CHAPTER III

THEORY AND CONCEPT

There are 5 types of petroleum hydrocarbon fluids. They are black oils, volatile oils, gas condensate (retrograde gases), wet gases, and dry gases. Typically, hydrocarbon-fluid compositions include hundreds or thousands of hydrocarbon and some of non-hydrocarbons e.g. carbon dioxide, nitrogen, and hydrogen sulfide. The physical properties of these mixtures depend on reservoir composition, temperature, and pressure. The reservoir temperature can be assumed to be constant due to a small temperature gradient as a function of depth. Each type of reservoir fluids requires different approaches by reservoir engineers and production engineers to maximize the production.

Gas-condensate reservoirs are one of the reservoirs which can be considered as the most complex reservoir. This reservoir has an unusual phase behaviors of reservoir fluids such as the condensing and vaporizing mechanism within the reservoirs. And when the reservoir pressure reaches the dew point pressure, the compositions, volumetric properties, and phase behavior of the hydrocarbon mixtures in gas condensate are changed. Gas cycling process can also change reservoir fluid composition and properties. Both the changes and characteristics have been simultaneously studied and described as theories, mathematics, and thermodynamic correlations which apply for compositional reservoir simulation.

3.1 Review of Gas-Condensate Reservoirs

Gas-condensate reservoirs have unique characteristics of phase behavior as illustrated in Figure 3.1, schematically. This reservoir initially contains single-phase gas (point A) where its pressure is above the dew point (point B). When the reservoirs are on production and its pressure declines continuously until reaches the dew point pressure, at this point, liquid begins to condense and dropout. At the envelope entry, the condensate liquid leaves out from gas phase with low liquid content. This phenomenon continues until the liquid saturation or maximum volume is reached (point C). When the reservoir fluid reaches the surface, its pressure will drop due to

hydrostatic and friction pressure loss and so does the temperature (point A). Gas-condensate reservoirs are important since condensate has higher value than natural gas. The production from these reservoir yields certain amounts of gas and condensate to supply worldwide consumption.

While the gas-condensate reservoirs are put on production, the reservoir pressure continuously declines until reaching the dew point pressure. Subsequently, gas starts condense and dropout in the reservoirs. This phenomenon will create a certain quantity of liquid in the reservoirs leading to a condensate blockage problem. This condensate blockage drastically reduces the well productivity and deliverability. Furthermore, some of the valuable condensate will be left in the reservoirs as residual oil. The pressure declining below the dew point pressure and the reduction in well productivity by condensate bank is predominantly a challenge to be avoided. And after that, reducing the reservoir pressure, from point C to point D, will cause liquid to vaporize. The composition of hydrocarbon gas will change during the retrograde condensation process. The condensate to gas ratio on the surface will change due to liquid dropout and re-vaporization in the reservoirs.

