CHAPTER II

CLIQUE PARAMETERS OF THE K-POWER OF PATHS AND CYCLES

Our purpose in this chapter is to investigate the values or bounds of the clique covering numbers and the clique partition numbers of the k-power of paths and cycles. We separate this chapter into two sections. The first section contains results of paths and the other contains results of cycles.

2.1 Clique Parameters of the k-power of Paths

First, we recall a definition of a path and find the number of edges of the k-power of paths.

Definition 2.1.1. A *path* is a simple graph whose vertices can be ordered so that two vertices are adjacent if and only if they are consecutive in the list. The path with n vertices is denoted by P_n.

Figure 2.1: Path of n vertices (P_n)

Remark 2.1.2. For $n, k \in \mathbf{N}$ where $k \geq n-1$.

Since $diam(P_n) = n-1$, we have $k \geq diam(P_n)$. Hence P_n^k is a complete graph.

Proposition 2.1.3. *For $n, k \in \boldsymbol{N}$ where $1 \leq k < n-1$,*

$$|E(P_n^k)| = kn - \frac{k(k+1)}{2}.$$

Proof. Let $V(P_n) = \{v_1, v_2, ..., v_n\}$ and $E(P_n) = \{v_i v_{i+1} \mid i = 1, 2, ..., n-1\}$. For $i = 2, 3, ..., k$, edge set $E(P_n^i \backslash P_n^{i-1})$ is compossed of all edges in P_n^i connecting two vertices of distance i in P_n. Thus $E(P_n^i \backslash P_n^{i-1}) = \{v_1 v_{1+i}, v_2 v_{2+i}, ..., v_{n-i} v_n\}$, and we get $|E(P_n^i \backslash P_n^{i-1})| = n - i$. Note that

$$P_n^k = P_n + (P_n^2 \backslash P_n) + (P_n^3 \backslash P_n^2) + ... + (P_n^k \backslash P_n^{k-1}).$$

Hence,

$$\begin{aligned} |E(P_n^k)| &= |E(P_n)| + |E(P_n^2 \backslash P_n)| + |E(P_n^3 \backslash P_n^2)| + ... + |E(P_n^k \backslash P_n^{k-1})| \\ &= (n-1) + (n-2) + (n-3) + ... + (n-k) \\ &= kn - (1 + 2 + ... + k) \\ &= kn - \frac{k(k+1)}{2}. \end{aligned}$$

□

Example 2.1.4. The set $\mathcal{C} := \{P_5^2[\{v_1, v_2, v_3\}], P_5^2[\{v_2, v_3, v_4\}], P_5^2[\{v_3, v_4, v_5\}]\}$ forms a clique covering of P_5^2. Thus $cc(P_5^2) \leq |\mathcal{C}| = 3$.

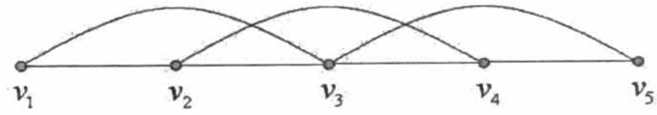

Figure 2.2: P_5^2

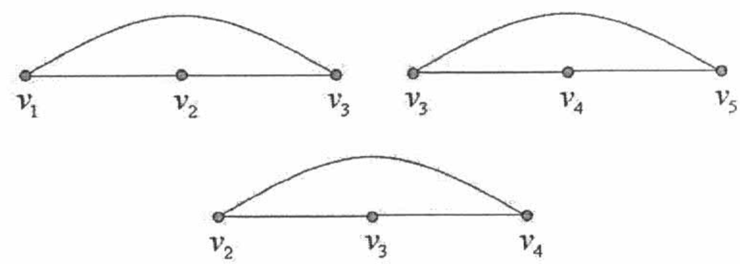

Figure 2.3: $\mathcal{C}$ in Example 2.1.4

Let $I = \{v_1v_3, v_2v_4, v_3v_5\}$. We have that I is a clique-independent set of P_5^2. Thus $cc(P_5^2) \geq |I| = 3$. Hence $cc(P_5^2) = 3$.

