CHAPTER I

INTRODUCTION

General Background

In has been known that some engineering problems, on structural basis, cannot be solved by the elementary method of strength of materials. That is the elementary theory is insufficient to give information regarding stress distribution in engineering structure. During recent years the theory of elasticity has found considerable application in the solution of structural problems, especially for two dimensional cases. Even if the elastic method is employed, many problems cannot be solved. For this reason three forms of solutions to the engineering problems in the field of solid mechanics are developed. These are analytical, experimental and approximate methods. In this paper the study of analytical and experimental methods are intentionally worked out in order to show that they agree with each other or not. The main topic of analytical solution being considered here will lead to satisfy the problem concerning a narrow slit in an infinite plate in a field of tension. The study of a narrow slit will be somewhat useful to current fracture mechanics. The experimental method that is used to verify the analytical solution is the photoelastic method which has been based on the discovery of Sir David Brewster Since 1816. The photoelastic theory was well developed and formulated by investigators and physicists based on the concept that the optical retardation producing the color effects is proportional to the difference of the principal stresses existing in the transparent material such as glass. The analytical solution leading to the plane problem concerning a narrow slit in an infinite plate with external uniform tension would be a problem of an elliptic hole in the same configuration of plate and loading. Such a problem, elliptic hole, Timoshenko and Goodier[(1)] suggested in the book of elasticity that it is important in current fracture mechanics or cracks.

Strictly speaking, the slits considered in this paper are the ones which have parallel sides and circular ends. It had been known that such narrow slits were studied solely where their directions perpendicular to the direction of external stresses. The effects on stress distribution due to the discontinuity are expressed in terms of stress concentration factor. The major part of this paper is also to find the points, at the edge of slits, where the maximum stress occurs and to evaluate the stress concentration at those points by the method of photoelasticity. These photoelastic results are compared with those of an elliptic hole theory. Therefore, the study in this paper is only concentrated on an elastic region of materials. Besides, the effects of the direction of slits on the maximum stress at its boundary are also studied at various inclinations of the slits.

Stress Concentration Factor

The following definitions are used in this paper.

Stress Concentration : A local increase in the intensity of a stress field due to a discontinuity.

Stress Concentration Factor : The ratio of the highest stress to a reference stress calculable from simple theory.

The reference stress calculated by simple theory may be based upon either the net cross section through the discontinuity or the gross cross section of the member ignoring the discontinuity. A reference stress based on the net cross section is preferable in most papers. However, in a number of cases considered it is not always possible to express in simple terms the nominal local stress based on the net cross section and in these cases the gross cross section has been used. The reference stress concerning with this paper exerts uniformly on an infinite plate, thus, it is preferable to use the reference stress based on the gross cross section area of the plate.

Previous Studies

The works previously studied and considered to be an important foundation to this paper may be divided into two groups. The results, as shown in Fig. 1-1 and Fig. 1-2, of both groups were carried out and confirmed by analytical studies and the experiments respectively.

