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APPENDIX A : List of Filter Coefficient of Daubechies Wavelet

Numerical data derived from results published by Daubechies[1988], restricted to even values of N in the range 2 to 20.

N = 2

C(1)	=	1	C(2)	=	1

N = 4

C(1)	=	(1+SQRT(3))/4	C(2)	=	(3 +SQRT(3))/4
C(3)	=	(3-SQRT(3))/4	C(4)	=	(1-SQRT(3)/4

N = 6

S	=	SQRT(5 + 2*SQRT(10))			
C(1)	=	(1+SQRT(10)+S)/16	C(2)	=	(5+SQRT(10)+3*S)/16
C(3)	=	(5-SQRT(10) +S) / 8	C(4)	=	(5-SQRT(10) - S) / 8
C(5)	=	(5+SQRT(10) -3*S) / 16	C(6)	=	(1-SQRT(10) - S) / 16

N = 8

C(1)	=	0.325803428051	C(2)	=	1.010945715092
C(3)	=	0.892200138246	C(4)	=	-0.039575026236
C(5)	=	-0.264507167396	C(6)	=	0.043616300475
C(7)	=	0.04503601071	C(8)	=	-0.014986989330

N = 10

C(1)	=	0.226418982583	C(2)	=	0.853943542705
C(3)	=	1.024326944260	C(4)	=	0.195766961347
C(5)	=	-0.342656715382	C(6)	=	-0.45601131884
C(7)	=	0.109702658642	C(8)	=	-0.008826800109
C(9)	=	-0.017791870102	C(10)	=	0.004717427938

N = 12

C(1)	=	0.157742432003	C(2)	=	0.699503814075
C(3)	=	1.062263759882	C(4)	=	0.445831322930
C(5)	=	-0.319986598891	C(6)	=	-0.183518064060
C(7)	=	0.137888092974	C(8)	=	0.03893209708
C(9)	=	-0.44663748331	C(10)	=	0.000783251152
C(11)	=	0.006756062363	C(12)	=	-0.001523533805

N = 14

C(1)	=	0.110099430746	C(2)	=	0.560791283626
C(3)	=	1.031148491636	C(4)	=	0.664372482211
C(5)	=	-0.203513822463	C(6)	=	-0.316835011281
C(7)	=	0.100846465010	C(8)	=	0.114003445160
C(9)	=	-0.053782452590	C(10)	=	-0.23439941565
C(11)	=	0.17749792379	C(12)	=	0.000607514996
C(13)	=	-0.002547904718	C(14)	=	0.000500226853

N=16

C(1)	=	0.76955622108	C(2)	=	0.442467247152
C(3)	=	0.955486150427	C(4)	=	0.827816532422
C(5)	=	-0.022385735333	C(6)	=	-0.401658632782
C(7)	=	0.000668194093	C(8)	=	0.182076356847
C(9)	=	-0.024563901046	C(10)	=	-0.062350206651
C(11)	=	0.019772159296	C(12)	=	0.012368844819
C(13)	=	-0.0068877192556	C(14)	=	-0.000554004548
C(15)	=	0.000955229711	C(16)	=	-0.000166137261

N=18

C(1)	=	0.053850349589	C(2)	=	0.344834303815
C(3)	=	0.855349064359	C(4)	=	0.929545714366
C(5)	=	0.188369549509	C(6)	=	-0.414751761802
C(7)	=	-0.136953549025	C(8)	=	0.210068342279
C(9)	=	0.43452675461	C(10)	=	-0.095647264120
C(11)	=	0.000354892813	C(12)	=	0.031624165853
C(13)	=	-0.006679620227	C(14)	=	-0.006054960574
C(15)	=	0.002612967280	C(16)	=	0.000325814672
C(17)	=	-0.000356329759	C(18)	=	0.000055645514

N = 20

C(1)	= 0.037717157593	C(2)	= 0.266122182794
C(3)	= 0.745575071487	C(4)	= 0.973628110734
C(5)	= 0.397637741770	C(6)	= -0.353336201794
C(7)	= -0.277109878720	C(8)	= 0.180127448534
C(9)	= 0.131602987102	C(10)	= -0.100966571196
C(11)	= -0.041659248088	C(12)	= 0.046969814097
C(13)	= 0.005100436968	C(14)	= -0.015179002335
C(15)	= 0.001973325365	C(16)	= 0.002817686590
C(17)	= -0.000969947840	C(18)	= -0.000164709006
C(19)	= 0.000132354366	C(20)	= -0.000018758416

