CHAPTER II

PRINCIPLE OF NUCLEAR MAGNETIC INDUCTION

2.1 Motion of a Free Spin(2, 8,9)

Consider a free spin whose magnetic moment is $\vec{\mu}$ placed in a field $\vec{H} = H_0\hat{z}$. A magnetic dipole in a magnetic field would experience a torque:

$$\vec{\Gamma} = \vec{\mu} \times \vec{H}. \qquad (2.1)$$

This torque gives rise to a rate of change of angular momentem $\frac{d\vec{I}}{dt}$, and we have

$$\frac{d\vec{I}}{dt} = \vec{\mu} \times \vec{H}\ . \qquad (2.2)$$

With $\vec{\mu} = \gamma\vec{I}$ we have

$$\frac{d\vec{\mu}}{dt} = \gamma\,(\vec{\mu} \times \vec{H}), \qquad (2.3)$$

which is the gyroscopic equation.

The rate of change of the magnetic moment in eq.(2.3) is along $(\vec{\mu} \times \vec{H})$ which is perpendicular to both $\vec{H}$ and $\vec{\mu}$ itself. The magnitide of the magnetic moment is unchanged. Only its direction is changed. The gyroscopic equation eq.(2.3) says that the magnetic moment precess about $\vec{H}$ with angular velocity $\omega = \gamma H_0$.

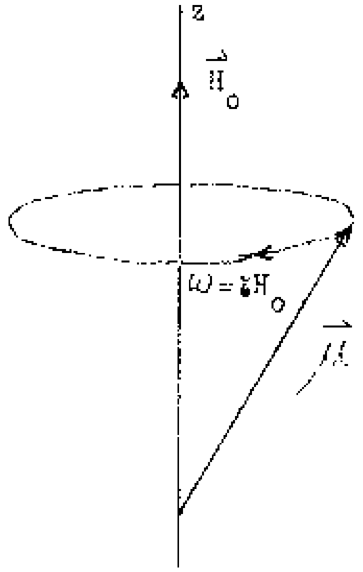


Fig. 2. Precession of nuclear moments.

It is fruitful however to look at eq.(2.3) in a suitable rotating coordinate system. Let S' be a frame of reference rotating with respect to the laboratory frame S with an angular velocity $\vec{\omega}$. The time derivative $\frac{d\vec{A}}{dt}$ of any time dependent quantity $\vec{A}(t)$, computed in the laboratory frame S and its derivative $\frac{\delta\vec{A}}{\delta t}$ computed in the rotating frame S', are related through

$$\frac{d\vec{A}}{dt} = \frac{\delta\vec{A}}{\delta t} + \vec{\omega} \times \vec{A}. \tag{2.4}$$

By making use of eq.(2.4), we can write eq.(2.3) in terms of a coordinate system rotating with angular velocity $\vec{\omega}$ with respect to laboratory frame:

$$\frac{\delta\vec{\mu}}{\delta t} + \vec{\omega} \times \vec{\mu} = \gamma\vec{\mu} \times \vec{H}, \tag{2.5}$$

or

$$\frac{\delta\vec{\mu}}{\delta t} = \gamma\vec{\mu} \times (\vec{H} + \frac{\vec{\omega}}{\gamma}). \tag{2.6}$$

Eq.(2.6) tells us that the motion of $\vec{\mu}$ in the rotating coordinate system obeys the the same equation as in the laboratory system, provided the magnetic field $\vec{H}$ is replaced by an effective field $\vec{H}_e = \vec{H} + \frac{\vec{\omega}}{\gamma}$, the sum of the laboratory field $\vec{H}$ and a fictitious field $\frac{\vec{\omega}}{\gamma}$.

We can now solve for the motion of $\vec{\mu}$ in a static field $\vec{H} = H_o\hat{z}$. By choosing a rotating frame with $\vec{\omega} = -\gamma H_o\hat{z}$ with respect to laboratry frame, the effective field $\vec{H}_e$ vanishes. In this reference frame $\frac{\delta\vec{\mu}}{\delta t} = 0$ and the magnetic moment is a fixed vector. Therefore, with respect to the laboratry frame it precesses with an angular velocity $\vec{\omega} = -\gamma H_o\hat{z}$. The minus sign only tell the sense of rotation i.e. clockwise if looking down toward -z direction. The value of the precessional frequency $\omega = \gamma H_o$ of the spin in a static field H_o is called the Larmor frequency. The motion of expectation value in quantum mechanical description is the same as the classical description.

