CHAPTER V

CONCLUSION AND DISCUSSIONS

An approximated solution of the deflection which is in the form of a polynomial function gives satisfactory results as seen in the graphs, although the values of the bending moment are scatter at some points. This is because of one of the boundary conditions has been approximated. That is, the total bending moment along the edge is equal to zero. Another cause of error is due to the chosen form of the deflection function. In using the value of bending moment near the vicinity of the applied load, caution must be taken because of the large error in this area.

The form of the assumed deflection function can also cause the discripancies between the theory and experiment. Were more terms in the polynomial employed, the theoretical results should show better agreement with the experimental ones.

When this proposed solution is extended to the case of distributed load over the area of the plate and is compared to the work of C. Vijakhana and Lee, the value of deflections, are 1.035 times their solution. Also the values of bending moments are 1.035 times their solution.

APPENDIX I

STRAIN GAGE TECHNIQUE

Principle

The strain gage technique is based on the characteristic of metal which changes the electrical resistance with the strain caused.

The electric resistance R is given by

$$R = \rho \frac{l}{A} \qquad (1)$$

where ρ = resistivity of wire material

To obtain a unit change in resistance, the logarithms for both the right and left members are differentiated,

$$\frac{\Delta R}{R} = \frac{\Delta \rho}{\rho} + \frac{\Delta l}{l} - \frac{\Delta A}{A} \qquad (2)$$

If a wire of length l is elongated by a tension and the initial diameter d is changed to d' effected by Poisson's ratio, therefore

$$\frac{\Delta A}{A} = 2\,\frac{d' - d}{d} = -\,\frac{2\nu\,\Delta l}{l}$$

Substituing into eq. (2)

$$\frac{\Delta R}{R} / \epsilon = (1 + 2\nu) + \frac{\Delta \rho}{\rho} / \epsilon \quad (3)$$

where ϵ = strain = $\Delta 1/1$

The change in resistivity ρ is in proportion to the volumetric change of the material

$$\Delta \rho = m\rho \frac{\Delta V}{V} \quad (4)$$

where m is a proportional constant

and $$\frac{\Delta V}{V} = (1 - 2\nu) \frac{\Delta 1}{1} \quad (5)$$

Substituting eq. (5) into eq. (4) and (3), then eq. (3) becomes

$$\frac{\Delta R}{R} / \epsilon = (1 + m) + 2\nu (1 - m) \quad (6)$$

Most of the resistive materials used for the strain gage is an alloy wire or foil of copper and nickel, where $m \simeq 1$ in value.

Therefore, the right side of eq. (6) becomes 2, ie.

$$\frac{\Delta R}{R} / \epsilon = 2 \quad (7)$$

The valve defined by eq. (7) is called strain sensitivity of metal material (K) which depends on the resistive material used for strain gage,

$$\frac{\Delta R}{R} = K \quad (8)$$

It is seen that the unit change in resistance is proportional to the magnitude of strain sensed. Therefore, it is desirable that resistive materials have a const. K value irrespective of strain magnitude.

For the circuit, it is generally used Wheatstone bridge circuit. This is the most frequently used, high accuracy measuring circuit for measurement of small resistance changes.

In Fig. 15 R is a gage resistance, R_2, R_3 and R_4 are fixed resistors. E is applied on the bridge circuit. If the resistance changes to $R + \Delta R$, then there will be a voltage e at the output terminals of the bridge as.

$$e = \frac{R_3}{R_3 + R_4} - \frac{R \pm \Delta R}{R \pm \Delta R + R_2}$$

If $R = R_2 = R_3 = R_4$, one has

$$e = \frac{\pm \Delta R}{2(2R \pm \Delta R)} E \qquad (9)$$

Here $2R \gg \Delta R$, therefore

$$e \approx \frac{E}{4} \cdot \frac{\Delta R}{R} \qquad (10)$$

From this, it is seen that the bridge output voltage, e is proportional to the unit change in resistance of the gage.

Substituting eq. (10) into eq. (8)

$$e \simeq \frac{E}{4} \cdot K \cdot \epsilon \qquad (11)$$

Then it is clear that e is proportional to strain, ϵ ,. Therefore, to find out the value of strain is to measure the bridge output voltage.

If two gages are used at right angles to each other, the value of strains will be recorded as apparent strains $\bar{\epsilon}_x$, $\bar{\epsilon}_y$. To obtain the true strains ϵ_x and ϵ_y, using the relationship

$$\epsilon_x = \frac{1 - \nu K}{1 - K^2} (\bar{\epsilon}_x - K \bar{\epsilon}_y) \qquad (12)$$

$$\epsilon_y = \frac{1 - \nu K}{1 - K^2} (\bar{\epsilon}_y - K \bar{\epsilon}_x) \qquad (13)$$

where K is the transverse sensitivity.

If the value of K is very small, the value of apparent strain and true strain will be the same.

The bending moment M_x and M_y can be obtained by the following expressions

$$M_x = \frac{2D}{h} (\epsilon x + \nu \epsilon y) \qquad (14)$$

$$M_y = \frac{2D}{h} (\epsilon y + \nu \epsilon x) \qquad (15)$$

where D is the flexural rigidity and h is the thickness of the plate.

Instrumentation

1. Equipments

(a) Portable Strain Gage Bridge Type 5580, Tinsley

Range	: ± 10,000 units of microstrain upon a 10 - turn slidewire.
Sensitivity	: The detector is sufficiently sensitive for balancing to 1 unit of microstrain with 100 ohm gages.
Gage Resistance	: Gages with any resistance between 50 and 2,000 ohms may be used.
Limit of Error	: ± .1 % of reading or 5 units of microstrain whichever is the greater.
External Circuits	: Suitable for single gage with dummy gage, 2 gage and 4 gage bridge.
Gage Factor Range	: The gage factor may be varied in steps of 0.01 from 1.8 to 4.5
Reactive Effect	: Lead capacitance up to .05 μF will not effect the balance or sensitivity of the bridge.

Output	: The output can be examined with an oscilloscope by means of a connection to the output jack.
Power Supply	: DEAC 13.5 volt chargeable battery.

(b) Apex Unit

Type	: 4907 J
Selector Switch	: 25 - point manual, 26 position selector switch.
Connection	: Two plugs and sockets for each resistor connection.

(c) Strain Gage Kyowa

Type	: KFC - 5 - D 16 - 11.
Resistance	: 120 $\sim$ 120.9 ohm.
Gage Factor	: 2.09

(d) Dial Cage Mitutoyo

Range	: 20 mm.
No.	: 2050 E
Graduation	: 0.01 mm.

2. Procedure For Bonding Gages

For the experiment, Kyowa gages (KFC - 11 - D16) are used. The gage factor is 2.09 and gage resistance is in the range 120 $\sim$ 120.9 ohms. For bonding gages the surface of specimen is finished by sand - paper. Remove oil and grease by using acetone. The adhesive (CC - 15A) is applied at the area of specimen then set the gage on the bonding position. After pressing with a finger, leave the gage in the air about 24 hours. To protect the moistive, the gages are coated with wax (C - 1A). Then the gages are connected to the strain gage bridge and an apex unit. The bridge circuit, connection and wiring diagram are shown in Fig. 16, 17 and 18.

APPENDIX II

TABLES

This content is composed of the results obtained from the proposed solution and the experimental investigation. The results are shown in tabular form from table 1 through table 9.

Table 1 The co-ordinates x, y of the position for measuring the deflections and strains

Point	Positions for measuring deflection		Positions for measuring strain	
	x(mm.)	y(mm.)	x(mm.)	y(mm.)
1	395.00	0	460.83	0
2	263.33	0	329.17	0
3	131.67	0	197.50	0
4	0	0	65.83	0
5	-131.67	0	-65.83	0
6	263.33	0	-197.50	0
7	-257.33	75.83	-257.33	37.92
8	-257.33	151.67	-257.33	113.75
9	-257.33	227.50	-257.33	189.58
10	-257.33	303.33	-257.33	265.42
11	-257.33	379.17	-257.33	341.25
12	-	-	-257.33	417.08

Table 2 Values of wD/Pa^2 and w/P from proposed solution

Point	wD/Pa^2		w/P	
	$\nu = 0.3$	$\nu = 0.1$	$\nu = 0.3$	$\nu = 0.1$
1	0.030	0.029	0.047	0.046
2	0.045	0.049	0.076	0.077
3	0.060	0.059	0.094	0.093
4	0.063	0.062	0.098	0.098
5	0.061	0.060	0.095	0.095
6	0.057	0.055	0.090	0.086
7	0.056	0.054	0.087	0.084
8	0.051	0.049	0.080	0.077
9	0.043	0.041	0.067	0.064
10	0.031	0.030	0.049	0.047
11	0.017	0.017	0.027	0.026

Table 3 Deflections by variation of load from proposed solution, $\nu = 0.3$

Point	w (mm.)					
	3.325 (kg)	6.225 (kg)	10.125 (kg)	12.425 (kg)	16.425 (kg)	20.625 (kg)
1	0.164	0.294	0.478	0.586	0.775	0.973
2	0.259	0.484	0.788	0.966	1.276	1.604
3	0.313	0.585	0.952	1.168	1.545	1.940
4	0.327	0.612	0.996	1.222	1.615	2.028
5	0.317	0.594	0.965	1.185	1.566	1.966
6	0.299	0.560	0.910	1.117	1.476	1.854
7	0.291	0.545	0.886	1.087	1.437	1.804
8	0.265	0.496	0.807	0.990	1.309	1.643
9	0.222	0.416	0.677	0.831	1.098	1.379
10	0.164	0.307	0.499	0.613	0.810	1.017
11	0.091	0.171	0.278	0.340	0.450	0.566

Table 4 Data of deflections from experimentation

Weight (kg)	5	10	15	20	25	30	Initial rdg. of dial gauge
Spring scale rdg. (kg)	1.8	3.9	5.0	7.7	8.7	9.5	
Dial gauge rdg.							
No. 1	10.8	16.5	23.6	28.0	35.5	42.9	5.1
2	36.1	45.8	58.0	65.6	78.0	90.4	26.2
3	75.0	5.4	43.3	66.5	5.3	44.0	43.0
4	0.8	32.1	72.5	97.5	38.0	80.0	66.2
5	86.7	18.7	55.4	78.4	16.3	55.1	55.0
6	34.5	62.7	94.2	14.8	49.3	85.0	5.6
7	28.1	55.8	86.4	6.1	40.3	75.2	0.5
8	27.4	52.6	79.6	98.5	30.2	60.5	1.9
9	22.5	43.8	65.8	81.0	7.8	33.2	1.9
10	18.1	33.7	49.6	60.4	79.0	3.0	3.0
11	9.0	17.2	25.8	31.4	41.1	50.8	1.2

Note. Dial gauge No. 1, 2 (in) Graduation 0.001 in

Dial gauge No. 3 – 11 (mm) Graduation 0.01 mm.

