Chapter IV

Multi-Objective Benchmark Problems, Results and Discussion

As previously stated, the purposes of a multi-objective evolutionary algorithm are to find good non-dominated solutions which must possess 2 qualities – they should be close to the true Pareto-optimal front and are diverse along their front [5]. Similarly, Deb [60] stated that there are two tasks that a multi-objective evolutionary algorithm (MOEA) should do well in solving a multi-objective optimization problem (MOOP). The first task is to guide the search towards the true Pareto-optimal region, and the second task is to maintain the population diversity in the current non-dominated front. He purposed 6 necessary difficult features, multimodality, deception, isolated optimum, convexity or non-convexity, discontinuity in true Pareto-optimal front, and non-uniform distribution of solutions in Pareto-optimal front, multi-objective optimization problems should have. The first 3 features, multimodality, deception, and isolated optimum, cause difficulty in true Pareto front convergence, while the last 3 features, convexity or non-convexity, discontinuity in true Pareto-optimal front, and non-uniform distribution of solutions in Pareto-optimal front, of the diversity maintenance. Deb [60] also mentioned another difficult feature, collateral noise, which is a difficult feature of true Pareto front convergence, comes from the improper evaluation of low-order building blocks (partial solutions which may lead towards the true Pareto-optimal front) due to the excessive noise that may come from other part of the solution vector. However, such problems can be solved by using adequate population size [61]. Therefore the collateral noise may not be necessary for a benchmark multi-objective optimization problem.

The necessary difficult features for benchmark multi-objective optimization problems are described as follows. The first feature, multimodality, represents many local Pareto-optimal fronts a multi-objective evolutionary algorithm may get struck to a local optimal front. The second feature, deception, causes a multi-objective evolutionary algorithm to get misled towards deceptive attractors. For the third feature, isolated optimum true Pareto-optimal front is placed isolated from the rest of the search space. The multi-objective optimization problem that represents this feature has the significantly low density of solutions near the true Pareto-optimal front compared to other regions in the search space. The next feature, the convexity or non-convexity in the true Pareto-optimal front, causes the difficulty to a multi-objective evolutionary algorithm having fitness of each solution is assigned proportional to the number of solutions it dominates [60]. For the fifth feature, true Pareto-optimal front is not continuous in which it contains a set of discrete sub-regions of the front. By this feature, the multi-objective evolutionary algorithm may not obtain solutions of all sub-regions since it may lose solutions within sub-regions in some generations due to the competition among solutions in current population. The last difficult feature is the non-uniform distribution of solutions in the true Pareto-optimal front. If the true Pareto-optimal front is not uniformly represented by feasible solutions, some regions in the front may be represented by a higher density of solutions than other regions. In such cases, there is a natural tendency for a multi-objective evolutionary algorithm to find a biased distribution in the true Pareto-optimal region.

The benchmark problems for multi-objective evolutionary algorithm evaluations in this thesis should represent all of the difficult features. In addition, they should have a tunable number of variables which then creates the adjustable search space. Many well-established multi-objective optimization problems had been proposed, most of them are the two-objective optimization problems. The characteristics of the well-established multi-objective optimization problems, SCH1-2 [62], KUR [63], FON [64], POL [65], ZDT1-6 [38], VNT1-2 [66], and DTLZ1-7 [39], are displayed in the Table 4.1.

Table 4.1 Characteristics summation of well-known multi-objective benchmark problems.

Prob.	NOJ	TND	Difficulty features						
			MM	DT	ISL	CV	NCV	DCT	NUF
SCH1-2	2	×	×	✓	×	✓	×	×	×
KUR	2	×	×	×	×	×	✓	✓	×
FON	2	✓	×	×	×	×	✓	×	×
POL	2	×	×	×	×	×	✓	✓	×
ZDT1-6	2	✓	✓	✓	✓	✓	✓	✓	✓
VNT1-2	3	×	×	×	×	✓	✓	✓	×
DTZL1-7	≥ 2	✓	✓	×	✓	✓	✓	✓	✓

where NOJ and TND represent number of objective and a tunable number of variables, respectively, while MM, DT, ISL, CV, NCV, DCT, and NUF represent multimodality, deceptive, isolated, convex, non-convex, discontinuous, and non-uniformly true Pareto-optimal fronts, respectively.

Table 4.1 shows that for two-objective optimization problems, the problems ZDT1-6 represent all features that cause difficulties to multi-objective evolutionary algorithms and also have a tunable number of decision variables. While the other well-known two-objective optimization problems represent only some features and most of them have a fixed number of decision variables. For three-or-more objectives optimization, the problems DTZL1-7 represent most of necessary difficult features, only the deceptive true Pareto-optimal front is not include in the problems, and also have a tunable number of decision variables. On the other hand, the problems VNT1-2 represent only convex or non-convex and discontinuous true Pareto-optimal front and cannot tune the number of decision variables which is fixed with two. It can be then noted that DTZL1-7 are more suitable than VNT1-2. Therefore, from the above discussion this thesis will employ the problems ZDT1-6 and DTLZ1-7 as the benchmark two-objective optimization problems and three-or-more-objective optimization problems, respectively. However, there is no coupling among decision variable of the ZDT1-6 and DTLZ1-7 problems, since most real-world problems have linkage between decision variables, then the problems may not be appropriate for to test MOEAs because this thesis uses MOEAs to solve continuum topology optimization problems which are the real-world problems having the linkage among decision variables. Deb et. al [67] proposed linked problems by introduce linkage among decision variables into problems ZDT1-6 and DTLZ1-7. This thesis will also employ linked DTLZ1-7 problems as the benchmark problems. The two objectives optimization benchmark ZDT1-6 problems, three-or-more objectives optimization benchmark DTLZ1-7 problems, linked DTLZ1-7 problems, performance evaluation criteria, and simulation results and discussions will be described as the following topics.

4.1. Two Objectives Benchmark Problems – ZDT1-6

Zitzler et. al [38] introduced the six two-objective test functions ZDT1-6 with a tunable number of decision variable. The problems ZDT1-6 represent the different difficult features identified by Deb [60]. ZDT1 has a convex true Pareto-optimal set. ZDT2 is the non-convex counterpart to ZDT1. ZDT3 has several convex disconnected regions. ZDT4 has a multi-modal Pareto optimal set. ZDT5 describes a deceptive problem and distinguishes itself from the other test functions. ZDT6 includes two difficulties caused by the non-uniformly of the search space; first, the Pareto-optimal solutions are non-uniformity distributed along the global Pareto-optimal set and second, the density of the solutions is lowest near the Pareto-optimal front and highest away from the front. For the test problems ZDT1-6, each of the test functions is structured in the same manner and consist itself of three functions f_1, g, h [60] :

$$\begin{aligned} &\text{Minimise} && T(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x})) \\ &\text{subject to} && f_1(\mathbf{x}) = f(x_1, x_2, \ldots, x_q) \\ &&& f_2(\mathbf{x}) = g(x_{q+1}, x_{q+2}, \ldots, x_n) \times h(f_1, g) \end{aligned} \qquad (4.1)$$

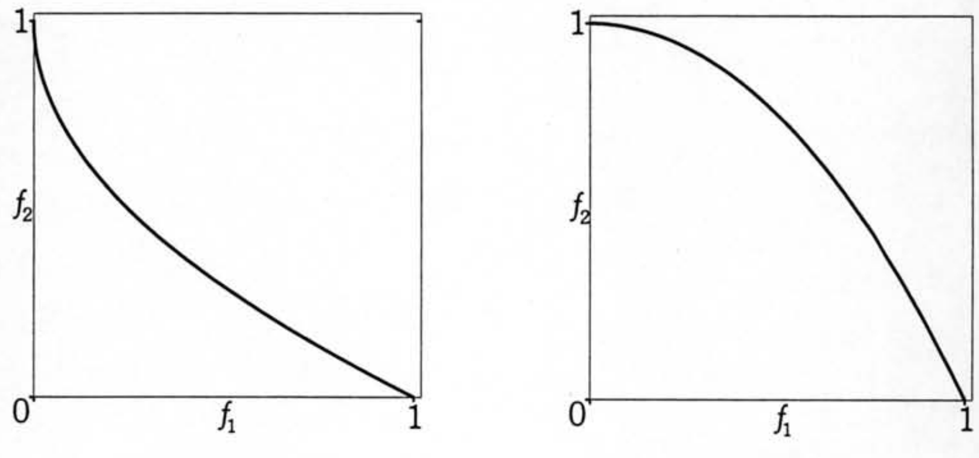
where n is number of variables The function f_1 is a function of the first q variables ($q < n$), the function g is a function of the other n-q variables, while the function h is a function of g and f_1. The benchmark problems ZDT1-6 employ $q = 1$ for all problems and are described as follows.

4.1.1. Test Problem ZDT1

The problem ZDT1 has a convex Pareto-optimal front:

$$f_1(x_1) = x_1$$
$$g(x_2, x_3, \ldots, x_n) = 1 + 9\sum_{i=2}^{n} x_i / (n-1)$$
$$h(f_1, g) = 1 - \sqrt{f_1 / g} \tag{4.2}$$

where $n = 30$, and $x_i \in [0,1]$ for all i. The true Pareto-optimal front (Figure 4.1a) is formed with $g(\mathbf{x}) = 1$.



(a) ZDT1 (b) ZDT2

Figure 4.1 True Pareto-optimal front of (a) ZDT1, and (b) ZDT2.

4.1.2. Test Problem ZDT2

The problem ZDT2 has a non-convex Pareto-optimal front:

$$f_1(x_1) = x_1$$
$$g(x_2, x_3, \ldots, x_n) = 1 + 9\sum_{i=2}^{q} x_i / (q-1)$$
$$h(f_1, g) = 1 - (f_1 / g)^2 \tag{4.3}$$

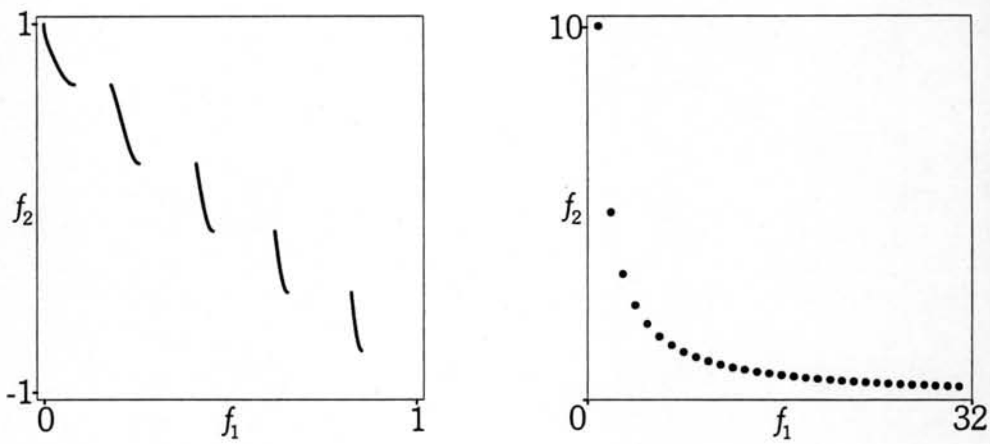
where $n = 30$, and $x_i \in [0,1]$ for all i. The true Pareto-optimal front (Figure 4.1b) is formed with $g(\mathbf{x}) = 1$.

