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DETERMINATION OF CLOSED LOOP FREQUENCY RESPONSE OF NONLINEAR SYSTEM BY GRAPHICAL METHOD

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บทคัดย่อ

วิทยานิพนธ์ฉบับนี้แสดงการหาฟรีเควันซี่เรสปอนส์ของระบบควบคุมนอนลิเนียร์โดยวิธีกราฟ กราฟมาตราฐานของสมการวงกลมสำหรับนอนลิเนียร์ทรานส์เฟอร์ฟังช์ชั่นหลาย ๆ ชนิดถูกสร้างขึ้นเพื่อการใช้งานทางปฏิบัติ และผลการวิเคราะห์หาฟรีเควันซี่เรสปอนส์ของระบบนอนลิเนียร์แบบเฉพาะต่าง ๆ ได้แสดงเป็นตัวอย่างการคำนวณไว้ด้วย ทั้งนี้ยังได้พิจารณาถึงวิธีการหาฟรีเควันซี่เรสปอนส์ของระบบควบคุมที่มีนอนลิเนียร์ตรี้เฉพาะแบบ และแอมปลิจูดเรสปอนส์อาจสามารถคำนวณอย่างคล้ายคลึงด้วยวิธีกราฟได้

Thesis Title : Determination of Closed Loop Frequency Responses of Nonlinear Systems by Graphical Method.

Name : Mr. Boonsook Sunkrasmee: Department of Electrical Engineer

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ABSTRACT

A graphical method for obtaining the frequency response of a typical nonlinear system is presented. The standard graphs of the circle equations for various types of nonlinear tranfer functions are derived and constructed for practical purpose. Analytical solutions of particular nonlinearities have been evaluated and the results have been also given as various applications. The determination of the frequency response of a special nonlinear system is also discussed and the amplitude response may be obtained by similar graphical technique.

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List of Symbols

r..... a sinusoidal input signal of the nonlinear control system .

R..... amplitude of the sinusoidal input signal of the nonlinear control system.

w..... the input signal frequency.

e..... error between the input and the output signal.

E..... amplitude of error signal.

ϕ..... phase shift of error signal from the input signal.

Keq(E)..... transfer function of nonlinear portion, always called "describing function".

g(E)..... real part of describing function.

b(E)..... imaginery part of describing function.

m..... output signal from nonlinear portion.

G(jw)..... transfer function of linear portion or the equivaleat gain of the linear portion.

$g_1(w)$..... real part of linear partion transfer function.

$g_2(w)$..... imaginery part of linear portion transfer function.

c..... output signal of nonlinear control system.

C..... output amplitude of nonlinear control system.

Θ..... phase shift of output signal from input signal.

a, b, d, n, M ..constant values.