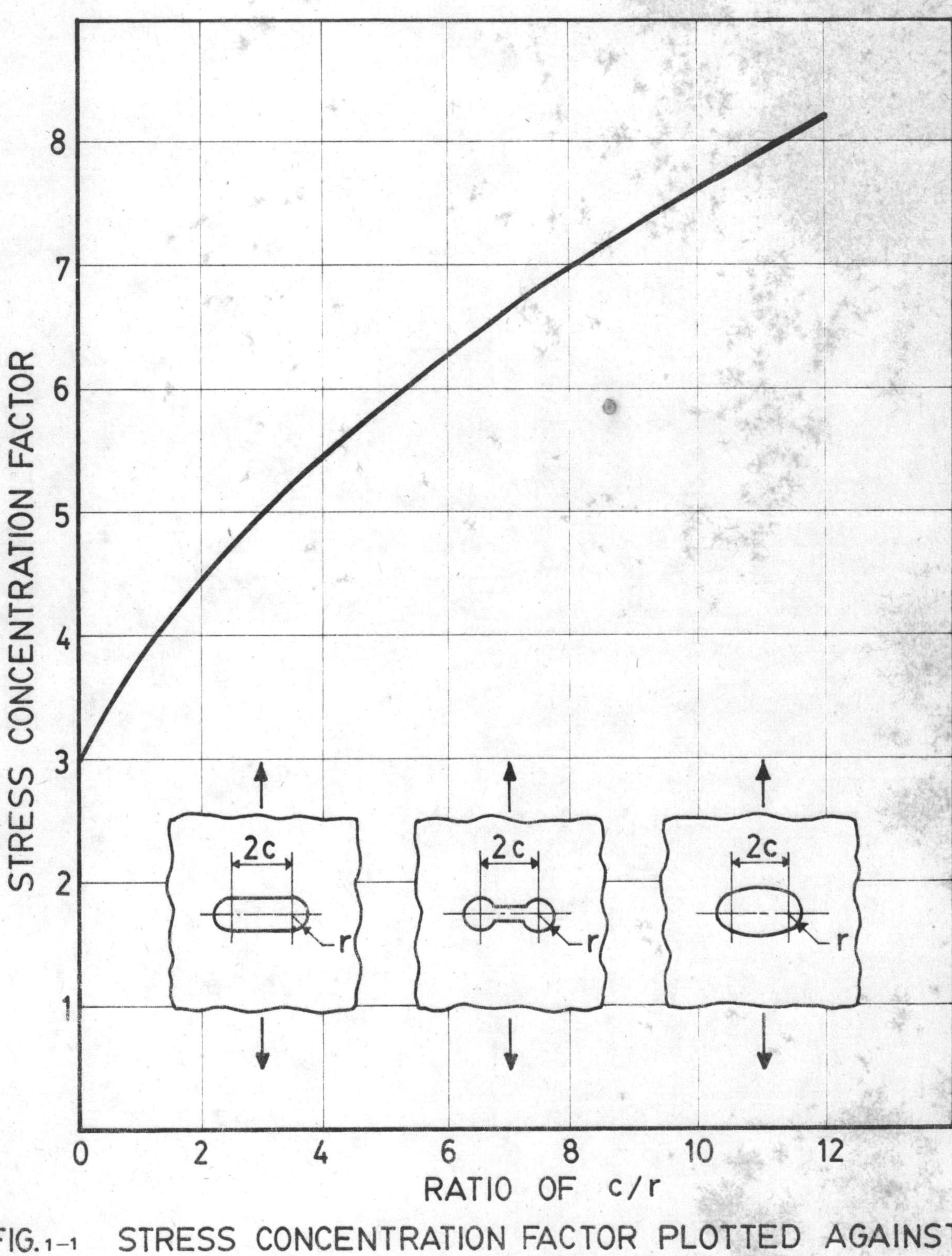

FIG. 1-1 STRESS CONCENTRATION FACTOR PLOTTED AGAINST THE RATIO OF c/r , PRESENTED BY COX

Fig. 1-1 is a curve of stress concentration factor plotted against c/r for a transverse slit centrally placed in an infinite flat bar or strip in simple tension. The curve originally presented by Cox[2] was produced by compromising between the mathematical results which were derived by Inglis[3] and Wigglesworth[4] and the photoelastic results which were conducted by Frocht and Leven[5], and Dixon.[6] The curve, as mentioned by Cox, may be applied to any of the three types of slit shown in the graph of Fig. 1-1, i.e., parallel - side slit with semi-circular ends, narrow slit with circular ends and elliptic slit, with sufficient accuracy for practical purpose. In each case, it is concluded that the maximum stress occurs at the ends of the slit.

In Fig. 1-2 the values of stress concentration factor are also plolted against c/r for a rectangular hole with rounded corners placed in an infinite flat plate loaded by uniformly distributed forces in tension. The geometrical dimensions of the rectangular hole are shown in the graph of Fig. 1-2. The curve appeared on the figure is concerned solely when the dimension "a′" equals to "r", i.e., the rectangular hole becomes a parallel - side slit with Semi-circular ends; and is the result of a combination of the two analytical solutions which were studied by Heller, Brock and Bart[17]; and Sobey.[8] In addition, they analytically studied the effect of a′/b′ on the stress concentration factor for unreinforced rectangular hole with rounded

