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MULTI-COMMODITY FLOW MODEL APPROACH TO A CREW ROSTERING PROBLEM

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A Thesis Submitted in Partial Fulfillment of the Requirements  
for the Degree of Master of Science Program in Applied Mathematics and  
Computational Science

Department of Mathematics and Computer Science

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Thesis Title                      MULTI-COMMODITY FLOW MODEL APPROACH TO A CREW  
 ROSTERING PROBLEM

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วาริยา พุทธปฏิโมกษ์ : แนวคิดตัวแบบการไหลของสินค้าโภคภัณฑ์หลายชนิดเพื่อแก้ปัญหาการมอบหมายงานให้กับพนักงาน. (MULTI-COMMODITY FLOW MODEL APPROACH TO A CREW ROSTERING PROBLEM ) อ.ที่ปรึกษาวิทยานิพนธ์หลัก: ดร.บุญฤทธิ์ อินทียศ, อ.ที่ปรึกษาวิทยานิพนธ์ร่วม: ผศ. ดร. ชวลิต จินอนันต์, 52 หน้า.

ปัญหาการจัดตารางงานของพนักงานสายการบิน เป็นการมอบหมายงานให้กับพนักงานไปยังเส้นทางการบิน(งาน)และต้องสอดคล้องกับกฎหมายแรงงาน และกฎเกณฑ์ของบริษัทการบินนั้นๆ ซึ่งปัญหาดังกล่าวเป็นปัญหาที่มีขนาดใหญ่และมีความซับซ้อน วิธีการแก้ปัญหานี้ส่วนใหญ่จะพิจารณาการลดเงินส่วนที่จ่ายให้กับพนักงานเท่านั้นเพื่อที่จะลดรายจ่ายของบริษัทการบิน สำหรับงานวิจัยนี้ได้นำเสนอการแก้ปัญหาการจัดตารางงานสำหรับเที่ยวบินต่างประเทศของสายการบินไทย โดยใช้ตัวแบบปัญหาการไหลของโภคภัณฑ์หลายชนิด โดยวัตถุประสงค์เพื่อทำให้ผลรวมขอบเขตบนของรายได้และค่าภาระงานของพนักงานมีค่าต่ำสุด ในกรณีศึกษาวางแผนการจัดตาราง ได้ใช้ข้อมูลจากบริษัทการบินไทยในการสร้างข้อมูลตัวอย่างในหลายกรณี และแก้ปัญหาของแต่ละกรณีโดยใช้โปรแกรม ไอบีเอ็ม ไอล๊อค ซีเพิลิก จากผลการทดลองพบว่าตัวแบบการไหลของสินค้าโภคภัณฑ์หลายชนิด สามารถหาผลเฉลยของตัวอย่างที่มีขนาดเล็กและกลางได้ภายในเวลา 60 ชั่วโมง อย่างไรก็ตามการกระจายรายได้และค่าภาระงานไม่ดีเท่าที่ควร เนื่องจากค่าส่วนเบี่ยงเบนมาตรฐานของรายได้อยู่ระหว่าง 1985.415 กับ 3855.295และค่าส่วนเบี่ยงเบนมาตรฐานของค่าภาระงานอยู่ระหว่าง 8.7595 กับ 28.22391 นอกจากนี้สำหรับปัญหาที่มีขนาดใหญ่ โปรแกรมไม่สามารถหาผลเฉลยได้ภายใน 60 ชั่วโมง

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WARIYA PUTTAPATIMOK: MULTI-COMMODITY FLOW MODEL APPROACH TO A  
CREW ROSTERING PROBLEM.

ADVISOR: BOONYARIT INTIYOT, Ph.D.

CO-ADVISOR: ASST. PROF. CHAWALIT JEENANUNTA, Ph.D., 52 pp.

An airline crew rostering problem is a large-scaled and complex optimization problem that assigns crew members to the flight duties while satisfying agreements with the labor union, the government regulations, the carrier's own policies, and other requirements. The traditional crew rostering problem considers only minimizing the total per-diem in order to reduce the airline expense. This paper presents the crew rostering problem for the international flights of Thai Airways. We propose a 0-1 multi-commodity flow problem whose objective function is to minimize the sum of the maximum of the per-diem and workloads among the crew members. Various test cases are generated from Thai Airways data set and solved by using the commercial optimizer IBM ILOG CPLEX. From the experiment results, the optimizer can solve the multi-commodity flow problems of small and medium sizes within 60 hours. However, the solutions give poor distribution of the per-diem and workloads among crew members where the standard deviations of the per-diem ranging from 1985.415 to 3855.295 and standard deviations of the workloads ranging from 8.7595 to 28.22391. The optimizer cannot solve the problems of large size within 60 hours.

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# CHAPTER I

## INTRODUCTION

### 1.1 Motivation

Nowadays, commercial air transports are highly competitive. Both premium airlines and low-cost airlines emerge in order to support the advancement of business. Therefore, the airline companies need to adjust themselves because of the advent of innovation. The increasing number of customers leads to the increasing number of planes, routes and crew (pilots and flight attendants). Consequently, the cost of fuel, cost of operation and crew cost increase. According to Anbil [1] after costs for fuel, crew costs are the second largest expenses of an airline. For this reason, cost-efficient crew scheduling plays an important role in cost saving. A few percent of crew reduction usually has an impact on annual savings of tens of millions US dollars for large airlines [2].

Efficient crew scheduling results in a significant cost saving. As a consequence, there have been a considerable number of researchers who were interested in solving a crew scheduling problem for the past decades. Crew scheduling method has been applied to transportation systems such as airlines railway and bus; emergency service such as police, ambulance and fire brigade; nurse scheduling in health care systems and many other service organizations such as hotels, restaurants and retail stores [3].

The planning and scheduling of aircraft and crews in airline industry is considered to be one of the largest and the most complex. For example one major U.S. airline must schedule about thousands of flight segments per day [4]. The airline must assign aircraft and specific cockpit crew and flight attendant crew to all of these

thousands of flight segments and determine a monthly schedule. Therefore, the planning process is usually separated into several planning steps including fleet assignment, scheduling design, aircraft maintenance routing, and crew scheduling [5]. However, this work focuses on the crew scheduling problem for the Thai Airways in case of international flights. Conventionally, airlines divide the construction of crew scheduling into two parts: a crew pairing problem and a crew rostering problem. The objective of two stages is mostly to find the minimum-cost schedule that also satisfies all given rules and regulations. We will elaborate each of these two stages in section 2.2.

In our research study, we assume that the crew pairing problem has been solved because the data of the crew pairing has already given by Thai Airways. This thesis presents the crew rostering problem for the international flights of Thai Airways. We propose a 0-1 multi-commodity flow model whose objective function is to minimize the sum of the upper bound of workload and per-diem. Ultimately, from this model, we wish to obtain the optimal monthly schedule determining a monthly schedule (for in-flight managers only).

## **1.2 Research objective**

The objective of a crew rostering is to assign each individual crew member to a flight schedule while satisfying work rules and regulations. A flight pairings, which composes of flight segments and rest periods, can be constructed to cover one workday or several workdays. In this research, we consider only the pairings that originate and terminate at the base airport in Bangkok and construct a monthly roster for each In-flight manager crew of the Thai Airways. The rosters are constructed so that the sum of the upper bound of per-diem and workloads are minimized.

### 1.3 Structure of the thesis

The rest of the thesis is described as follows.

In Chapter II, The backgrounds and related research in the literature are described. This includes airline crew scheduling problem, some well-known combinatorial optimization problems, the heuristic approaches, and the solution approaches. The network flow problems and their appreciations are also reviewed.

In Chapter III, rules and regulations of Thai Airways is presented and then the characteristics of multi-commodity flow model correspond to Thai Airways crew rostering problem are described.

In Chapter IV, the experiments and results are presented.

In Chapter V, the results are discussed and analyzed and the conclusion from the study is drawn.

## CHAPTER II

### LITERATURE REVIEW

#### 2.1 Airline Crew Scheduling Problem

The propose of this chapter is to provide the background of the airline scheduling problem and review some approaches for solving crew scheduling problem.

##### 2.1.1 Crew pairing problem

The airline crew scheduling problem is usually divided into two problems. The first problem in the crew scheduling is called crew pairing and the second is called crew rostering. The crew pairing problem involves construction of flight pairings for an unspecified crew member. Each flight pairing begins and ends at the same base station for the crew. Moreover, all flight pairings must satisfy all of the governmental regulations and collective agreements which vary from airline to airline. The main objective of the crew pairing is to find a minimum cost of pairings that cover all flights for the scheduling period (usually one month). Traditionally, during the past decades, the crew pairing problem has been formulated as a set covering problem or a set partitioning problem [6] [7] [8] [9]. It is usually solved by the generate-and-optimize principle [6]. Unfortunately very large set partitioning problems are generated (400 rows, 30,000 columns) and found to be unsolvable by any available techniques [7] [8]. Consequently, there are several other approaches such as a network model [9], a nonlinear multi-commodity network flow [10], heuristics approach [11] [12].

### 2.1.2 Crew rostering problem

Once the crew pairing problem is solved, the second problem is the crew rostering problem. The output of crew pairing will become the input of the crew rostering. Crew rostering assigns individual crew members to each pairing that constructs in the crew pairing process. In a process of the crew rostering one needs to consider other pre-assignment activities such as training periods, annual leaves, and holidays as well as government regulations and contractual agreements. Generally, the aim of crew rostering is to minimize cost or maximize the quality of life among the crew members.

All of these constraints add more complexity to the crew rostering problem, and this explains why most researchers in the literature proposed solution methods based on heuristics [5] [13] [14]. For non-heuristics approaches, many researchers constructed combinatorial optimization models, such as a set partitioning or a set covering and a network flow model for modeling a crew rostering problem.

The set partitioning problem or the set covering problem has traditionally been applied to the crew rostering for almost two decades [5] [11]. Methods for solving the set partitioning(covering) problem are often based on branch-and-bound, branch-and-cut, etc.[11] [15]. When the numbers of decision variables are small, most integer programming solvers can handle these problems. However, when numbers of decision variables are large, other techniques must be used. A popular choice of such techniques is the column generation approach.

A set partitioning model was proposed by G. Yu [15] for a rostering problem. The problem was solved by a column generation technique and a branch-and-bound algorithm. The author proposed a strategy for decreasing a solution time. The strategy



was called "disjoint columns". Details of this strategy can be found in [15]. It could decrease solution time by at least 40% in their test problems.

Kato and Jeenanunta [21] presented the set partitioning model for solving a crew rostering problem using Thai domestic flight data from Nok Air and the main goal was the balancing the quality of life among crew members in terms of the fairness of workloads, destinations, and holidays.

Desaulniers et al [10] have proposed two models for Daily Aircraft Routing and Scheduling. The first model was constructed as a set partitioning problem and a column generation method was employed to solve the linear relaxation of the set partitioning problem. The second model was constructed as a time constrained multicommodity flow formulation and a Dantzig-Wolfe decomposition method was used to solve the linear relaxation of the problem. Finally a branch-and bound algorithm was used to obtain integer solutions of the two models.

Yan, S. et al [8] presented models that incorporate three factors: home bases, aircraft, or cabin classes, into the crew scheduling problem in order to improve the construction of cabin crew schedules. The authors developed eight models for minimizing crew cost and planning proper pairings under the real constraints for a Taiwan airline. The networks were constructed using weekly flight schedules and cabin crew information. Eight models were formulated as integer programs which are solved by a column-generation-based algorithm developed by the authors.

Yan S. & Tu Y.-P. [9] constructed pure network models which can both efficiently and effectively solve crew scheduling problems for a Taiwan airline. The flow decomposition method [18] was used to generate pairings that cover all duties.

Cappanera and Gallo [22] formulated the airline crew rostering problem as a 0-1 multicommodity flow problem and focused on minimizing the number of noncovered activities in the objective function. They used a preprocessing phase in order to reduce the size of the network and proposed some families of valid inequalities that had proved to be computationally effective.