Weight of the tray 0.125 kg.

Table 5 Deflections by variation of load from experimentation

Load (kg)	3.325	6.225	10.125	12.425	16.425	20.625
Deflection (mm) of point No. 1	0.144	0.290	0.470	0.582	0.772	0.960
2	0.251	0.498	0.808	1.001	1.316	1.631
3	0.320	0.624	1.003	1.235	1.623	2.010
4	0.346	0.659	1.063	1.313	1.718	2.158
5	0.337	0.637	1.004	1.234	1.613	2.001
6	0.287	0.569	0.984	1.090	1.435	1.792
7	0.276	0.553	0.859	1.056	1.398	1.747
8	0.249	0.503	0.771	0.960	1.277	1.560
9	0.206	0.419	0.639	0.791	1.059	1.313
10	0.151	0.307	0.466	0.574	0.760	1.000
11	0.078	0.160	0.240	0.302	0.399	0.496

Table 6 Values of M_x/P and M_y/P from proposed solution

Point	$\nu = 0.3$		$\nu = 0.1$	
	M_x/P	M_y/P	M_x/P	M_y/P
1	0.324	-0.119	0.338	-0.107
2	0.326	-0.002	0.334	-0.001
3	0.306	0.100	0.306	0.095
4	0.259	0.188	0.255	0.181
5	0.186	0.260	0.160	0.256
6	0.088	0.318	0.079	0.322
7	0.034	0.339	0.025	0.348
8	0.030	0.330	0.022	0.340
9	0.020	0.313	0.015	0.324
10	-0.014	0.254	-0.006	0.268
11	-0.039	0.212	-0.025	0.228

Table 7 Bending moments M_x and M_y by variation of load from proposed solution, $\nu = 0.3$

Load (kg)	3.125	7.125	11.125	14.125	17.125
Bending moment (N-m/m)					
M_{1x}	9.933	22.646	35.360	44.895	54.431
M_{1y}	-3.648	-8.318	-12.987	-16.489	-19.992
M_{2x}	10.055	22.926	35.797	45.450	55.103
M_{2y}	-0.061	-0.140	-0.128	-0.277	-0.336
M_{3x}	9.3[illegible]1	21.388	33.396	42.401	51.407
M_{3y}	3.066	6.990	10.913	13.857	16.780
M_{4x}	7.940	18.103	28.266	35.889	43.511
M_{4y}	5.763	13.140	20.518	26.050	31.583
M_{5x}	5.702	13.001	20.299	25.773	31.247
M_{5y}	7.971	18.173	28.375	36.027	45.679
M_{6x}	2.698	6.151	9.604	12.194	14.784
M_{6y}	9.749	22.227	34.705	44.064	53.423

Table 7 (continued)

Load (kg)	3.125	7.125	11.125	14.125	17.125
Bending moment (N-m/m)					
M_{7x}	1.042	2.376	3.711	4.711	5.712
M_{7y}	10.392	23.695	36.997	46.974	56.951
M_{8x}	0.920	2.097	3.274	4.157	5.040
M_{8y}	10.116	23.066	36.015	45.727	55.439
M_{9x}	0.613	1.398	2.183	2.771	3.360
M_{9y}	9.595	21.877	34.160	43.371	52.582
M_{10x}	0.153	0.349	0.546	0.693	0.840
M_{10y}	8.829	20.130	31.431	39.907	48.383
M_{11x}	-0.429	-0.979	-1.528	-1.940	-2.352
M_{11y}	7.787	17.754	27.721	35.196	42.671
M_{12x}	-1.196	- 2.726	-4.256	-5.404	-6.552
M_{12y}	6.449	14.818	23.137	29.376	35.615

Table 8 Strains at various point by variation of load

(experimentation)

Weight (kg)	5	10	15	20	25
Spring scale rdg. (kg)	2	3	4	6	8
strain $x10^6$					
Point 1x	18.75	41.00	65.00	82.25	100.00
1y	-13.00	-18.00	-28.50	-42.50	-68.00
2x	13.25	32.75	51.50	67.25	85.50
2y	-5.75	-13.00	-17.25	-20.75	-25.50
3x	9.50	33.00	54.5	72.00	88.00
3y	3.00	6.75	9.75	12.00	13.75
4x	14.00	33.00	52.00	70.00	84.00
4y	13.00	20.50	44.00	57.00	68.00
5x	5.00	20.00	38.00	50.00	62.00
5y	8.00	30.00	50.00	67.50	87.25
6x	-2.00	-3.75	-60.00	-7.00	-8.00
6y	17.00	43.00	70.00	89.75	113.00

Table 8 (continued)

Weight (kg)	5	10	15	20	25
Spring scale rdg. (kg)	2	3	4	6	8
strain $\times 10^6$					
point 7x	-3.75	-11.0	-13.00	-15.75	-21.25
7y	24.00	50.25	65.00	81.00	105.00
8x	-5.25	-9.50	-15.00	-19.75	-24.75
8y	24.50	41.00	67.50	87.00	108.00
9x	-6.0	-6.50	-10.25	-14.75	-17.25
9y	20.00	39.25	58.00	77.0	91.75
10x	-2.00	-7.00	-10.25	-14.75	-17.25
10y	13.25	31.00	48.25	61.00	73.75
11x	-4.0	-6.75	-9.50	-12.00	-15.00
11y	16.00	31.30	51.00	62.75	72.50
12x	-1.75	-5.00	-9.00	-11.75	-13.75
12y	9.25	21.75	35.25	46.25	59.00

Table 9 Bending moments at various point by variation of load (experimentation), $\nu = 0.3$

Load (kg)	3.125	7.125	11.125	14.125	17.125
Bending moment (N-m/m)					
M_{1x}	9.795	23.480	37.234	45.842	52.504
M_{1y}	-4.865	-3.750	-5.936	-11.757	-25.065
M_{2x}	7.602	19.029	30.556	40.252	51.350
M_{2y}	-1.171	-2.094	-1.187	-0.379	0.099
M_{3x}	0.860	23.112	37.848	49.866	60.756
M_{3y}	3.859	11.015	17.117	22.162	26.450
M_{4x}	11.807	27.406	43.006	57.451	68.862
M_{4y}	11.345	25.329	39.312	51.449	61.475
M_{5x}	4.881	19.128	34.959	46.337	58.160
M_{5y}	6.266	23.746	40.499	54.417	69.819
M_{6x}	2.045	6.035	9.894	13.143	17.004
M_{6y}	10.817	27.621	44.985	57.814	72.952

Table 9 (continued)

Load (kg)	3.125	7.125	11.125	14.125	17.125
Bending moment (N-m/m)					
M_{7x}	2.276	2.688	4.287	5.6[illegible]9	6.761
M_{7y}	15.088	30.969	40.302	50.311	65.053
M_{8x}	1.385	1.847	3.463	4.188	5.013
M_{8y}	15.121	25.164	41.555	53.477	66.33[illegible]
M_{9x}	0.000	3.479	4.716	6.167	6.777
M_{9y}	12.005	24.603	36.229	48.068	57.105
M_{10x}	1.303	1.517	2.787	2.342	3.216
M_{10y}	8.[illegible]44	19.062	29.600	37.317	45.2[illegible]
M_{11x}	0.528	1.541	0.826	4.502	4.452
M_{11y}	9.762	19.310	31.760	39.015	44.853
M_{12x}	0.676	1.006	1.0[illegible]9	1.402	2.605
M_{12y}	5.755	13.357	21.470	28.181	36.196