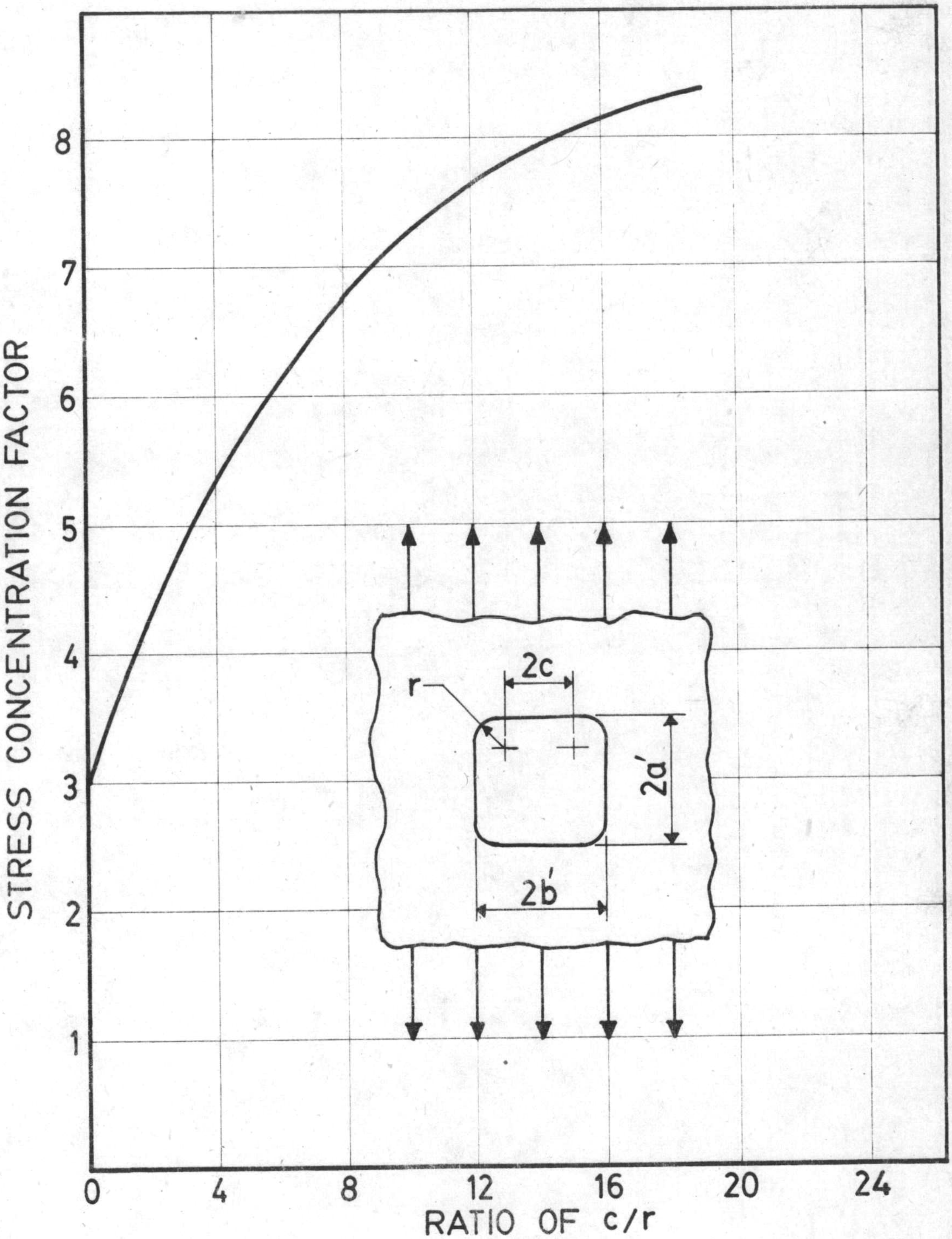

FIG. 1–2 STRESS CONCENTRATION FACTOR OF A ROUNDED-END RECTANGULAR HOLE PLOTTED AGAINST c/r WHERE a′ = r

corners for the biaxial loading and pure shear. It is stated that the criterion adopted for the stress concentration factor is the Mises - Hencky criterion which is based on the strain energy of distortion of the material. The curve have been plotted for a value of Poisson's ratio equal to 1/3, and have been calculated when the external stress is uniform in a region remote from the hole. It is suggested that the curve strictly apply to infinite plates. However, insignificant error will result when using a reference stress based on the gross cross section providing that the total width of plate is greater than 8 b'.

Consequently, the conclusions can be drawn from the previous studies that

(a) three types of slits : parallel - side slit with semi - circular ends, narrow slit with circular ends and elliptic slit were studied especially where their directions perpendicular to the direction of the external stresses,

(b) the stress concentration factors are determined and plotted sgainst c/r for the case stated in (a) and the maximum value of c/r considered is 12, and

(c) in the case of transverse slits the stress concentration factor of an elliptic slit notably equals to that of a parallel - side slit with semi - circular ends and that of a narrow slit with circular ends.

Because of the above conclusions, the study in this paper is aimed to find out the effect of slits' directions on stress concentration factors and to show whether the stress concentration factors of an elliptic slit agree with those of a parallel - side slit with semi - circular ends and with those of a narrow slit with circular ends at any inclination angles of slits.