4.1.3. Test Problem ZDT3

The problem ZDT3 has several convex disconnected regions:

$$f_1(x_1) = x_1$$
$$g(x_2, x_3, \ldots, x_n) = 1 + 9\sum_{i=2}^{n} x_i/(n-1)$$
$$h(f_1, g) = 1 - \sqrt{f_1/g} - (f_1/g)\sin(10\pi f_1) \tag{4.4}$$

where $n = 30$, and $x_i \in [0,1]$ for all i. The true Pareto-optimal front (Figure 4.2a) is formed with $g(\mathbf{x}) = 1$. The introduced sine function in h causes discontinuity in the Pareto-optimal front. However, there is no discontinuity in the solution space.



(a) ZDT3 (b) ZDT5

Figure 4.2 True Pareto-optimal front of (a) ZDT3, and (b) ZDT5.

4.1.4. Test Problem ZDT4

The problem ZDT4 has a multi-modality Pareto optimal set which contains 21^9 local Pareto fronts:

$$f_1(x_1) = x_1$$
$$g(x_2, x_3, \ldots, x_k) = 1 + 10(m-1) + \sum_{i=2}^{m}\left(x_i^2 - 10\cos(4\pi x_i)\right)$$
$$h(f_1, g) = 1 - \sqrt{f_1/g} \tag{4.5}$$

where $n = 10$, $x_1 \in [0,1]$, and $x_2,x_3,\ldots,x_n \in [-5,5]$. The global Pareto-optimal front, which is the same as that of ZDT2 (Figure 4.1b), is formed with $g(\mathbf{x}) = 1$ and convex.

4.1.5. Test Problem ZDT5

The problem ZDT5 is a deceptive problem:

$$\begin{aligned} f_1(x_1) &= 1 + u(x_1) \\ g(x_2,\ldots,x_n) &= \sum\nolimits_{i=2}^{n} \upsilon(u(x_i)) \\ h(f_1,g) &= 1/f_1 \end{aligned} \tag{4.6}$$

where $u(x_i)$ gives the number of ones in the bit vector,

$$\upsilon(u(x_i)) = \begin{cases} 2+u(x_i) & \text{if } u(x_i) < 5 \\ 1 & \text{if } u(x_i) = 5 \end{cases} \tag{4.7}$$

and $n = 11$, $x_1 \in \{0,1\}^{30}$, and $x_2,\ldots,x_m \in \{0,1\}^5$. The true Pareto-optimal front (Figure 4.2a), is formed with $g(x) = 10$ and convex.

4.1.6. Test Problem ZDT6

The problem ZDT6 presents two difficulties of the non-uniform search space which are described as follows. First, the true Pareto-optimal solutions are non-uniformly distributed along the true Pareto front. Second, the density of the solutions in search space is lowest near the true Pareto front and highest away from the front

$$\begin{aligned} f_1(x_1) &= 1\text{-}e^{-4x_1}\sin^6(6\pi x_1) \\ g(x_2,\ldots,x_n) &= 1+9\left[\left(\sum\nolimits_{i=2}^{n} x_i\right)/(n-1)\right]^{0.25} \\ h(f_1,g) &= 1-(f_1/g)^2 \end{aligned} \tag{4.8}$$

where $n = 10$, $x_i \in [0,1]$ for all i. The global Pareto-optimal front, which is identical to that of ZDT2 (Figure 4.2a), is formed with $g(\mathbf{x}) = 1$ and non-convex.

4.2. Three-Or-More Objectives Benchmark Problems – DTLZ1-7

The seven optimization test cases DTLZ1-7 are developed by Deb et al. [39]. The problems are minimisation problems with n decision variables represented by a decision variable vector $\mathbf{x}$ and m objectives. The decision variable vector $\mathbf{x}$ can be partitioned into 2 non-overlapping variable vectors such that $\mathbf{x} = [\mathbf{x}_l, \mathbf{x}_m]$. For a decision variable vector $\mathbf{x} = [x_1, x_2, \ldots, x_n]$, its partitioned vectors $\mathbf{x}_l = [x_1, x_2, \ldots, x_{m-1}]$ and $\mathbf{x}_m = [x_m, x_{m+1}, \ldots, x_n]$, then $|\mathbf{x}_l|$ and $|\mathbf{x}_m|$ are m-1 and k = (n-m+1) respectively. The first six problems – DTLZ1-6 – can be described in the following form:

$$
\begin{aligned}
&\text{Minimize} && T(\mathbf{x}) = (f_1(\mathbf{x}),\ldots,f_m(\mathbf{x})) \\
&\text{subject to} && f_1(\mathbf{x}) = h_1(x_1,\ldots,x_{m-1},g(\mathbf{x}_m)), \\
& && f_2(\mathbf{x}) = h_2(x_1,\ldots,x_{m-1},g(\mathbf{x}_m)), \\
& && f_3(\mathbf{x}) = h_3(x_1,\ldots,x_{m-2},g(\mathbf{x}_m)), \\
& && \vdots \\
& && f_{m-1}(\mathbf{x}) = h_{m-1}(x_1,x_2,g(\mathbf{x}_m)) \\
&\text{and} && f_m(\mathbf{x}) = h_m(x_1,g(\mathbf{x}_m))
\end{aligned}
\tag{4.9}
$$

These test problems reflect various characteristics that can be found in real-world problems. In addition, the functions that are used to construct all test problems are also interchangeable. This means that other variants of the test problems can be easily constructed using the existing functions. Furthermore, these problems are also scalable in terms of the number of decision variables n = m+k-1 and the number of objectives m. As a result, a change in the algorithm performance due to an increase or decrease of the number of decision variables and/or objectives can be easily interpreted. In this thesis, the problems will be scaled by changing the number of objectives only where the interested numbers of objectives are 3-6. By increasing the number of objectives, the difficulty level of the problem will also increase. Detailed description of each test problem follows.

4.2.1. Test Problem DTLZ1

The problem DTLZ1 has a linear Pareto front. The true Pareto front for 3 objectives DTLZ1 is shown in Figure 4.3. The functions that form DTLZ1 are given by

$$
\begin{aligned}
f_1(\mathbf{x}) &= 0.5x_1x_2\cdots x_{m-1}(1+g(\mathbf{x}_m)),\\
f_2(\mathbf{x}) &= 0.5x_1x_2\cdots(1-x_{m-1})(1+g(\mathbf{x}_m)),\\
&\vdots\\
f_{m-1}(\mathbf{x}) &= 0.5x_1(1-x_2)(1+g(\mathbf{x}_m)),\\
f_m(\mathbf{x}) &= 0.5(1-x_1)(1+g(\mathbf{x}_m))
\end{aligned}
$$

and $$g(\mathbf{x}_m) = 100\left[|\mathbf{x}_m| + \sum_{x_i\in\mathbf{x}_m}(x_i-0.5)^2 - \cos(20\pi(x_i-0.5))\right] \tag{4.10}$$

where $x_i \in [0,1]$ for all i. The Pareto front is formed with $g(\mathbf{x}_m) = 0$ where the Pareto optimal solutions correspond to $x_i = 0.5$ for all $x_i \in \mathbf{x}_m$ and the objective values lie on the linear hyper-surface $\sum_{i=1}^{m} f_i = 0.5$. A convergence of solutions to the true Pareto front is difficult to achieve since the search space contains 11^{n-m+1}-1 local Pareto fronts. This means that the solutions generated by a genetic algorithm can be attracted to the local fronts prior to reaching the true front.

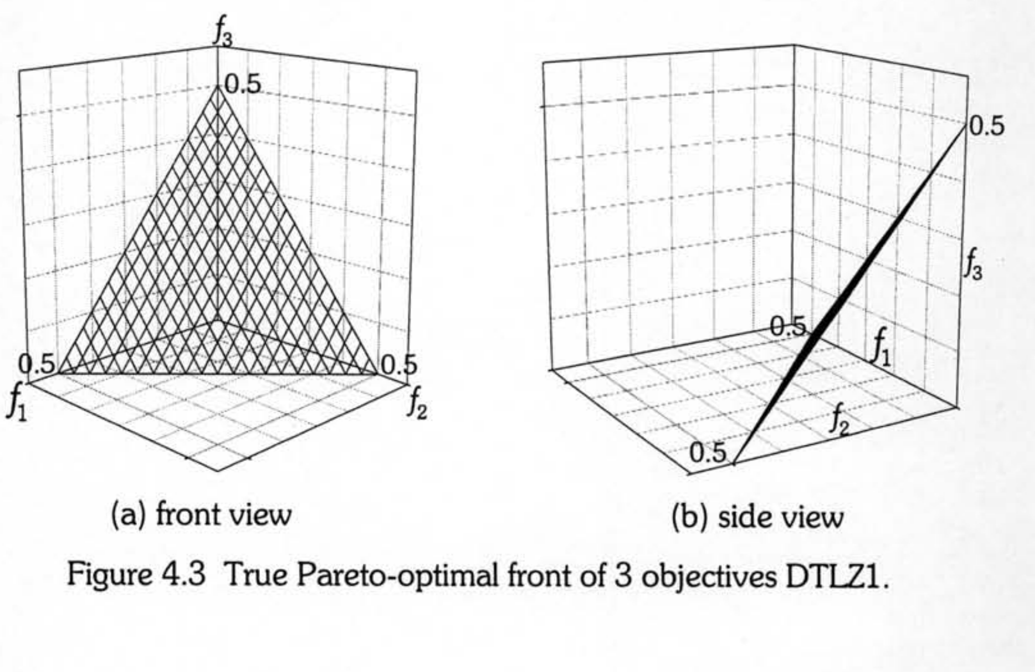


Figure 4.3 True Pareto-optimal front of 3 objectives DTLZ1.