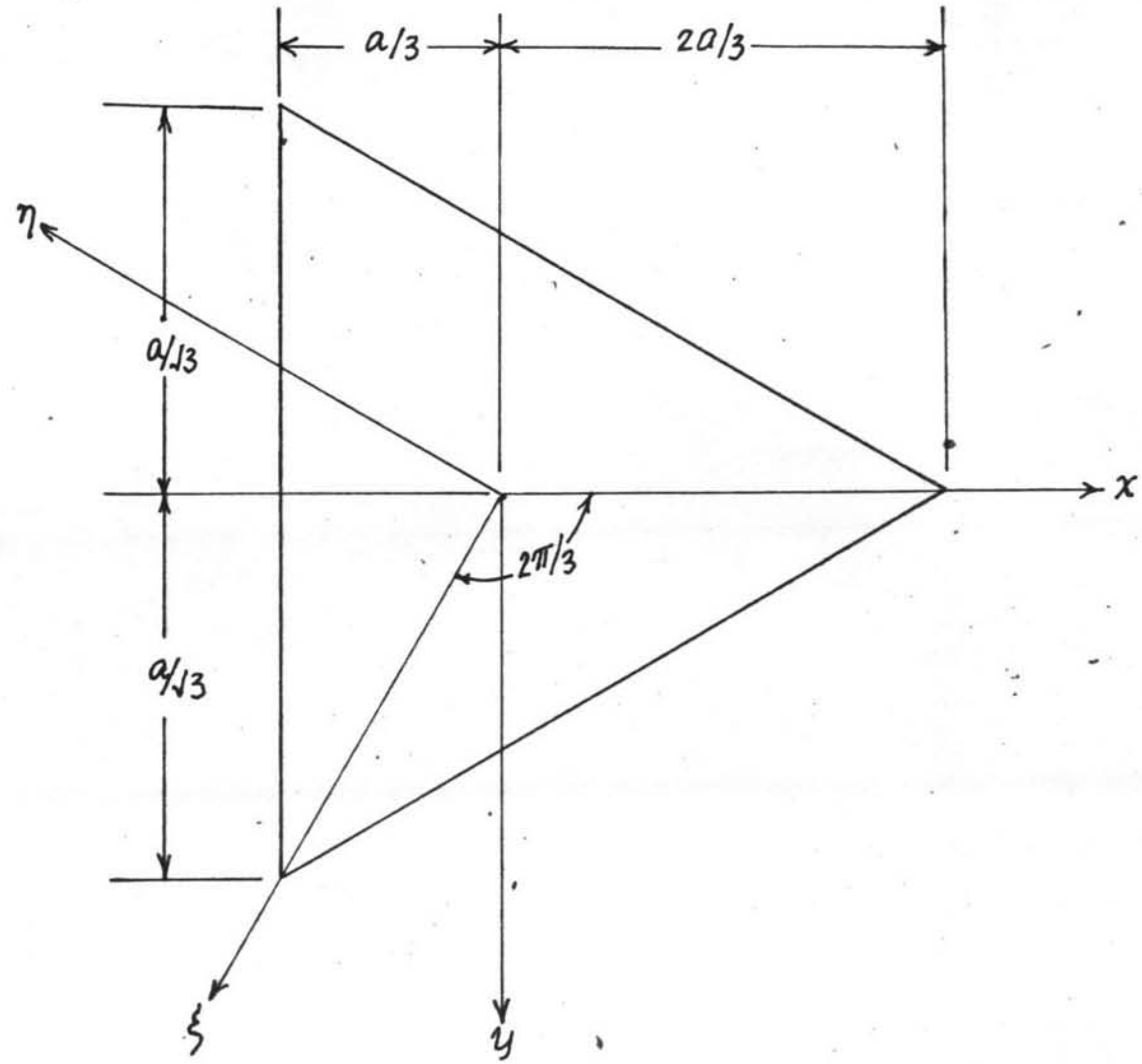


Fig. 1. Equilateral triangular plate

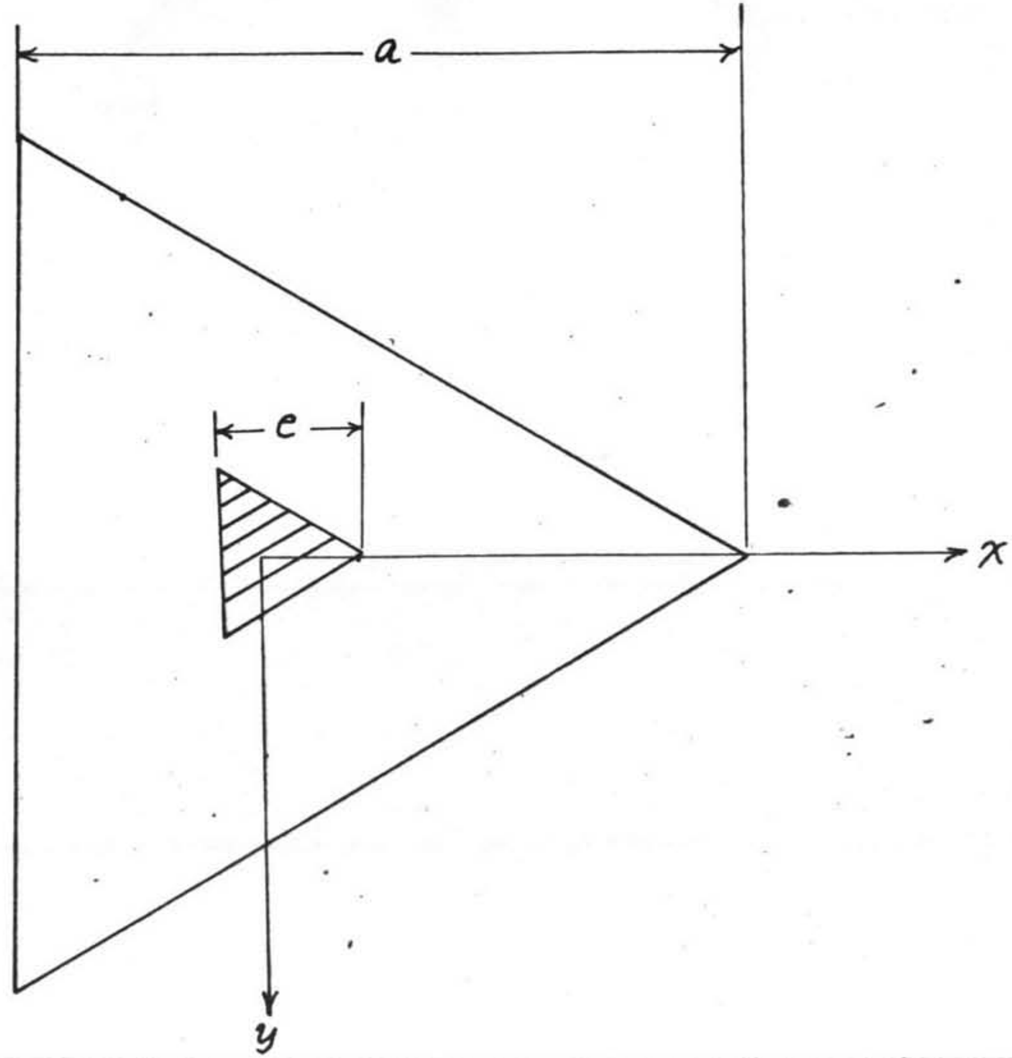


Fig 2. Triangular plate loaded with uniformly distributed load q (partially distributed load)

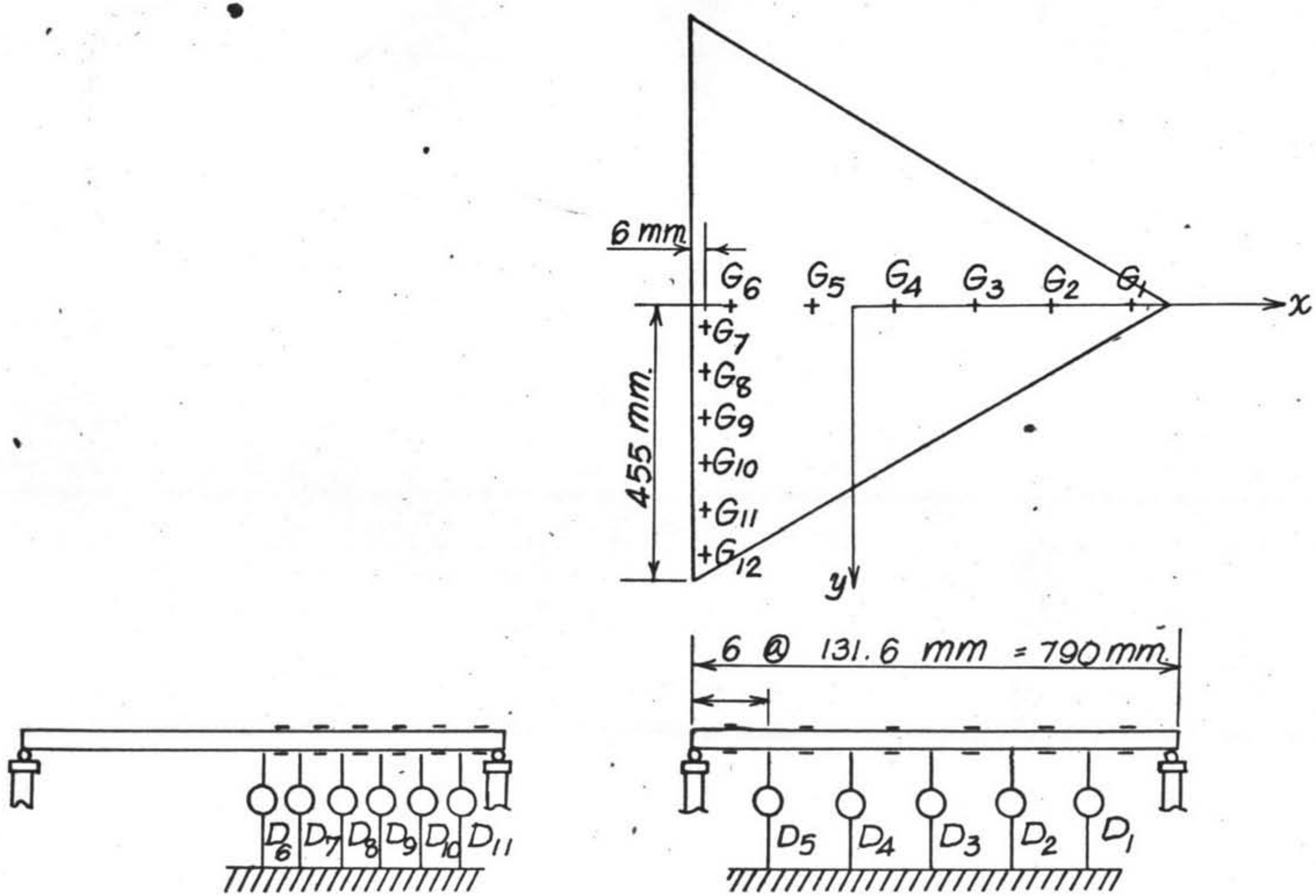


Fig. 3. Schematic Diagram of supports and instrumentations

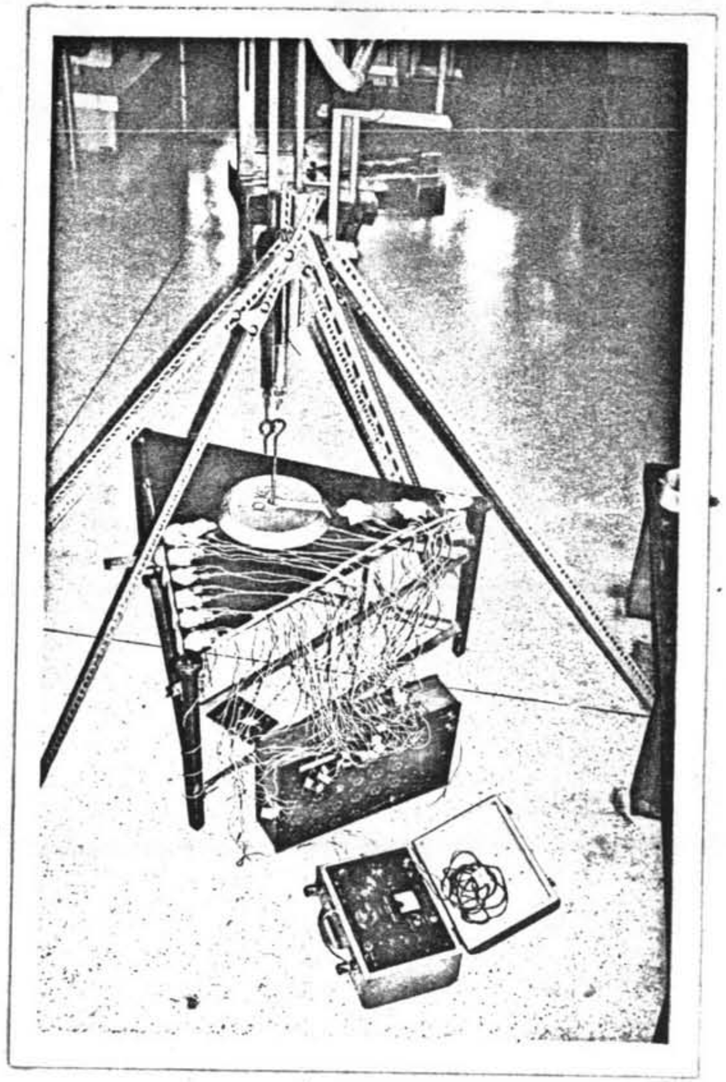


Fig. 4. Equipments used in the experimentation.