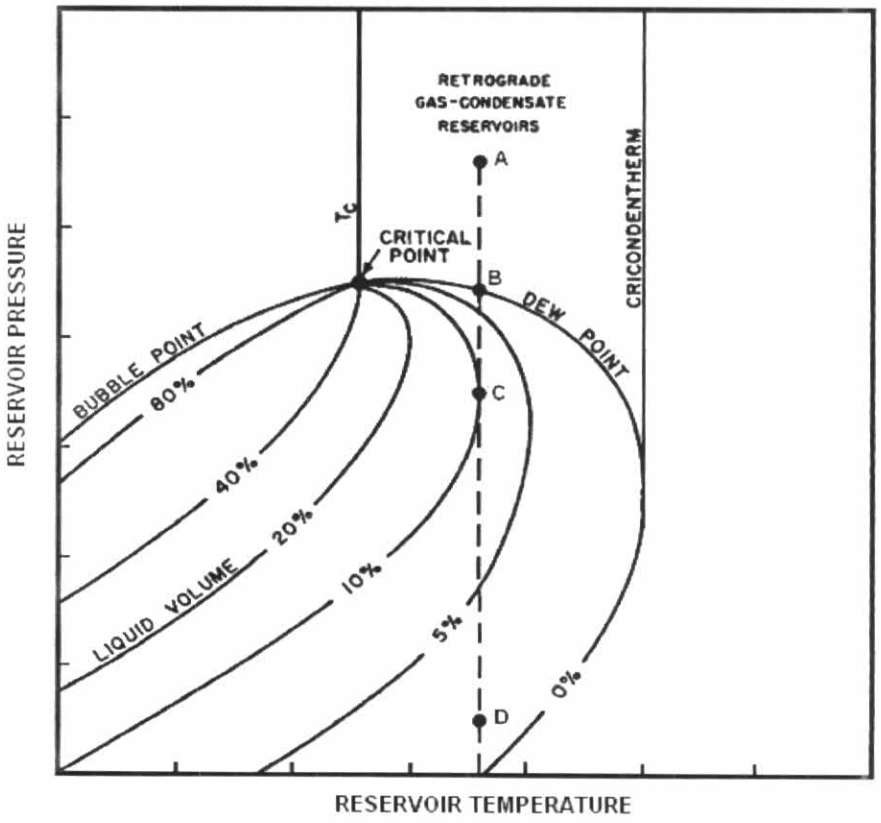

Figure 3.1: Pressure-Temperature phase diagram of gas-condensate.

To describe phase behavior and the changing of hydrocarbon composition within gas-condensate reservoirs, the correlations for PVT of gas and equations of state (EOS's) are presented.

3.1.1 Correlations for PVT Properties of Gas

The behavior of ideal gas is described by the experiments of Charles and Boyle, which is called ideal gas law, general gas law, or the perfect gas law as follows:

$$pV = nRT \tag{3.1}$$

where

p = pressure (psia)

V = volume (cu.ft)

n = number of pound moles

R = universal gas constant

T = absolute temperature (°R)

For gas mixture, Equation 3.1 does not give accurate prediction of gas behavior. So, an equation of state can be written by inserting a correction factor which is the compressibility factor, z.

$$z = \frac{V_{actual}}{V_{ideal}} \tag{3.2}$$

Then, the correct equation of state is the compressibility equation of state which can be written as:

$$pV = znRT \tag{3.3}$$

The volumetric properties of gas can be derived from the above equation.

Gas density is given by

$$\rho_g = \frac{pM_g}{zRT} \tag{3.4}$$

where

ρ_g = density of gas (lb/cu.ft)

M_g = apparent molecular weight of gas (lb/lb.mole)

The gas coefficient of isothermal compressibility is given by

$$c_g = -\frac{1}{V_g}\left(\frac{\partial V_g}{\partial p}\right) = \frac{1}{p} - \frac{1}{z}\left(\frac{\partial z}{\partial p}\right)_T \tag{3.5}$$

where

c_g = gas coefficient of isothermal compressibility (psi $^{-1}$)

V_g = volume of gas (cu.ft)

The dew point pressure can be found from a correlation based on composition and C_{7+} properties as proposed by Nemeth and Kennedy [10].

$$\begin{aligned}\ln p_d = {} & A_1\left[z_{c_2} + z_{co_2} + z_{H_2S} + z_{c_6} + 2\left(z_{c_3} + z_{c_4}\right) + z_{c_5} + 0.4z_{c_1} + 0.2z_{N_2}\right] \\ & A_1\gamma_{c_{7+}} + A_3\left[\frac{z_{c_1}}{\left(z_{c_{7+}} + 0.002\right)}\right] + A_4T + \left(A_5 z_{c_{7+}} M_{c_{7+}}\right) + \\ & A_6\left(z_{c_{7+}} M_{c_{7+}}\right)^2 + A_7\left(z_{c_{7+}} M_{c_{7+}}\right)^3 + A_8\left[\frac{M_{c_{7+}}}{\gamma_{c_{7+}} + 0.0001}\right] + \\ & A_9\left[\frac{M_{c_{7+}}}{\gamma_{c_{7+}} + 0.0001}\right]^2 + A_{10}\left[\frac{M_{c_{7+}}}{\gamma_{c_{7+}} + 0.0001}\right]^3 + A_{11}\end{aligned} \tag{3.6}$$

where

z	=	gas compressibility factor			
γ	=	specific gravity			
A_1	=	- 2.0623054,	A_2	=	6.6259728,
A_3	=	- 4.4670559 x 10^{-3}	A_4	=	1.0448346 x 10^{-4}
A_5	=	3.2673714 x 10^{-2}	A_6	=	- 3.6453277 x 10^{-3}
A_7	=	7.4299951 x 10^{-5}	A_8	=	- 1.1381195 x 10^{-1}
A_9	=	6.2476497 x 10^{-4}	A_{10}	=	- 1.0716866 x 10^{-6}
A_{11}	=	1.0746622 x 10^{1}			