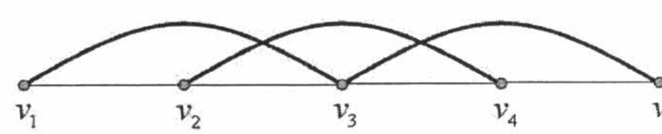

Figure 2.4: I in Example 2.1.4

The set $\mathcal{P} := \{P_5^2[\{v_1, v_2, v_3\}], v_2v_4, P_5^2[\{v_3, v_4, v_5\}]\}$ forms a clique partition of P_5^2. Thus $cp(P_5^2) \leq |\mathcal{P}| = 3$. Since $cp(P_5^2) \geq cc(P_5^2) = 3$, $cp(P_5^2) = 3$.

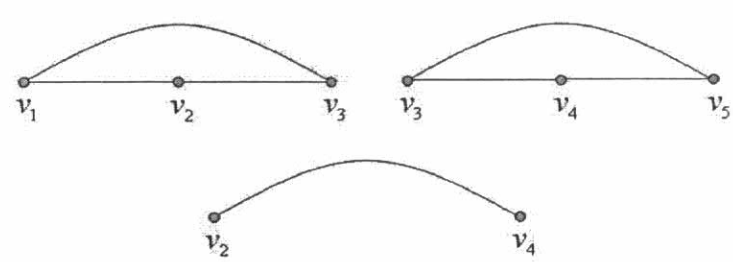

Figure 2.5: $\mathcal{P}$ in Example 2.1.4

Next, we give the values of the clique covering numbers of the k-power of paths.

Theorem 2.1.5. *For $n, k \in \boldsymbol{N}$,*

$$cc(P_n^k) = \begin{cases} 1 & \text{if} \quad k \geq n-1, \\ n-k & \text{if} \quad 1 \leq k < n-1. \end{cases}$$

Proof. Let $V(P_n) = \{v_1, v_2, ..., v_n\}$ and $E(P_n) = \{v_i v_{i+1} \mid i = 1, 2, ..., n-1\}$.

Case 1 : $k \geq n-1$.

Since $diam(P_n) = n-1$, P_n^k is a complete graph. Hence $cc(P_n^k) = 1$.

Case 2 : $1 \leq k < n-1$.

Consider a subset of the edge set of P_n^k, let $I_k = \{v_i v_{i+k} \mid i = 1, 2, ..., n-k\}$. Then $|I_k| = n-k$.

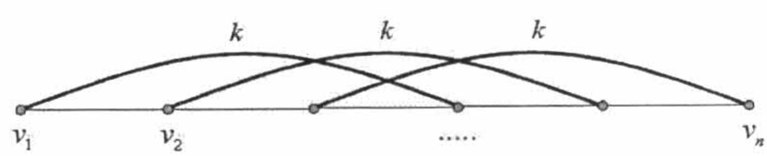

Figure 2.6: I_k in Theorem 2.1.5

Next, we show that I_k is a clique-independent set of P_n^k. Let $e_1, e_2 \in E(I_k)$ where $e_1 \neq e_2$, say $e_1 = v_p v_{p+k}$ and $e_2 = v_q v_{q+k}$ where $p < q$. Since the distance between v_p and v_{q+k} in P_n is more than k , v_p is not adjacent to v_{q+k} in P_n^k. By Remark 1.1.15, we have that e_1 and e_2 are clique-independent edges of P_n^k. Thus I_k is a clique-independent set of P_n^k. Hence $cc(P_n^k) \geq |I_k| = n-k$.

Consider subsets of the vertex set of P_n^k. For $i = 1, 2, ..., n-k$, let $V_i = \{v_i, v_{i+1}, ..., v_{i+k}\}$. Note that the distance between two vertices of V_i in P_n^k is at most k. Thus $B_i := P_n^k[V_i]$, an induced subgraph of P_n^k, is a clique in P_n^k.