APPENDIX B : About Eigenvalue Matrix for Generating Wavelet Recursion and Diadic Interpolation Method(Kaiser 1994)

To obtain the scaling function ϕ at argument being integers, note that for $x=t$ the dilation equation becomes

$$\phi(t) \quad = \quad \sum_{n=0}^{N-1} c_n \phi(2t-n) \quad = \quad \sum_{n=0}^{N-1} c_{2t-n}\phi(n) \qquad \text{(B1)}$$

where N is an even number. and with convention that c_m =0 unless $0 \le m \le N-1$. Both sides involve the value of ϕ at argument being integers, hence (B1) is a linear equation for the unknown $\phi(t)$. If ϕ is supported in [0, N-1] and continuous, then only $\phi(1)$, $\phi(2)$,..., $\phi(N-2)$ are (possible) nonzero. Thus we have M=N-2 unknowns, and (B1) becomes an M*M matrix equation, stating that the column vector u= $[\phi(1)...\phi(M)]^{*}$ is an eigenvector of the matrix $A_{t,n} \equiv c_{2t-n}$ n,k =1,...,M. The normalization of u is determined by partition of unity. This gives the exact values of ϕ at argument being integers. From here the computation is completely straightforward. At $t=n+0.5$, the dilation equation gives

$$\phi(t+0.5) \quad = \quad \sum_{n=0}^{N-1} c_n \phi(2t+1-n) \qquad \text{(B2)}$$

Which determines the exact values of ϕ at the half-integers, and so on. Any solution of $Au=u$ thus determine ϕ at all diadic rational $t=n/2^m (t,m \in Z, m \ge 0)$, which form a dense set. If ϕ is continuous, so we can calculate ϕ everywhere. Conversely, any solutions of the dilation equation determines a solution of $Au=u$. Since we already know that the dilation equation has a unique normalized solution, it follows that u is unique as well, i.e., the eigenvalue 1 of A is nondegenerate.

For instance the scaling function has $N=4$, then the set of c_n is

$$c_0=\frac{1+\sqrt{3}}{4}, \qquad c_1=\frac{3+\sqrt{3}}{4}, \qquad c_2=\frac{3-\sqrt{3}}{4}, \qquad c_3=\frac{1-\sqrt{3}}{4} \tag{B3}$$

$Au=u$ becomes

$$\begin{bmatrix} c_1 & c_0 \\ c_3 & c_2 \end{bmatrix}\begin{bmatrix} \phi(1) \\ \phi(2) \end{bmatrix} = \begin{bmatrix} \phi(1) \\ \phi(2) \end{bmatrix} \tag{B4}$$

Together with normalization $\sum_n \phi(n)=1$, this gives the unique solution

$$\phi(1)=\frac{1+\sqrt{3}}{2}, \qquad \phi(2)=\frac{1-\sqrt{3}}{2} \tag{B5}$$

Hence

$$\begin{aligned} \phi(0.5) &= \sum_{n=0}^{3} c_n\phi(1-n) = c_0\phi(1) = \frac{2+\sqrt{3}}{4} \\ \phi(1.5) &= \sum_{n=0}^{3} c_n\phi(3-n) = c_1\phi(2)+c_2\phi(1) = 0 \\ \phi(2.5) &= \sum_{n=0}^{3} c_n\phi(5-n) = c_3\phi(2) = \frac{2-\sqrt{3}}{4} \end{aligned} \tag{B6}$$

APPENDIX C : The result of forward discrete wavelet transform

Input signal are two cycle square wave (128 points) data and the result of forward discrete wavelet transform by using DAUB4 has been shown.

