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ภาคผนวก

ภาคผนวก ก.

แสดงค่าคงที่ของธาตุต่าง ๆ และค่าภาคตัดขวางต่าง ๆ สำหรับเทอร์มัลนิวตรอน

CROSS SECTIONS FOR NATURALLY OCCURRING ELEMENTS
(2200-METERS/SEC NEUTRONS)

Atomic No.	Element or Compound	Atomic or Mol. Wt.	Density, g/cm ³	Nuclei per Unit Vol. × 10 ⁻²⁴	ξ	Microscopic Cross Section, barns		Macroscopic Cross Section, cm ⁻¹	
						σ _a	σ _s	Σ _a	Σ _s
1	H	1.008	8.9 ^a	5.3 ^a	1.000	0.33	38	1.7 ^a	0.002
	H ₂ O	18.016	1	0.0335 ^b	0.948	0.66	103	0.022	3.45
	D ₂ O	20.030	1.10	0.0331 ^b	0.570	0.001	13.6	3.3 ^a	0.449
2	He	4.003	17.8 ^a	2.6 ^a	0.425	0.007	0.8	0.02 ^a	2.1 ^a
3	Li	6.940	0.534	0.0463	0.268	71	1.4	3.29	0.065
4	Be	9.013	1.85	0.1236	0.209	0.010	7.0	124 ^a	0.865
	BeO	25.02	3.025	0.0728 ^b	0.173	0.010	6.8	73 ^a	0.501
5	B	10.82	2.45	0.1364	0.171	755	4	103	0.346
6	C	12.011	1.60	0.0803	0.158	0.004	4.8	32 ^a	0.385
7	N	14.008	0.0013	5.3 ^a	0.136	1.88	10	9.9 ^a	50 ^a
8	O	16.000	0.0014	5.3 ^a	0.120	20 ^a	4.2	0.000	21 ^a
9	F	19.00	0.0017	5.3 ^a	0.102	0.01	3.9	0.05 ^a	20 ^a
10	Ne	20.183	0.0009	2.6 ^a	0.0968	<2.8	2.4	7.3 ^a	6.2 ^a
11	Na	22.991	0.971	0.0254	0.0845	0.525	4	0.013	0.102
12	Mg	24.32	1.74	0.0431	0.0811	0.069	3.6	0.003	0.155
13	Al	26.98	2.699	0.0602	0.0723	0.241	1.4	0.015	0.084
14	Si	28.09	2.42	0.0522	0.0698	0.16	1.7	0.008	0.089
15	P	30.975	1.82	0.0354	0.0632	0.20	5	0.007	0.177
16	S	32.066	2.07	0.0389	0.0612	0.52	1.1	0.020	0.043
17	Cl	35.457	0.0032	5.3 ^a	0.0561	33.8	16	0.002	80 ^a
18	Ar	39.944	0.0018	2.6 ^a	0.0492	0.66	1.5	1.7 ^a	3.9
19	K	39.100	0.87	0.0134	0.0504	2.07	1.5	0.028	0.020
20	Ca	40.08	1.55	0.0233	0.0492	0.44	3.0	0.010	0.070
21	Sc	44.96	2.5	0.0335	0.0438	24	24	0.804	0.804
22	Ti	47.90	4.5	0.0566	0.0411	5.8	4	0.328	0.226
23	V	50.95	5.96	0.0704	0.0387	5	5	0.352	0.352
24	Cr	52.01	7.1	0.0822	0.0385	3.1	3	0.255	0.247
25	Mn	54.94	7.2	0.0789	0.0359	13.2	2.3	1.04	0.181
26	Fe	55.85	7.86	0.0848	0.0353	2.62	11	0.222	0.933
27	Co	58.94	8.9	0.0190	0.0335	38	7	3.46	0.637
28	Ni	58.71	8.90	0.0913	0.0335	4.6	17.5	0.420	1.60