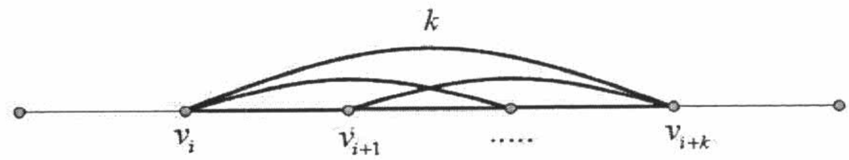

Figure 2.7: B_i in Theorem 2.1.5

Let $\mathcal{C} = \{B_1, B_2, ..., B_{n-k}\}$. We show that $\mathcal{C}$ is a clique covering of P_n^k. Let $e \in E(P_n^k)$. Then $e = v_iv_j$ for some $i, j \in \{1, 2, ..., n\}$. WLOG, assume $i < j$. Since $e \in E(P_n^k)$, the distance between v_i and v_j in P_n is at most k. If $1 \leq i < n-k$, then $e \in E(B_i)$. Otherwise $n - k \leq i \leq n - 1$, then $e \in E(B_{n-k})$. Thus, $\mathcal{C}$ is a clique covering of P_n^k. Hence $cc(P_n^k) \leq |\mathcal{C}| = n-k$. Therefore, $cc(P_n^k) = n-k$. □

We next show the values of the clique partition numbers of the square of paths.

Theorem 2.1.6. *For $n \in \mathbb{N}$,*

$$cp(P_n^2) = \begin{cases} 1 & \text{if } n = 1, 2, 3, \\ n-1 & \text{if } n = 2r \ , \ r \geq 2, \\ n-2 & \text{if } n = 2r+1 \ , \ r \geq 2. \end{cases}$$

Proof. Let $V(P_n) = \{v_1, v_2, ..., v_n\}$ and $E(P_n) = \{v_iv_{i+1} \mid i = 1, 2, ..., n-1\}$.

Case 1 : $n = 1, 2, 3$.

We have that P_n^2 is a complete graph. Hence $cp(P_n^2) = 1$.

Case 2 : $n = 2r$ where $r \geq 2$.

For $i = 1, 3, 5..., 2r-3$, let $B_i = P_n^2[\{v_i, v_{i+1}, v_{i+2}\}]$. Then B_i is a copy of K_3 and $|E(B_i)| = 3$.

Figure 2.8: B_i in Theorem 2.1.6

By Proposition 2.1.3, $|E(P_n^2)| = 2n - \frac{2(2+1)}{2} = 2n - 3 = 2(2r) - 3 = 4r - 3$. Let $H = P_n^2 \backslash (B_1 + B_3 + ... + B_{2r-3})$. Then $|E(H)| = (4r-3) - 3(r-1) = r$. We have that $\{B_1, B_3, ..., B_{2r-3}\} \cup E(H)$ forms a clique partition $\mathcal{P}$ of P_n^2 such that $|\mathcal{P}| = (r-1) + (r) = 2r - 1 = n - 1$. Thus $cp(P_n^2) \leq |\mathcal{P}| = n - 1$. By Theorem 2.1.5, $cc(P_n^2) = n - 2$. Hence $cp(P_n^2) \geq n - 2$. Next, we show that $cp(P_n^2) \neq n - 2$. Suppose that there exists $\mathcal{P}' = \{A_1, A_2, ...A_{n-2}\}$ a clique partition of P_n^2. For each $v_i, v_j \in V(P_n^2)$, v_i is not adjacent to v_j if the distance between v_i and v_j in P_n is more than 2. Thus P_n^2 does not contain a copy of K_m for all $m \geq 4$. We have that A_i is a copy of K_2 or K_3 for all $i = 1, 2, ..., n-2$.

From $|E(P_n^2)| = 2n - 3$, we get

$$3k_1 + k_2 = 2n - 3 \qquad (2.1)$$

$$k_1 + k_2 = n - 2 \qquad (2.2)$$

where k_1 is the number of copies of K_3's in $\mathcal{P}'$ and k_2 is the number of copies of K_2's in $\mathcal{P}'$.

From (2.1) - (2.2), we have that $2k_1 = n - 1 = 2r - 1$. This is a contradiction.