Moz and Pato [14] presented two new integer multicommodity flow formulations for solving the problem of rostering nurse schedules. The authors did not specify the technique used for solving this problem, but focused on the comparison of the two models in terms of solution quality and computational time. The first model was based on a directed multilevel acyclic network. The second model was obtained by aggregating some nodes in the first model.

Although exact methods have been applied in the rostering problem for past decades, a nature of this problem is a large-scaled and complex optimization. Therefore, many literatures have proposed heuristic approaches to solve the rostering problem. A heuristic is an approach that can seek a good approximate result in a reasonable time. Examples of heuristic approaches used in crew rostering are a simulated annealing such as [16], and Genetic algorithm such as [17], etc.

## 2.2 Network Flow Problems

In this section, we review the features of a single-commodity network and a multi-commodity network that we use in this thesis. Their applications are also review in the end of this section.

The single- commodity flow problem is the minimum cost flow problem. It is a rudimentary network flow problem. The objection of this problem aims to find a minimum cost. A flow in a network has to satisfy some constraints which explained below.

Let  $G= (N, A)$  be a directed graph, where  $N$  is the set of nodes and  $A$  is the set of arcs.

$c_{ij}$  be the per-unit cost on arc  $(i, j)$

$x_{ij}$  be the flow on arc  $(i, j)$

$u_{ij}$  be the upper limit on arc  $(i, j)$

The formulation of the single-commodity can be shown as follows:

$$\min \sum_{(i,j) \in A} c_{ij} x_{ij}$$

subject to

$$\sum_{j:(i,j) \in A} x_{ij} = \sum_{l:(l,i) \in A} x_{li} \quad \forall i \in N \quad (1)$$

$$0 \leq x_{ij} \leq u_{ij} \quad \forall (i, j) \in A \quad (2)$$

The objective is to minimize total cost such that the amount of flow into and out of each node must be equal. The flow on each arc  $(i, j)$  cannot exceed its upper limit  $u_{ij}$ . Condition (1) is called a flow conservation constraint and (2) is called an arc capacity constraint.

For multi-commodity, all types of commodities do share the same arc capacity  $c$ . Let  $t$  define the number of the different commodities,

$c_{ij}^k$  be the per-unit cost on arc  $(i, j)$  for commodity  $k$ ,

$x_{ij}^k$  be the flow on arc  $(i, j)$  for commodity  $k$ ,

$u_{ij}$  be the upper limit on the sum of all commodities on arc  $(i, j)$ .

The formulation of the multi-commodity can be express as follows:

$$\min \sum_{k=1}^t \sum_{(i,j) \in A} c_{ij}^k x_{ij}^k$$

subject to

$$\sum_{j:(i,j) \in A} x_{ij}^k = \sum_{l:(l,i) \in A} x_{li}^k \quad \forall i \in N, \forall k = 1, \dots, t \quad (3)$$

$$0 \leq \sum_{k=1}^t x_{ij}^k \leq u_{ij} \quad \forall (i, j) \in A, \forall k = 1, \dots, t \quad (4)$$

The objective is to minimize total cost. Condition (3) is called a *flow conservation constraint*. The amount of flow into and out of each node must be equal Condition (4) is called an *arc capacity constraint*. The total of the flow from all commodities cannot exceed its upper limit  $u_{ij}$ .

The multi-commodity flow approach is widely used in routing of multiple commodities, warehousing of seasonal products, train scheduling and airline scheduling. The size of the airline scheduling problem makes the multi-commodity network flow problem difficult to solve. Therefore, many approaches such as Lagrangian relaxation, column generation, and Dantzig-Wolfe decomposition are used for solving the LP relaxation of the corresponding multi-commodity network flow problems.

## CHAPTER III

### THAI AIRWAYS CREW ROSTERING PROBLEM

In this chapter, we describe the rules and regulations of the Thai Airways which are important in the network construction used in our network flow model. The formulation of the multi-commodity flow problem as a 0-1 programming model is also described.

#### 3.1 Rules and Regulations in Thai Airways

There are two types of services in Thai Airways: International and Domestic services. Their routes can be grouped as North America routes, Europe routes, Africa routes, Australia and New Zealand routes, Regional routes, and Domestic routes. There are six categories of crew members: In-flight manager (IM), Air purser (AP), F (who works only on the first class), E (who works on the business class), R (who works on the business class) and, Y (who works on the economic class).

First, we make a note on our definition of some important terms:

1. A Block Time/Flight Time is the time period from which the aircraft starts moving from the parking at the departure airport until the aircraft stops moving at the arrival airport.
2. A Flight Duty Period is the period of the time starting from one hour before the block time until thirty minutes after the end of the block time.

Table 3.1 and 3.2 show the regulations for cabin attendants of Thai Airways which are considered in our model.

Table 3.1: Block Time Constraints for each cabin crew.

Number of Days	Total Block Time
Every 7 consecutive days	should not exceed 34 hours
Every 28 consecutive days	should not exceed 110 hours
Every 365 consecutive days	should not exceed 1000 hours

Table 3.2: Rest Period Constraints for each cabin crew.

Flight Duty Period	Rest Period
< 8 hrs	$\geq 8$ hrs
8 - 10 hrs	$\geq 10$ hrs
10 - 12 hrs	$\geq 12$ hrs
12 - 14 hrs	$\geq 14$ hrs
14 - 16 hrs	$\geq 16$ hrs
16 - 20 hrs	$\geq 24$ hrs

## 3.2 Multi-commodity Flow Model Construction

In this section we represent the crew rostering problem for Thai Airways with a multi-commodity network flow model. First, the construction of the network model is described. Then we discuss the multi-commodity flow constraints and the model objective function.

### 3.2.1 Network Construction

We construct a network that satisfies flight duty period constraints. Moreover, we assume that the crew pairing problem has been solved. In our case study, we consider the crew rostering problem for the in-flight managers only.

#### 3.2.1.1 Model I

This is a preliminary model that relaxes the rest period constraints and is presented in [23]. In this model, we formulate the crew rostering problem as a multicommodity network, which is a directed graph  $G = (N, A)$ , where  $N$  is the set of nodes and  $A$  is the set of arcs.

We describe the network using a small example shown in Figure 3.1. In this example, the multi-commodity network flow model represents possible scheduling for 2 working days and 4 pairings. Every node represents a point in time. There are four types of arcs which are day-off arcs, duty arcs, rest arcs, and a cyclic arc.

1. *Day-off arcs*: a day-off arc represents a day off. The head node and the tail node of a day-off arc indicate the beginning time of the day, which is 0:00 and the ending time of the day, which is 24:00, respectively. The arc cost is zero. The lower bound is zero and the arc upper bound is infinite.



2. *Duty arcs*: a duty arc represents a work duty for a pairing. The arc cost is given and depends on the corresponding pairing. The arc lower bound and upper bound are equal to one, meaning that exactly one crew member serves as IM for that duty.
3. *Rest arcs*: the rest arcs connect the head node of a day-off arc with all starting nodes of the duty arcs of the same day. The rest arcs also connect all ending nodes of the duty arcs with tail nodes of the day-off arc of the same day. The arc cost is zero. The arc lower bound is zero and the arc upper bound is infinite.
4. *Cyclic arc*: a cyclic arc connects the end schedule node to the start schedule node. The arc cost is zero. The arc lower bound is zero and the arc upper bound is infinite.

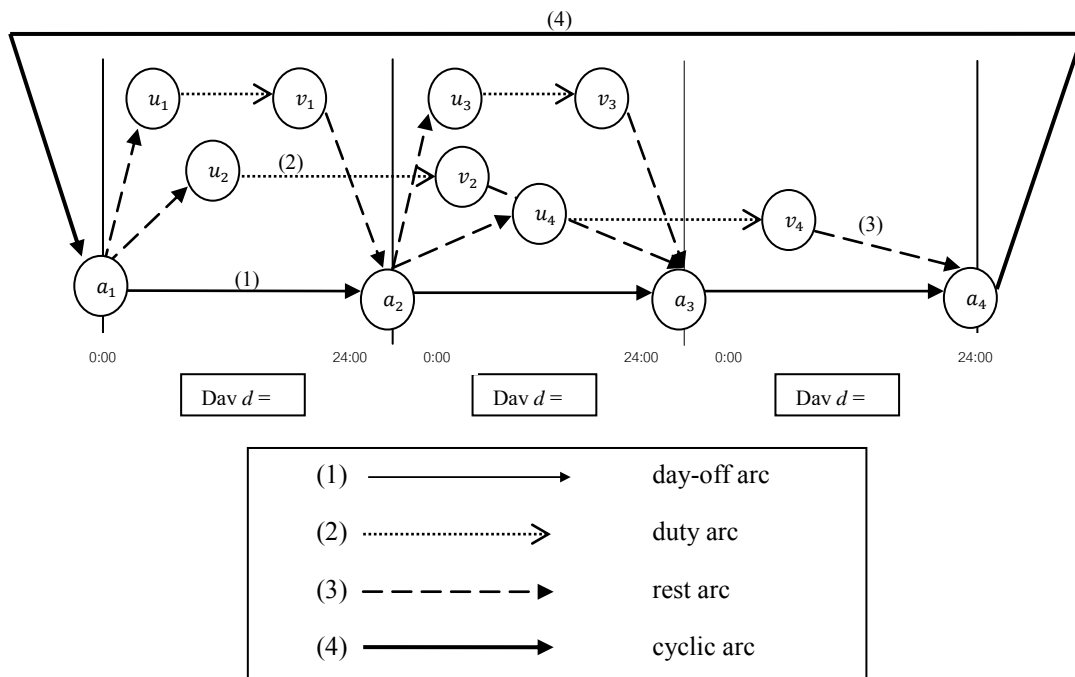
The set of nodes  $N$  contains the following types of node:

1. The node of a day-off arc on the day  $d$ , denoted by  $a_d$ .
2. The starting node of a duty arc associated with the pairing number  $p$ , denoted by  $u_p$ .
3. The ending node of a duty arc associated with the pairing number  $p$ , denoted by  $v_p$ .

We build the crew rostering network as follows:

1. The starting nodes of a day-off arc are constructed for each day and the nodes are linked by the day-off arcs. The number of day-off arcs is exactly the total number of days covered by all pairings.
2. The duty arcs are constructed according to the pairing data.
3. The rest arcs are constructed by joining the head node of a day-off arc with all starting nodes of the duty arcs of the same day. The rest arcs also connect all ending nodes of the duty arcs with tail nodes of the day-off arc of the same day.
4. The cyclic arc connects the sink node to the source node.

Figure 3.1: An example of the crew rostering network for Model I.



### 3.2.1.2 Model II

In this model, we improve Model I by adding the rest period constraints. The previous network must be modified since it can generate infeasible solution in this model. For example, Table 3.3 shows the data set that we use for formulating the network in Figure 3.2. The next possible departure day and the next possible departure time indicate the day and the time that the next possible pairing can start in a feasible schedule. They are calculated by incorporating the appropriate rest period.