29	Cu	63.54	8.94	0.0848	0.0309	3.85	7.2	0.326	0.611
30	Zn	65.38	7.14	0.0658	0.0304	1.10	3.6	0.072	0.237
31	Ga	69.72	5.91	0.0511	0.0283	2.80	4	0.143	0.204
32	Ge	72.60	5.36	0.0445	0.0271	2.45	3	0.109	0.134
33	As	74.91	5.73	0.0461	0.0264	4.3	6	0.198	0.277
34	Se	78.96	4.8	0.0366	0.0251	12.3	11	0.450	0.403
35	Br	79.916	3.12	0.0235	0.0247	6.7	6	0.157	0.141
36	Kr	83.80	0.0037	2.6 ^a	0.0236	31	7.2	81 ^a	19 ^a
37	Rb	85.48	1.53	0.0108	0.0233	0.73	12	0.008	0.130
38	Sr	87.63	2.54	0.0175	0.0226	1.21	10	0.021	0.175
39	Y	88.92	5.51	0.0373	0.0223	1.31	3	0.049	0.112
40	Zr	91.22	6.4	0.0423	0.0218	0.185	8	0.008	0.338
41	Nb	92.91	8.4	0.0545	0.0214	1.16	5	0.063	0.273
42	Mo	95.95	10.2	0.0640	0.0207	2.70	7	0.173	0.448
43	Tc	98			0.0203	22	6	0.186	0.436
44	Ru	101.1	12.2	0.0727	0.0197	2.56	5	0.109	0.366
45	Rh	102.91	12.5	0.0732	0.0193	149	5	0.551	0.248
46	Pd	106.4	12.16	0.0689	0.0187	8	3.6	3.69	0.352
47	Ag	107.88	10.5	0.0586	0.0184	63	6	114	0.325
48	Cd	112.41	8.65	0.0464	0.0178	2450	7	7.30	0.084
49	In	114.82	7.28	0.0382	0.0173	191	2.2	0.021	0.132
50	Sn	118.70	6.5	0.0330	0.0167	0.625	4	0.189	0.142
51	Sb	121.76	6.69	0.0331	0.0163	5.7	4.3	0.139	0.148
52	Te	127.61	6.24	0.0295	0.0155	4.7	5	0.164	0.084
53	I	126.91	4.93	0.0234	0.0157	7.0	3.6	95 ^a	12 ^a
54	Xe	131.30	0.0059	2.7 ^a	0.0152	35	4.3	0.238	0.170
55	Cs	132.91	1.873	0.0085	0.0150	28	20	0.018	0.123
56	Ba	137.36 ^b	3.5	0.0154	0.0145	1.2	8	0.239	0.403
57	La	138.92	6.19	0.0268	0.0143	8.9	15	0.021	0.263
58	Ce	140.13	6.78	0.0292	0.0142	0.73	9	0.328	0.116
59	Pr	140.92	6.78	0.0290	0.0141	11.3	4	1.33	0.464
60	Nd	144.27	6.95	0.0290	0.0138	46	16		
61	Pm	145			0.0137	60			
62	Sm	150.35	7.7	0.0309	0.0133	5600	5	173	0.155
	Sm ₂ O ₃	348.70	7.43	0.0128 ^b	0.076	11,200	22.6	143	0.289
	Eu	152	5.22	0.0207	0.0131	4300	8	89.0	0.166
63	Eu ₂ O ₃	352.00	7.42	0.0127 ^b	0.063	8600	30.2	109	0.383



CROSS SECTIONS FOR NATURALLY OCCURRING ELEMENTS (continued)

Atomic No.	Element or Compound	Atomic or Mol. Wt.	Density, g/cm ³	Nuclei per Unit Vol. × 10 ⁻²⁴	ξ	Microscopic Cross Section, barns		Macroscopic Cross Section, cm ⁻¹	
						σ _a	σ _t	Σ _a	Σ _t
64	Gd	157.26	7.95	0.0305	0.0127	46,000		1403	
65	Tb	158.93	8.33	0.0316	0.0125	46		1.45	
66	Dy	162.51	8.56	0.0317	0.0122	950	100	30.1	3.17
	Dy ₂ O ₃	372.92	7.81	0.0126 ^b	0.019	2200	214	27.7	2.7
67	Ho	164.94	8.76	0.0320	0.0121	65		2.08	
68	Er	167.27	9.16	0.0330	0.0119	173	15	5.71	0.495
69	Tm	168.94	9.35	0.0333	0.0118	127	7	4.23	0.233
70	Yb	173.04	7.01	0.0244	0.0115	37	12	0.903	0.293
71	Lu	174.99	9.74	0.0335	0.0114	112		3.75	
72	Hf	178.5	13.3	0.0449	0.0112	105	8	4.71	0.359
73	Ta	180.95	16.6	0.0553	0.0110	21	5	1.16	0.277
74	W	183.86	19.3	0.0632	0.0108	19.2	5	1.21	0.316
75	Re	186.22	20.53	0.0664	0.0107	86	14	5.71	0.930
76	Os	192.2	22.48	0.0712	0.0105	15.3	11	1.09	0.783
77	Ir	192.2	22.42	0.0703	0.0104	440		30.9	
78	Pt	195.09	21.37	0.0660	0.0102	8.8	10	0.581	0.660
79	Au	197	19.32	0.0591	0.0101	98.8	9.3	5.79	0.550
80	Hg	200.61	13.55	0.0407	0.0099	380	20	15.5	0.814
81	Tl	204.39	11.85	0.0349	0.0098	3.4	14	0.119	0.489
82	Pb	207.21	11.35	0.0330	0.0096	0.170	11	0.006	0.363
83	Bi	209	9.747	0.0281	0.0095	0.034	9	0.001	0.253
84	Po	210	9.24	0.0265	0.0095				
85	At	211		2.6 ^a	0.0094				
86	Rn	222	0.0097		0.0090				
87	Fr	223			0.0089				
88	Ra	226.05	5	0.0133	0.0088	20		0.266	
89	Ac	227			0.0088				
90	Th	232.05	11.3	0.0293	0.0086	7.56	12.6	0.222	0.369
91	Pa	231	15.4	0.0402	0.0086	200		8.04	0.397
92	U	238.07	18.9	0.04783	0.0084	7.68	8.3	0.367	0.372
	UO ₂	270.07	10	0.0223 ^b	0.036	7.7	16.7	0.17	
93	Np	237			0.0084	170			
94	Pu	239	19.7	0.0497	0.0083	1029	9.6	51.1	0.478