It happens that the Larmor frequency is exactly the same as the angular frequency needed for the magnetic resonance absorption mentioned in section 1.4 . From measurement of the Larmor frequency, H_o can be computed when γ is known with precission.

2.2 Bloch Equations and Relaxation Times(5)

The nuclear magnetization $\vec{M}$ is the sum $\sum_i \mu_i$ (i denotes the i th nucleus) over all the nuclei in a unit volume. When the nuclei are placed in a static field $\vec{H} = H_o\hat{z}$, the equation of motion for M is exactly the same as that for individual spin: that is

$$\frac{d\vec{M}}{dt} = \gamma \vec{M} \times \vec{H}. \qquad (2.7)$$

In thermal equilibrium at temperature T the magnetization will be along $\hat{z}$, even though individual $\vec{\mu}$ can not be along $\vec{H}$ quantum mechanically. From knowledge of paramagnetism it is expected that at thermal equilibrium

$$M_x = 0, \qquad (2.8)$$

$$M_y = 0, \qquad (2.9)$$

$$M_z = M_o = \chi H_o. \qquad (2.10)$$

Suppose the magnetization component M_z is not in thermal equilibrium. We assume that M_z approaches equilibrium at a rate proportional to the departure from the equilibrium value M_o,

$$\frac{d\vec{M}_z}{dt} = (M_o - M_z)/T_1. \qquad (2.11)$$

T_1 is called the spin-lattice relaxation time or longitudinal relaxation time. The spin-lattice relaxation time T_1 is a time constant of M which approaches exponentially to the equilibrium value M_o.

If at $t = 0$ an unmagnetized specimen is placed in a magnetic field $H_o\hat{z}$, the magnetization will increase from the initial value $M_z = 0$ to a final value $M_z = M_o$. The eq.(2.11) can be integrated;

$$\int_{0}^{M_z} \frac{dM_z}{M_o - M_z} = \frac{1}{T_1}\int_{0}^{t} dt, \tag{2.12}$$

then $$M_z(t) = M_o(1 - e^{-t/T_1}). \tag{2.13}$$

Taking account of eq.(2.11), the z-component of the equation of motion eq.(2.7) becomes

$$\frac{dM_z}{dt} = (\vec{M} \times \vec{H})_z + (M_o - M_z)/T_1 , \tag{2.14}$$

where $(M_o - M_z)/T_1$ is an extra trem in the equation of motion, ariseing from interactions not included in the magnetic field $\vec{H}$. That is, besides processing about the magnetic field, $\vec{M}$ will relax to the equilibrium value $\vec{M}_o$.

If in a static field $H_o\hat{z}$ the transverse magnetization component M_x is not zero, then M_x will decay to zero, and similarly for M_y. The decay occurs because in thermal equilibrium the transverse components are zero. We can modify the equations to provide for transverae relaxation;

$$\frac{dM_x}{dt} = \gamma(\vec{M} \times \vec{H})_x - \frac{M_x}{T_2} , \tag{2.15}$$

$$\frac{dM_y}{dt} = \gamma(\vec{M} \times \vec{H})_y - \frac{M_y}{T_2} , \tag{2.16}$$

where T_2 is called the transverse relaxation time.

The transverse relaxation time is a time constant of M_x or M_y which decays to zero.

The magnetic energy $-\vec{H}.\vec{M}$ does not change as M_x or M_y changes, provided that H is along z. No energy flow out of the spin system for relaxation of M_x or M_y to occur, so that the conditions which determine T_2 may be less than for T_1. Sometimes the two times are nearly equal, but usually $T_1 \gg T_2$, depending on local conditions. Two separate kinds of relaxation processes must be considered.

The eq.(2.14), eq.(2.15) and eq.(2.16) are called the Bloch equations.

