CHAPTER II

PHOTOELASTIC THEORY

Fundamental Optical Laws of Photoelasticity

Almost all transparent materials such as glass, Celluloid, Bakelite, and many other synthetic resins temporarily have to some extent the same optical effect on a beam of light as a crystal when these materials are subjected to stress. For normal incidence on flat plates subjected to plane stress within the elastic limit, the transmission of light obeys two laws which form the basis of photoelastic stress determination.

(a) The light is "polarized" in the directions of the principal stress axes and is transmitted only on the plane of principal axis.

(b) The velocity of transmission in each principal plane is dependent on the intensities of the principal stresses in these two planes and obeys the following equations which have been simplified, by Hetenyi[9], from the general case to terms of plane stress

$$\delta_1 = N_1 - N_0 = K_a \sigma_1 - K_b \sigma_2 \qquad (1)$$

$$\delta_2 = N_2 - N_0 = K_b \sigma_1 - K_a \sigma_2 \qquad (2)$$

By subtracting, one finds

$$\delta_1 - \delta_2 = N_1 - N_2 = (K_a - K_b)(\sigma_1 - \sigma_2)$$
$$= k(\sigma_1 - \sigma_2) \qquad (3)$$

where k is the differential - stress optical constant. This relation may be expressed in terms of the velocity of transmission of the light, as

$$\frac{V}{V_1} - \frac{V}{V_2} = \frac{V(V_2 - V_1)}{V_1 V_2} = k(\sigma_1 - \sigma_2) \qquad (4)$$

Thus, the difference in velocities of transmission $(V_1 - V_2)$ and the resulting phase difference is seen to be directly related to the difference of the two principal stress $(\sigma_1 - \sigma_2)$

The Plane Polariscope and the Photoelastic Effect

The device or optical system most frequently employed to produce the necessary polarized beam of light and to interpret the photoelastic effect in terms of stress is called a "polariscope." In general, it consists of a light source, a polarizing device called the polarizer, the photoelastic model, and a second polarizing device known as the analyzer. In addition, there may be a system of lenses, a viewing screen, and other adjuncts for convenient visual observation or photographic recording.

The relation between the optical effects and the stress prevailing in the model may be illustrated by analyzing the passage of light through a plane polariscope. Although this is the simplest case, the corresponding analysis for other more complicated arrangements of the polariscope can be made in a similar manner.

Fig. 2-1.A shows diagramatically how light directed from the source is plane - polarized by the polarizer which is usually a Polaroid disk, then resolved by the model into two components in the direction of the principal stress axes, and transmitted on the principal planes. If the principal stress intensities are not equal, then the optical properties on the two principal planes will be different, and the velocity of transmission on one principal plane will be greater than on the other, as indicated by eq. (1) to eq. (4). This result in a phase difference between the two component vibrations as they emerge from the model.

This phase difference is proportional to the difference between the principal stresses and is measured by introducing the analyzer which brings part of each component vibration into interference in a single plane. Since white light consists of many wavelengths, each of which will be influenced in a similar manner, the analysis will be made on the basis of monochromatic light using the very simplest form of mathematical representation.

Assume a source of monochromatic light as shown in Fig. 2-1.A, and investigate the effect produced as the light passes,