4.2.2. Test Problem DTLZ2

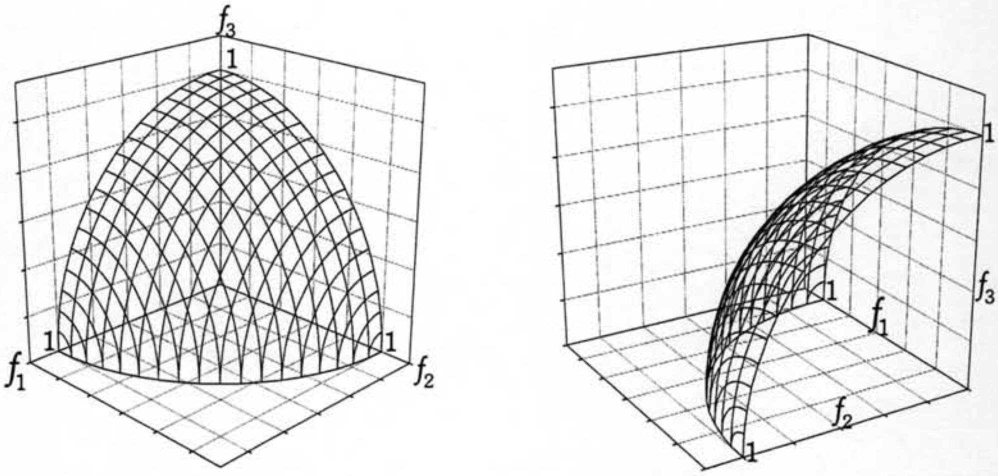
The problem DTLZ2 has a spherical Pareto front. The true Pareto front of DTLZ2, which is the same as those of DTLZ3 and DTLZ4, is shown in Figure 4.4. The functions that form DTLZ2 are given by

$$
\begin{aligned}
f_1(\mathbf{x}) &= (1+g(\mathbf{x}_m))\cos(x_1\pi/2)\cdots\cos(x_{m-2}\pi/2)\cos(x_{m-1}\pi/2),\\
f_2(\mathbf{x}) &= (1+g(\mathbf{x}_m))\cos(x_1\pi/2)\cdots\cos(x_{m-2}\pi/2)\sin(x_{m-1}\pi/2),\\
f_3(\mathbf{x}) &= (1+g(\mathbf{x}_m))\cos(x_1\pi/2)\cdots\sin(x_{m-2}\pi/2),\\
&\vdots\\
f_m(\mathbf{x}) &= (1+g(\mathbf{x}_m))\sin(x_1\pi/2)
\end{aligned}
$$

and

$$g(\mathbf{x}_m) = \sum_{x_i \in \mathbf{x}_m} (x_i - 0.5)^2 \tag{4.11}$$

where $x_i \in [0,1]$ for all i. The Pareto front is formed with $g(\mathbf{x}_m) = 0$ where the Pareto optimal solutions correspond to $x_i = 0.5$ for all $x_i \in \mathbf{x}_m$ and the objective values lie on the spherical hyper-surface $\sum_{i=1}^{m} f_i^2 = 1$. This problem can be used to identify the ability of the algorithm to scale up its performance when the number of objectives is large.



(a) front view (b) side view

Figure 4.4 True Pareto front for three-objective DTLZ2-4.

4.2.3. Test Problem DTLZ3

The problem DTLZ3 also has a spherical Pareto front (Figure 4.4). However, the problem contains multiple local Pareto fronts. The functions that form DTLZ3 are given by

$$f_1(\mathbf{x}) = (1 + g(\mathbf{x}_m))\cos(x_1\pi/2)\cdots\cos(x_{m-2}\pi/2)\cos(x_{m-1}\pi/2),$$
$$f_2(\mathbf{x}) = (1 + g(\mathbf{x}_m))\cos(x_1\pi/2)\cdots\cos(x_{m-2}\pi/2)\sin(x_{m-1}\pi/2),$$
$$f_3(\mathbf{x}) = (1 + g(\mathbf{x}_m))\cos(x_1\pi/2)\cdots\sin(x_{m-2}\pi/2),$$
$$\vdots$$
$$f_m(\mathbf{x}) = (1 + g(\mathbf{x}_m))\sin(x_1\pi/2)$$

and
$$g(\mathbf{x}_m) = 100\left[|\mathbf{x}_m| + \sum_{x_i \in \mathbf{x}_m}([x_i - 0.5]^2 - \cos[20\pi(x_i - 0.5)])\right] \qquad (4.12)$$

where $x_i \in [0,1]$ for all i. From equation (4.12), the structure of objective functions in the DTLZ3 problem is similar to that from the DTLZ2 problem while $g(\mathbf{x}_m)$ in the DTLZ3 problem and $g(\mathbf{x}_m)$ in the DTLZ1 problem are the same functions. As a result, both the global Pareto front of the DTLZ3 problem and the Pareto front of the DTLZ2 problem has the same shape. However, the use of $g(\mathbf{x}_m)$ as described in equation (10) makes the DTLZ3 problem more difficult than the DTLZ2 problem since 3^{n-m+1}-1 local Pareto fronts are now introduced to the problem. It is noted that the global Pareto front of the DTLZ3 problem is formed with $g(\mathbf{x}_m) = 0$ where the global Pareto optimal solutions correspond to $x_i = 0.5$ for all $x_i \in \mathbf{x}_m$ and the objective values lie on the spherical hyper-surface $\sum_{i=1}^{m} f_i^2 = 1$.

4.2.4. Test Problem DTLZ4

The test problem DTLZ4 also has a spherical Pareto front with the shape similar to that from the DTLZ2 problem (Figure 4.4). The functions that form DTLZ4 are given by

$$f_1(\mathbf{x}) = (1 + g(\mathbf{x}_m))\cos(x_1^{\beta}\pi/2)\cdots\cos(x_{m-2}^{\beta}\pi/2)\cos(x_{m-1}^{\beta}\pi/2),$$

$$f_2(\mathbf{x}) = (1+g(\mathbf{x}_m))\cos(x_1^{\beta}\pi/2)\cdots\cos(x_{m-2}^{\beta}\pi/2)\sin(x_{m-1}^{\beta}\pi/2),$$
$$f_3(\mathbf{x}) = (1+g(\mathbf{x}_m))\cos(x_1^{\beta}\pi/2)\cdots\sin(x_{m-2}^{\beta}\pi/2),$$
$$\vdots$$
$$f_m(\mathbf{x}) = (1+g(\mathbf{x}_m))\sin(x_1^{\beta}\pi/2)$$

and $$g(\mathbf{x}_m) = \sum_{x_i \in \mathbf{x}_m} (x_i - 0.5)^2 \qquad (4.13)$$

where $x_i \in [0,1]$ for i = 1, ..., n. It can be seen that the objective function in equation (11) are created by replacing x_i for i = 1, ..., m-1 in equation (4.11) with x_i^{β}. In this investigation, β is set to 100; this modification allows a dense set of solutions to exist near the f_m-f_1 plane. In other words, the Pareto optimal solutions are non-uniformly distributed along the Pareto front. The condition for a solution to be Pareto optimal for this problem is the same as that given for the DTLZ2 problem.

4.2.5. Test Problem DTLZ5

The Pareto front of the problem DTLZ5, which is identical to that of DTLZ6, can be visually displayed as a curve. The true-Pareto front of 3 objectives DTLZ5 is shown in Figure 4.5.The functions that form DTLZ5 are given by

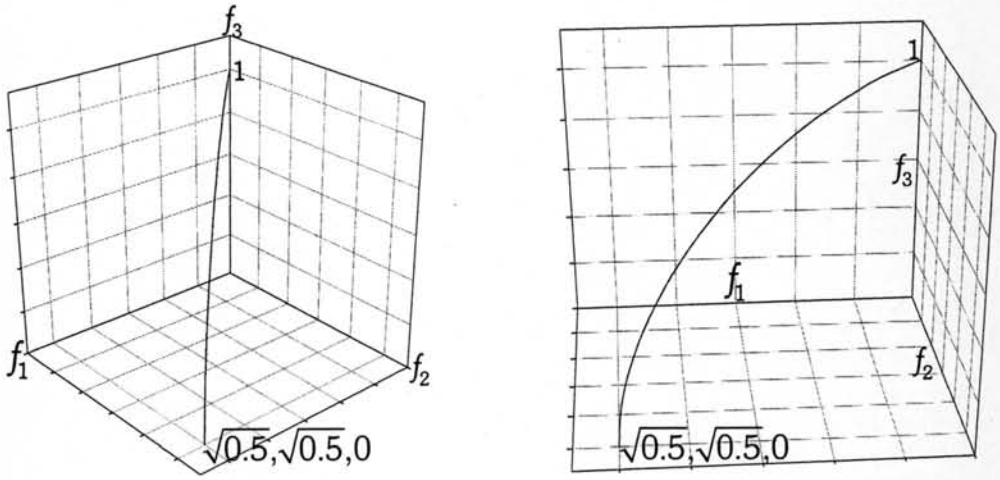
$$f_1(\mathbf{x}) = (1+g(\mathbf{x}_m))\cos(\theta_1)\cdots\cos(\theta_{m-2})\cos(\theta_{m-1}),$$
$$f_2(\mathbf{x}) = (1+g(\mathbf{x}_m))\cos(\theta_1)\cdots\cos(\theta_{m-2})\sin(\theta_{m-1}),$$
$$f_3(\mathbf{x}) = (1+g(\mathbf{x}_m))\cos(\theta_1)\cdots\sin(\theta_{m-2}),$$
$$\vdots$$
$$f_m(\mathbf{x}) = (1+g(\mathbf{x}_m))\sin(\theta_1)$$

with $$g(\mathbf{x}_m) = \sum_{x_i \in \mathbf{x}_m} (x_i - 0.5)^2$$

where $$\theta_1 = x_1\pi/2$$

and $$\theta_i = \frac{\pi}{4(1+g(\mathbf{x}_m))}(1+2g(\mathbf{x}_m)x_i) \text{ for } i = 2, \ldots, m-1 \qquad (4.14)$$

where $x_i \in [0,1]$ for all i. This test problem is created by changing the argument of the sinusoidal functions within the DTLZ2 objective functions from $\pi x_i/2$ to θ_i. As a result, this problem will test the ability of the search algorithm to produce converged solutions that form a curve. The performance can be visually observed by plotting the f_m objective with any other objectives. The conditions for a solution to be Pareto optimal for this problem are also the same as that given for the DTLZ2 problem.



(a) front view (b) side view

Figure 4.5 True Pareto front for three-objective DTLZ5 and DTLZ6.

4.2.6. Test Problem DTLZ6

The problem DTLZ6, of which 3 objectives true Pareto front is shown in the Figure 4.5, is a harder version of the test problem DTLZ5. Similar to the modification done on the DTLZ2 problem in order to create the DTLZ3 problem, the modification made to the DTLZ5 also involves the use of a different $g(\mathbf{x}_m)$ function, which leads to the introduction of local Pareto fronts. However, in the test problem DTLZ6 the $g(\mathbf{x}_m)$ function is given by

$$g(\mathbf{x}_m) = \sum_{x_i \in \mathbf{x}_m} x_i^{0.1} \tag{4.15}$$

The true Pareto front is formed with $g(\mathbf{x}_m) = 0$ where the Pareto optimal solutions correspond to $x_i = 0$ for all $x_i \in \mathbf{x}_m$. Multiple local Pareto fronts make this problem a hard problem.