^a Value has been multiplied by 10⁶.^b Molecules/cm³

ภาคผนวก ข.

แสดงค่าคงที่ต่าง ๆ ของตัวหน่วงนิวตรอน

PROPERTIES OF MODERATORS AT 78°F (20°C)

Material	Ordinary Water	Heavy Water (99.75% D ₂ O)	Beryllium	Beryllium Oxide	Graphite (Reactor grade)	Diphenyl (at 200°F)	Zirconium Hydride (ZrH ₂)
Atomic (or molecular) weight.....	18	20	9	25	12	154	93.2
Density (g/cm ³).....	1.00	1.10	1.84	3.0	1.70	0.96	5.61
N (atoms or molecules per cm ³) × 10 ⁻²⁴	0.0334	0.0332	0.124	0.072	0.0855	0.0039	0.036
Thermal (2200 m/sec)							
σ _s (barns).....	0.66	0.003	0.010	0.010	0.0037	2.5	0.84
σ _a (barns).....	103	13.6	7.0	6.8	4.8	600	—
*Σ _s (cm ⁻¹).....	0.022	8.5 × 10 ⁻⁴	0.00123	0.00073	0.00032	0.01	0.030
Σ _a (cm ⁻¹).....	3.45	0.45	0.86	0.50	0.41	2.3	—
D (cm).....	0.17	0.85	0.54	0.66	0.94	0.24	~0.2
L (cm).....	2.76	100	21	30	54.2	4.9	~3.0
Epithermal							
ξ.....	0.93	0.51	0.206	0.17	0.158	0.81	0.84
σ _s (barns).....	42	10.5	6.1	9.9	4.8	260	48.6
Σ _s (cm ⁻¹).....	1.40	0.35	0.75	0.72	0.41	1.0	1.75
ξΣ _s (cm ⁻¹).....	1.28	0.18	0.16	0.12	0.065	0.8	1.47
ξΣ _s /Σ _a	58	21,000	130	163	200	80	49
Age to thermal τ (cm ²).....	31	120	85	100	350	58	27

ภาคผนวก ค.

ค่าคงที่ในการฟุ้งกระจายของตัวหน่วงนิวตรอนสำหรับ เทอร์มัลนิวตรอน

DIFFUSION PROPERTIES OF MODERATORS FOR THERMAL NEUTRONS

Moderator	Density (g/cm ³)	<i>L</i> (cm)	Σ_a (cm ⁻¹)	<i>D</i> (cm)
Water.....*	1.00	2.76	2.2×10^{-2}	0.17
Heavy water (99.75% D ₂ O).....	1.10	100	8.5×10^{-6}	0.85
Beryllium.....	1.84	21	1.2×10^{-3}	0.54
Carbon (graphite)*.....	1.70	54.2	3.2×10^{-4}	0.94

* High-purity commercial (reactor grade) product.

ภาคผนวก ง.