3.1.2 Equation of State [EOS]

One of the important things in the calculation of Thermodynamics is the relation between pressure, temperature, and volume (density) of the system. This relation is called "Equation of State" which can be shown as:

$$f(P,V,T) = 0 \tag{3.7}$$

However, from Gibb's Phase Rule, pressure, temperature, and specific volume of system cannot be fixed at the same time. Then, the equation of state can be re-written in the single phase as below

$$P = f(T,V) \tag{3.8a}$$

or

$$V = f(P,T) \tag{3.8b}$$

or

$$T = f(P,V) \tag{3.8c}$$

The development of equation from ideal gas to real gas starts from compressibility equation of state which is Equation 3.3. From that, the volumetric behavior is calculated by solving that equation expressed in term of compressibility factor, z.

$$z^3 + E_2 z^2 + E_1 z + E_0 = 0 \tag{3.9}$$

where E_0, E_1, and E_2 are the constant as a functions of pressure, temperature, and phase composition.

3.1.2.1 van der Waals Equation of State

Van der Waals proposed a simple, qualitatively accurate relation between pressure, temperature and molar volume.

$$\left(p + \frac{a}{v_m^2}\right)(v_m - b) = RT \tag{3.10}$$

where V_m is molar volume.

The constants a and b in Van der Waals equation can be calculated from the critical properties as

$$a = 3p_c v_c^2 \tag{3.11a}$$

$$b = \frac{v_c}{3} \tag{3.11b}$$

Also written as

$$a = \frac{27}{64}\frac{R^2 T_c^2}{p_c} \tag{3.11c}$$

and $$b = \frac{1}{8}\frac{RT_c}{p_c} \tag{3.11d}$$

where

T_c = critical temperature (°R)

p_c = critical pressure (psia)

The Van der Waals equation of state was one of the first to perform markedly better than the ideal gas law. In this landmark equation, "a" is called the *attraction* parameter and "b" the *repulsion* parameter or the *effective molecular volume*. While the equation is definitely superior to the ideal gas law and does predict the formation of a liquid phase, the agreement with experimental data is limited for conditions where the liquid forms. The Van der Waals equation may be considered as the ideal gas law.

3.1.2.2 Redlich-Kwong Equation of State

Redlish and Kwong proposed an equation of state which takes the temperature dependency if molecular attraction term (a_T) is taken into account

$$\left[p+\frac{a_T}{(V_M+b)}\right](V_M-b) = RT \tag{3.12}$$

The equation for a_T is

$$a_T = a_C\alpha \tag{3.13}$$

where a_c is the value of a_T at the critical temperature and α is a nondimensional temperature-dependent term which has a value of 1.0 at the critical pressure. Setting the first and second derivatives of Equation 3.12 equal to zero at the critical point results in

$$b = 0.08664\frac{RT_c}{P_c} \tag{3.14a}$$

and

$$a_c = 0.42747\frac{R^2T_c^2}{P_c} \tag{3.14b}$$

3.1.2.3 Peng-Robinson Equation of State

Peng-Robinson equation is the one most widely used in petroleum calculations. Peng and Robinson proposed a different term for molecular attraction which improved the fluid density prediction. The equation is given as

$$p = \frac{RT}{v_m-b}-\frac{a\alpha}{v_m^2+2bv_m-b^2} \tag{3.15}$$

The constants in the equation are given by

$$a = \frac{0.45724 R^2 T_c^2}{p_c} \tag{3.15a}$$

$$b = \frac{0.07780 R T_c}{p_c} \tag{3.15b}$$

$$\alpha = \left(1 + \left(0.37464 + 1.54226\omega - 0.26992\omega^2\right)\left(1 - T_r^{0.5}\right)\right)^2 \tag{3.15c}$$

$$T_r = \frac{T}{T_c} \tag{3.15d}$$

where ω is the acentric factor of the species, and R is the universal gas constant.

The Peng-Robinson equation was developed in 1976 in order to satisfy the following goals:

The parameters should be expressible in terms of the critical properties and the acentric factor.

1. The model should provide reasonable accuracy near the critical point, particularly for calculations of the compressibility factor and liquid density.
2. The mixing rules should not employ more than a single binary interaction parameter, which should be independent of temperature, pressure, and composition.
3. The equation should be applicable to all calculations of all fluid properties in natural gas processes.