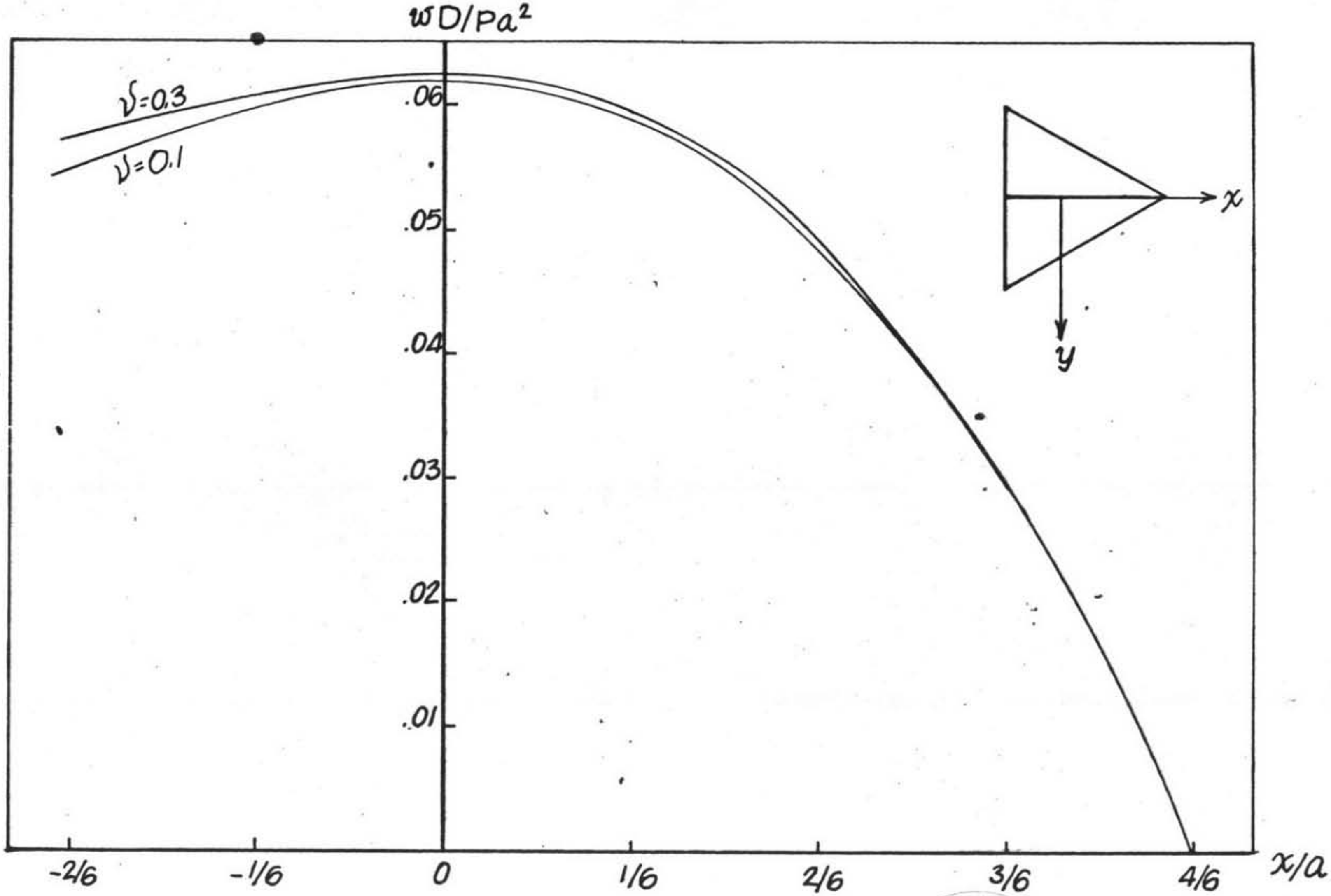


Fig. 5. Deflection plotted against x/a , $y/a \doteq 0$

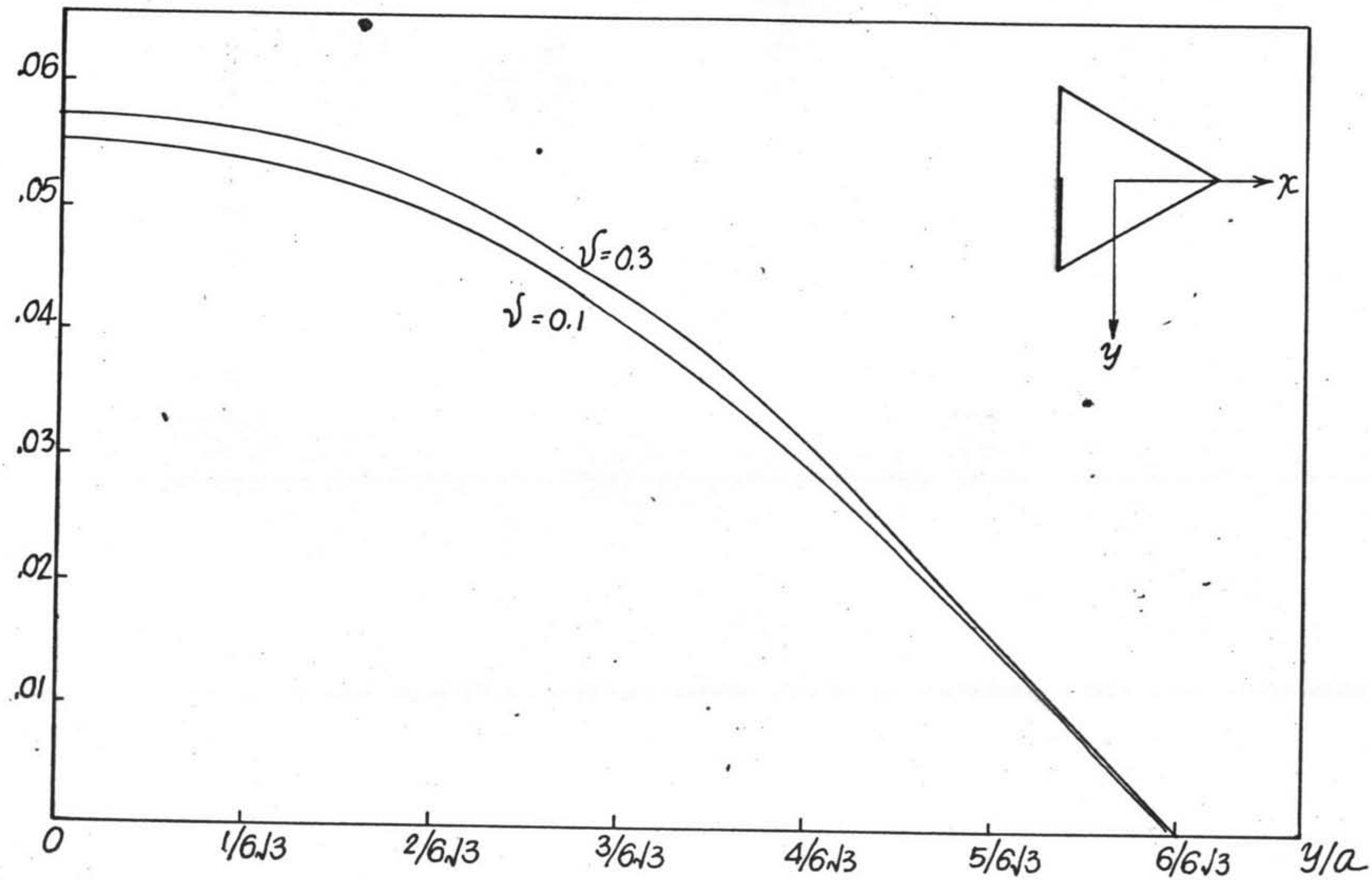


Fig. 6. Deflection plotted against y/a, $x/a = -1/3$

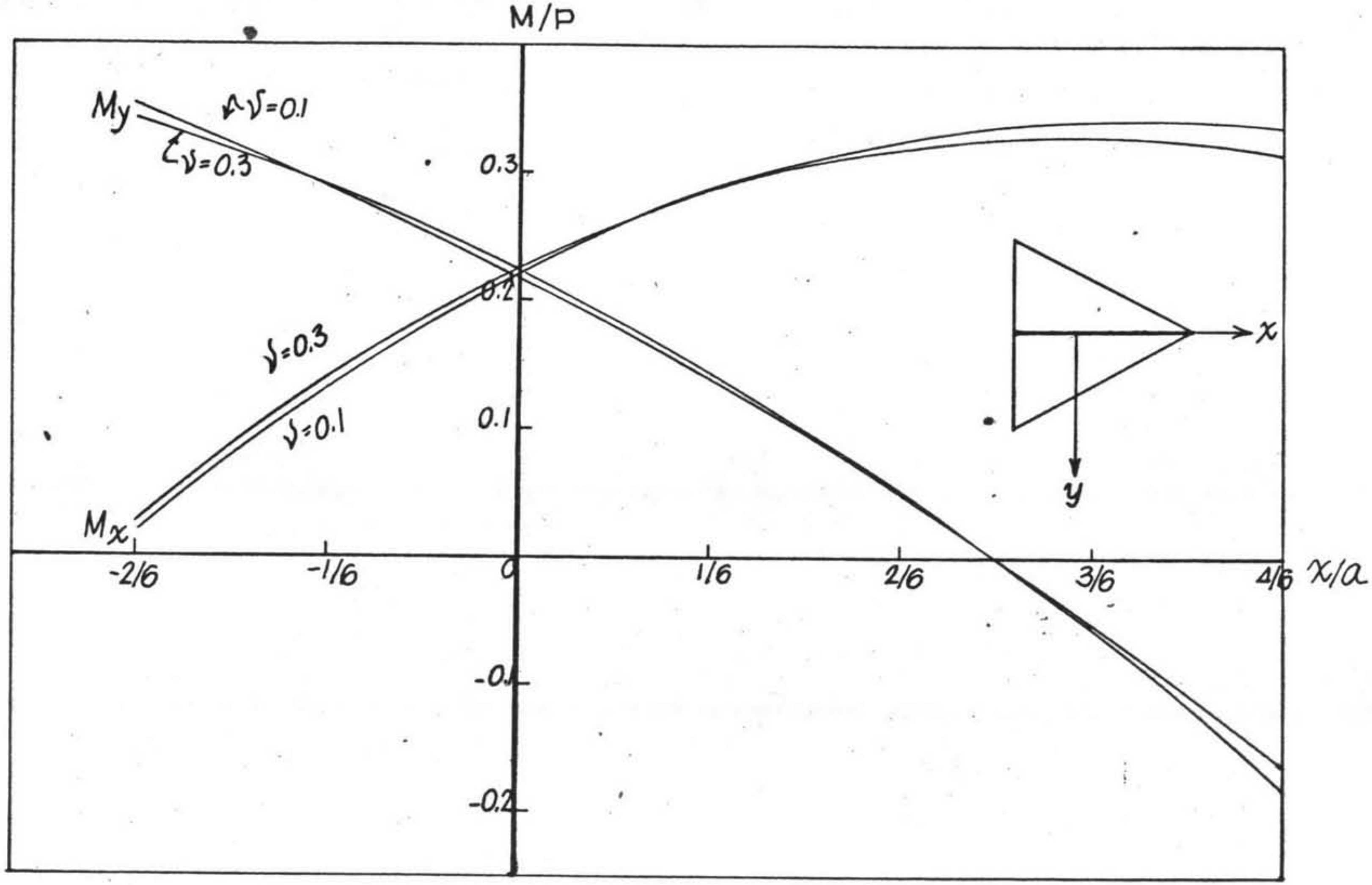