4.2.7. Test Problem DTLZ7

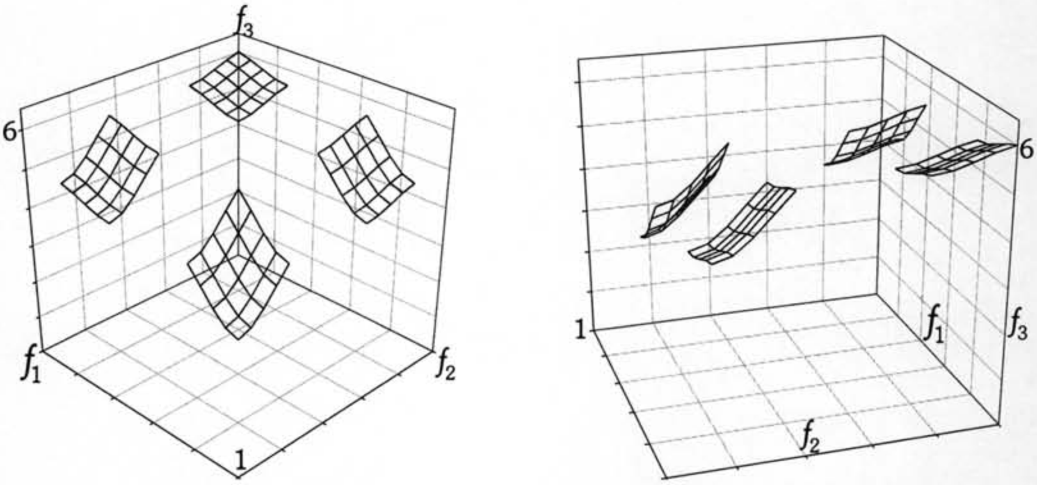
This problem is constructs using the problem stated in equation (4.9). This problem has a disconnected set of Pareto-optimal regions:

$$\begin{aligned} f_1(\mathbf{x}) &= x_1, \\ f_2(\mathbf{x}) &= x_2, \\ f_3(\mathbf{x}) &= x_3, \\ &\vdots \\ f_m(\mathbf{x}) &= (1+g(\mathbf{x}_m))h(f_1, f_2, \ldots, f_{M-1}, g) \end{aligned}$$

where $$g(\mathbf{x}_m) = 1 + \frac{9}{|\mathbf{x}_m|} \sum_{x_i \in \mathbf{x}_m} x_i$$

and $$h(f_1, f_2, \ldots, f_{M-1}, g) = m - \sum_{i=1}^{m-1} \left[\frac{f_i}{1+g} \left(1 + \sin(3\pi f_i)\right) \right] \tag{4.16}$$

where $x_i \in [0,1]$ for all i. This test problem has 2^{m-1} disconnected Pareto-optimal regions in the search space. The true-Pareto front of 3 objectives DTLZ7 is shown in the Figure 4.6. The function g requires $|\mathbf{x}_m| = k$ decision variables in $n = m + k-1$. The Pareto-optimal solutions corresponds to $\mathbf{x}_m = 0$.



(a) front view (b) side view

Figure 4.6 True Pareto front for three-objective DTLZ7.

There are no linkage among decision variables of DTLZ1-4, and DTLZ7 problems, while DTLZ5-6 problems have linkage among decision variables. However the DTLZ5-6 problems are not enough for the evaluation of performance of MOEAs. The linked benchmark problems, which are linked DTLZ1-7 problems, will be presented in the following topic.

4.3. Linked Benchmark Problems – Linked DTLZ1-7

Although the ZDT and DTLZ problems have suitable difficult features, there are no linkages among variables or bits representing any variable, which can cause difficulty for MOEA, especially for MOEAs employs co-operative co-evolution [43] such as [31], [32], [35]. These problems may not be appropriate, since in most real-world optimization problems, there is linkage among decision variables in encoded chromosome of any solution. Deb et al. [67] introduced explicit linkages among variables so as to develop difficult to the benchmark ZDT and DTLZ problems, this thesis will use the linked DTLZ problems to test performance of the proposed MOEAs, improved compressed genetic algorithm (COGA-II), co-operative co-evolutionary multi-objective algorithm (CCMOA), and co-operative co-evolutionary improved compressed-objective genetic algorithm (CCCOGA-II). The linked DTLZ problems are necessary for the last two proposed MOEAs, CCMOA and CCCOGA-II, for the reason that these MOEAs employ co-operative co-evolution [43].

For a linked DTLZ problem with an encoded decision variable vector $\mathbf{x} = (x_1, \ldots, x_n)$, a temporary variable vector $\mathbf{y} = (y_1, \ldots, y_n)$ is a function of $\mathbf{x}$, and another variable vector $\mathbf{z} = (z_1, \ldots, z_n)$ of which each representative variable $z_i = y_i^2$. Similar to that of normal DTLZ problems, the variable vectors $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$ are partitioned into two non-overlapping vectors such that $\mathbf{x} = [\mathbf{x}_l, \mathbf{x}_m]$, $\mathbf{y} = [\mathbf{y}_l, \mathbf{y}_m]$ and $\mathbf{z} = [\mathbf{z}_l, \mathbf{z}_m]$ in which the first partitioned vector, $\mathbf{x}_l$, $\mathbf{y}_l$, and $\mathbf{z}_l$, are size of $n-k$ and the second partitioned vectors are size of k. By the variable vector $\mathbf{z}$, linked DTLZ problems can be described in the following form:

Minimise $$T(\mathbf{z}) = (f_1(\mathbf{z}), \dots, f_m(\mathbf{z}))$$

subject to
$$f_1(\mathbf{z}) = h_1(z_1, \dots, z_{m-1}, g(\mathbf{z}_m)),$$
$$f_2(\mathbf{z}) = h_2(z_1, \dots, z_{m-1}, g(\mathbf{z}_m)),$$
$$f_3(\mathbf{z}) = h_3(z_1, \dots, z_{m-2}, g(\mathbf{z}_m)),$$
$$\vdots$$
$$f_{m-1}(\mathbf{z}) = h_{m-1}(z_1, z_2, g(\mathbf{z}_m)),$$
and
$$f_m(\mathbf{z}) = h_m(z_1, g(\mathbf{z}_m)) \qquad (4.17)$$

There are there types of linked problems, which identified by the function representing for the variable vector $\mathbf{y}$. For a mapping function, by a constant $n \times n$ metric $\mathbf{M}$, a linear mapping function, which is employed for the first and second types of the linked problems, is given by $\mathbf{y} = \mathbf{M} \cdot \mathbf{x}$. The metric $\mathbf{M}$ can be partitioned into 4 non-overlapping metrics $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$, thereafter the linear mapping function can be written in the following form:

$$\begin{bmatrix} \mathbf{y}_l \\ \mathbf{y}_m \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{x}_l \\ \mathbf{x}_m \end{bmatrix} \qquad (4.18)$$

where the size of the metrics $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ are $(n-k) \times (n-k)$, $(n-k) \times k$, $k \times (n-k)$, and $k \times k$ respectively.

For a first type of linked DTLZ, L_1-DTLZ, by setting $\mathbf{B} = \mathbf{0}$ and $\mathbf{C} = \mathbf{0}$, the mapping function of $\mathbf{y}$ is given by

$$\begin{bmatrix} \mathbf{y}_l \\ \mathbf{y}_m \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{x}_l \\ \mathbf{x}_m \end{bmatrix} \qquad (4.19)$$

where $\mathbf{A} = [a_{ij}]$, a_{ij} is a random number $\in$ [-1,1], and $\mathbf{D} = [d_{ij}]$, d_{ij} is a random number $\in$ [-1,1].

For the L_1-DTLZ, a partitioned vector of $\mathbf{y}$ is a function of only its corresponding partitioned vector of $\mathbf{x}$ only, subsequently there are no coupling between $\mathbf{x}_l$ and $\mathbf{x}_m$. On the other hand, in order to make coupling between variable vectors $\mathbf{x}_l$ and $\mathbf{x}_m$, the second type of DTLZ, L_2-DTLZ, employs non-zero metrics **B** and **C**, therefore the mapping function for L_2-DTLZ can be written by the following equation.

$$\mathbf{y} = \mathbf{M} \cdot \mathbf{x} \tag{4.20}$$

where $\mathbf{M} = [M_{ij}]$, of which M_{ij} is a random number $\in [-1,1]$.

The L_1-DTLZ, and L_2-DTLZ employ linear mapping functions. On the other hand, the third type of linked DTLZ, L_3-DTLZ, uses a non-linear mapping function. This thesis employs a quadratic mapping function as introduced in [67], by using the metric **M** as that of L_2-DTLZ, the mapping function of L_3-DTLZ is given by

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \mathbf{M} \cdot \begin{bmatrix} x_1^2 \\ x_2^2 \\ \vdots \\ x_n^2 \end{bmatrix} \tag{4.21}$$

It should note that true Pareto-optimal front of linked DTLZ problems cannot be represented by any explicit functions and also are varied by a constant randomized metric **M**. This thesis will use linked DTLZ2 and DTLZ6 problems of all three linked types for performance evaluation of any employed MOEAs. Since the normal DTLZ6 problem has linkage among decision variables while the normal DTLZ2 problem does not have such linkage, therefore, in the same linked type, a linked DTLZ6 problem has more linkage among decision variables than that of a linked DTLZ2 problem.

4.4. Performance Evaluation Criteria

As previously stated, good non-dominated solutions should be close to the true Pareto optimal front and diverse along their front. There are many performance metrics for the evaluation of a multi-objective evolutionary algorithm (MOEA) [68]. This thesis will employ two performance metrics; the first metric, M_1, which is proposed by Zitzler et al [38], is average distance of non-dominated solutions to the true Pareto optimal front, while the second metric is the clustering index (*CI*), a diversity metric which is firstly proposed in this thesis, indicates the distribution of any non-dominated solution set on its front. These two performance metrics are described as follows.