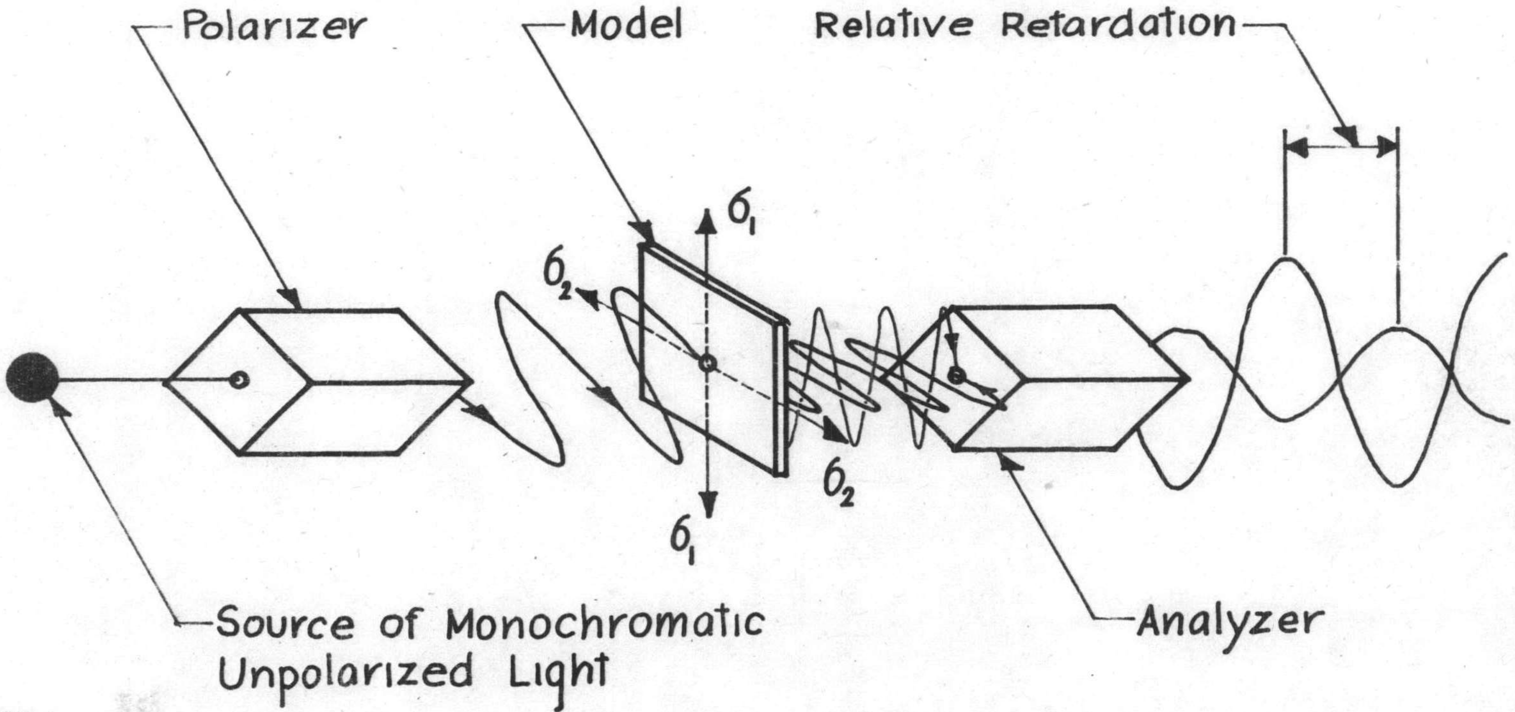

FIG. 2-1.A MODEL IN A PLANE POLARISCOPE

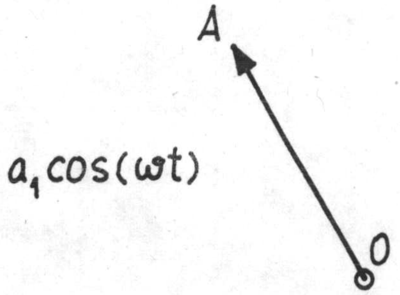

FIG. 2–1.B LIGHT LEAVING POLARIZER

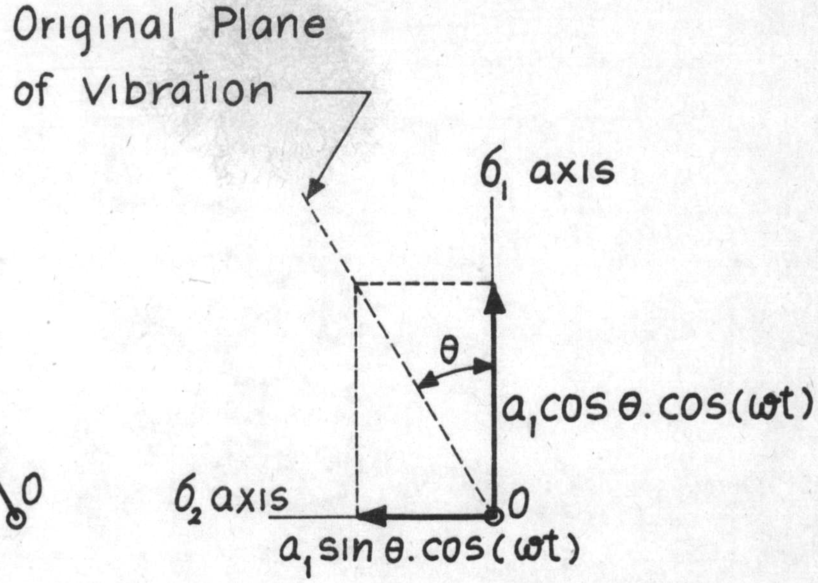

FIG. 2–1.C ENTERING THE MODEL

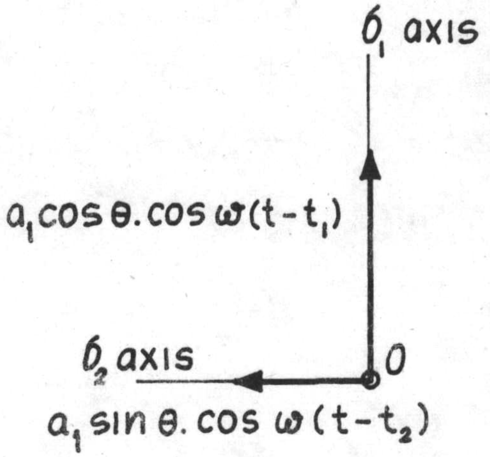

FIG. 2–1.D LEAVING THE MODEL

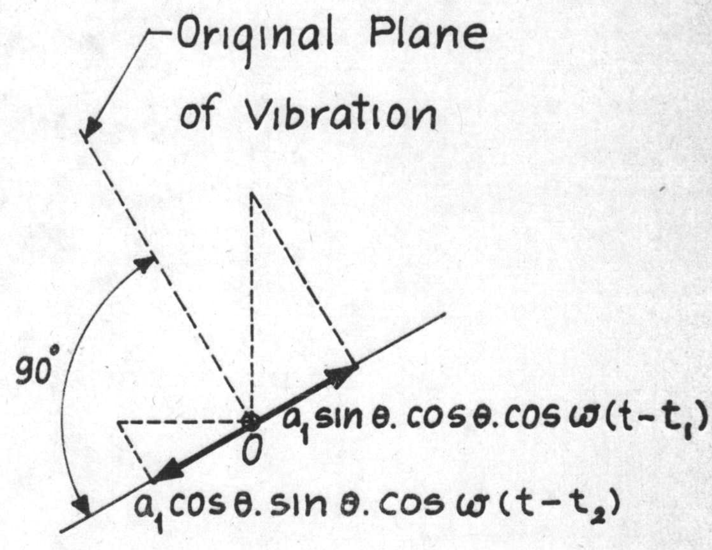

FIG. 2–1.E LEAVING THE ANALYZER

FIG. 2–1 LIGHT THROUGH A PLANE POLARISCOPE at normal incidence, through a point in the photoelastic model. When the polarizer has been traversed, the vibration has been confined to a single plane in the direction of and with amplitude proportional to OA, Fig. 2-1.B. In symblos this is represented by the simple equation

$$S_0 = a_1 \cos \omega t \tag{5}$$

When the light arrives at the model, in general, its plane of vibration will not coincide with either principal plane of stress. Therefore, since the stressed model only transmits light on the principal planes, the original vibration is immediately resolved into two components as it enters the model. These will be

$$a_1 \cos \theta \cos \omega t \tag{6}$$

and

$$a_1 \sin \theta \cos \omega t \tag{7}$$

which are parallel to the no. 1 and no. 2 principal planes, respectively, where θ is the angle between the original plane of vibration and the no. 1 principal plane, see Fig. 2-1.C