For the most part, the Peng-Robinson equation exhibits performance similar to the Soave equation, although it is generally superior in predicting the liquid densities of many materials, especially nonpolar ones.

For Peng-Robinson EOS, the mixing rule is used to determine the properties of the mixture as

$$b = \sum_k y_k b_k \tag{3.15e}$$

and

$$a_T = \sum_j \sum_k y_j y_k (a_{Tj}\, a_{Tk})^{1/2} (1 - \delta_{jk}) \tag{3.15f}$$

where the term δ_{jk} are binary interaction coefficients, which are assumed to be independent of pressure and temperature.

3.1.2.4 Martin-Volume Translation

Equation 3.9 is the generalized form using Martin's equation,

$$z^3 + E_2 z^2 + E_1 z + E_0 = 0$$

with

$$E_2 = (m_1 + m_2 - 1)B - 1 \tag{3.16}$$

$$E_1 = A - (m_1 + m_2 - m_1 m_2)B^2 - (m_1 + m_2)B \tag{3.17}$$

$$E_0 = -[AB + m_1 m_2 B^2 (B + 1)] \tag{3.18}$$

The coefficients m_1 and m_2 depend upon the equation used. Their values are tabulated in Table 3.1.

Table 3.1: The value coefficients, m_1 and m_2, for different equations of state.

Equation of state	m_1	m_2
Redlich-Kwong	0	1
Soave-Redlich-Kwong	0	1
Zudkevitch-Joffe-Redlich-Kwong	0	1
Peng-Robinson	$1+\sqrt{2}$	$1+\sqrt{2}$

To find the coefficients A and B, the fugacity coefficients are calculated using

$$\ln\left(\frac{f_i}{(px_i)}\right) = -\ln(z - B) + \frac{A}{(m_1 - m_2)B}\left(\frac{2\sum_i}{A} - \frac{B_i}{B}\right)\ln\left(\frac{z + m_2 B}{z + m_1 B}\right) + \frac{B_i}{B}(z - 1) \tag{3.19}$$

where

$$\sum_i = \sum_j A_{ij} x_j \tag{3.20}$$

$$A = \sum_{j=1}^{n}\sum_{k=1}^{n} x_j x_k A_{jk} \tag{3.21}$$

$$B = \sum_{j=1}^{n} x_j B_j \tag{3.22}$$

$$A_{jk} = \left(1-\delta_{jk}\right)\left(A_j A_k\right)^{1/2} \tag{3.23}$$

$$A_j = \Omega_a(T,j)\frac{P_{rj}}{T_{rj}^2} \tag{3.24}$$

$$B_j = \Omega_b(T,j)\frac{P_{rj}}{T_{rj}} \tag{3.25}$$

δ_{jk} are the binary interaction parameter (BIP's) where normally $\delta_{jj} = 0$ and $\delta_{jk} = \delta_{kj}$. To find the value of the constant, $\Omega_a(T,j)$, different equations are needed for different equations of state as shown below. Table 3.2 shows these constants used in different EOSs.

a) For Redlich-Kwong

$$\Omega_a(T,j) = \Omega_{a_0} T_{rj}^{-1/2} \tag{3.26a}$$

$$\Omega_b(T,j) = \Omega_{b_0} \tag{3.26b}$$

b) For Soave-Redlich-Kwong

$$\Omega_a(T,j) = \Omega_{a_0}\left[1+(0.48+1.574\omega_j - 0.176\omega_j^2)(1-T_{rj}^{1/2})\right]^2 \tag{3.27a}$$

$$\Omega_b(T,j) = \Omega_{b_0} \tag{3.27b}$$

c) For Zudkevitch-Joffe

$$\Omega_a(T,j) = \Omega_{a_0} F_{aj}(T) T_{rj}^{-1/2} \tag{3.28a}$$

$$\Omega_b(T,j) = \Omega_{b_0} F_{bj}(T) \tag{3.28b}$$

d) For Peng-Robinson

$$\Omega_a(T,j) = \Omega_{a_0}\left[1+(0.37464+1.54226\omega_j - 0.26992\omega_j^2)(1-T_{rj}^{1/2})\right]^2 \tag{3.29a}$$

$$\Omega_b(T,j) = \Omega_{b_0} \tag{3.29b}$$

Table 3.2: The value of Ω_a and Ω_b for different equations of state.

