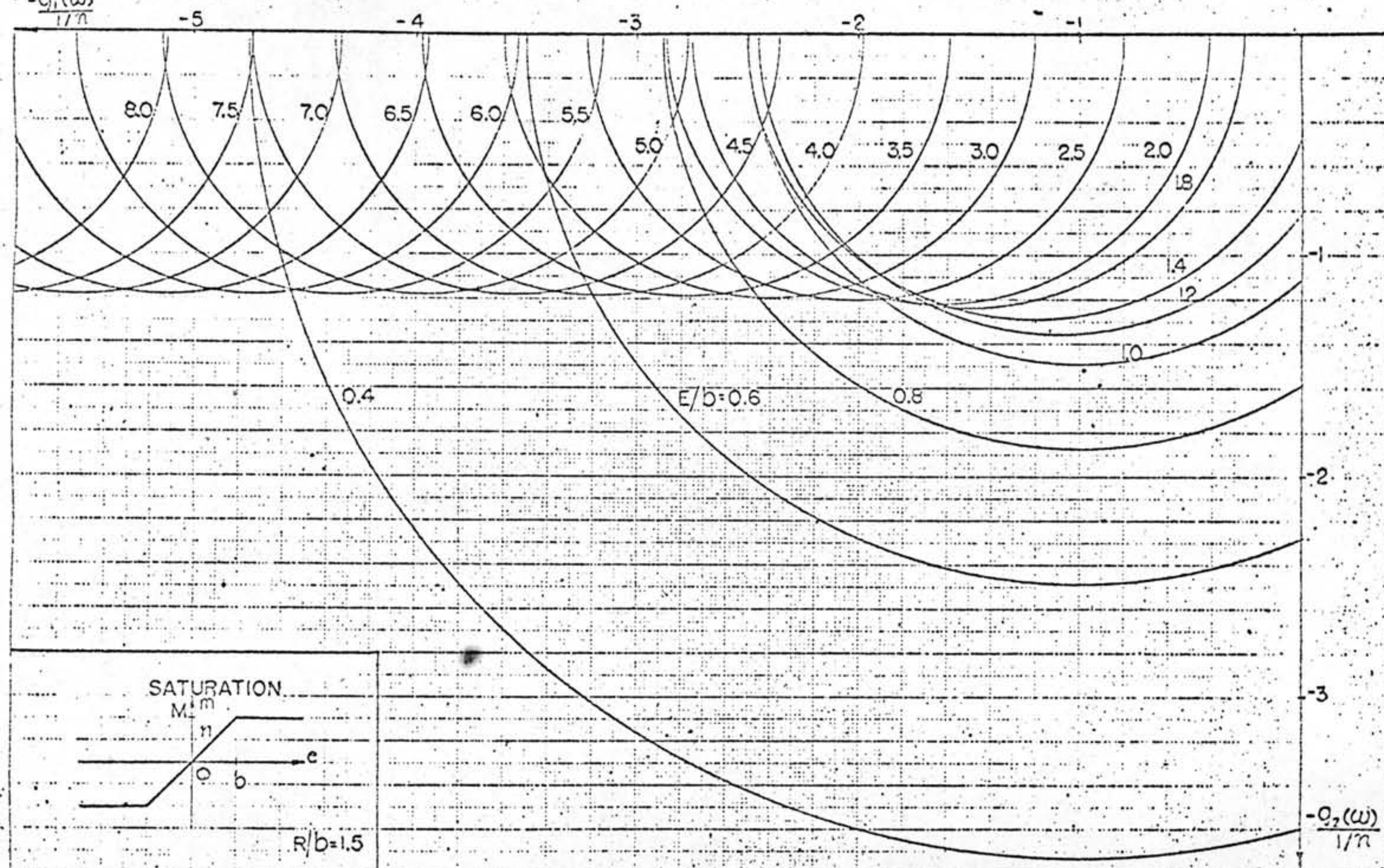

FIG.C.8.1 Normalized circle curves for Saturation with R/b = 1.5

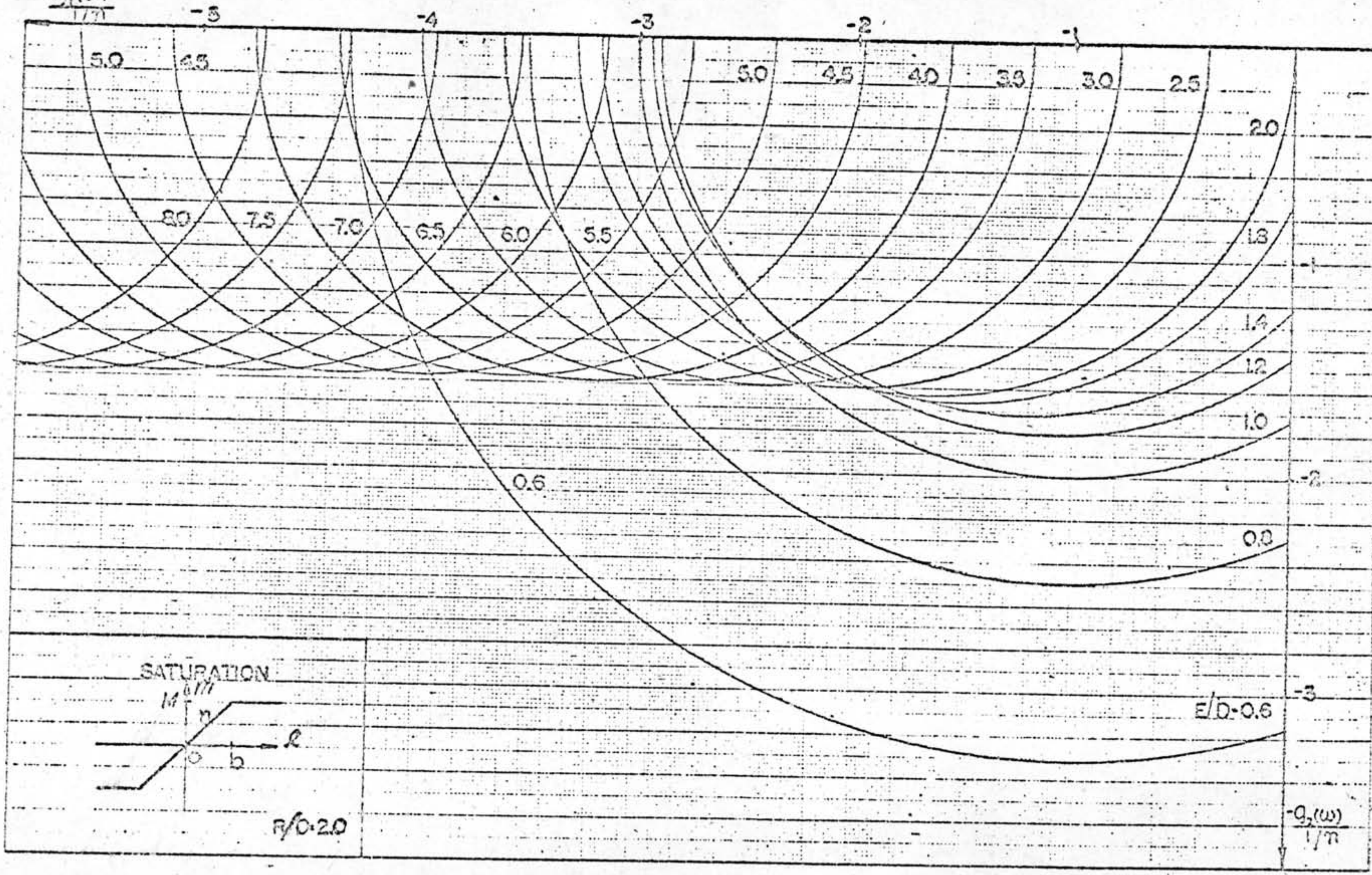

FIG. C.8.2 Normalized circle curves for saturation with R/b = 2.0