Now, if t_1 and t_2 represent the time required for transmission on the no. 1 and no. 2 principal planes, respectively, then the two components vibrations leaving the model will be represented by the equations

$$a_1 \cos \theta \cos \omega (t - t_1) \tag{8}$$

and

$$a_1 \sin \theta \cos \omega (t - t_2) \tag{9}$$

This will be observed to have a phase difference, $\omega (t_1 - t_2)$ which can be shown to be proportional to the difference between the principal stresses σ_1 and σ_2

If h represents the thickness of the photoelastic model along the path of the light, then

$$t_1 = \frac{h}{V_1} \quad \text{and} \quad t_2 = \frac{h}{V_2} \tag{10}$$

whence

$$t_1 - t_2 = h \frac{V_2 - V_1}{V_1 V_2} \tag{11}$$

and, by subtracting for the velocities the value from eq. (4), we have

$$t_1 - t_2 = \frac{hk}{V} (\sigma_1 - \sigma_2) \tag{12}$$

Therefore, the phase difference, $\omega (t_1 - t_2)$, of the waves emerging from the model is seen to be directly proportional to the difference between the principal stresses $(\sigma_1 - \sigma_2)$; the phase difference is also proportional to the model thickness h and to the optical constant k/V for the material and surrounding medium. Hence, any method that can be employed to determine this phase difference can be used as a measure of the difference between the principal stresses.