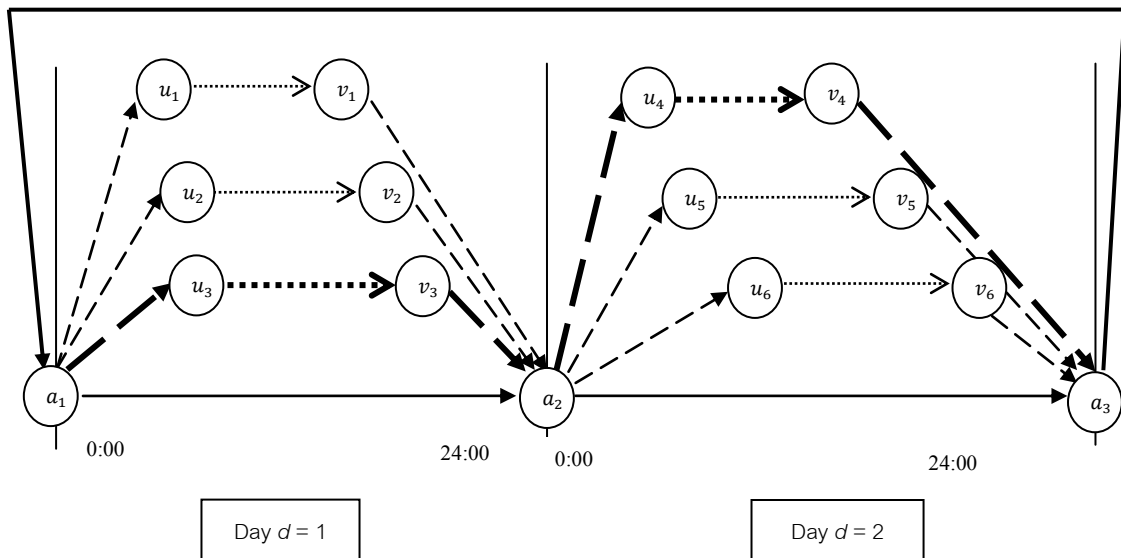
Table 3.3: The example of the data set

Pairing number	1	2	3	4	5	6
Departure Flight	403	319	664	403	319	664
Arrival Flight	404	320	665	404	320	665
Departure Day	1	1	1	2	2	2
Departure Time	8:00	10:35	11:00	8:00	10:35	11:00
Arrival Day	1	1	1	2	2	2
Arrival Time	13:45	18:35	21:00	13:45	18:35	21:00
Block Time(hrs)	5:45	8:00	10:00	5:45	8:00	10:00
Flight Duty Period (hrs)	7:15	9:30	11:30	7:15	9:30	11:30
Rest period	$\geq 8$ hrs	$\geq 10$ hrs	$\geq 12$ hrs	$\geq 8$ hrs	$\geq 10$ hrs	$\geq 12$ hrs
Next possible Departure Day	1	2	2	2	3	3
Next possible Departure Time	21:45	4:35	9:00	21:45	4:35	9:00

From Table 3.3, the pairing number 1 has the departure flight 403, departure day 1 and departure time 8:00. This pairing has the arrival flight 403, arrival day 1 and arrival time 13:45. The block time is the arrival time – the departure time = 13:45 – 8:00 = 5:45. Therefore, the flight duty period of the pairing number 1 is the block time (hrs) +1:30 (hrs) = 5:45+1:30 = 7:15 (hrs). The rest period must be at least 8 consecutive hours. The next possible departure time is the arrival time + the rest period. Thus the next possible

departure time is  $13:45+8:00 = 21:45$  and the next possible departure day is 1. The data set of other pairings is obtained in a similar manner.

Figure 3.2: The crew rostering network form Model I using the data set from Table 3.3



If we use Model I to construct the network using data from Table 3.3, we will obtain a network show in Figure 3.2. Every path from node  $a_1$  to node  $a_3$  represents a possible 2-day schedule for a particular crew member. However, if the path is  $a_1, u_3, v_3, a_2, u_4, v_4, a_3, a_1$ , it will be infeasible schedule because the pairing 3 has the next possible departure time = 9:00 and the next possible depart day = 2 but the pairing 4 has departure time = 8:00, which cause the violation on the rest time period constraints. Thus, this thesis proposes a new network construction to the rostering network in order to eliminate the shortcomings of the previous model.

The multi-commodity network flow model for Thai Airways crew rostering problem in this thesis is a directed graph  $G = (N, A)$ , where  $N$  is the set of nodes and  $A$  is the set of arcs. Figure 3.3 shows an example of the crew rostering network. The set of nodes  $N$  contains the following types of node:

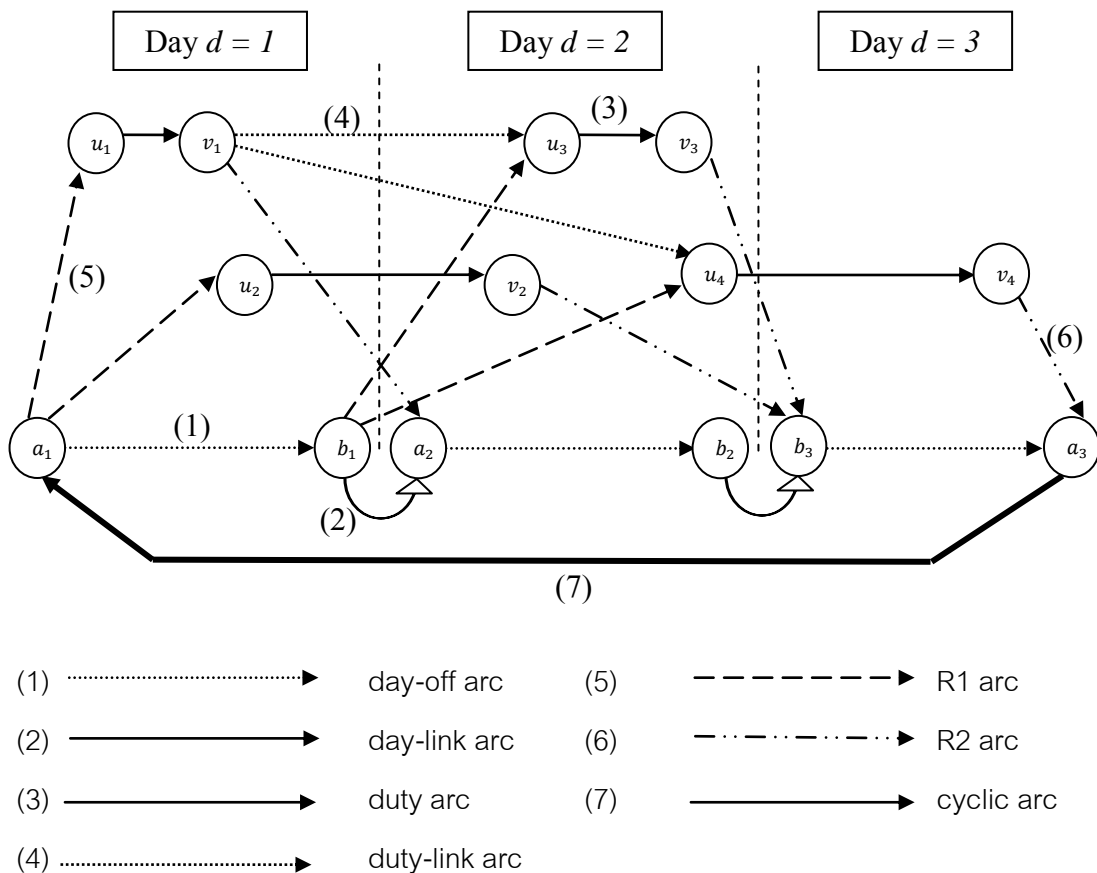
1. The starting node of a day-off arc on the day  $d$ , denoted by  $a_d$ .
2. The ending node of a day-off arc on the day  $d$ , denoted by  $b_d$ .
3. The starting node of a duty arc associated with the pairing number  $p$ , denoted by  $u_p$ .
4. The ending node of a duty arc associated with the pairing number  $p$ , denoted by  $v_p$ .

In the network, the nodes defined above can be linked by one of the following arc types. Figure 3.3 shows each type of arc with the label indicating the type number:

1. Day-off arcs: a day-off arc represents a day off. The arc cost is zero. The lower bound is zero and the upper bound is infinite.
2. Day-link arcs: a day-link arc links the ending day-off node of the day  $d$  to the beginning day-off node of the day  $d+1$ . The arc cost is zero. The lower bound is zero and the upper bound is infinite.
3. Duty arcs: a duty arc represents a work duty for a pairing. The arc cost is given and depends on the corresponding pairing. The arc lower bound and upper bound are equal to one, meaning that exactly one crew member serves as IM for that duty.
4. Duty-link arcs: If a pairing that can be followed by another pairing within 24-hour period without violating the rest period constraints, those two pairing will be joined by a duty-link arc. The arc cost is zero. The arc lower bound is zero and the arc upper bound is infinite.

5. R1 arcs: Some R1 arcs are the rest arcs that join the head node of the day-off arc on the first day with all pairings on the same day. The other R1 arcs join the tail node  $b_d$  of a day-off arc with all starting nodes of the duty arcs in the  $day = d+1$ . The arc cost is zero. The arc lower bound is zero and the arc upper bound is infinite.
6. R2 arcs: R2 are the rest arcs that connect all ending nodes of the duty arcs on the day  $d$  with starting nodes of the day-off arc on the day  $d+1$ . The arc cost is zero. The arc lower bound is zero and the arc upper bound is infinite.
7. Cyclic arc: a cyclic arc connects the end-of-schedule node to the start-of-schedule node. The arc cost is zero. The arc lower bound is zero and the arc upper bound is infinite.

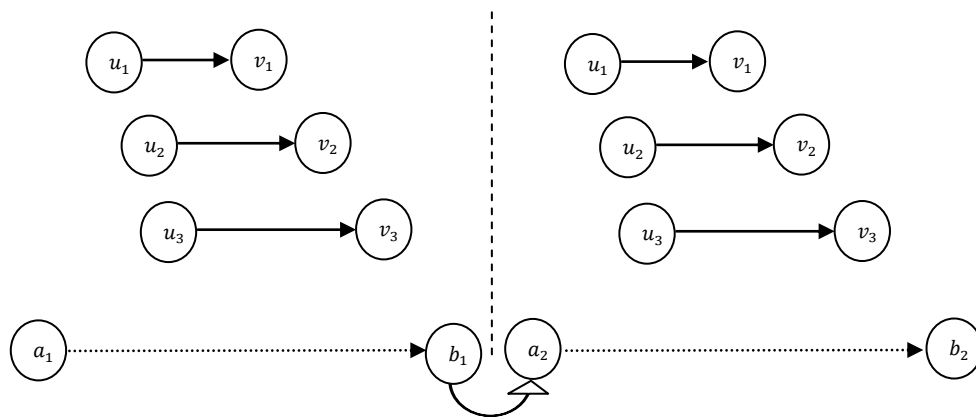
Figure 3.3: Each type of arc



We demonstrate the network construction using an example from Table 3.3, which consists of 6 pairings that must be covered.

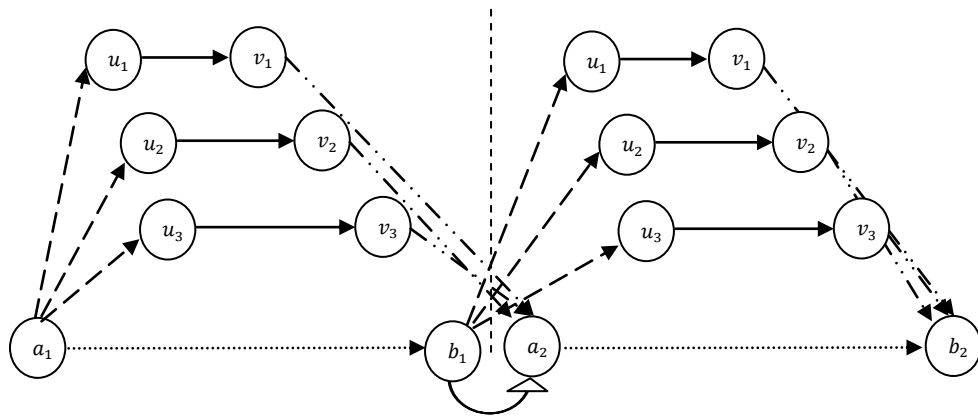
1. Two day-off nodes are constructed for each day representing the start and the end of the day. The nodes in the same day will be linked by the day-off arcs. A day-link arc will link the ending node  $j = b_d$  and beginning node  $i = a_{d+1}$ . The number of day-off arcs is exactly the total number of days covered by all pairings. Since all six pairings together span 3 working days, we must have 3 day-off arcs.
2. The duty arcs are constructed according to the pairing data. Figure 3.4 displays the day-off arcs and the duty arcs constructed from the data table.