Equation of state	Ω_{a_0}	Ω_{b_0}
RK, SRK, ZJRK	0.4274802	0.08664035
PR	0.457235529	0.07796074

3.1.2.5 Mixing Rules

For the prevoius equation, Soave-Redlich-Kwong and Peng-Robinson, the mixture rules are recommended to be used.

$$b = \sum_j y_j b_j \tag{3.30a}$$

and

$$a_T = \sum_i \sum_j y_i y_j a_{Tij} \tag{3.30b}$$

where

$$a_{Tij} = (1-\delta_{ij})(a_{Ti}a_{Tj})^{1/2} \tag{3.31}$$

Then,

$$a_T = \sum_i \sum_j y_i y_j (1-\delta_{ij})(a_{Ti}a_{Tj})^{1/2} \tag{3.32}$$

where the term δ_{ij} are binary interaction coefficients, and they are supposed to be independent of pressure and temperature. For each binary pair, each equation of state has a different value of the binary interaction coefficient.

3.1.3 Flash Calculation

For a thermodynamic system, the fugacity in the liquid and vapor phase must be equal for each component. For a thermodynamic system at equilibrium, the equilibrium ratio or equilibrium constant (*K*-value) for each component can be defined as

$$K_i = \frac{y_i}{x_i} \tag{3.33}$$

The mole fractions of each component in the liquid phase (x_i) and vapor phase (y_i) are given as

$$x_i = \frac{z_i}{1+(K_i-1)\alpha} \tag{3.34}$$

$$y_i = \frac{K_i z_i}{1+(K_i-1)\alpha} \tag{3.35}$$

where

$$\alpha = \frac{lb\ mole\ of\ the\ vapor\ leaving\ the\ separator\ ,(v)}{lb\ mole\ of\ the\ fluid\ stream\ entering\ the\ separator\ ,(n)}$$

The K-values are used to specify inter-phase equilibrium. Equations 3.34 and 3.35 are used to specify the liquid and vapor composition directly.

In conclusion, the equations of states (EOS's) are used to calculate and describe the volumetric and phase behavior of gas-condensate reservoirs. The compositional simulator used in this study incorporates the equation of state with generalized form using Martin's equation. Flash calculation is used to specify liquid and gas composition obtained from the surface separator. The formula including equation of state are computed and processed till the final timestep. After that, the results from simulation are analyzed for optimal injection/production strategy.

3.2 Gas Injection in Gas-Condensate Reservoirs

The kinds of development of gas-condensate reservoirs are natural depletion and gas cycling which is described below:

(a) The reservoirs are produced with natural depletion. The produced fluids are gas and condensate liquid hydrocarbon that are ready to be sold.

(b) Full or partial pressure maintenance by re-injecting the produced gas back to the reservoirs. This process is called gas cycling. The main objective of gas injection is tried to maintain the reservoir pressure at a level that would avoid vapor phase to change to liquid in the gas-condensate reservoirs. In gas-condensate reservoirs, injection of produced gas can be a miscible process if the reservoir pressure is above the dew point; otherwise, the injected gas can revaporize liquids that dropout by retrograde condensation, which occurs when the reservoir pressure is below the dew point. A gas recycling project generally ends with a natural depletion after the dry gas breaks through the production wells and is continued until the economic limit is reached.

3.2.1 Flooding Pattern and Sweep Efficiency

Production wells and injection wells are typically arranged in a certain pattern for an EOR project. Normally patterns are,

(a) Two-spot

(b) Three-spot

(c) Regular four-spot and skewed four-spot

(d) Normal five-spot and inverted five-spot

(e) Normal seven-spot and inverted seven-spot

(f) Normal nine-spot and inverted nine-spot

(g) Direct line drive

(h) Staggered line drive

Different flooding patterns will result in different areal sweep efficiencies. The areal sweep efficiency at breakthrough was determined by various experimental

techniques. The value of such areal sweep efficiency was calculated for a mobility ratio of unity. The most popular and highest efficiency pattern is normal five spot.

The overall efficiency at breakthrough is defined as

$$E = E_A \times E_i \times E_d \tag{3.36}$$

where E_A is areal sweep efficiency, E_i is invasion or vertical sweep efficiency, and E_d is displacement efficiency.

Gas cycling is performed by injecting produced gas from production wells into injection wells. In this study, injection-production well arrangement is selected by considering the highest areal sweep efficiency. Normal five-spot flooding patterns have been studied and reported to have the highest sweep efficiency at breakthrough. Figure 3.2 shows the schematic of normal five-spot flooding pattern. There are four production wells at the corners and one injector at the center of a square.