By introducing the analyzer in the system, being like Fig. 2-1.A, in the proper orientation, the phase difference of the two waves can be made evident by the interference effects of their components in the plane of the analyzer; the amplitude of the resultant vibration is a function of $\omega\sigma(t_1 - t_2)$. If the analyzer's plane of transmission is at right angle to that of the polarizer, the components of the two vibrations emerging from the model that will be transmitted by the analyzer may be represented by

$$a_1 \cos\theta \sin\theta \cos \omega\sigma(t - t_1) \tag{13}$$

and

$$a_1 \sin\theta \cos\theta \cos \omega\sigma(t - t_2) \tag{14}$$

both of which have the same amplitude. Since the two vibrations lie in the same plane, they may be added algebraically, or subtracted arithmatically since the vectors are opposed in directions, to give the expression for the resultant vibration

$$a_1 \cos\theta \sin\theta \left[\cos\omega\sigma(t-t_1) - \cos\omega\sigma(t-t_2)\right]$$

or

$$a_1 \sin 2\theta \sin\omega\sigma\left(\frac{t_1-t_2}{2}\right) \sin \omega\sigma\left(t - \frac{t_1+t_2}{2}\right) \tag{15}$$

Thus, the amplitude of the resultant vibration leaving the analyzer is a function of both the angle θ and the phase difference $\omega\sigma(t_1 - t_2)$, and, hence, it is influenced by the directions of the principal stresses and by the difference between the principal stresses at the given point in the model.

The intensity of light transmitted through any given point in the model is proportional to the square of the amplitude of vibration, and a dark spot will be observed on the model's image for every point at which

$$a_1 \sin 2\theta \sin \omega\left(\frac{t_1 - t_2}{2}\right) = 0 \qquad (16)$$

Such dark points are, in general, linked together to form loci representing one of two conditions, namely : (a) loci of constant stress direction called "isoclinics", when $\theta = 0^\circ$ or 90°, or (b) loci of constant difference ($\sigma_1 - \sigma_2$) between the principal stresses and referred to as "isochromatics", for those cases in which $\omega\left(\frac{t_1 - t_2}{2}\right) = 0^\circ$ or 180° etc.

It is noted that "a_1" which represents the amplitude of light source never be zero.

<u>Isoclinic Lines</u>

For a first condition let us assume that $\sin \omega\left(\frac{t_1 - t_2}{2}\right)$ is not zero. Then if $\sin 2\theta = 0$, θ must be 0° or 90°. This means that, if the planes of transmission of the polarizer and analyzer are parallel to the directions of the principal planes of stresses, on the image of the model there will be dark spots corresponding to all points at which the principal - stress directions coincide with the planes of transmission of the palarizer and analyzer. Such points are linked up and form a locus of all points having the same directions for the principal stresses, that is, with a dark field arrangement, an isoclinic line will be represented by a black rather broad line on the image of the model.

By adjusting the orientation of the polarizer and analyzer, but always locking them, respectively, at right angles, the isoclinic line corresponding to any desired stress inclination may be determined.

White light is generally used when tracing the isoclinic lines so that the black bands may be readily distinguishable from the colored isochromatics which remain stationary as the polarizer and analyzer are rotated.

Isochromatic Lines

This time assume that both a_1 and sin 2θ are other than zero. Under these conditions, at a point in the photoelastic model, no light will be transmitted through a plane polariscope when

$$\sin \omega \left(\frac{t_1 - t_2}{2}\right) = 0 \tag{17}$$

in which case a dark spot will be shown on the image. This condition will prevail for all points at which

$$\omega \left(\frac{t_1 - t_2}{2}\right) = 0 \quad \text{or} \quad n\pi \tag{18}$$

where n is any integer.

Conversely, a maximum intensity of transmitted light will take place at all points for which

$$\omega \left(\frac{t_1 - t_2}{2}\right) = \frac{\pi}{2} \quad \text{or} \quad (n + \tfrac{1}{2})\pi \tag{19}$$

In general, all points of a model having constant retardation $\omega\vartheta\,(t_1 - t_2)$ form a continuous band or line. Thus, a dark line or locus appears on the image on the model for each value of n in eq. (18), and, similarly, a bright band or locus appears for each value of n in eq. (19). When examine in white light the various fractional orders of retardation are each made evident by a brilliant band of a particular color or hue, and hence, they have been designated by the name "isochromatics". The alternate bright and dark lines formed in monochromatic light are also isochromatics and are distinguihed from one another according to the value of n ; consequently, they are often referred to as the isochromatic of zero, first, second order of interference and so on.