Figure 3.4: Construction of the day-off and duty arcs.



3. For the first day, the R1 arcs are constructed by joining the head node of the day-off arc with all starting nodes of the duty arcs of the same day. For the following days, the R1 arcs join the tail node  $j = b_d$  of a day-off arc with all starting nodes of the duty arcs in the day  $d+1$ . The R2 arcs connect all ending nodes of the duty arcs with starting nodes of the day-off arc in the next day. (See Figure 3.5.)

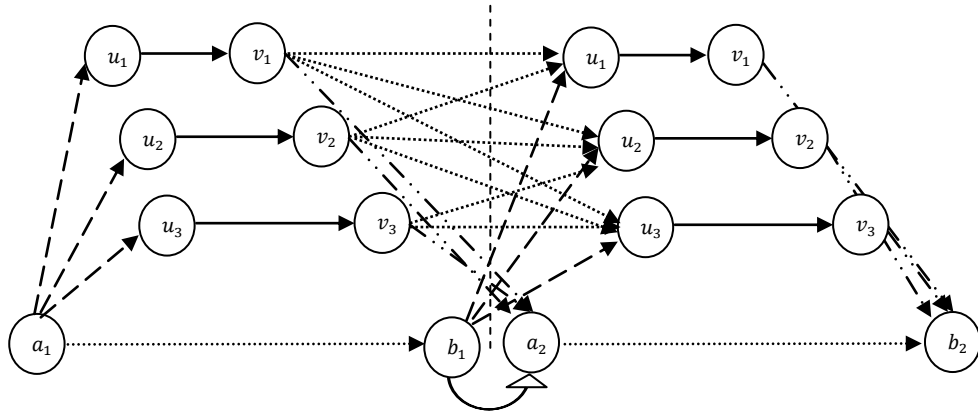
Figure 3.5: Construction of the R1 and R2 arcs.



4. The next possible departure time and the next possible departure day from Table 3.3 are used for constructing duty-link arcs. For any two pairing, if one pairing can follow another pairing within a 24-hour period without violating the rest period constraints, they will be linked by a duty-link arc. Figure 3.6 illustrates all duty-link arcs using data from Table 3.3.

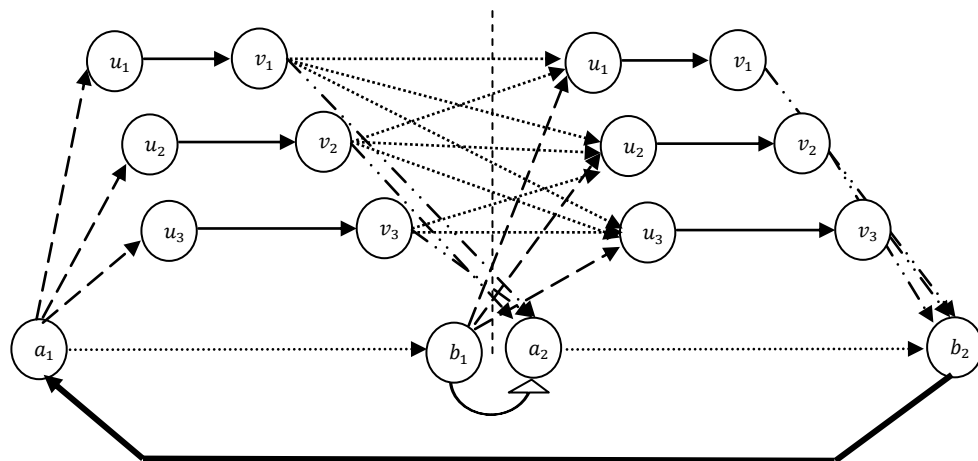


Figure 3.6: Construction of the duty-link arcs.



5. The cyclic arc connects the sink node to the source node. (See Figure 3.7.)

Figure 3.7: Construction the cyclic arc.



### 3.2.2 Notations

The formulation of the crew rostering problem is presented in this subsection.

First we introduce the notations then the formulation.

The following variables are used in the model:

$x_{ij}^C$  is the binary decision variable where  
 $x_{ij}^C = 1$  if a crew member  $C$  is assigned to arc  $(i, j)$   
and  $x_{ij}^C = 0$  otherwise

$MW$	is the variable representing the maximal total workload assigned among the crew members.
$MP$	is the variable representing the maximal total per-diem assigned among the crew members.

The following parameter sets are to be used:

$w_{ij}$	is the workload rating for arc $(i, j)$
$p_{ij}$	is the per-diem for arc $(i, j)$
$b_{ij}$	is the block time (in minutes) for arc $(i, j)$
$L_{ij}$	is the lower bound for arc $(i, j)$
$U_{ij}$	is the upper bound for arc $(i, j)$
$N$	is the set of all nodes.
$A$	is the set of all arcs.
$C$	is the set of crew members
$DAY$	is the set of valid departure days = $\{7, 8, 9, \dots, \text{last day}\}$ .
$A^D$	is the set of arcs where the starting node lies in day $D$
$P^c$	is the set of pre-assignments arc for crew member $c \in C$ .

Note:  $w_{ij} = p_{ij} = b_{ij} = 0$  for all arcs  $(i, j)$  which are not duty arcs.

### 3.2.3 Model Formulation

The multi-commodity flow model for Thai Airways is given by:

$$\text{Minimize } MW + MP \quad (1)$$

Subject to

$$\sum_{j:(i,j) \in A} x_{ij}^c - \sum_{k:(k,i) \in A} x_{ki}^c = 0 \quad \forall i \in N, \forall c \in C \quad (2)$$

$$\sum_{(i,j) \in A} w_{ij} x_{ij}^c \leq MW \quad \forall c \in C \quad (3)$$

$$\sum_{(i,j) \in A} p_{ij} x_{ij}^c \leq MP \quad \forall c \in C \quad (4)$$

$$\sum_{D=d-6}^d \sum_{(i,j) \in A^D} b_{ij} x_{ij}^c \leq 2040 \quad \forall d \in DAY, \forall c \in C \quad (5)$$

$$L_{ij} \leq \sum_{(i,j) \in A} x_{ij}^c \leq U_{ij} \quad \forall (i,j) \in A \quad (6)$$

$$x_{ij}^c = 1 \quad \forall (i,j) \in \text{Cyclic arc}, \forall c \in C \quad (7)$$

$$x_{ij}^c = 1 \quad \forall (i,j) \in P^c, \forall c \in C \quad (8)$$

$$x_{ij}^c \in \{0,1\} \quad \forall (i,j) \in A, \forall c \in C \quad (9)$$

The objective of this model is to minimize the sum of the upper bound of workload and per-diem, which is a linear function. The set of constraints can be explained as follows:

*Flow Conservation Constraints* (2) state that the amount of flow into and out of each node  $i$  must be equal for each crew member.

*Upper bound Workload Constraints* (3) ensure that the workload of each crew member does not exceed the upper bound.

*Upper bound Per-diem Constraints* (4) ensure that the per-diem of each crew member does not exceed the upper bound.

*Block Time Constraints* (5) ensure that for each crew member  $c$  cannot work more than 2040 minutes (34 hours) a week on any 7 consecutive days.

*Arc Capacity Constraints* (6) demonstrate the minimum number and maximum number of crew members required for each arc.

*Cyclic Arc Constraints* (7) show that the flow on the cyclic arc for each crew member must be one in order to force a flow from the end-of- schedule node to the start-of-schedule node.

*Pre-assignments Constraints* (8) can be different events, such as annual leave, training period, etc that are pre-assigned to each crew member.

## CHAPTER IV

### EXPERIMENTS AND RESULTS

To evaluate how well the model can be applied in the real situations, we performed 3 sets of case study using data from some international flights of the Thai Airways and only focused on the in-flight manager (IM) crew members. To construct the rostering network, we used the R scripting language. The multi-commodity flow problem is solved by using IBM ILOG CPLEX 12.10. running on Intel Core 2 Duo 2.67 GHz, RAM 2.00 GB

#### 4.1 Data Set

The model proposed in this work is tested using 3 sets of flight data of Thai Airways. Data set 1, 2, and 3 consist of the pairing data for 7, 14, and 28 working days respectively. Each data set is composed of 5 instances, which are varied by the numbers of pairings and the numbers of crew members being considered.

Table 4.1: The details of the data set 1.

Instances' details	Instances				
	7A	7B	7C	7D	7E
Number of days	7	7	7	7	7
Number of pairings/day	7	7	14	14	26
Total number of pairings	49	49	98	98	182
Number of crew members	15	15	45	45	56

Table 4.1 shows data set 1, which contains pairing data for 7 consecutive working days. There are 5 different instances in this data set, namely, 7A, 7B, 7C, 7D, and 7E. The Instance 7E is a scaled-down 7-day version of the real data, which contains 728 pairings from 28 days.

The instances 7A and 7B comprise of 7 pairings per day with different sets of flight legs. Each of these two instances contains the combination of pairings for each day shown in Table 4.2. The instances 7C and 7D comprise of 14 pairings per day using the pairing combination whose number of each pairing type is twice as many as that of Table 4.2.

Table 4.2: The combination of pairings for each day of 7A and 7B instances.

Pairings with flight duty period	Number of pairings
< 8 hours	2
8 - 10 hours	1
10-12 hours	1
12-14 hours	1
14-16 hours	1
16-20 hours	1

Table 4.3 shows data set 2, which contains pairings for 14 consecutive working days. There are 5 instances in this data set, namely, 14A-14E. The pairings in these instances are chosen using the same combination of types of pairing as data set 1.

Table 4.3: The details of the data set 2.

Instances' details	Instances				
	14A	14B	14C	14D	14E
Number of days	14	14	14	14	14
Number of pairings/day	7	7	14	14	26
Total number of pairings	98	98	196	196	364
Number of crew members	30	30	61	61	113

Table 4.4 shows data set 3, which contains pairings for 28 consecutive working days. There are 5 instances in this data set, namely, 28A-28E. The pairings in these instances are chosen using the same combination of types of pairing as data set 1.

Table 4.4: The details of the data set 3.

Instances' details	Instances				
	28A	28B	28C	28D	28E
Number of days	28	28	28	28	28
Number of pairings/day	7	7	14	14	26
Total number of pairings	196	196	392	392	728
Number of crew members	61	61	122	122	227

## 4.2 Result and Discussion

Table 4.5: Computational results of the data set 1.

	7A	7B	7C	7D	7E
Computational Time (hh:mm:ss)	00:00:23	00:00:21	09:15:42	09:57:12	12:16:01
Number of Nodes	114	114	212	214	384
Number of Arcs	375	410	1226	1236	3507
Number of Variables	5628	6153	55173	554443	196395
Number of Constraints	7785	8345	66161	66572	221795
Non-Zero Coefficients	25485	27585	238455	242055	842520
Maximum Per-diem (MP)	29260	36024.24	24264.32	24864.22	40975.71
Maximum Workload (MW)	200.36	184.5	142.83	135.01	228.08



Our main objective is to minimize the sum of the upper bound of workload and per-diem. That means the maximum of the workload and per-diem assigned to the crew members should be close to the average value. Table 4.6 displays the detailed statistical values of the per-diem and the workload that are assigned to all crew members by the optimal solution in each example.

Table 4.6: Solution Quality of the data set 1.

		7A	7B	7C	7D	7E
Per-diem	Standard Deviation	1985.415	2950.769	3282.541	3621.547	3841.751
	Mean	26458.24	33044.11	19834.12	19990.929	33808.235
	Max	29260.47	36024.24	24264.3	24864.22	40975.71
	Min	23694.21	24452.63	14628	12696.35	27162.19
Workload	Standard Deviation	28.22391	15.4773	19.15785	15.21	8.7595
	Mean	155.0173	157.6167	104.2113	106.1169	175.19
	Max	200.36	184.5	142.83	135.01	228.08
	Min	85.34	130.53	74	63.92	140.17

From the Table 4.6, the standard deviation of a per-diem is calculated from

$$\sqrt{\frac{\sum(x-\bar{x})^2}{n}}$$

when  $n$  = the numbers of crew members,  $x$  = a per-diem and  $\bar{x}$  = the average

per-diem. The standard deviation of the workload is also calculated in the same way.