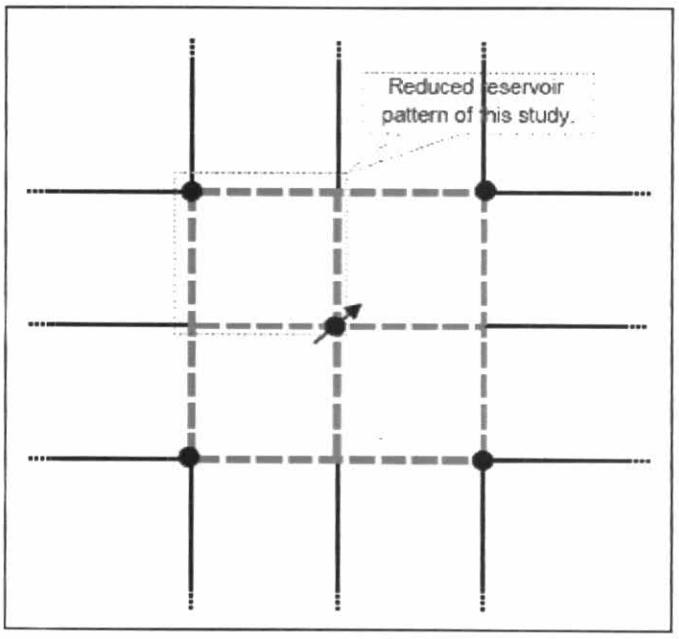

Figure 3.2: Flooding pattern (Normal Five-Spot).

3.2.2 Miscible Fluid Displacement

A miscible fluid displacement is defined as a displacement process where no phase boundary or interface exists between the displaced and displacing fluids. In this process, the displacing fluid is miscible, or will mix in all proportions with the displaced fluid. According to the definition described above, the main miscible fluid displacement processes are as follows: [11]

(1) High pressure dry gas miscible displacement

(2) Enriched gas miscible displacement

(3) Miscible slug flooding, where the leading edge of the slug is miscible with the displaced fluid

(4) Aqueous and oleic miscible slug flooding (such as several of the alcohols)

(5) Carbon dioxide, flue or inert gas displacements

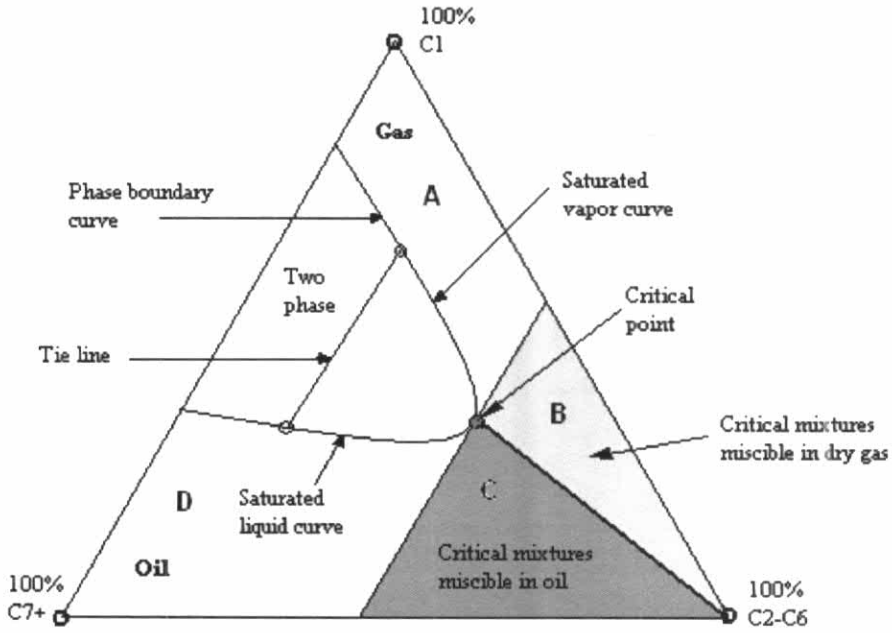

Figure 3.3: Ternary diagram for a hydrocarbon system.