By comparing eq. (12) and eq. (18) or eq. (19) it may be observed that, since

$$\omega\vartheta\left(\frac{t_1 - t_2}{2}\right) \text{ is proportional to } (\sigma_1 - \sigma_2) \qquad (20)$$

the order of interference is, therefore, directly proportional to the difference between the principal stresses; consequently, the isochromatic line may be defined as the locus of all points having a constant value for the difference between the two principal stresses.

From eq. (12) this may be written in the form:

$$\frac{\sigma_1 - \sigma_2}{2} = \frac{V\,(t_1 - t_2)}{2\,kh} = Fn \qquad (21)$$

in which n is the fringe order and F is a constant for any particular model and light source. This constant F is defined by Frocht[10] as the "model fringe value" and is given by

$$F = \frac{V\pi}{kh\omega} = \frac{f}{h} \qquad (22)$$

where f is another constant equal to $V\pi/k\omega$.

The fringe order n is of utmost important in photoelasticity since the maximum shear at any point, $(\sigma_1 - \sigma_2)/2$, is directly proportional to it, and it must be known before the numerical value of the shear corresponding to any point can be determined.

The simplest and most direct way to determine the fringe order is by carefully observing the formation of the stress pattern during repeated slow loading and unloading of the model placed in the polariscope.

The Standard Plane Polariscope

This is the simplest form of instrument, and it can be used to determine the following quantities in the photoelastic model:

(a) The directions of the principal stress at all points. i.e., isoclinics.

(b) The maximum shear stress at all points, i.e., isochromatics.

(c) The individual values of the principal stresses along free boundary where the directions are either normal or tangent to the edge.

The polarizer and analyzer may be cartooned as a pair of slots set at right angles to each other, each of which will pass only those components of the light wave which are parallel to the slot. Thus, with no specimen in the polariscope, and the transmission planes of the polarizer and analyzer at 90°, the light is completely extinguished. When a stressed model is placed in the field, however the plane polarized light passing through the first "slot" is broken up into two rays, the planes of which coincide with the directions of principal stresses in the model, as illustrated in Fig. 2-1. The rays in these new planes may have components parallel to the "slot" in the analyzer, and thus some light is transmitted through the analyzer and produces the interference fringes, or isochromatic color effects in white light, on the screen. At some points in the model one of the planes of principal stresses will coincide with the plane of polarization of the incident light passed through the first "slot". At these points the light is not rotated or split up in passing through the model and will be completely extinguished by the analyzer, leaving black spots on the image of the model. In general, the stress directions in a loaded member vary continuously so that these dark points will form one or more continuous lines on the image. These lines are known as isoclinics or loci of all points having their principal stresses acting on planes of equal inclination, or parallel to the planes of polarization.

Unfortunately, the isoclinics lines representing stress direction and the isochromatic lines representing stress magnitude are superimposed on each other. If monochromatic light is used, the resulting combination of black lines representing two different conditions may be confusing. However, when white light is used, the isoclinic lines will be represented by black whereas the isochromatic lines will be color.

The Standard Circular Polariscope

The use of quarter - wave plates to convert the field into circular polarized light is the only difference between the plane and the circular polariscopes. Fig. 2-2 shows such a standard circular polariscope. In order for the quarter - wave plates to be effective, they must be used with monochromatic light, and each plate must match the other. With this equipment one may determine.

(a) The maximum shear stress at all points.

(b) The individual values of the principal stresses along free boundaries.