Fig. 7. Moment plotted against x/a , $y/a = 0$

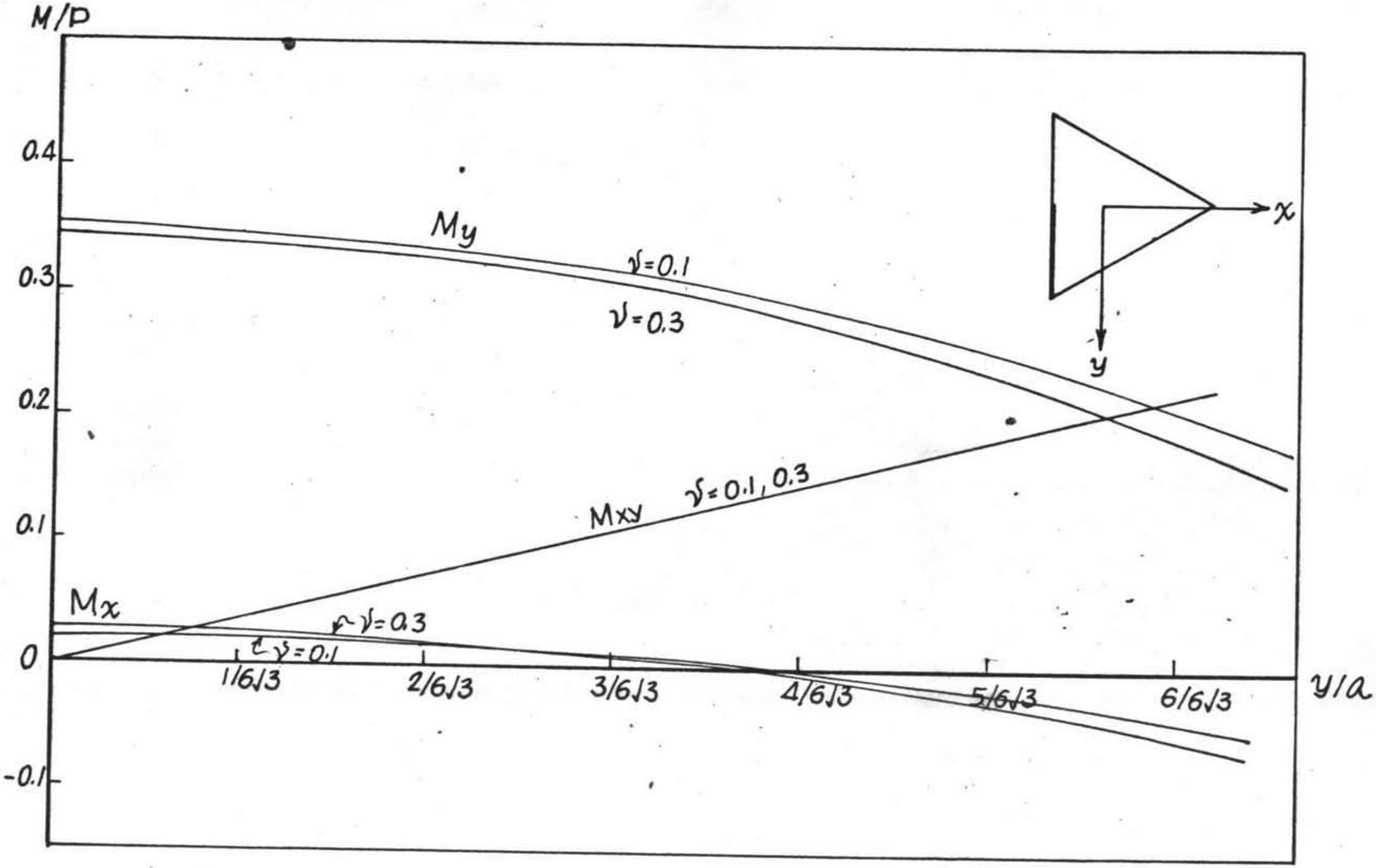


Fig. 8. Moment plotted against y/a, $x/a = -1/3$

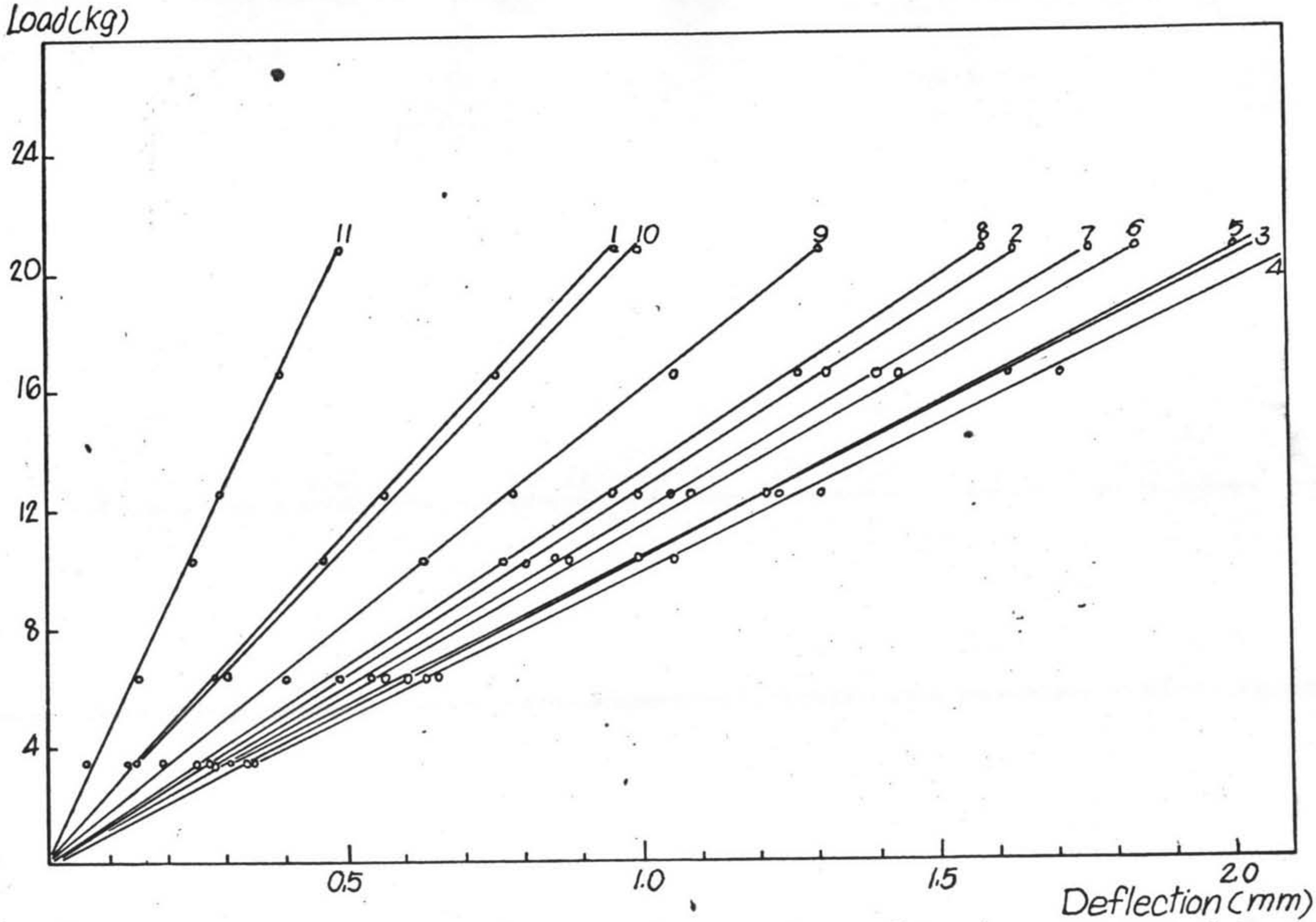


Fig. 9. Deflections at each point by variation of load. (experimentation)