แสดงค่าเบสเซล

Bessel Functions: $J_0(x)$ and $J_1(x)$
 $J_0(x)$

x	0	1	2	3	4	5	6	7	8	9
0.0	+1.0000	1.0000	0.9999	9998	9996	9994	9991	9988	9984	9980
1	+0.9975	9970	9964	9958	9951	9944	9936	9928	9919	9910
2	9900	9890	9879	9868	9857	9844	9832	9819	9805	9791
3	9776	9761	9746	9730	9713	9696	9679	9661	9642	9623
4	9604	9584	9564	9543	9522	9500	9478	9455	9432	9409
5	9385	9360	9335	9310	9284	9258	9231	9204	9177	9149
6	9120	9091	9062	9032	9002	8971	8940	8909	8877	8845
7	8812	8779	8745	8711	8677	8642	8607	8572	8536	8500
8	8463	8426	8388	8350	8312	8274	8235	8195	8156	8116
9	8075	8034	7993	7952	7910	7868	7825	7783	7739	7696
1.0	7652	7608	7563	7519	7473	7428	7382	7336	7290	7243
1	7196	7149	7101	7054	7006	6957	6909	6860	6810	6761
2	6711	6661	6611	6561	6510	6459	6408	6356	6305	6253
3	6201	6149	6096	6043	5990	5937	5884	5830	5777	5723
4	5669	5614	5560	5505	5450	5395	5340	5285	5230	5174
5	5118	5062	5006	4950	4894	4838	4781	4725	4668	4611
6	4554	4497	4440	4383	4325	4268	4210	4153	4095	4038
7	3980	3922	3864	3806	3748	3690	3632	3574	3516	3458
8	3400	3342	3284	3225	3167	3109	3051	2993	2934	2876
9	2818	2760	2702	2644	2586	2528	2470	2412	2354	2297
2.0	2239	2181	2124	2066	2009	1951	1894	1837	1780	1723
1	1666	1609	1553	1496	1440	1383	1327	1271	1215	1159
2	1104	1048	0993	0937	0882	0827	0773	0718	0664	0609
3	0555	0502	0448	0394	0341	0288	0235	0182	0130	0077
4	+0025	*0027	*0079	*0130	*0181	*0232	*0283	*0334	*0384	*0434
5	-0484	0533	0583	0632	0681	0729	0778	0826	0873	0921
6	-0968	1015	1062	1108	1154	1200	1245	1291	1336	1380
7	-1424	1469	1512	1556	1599	1641	1684	1726	1768	1809
8	-1854	1891	1932	1972	2012	2051	2090	2129	2167	2205
9	-2243	2280	2317	2354	2390	2426	2462	2497	2532	2566
3.0	-2601	2634	2668	2701	2733	2765	2797	2829	2860	2890
1	-2921	2951	2980	3009	3038	3066	3094	3122	3149	3176
2	-3202	3228	3253	3278	3303	3328	3351	3375	3398	3421
3	-3443	3465	3486	3507	3528	3548	3568	3587	3606	3625
4	-3643	3661	3678	3695	3711	3727	3743	3758	3773	3787
5	-3801	3815	3828	3841	3853	3865	3876	3887	3898	3908
6	-3918	3927	3936	3944	3953	3960	3967	3974	3981	3987
7	-3992	3997	4002	4007	4011	4014	4017	4020	4022	4024
8	-4026	4027	4027	4028	4027	4027	4026	4025	4023	4021
9	-4018	4015	4012	4008	4004	4000	3995	3990	3984	3978
4.0	-3971	3965	3958	3950	3942	3934	3925	3916	3907	3897
1	-3887	3876	3865	3854	3842	3831	3818	3806	3793	3779
2	-3766	3752	3737	3722	3707	3692	3676	3660	3644	3627
3	-3610	3593	3575	3557	3539	3520	3501	3482	3463	3443
4	-3423	3402	3381	3360	3339	3318	3296	3274	3251	3228
5	-3205	3182	3159	3135	3111	3087	3062	3037	3012	2987
6	-2961	2936	2910	2883	2857	2830	2803	2776	2749	2721
7	-2693	2665	2637	2609	2580	2551	2522	2493	2464	2434
8	-2404	2374	2344	2314	2283	2253	2222	2191	2160	2129
9	-2097	2066	2034	2002	1970	1938	1906	1874	1841	1809
5.0	-1776	1743	1710	1677	1644	1611	1578	1544	1511	1477

$$J_0(x) = 1 - \frac{x^2}{2^2(1!)^2} + \frac{x^4}{2^4(2!)^2} - \frac{x^6}{2^6(3!)^2} + \dots$$

$$J_0'(x) = -J_1(x)$$

Bessel Functions: $J_0(x)$ and $J_1(x)$. (Continued)