The action of the quarter - wave plate is shown in Fig. 2-3. The light approaching the first quarter - wave plate is inclined 45° to the plane of polarization. As the light passes through the plate, it is given a "twist" that advances it in a

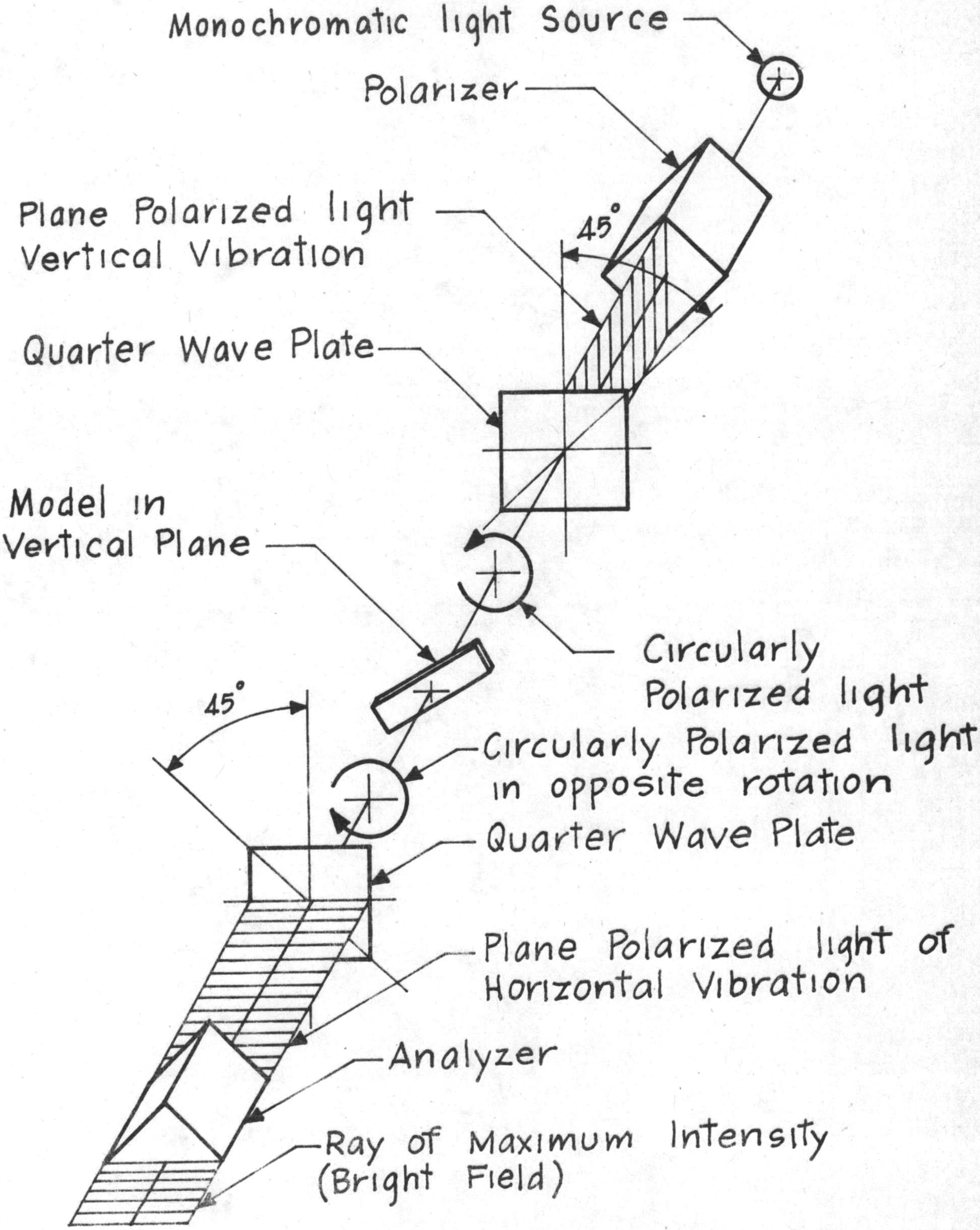

FIG. 2-2.A LIGHT IN CIRCULAR POLARISCOPE

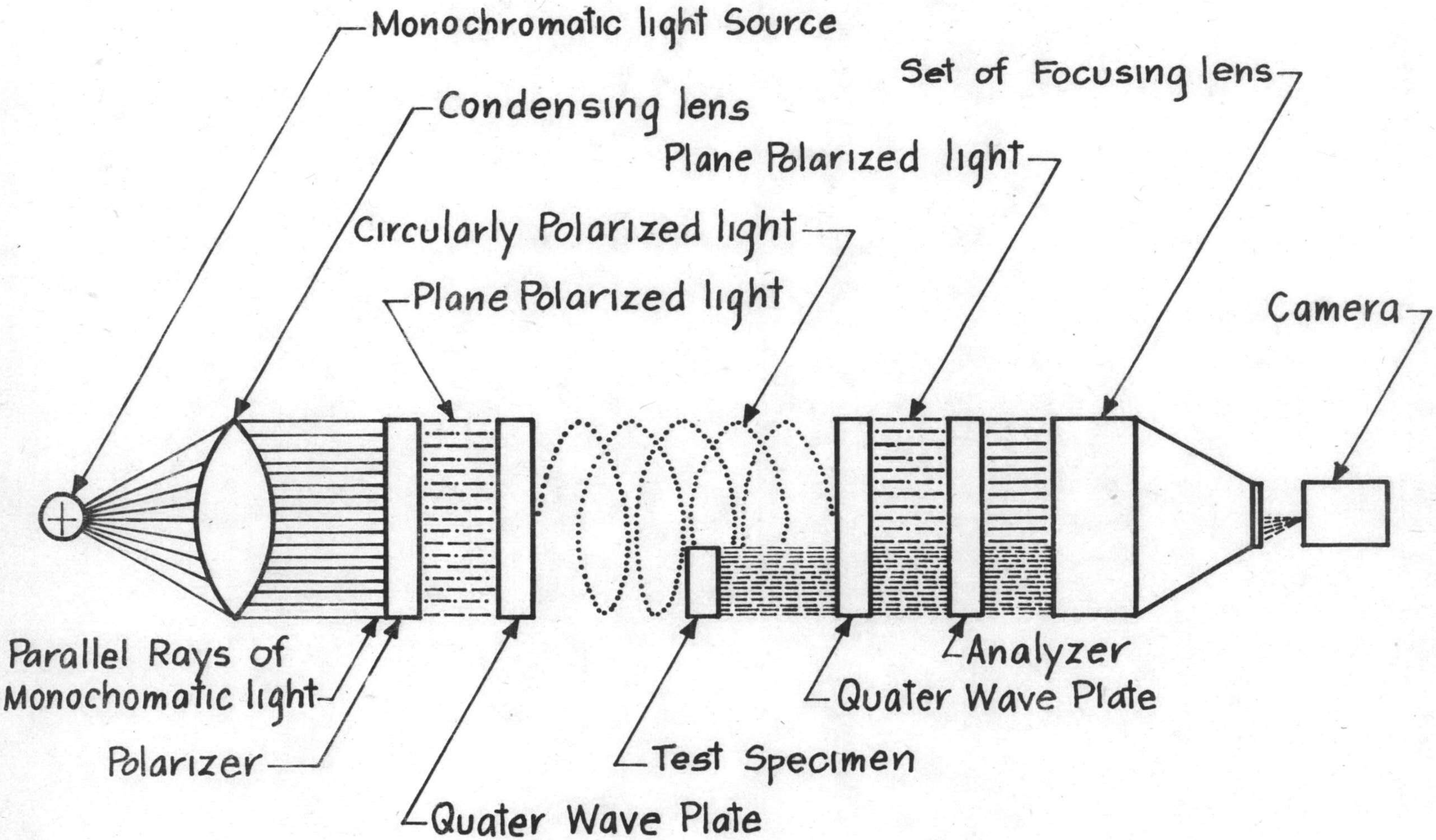

FIG. 2—2.B DIAGRAM OF CIRCULAR POLARISCOPE

FIG. 2—2 STANDARD CIRCULAR POLARISCOPE

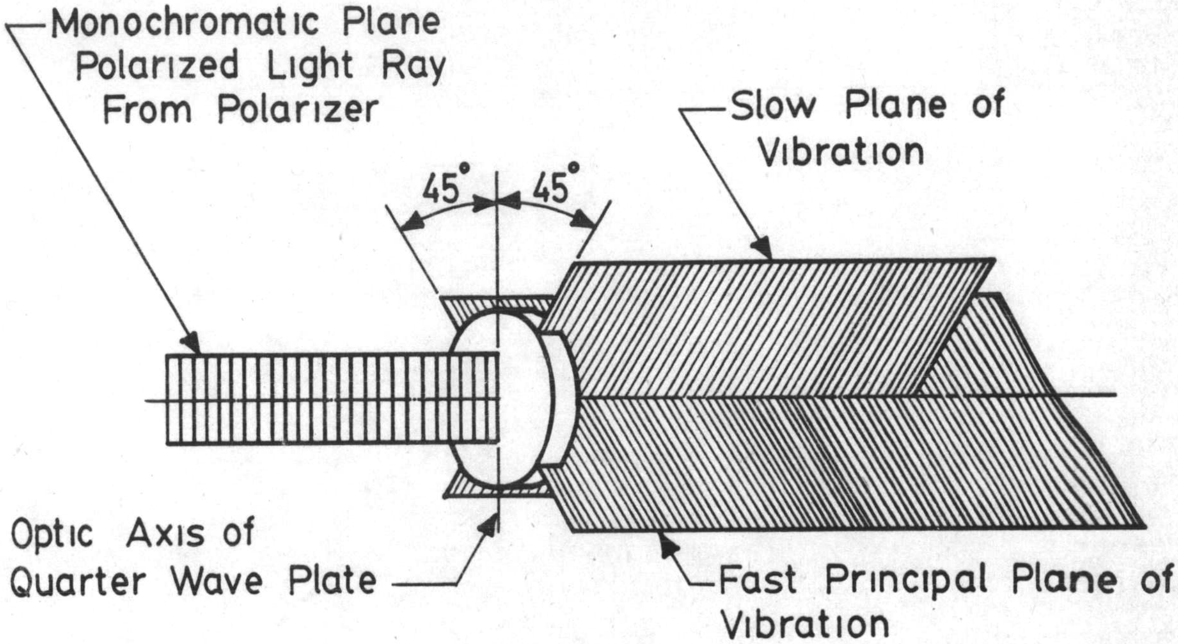

FIG. 2—3.A SHOW TWO PLANES OF VIBRATION