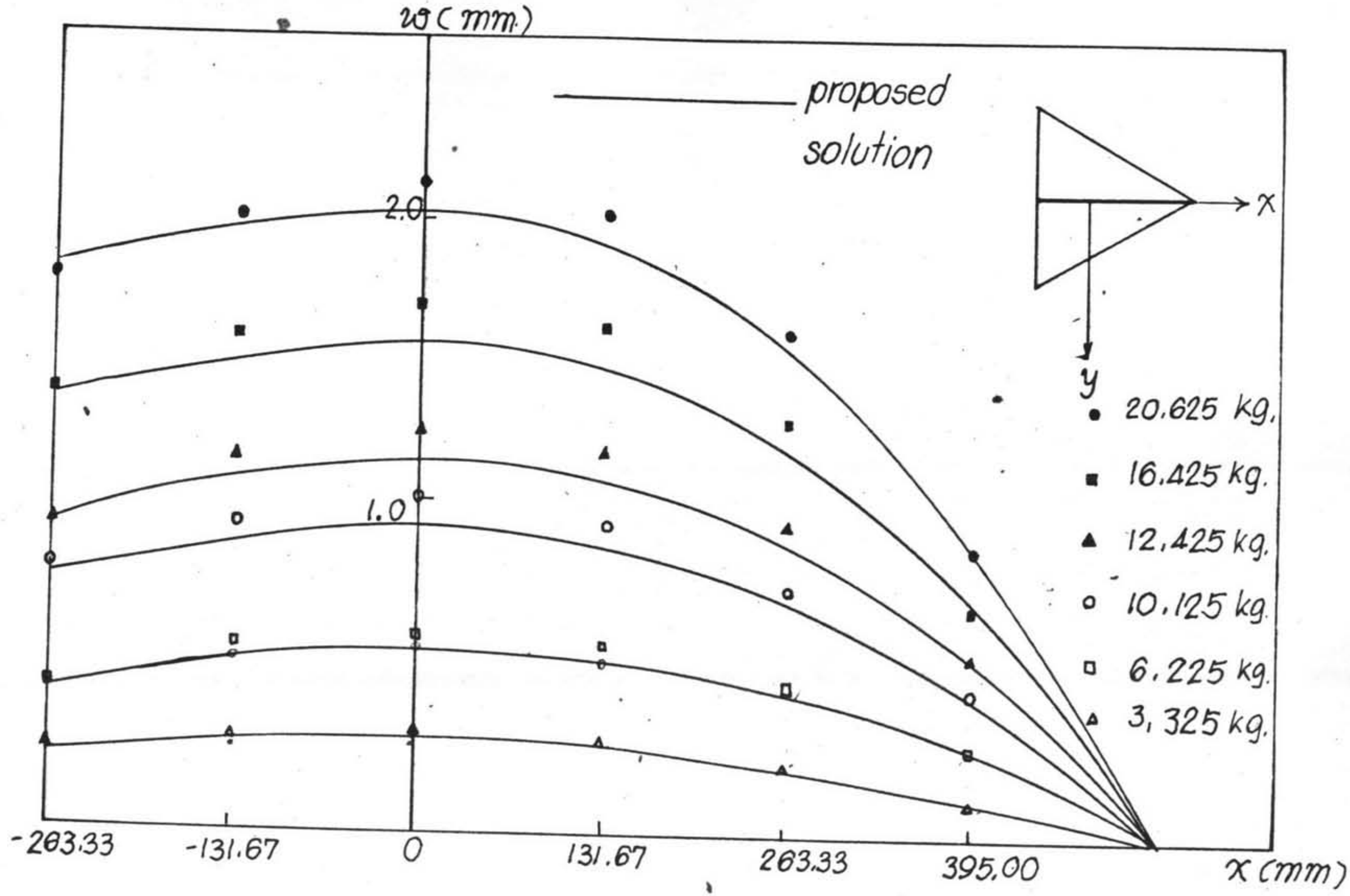


Fig. 10. Deflection plotted against x, $y=0$

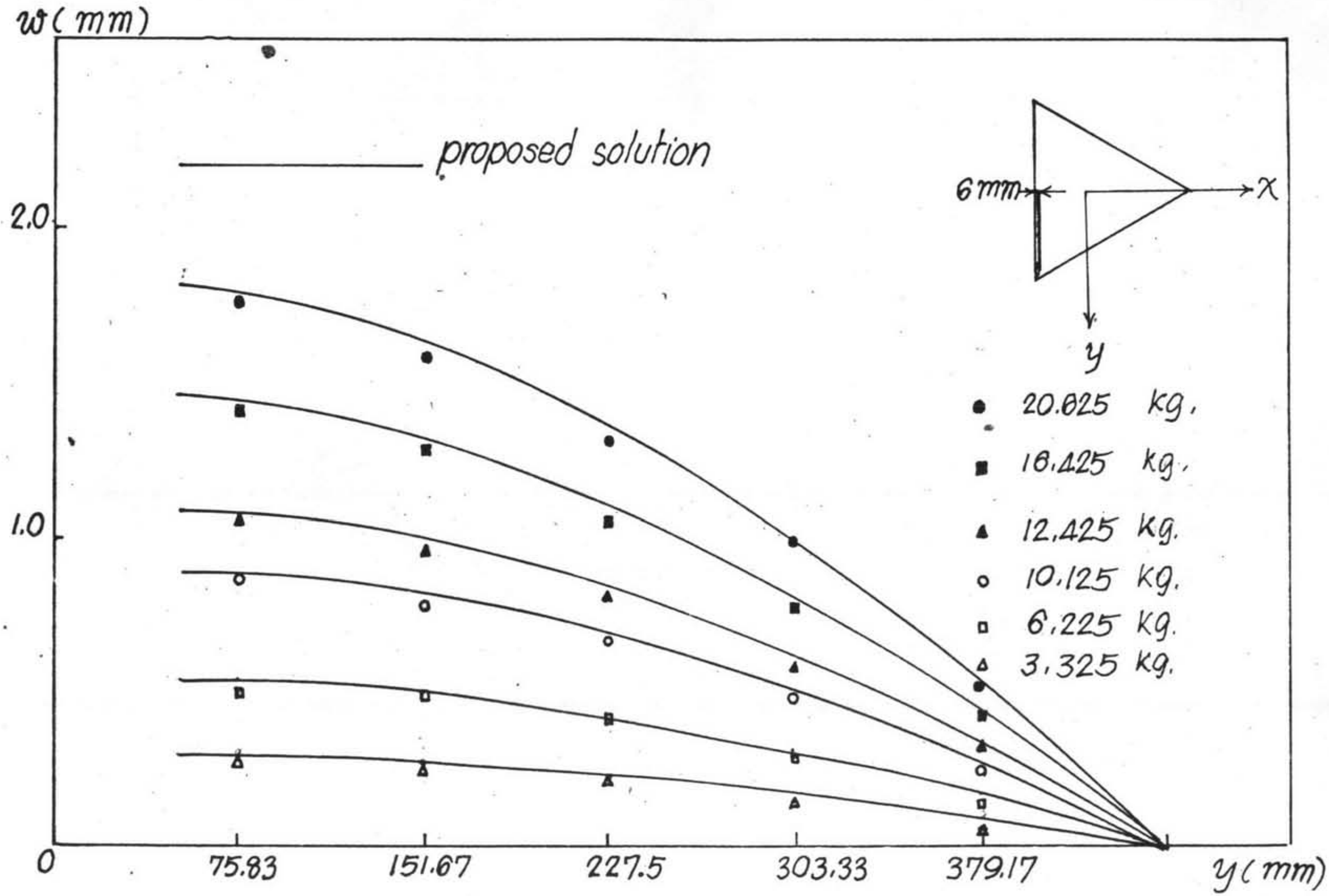


Fig. 11. Deflection plotted against y, x = -257.33 mm.