x	0	1	2	3	4	5	6	7	8	9
5.0	-0.1776	1743	1710	1677	1644	1611	1578	1544	1511	1477
1	-1443	1410	1376	1342	1308	1274	1240	1206	1171	1137
2	-1103	1069	1034	1000	0965	0931	0896	0862	0827	0793
3	-0758	0723	0689	0654	0620	0585	0550	0516	0481	0447
4	-0412	0378	0343	0309	0274	0240	0205	0171	0137	0103
5	-0068	0034	0000	*0034	*0068	*0102	*0135	*0169	*0203	*0236
6	+0270	0303	0336	0370	0403	0436	0469	0501	0534	0567
7	0599	0632	0664	0696	0728	0760	0792	0823	0855	0886
8	0917	0948	0979	1010	1040	1071	1101	1131	1161	1191
9	1220	1250	1279	1308	1337	1366	1394	1423	1451	1479
6.0	1506	1534	1561	1589	1616	1642	1669	1695	1721	1747
1	1773	1798	1824	1849	1873	1898	1922	1947	1970	1994
2	2017	2041	2064	2086	2109	2131	2153	2175	2196	2217
3	2238	2259	2279	2299	2319	2339	2358	2377	2396	2415
4	2433	2451	2469	2486	2504	2521	2537	2554	2570	2585
5	2601	2616	2631	2646	2660	2674	2688	2702	2715	2728
6	2740	2753	2765	2777	2788	2799	2810	2821	2831	2841
7	2851	2860	2869	2878	2886	2895	2902	2910	2917	2924
8	2931	2937	2943	2949	2955	2960	2965	2969	2973	2977
9	2981	2984	2987	2990	2993	2995	2997	2998	2999	3000
7.0	3001	3001	3001	3001	3000	2999	2998	2997	2995	2993
1	2991	2988	2985	2982	2978	2974	2970	2966	2961	2956
2	2951	2945	2939	2933	2927	2920	2913	2906	2898	2890
3	2882	2874	2865	2856	2847	2837	2828	2818	2807	2797
4	2786	2775	2764	2752	2740	2728	2715	2703	2690	2677
5	2663	2650	2636	2622	2607	2593	2578	2563	2547	2532
6	2516	2500	2484	2467	2451	2434	2416	2399	2381	2364
7	2346	2327	2309	2290	2271	2252	2233	2214	2194	2174
8	2154	2134	2113	2093	2072	2051	2030	2009	1987	1965
9	1944	1922	1899	1877	1855	1832	1809	1786	1763	1740
8.0	1717	1693	1669	1645	1622	1597	1573	1549	1524	1500
1	1475	1450	1425	1400	1375	1350	1325	1299	1274	1248
2	1222	1196	1170	1144	1118	1092	1066	1039	1013	0987
3	0960	0934	0907	0880	0853	0826	0800	0773	0745	0719
4	0692	0665	0637	0610	0583	0556	0529	0501	0474	0447
5	0419	0392	0365	0337	0310	0283	0255	0228	0201	0174
6	+0146	0119	0092	0065	0037	0010	*0017	*0044	*0071	*0100
7	-0125	0152	0179	0206	0233	0260	0286	0313	0339	0366
8	-0392	0419	0445	0471	0497	0524	0549	0575	0601	0627
9	-0653	0678	0704	0729	0754	0779	0804	0829	0854	0879
9.0	-0903	0928	0952	0976	1000	1024	1048	1072	1096	1119
1	-1142	1166	1189	1211	1234	1257	1279	1302	1324	1346
2	-1367	1389	1411	1432	1453	1474	1495	1516	1536	1556
3	-1577	1597	1616	1636	1655	1674	1694	1712	1731	1749
4	-1768	1786	1804	1821	1839	1856	1873	1890	1907	1923
5	-1939	1955	1971	1987	2002	2017	2032	2047	2061	2076
6	-2090	2104	2117	2131	2144	2157	2169	2182	2194	2206
7	-2218	2230	2241	2252	2263	2273	2284	2294	2304	2313
8	-2323	2332	2341	2350	2358	2366	2374	2382	2389	2396
9	-2403	2410	2417	2423	2429	2434	2440	2445	2450	2455
10.0	-2459	2464	2468	2471	2475	2478	2481	2484	2486	2488
Zeros of $J_0(x)$	1	2	3	4	5	6	7	8		
$x =$	2.4048	5.5201	8.6537	11.7915	14.9309	18.0711	21.2116	24.3525		