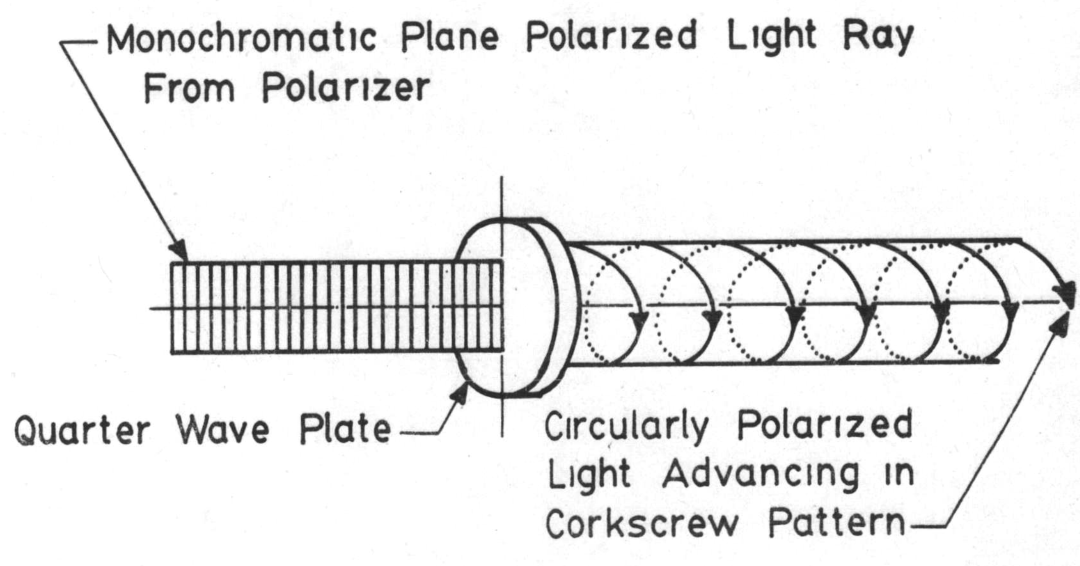

FIG. 2—3.B SHOW CIRCULARLY POLARIZED LIGHT

FIG. 2—3 ACTION OF A QUARTER WAVE PLATE
UPON ONE LIGHT RAY

spiral, somewhat like a corckscrew. If the second quarter - wave plate is set with its axis 90° inclined to the first plate, the circular polarized light will be changed back into plane polarized light, and it will then approach the analyzer in the same manner as before, i.e., as plane polarized light with the plane of polarization perpendicular to the axis of the analyzer. The final result will be complete extinction of the field.

The main adventage of using a circular polariscope is that directional effects in the pattern are eliminated. This makes for convenience when the directions of the principal stresses are known, and the resulting pattern frequently is much more sharply defined than when plane polarized is used. As a side issue, the use of monochromatic ligh makes the observation of interference bands easier, since there are no prism effects with resultant profusion of colors. In this case, there is only a light band followed by a dark contour band.