Thus there is no clique partition $\mathcal{P}'$ of P_n^2 such that $|\mathcal{P}'| = n - 2$. Hence $n - 2 < cp(P_n^2) \leq n - 1$. Therefore, $cp(P_n^2) = n - 1$.

Case 3 : $n = 2r + 1$ where $r \geq 2$.

For $i = 1, 3, 5..., 2r - 1$, let $B_i = P_n^2[\{v_i, v_{i+1}, v_{i+2}\}]$. Then B_i is a copy of K_3 and $|E(B_i)| = 3$. By Proposition 2.1.3, $|E(P_n^2)| = 2n - \frac{2(2+1)}{2} = 2n - 3 =$

$2(2r+1)-3 = 4r-1$. Let $H = P_n^2 \backslash (B_1 + B_3 + ... + B_{2r-1})$. Then $|E(H)| = (4r-1)-(3r) = r-1$. We have that $\{B_1, B_3, ..., B_{2r-1}\} \cup E(H)$ forms a clique partition $\mathcal{P}$ of P_n^2 such that $|\mathcal{P}| = (r)+(r-1) = 2r-1 = 2(\frac{n-1}{2})-1 = n-2$. Thus $cp(P_n^2) \leq |\mathcal{P}| = n-2$. By Theorem 2.1.5, $cc(P_n^2) = n-2$. Hence $cp(P_n^2) \geq n-2$. Therefore, $cp(P_n^2) = n-2$. □

To find the values of the clique covering numbers of the k-power of paths, we get the complete results for all $k \in \mathbf{N}$. However, for the values of the clique partition numbers of the k-power of paths, we have the results in case $k = 2$. Finding the values of the clique partition numbers of the k-power of paths where $k \geq 3$ is still an open problem.

2.2 Clique Parameters of the k-power of Cycles

Now, we recall a definition of a cycle and find the number of edges of the k-power of cycles.

Definition 2.2.1. A *cycle* is a graph with an equal number of vertices and edges whose vertices can be placed around a circle so that two vertices are adjacent if and only if they appear consecutively along the circle. The cycle with n vertices is denoted by C_n.

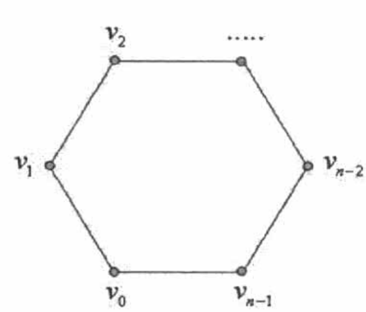

Figure 2.9: Cycle of n vertices (C_n)

Remark 2.2.2. For $n, k \in \mathbf{N}$ where $k > \frac{n-1}{2}$.

We have $k \geq \frac{n-1}{2} + 1 = \frac{n+1}{2} > \lfloor \frac{n}{2} \rfloor = diam(C_n)$. Hence C_n^k is a complete graph.

Proposition 2.2.3. *For $n, k \in \boldsymbol{N}$ where $k \leq \frac{n-1}{2}$,*

$$|E(C_n^k)| = kn.$$

Proof. Let $v_0 \in V(C_n^k)$. Let $V_1 = \{v_1, v_2, ..., v_k\}$ be a subset of $V(C_n^k)$ whose all elements are adjacent to v_0 along clockwise direction and $V_2 = \{v_{n-1}, v_{n-2}, ..., v_{n-k}\}$ a subset of $V(C_n^k)$ whose all elements are adjacent to v_0 along anticlockwise direction.