4.4.1. Average Distance to True Pareto-Optimal Front (M_1)

The average distance to true Pareto-optimal solutions (M_1) [38] can be evaluated in solution space or objective space. For this thesis, the metric M_1 will be measured by in objective space, a distance of a solution i to true Pareto-optimal front, d_i, which is the Euclidean distance of the solution i to its nearest solution j on the true Pareto-optimal front, is evaluated by

$$d_i = \sqrt{\sum_{k=1}^{m}\left(\frac{f_{ik} - f_{jk}}{(f_k)_{\max} - (f_k)_{\min}}\right)^2} \tag{4.22}$$

where f_{ik} and f_{jk} are objectives k of solutions i and j, respectively while $(f_k)_{\min}$ and $(f_k)_{\max}$ are minimum and maximum value of an objective k of the true Pareto-optimal solutions. For the true Pareto-optimal front that can be written in explicit form, the distance d_i can be calculated by Lagrange multiplication method [69]. The metric M_1 of a solution set F is therefore equal to average d_i of solutions in the set F. Since true Pareto-optimal front of a linked DTLZ problem cannot be represented by explicit form, therefore M_1 of the problem cannot be solved by the Lagrange multiplication method [69]. To solve this problem, artificial true Pareto optimal front which is obtained from the non-dominated individuals of merged individuals of all runs of any multi-objective evolutionary algorithms (MOEAs) is used instead of the true Pareto front. Similar to that of the normal DTLZ problems, the distance d_i, of a solution i is the Euclidean distance of the solution i to its nearest solution j on the artificial true Pareto-optimal front. However, it should be noted that M_1 which is obtained from artificial true Pareto-optimal front is not the exact value for a linked DTLZ problem. It can only be used to compare closeness to Pareto-optimal front of solutions by the employed MOEAs.

4.4.2. Clustering Index

A clustering index (*CI*) is diversity metric which indicates distribution of a non-dominated solution set on its hyper-surface. The clustering index does not need grid division of an objective *i*, which is hard to identify a suitable number of divided grids of the objective *i*, as the grid diversity metric presented by Deb and Jain [68]. For a non-dominated solution set *A* of size *Q*, the clustering index, *CI*, of the set *A* is evaluated from a generated non-dominated solution set *B* of size *R*, in which $R \gg Q$. The evaluations of the *CI* of the solutions in set *A* are as follows.

1) Copy all solutions in set *A* to the generated solution set *B*.

2) Randomize the first parent, p_1, from the set *A*.

3) Randomize the second parent, p_2, from the set *B*.

4) Perform crossover and mutation to these two parents in order to obtained two children, c_1 and c_2, and then calculate objectives of the children.

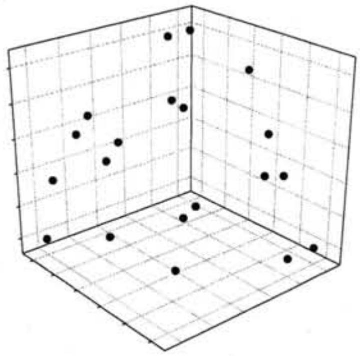
5) Check whether each child individual neither dominates nor is dominated by any solution in the set *A* or not. If both children are not satisfied this condition, go back to step 3). If only one child individual is satisfied, it is put into the set *B*, else if both individuals are satisfied the condition, only one individual is selected at random to put into the set *B*.

6) Increase the number of members of *B* by one, if the set *B* is not yet fulfilled, go back to step 2), else if the set *B* is fulfilled, go to the next step.

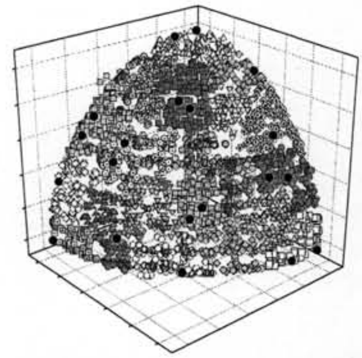
7) Divide *R* solutions in set *B* into *Q* groups by the clustering method [27], then find the number of groups, *S*, that contains the first *Q* solutions, which are identical to solutions in set *A*. The clustering index, *CI*, of the set *A* is equal to the quotient of *S* and *Q*.

In each time of addition of solution into the solution set B, a first parent, p_1, which is randomly selected from the solution set A, is not changed, only a second parent is randomly re-selected if their children do not satisfy the condition to put into the set B and only one solution is randomly selected from children, which are both satisfied the condition. Therefore for each time of addition, any first randomly selected parent, p_1, contributes one solution to put into the set B. Since the number of generated solution in set B, R, is much more than the number of solutions in set A, Q, it can conclude that the contribution of solutions in set A to the generated solutions in set B are close to each other. For a good generated non-dominated solution set B, if hyper-surface of its non-dominated solutions is divided into Q equal portions, it should have at least one solution in any divided portion. Therefore, a defined size of set B, R, should be significantly large as possible.

For example, a set A with 20 non-dominated solutions of the DTLZ2 problem [39] with 3 objectives (Figure 4.7a), after the generation of a set B with 1000 non-dominated solutions (Figure 4.7b), by clustering method [27], 20 divided solution groups, which are represented by different scatter plots, are obtained. Thereafter, the clustering index (*CI*) is equal to the number of groups containing a solution from the set A, which is equal to 13, divide the total number of group, which is equal to 20, thus the *CI* of the set A is equal to 0.65.



(a) Solution set A



(b) Generated solution set B

Figure 4.7 An example *CI* evaluation of non-dominated solutions of the problem DTLZ2 with 3 objectives.

However, by restriction of time the number of generated solutions in a set B is limited, then the generated solutions may not cover their hyper-surface especially for an optimization problem with the large number of objectives. In addition, the clustering index (CI) needs many generated solutions; therefore, by limit of time it is suitable for optimization problems which can be written in explicit forms. In this thesis, the number of generated solutions in CI evaluation for any problems is equal to 4000.

4.5. Simulation Results and Discussions

The average distance to true Pareto-optimal front (M_1) and clustering index (CI) of all employed multi-objective evolutionary algorithms (MOEAs) for any benchmark problems, will be described in the following topics.

4.5.1. ZDT1-6

The problems ZDT1-6 are two-objective optimization problems, therefore only 3 MOEAs, which are NSGA-II, SPEA-II, and CCMOA are used for these problems.

In addition, mutation probability varies with characteristics of problems; for a multimodality problem, ZDT4, and a deceptive problem, ZDT5, the mutation probabilities for such problems should be more than the other problems. For CCMOA, only one part (species) in chromosome is performed mutation, then chromosome length of an individual to be performed mutation of CCMOA are much less than that of NSGA-II and SPEA-II. The mutation probabilities of CCMOA should be more than those of NSGA-II, SPEA-II.

The parameter setting for the MOEAs in all problems is shown in Table 4.2. Thereafter, the obtained results of M_1 and CI of all MOEAs of the problems ZDT1-6 are displayed in Table 4.3 and Table 4.4, respectively.

Table 4.2 Parameter setting of MOEAs for ZDT problems.

Parameter	**Setting and Value**
Chromosome Coding	Binary chromosome with chromosome length of a 900 (ZDT1-3); 300 (ZDT4; ZDT6); 80 (ZDT5)
Species Specification	An species represents of a decision variable (chromosome length = 30 for species of ZDT1-4,6, first species of ZDT5, 5 for other species of ZDT5)
Crossover method	Uniform crossover with probability = 1.0
Mutation method	Bit-flip mutation
Mutation probability	For NSGA-II and SPEA-II, mutation probability = 0.01 (ZDT1-3,6), 0.04 (ZDT4-5) For CCMOA, mutation probability = 0.033 (ZDT1-3,6, first species of ZDT5), 0.133 (ZDT4), 0.2 (other species of ZDT5).
Population size	100
Archive size (for SPEA-II, CCMOA)	100
No. of generations	600
No. of repeated runs	30

Table 4.3 Comparisons of average (Avg) and standard deviation (SD) values of M_1 of ZDT1-6.

M_1	NSGA-II		SPEA-II		CCMOA	
	Avg	SD	Avg	SD	Avg	SD
ZDT1	<u>0.0048</u>	0.0011	0.0052	0.0009	**0.0008**	0.0002
ZDT2	<u>0.0039</u>	0.0011	0.0041	0.0008	**0.0016**	0.0004
ZDT3	<u>0.0028</u>	0.0004	0.0032	0.0006	**0.0004**	0.0002
ZDT4	14.119	3.9287	<u>13.857</u>	4.5690	**0.0033**	0.0096
ZDT5	<u>0.0701</u>	0.0074	0.0835	0.0092	**0.0063**	0.0076
ZDT6	<u>0.0248</u>	0.0163	0.0383	0.0133	**0.0000**	0.0000

Table 4.4 Comparisons of average (Avg) and standard deviation (SD) values of *CI* of ZDT1-6.

CI	NSGA-II		SPEA-II		CCMOA	
	Avg	SD	Avg	SD	Avg	SD
ZDT1	0.8250	0.0281	**0.8880**	0.0211	0.8553	0.0196
ZDT2	0.8703	0.0366	**0.9140**	0.0225	0.8630	0.0256
ZDT3	0.8143	0.0292	**0.8680**	0.0339	*0.8147*	0.0416
ZDT4	0.7723	0.0898	0.7817	0.2204	**0.8320**	0.0552
ZDT5	0.9989	0.0059	**1.0000**	0.0000	0.9613	0.0230
ZDT6	*0.7842*	0.0481	0.7572	0.0993	**0.8660**	0.0148

From values of M_1, which is a minimization criterion, in Table 4.3, CCMOA outperforms NSGA-II and SPEA-II for any ZDT problems. It can also obtain solutions which close to true Pareto-optimal fronts in which the most average value of M_1 for CCMOA is only 0.0063 which is very close to 0. In the other hand, for *CI*, a maximum criterion, in Table 4.4, all MOEAs give good results for this criterion. SPEA-II gives the best average values of *CI* for 4 problems – ZDT1-3, and ZDT5, while CCMOA gives the best average values of *CI* for other problems – ZDT4, and ZDT6. The *CI* values of SPEA-II and CCMOA are better than those of NSGA-II. In overall, by this criterion, SPEA-II gives the best results; however it is marginally better than NSGA-II and CCMOA.

4.5.2. DTLZ1-7

The parameter setting for all employed MOEAs in all problems is shown in Table 4.5. The results of M_1 and *CI* of the DTLZ1-7 problems are displayed in Table 4.6-Table 4.13. Table 4.6-Table 4.9 display values of M_1 for the problems with 3-6 objectives respectively, while Table 4.10-Table 4.13 display values of *CI* for the problems with 3-6 objectives respectively. Figure 4.8 shows distribution of obtained non-dominated solutions of the three-objective DTLZ4 problem, of which Pareto optimal solutions are non-uniformly distributed along the Pareto front, of all employed MOEAs from one selected run.

Table 4.5 Parameter setting of MOEAs for DTLZ problems.

Parameter	**Setting and Value**
Chromosome Coding	Real-value representation
Species Specification	An species represents of a decision variable (chromosome length for a species = 1)
Crossover method	SBX recombination with probability = 1 [11]
Mutation method	Variable-wise polynomial mutation
Mutation probability	1/number of decision variables for NSGA-II, SPEA-II and COGA-II, 0.5 for CCMOA, and CCCOGA-II
Population size	100
Archive size[1]	100
No. of generations	800
No. of repeated runs	30

[1]for SPEA-II, CCMOA, COGA-II, and CCCOGA-II

Table 4.6 Comparisons of average (Avg) and standard deviation (SD) values of M_1 of DTLZ1-7 with 3 objectives.