Bessel Functions: $J_0(x)$ and $J_1(x)$. (Continued)
 $J_0(x)$

x	0	1	2	3	4	5	6	7	8	9
10.0	-0.2459	2464	2468	2471	2475	2478	2481	2484	2486	2488
1	-2490	2492	2493	2495	2496	2496	2497	2497	2497	2497
2	-2496	2495	2494	2493	2492	2490	2488	2485	2483	2480
3	-2477	2474	2470	2467	2463	2458	2454	2449	2444	2439
4	-2434	2428	2422	2416	2410	2403	2396	2389	2382	2374
5	-2366	2358	2350	2342	2333	2324	2315	2306	2296	2286
6	-2276	2266	2256	2245	2234	2223	2212	2200	2188	2177
7	-2164	2152	2140	2127	2114	2101	2087	2074	2060	2046
8	-2032	2018	2003	1989	1974	1959	1943	1928	1912	1897
9	-1881	1865	1848	1832	1815	1798	1781	1764	1747	1730
11.0	-1712	1694	1676	1658	1640	1622	1603	1584	1566	1547
1	-1528	1508	1489	1470	1450	1430	1411	1391	1370	1350
2	-1330	1309	1289	1268	1247	1227	1206	1185	1163	1142
3	-1121	1099	1078	1056	1034	1012	991	969	946	924
4	-0902	0880	0858	0835	0813	0790	0767	0745	0722	0699
5	-0677	0654	0631	0608	0585	0562	0539	0516	0493	0469
6	-0446	0423	0400	0376	0353	0330	0307	0283	0260	0237
7	-0213	0190	0167	0143	0120	0097	0073	0050	0027	0004
8	+0020	0043	0066	0089	0112	0135	0159	0182	0205	0228
9	0250	0273	0296	0319	0342	0364	0387	0410	0432	0455
12.0	0477	0499	0521	0544	0566	0588	0610	0632	0653	0675
1	0697	0718	0740	0761	0782	0803	0824	0845	0866	0887
2	0908	0928	0949	0969	0989	1009	1029	1049	1069	1088
3	1108	1127	1147	1166	1185	1203	1222	1241	1259	1277
4	1296	1314	1331	1349	1367	1384	1401	1418	1435	1452
5	1469	1485	1502	1518	1534	1550	1565	1581	1596	1611
6	1626	1641	1655	1670	1684	1698	1712	1726	1739	1753
7	1766	1779	1792	1804	1817	1829	1841	1853	1864	1876
8	1887	1898	1909	1920	1930	1940	1950	1960	1970	1979
9	1988	1997	2006	2015	2023	2031	2039	2047	2055	2062
13.0	2069	2076	2083	2089	2096	2102	2108	2113	2119	2124
1	2129	2134	2138	2143	2147	2151	2154	2158	2161	2164
2	2167	2169	2172	2174	2176	2178	2179	2180	2182	2182
3	2183	2183	2184	2184	2183	2183	2182	2181	2180	2179
4	2177	2175	2173	2171	2169	2166	2163	2160	2157	2154
5	2150	2146	2142	2138	2133	2128	2123	2118	2113	2107
6	2101	2095	2089	2083	2076	2069	2062	2055	2048	2040
7	2032	2024	2016	2008	1999	1990	1981	1972	1963	1953
8	1943	1933	1923	1913	1903	1892	1881	1870	1859	1847
9	1836	1824	1812	1800	1788	1775	1763	1750	1737	1724
14.0	1711	1697	1684	1670	1656	1642	1628	1613	1599	1584
1	1570	1555	1539	1524	1509	1493	1478	1462	1446	1430
2	1414	1397	1381	1364	1348	1331	1314	1297	1280	1262
3	1245	1227	1210	1192	1174	1156	1138	1120	1102	1083
4	1065	1046	1028	1009	0990	0971	0952	0933	0914	0895
5	0875	0856	0837	0817	0798	0778	0758	0738	0719	0699
6	0679	0659	0639	0618	0598	0578	0558	0538	0517	0497
7	0476	0456	0436	0415	0394	0374	0353	0333	0312	0291
8	0271	0250	0229	0209	0188	0167	0147	0126	0105	0085
9	+0064	0043	0023	+0002	-0019	-0039	-0060	-0081	-0101	-0122

For $x > 15$

$$J_0(x) \approx \sqrt{\frac{2}{\pi x}} \left[\cos\left(x - \frac{\pi}{4}\right) + \frac{1}{8x} \sin\left(x - \frac{\pi}{4}\right) \right], \text{ error} < 0.0001$$

$$\sqrt{\frac{2}{\pi}} = 0.7979 \quad \frac{\pi}{4} = 0.7854$$

Bessel Functions: $J_0(x)$ and $J_1(x)$. (Continued)