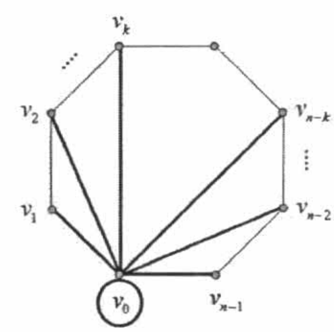

Figure 2.10: V_1 and V_2 of v_0 in Proposition 2.2.3

Since $k \leq \frac{n-1}{2}$, $n \geq 2k+1$. Then $n-k \geq k+1 > k$. We have that $V_1 \cap V_2 = \phi$. Thus $d(v_0) = 2k$. We have that $d(v) = 2k$ for all $v \in V(P_n^k)$. Hence

$$|E(C_n^k)| = \frac{\sum d(v)}{2} = \frac{2kn}{2} = kn.$$

□

Example 2.2.4. The set $\mathcal{C} := \{C_6^2[\{v_1, v_2, v_3\}], C_6^2[\{v_3, v_4, v_5\}], C_6^2[\{v_5, v_6, v_1\}], C_6^2[\{v_2, v_4, v_6\}]\}$ forms a clique covering of C_6^2. Thus $cc(C_6^2) \leq |\mathcal{C}| = 4$.

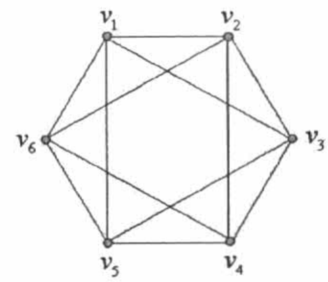

Figure 2.11: C_6^2

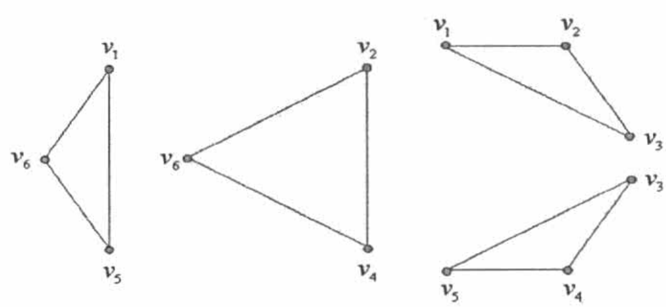

Figure 2.12: $\mathcal{C}$ in Example 2.2.4

Let $I = \{v_1v_2, v_2v_4, v_4v_5, v_5v_1\}$. Since $v_1v_4, v_2v_5 \notin E(C_6^2)$, I is a clique-independent set of C_6^2. Thus $cc(C_6^2) \geq |I| = 4$. Hence $cc(C_6^2) = 4$.

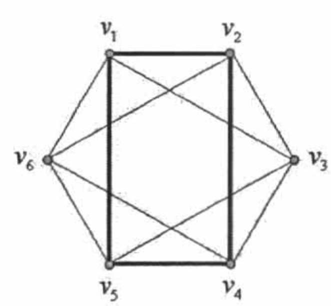

Figure 2.13: I in Example 2.2.4

The clique covering $\mathcal{C}$ in Example 2.2.4 is a clique partition of C_6^2 because each pair of clique in $\mathcal{C}$ do not share an edge. Thus $cp(C_6^2) \leq |\mathcal{C}| = 4$. Since $cp(C_6^2) \geq cc(C_6^2) = 4$, $cp(C_6^2) = 4$.

Next, we show upper bounds of the clique covering numbers of the k-power of cycles.

Lemma 2.2.5. *For $n, k \in \boldsymbol{N}$, $cc(C_n^k) \leq n$.*

Proof. **Case 1** : $k \geq \lfloor \frac{n}{2} \rfloor$.

Since $diam(C_n) = \lfloor \frac{n}{2} \rfloor$, C_n^k is a complete graph. Hence $cc(C_n^k) = 1 \leq n$.

Case 2 : $k < \lfloor \frac{n}{2} \rfloor$.

Let $v \in V(C_n^k)$. Let V_v be the subset of $V(C_n^k)$ containing v and all elements adjacent to v along clockwise direction. Note that the distance between two vertices of V_v in C_n is at most k. Thus $B_v := C_n^k[V_v]$, an induced subgraph of C_n^k, is a clique in C_n^k. Let $\mathcal{C} = \{B_v \mid v \in V(C_n^k)\}$. Then $|\mathcal{C}| = n$. We will next show that $\mathcal{C}$ is a clique covering of C_n^k. Let $e \in E(C_n^k)$. Assume that $e = vv'$ for some $v, v' \in V(C_n^k)$. Since $e \in E(C_n^k)$, the distance between v and v' in C_n is at most k. We have that $e \in E(B_v)$ or $e \in E(B_{v'})$. Thus $\mathcal{C}$ is a clique covering of C_n^k. Hence $cc(C_n^k) \leq |\mathcal{C}| = n$. □

In Theorem 2.2.6, we give the values of the clique covering numbers of the k-power of cycles where $k < \frac{n}{3}$.