Problems		NSGA-II	SPEA-II	CCMOA	COGA-II	CCCOGA-II
DTLZ1	Avg	0.0184	0.0197	<u>0.0012</u>	0.0099	**0.0009**
	SD	0.0494	0.0392	0.0012	0.0300	0.0002
DTLZ2	Avg	0.0090	0.0089	**0.0000**	0.0033	**0.0000**
	SD	0.0018	0.0014	0.0000	0.0006	0.0000
DTLZ3	Avg	0.0102	0.0324	<u>0.0025</u>	0.0079	**0.0018**
	SD	0.0133	0.0521	0.0024	0.0115	0.0022
DTLZ4	Avg	0.0088	0.0094	**0.0000**	0.0026	**0.0000**
	SD	0.0015	0.0018	0.0000	0.0008	0.0000
DTLZ5	Avg	0.0012	0.0010	**0.0000**	0.0004	**0.0000**
	SD	0.0003	0.0003	0.0000	0.0001	0.0000
DTLZ6	Avg	0.0641	0.1269	<u>0.0126</u>	0.0369	**0.0082**
	SD	0.0168	0.0331	0.0071	0.0097	0.0049
DTLZ7	Avg	0.0163	0.0162	<u>0.0062</u>	0.0069	**0.0038**
	SD	0.0034	0.0026	0.0019	0.0014	0.0016

Table 4.7 Comparisons of average (Avg) and standard deviation (SD) values of M_1 of DTLZ1-7 with 4 objectives.

Problems		NSGA-II	SPEA-II	CCMOA	COGA-II	CCCOGA-II
DTLZ1	Avg	228.41	343.97	1.0701	3.3139	**0.0046**
	SD	105.49	56.344	0.7916	4.8817	0.0074
DTLZ2	Avg	0.0395	0.0687	**0.0000**	0.0054	**0.0000**
	SD	0.0120	0.0201	0.0000	0.0015	0.0000
DTLZ3	Avg	351.18	316.23	0.6721	15.454	**0.0059**
	SD	59.885	49.012	0.6096	10.236	0.0259
DTLZ4	Avg	0.0416	0.1200	**0.0000**	0.0043	**0.0000**
	SD	0.0204	0.0276	0.0000	0.0018	0.0000
DTLZ5	Avg	1.5054	1.5696	1.7562	**1.4583**	1.4608
	SD	0.0643	0.0627	0.0709	0.0751	0.0994
DTLZ6	Avg	10.166	7.2514	5.7229	4.6022	**2.9480**
	SD	0.6209	0.4098	0.5276	0.4689	0.2493
DTLZ7	Avg	0.1076	0.1082	0.0426	0.0306	**0.0242**
	SD	0.0093	0.0136	0.0062	0.0047	0.0057

Table 4.8 Comparisons of average (Avg) and standard deviation (SD) values of M_1 of DTLZ1-7 with 5 objectives.

Problems		NSGA-II	SPEA-II	CCMOA	COGA-II	CCCOGA-II
DTLZ1	Avg	836.17	956.50	16.645	28.118	**0.2602**
	SD	89.932	68.417	5.9984	24.311	0.1715
DTLZ2	Avg	0.4600	1.3523	**0.0001**	0.0108	**0.0001**
	SD	0.0987	0.1092	0.0004	0.0034	0.0000
DTLZ3	Avg	843.19	1024.7	20.545	254.63	**0.3680**
	SD	96.118	88.152	6.6199	68.619	0.4140
DTLZ4	Avg	1.2253	1.5949	0.0003	0.0054	**0.0000**
	SD	0.2204	0.0742	0.0014	0.0017	0.0002
DTLZ5	Avg	2.2099	2.3259	2.5867	2.1381	**2.0071**
	SD	0.0863	0.1088	0.1207	0.0593	0.0289
DTLZ6	Avg	14.370	15.287	10.9600	6.5050	**6.0440**
	SD	0.2809	0.1743	0.3813	0.3127	0.2904
DTLZ7	Avg	0.2236	0.3917	0.0866	0.0652	**0.0497**
	SD	0.0328	0.0590	0.0096	0.0078	0.0079

Table 4.9 Comparisons of average (Avg) and standard deviation (SD) values of M_1 of DTLZ1-7 with 6 objectives.

Problems		NSGA-II	SPEA-II	CCMOA	COGA-II	CCCOGA-II
DTLZ1	Avg	1114.0	1230.98	<u>33.202</u>	161.84	**2.0919**
	SD	62.783	23.460	7.3610	78.768	2.9743
DTLZ2	Avg	1.6093	2.2346	<u>0.0169</u>	0.0237	**0.0001**
	SD	0.1630	0.0308	0.0195	0.0079	0.0000
DTLZ3	Avg	1223.3	1674.4	<u>41.537</u>	482.55	**2.1483**
	SD	74.211	64.004	15.012	57.373	2.0233
DTLZ4	Avg	1.9653	2.2703	<u>0.0458</u>	0.0088	**0.0000**
	SD	0.0834	0.0268	0.0300	0.0045	0.0000
DTLZ5	Avg	3.2738	4.5426	3.9159	<u>2.5996</u>	**2.5490**
	SD	0.2849	0.1072	0.1183	0.0794	0.0721
DTLZ6	Avg	19.451	20.362	18.273	<u>10.495</u>	**9.6950**
	SD	0.3376	0.2201	0.3880	0.4016	1.0441
DTLZ7	Avg	0.4441	0.9042	0.1492	<u>0.0836</u>	**0.0703**
	SD	0.0664	0.1348	0.0124	0.0127	0.0139

Table 4.10 Comparisons of average (Avg) and standard deviation (SD) values of *CI* of DTLZ1-7 with 3 objectives.

Problems		NSGA-II	SPEA-II	CCMOA	COGA-II	CCCOGA-II
DTLZ1	Avg	0.5467	<u>0.8420</u>	**0.8633**	0.8080	0.8297
	SD	0.0411	0.0816	0.0195	0.0551	0.0283
DTLZ2	Avg	0.5880	**0.8967**	<u>0.8713</u>	0.8440	0.8400
	SD	0.0322	0.0234	0.0234	0.0304	0.0232
DTLZ3	Avg	0.5573	0.7673	0.7773	<u>0.7900</u>	**0.8037**
	SD	0.0264	0.0571	0.0675	0.0574	0.0367
DTLZ4	Avg	0.6153	**0.8847**	<u>0.8620</u>	0.8473	0.8360
	SD	0.0229	0.0249	0.0743	0.0277	0.0356
DTLZ5	Avg	0.7940	**0.9200**	0.8420	<u>0.8900</u>	0.8797
	SD	0.0267	0.0174	0.0278	0.0217	0.0154
DTLZ6	Avg	0.6533	0.7833	0.8353	<u>0.8360</u>	**0.8700**
	SD	0.0416	0.1088	0.0243	0.0251	0.0373
DTLZ7	Avg	0.5633	**0.8340**	<u>0.8287</u>	0.8180	0.8173
	SD	0.0282	0.0243	0.0299	0.0307	0.0198

Table 4.11 Comparisons of average (Avg) and standard deviation (SD) values of *CI* of DTLZ1-7 with 4 objectives.

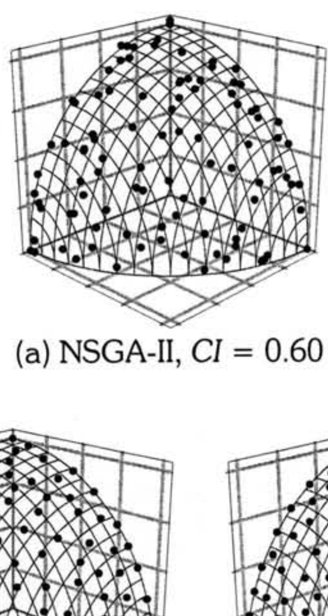
Problems		NSGA-II	SPEA-II	CCMOA	COGA-II	CCCOGA-II
DTLZ1	Avg	0.4053	0.7253	0.7473	**0.7847**	<u>0.7767</u>
	SD	0.0588	0.0226	0.0825	0.0646	0.0511
DTLZ2	Avg	0.5213	<u>0.8253</u>	**0.8327**	0.8220	0.8073
	SD	0.0352	0.0326	0.0215	0.0250	0.0250
DTLZ3	Avg	0.4720	0.7427	0.7580	<u>0.7800</u>	**0.8053**
	SD	0.0509	0.0314	0.0722	0.0638	0.0581
DTLZ4	Avg	0.5513	0.7973	**0.8360**	<u>0.8067</u>	0.7960
	SD	0.0415	0.0289	0.0157	0.0192	0.0265
DTLZ5	Avg	0.5007	0.7693	0.7640	<u>0.7913</u>	**0.7933**
	SD	0.0284	0.0291	0.0237	0.0252	0.0292
DTLZ6	Avg	0.4980	**0.7880**	0.7553	<u>0.7587</u>	0.7447
	SD	0.0300	0.0307	0.0756	0.0249	0.0478
DTLZ7	Avg	0.5400	**0.8187**	<u>0.7960</u>	0.7747	0.7580
	SD	0.0270	0.0270	0.0270	0.0344	0.0451

Table 4.12 Comparisons of average (Avg) and standard deviation (SD) values of *CI* of DTLZ1-7 with 5 objectives.

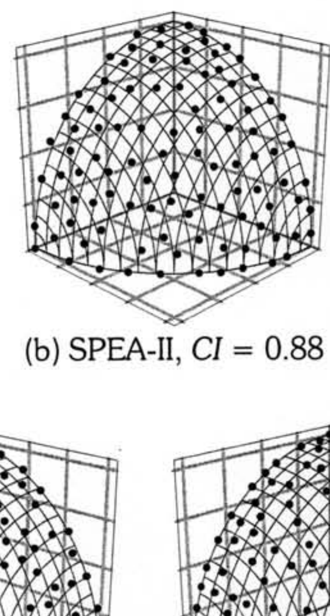
Problems		NSGA-II	SPEA-II	CCMOA	COGA-II	CCCOGA-II
DTLZ1	Avg	0.4700	**0.7400**	0.7087	<u>0.7353</u>	0.7207
	SD	0.0358	0.0507	0.0739	0.0557	0.0684
DTLZ2	Avg	0.4367	0.7780	<u>0.8287</u>	**0.8327**	0.7900
	SD	0.0358	0.0351	0.0287	0.0272	0.0330
DTLZ3	Avg	0.4527	0.6813	0.6747	**0.7033**	**0.7033**
	SD	0.0427	0.0310	0.0243	0.0976	0.0765
DTLZ4	Avg	0.4827	<u>0.7867</u>	0.7460	**0.8153**	0.7100
	SD	0.0282	0.0270	0.1040	0.0219	0.0708
DTLZ5	Avg	0.4740	**0.7620**	0.7253	<u>0.7613</u>	0.7093
	SD	0.0321	0.0313	0.0410	0.0292	0.0245
DTLZ6	Avg	0.5053	**0.8880**	0.7440	<u>0.7607</u>	0.7373
	SD	0.0323	0.0175	0.0447	0.0227	0.0539
DTLZ7	Avg	0.5033	**0.7793**	<u>0.7780</u>	0.7413	0.7293
	SD	0.0209	0.0347	0.0313	0.0373	0.0355

Table 4.13 Comparisons of average (Avg) and standard deviation (SD) values of *CI* of DTLZ1-7 with 6 objectives.