2.3 Solution to Bloch Equations

We are interested in the behavior of the transverse components M_x or M_y of the magnetization. Solution to the longitudinal component has been given by eq.(2.13). The first term on the right of eq.(2.15) or eq.(2.16) implies a precession of the transverse components of the magnetization around the applied field $\vec{H}$. Hence in a frame of reference rotating with an angular velocity $\vec{\omega} = -\gamma H_0 \hat{z}$ with respect to laboratory frame, M_x will be considered as a constant vector. Let us consider the second term on the right of eq.(2.15), this term is not included in the magnetic field M. We can write the rate of change of M_x as

$$\frac{dM_x}{dt} = -\frac{M_x}{T_2} \qquad (2.17)$$

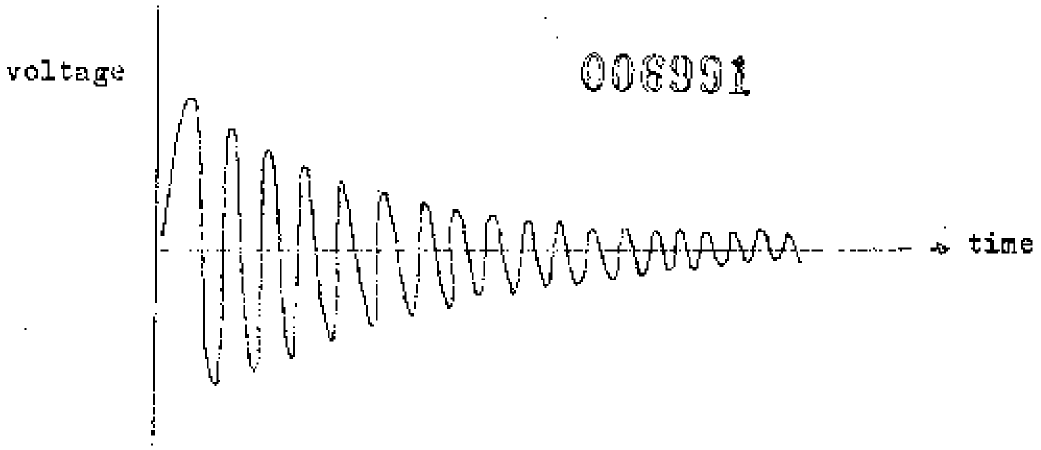
Suppose at $t = 0$ M_x is not zero. In thermal equilibrium

M_x is zero. The eq.(2.17) can be integrated

$$\int_{M_x(0)}^{M_x(t)} \frac{dM_x}{M_x} = -\frac{1}{T_2}\int_0^t dt \, , \qquad (2.18)$$

then $$M_x(t) = M_x(0)e^{-t/T_2} \, . \qquad (2.19)$$

Eq.(2.19) tells us that $M_x(t)$ decays exponentially at a rate determine by T_2. That is, in the frame rotating with angular velocity of $\vec{\omega} = -\gamma H_0 \hat{z}$ with respect to laboratory frame, M_x precesses around the field $H_0 \hat{z}$ and simultaneously decays with a time constant T_2. M_x will induce a sinusoidal voltage in a coil oriented in the x-axis of the laboratory frame. The amplitude of a sinusoidal voltage will also decays exponentially with a time constant T_2. Hence the signal of the type of Fig. 3 should be observed. T_2 determines the amplitude of the induced voltage.



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F . 3. Decaying signal.

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2.4 Effect of Field Inhomogeneity(3)

We have seen in section 2.3 that the precessing transverse magnetization have to decay to zero at a rate determines by T_2 and signal induced in a coil is damped at the same rate. In practice T_2 is not the only source of damping of the induced signal. Another important source of damping is due to the field inhomogeneity. This arises from the fact that magnetization at various part of the sample volume are often in slightly different field and precess at slightly different frequencies. The precession of the magnetization in various parts of the sample volume will be out of phase in time $(\gamma\Delta H)^{-1}$, where ΔH denotes the average deviation of the field at various points over the sample. If $(\gamma\Delta H)^{-1}$ is shorter than T_2, the controlling factor of signal amplitude is due to ΔH, the field inhomogeneity. The decaying curve is not necessarily exponential.