Figure 4.1: Per-diem distribution of 7A

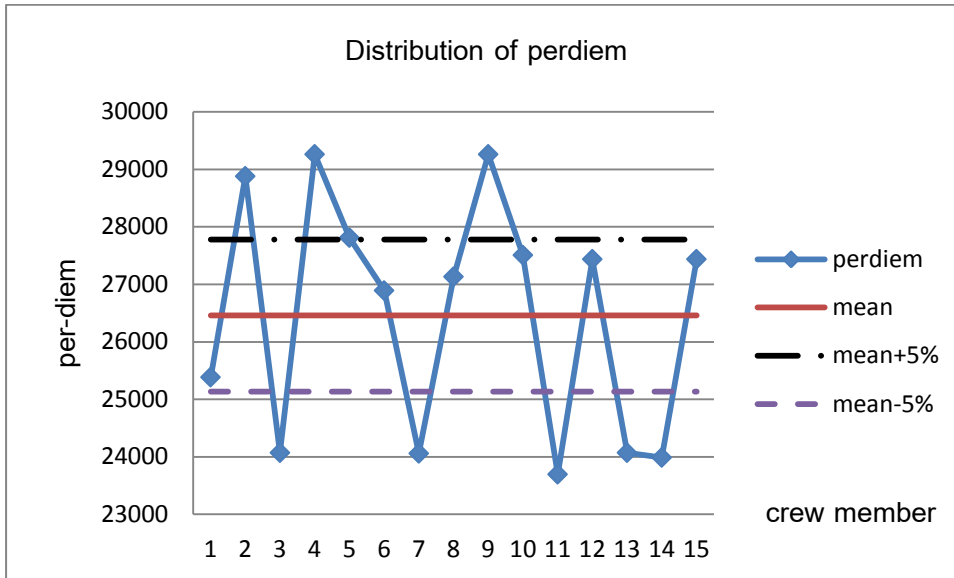


Figure 4.2: Workload distribution of 7A

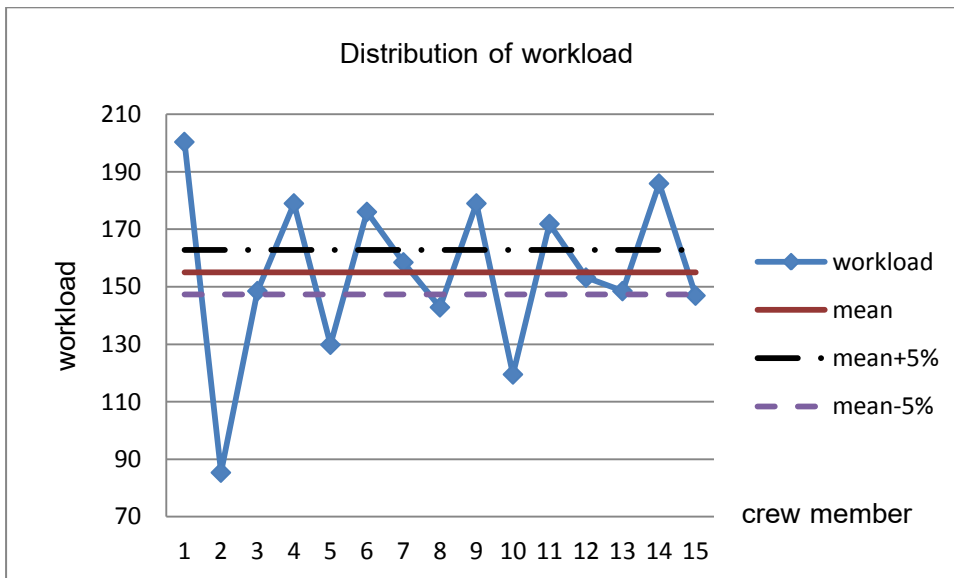


Figure 4.3: Per-diem distribution of 7B

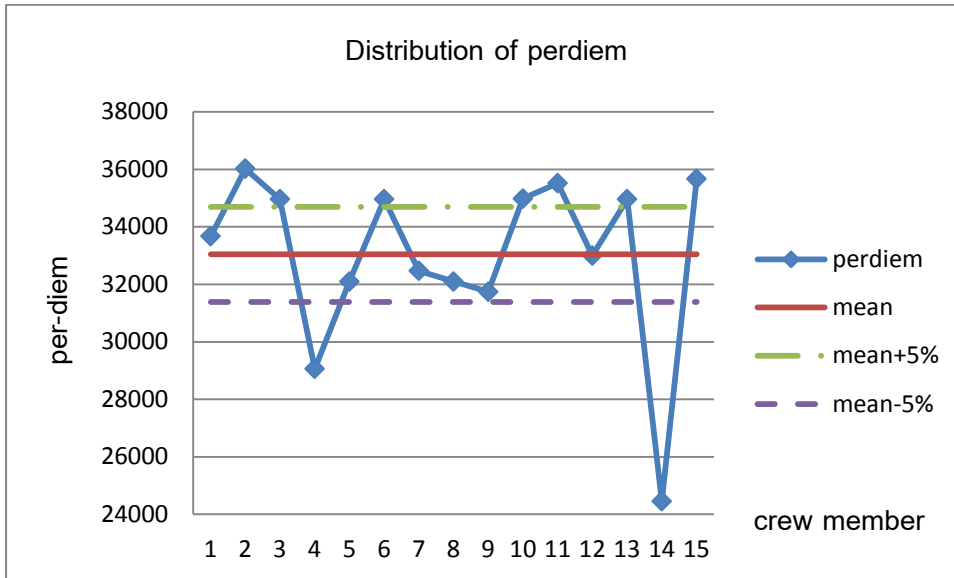


Figure 4.4: Workload distribution of 7B

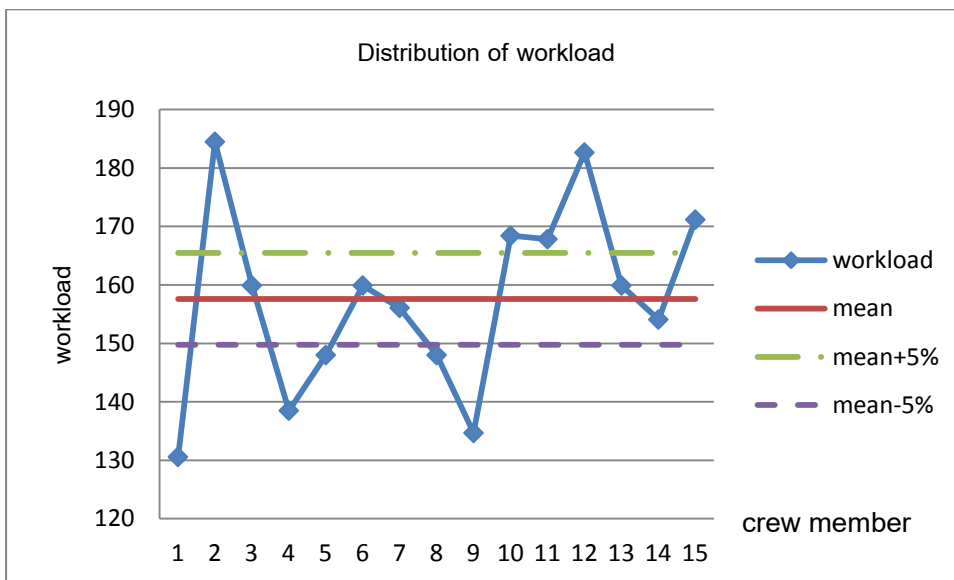


Figure 4.5: Per-diem distribution of 7C

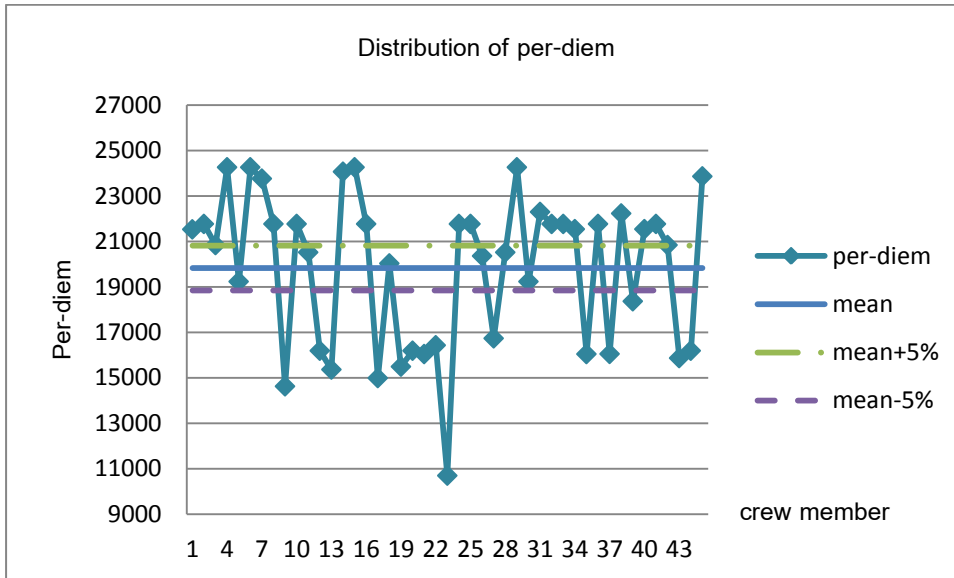


Figure 4.6: Workload distribution of 7C

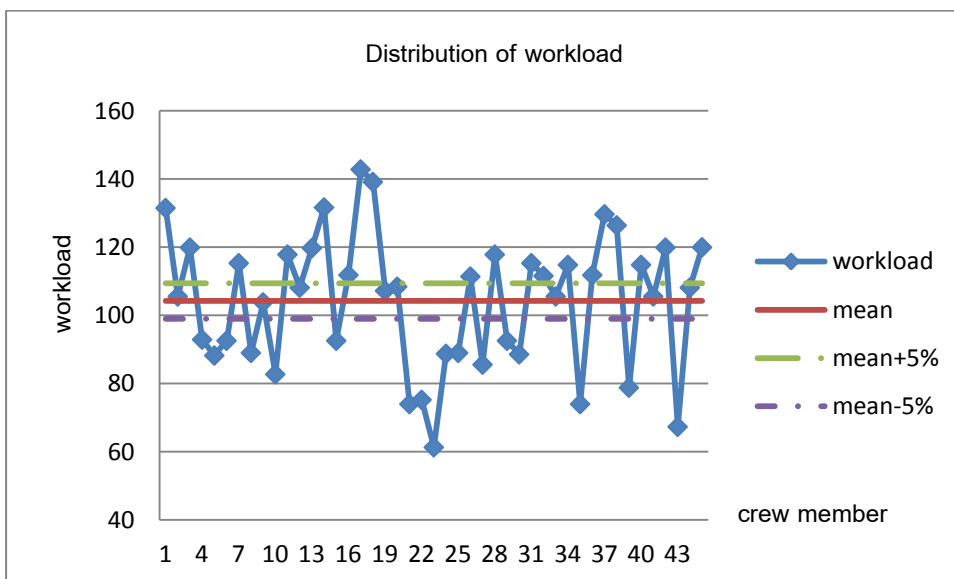


Figure 4.7: Per-diem distribution of 7D

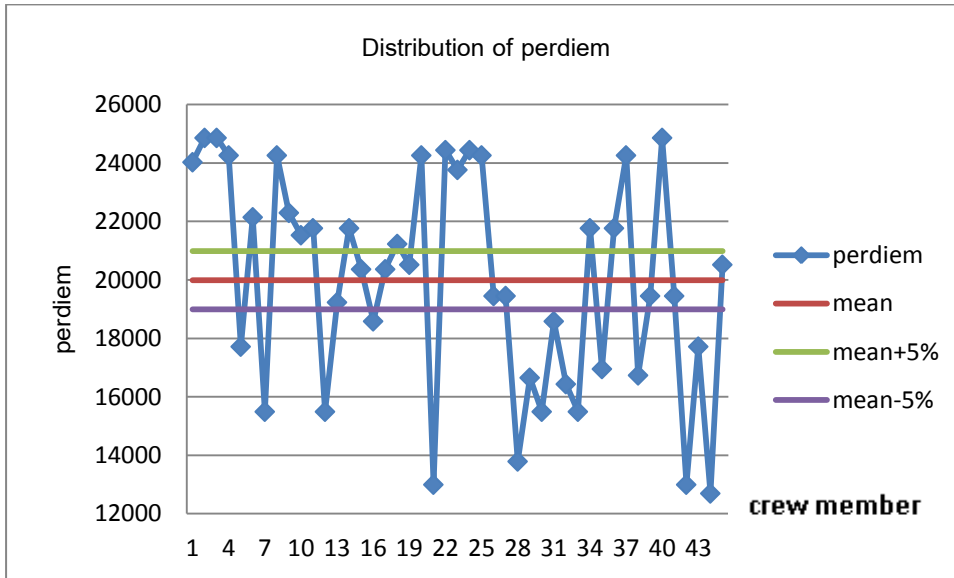


Figure 4.8: Workload distribution of 7D

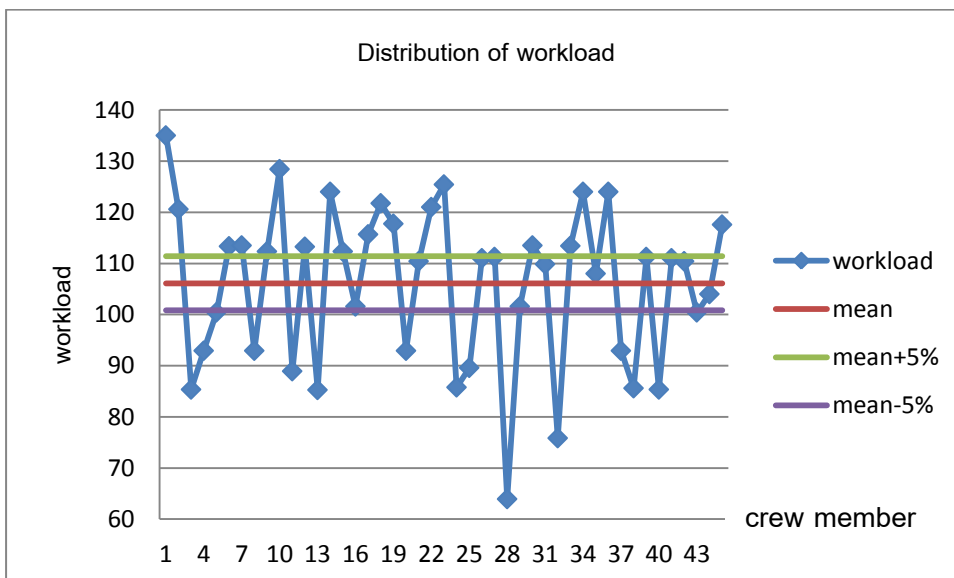


Figure 4.9: Per-diem distribution of 7E

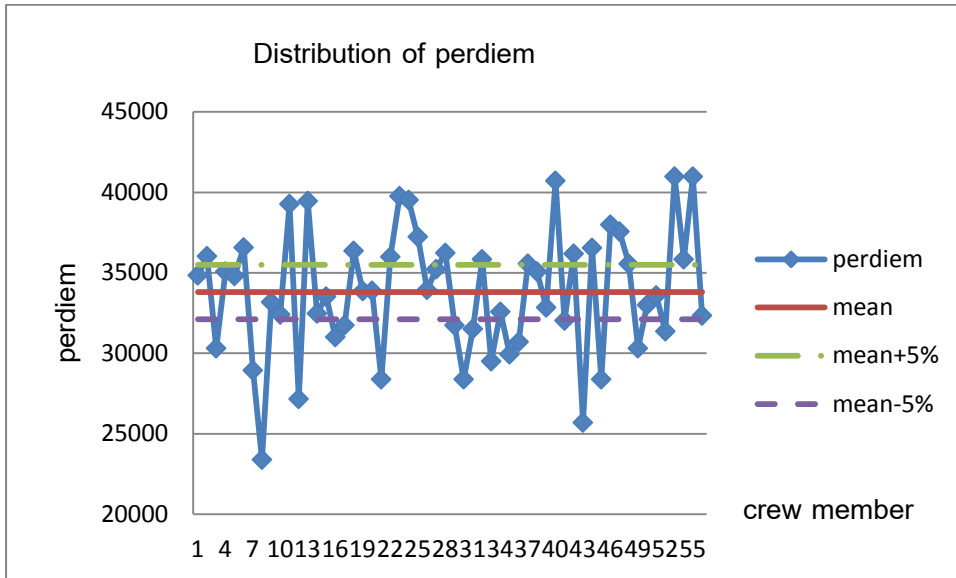


Figure 4.10: Workload distribution of 7E

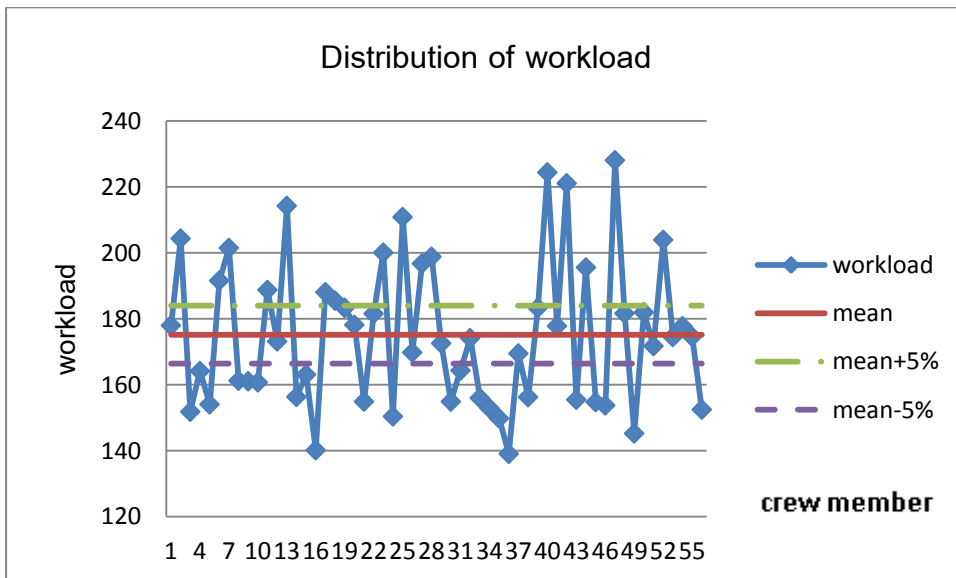


Table 4.7: Computational results of the data set 2.

	14A	14B	14C	14D	14E
Computational Time (hh:mm:ss)	00:01:33	00:01:58	31:53:54	34:15:07	Not done within 60 hours
Number of Nodes	226	226	422	421	762
Number of Arcs	802	900	2724	2790	8127
Number of Variables	24063	27003	166167	169949	918354
Number of Constraints	32002	35040	195362	199389	1014166
Non-Zero Coefficients	116910	128670	748531	761097	3993985
Maximum Per-diem( <i>MP</i> )	32930.9	40172.07	35901.61	36739.1	-
Maximum Workload( <i>MW</i> )	191.58	218.08	220.1	221.51	-

Table 4.8: Solution Quality of the data set 2.

		14A	14B	14C	14D	14E
Per-diem	Standard Deviation	3328.92	3142.088	3544.122	3443.324	-
	Mean	26458.241	33044.107	29263.45	29494.81	-
	Max	32930.9	40172.07	35901.61	36739.1	-
	Min	21396.24	25137.79	20522.77	19718.76	-
Workload	Standard Deviation	25.34	19.624376	28.07855	27.59304	-
	Mean	155.0173	157.61666	153.7544	156.5659	-
	Max	191.58	218.08	220.1	221.51	-
	Min	105.58	109.66	84.6	85.34	-

