CHAPTER II

THEORETICAL ANALYSIS

2.1 Assumptions

The assumptions for the analysis of inelastic bending of wood beam are as follows :

1) Plane section of the beam, normal to the axis, remains plane during bending.
2) Poisson's ratio is neglected. This due to the structure of wood that composed of many longitudinal fiber, so each of them can be considered as infinitesimal axially loaded rod and stressed up to the correspond strains only.
3) The moduli of elasticity for tension and compression of the wood are the same.
4) On the compression side, stress distribution at failure loading on the cross-section of the beam is assumed to be "Trapezoidal" distribution or "Second degree parabola" distribution.

2.2 Trapezoidal Distribution

In 1940 the "Trapezoidal distribution" of the compressive stresses has been assumed by Thunell (7).

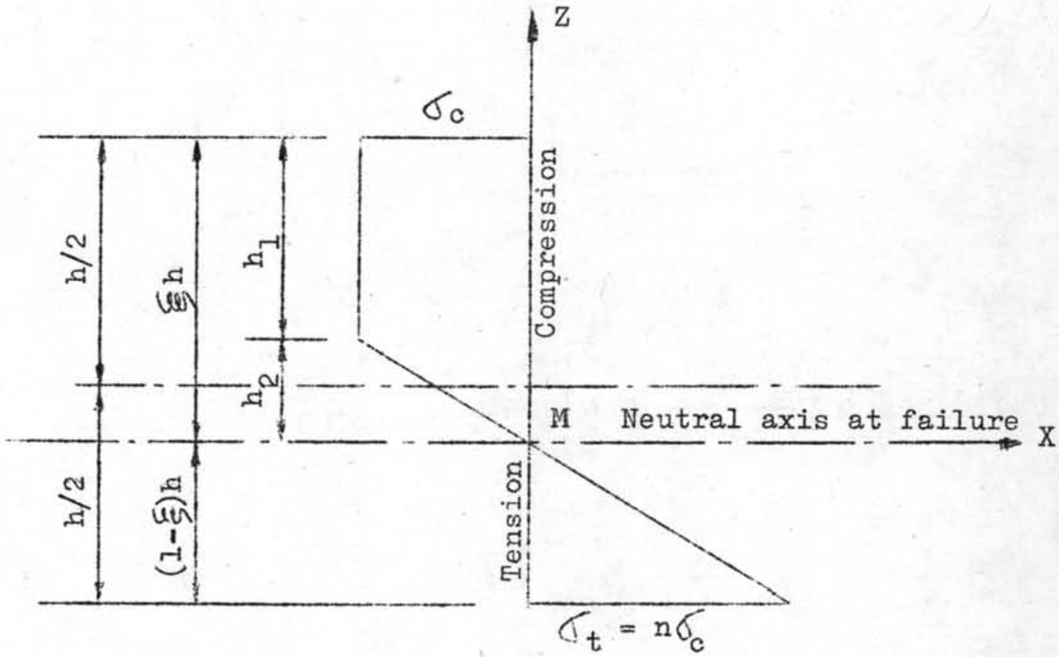

Fig. 2.1 Stress diagram for trapezoidal distribution of wood beam at failure.

From figure 2.1

$$\frac{h_2}{(1-\xi)h} = \frac{\sigma_c}{n\sigma_c}$$

$$h_2 = \frac{(1-\xi)h}{n} \qquad \text{------------(1)}$$

$$h_1 = \xi h - h_2$$

$$h_1 = \left[\xi - \frac{(1-\xi)}{n}\right] h \qquad \text{--------(2)}$$

At any stage of equilibrium, the internal compression force is equal to the internal tensile force.

$$\sum \text{Compression force} = \sum \text{Tension force}$$

$$\frac{1}{2} h_2.\sigma_c + h_1.\sigma_c = \frac{1}{2}(1-\xi)h.\sigma_t$$

$$\frac{1(1-\xi)}{2n}h.\sigma_c + \left[\xi - \frac{(1-\xi)}{n}\right]h.\sigma_c = \frac{1}{2}(1-\xi)h.n.\sigma_c$$

$$\frac{n(1-\xi)}{2} + \frac{(1-\xi)}{2n} = \xi$$

$$\frac{(1-\xi)}{\xi} = \frac{2n}{1+n^2}$$

$$\frac{1}{\xi} = \frac{1+2n+n^2}{1+n^2}$$

$$\xi = \frac{1+n^2}{(1+n)^2} \quad \text{------------(3)}$$

Equilibrium of moment.