Input data	Wavelet Coefficients
1	-1.000000000000000E-14
1	0.000000000000000E+00
1	5.448337078094480E+00
1	5.448337078094480E+00
1	-3.219870328903190E+00
1	3.219870328903190E+00
1	-3.219870328903190E+00
1	3.219870328903190E+00
1	4.003402590751640E-01
1	-1.845612525939940E+00
1	-4.003402590751640E-01
1	1.845612525939940E+00
1	4.003402590751640E-01
1	-1.845612525939940E+00
1	-4.003402590751640E-01
1	1.845612525939940E+00
1	3.700000000000000E-15
1	3.700000000000000E-15
1	2.264665961265560E-01
1	-1.321802020072930E+00

1	-3.700000000000000E-15
1	-3.700000000000000E-15
1	-2.264665961265560E-01
1	1.321802020072930E+00
1	3.700000000000000E-15
1	3.700000000000000E-15
1	2.264665961265560E-01
1	-1.321802020072930E+00
1	-3.700000000000000E-15
1	-3.700000000000000E-15
1	-2.264665961265560E-01
1	1.321802020072930E+00
-1	2.700000000000000E-15
-1	2.700000000000000E-15
-1	2.700000000000000E-15
-1	2.700000000000000E-15
-1	2.700000000000000E-15
-1	2.700000000000000E-15
-1	9.150631725788120E-02
-1	-9.575317502021780E-01
-1	-2.700000000000000E-15
-1	-2.700000000000000E-15
-1	-2.700000000000000E-15
-1	-2.700000000000000E-15
-1	-2.700000000000000E-15
-1	-2.700000000000000E-15
-1	-9.150631725788120E-02

-1	9.575317502021780E-01
-1	2.700000000000000E-15
-1	2.700000000000000E-15
-1	2.700000000000000E-15
-1	2.700000000000000E-15
-1	2.700000000000000E-15
-1	2.700000000000000E-15
-1	9.150631725788120E-02
-1	-9.575317502021780E-01
-1	-2.700000000000000E-15
-1	-2.700000000000000E-15
-1	-2.700000000000000E-15
-1	-2.700000000000000E-15
-1	-2.700000000000000E-15
-1	-2.700000000000000E-15
-1	-9.150631725788120E-02
-1	9.575317502021780E-01
1	1.800000000000000E-15
1	1.800000000000000E-15
1	1.800000000000000E-15
1	1.800000000000000E-15
1	1.800000000000000E-15
1	1.800000000000000E-15
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1	1.800000000000000E-15

1	1.800000000000000E-15
1	1.800000000000000E-15
1	1.800000000000000E-15
1	1.800000000000000E-15
1	1.800000000000000E-15
1	-7.071067690849300E-01
1	-1.800000000000000E-15
1	-1.800000000000000E-15
1	-1.800000000000000E-15
1	-1.800000000000000E-15
1	-1.800000000000000E-15
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1	-1.800000000000000E-15
1	-1.800000000000000E-15
1	-1.800000000000000E-15
1	-1.800000000000000E-15
1	7.071067690849300E-01
-1	1.800000000000000E-15
-1	1.800000000000000E-15
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-1	1.800000000000000E-15
-1	-7.071067690849300E-01
-1	-1.800000000000000E-15
-1	-1.800000000000000E-15
-1	-1.800000000000000E-15
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-1	-1.800000000000000E-15
-1	-1.800000000000000E-15
-1	-1.800000000000000E-15

APPENDIX D : Detail in Calculating the Landau-Ginzburg Hamiltonian

In our system we choose a Gaussian distribution in wavelet space, charcaterized by the expectation values

$$< \hat{S}_{t_1}^{(n_1)}(x_1')\hat{S}_{t_2}^{(n_2)}(x_2') > \quad = \quad \delta_{n_1,n_2}\delta_{x_1',x_2'}A_{t_1t_2}^{(n_1)} \qquad \text{(D1)}$$

and the wavelet expansion of the field $S(x)$ in wavelet basis is defined by

$$S(x) \quad = \quad \sum_{n}\sum_{t}^{n}\sum_{x\in\Gamma^n}\hat{S}^{(n)}(x')\psi_t^{(n)}(x')(x)+S \qquad \text{(D2)}$$

when substituting this into the Landau Ginzburg Hamiltonian for the kinetic term we get

$$H_1 \quad = \quad \int d^D x[\nabla S]^2$$

$$= \quad \sum_{n_1n_2}\sum_{t_1t_2}\sum_{x_1'x_2'}\hat{S}_{t_1}^{n_1}(x_1')\hat{S}_{t_2}^{n_2}(x_2')\left(-\hat{\Delta}_{t_1t_2}^{(n_1n_2)}(x_1'x_2')\right) \qquad \text{(D3)}$$

with the representation of Laplace operator in wavelet space.