Theorem 2.2.6. *For $n, k \in \boldsymbol{N}$ where $k < \frac{n}{3}$,*

$$cc(C_n^k) = n.$$

Proof. For $k = 1, 2, ..., \lfloor \frac{n}{3} \rfloor$, let $I_k = \{v_i v_j \mid v_i, v_j \in V(C_n^k) \text{ and } d_{C_n}(v_i, v_j) = k\}$. Note that $I_k \subseteq E(C_n^k)$. Since $v_i v_j = v_j v_i$ for all $v_i, v_j \in V(C_n^k)$, we have $|I_k| = \frac{2n}{2} = n$.

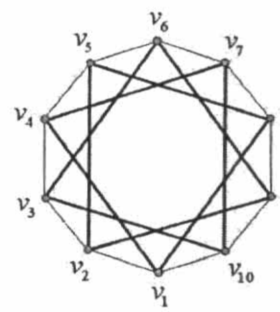

Figure 2.14: I_k in Theorem 2.2.6 where $k = 3$ and $n = 10$

We claim that I_k is a clique-independent set of C_n^k. Let $e_1, e_2 \in I_k$ where $e_1 \neq e_2$. Assume that $e_1 = v_{i_1}v_{j_1}$ and $e_2 = v_{i_2}v_{j_2}$ for some distinct vertices $v_{i_1}, v_{j_1}, v_{i_2}, v_{j_2} \in V(C_n^k)$.

Case 1 : e_1 is incident to e_2.

Assume $v_{i_1} = v_{i_2}$. Suppose that there exists a clique in C_n^k which is containing e_1 and e_2. We have $d_{C_n}(v_{j_1}, v_{i_1}) = k$, $d_{C_n}(v_{i_1}, v_{j_2}) = k$ and $d_{C_n}(v_{j_2}, v_{i_1}) \leq k$. Then $n = d_{C_n}(v_{i_1}, v_{j_1}) + d_{C_n}(v_{j_1}, v_{j_2}) + d_{C_n}(v_{j_2}, v_{i_1}) \leq 3k$. This is a contradiction because $n > 3k$. Hence e_1 and e_2 do not be contained in a clique in C_n^k.

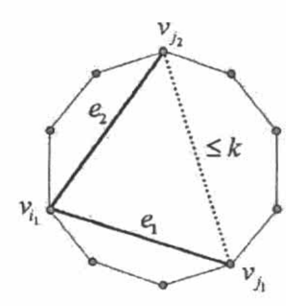

Figure 2.15: Case 1 in Theorem 2.2.6

Case 2 : e_1 is not incident to e_2.

Let A and B be the set of vertices from v_{i_1} along the cycle to v_{j_1} not include v_{i_1} and v_{j_1} in clockwise direction and anticlockwise direction, respectively.

Case 2.1 : v_{i_2} and v_{j_2} are in the different set of A and B.

Suppose that there exists a clique in C_n^k which is containing e_1 and e_2. We have $d_{C_n}(v_{i_1}, v_{j_1}) = k,\ d_{C_n}(v_{j_1}, v_{j_2}) \le k$ and $d_{C_n}(v_{j_2}, v_{i_1}) \le k$. Then $n = d_{C_n}(v_{i_1}, v_{j_1}) + d_{C_n}(v_{j_1}, v_{j_2}) + d_{C_n}(v_{j_2}, v_{i_1}) \le 3k$. This is a contradiction because $n > 3k$. Hence e_1 and e_2 do not be contained in a clique in C_n^k.