The basic process of hydrocarbon miscible, fluid-fluid displacement can be described by using a ternary diagram for a hydrocarbon system as shown in Figure 3.3. The diagram is a visual picture of phase behavior for a system consisting of three components: methane (C_1), the intermediates ($C_2 - C_6$), and the heavier components (C_{7+}). The phase behavior of gases and liquids are a function of pressure, temperature, and composition. Region A is gaseous phase which is mostly methane (C_1), and region D is liquid which is mostly heptanes plus (C_{7+}). The intermediate component (region B and C), is possibly both phases, gaseous and liquid, depending on particular temperature and pressure. The phase boundary curve enfolds two phase region that gas and liquid exist. The tie line shows two points connection of saturated vapor curve and saturated liquid curve that represent the equilibrium composition of gas and liquid. At high pressure, dry gas displacement (lean gas injection), the injected gas represented by region A will be miscible with the displaced fluid that is rich in intermediate components represented by region C. The injected gas will vaporize the intermediate components in the displaced fluid until a zone is totally miscible.

3.3 Reservoirs Simulation

Reservoirs simulation is an efficiency tool which can describe the flow of multiple phases quantitatively in heterogeneous reservoirs. It is, therefore, used and applied widely in order to determine reservoirs performance, investment strategy, and reservoirs management.

The reservoirs model is constructed by an amount of established volume elements namely 'grid blocks' that represent the geological reservoirs construction. Appropriate equations were used to replace the partial differential equations that describe fluid flow in the reservoirs and can be solved numerically. Input data are required for each grid block. Similarly, well locations and well conditions have to be specified. The required flow in/out rate also has to be specified. The appropriate equations are solved for pressures and saturations for each block as well as the production of each phase from each well.

3.3.1 Compositional Simulation

The compositional simulator is designed to describe hydrocarbons fluid behavior when the composition of the hydrocarbons is changing with respect to temperature and pressure. The compositional simulator may employ the cubic equation of state, pressure dependent K-value, and black oil fluid treatment. It has several equations of state where is implemented from Martin's generalized equation. These include the Redlich-Kwong, Soave-Redlich-Kwong, Zudkevitch-Joffe, and Peng-Robinson.

The compositional simulation requires data which are phase-equilibrium information, phase densities, phase viscosities, and compositions of reservoirs hydrocarbons and injected hydrocarbons. Moreover, the separator conditions through which hydrocarbons are produced are also required.

3.3.2 Formulation of Simulation Equations

The basic equations of reservoirs simulation are obtained by combining material balance equation with conservation of momentum (Darcy's Law). These equations along with appropriate constraints, constitutive relations, and initial conditions can be solved by approximate numerical techniques to predict the performance of reservoirs under different operating conditions.

3.3.2.1 Mass Conservation

The differential equation in Cartesian coordinates is developed by considering a small block, depicted in Figure 3.4.

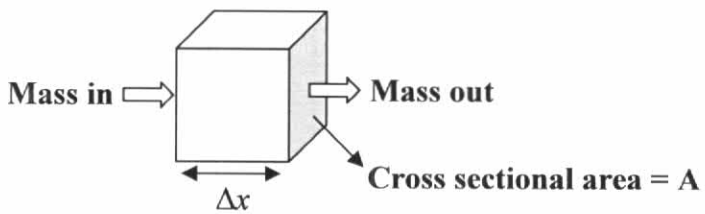

Figure 3.4: Mass balance on element

Consider an element of reservoirs through which a single phase is flowing in the x-direction. Then, at any instant:

Mass rate in – Mass rate out = Mass rate of accumulation

$$(v_x \rho_x \Delta y \Delta z) - (v_{x+\Delta x} \rho_{x+\Delta x} \Delta y \Delta z) = (\Delta x \Delta y \Delta z) \frac{\phi(\rho_{t+\Delta t} - \rho_t)}{\Delta t} \tag{3.37}$$

Dividing Equation (3.37) by Δx, Δy, Δz,

$$-\frac{(v_{x+\Delta x} \rho_{x+\Delta x}) - (v_x \rho_x)}{\Delta x} = \frac{\phi(\rho_{t+\Delta t} - \rho_t)}{\Delta t} \tag{3.38}$$

Taking the limit as $\{\Delta x, \Delta t\}$ go to zero simultaneously,

$$\frac{\partial(v\rho)}{\partial x} = -\phi\frac{\partial\rho}{\partial t} \tag{3.39}$$

This is the *continuity equation* in a linear system. Similarly:

$$\frac{\partial(v\rho)}{\partial y} = -\phi\frac{\partial\rho}{\partial t} \tag{3.40}$$

$$\frac{\partial(v\rho)}{\partial z} = -\phi\frac{\partial\rho}{\partial t} \tag{3.41}$$

Then, for three-dimensional flow,

$$\frac{\partial(v\rho)}{\partial x} + \frac{\partial(v\rho)}{\partial y} + \frac{\partial(v\rho)}{\partial z} = -\phi\frac{\partial\rho}{\partial t} \tag{3.42}$$

3.4 Economics Evaluation

Economic analysis will be performed for each specific scenario in order to assess pros and cons in term of monetary value among the studied cases. The result can be used as criteria for production making decision.