$$\hat{\Delta}_{t_1t_2}^{(n_1n_2)}(x_1'x_2') \quad = \quad \int d^D x\Psi_{t_1}^{(n_1)}(x_1')(x)\Delta\Psi_{t_2}^{n_2}(x_2')(x) \qquad \text{(D4)}$$

then we find

$$< H_1 >_0 \quad = \quad \sum_{n_1n_2}\sum_{t_1t_2}\sum_{x_1'x_2'}< \hat{S}_{t_1}^{n_1}(x_1')\hat{S}_{t_2}^{n_2}(x_2') >\left(-\hat{\Delta}_{t_1t_2}^{(n_1n_2)}(x_1'x_2')\right)$$

$$= \sum_n \sum_{l_1 l_2} \sum_{x'} A_{l_1 l_2}^{(n)} \left(-\hat{\Delta}_{l_1 l_2}^{(nn)}(x'x')\right)$$

Assuming that the system are translational invariance

$$= \sum_n \sum_{l_1 l_2} 2^{-nD} N_0 A_{l_1 l_2}^{(n)} \left(-\hat{\Delta}_{l_1 l_2}^{(nn)}(00)\right)$$

Using the scaling form of matrix element (see section 6.1.2)) then we obtain

$$= \sum_n \sum_{l_1 l_2} 2^{-nD} N_0 A_{l_1 l_2}^{(n)} 2^{-2nD} \left(-\hat{\Delta}_{l_1 l_2}^{(00)}(00)\right)$$

$$= \sum_n \sum_{l_1 l_2} 2^{-3nD} N_0 A_{l_1 l_2}^{(n)} \left(-\hat{\Delta}_{l_1 l_2}^{(00)}(00)\right)$$

$$= \sum_n \sum_{l_1 l_2} 2^{-3nD} N_0 A_{l_1 l_2}^{(n)} (-\Delta_{l_1 l_2})$$

then

$$\frac{< H_1 >_0}{N_0} = \sum_n \sum_{l_1 l_2} 2^{-3nD} A_{l_1 l_2}^{(n)} (-\Delta_{l_1 l_2}) \qquad \text{(D5)}$$

For the quadratic term defined by

$$H_2 = \int d^D x S(x)^2 \qquad \text{(D6)}$$

substituting the wavelet expansion we get

$$= \int d^D x \left[\sum_{n_1} \sum_{x_1'} \sum_{l_1}^{n_1} \hat{S}_{l_1}^{(n_1)}(x_1') \psi_{l_1}^{(n_1)}(x_1')(x) + S\right] * \left[\sum_{n_2} \sum_{x_2'} \sum_{l_2}^{n_2} \hat{S}_{l_2}^{(n_2)}(x_2') \psi_{l_2}^{(n_2)}(x_2')(x) + S\right] \qquad \text{(D7)}$$

$$= \int d^D x \left[\sum_{n_1,n_2} \sum_{x_1', x_2'} \sum_{\ell_1,\ell_2} \hat{S}_{\ell_1}^{(n_1)}(x_1') \hat{S}_{\ell_2}^{(n_2)}(x_2') \psi_{\ell_1}^{(n_1)}(x_1')(x) \psi_{\ell_2}^{(n_2)}(x_2')(x) \right]$$

$$+ 2\int d^D x \hat{S}_{\ell_1}^{(n_1)}(x_1') \psi_{\ell_1}^{(n_1)}(x_1')(x) + \int d^D x S^2 \qquad \text{(D8)}$$

Then we find

$$< H_2 >_0 = \int d^D x \left[\sum_{n_1,n_2} \sum_{x_1', x_2'} \sum_{\ell_1,\ell_2} < \hat{S}_{\ell_1}^{(n_1)}(x_1') \hat{S}_{\ell_2}^{(n_2)}(x_2') > \psi_{\ell_1}^{(n_1)}(x_1')(x) \psi_{\ell_2}^{(n_2)}(x_2')(x) \right]$$

$$+ 2\int d^D x < \hat{S}_{\ell_2}^{(n_2)} > (x_2') \psi_{\ell_1}^{(n_1)}(x_1')(x) + \int d^D x < S^2 > \qquad \text{(D9)}$$

$$= \int d^D x \left[\sum_{n} \sum_{x'} \sum_{\ell_1,\ell_2} A_{\ell_1 \ell_2}^{(n)} \psi_{\ell_1}^{(n)}(x')(x) \psi_{\ell_2}^{(n)}(x')(x) \right] + N_0 \bar{S}^2 \qquad \text{(D10)}$$