$$\sum \text{External moment} = \sum \text{Internal moment}$$

$$M_{UL} = \frac{1}{2}(1-\xi)h.n.\sigma_c \times \frac{2}{3}(1-\xi)h$$

$$+ \frac{1}{2}\frac{(1-\xi)}{n}h.\sigma_c \times \frac{2}{3}\frac{(1-\xi)}{n}h$$

$$+ \left[\xi - \frac{(1-\xi)}{n}\right]h.\sigma_c \times \left[\frac{\xi}{2} - \frac{(1-\xi)}{2n} + \frac{(1-\xi)}{n}\right]h$$

$$M_{UL} = \sigma_c h^2 \left[\frac{n}{3}(1-\xi)^2 + \frac{1}{3}\frac{(1-\xi)^2}{n^2} + \frac{1}{2}\left(\xi - \frac{(1-\xi)}{n}\right)\left(\xi + \frac{(1-\xi)}{n}\right) \right]$$

$$= \sigma_c h^2 \left[\frac{1}{3}n(1-\xi)^2 + \frac{1}{3}\left(\frac{1-\xi}{n}\right)^2 + \frac{1}{2}\left(\xi^2 - \left(\frac{1-\xi}{n}\right)^2\right) \right]$$

$$= \sigma_c h^2 \left[\frac{1}{2}\xi^2 - \frac{1}{6}\left(\frac{1-\xi}{n}\right)^2 + \frac{1}{3}n(1-\xi)^2 \right] \text{-----(4)}$$

2.3 Second Degree Parabola Distribution

In 1941 Suenson (7) assumed a parabola as a curve for the compressive stresses and a linear distribution for tensile stresses. Later in 1960 Brochard (4) found the second degree parabola was the best approximation, also by Borislav D. Zakic (14) in 1973.

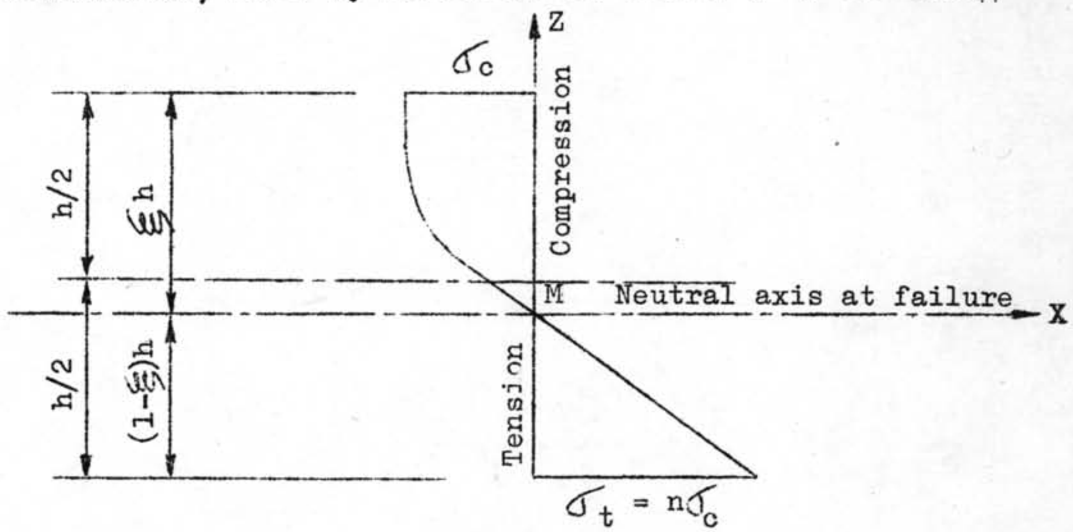

Fig. 2.2 Stress diagram for second degree parabola distribution of wood beam at failure.

Assume stress distribution in compression side in general form :

$$X = AZ^2 + BZ + C \qquad \text{------------------}(5)$$

first derivative being

$$X' = 2AZ + B \qquad \text{------------------}(6)$$

The constants of the second - degree parabola A, B and C can be solved from the boundary condition.