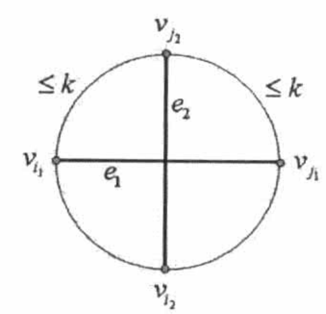

Figure 2.16: Case 2.1 in Theorem 2.2.6

Case 2.2 : v_{i_2} and v_{j_2} are in the same set of A or B.

Assume $d_{C_n}(v_{i_1}, v_{i_2}) \le d_{C_n}(v_{i_1}, v_{j_2})$. We have $d_{C_n}(v_{i_1}, v_{j_2}) \ge k+1 > k$. Thus v_{i_1} is not adjacent to v_{j_2} in C_n^k.

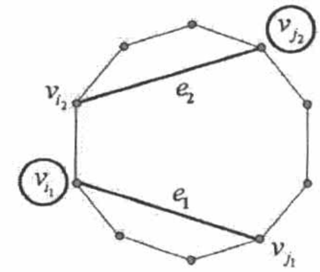

Figure 2.17: Case 2.2 in Theorem 2.2.6

By all cases, we can conclude that e_1 and e_2 are clique-independent edges of C_n^k. Hence, I_k is a clique-independent set of C_n^k. Thus $cc(C_n^k) \geq n$. By Lemma 2.2.5, $cc(C_n^k) \leq n$. Therefore, $cc(C_n^k) = n$. □

In Example 2.2.4, we have $cc(C_6^2) = 4$ ($n = 6$ and $k = 2$ i.e, $k = \frac{n}{3}$). By Theorem 2.2.5, we have $cc(C_n^k) \leq n$ for all $n, k \in \mathbf{N}$. Hence, the values of the clique covering numbers of the k-power of cycles maybe less than n where $k \geq \frac{n}{3}$.

In Theorem 2.2.7, we give the values of the clique covering numbers of the square of cycles.

Theorem 2.2.7. *For $n \in \mathbf{N}$,*

$$cc(C_n^2) = \begin{cases} 1 & \text{if } 1 \leq n \leq 5, \\ 4 & \text{if } n = 6, \\ n & \text{if } n \geq 7. \end{cases}$$

Proof. **Case 1** : $1 \leq n \leq 5$.

Since $diam(C_n) = \lfloor \frac{n}{2} \rfloor \leq 2$, C_n^2 is a complete graph. Hence $cc(C_n^2) = 1$.

Case 2 : $n = 6$.

From Example 2.2.4, $cc(C_6^2) = 4$.

Case 3 : $n \geq 7$.

Since $n \geq 7$, we have $\frac{n}{3} \geq \frac{7}{3} > 2$. By Theorem 2.2.6, $cc(C_n^2) = n$. □

We show the values of the clique partition numbers of the square of cycles in the next theorem.

Theorem 2.2.8. *For $n \in \boldsymbol{N}$,*

$$cp(C_n^2) = \begin{cases} 1 & if \;\; 1 \leq n \leq 5, \\ 4 & if \;\; n = 6, \\ n+1 & if \;\; n = 2r+1, r \geq 3, \\ n & if \;\; n = 2r, r \geq 4. \end{cases}$$

Proof. Let $V(C_n) = \{v_0, v_1, ..., v_{n-1}\}$ and $E(C_n) = \{v_iv_j \mid j \equiv i+1(mod\ n)\}$.

Case 1 : $1 \leq n \leq 5$.

Since $diam(C_n) = \lfloor \frac{n}{2} \rfloor \leq 2$, C_n^2 is a complete graph. Hence $cp(C_n^2) = 1$.

Case 2 : $n = 6$.

From Example 2.2.4, $cp(C_6^2) = 4$.

Case 3 : $n = 2r + 1$ where $r \geq 3$.

For $i = 0, 2, 4..., 2r-2$, let $B_i = C_n^2[\{v_i, v_{i+1}, v_{i+2}\}]$. Then B_i is a copy of K_3 and $|E(B_i)| = 3$. Since $n \geq 7$, $\frac{n-1}{2} \geq \frac{7-1}{2} = 3 > 2$. By Proposition 2.2.3, $|E(C_n^2)| = 2n = 2(2r+1) = 4r+2$. Let $H = C_n^2 \backslash (B_0 + B_2 + ... + B_{2r-2})$. Then $|E(H)| = (4r+2) - (3r) = r+2$.