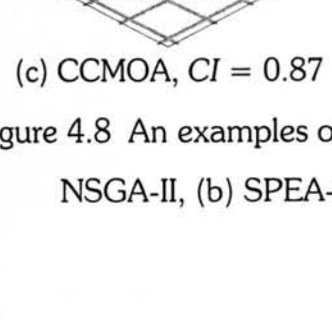
Problems		NSGA-II	SPEA-II	CCMOA	COGA-II	CCCOGA-II
DTLZ1	Avg	0.4740	**0.8353**	0.7260	0.7627	0.7547
	SD	0.0478	0.0358	0.0492	0.0432	0.0389
DTLZ2	Avg	0.4653	**0.8653**	0.8060	0.8320	0.8047
	SD	0.0426	0.0213	0.0317	0.0316	0.0350
DTLZ3	Avg	0.4413	**0.8047**	0.7387	0.7420	0.7407
	SD	0.0240	0.0371	0.0352	0.0448	0.0372
DTLZ4	Avg	0.5077	0.8120	0.7993	**0.8147**	0.7987
	SD	0.0233	0.0263	0.0248	0.0273	0.0254
DTLZ5	Avg	0.4340	**0.8420**	0.7493	0.8113	0.7407
	SD	0.0228	0.0167	0.0351	0.0326	0.0715
DTLZ6	Avg	0.4973	**0.8653**	0.7580	0.7593	0.7493
	SD	0.0221	0.0265	0.0404	0.0353	0.0744
DTLZ7	Avg	0.5157	0.7620	0.7613	**0.7727**	0.7273
	SD	0.0327	0.0291	0.0153	0.0307	0.0253



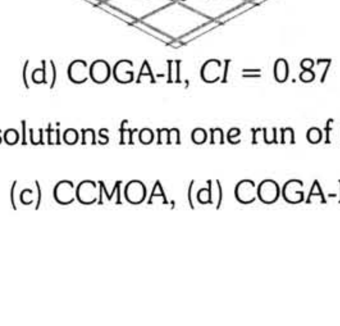
(a) NSGA-II, *CI* = 0.60



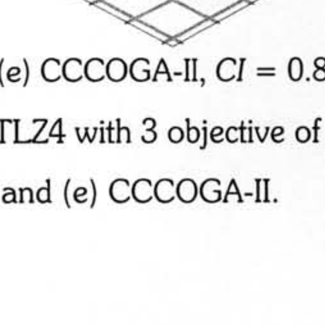
(b) SPEA-II, *CI* = 0.88



(c) CCMOA, *CI* = 0.87



(d) COGA-II, *CI* = 0.87



(e) CCCOGA-II, *CI* = 0.86

Figure 4.8 An examples of solutions from one run of DTLZ4 with 3 objective of (a) NSGA-II, (b) SPEA-II, (c) CCMOA, (d) COGA-II, and (e) CCCOGA-II.

From values of M_1 in Table 4.6-Table 4.9, the purposed MOEAs – CCOMA, COGA-II, and CCCOGA-II outperform the well-established MOEAs – NSGA-II and SPEA-II in which CCCOGA-II, which employs both co-operative co-evolution and rank assignment by winning score, give the best results of this criterion. Due to the use of co-operative co-evolution, CCMOA is superior to NSGA-II, and SPEA-II, and CCCOGA-II is also superior to COGA-II. By the use of winning score in rank assignment, COGA-II outperforms NSGA-II, and SPEA-II, and CCCOGA-II outperforms CCMOA. For DTLZ1-4 problems, which have no linkage among decision variables, CCMOA outperforms COGA-II. This shows that the use of co-operative co-evolution is more successful than the rank assignment by winning score in COGA-II for these problems. In the other hand, for problems having linkage among decision variables, DTLZ5-6 problems with 4-6 objectives, due to the linkage among decision variables, which diminishes performance of the co-operative co-evolution, rank assignment by winning score has more impact for these problems, COGA-II is therefore superior to CCMOA. Performance of NSGA-II and SPEA-II are comparable when the number of considering objectives is less than five. Once the number of objectives exceeds four, the performance of NSGA-II is noticeably better than that of SPEA-II.

From values of *CI* in Table 4.10-Table 4.13, in overall SPEA-II gives the best results of this criterion; however, it is marginally better than CCMOA, COGA-II, and CCCOGA-II, while NSGA-II give the worst results. Since NSGA-II uses crowding distance in selection and truncation, the crowding distance tries to uniformly spread solutions in each objective; it is suitable for problems of which Pareto-optimal front represents by a line such as two-objective problems which can be described by the good obtained results in Table 4.4. However, in problems of which Pareto-optimal front represents by a hyper-surface, in each objective *j* density of solutions is high near its best value, and low near its worst value. Although true Pareto-optimal fronts of DTLZ5 and DTLZ6 problems represent by a line, other optimal fronts far away from the true Pareto-optimal fronts of the problems represent by hyper-surfaces. *CI* of NSGA-II is very good for the three-objective DTLZ5 problem (Table 4.10) because NSGA-II can obtain solutions that close to true Pareto-optimal front (Table 4.6). However, for other DTLZ problems, NSGA-II gives the worst results. In the same way, for distribution of solutions of three-objective DTLZ4 problem in Figure 4.8, NSGA-II contributes the worst distribution; in the other hand, SPEA-II, CCMOA, COGA-II, and CCCOGA-II give good distributions of solutions.

4.5.3. Linked Problems – Linked DTLZ2 and Linked DTLZ6

There are six linked DTLZ problems, L_1-DTLZ2, L_2-DTLZ2, L_3-DTLZ2, L_1-DTLZ6, L_2-DTLZ6, and L_3-DTLZ6. The parameter setting for all employed MOEAs in the linked problems is the same as that of the normal DTLZ problems (Table 4.5). The results of M_1 and *CI* of all MOEAs of the problems are shown in Table 4.14-Table 4.17 and Table 4.18-Table 4.21, respectively.

Table 4.14 Comparisons of average (Avg) and standard deviation (SD) values of M_1 of linked DTLZ2 and linked DTLZ6 with 3 objectives.

Problems		NSGA-II	SPEA-II	CCMOA	COGA-II	CCCOGA-II
L_1-DTLZ2	Avg	0.0972	0.1409	<u>0.0515</u>	0.0938	**0.0493**
	SD	0.0994	0.0984	0.0377	0.0921	0.0249
L_2-DTLZ2	Avg	0.1534	0.0868	0.0944	<u>0.0850</u>	**0.0770**
	SD	0.0454	0.0167	0.0191	0.0249	0.0227
L_3-DTLZ2	Avg	0.2136	0.1422	**0.1154**	0.1168	<u>0.1165</u>
	SD	0.0616	0.0348	0.0321	0.0427	0.0636
L_1-DTLZ6	Avg	2.3566	<u>2.2415</u>	3.8050	**2.1992**	3.7519
	SD	0.5447	0.6758	0.2394	0.9213	0.1367
L_2-DTLZ6	Avg	0.2355	0.1505	0.1406	**0.1341**	<u>0.1376</u>
	SD	0.0330	0.0235	0.0295	0.0206	0.0306
L_3-DTLZ6	Avg	0.2676	0.1662	0.1615	<u>0.1473</u>	**0.1460**
	SD	0.0237	0.0185	0.0262	0.0140	0.0294

Table 4.15 Comparisons of average (Avg) and standard deviation (SD) values of M_1 of linked DTLZ2 and linked DTLZ6 with 4 objectives.

Problems		NSGA-II	SPEA-II	CCMOA	COGA-II	CCCOGA-II
L_1-DTLZ2	Avg	0.1158	0.0727	0.0435	0.0597	**0.0367**
	SD	0.1000	0.0641	0.0295	0.0850	0.0579
L_2-DTLZ2	Avg	0.6567	0.3758	0.3977	**0.2706**	0.3102
	SD	0.0646	0.0432	0.0461	0.0449	0.0472
L_3-DTLZ2	Avg	0.6198	0.3531	0.3272	0.2587	**0.2566**
	SD	0.0509	0.0444	0.0429	0.0269	0.0424
L_1-DTLZ6	Avg	0.7333	0.4089	0.6999	**0.3778**	0.6136
	SD	0.1478	0.2081	0.1092	0.1555	0.0992
L_2-DTLZ6	Avg	0.2136	0.1521	0.1775	0.1096	**0.1049**
	SD	0.0212	0.0132	0.0148	0.0094	0.0163
L_3-DTLZ6	Avg	0.2352	0.1649	0.1820	0.1136	**0.1117**
	SD	0.0252	0.0154	0.0174	0.0133	0.0095

Table 4.16 Comparisons of average (Avg) and standard deviation (SD) values of M_1 of linked DTLZ2 and linked DTLZ6 with 5 objectives.

Problems		NSGA-II	SPEA-II	CCMOA	COGA-II	CCCOGA-II
L_1-DTLZ2	Avg	0.5512	0.1977	0.1772	0.0691	**0.0455**
	SD	0.1683	0.0737	0.0624	0.0625	0.0309
L_2-DTLZ2	Avg	0.6494	0.4955	0.4824	**0.3028**	0.3072
	SD	0.0363	0.0463	0.0433	0.0269	0.0300
L_3-DTLZ2	Avg	0.6024	0.4974	0.4781	0.2931	**0.2920**
	SD	0.0605	0.0543	0.0409	0.0292	0.0262
L_1-DTLZ6	Avg	0.2781	0.1573	0.1561	0.0778	**0.0640**
	SD	0.0299	0.0351	0.0146	0.0224	0.0145
L_2-DTLZ6	Avg	0.2068	0.1883	0.1825	**0.0835**	0.0850
	SD	0.0126	0.0230	0.0111	0.0094	0.0058
L_3-DTLZ6	Avg	0.2232	0.1879	0.1967	0.0970	**0.0901**
	SD	0.0167	0.0273	0.0140	0.0077	0.0070

Table 4.17 Comparisons of average (Avg) and standard deviation (SD) values of M_1 of linked DTLZ2 and linked DTLZ6 with 6 objectives.