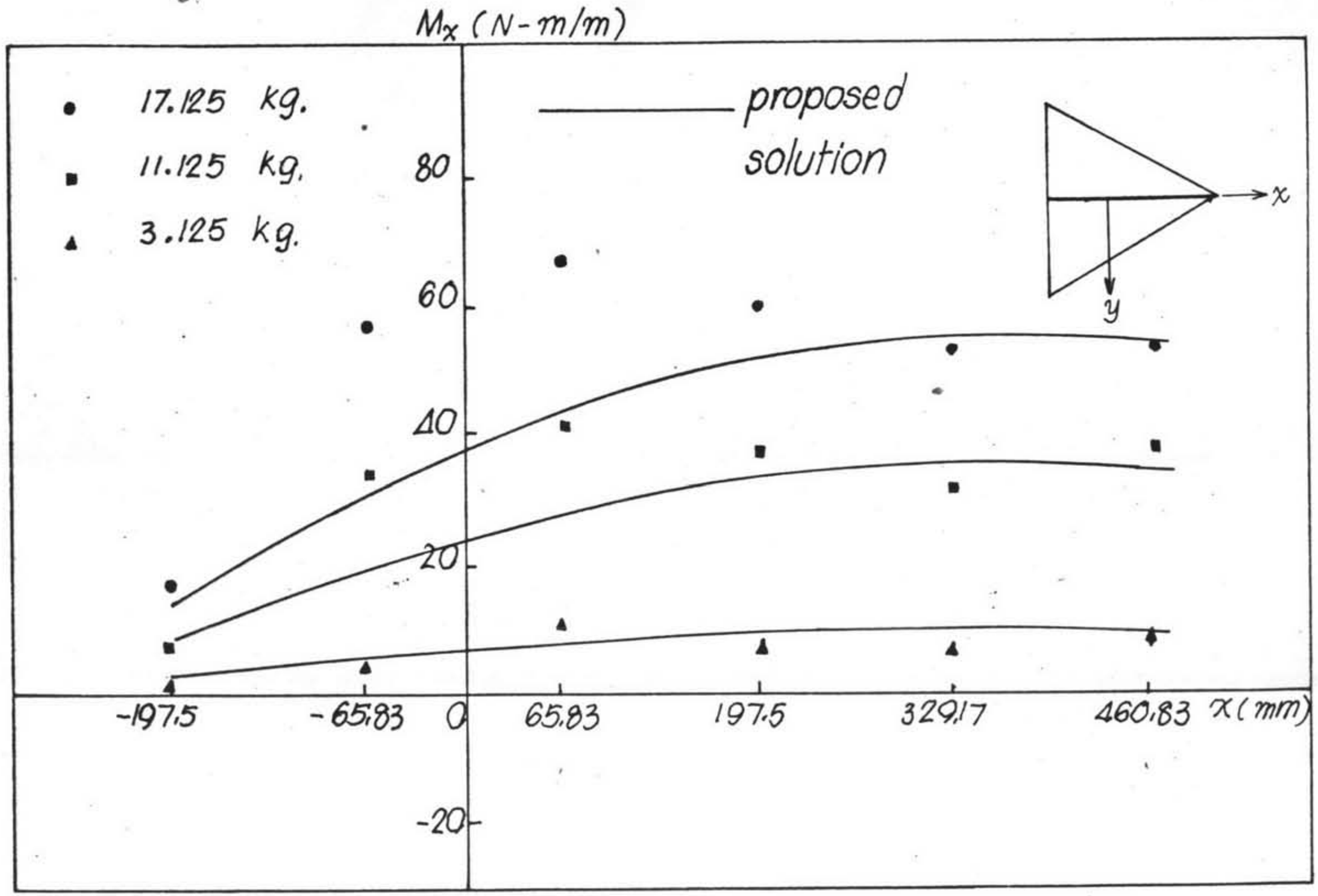


Fig. 12. Moment (M_x) plotted against x, $y = 0$

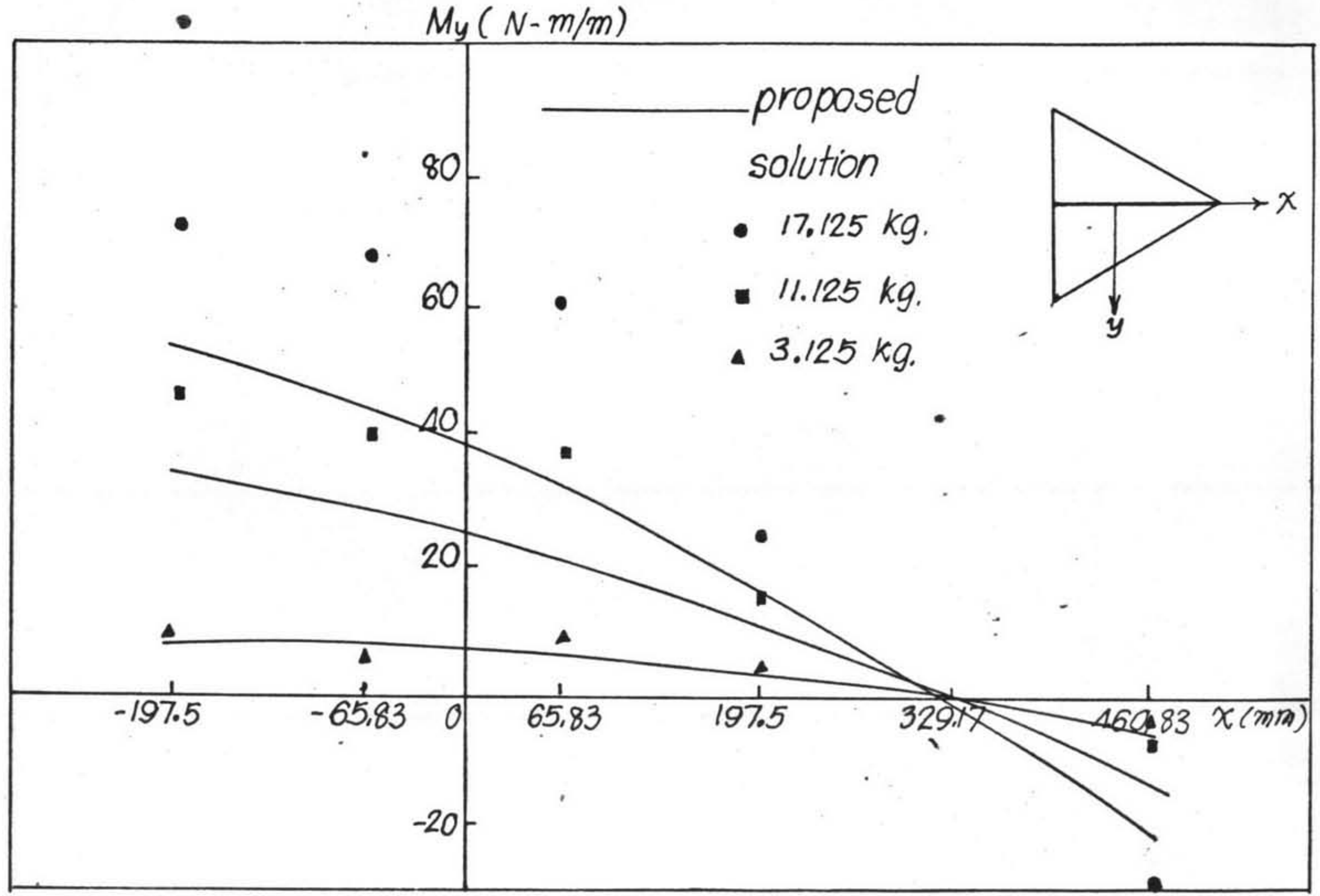


Fig. 13. Moment (M_y) plotted against x, $y=0$

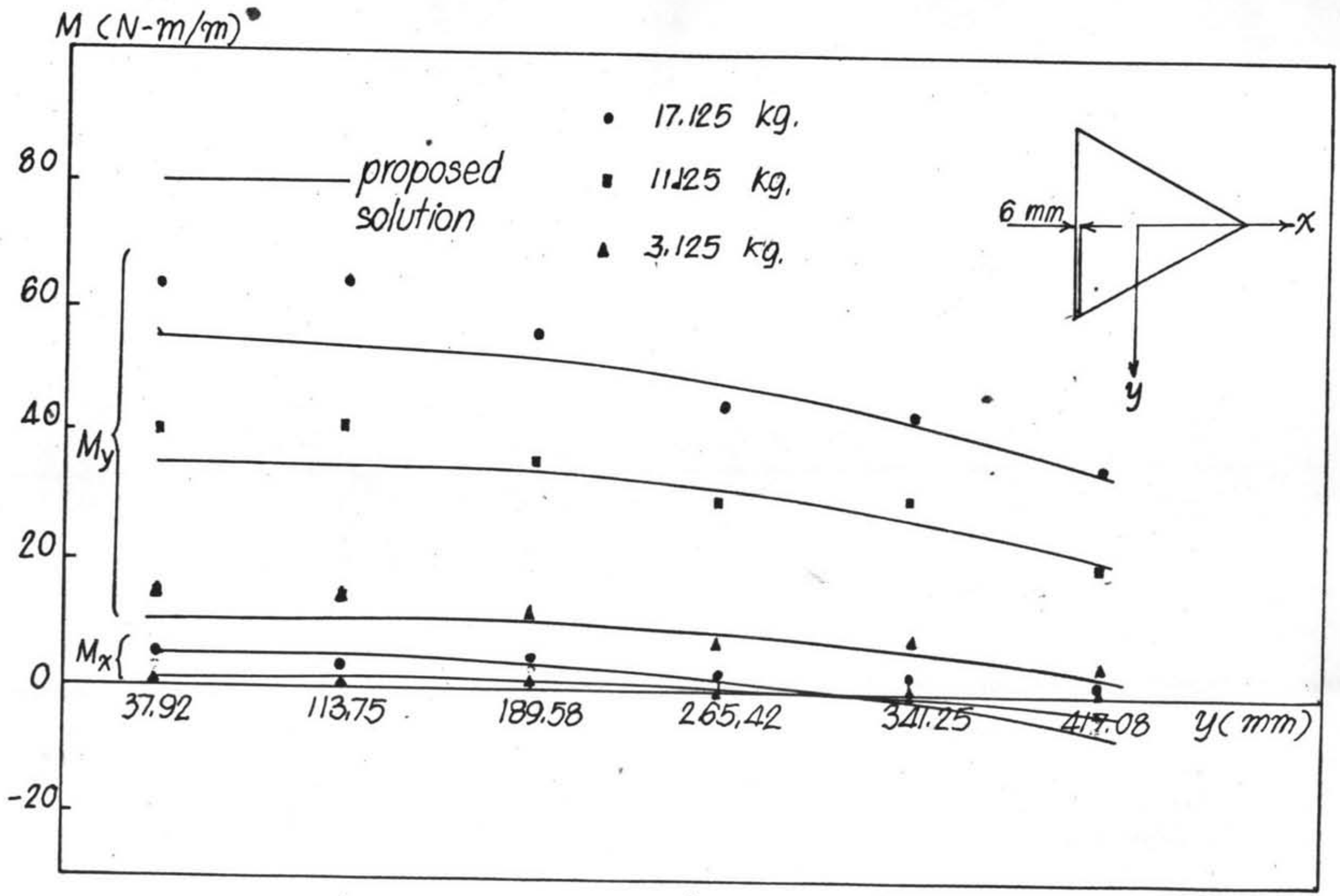


Fig. 14. Moment plotted against y, x = -257.33 mm.