Figure 4.11: Per-diem distribution of 14A

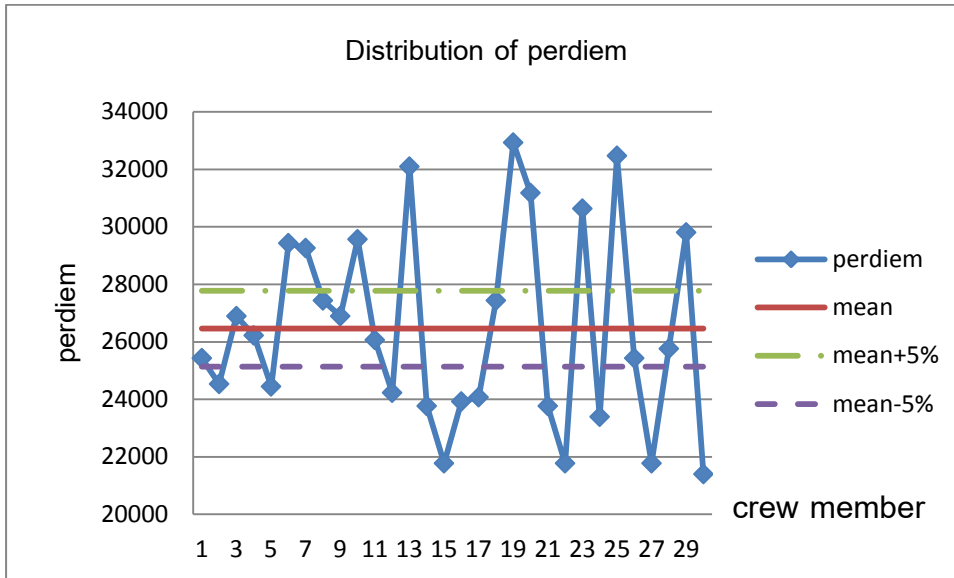


Figure 4.12: Workload distribution of 14A

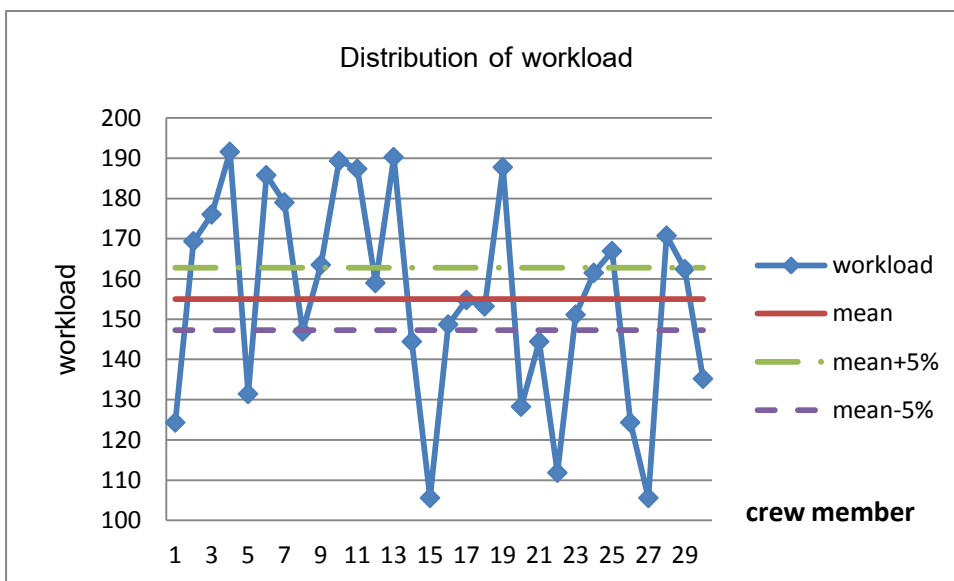




Figure 4.13: Per-diem distribution of 14B

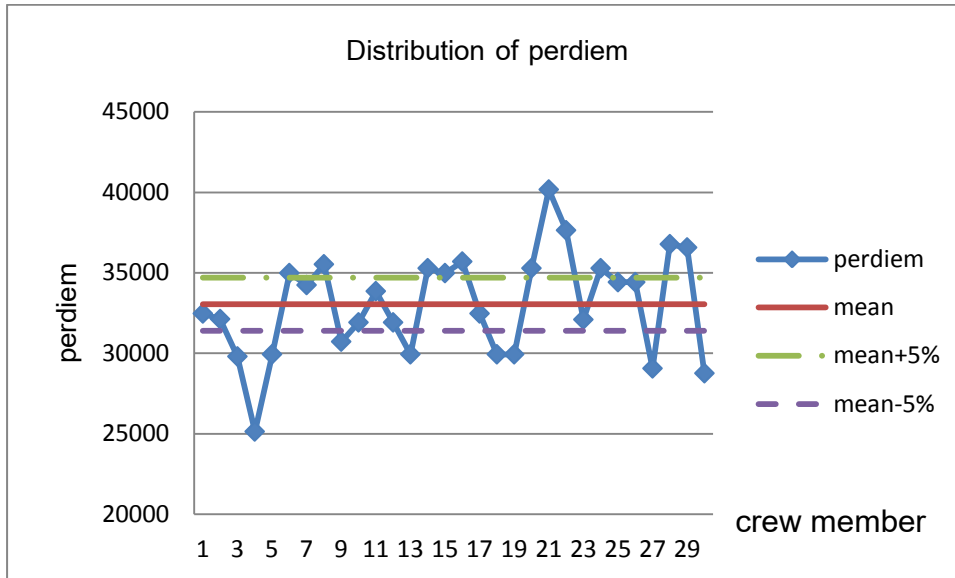


Figure 4.14: Workload distribution of 14B

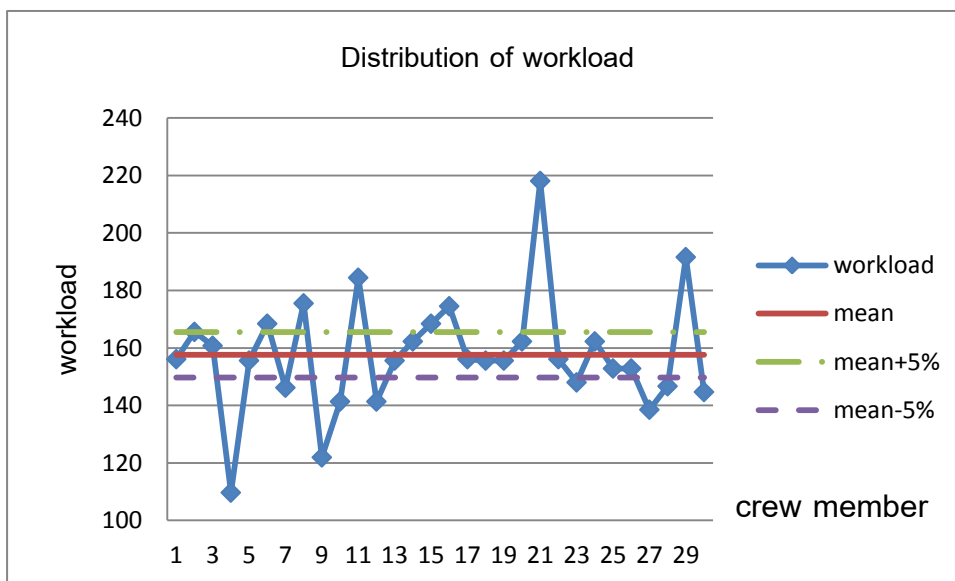


Figure 4.15: Per-diem distribution of 14C

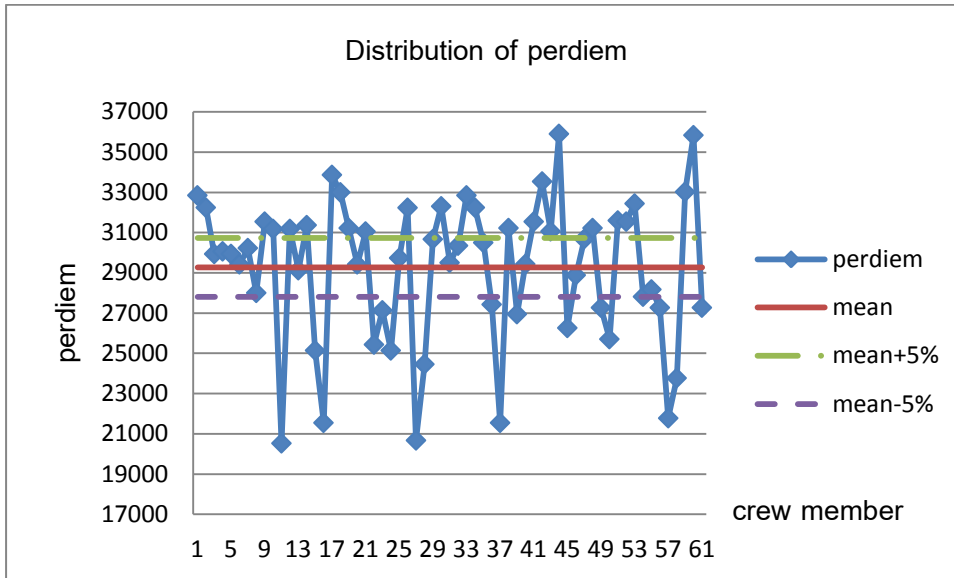


Figure 4.16: Workload distribution of 14C

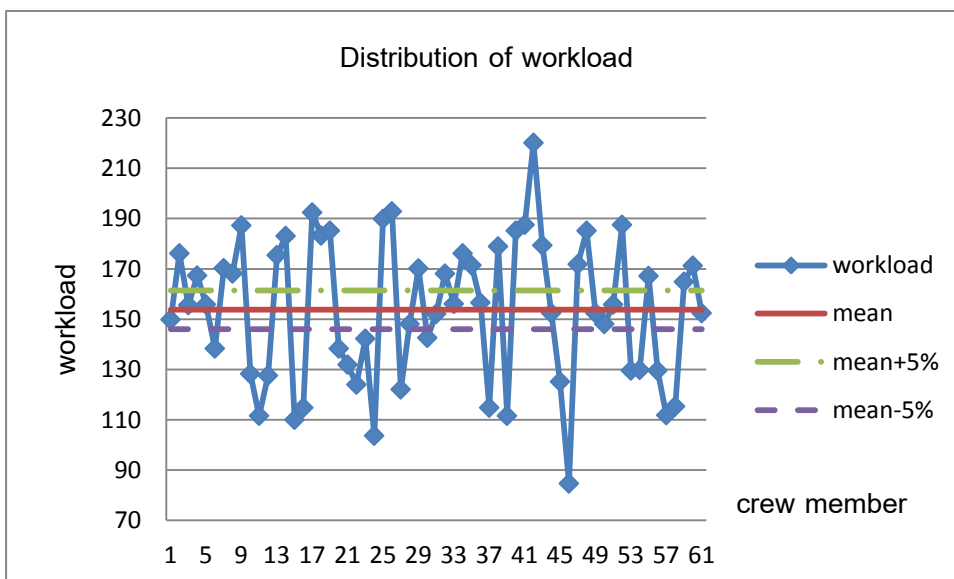


Figure 4.17: Per-diem distribution of 14D

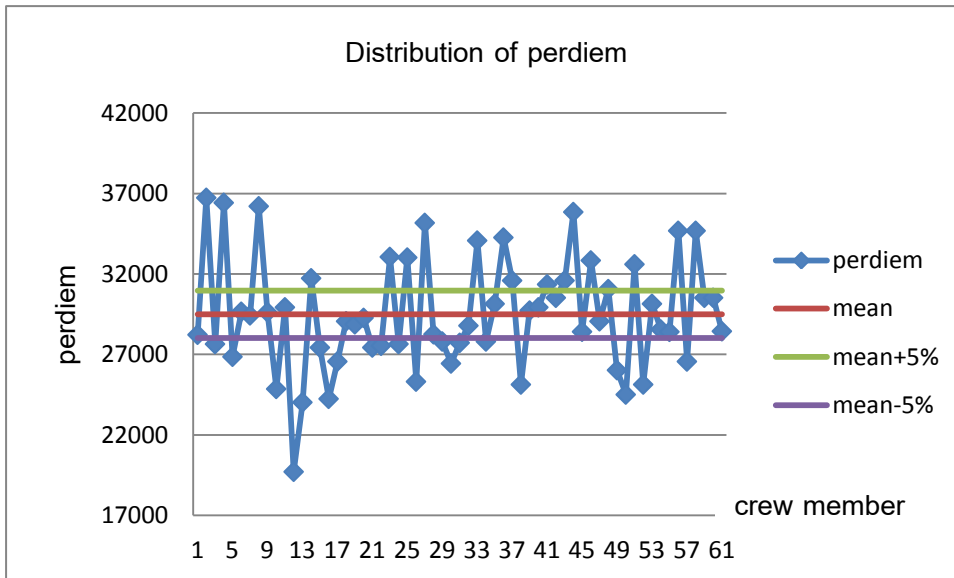


Figure 4.18: Workload distribution of 14D

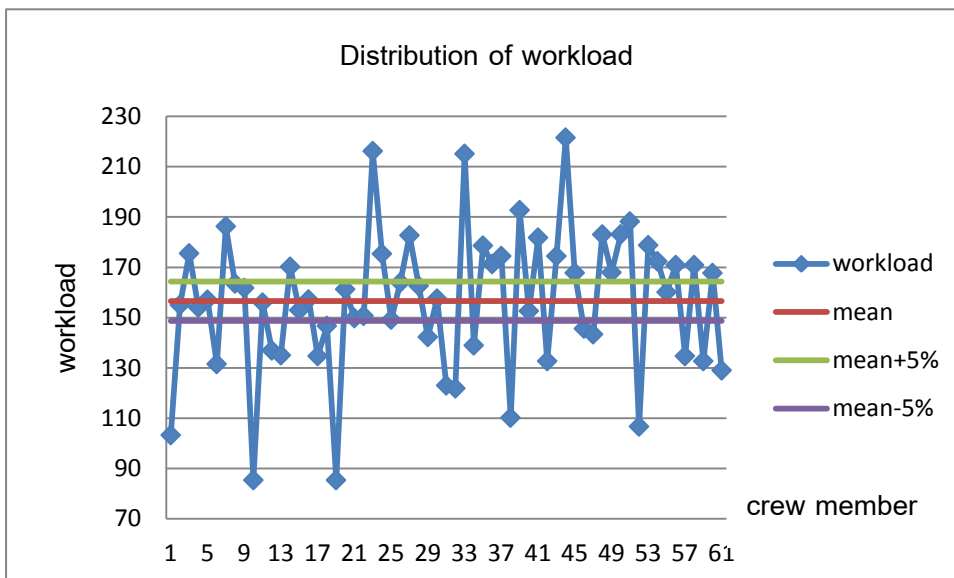


Table 4.9: Computational results of the data set 3.

	28A	28B	28C	28D	28E
Computational Time (hh:mm:ss)	00:38:47	00:29:00	Not done within 60 hours	Not done within 60 hours	Not done within 60 hours
Number of Nodes	450	450	842	844	1518
Number of Arcs	1656	1880	5720	5944	17367
Number of Variables	101019	114683	697843	725171	-
Number of Constraints	131708	145596	809456	837374	-
Non-Zero Coefficients	505873	560529	3198230	3317790	-
Maximum Per-diem( <i>MP</i> )	32469.79	39577.46	-	-	-
Maximum Workload( <i>MW</i> )	196.11	198.75	-	-	-

28C, 28D, and 28E are the three largest instances in all of our data sets.