3.4.1 Time Value of Money

One of the most fundamental concepts in Economics is that money has a "time value." That is to say that money in hand today is worth more than money that is expected to be received in the future. The reason is straightforward: A dollar that you receive today can be invested or by percent interest or inflation such that you will have more than a dollar at some future time. This leads to the saying that we often use to summarize the concept of time value: "A dollar today is worth more than a dollar tomorrow." The differences in the values of the flows are based on the **time value of money.** [13]

In capital budgeting calculations, cash flows are usually brought back from various points in the future to the beginning of the project-time zero. As we can summarize that all cash flows are **discounted** to the present to obtain a present value. The two methods that do discount cash flows to a present value are **net present value (NPV)** and **internal rate of return (IRR)**. Both of these techniques satisfy the two major criteria requirements for evaluating the project: use of cash flow and use of the time value of money.

3.4.2 Economic Decision Tools

Various methods are used to make capital budgeting decisions, that is, to evaluate the worth of investment projects. The used methods must have sufficient requirement in order to judge and cover all economic criteria.

3.4.2.1 Net Present Value (NPV)

The net present value (NPV) of the project investment is the difference between the sum of the discounted cash flows which are expected from the investment, and the amount which is initially invested. It is a traditional valuation method (often for a project) used in the "discounted cash flow" measurement methodology. Therefore, the net present value of an investment can be interpreted as

$$NPV = \sum_{t=1}^{n} \frac{R_t}{(1+k)^t} - \sum_{t=0}^{n} \frac{O_t}{(1+k)^t} \tag{3.43}$$

where t = time period (e.g.,year)

n = last period of project

R_t = cash inflow in period t

O_t = cash outflow in period t

k = discount rate (cost of capital)

The discount rate, k, is the interest rate or inflation used to evaluate the project. This rate represents the minimum required rate of return. A positive value of NPV is financially acceptable for project investment. On the other hand, a negative value of NPV indicates financially unacceptable investment. If NPV is exactly zero, the project appears to be acceptable since the return equals the required rate of return.

3.4.2.2 Internal Rate of Return (IRR)

The internal rate of return (IRR) is the break-even interest rate which equates the present worth of a project's cash outflows to the present worth of its cash inflows. It is an indicator of the efficiency of an investment, as opposed to net present value (NPV), which indicates value or magnitude. The equation for calculating the internal rate of return is simply setting the NPV formula equal to zero:

$$\sum_{t=1}^{n} \frac{R_t}{(1+r)^t} - \sum_{t=0}^{n} \frac{O_t}{(1+r)^t} = 0 = NPV \qquad (3.44)$$

where r = internal rate of return (IRR)

Using Equation 3.44, the solving rate of return (r) is the internal rate of return (IRR) to the project. So the decision criteria for the internal rate of return is based on the comparison of IRR with discount rate used in that certain project. A project is a good investment proposition if its IRR is greater than the discount rate (IRR > k) that could be earned by alternate investments (investing in other projects, buying bonds, even putting the money in a bank account). If both values are equal (IRR = k), the project should be accepted at the margin or remain indifferent. If internal rate of return is less than discount rate (IRR < k), reject the project. Thus, the IRR should be compared to any alternate costs of capital including an appropriate risk premium.

3.4.2.3 Payback Period Analysis

Determining the relative worth of new investment by calculating the time it will take to pay back what it costs is the single most popular method of project screening. If the investment decision is made based solely on the pay back period, it considers only those projects with a pay back period shorter than the maximum acceptable payback period. One consequence of insisting that each proposed investment has a short payback period is that investors can assure themselves of being restored to their initial position within a short span of time. In equation form, the payback period is

$$\text{Payback Period} = \frac{\text{Initial cost}}{\text{Uniform annual benefit}}$$

The shorter the payback period, the more attractive the investment is.