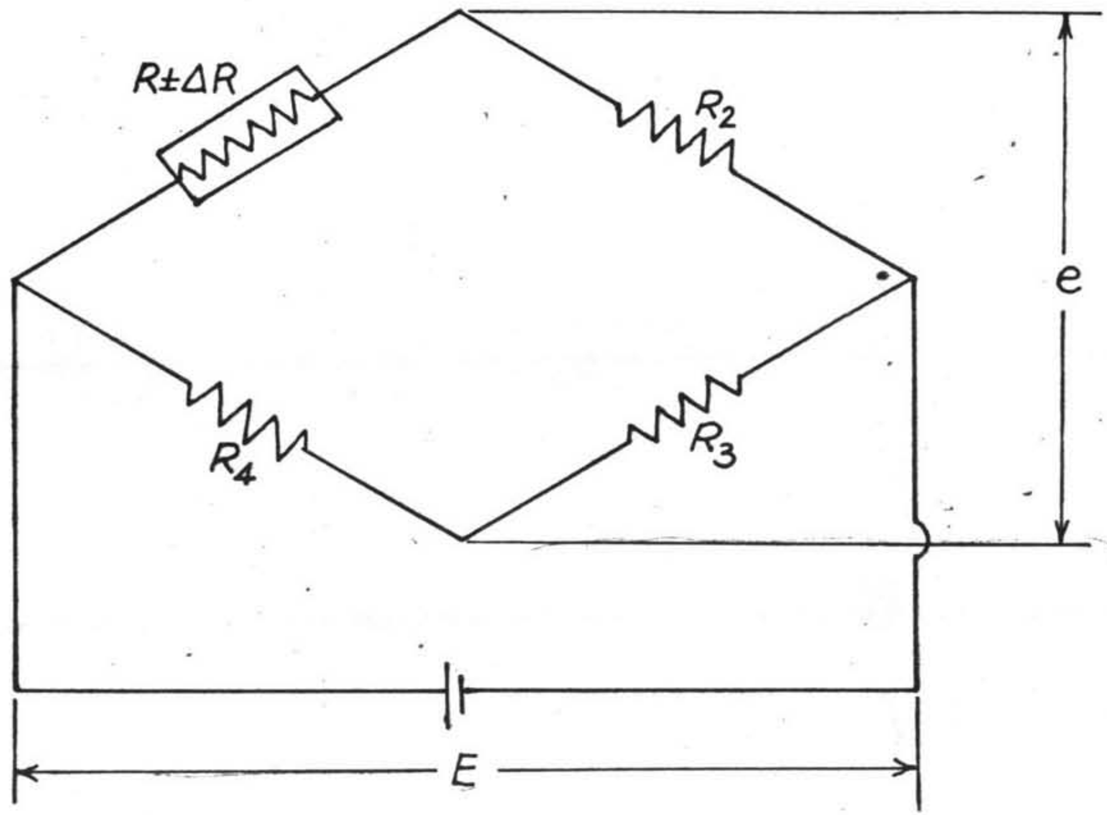
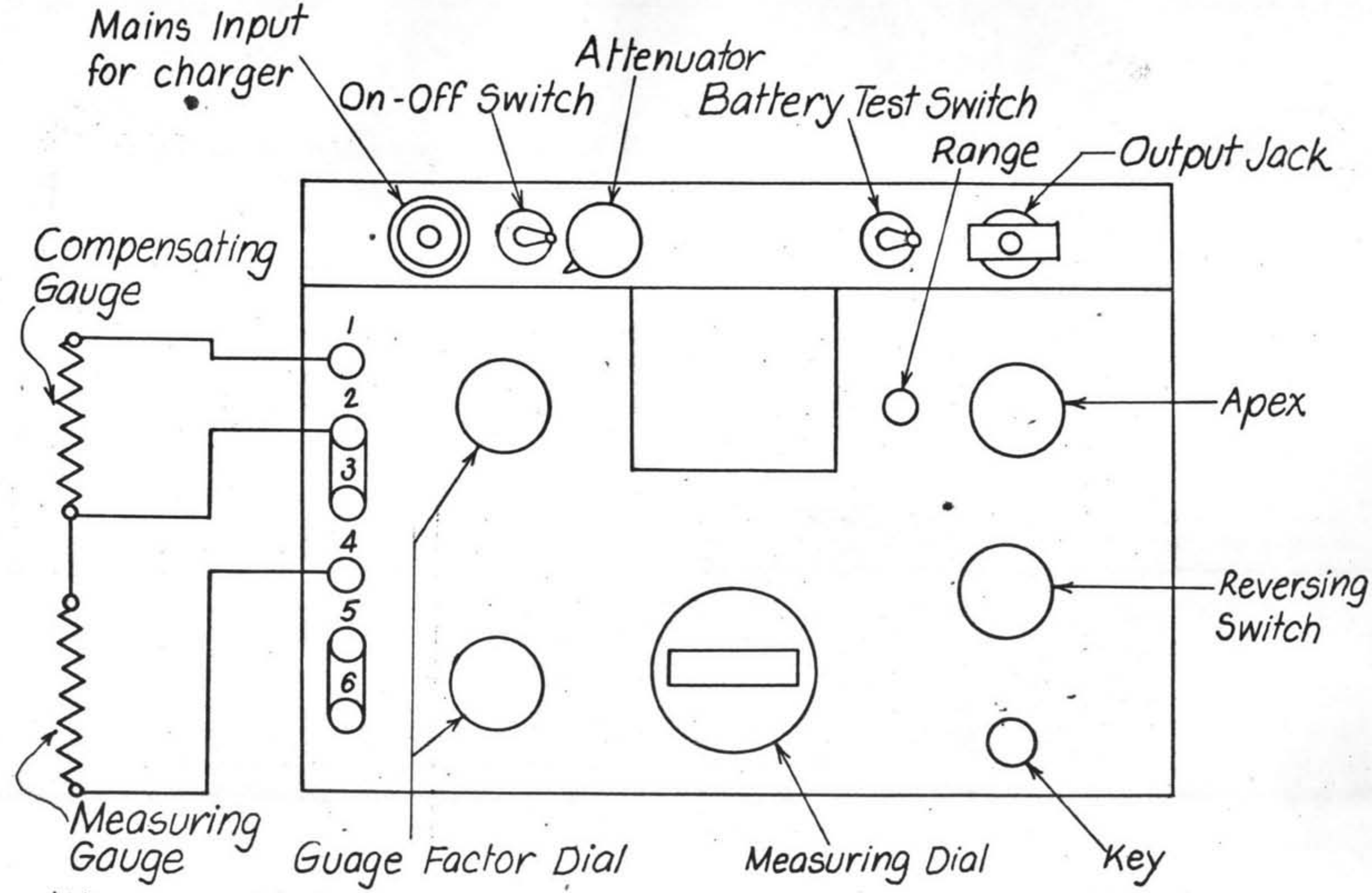
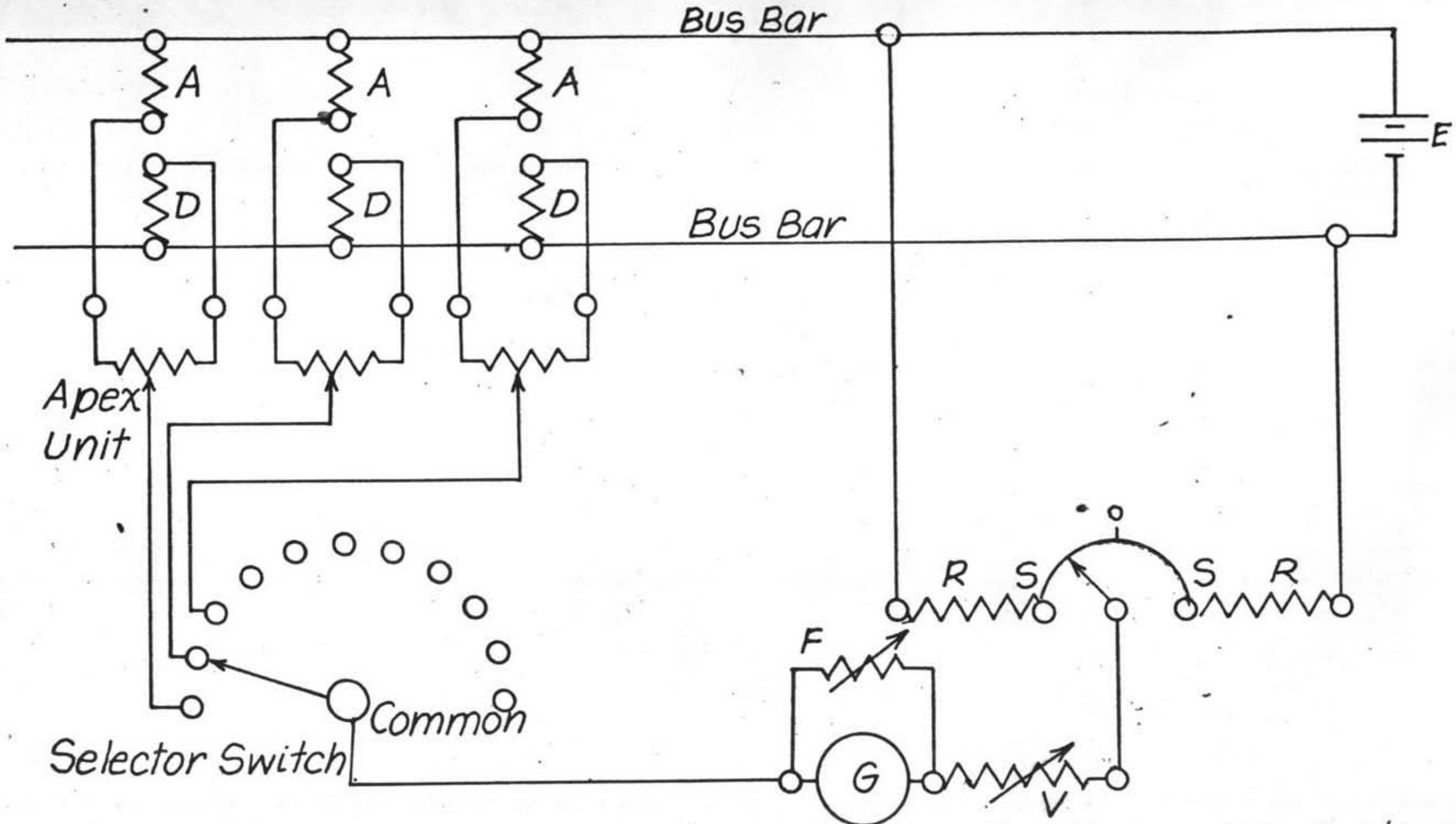


Fig. 15. Wheatstone bridge circuit



NOTE: When used with external Apex Unit open link between 2 and 3

Fig. 16. Strain Gauge Bridge connections for Two Arm Bridge



A = Active gauge , D = Dummy Gauge , G = Galv. , RR = Bridge ratio

E = Gauge resistance × 0.04 volts approx. = 4 V for 100 Ω gauges

F = Galv. shunt in Bridge. SS = Bridge slidewire.

V = Series sensitivity control (if used)

Fig. 17. Schematic Wiring Diagram.

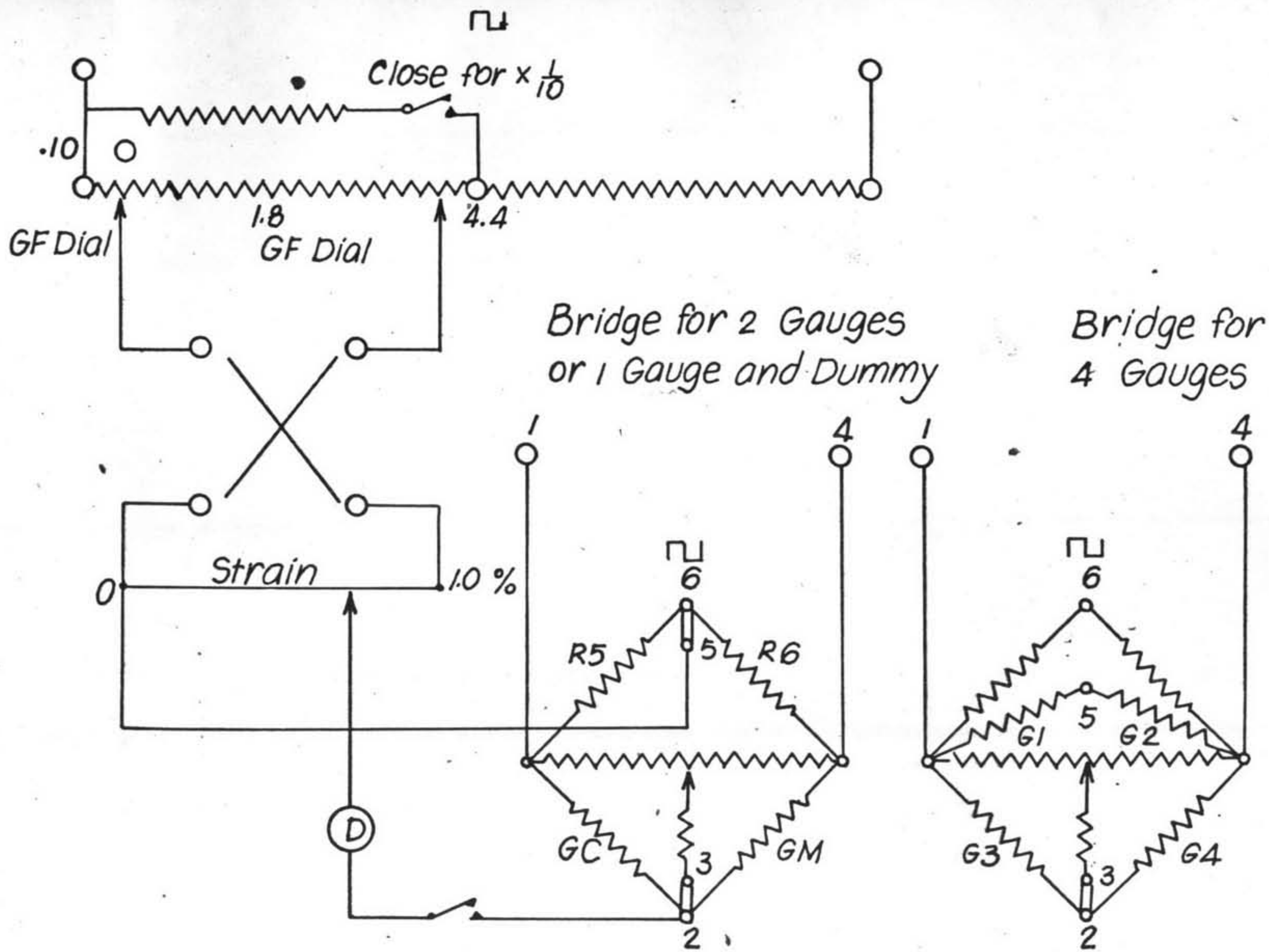


Fig. 18. Strain Gauge Bridge Type 5580