x	0	1	2	3	4	5	6	7	8	9
0.0	+0.00000	00500	01000	01500	02000	02499	02999	03498	03997	04495
1	0.04994	05492	05989	06486	06983	07479	07974	08469	08964	09457
2	0.09950	10442	1093	1142	1191	1240	1289	1338	1386	1435
3	0.1483	1531	1580	1628	1676	1723	1771	1819	1866	1913
4	1960	2007	2054	2101	2147	2194	2240	2286	2332	2377
5	2423	2468	2513	2558	2603	2647	2692	2736	2780	2823
6	2867	2910	2953	2996	3039	3081	3124	3166	3207	3249
7	3290	3331	3372	3412	3452	3492	3532	3572	3611	3650
8	3688	3727	3765	3803	3840	3878	3915	3951	3988	4024
9	4059	4095	4130	4165	4200	4234	4268	4302	4335	4368
1.0	4401	4433	4465	4497	4528	4559	4590	4620	4650	4680
1	4709	4738	4767	4795	4823	4850	4878	4904	4931	4957
2	4983	5008	5033	5058	5082	5106	5130	5153	5176	5198
3	5220	5242	5263	5284	5305	5325	5344	5364	5383	5401
4	5419	5437	5455	5472	5488	5504	5520	5536	5551	5565
5	5579	5593	5607	5620	5632	5644	5656	5667	5678	5689
6	5699	5709	5718	5727	5735	5743	5751	5758	5765	5772
7	5778	5783	5788	5793	5798	5802	5805	5808	5811	5813
8	5815	5817	5818	5818	5819	5818	5818	5817	5816	5814
9	5812	5809	5806	5803	5799	5794	5790	5785	5779	5773
2.0	5767	5761	5754	5746	5738	5730	5721	5712	5703	5693
1	5683	5672	5661	5650	5638	5626	5614	5601	5587	5574
2	5560	5545	5530	5515	5500	5484	5468	5451	5434	5416
3	5399	5381	5362	5343	5324	5305	5285	5265	5244	5223
4	5202	5180	5158	5136	5113	5091	5067	5044	5020	4996
5	4971	4946	4921	4895	4870	4843	4817	4790	4763	4736
6	4708	4680	4652	4624	4595	4566	4536	4507	4477	4446
7	4416	4385	4354	4323	4291	4260	4228	4195	4163	4130
8	4097	4064	4030	3997	3963	3928	3894	3859	3825	3790
9	3754	3719	3683	3647	3611	3575	3538	3502	3465	3428
3.0	3391	3353	3316	3278	3240	3202	3164	3125	3087	3048
1	3009	2970	2931	2892	2852	2813	2773	2733	2694	2654
2	2613	2573	2533	2492	2452	2411	2370	2330	2289	2248
3	2207	2165	2124	2083	2042	2000	1959	1917	1876	1834
4	1792	1751	1709	1667	1625	1583	1541	1500	1458	1416
5	1374	1332	1290	1248	1206	1164	1122	1080	1038	996
6	0955	0913	0871	0829	07876	0746	0704	0663	0621	0580
7	0538	0497	0456	04145	0373	0332	0291	0250	0210	0169
8	+0128	0088	0047	00069	*0033	*0074	*0114	*0153	*0193	*0233
9	-0272	0312	0351	0390	0429	0468	0507	0546	0584	0622
4.0	-0660	0698	0736	0774	0811	0849	0886	0923	0960	0996
1	-1033	1069	1105	1141	1177	1212	1247	1282	1317	1352
2	-1386	1421	1455	1489	1522	1556	1589	1622	1654	1687
3	-1719	1751	1783	1814	1845	1876	1907	1938	1968	1998
4	-2028	2057	2086	2115	2144	2173	2201	2229	2256	2284
5	-2311	2337	2364	2390	2416	2442	2467	2492	2517	2541
6	-2566	2589	2613	2636	2659	2682	2704	2726	2748	2770
7	-2791	2812	2832	2852	2872	2892	2911	2930	2949	2967
8	-2985	3003	3020	3037	3054	3070	3086	3102	3117	3132
9	-3147	3161	3175	3189	3202	3216	3228	3241	3253	3264
5.0	-3276	3287	3298	3308	3318	3328	3337	3346	3355	3363

$$J_1(x) = \frac{x}{2 \cdot 0! \cdot 1!} - \frac{x^3}{2^2 \cdot 1! \cdot 2!} + \frac{x^5}{2^5 \cdot 2! \cdot 3!} - \dots$$

$$J_1(x) = -J_0'(x)$$

Bessel Functions: $J_0(x)$ and $J_1(x)$. (Continued)