Problems		NSGA-II	SPEA-II	CCMOA	COGA-II	CCCOGA-II
L_1-DTLZ2	Avg	0.8975	0.8326	0.3077	0.1464	**0.1039**
	SD	0.0701	0.0552	0.0394	0.0584	0.0314
L_2-DTLZ2	Avg	0.5365	0.5291	0.4355	0.2739	**0.2543**
	SD	0.0395	0.0523	0.0273	0.0265	0.0225
L_3-DTLZ2	Avg	0.5723	0.6159	0.4969	**0.3022**	0.3118
	SD	0.0418	0.0474	0.0377	0.0212	0.0232
L_1-DTLZ6	Avg	0.2855	0.2878	0.2188	0.1116	**0.0830**
	SD	0.0157	0.0102	0.0215	0.0157	0.0147
L_2-DTLZ6	Avg	0.1358	0.1212	0.1048	0.0459	**0.0394**
	SD	0.0091	0.0063	0.0091	0.0068	0.0049
L_3-DTLZ6	Avg	0.1584	0.1559	0.1347	0.0620	**0.0575**
	SD	0.0165	0.0079	0.0076	0.0063	0.0058

Table 4.18 Comparisons of average (Avg) and standard deviation (SD) values of *CI* of linked DTLZ2 and linked DTLZ6 with 3 objectives.

Problems		NSGA-II	SPEA-II	CCMOA	COGA-II	CCCOGA-II
L_1-DTLZ2	Avg	0.4897	**0.8833**	0.8607	0.8287	0.8047
	SD	0.0329	0.0251	0.0345	0.0216	0.0854
L_2-DTLZ2	Avg	0.5440	**0.7873**	0.7273	0.7867	0.7627
	SD	0.0328	0.0279	0.0424	0.0331	0.0404
L_3-DTLZ2	Avg	0.5463	0.7647	0.7360	**0.7987**	0.7740
	SD	0.0334	0.0505	0.0287	0.0517	0.0508
L_1-DTLZ6	Avg	0.4723	0.7587	**0.8007**	0.7493	0.7420
	SD	0.0412	0.0582	0.0286	0.0488	0.0525
L_2-DTLZ6	Avg	0.5863	**0.7907**	0.7393	0.7620	0.7313
	SD	0.0253	0.0253	0.0239	0.0529	0.0262
L_3-DTLZ6	Avg	0.5647	**0.7760**	0.7227	0.7460	0.7320
	SD	0.0344	0.0306	0.0349	0.0514	0.0427

Table 4.19 Comparisons of average (Avg) and standard deviation (SD) values of *CI* of linked DTLZ2 and linked DTLZ6 with 4 objectives.

Problems		NSGA-II	SPEA-II	CCMOA	COGA-II	CCCOGA-II
L_1-DTLZ2	Avg	0.4680	**0.8407**	0.7290	<u>0.8147</u>	0.7573
	SD	0.0276	0.0248	0.0487	0.0328	0.0555
L_2-DTLZ2	Avg	0.4767	0.7107	<u>0.7260</u>	**0.7300**	0.7047
	SD	0.0359	0.0357	0.0356	0.0497	0.0459
L_3-DTLZ2	Avg	0.4833	<u>0.7280</u>	0.7020	**0.7347**	0.7113
	SD	0.0314	0.0313	0.0268	0.0393	0.0304
L_1-DTLZ6	Avg	0.4900	**0.8093**	0.7433	<u>0.7700</u>	0.7273
	SD	0.0326	0.0339	0.0314	0.0182	0.0261
L_2-DTLZ6	Avg	0.5380	**0.7887**	0.7300	<u>0.7427</u>	0.7247
	SD	0.0402	0.0196	0.0354	0.0286	0.0229
L_3-DTLZ6	Avg	0.5347	**0.7747**	0.7240	<u>0.7687</u>	0.7240
	SD	0.0369	0.0181	0.0251	0.0174	0.0294

Table 4.20 Comparisons of average (Avg) and standard deviation (SD) values of *CI* of linked DTLZ2 and linked DTLZ6 with 5 objectives.

Problems		NSGA-II	SPEA-II	CCMOA	COGA-II	CCCOGA-II
L_1-DTLZ2	Avg	0.4493	<u>0.7333</u>	0.7230	**0.7787**	0.7280
	SD	0.0357	0.0287	0.0445	0.0285	0.0432
L_2-DTLZ2	Avg	0.5003	**0.7347**	<u>0.7287</u>	0.7087	0.7073
	SD	0.0410	0.0388	0.0206	0.0287	0.0406
L_3-DTLZ2	Avg	0.4920	**0.7407**	0.7153	0.7213	<u>0.7227</u>
	SD	0.0316	0.0294	0.0315	0.0344	0.0221
L_1-DTLZ6	Avg	0.4660	**0.7860**	0.7367	<u>0.7573</u>	0.7053
	SD	0.0306	0.0304	0.0317	0.0316	0.0308
L_2-DTLZ6	Avg	0.5327	**0.7853**	<u>0.7660</u>	0.7193	0.7160
	SD	0.0279	0.0280	0.0287	0.0331	0.0257
L_3-DTLZ6	Avg	0.5340	**0.7767**	<u>0.7653</u>	0.7287	0.7107
	SD	0.0352	0.0336	0.0226	0.0344	0.0215

Table 4.21 Comparisons of average (Avg) and standard deviation (SD) values of *CI* of linked DTLZ2 and linked DTLZ6 with 6 objectives.

Problems		NSGA-II	SPEA-II	CCMOA	COGA-II	CCCOGA-II
L_1-DTLZ2	Avg	0.4417	**0.7727**	0.7227	<u>0.7253</u>	0.6933
	SD	0.0320	0.0333	0.0440	0.0285	0.0833
L_2-DTLZ2	Avg	0.4803	**0.7520**	0.7013	<u>0.7373</u>	0.7293
	SD	0.0378	0.0268	0.0243	0.0218	0.0381
L_3-DTLZ2	Avg	0.4557	**0.7507**	0.7067	<u>0.7313</u>	0.7153
	SD	0.0339	0.0272	0.0385	0.0185	0.0246
L_1-DTLZ6	Avg	0.4723	**0.8487**	0.7647	<u>0.7713</u>	0.7313
	SD	0.0339	0.0166	0.0360	0.0360	0.0453
L_2-DTLZ6	Avg	0.5183	**0.7753**	<u>0.7727</u>	0.7553	0.7237
	SD	0.0431	0.0334	0.0191	0.0373	0.0297
L_3-DTLZ6	Avg	0.4943	**0.7747**	<u>0.7737</u>	0.7347	0.7160
	SD	0.0370	0.0229	0.0300	0.0362	0.0270

The results of *CI* of the linked DTLZ problems in Table 4.18-Table 4.21 are similar to those of the normal DTLZ problems. In overall, SPEA-II gives the best results of *CI*; it marginally better than the proposed MOEAs, CCMOA, COGA-II, and CCCOGA-II, while NSGA-II give the worst results for this criterion.

On the other hand, the results of M_1 in Table 4.14-Table 4.17, are quite different from those of the normal DTLZ problems. Unlike in the normal DTLZ problems, performance of SPEA-II is better than that of NSGA-II, only the problems L_3-DTLZ2 and L_1-DTLZ6 with 6 objectives that average values of M_1 of NSGA-II are better than those of SPEA-II. By the use of winning score in rank assignment, COGA-II outperforms NSGA-II and SPEA-II for all problems. Similarly, mostly the results of M_1 of CCCOGA-II are better than those of CCMOA, only for L_3-DTLZ2 problem with three objectives, CCMOA gives result which is marginally better than that of CCCOGA-II. Therefore, these show that the use of winning score enhance performance of MOEAs not only for the normal DTLZ problems in the previous topic but also for the linked DTLZ problems.

For L_1-DTLZ2, the employed linked problem with lowest linkage among decision variables, MOEAs employing co-operative co-evolution, CCMOA and CCCOGA-II outperform NSGA-II and SPEA-II for any number of objectives. CCCOGA-II is better than COGA-II for the problem with any number of objectives, while CCMOA outperforms COGA-II only for the problem with three and four objectives. However, due to the effect of use of winning score, COGA-II outperforms CCMOA for the problem with larger numbers of objectives (5-6). For the problems with 5-6 objectives, mostly, CCMOA, and CCCOGA-II are superior to NSGA-II, and SPEA-II, only for L_3-DTLZ6 with 5 objectives that SPEA-II is slightly better than CCMOA. In addition, unlike the use of co-operative co-evolution of CCGA [43] for single-objective optimization problems, CCGA has only one best solution, any candidate solution, which may be new best solution, is the resulting solutions of combination of the solution with an individual any species in corresponding location of the species in objective calculation of the individual. For single-objective problems with linkage among decision variables, premature convergence may probably occurs due to that combination. Although CCGA-II [43] was proposed to solve this trouble by adding a candidate solution which is the resulting solution after combination of the individual with other randomly selected individuals from other species, it is impossible to obtain good candidate solution by this combination for problem with a solution is divided into a large number of species. In the other hand, the use of co-operative co-evolution for multi-objective optimization in this thesis is quite different from that of CCGA; a number of best solutions in archive are used to form candidate solutions. It, therefore, can be efficiently used in multi-objective optimization problems with linkage among decision variables which can be described by good obtained results for DTLZ5 with 6 objectives, and DTLZ6 and L_1-DTLZ2 with any number of objectives, and the other linked problems with 5-6 objectives.

This chapter presented the evaluation of performances of all employed multi-objective evolutionary algorithms (MOEAs), which are fast elitist non-dominated sorting genetic algorithm (NSGA-II) [28], improved strength Pareto evolutionary algorithm (SPEA-II) [30], co-operative co-evolutionary multi-objective algorithm (CCMOA), improved compressed-objective genetic algorithm (COGA-II, and co-operative co-evolutionary improved compressed-objective genetic algorithm (CCCOGA-II) in benchmark problems which are two-objective ZDT1-6 problems, DTLZ1-7 problems with 3-6 objectives and linked DTLZ2, linked DTLZ6 problems with 3-6 objectives. The proposed MOEAs – CCMOA, COGA-II, and CCCOGA-II, perform well on such test problems. The co-operative co-evolution which employed in CCMOA, and CCCOGA-II can successfully improve the performance of MOEAs not only in problems without linkage among decision variables but also in some problems with linkage among decision variables especially in such problems with large number of objectives. The rank assignment by winning score in COGA-II and CCCOGA-II, which is proposed for optimization problem with three-or-more objectives, can improve the performance of MOEAs for any problems with three-or-more objectives.

The next chapter will use MOEAs for multi-objective continuum topology optimization problems. It will show that the employed MOEAs can also improve the performance of MOEAs for such problems.