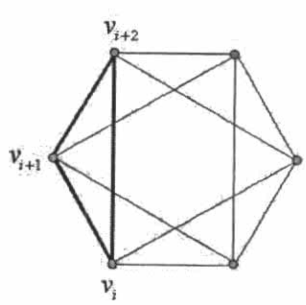

Figure 2.18: B_i in Theorem 2.2.8

We have that $\{B_0, B_2, ..., B_{2r-2}\} \cup E(H)$ forms a clique partition $\mathcal{P}$ of C_n^2 such that $|\mathcal{P}| = (r) + (r+2) = 2r + 2 = 2(\frac{n-1}{2}) + 2 = n + 1$. Thus $cp(C_n^2) \leq n+1$. By Theorem 2.2.7 and $n \geq 7$, $cc(C_n^2) = n$. Hence $cp(C_n^2) \geq n$. Next, we show that $cp(C_n^2) \neq n$. Suppose that there exists $\mathcal{P}' = \{A_1, A_2, ..., A_n\}$ a clique partition of C_n^2. For each $v_i, v_j \in V(C_n^2)$, v_i is not adjacent to v_j if the distance between v_i and v_j in C_n is more than 2. Thus C_n^2 does not contain a copy of K_m for all $m \geq 4$. We have that A_i is a copy of K_2 or K_3 for all $i = 1, 2, ..., n$.

From $|E(C_n^2)| = 2n$, we get

$$3k_1 + k_2 = 2n \tag{2.3}$$

$$k_1 + k_2 = n \tag{2.4}$$

where k_1 is the number of copies of K_3's in $\mathcal{P}'$ and k_2 is the number of copies of K_2's in $\mathcal{P}'$.

From (2.3) - (2.4), we have that $2k_1 = n = 2r+1$. This is a contradiction. Thus there is no a clique partition $\mathcal{P}'$ of C_n^2 such that $|\mathcal{P}'| = n$. We have $cp(C_n^2) \neq n$. Hence $n < cp(C_n^2) \leq n+1$. Therefore, $cp(C_n^2) = n+1$.

Case 4 : $n = 2r$ where $r \geq 4$.

For $i = 0, 2, 4..., 2r-2$, let $B_i = C_n^2[\{v_i, v_{i+1}, v_{i+2}\}]$. Then B_i is a copy of K_3 and $|E(B_i)| = 3$. Since $n \geq 8$, $\frac{n-1}{2} \geq \frac{7}{2} > 2$. By Proposition 2.2.3, $|E(C_n^2)| = 2n$ $= 2(2r) = 4r$. Let $H = C_n^2 \backslash (B_0 + B_2 + ... + B_{2r-2})$. Then $|E(H)| = (4r) - (3r) = r$. We have that $\{B_0, B_2, ..., B_{2r-2}\} \cup E(H)$ forms a clique partition $\mathcal{P}$ of C_n^2 such that $|\mathcal{P}| = r + r = 2r = n$. Thus $cp(C_n^2) \leq n$. By Theorem 2.2.7 and $n \geq 7$, $cc(C_n^2) = n$. Thus $cp(C_n^2) \geq n$. Hence $cp(C_n^2) = n$. $\square$

We have obtained the values of the clique covering numbers of the k-power of cycles where $k < \frac{n}{3}$. And since $diam(C_n) = \lfloor \frac{n}{2} \rfloor$, we have that C_n^k is a complete graph and $cc(C_n^k) = 1$ for all $k \geq \lfloor \frac{n}{2} \rfloor$. For the clique partition numbers of the k-power of cycles, we get the results for case $k = 2$. Open problems are to find values of the clique covering numbers of the k-power of cycles where $\frac{n}{3} \leq k < \lfloor \frac{n}{2} \rfloor$ and the values of the clique partition numbers of the k-power of cycles where $k \geq 3$.