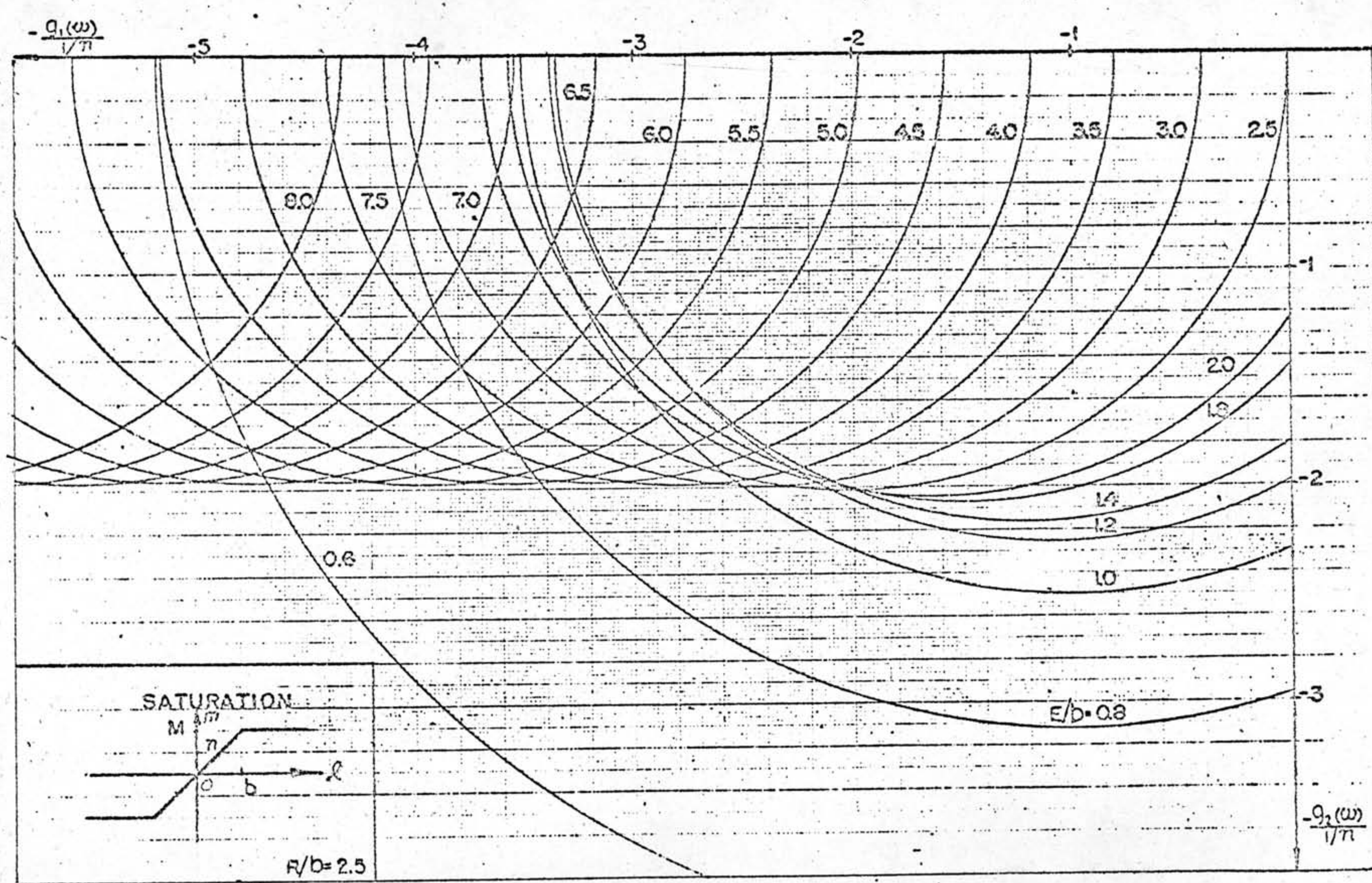

FIG. C.8.3 Normalized circle curves for saturation with R/b = 2.5

V I T A

Mr. Boonsook Sunkrasmee was born on November 1, 1949 in Bangkok, Thailand. He received the Bachelor Degree of Electrical Engineer from Chulalongkorn University in 1972. Follow his graduated, he has worked in Colgate Palmolive (Thailand) Co., Ltd. as maintenance engineer.