At point M ; $Z = 0$, $X = 0$

$$C = 0 \qquad \text{------------------}(7)$$

At $Z = \xi h$,

$$X' = 2A\xi h + B = 0$$

$$B = -2A\xi h$$

At $Z = \xi h$,

$$X = A\xi^2 h^2 + B\xi h = -\sigma_c$$

$$A = \frac{\sigma_c}{\xi^2 h^2} \qquad \text{------------------}(8)$$

$$B = -2\frac{\sigma_c}{\xi h} \qquad \text{------------------}(9)$$

Substituting equations (7), (8) and (9) into (5)

$$X = \frac{\sigma_c}{\xi^2 h^2} Z^2 - 2\frac{\sigma_c}{\xi h} Z \qquad \text{----------------}(10)$$

For tensile stress distribution, straight line equation be given in the form :

$$X = mZ + q \qquad \text{----------------}(11)$$

Constant m and q can be solved from the boundary condition

At $Z = 0$: $X = 0$

$$q = 0 \qquad \text{------------------} (12)$$

At $Z = -(1-\xi)h$: $X = \sigma_t$

$$-m(1-\xi)h = n.\sigma_c$$

$$m = -\frac{n.\sigma_c}{(1-\xi)h} \qquad \text{--------------}(13)$$

Substituting q and m from equations (12) and (13) into equation (11)

$$X = -\frac{n.\sigma_c}{(1-\xi)h} Z \qquad \text{-------------}(14)$$

At any stage of equilibrium ;

$$\Sigma F_x = 0, \quad \text{or} \quad \int_{A'} \sigma_x \, dA' = 0 \qquad \text{-------------}(15)$$

$$\Sigma M_z = 0, \quad \text{or} \quad -\int_{A'} \sigma_x y_d \, dA' = 0 \qquad \text{--------------}(16)$$

For the unit width of cross-section, the compressive stress diagram area is

$$A_c = \int_0^{\xi h} \left[\frac{\sigma_c}{\xi^2 h^2} Z^2 - 2 \frac{\sigma_c}{\xi h} Z \right] dZ \qquad \text{----------}(17)$$

$$A_t = -\int_{-(1-\xi)h}^{0} \frac{n.\sigma_c}{(1-\xi)h} Z dZ \qquad \text{----------------}(18)$$

According to equation (15) $F_x = 0$;

$$A_c + A_t = 0$$

$$\int_0^{\xi h}\left[\frac{\sigma_c}{\xi^2 h^2}Z^2 - 2\,\frac{\sigma_c}{\xi h}Z\right]dZ + \int_{-(1-\xi)h}^{0}\frac{-n.\sigma_c}{(1-\xi)h}Z\,dZ = 0$$

$$-\frac{2}{3}\cdot\sigma_c\,\xi h + \frac{1}{2}n.\sigma_c(1-\xi)h = 0$$

$$\frac{(1-\xi)}{\xi} = \frac{4}{3n}$$

$$\xi = \frac{3n}{3n+4} \quad \text{------------(19)}$$

According to equation (16) $M_z = 0$;

$$M_{UL} = -\int_0^{\xi h}\left[\frac{\sigma_c}{\xi^2 h^2}Z^2 - 2\frac{\sigma_c}{\xi h}Z\right]Z\,dZ - \int_{-(1-\xi)h}^{0} -\frac{n\sigma_c}{(1-\xi)h}Z^2\,dZ$$

$$= -\frac{1}{4}\,\frac{\sigma_c}{\xi^2 h^2}(\xi h)^4 + \frac{2}{3}\,\frac{\sigma_c}{\xi h}(\xi h)^3 + \frac{1}{3}\,\frac{n\sigma_c}{(1-\xi)h}\,(1-\xi)h$$

$$M_{UL} = \left[\frac{5}{12}\xi^2 + \frac{1}{3}\,n(1-\xi)^2\right]\sigma_c h^2 \quad \text{------------------------(20)}$$

For ultimate moment of over all cross-section

$$M_{UL} = \left[\frac{5}{12}\xi^2 + \frac{1}{3}\,n(1-\xi)^2\right]\sigma_c b h^2 \quad \text{--------------------(21)}$$

$$= R.bh^2 \quad \text{--------------------(22)}$$

$$\text{where } R = \left[\frac{5}{12}\xi^2 + \frac{1}{3}\,n(1-\xi)^2\right]\sigma_c \quad \text{--------------------(23)}$$