2.5 Induced Signal(8,10)

Only in a sample with long relaxation time T_1 is it possible to observe the free precession in very weak field such as the earth field. Distilled water is chosen as a sample. It is magnetized in a large field $\vec{H}_p + \vec{H}_1$, with $H_p \gg H_1$, where H_p is the polarizing field along z-direction and H_1 is the earth's total magnetic field along y-direction. A sample acquires the magnetization $M = \chi H_p$. When H_p is switched off immediately. a large magnetization M will precess around the earth field H_1 with angular velocity $\omega_o = -\gamma H_1$. Following a precession of

M, a component Msinθ precesses in the xz plane as shown in Fig. 4, and induced a sinusoidal voltage at the terminals of a pick-up coil oriented along z-direction. If the coil is tuned in parallel with a condenser C at a frequency ω_o, it is seen that an e.m.f. $\mathcal{E}$ of that frequency induced in the coil will be a voltage of $V' = Q\mathcal{E}$ across its terminals, where Q is the quality factor of the coil.

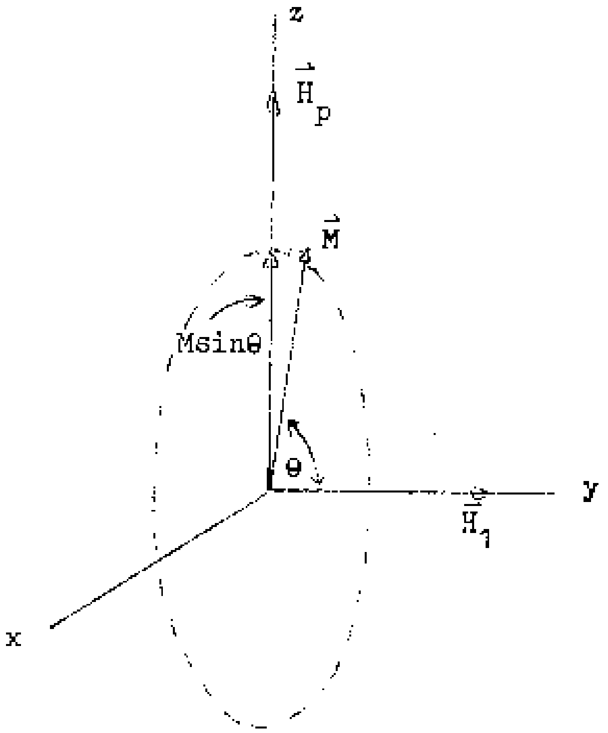

Fig. 4. Illustration of the position of $\vec{M}$ after polarizing field is switched off. A component Msinθ precesses in the xz plane.

The voltage $V = Q\mathcal{E}$ available at the terminals of the coil of n turns of diameter d is easily computed. The amplitude $\mathcal{E}$ of the e.m.f. induced in the coil by the precessing magnetization is given by the Faraday law

$$\mathcal{E} = 4\pi\omega_0 AM\sin\theta \ , \qquad (2.20)$$

where $A = \frac{1}{4}\pi d^2 n$ is the total coil area crossed by the magnetic flux. In suitable units, this is

$$\mathcal{E}_{volts} = 10^{-8}\pi^2\omega_0 d^2 nM\sin\theta \ . \qquad (2.21)$$

Hence, for $\theta = \frac{\pi}{2}$, $V = Q\mathcal{E}$,

$$V_{volts} = 10^{-8}\pi^2\omega_0 d^2 n\, Q\, M. \qquad (2.22)$$

It is of interest to calculate the magnitude of induced e.m.f. that might be expected in a practical case. Consider a 110 cm^3 sample of water, in which there are 6.6×10^{22} protons/cm^3, each with a magnetic moment μ equal to 1.4×10^{23} c.g.s. units. Since $I = \frac{1}{2}$ for the proton, if a polarizing field of 60 oersted is applied to the sample, when its temperature is about 300 ^{0}K, the nuclear magnetization M is equal to 3.4×10^{-8} c.g.s. units. In Bangkok, the magnetization M precesses in the earth field at frequency of approximately 1800 Hz . If $n = 520$ turns, $d = 5$ cm, $Q = 18$, then

$$V \simeq 5 \times 10^{-6} \text{ volt.}$$