Unfortunately, IBM ILOG CPLEX 21.10 cannot obtain the solutions of these instances within 60 hours of computation as shown in Table 4.9.

Table 4.10: Solution Quality of the data set 3.

		28A	28B	28C	28D	28E
Per-diem	Standard Deviation	3666.276	3855.295	-	-	-
	Mean	26024.5	32502.4	-	-	-
	Max	32469.79	39577.46	-	-	-
	Min	15294.06	23165.29	-	-	-
Workload	Standard Deviation	19.77472	23.92913	-	-	-
	Mean	152.4761	155.0328	-	-	-
	Max	196.11	198.75	-	-	-
	Min	85.34	109.66	-	-	-

Figure 4.19: Per-diem distribution of 28A

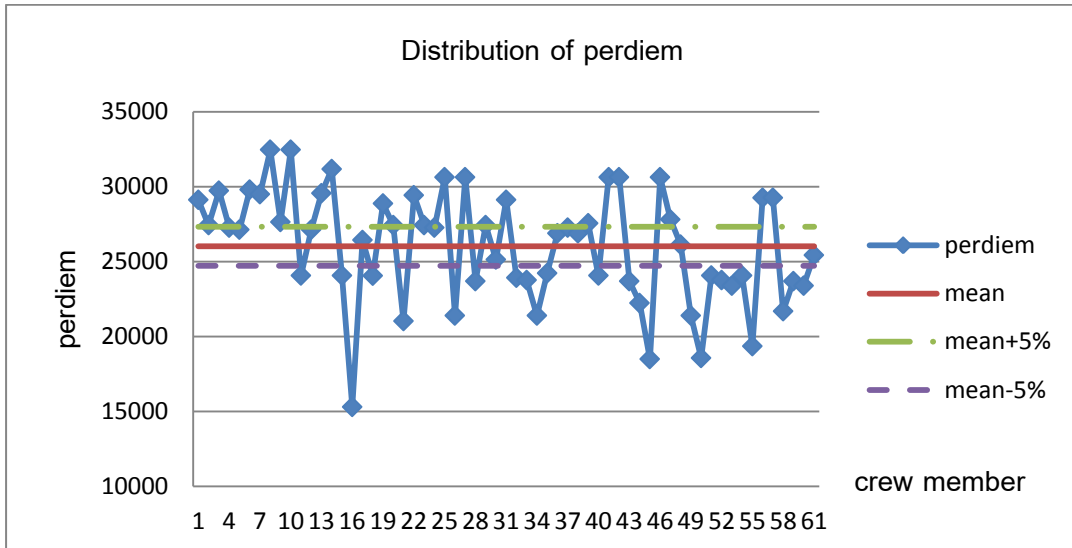


Figure 4.20: Workload distribution of 28A

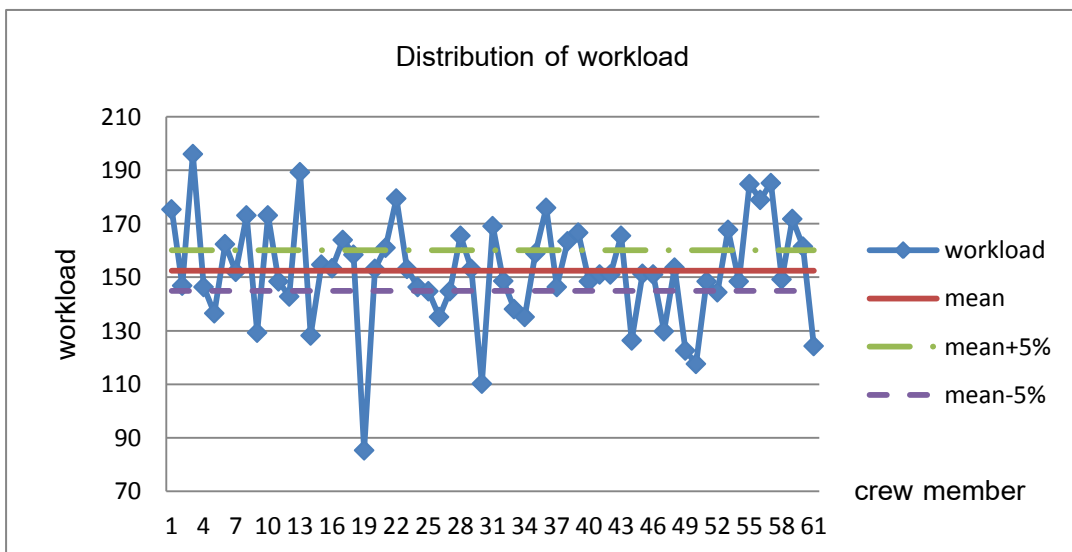


Figure 4.21: Per-diem distribution of 28B

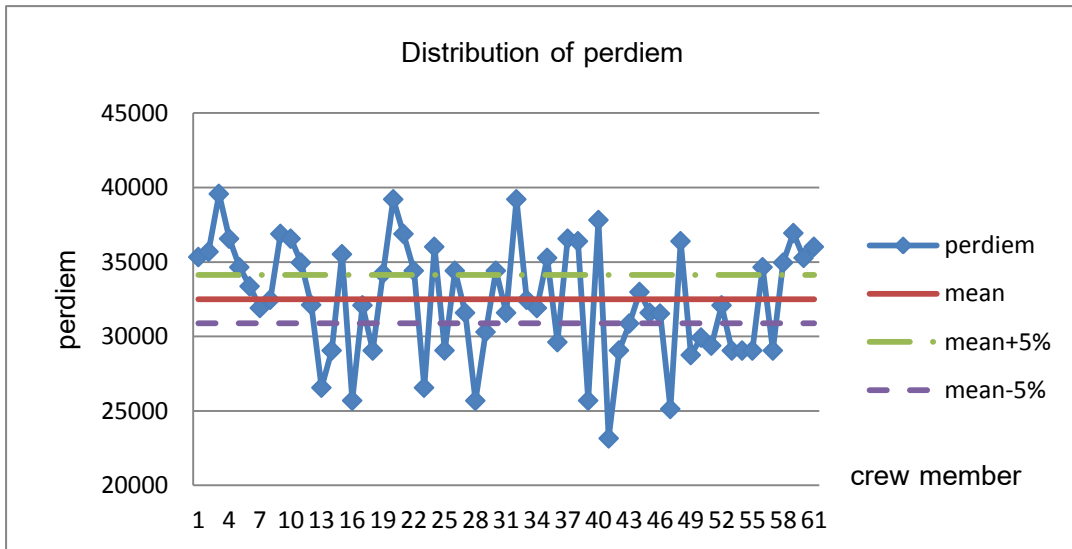
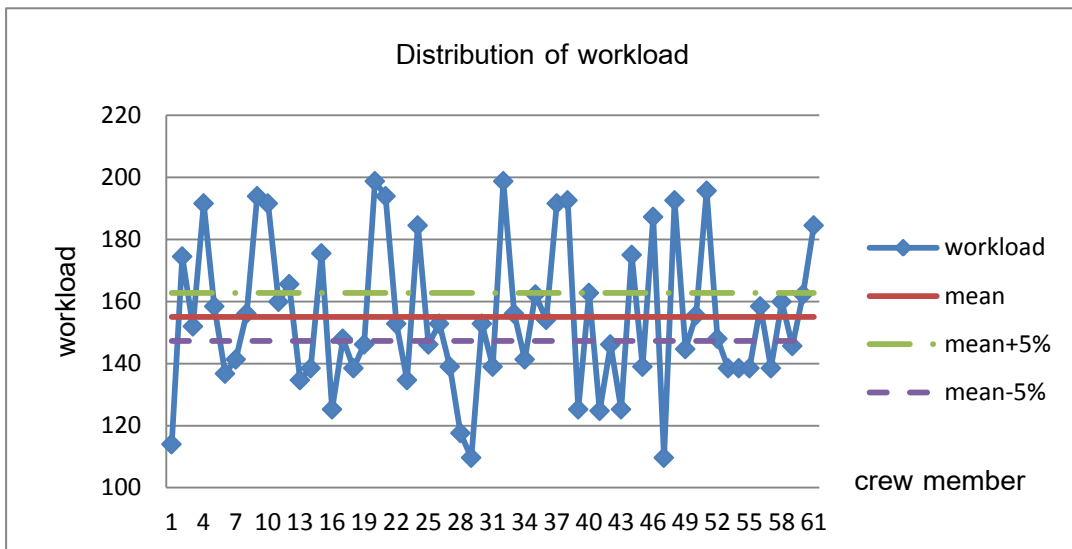


Figure 4.22: Workload distribution of 28B



From the result, the largest instance we can solve the matrix size  $169949 \times 199389$  which takes about under 35 hours to obtain the solution. However, solving the instance with the matrix size larger than  $169949 \times 199389$  cannot be done within 60 hours. For the instances that can be solved, we can minimize the sum of the upper bound of workload and per-diem but it is not guaranteed that the per-diem and the workload are distributed evenly among crew members as we can see from Figure 4.1- Figure 4.22.

The distribution of the per-diem and the workload from the 3 sets of example tends to vary beyond the range of the mean  $\pm 5\%$ , which are the preferred bounds. Ideally, the variation from the optimal solution should be small but our results show otherwise. This can be explained as follows:

By inspecting data, we find that the pairings in each day are the same. Moreover, for some examples such as 7D and 14D most pairs in each day are long (at least 2 days). This increases the chance of overlapping among these pairs resulting in smaller number of selections of the next pairs. Therefore, balancing the workload and the per-diem is difficult due to the limitation of choices of pairings.

In addition, the value of per-diem in each pair does not depend on the length of duty in the pair. Even if the lengths of two pairs are comparable, the per-diems can be very different. Therefore, such choices make balancing the workload and the per-diem very difficult.

Moreover, the value of workload rating is smaller than 100 while the value of per-diem ranges between 2,000 and 17,000. The difference in the scale of these two quantities is quite large. Hence, from the objective function, the significance of the



workload will be dominated by the per-diem. Therefore, this could cause the big variation in the workload distribution.

In summary, although our objective aims to minimize the upper bounds of the workload rating and the per-diem, it does not guarantee that the variation will be small. In fact the variation is quite large due to the reasons mentioned above.

## CHAPTER V

### CONCLUSION

In this thesis, we have studied the multi-commodity flow model for solving the Thai Airways crew rostering problem that minimizes the upper bound of the per-diem and the workload of each crew member. The main objective in crew rostering is to minimize the sum of the maximum of the per-diem and workloads among the crew members. By minimizing the objective function, we are hoping the solution will keep the upper bounds of the per-diem and workloads close to the average and, in turn, distribute the workload and per-diem somewhat evenly among crew members. In fact, if the workload and per-diem of all pairings are roughly the same, minimizing the objective function is equivalent to balancing the workload and per-diem among crew members.

In our case study, we focus on the in-flight manager crew members. The proposed model was tested for the solution quality on 3 sets of case study. The network structure of the crew rostering problem for each set is generated using R script language and the problem is solved by using IBM ILOG CPLEX 12.10. running on Intel Core 2 Duo 2.67 GHz, RAM 2.00 GB.

The results show that the instance with the matrix size at most  $169949 \times 199389$  can be solved under 35 hours but the instance with the larger matrix size at least cannot be solved within 60 hours. The sum of the upper bound of workload and per-diem is minimized for the instances that can be solved, but the distribution of the workloads and the per-diem varies greatly on each instance. It should be the result of the limitation of the choices of pairings available where most of them are long flight duty pairs. Moreover, the multi-objective function makes the structure of the problem more

complicated, making balancing both workload and per-diem difficult and the solving process slow.

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