The second term is equal to zero because the mean of Gaussian distribution is zero and by the orthonormality of Daubechies wavelet equation (D10) will lead to

$$= \sum_{n} \sum_{x'} \sum_{\ell} A_{\ell\ell}^{(n)} + N_0 \bar{S}^2$$

$$= 2^{-nD} N_0 \sum_{\ell} \sum_{n} A_{\ell\ell}^{(n)} + N_0 \bar{S}^2$$

then

$$\frac{< H_2 >_0}{N_0} = 2^{-nD} \sum_{\ell} \sum_{n} A_{\ell\ell}^{(n)} + \bar{S}^2 \qquad \text{(D11)}$$

In case of the quartic term, when we substitute the wavelet expansion and find the expectation in Gaussian distribution, we will obtain the zero to fourth order of wavelet coefficients. The odd order will be equal to zero and our main problem now is to simplify the fourth term which is defined by

$$H_3 \quad = \quad \int d^D x S(x)^4$$

$$= \quad \sum_{n_{1_1}\ldots n_4}\sum_{\ell_1\ldots\ell_4}\sum_{x_1'\ldots x_4'} \hat{S}_{\ell_1}^{(n_1)}(x_1')\ldots\hat{S}_{\ell_4}^{(n_4)}(x_4')\,\overline{M}_{\ell_1\ell_2\ell_3\ell_4}^{(n_1n_2n_3n_4)}(x_1'x_2'x_3'x_4') \qquad \text{(D12)}$$

The matrix $\overline{M}$ is obtained by Cartesian composition.

$$\overline{M}_{\ell_1\ell_2\ell_3\ell_4}^{(n_1n_2n_3n_4)}(x_1'x_2'x_3'x_4') \quad = \quad \prod_{j=1}^{D} M_{\ell_{1i}\ell_{2i}\ell_{3i}\ell_{4i}}^{(n_1n_2n_3n_4)}(x_{1i}', x_{2i}', x_{3i}', x_{4i}') \qquad \text{(D13)}$$

from the matrix element

$$M_{\ell_1\ell_2\ell_3\ell_4}^{(n_1n_2n_3n_4)}(x_1', x_2', x_3', x_4') \quad = \quad \int dx \Psi_{\ell_{1_1}}^{(n_1)}(x_1')(x)\ldots\Psi_{\ell_4}^{(n_4)}(x_4')(x)$$

when we find $< H_3 >_0$

$$= \quad \sum_{n_{1_1}\ldots n_4}\sum_{\ell_1\ldots\ell_4}\sum_{x_1'\ldots x_4'} < \hat{S}_{\ell_1}^{(n_1)}(x_1')\ldots\hat{S}_{\ell_4}^{(n_4)}(x_4') > \overline{M}_{\ell_1\ell_2\ell_3\ell_4}^{(n_1n_2n_3n_4)}(x_1'x_2'x_3'x_4') \qquad \text{(D14)}$$

Using the wick's theorem say

$$< abcd > \quad = \quad < ab >< cd > + < ac >< bd > + < ad >< bc > \qquad \text{(D15)}$$

In equation (D14) and using equation (D1), we will get three terms which have the same magnitude, so we can write the result of (D14) as

$$= \quad 3\sum_{n_1 n_2}\sum_{l_1 \ldots l_4}\sum_{x_1' x_2'} A_{l_1 l_2}^{(n_1)} A_{l_3 l_4}^{(n_2)} \overline{M}_{l_1 l_2 l_3 l_4}^{(n_1 n_1 n_2 n_2)}\left(x_1' x_1' x_2' x_2'\right) \qquad \text{(D16)}$$

From scaling forms of four point matrix elements, the result becomes

$$= \quad 3\sum_{n_1 n_2}\sum_{l_1 l_2} A_{l_1 l_1}^{(n_1)} A_{l_2 l_2}^{(n_2)} N_0 2^{-(n_1+n_2)D} \qquad \text{(D17)}$$

therefore

$$\frac{< H_3 >_0}{N_0} \quad = \quad 3\sum_{n_1 n_2}\sum_{l_1 l_2} A_{l_1 l_1}^{(n_1)} A_{l_2 l_2}^{(n_2)} 2^{-(n_1+n_2)D} \qquad \text{(D18)}$$

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