x	0	1	2	3	4	5	6	7	8	9
5.0	-0.3276	3287	3298	3308	3318	3328	3337	3346	3355	3363
1	-3371	3379	3386	3393	3400	3406	3412	3417	3423	3428
2	-3432	3436	3440	3444	3447	3450	3453	3455	3457	3458
3	-3460	3460	3461	3461	3461	3461	3460	3459	3457	3456
4	-3453	3451	3448	3445	3442	3438	3434	3430	3425	3420
5	-3414	3409	3403	3396	3390	3383	3376	3368	3360	3352
6	-3343	3335	3325	3316	3306	3296	3286	3275	3264	3253
7	-3241	3230	3218	3205	3192	3179	3166	3153	3139	3125
8	-3110	3096	3081	3065	3050	3034	3018	3002	2985	2969
9	-2951	2934	2917	2899	2881	2862	2844	2825	2806	2786
6.0	-2767	2747	2727	2707	2686	2666	2645	2623	2602	2580
1	-2559	2537	2514	2492	2469	2446	2423	2400	2377	2353
2	-2329	2305	2281	2257	2232	2207	2182	2157	2132	2106
3	-2081	2055	2029	2003	1977	1950	1924	1897	1870	1843
4	-1816	1789	1762	1734	1707	1679	1651	1623	1595	1567
5	-1538	1510	1481	1453	1424	1395	1366	1337	1308	1279
6	-1250	1220	1191	1162	1132	1102	1073	1043	1013	0983
7	-0953	0923	0893	0863	0833	0803	0773	0743	0713	0682
8	-0652	0622	0592	0561	0531	0501	0470	0440	0410	0379
9	-0349	0319	0288	0258	0228	0198	0167	0137	0107	0077
7.0	-0047	0017	*0013	*0043	*0073	*0103	*0133	*0163	*0192	*0222
1	+0251	0282	0310	0340	0369	0398	0428	0457	0486	0514
2	0543	0572	0601	0629	0658	0686	0714	0742	0770	0798
3	0826	0853	0881	0908	0935	0963	0990	1016	1043	1070
4	1096	1123	1149	1175	1201	1226	1252	1277	1302	1328
5	1352	1377	1402	1426	1450	1475	1498	1522	1546	1569
6	1592	1615	1638	1660	1683	1705	1727	1749	1771	1792
7	1813	1834	1855	1875	1896	1916	1936	1956	1975	1994
8	2014	2032	2051	2069	2088	2106	2123	2141	2158	2175
9	2192	2208	2225	2241	2257	2272	2287	2303	2317	2332
8.0	2346	2360	2374	2388	2401	2414	2427	2440	2452	2464
1	2476	2488	2499	2510	2521	2531	2542	2552	2561	2571
2	2580	2589	2598	2606	2614	2622	2630	2637	2644	2651
3	2657	2664	2670	2675	2681	2686	2691	2696	2700	2704
4	2708	2711	2715	2718	2720	2723	2725	2727	2729	2730
5	2731	2732	2733	2733	2733	2733	2732	2731	2730	2729
6	2728	2726	2724	2721	2719	2716	2713	2709	2705	2701
7	2697	2693	2688	2683	2678	2672	2666	2660	2654	2648
8	2641	2634	2626	2619	2611	2603	2595	2586	2577	2568
9	2559	2550	2540	2530	2519	2509	2498	2487	2476	2465
9.0	2453	2441	2429	2417	2404	2391	2378	2365	2352	2338
1	2324	2310	2296	2281	2267	2252	2237	2221	2206	2190
2	2174	2158	2142	2125	2108	2091	2074	2057	2040	2022
3	2004	1986	1968	1950	1931	1912	1893	1874	1855	1836
4	1816	1797	1777	1757	1737	1716	1696	1675	1655	1634
5	1613	1591	1570	1549	1527	1506	1484	1462	1440	1418
6	1395	1373	1350	1328	1305	1282	1259	1236	1213	1190
7	1166	1143	1119	1096	1072	1048	1025	10006	0977	0953
8	0928	0904	0880	0856	0831	0807	0782	0758	0733	0708
9	0684	0659	0634	0609	0585	0560	0535	0510	0485	0460
10.0	+0435	0410	0385	0360	0334	0309	0284	0259	0234	0209
Zeros of $J_1(x)$	1	2	3	4	5	6	7	8		
$x =$	3.8317	7.0156	10.1735	13.3237	16.4706	19.6159	22.7601	25.9037		

Bessel Functions: $J_0(x)$ and $J_1(x)$. (Continued)
 $J_1(x)$

x	0	1	2	3	4	5	6	7	8	9
10.0	+0.0435	0410	0385	0360	0334	0309	0284	0259	0234	0209
1	+0184	0159	0134	0109	0084	0059	0034	0009	*0016	*0041
2	-0066	0091	0116	0141	0165	0190	0215	0240	0264	0289
3	-0313	0338	0362	0386	0411	0435	0459	0483	0507	0531
4	-0555	0578	0602	0626	0649	0673	0696	0719	0742	0766
5	-0789	0811	0834	0857	0879	0902	0924	0946	0968	0990
6	-1012	1034	1056	1077	1099	1120	1141	1162	1183	1203
7	-1224	1244	1265	1285	1305	1325	1344	1364	1383	1403
8	-1422	1441	1459	1478	1496	1515	1533	1551	1568	1586
9	-1603	1621	1638	1655	1671	1688	1704	1720	1736	1752
11.0	-1768	1783	1798	1814	1828	1843	1857	1872	1886	1900
1	-1913	1927	1940	1953	1966	1979	1991	2003	2015	2027
2	-2039	2050	2061	2072	2083	2093	2104	2114	2123	2133
3	-2143	2152	2161	2169	2178	2186	2194	2202	2210	2217
4	-2225	2231	2238	2245	2251	2257	2263	2268	2274	2279
5	-2284	2288	2293	2297	2301	2305	2308	2312	2315	2317
6	-2320	2322	2324	2326	2328	2329	2331	2332	2332	2333
7	-2333	2333	2333	2332	2332	2331	2330	2328	2327	2325
8	-2323	2321	2318	2315	2312	2309	2306	2302	2298	2294
9	-2290	2285	2281	2276	2270	2265	2259	2253	2247	2241
12.0	-2234	2228	2221	2214	2206	2199	2191	2183	2175	2166
1	-2157	2149	2140	2130	2121	2111	2101	2091	2081	2070
2	-2060	2049	2038	2027	2015	2004	1992	1980	1968	1955
3	-1943	1930	1917	1904	1891	1877	1863	1850	1836	1821
4	-1807	1793	1778	1763	1748	1733	1718	1702	1687	1671
5	-1655	1639	1623	1606	1590	1573	1556	1539	1522	1505
6	-1487	1470	1452	1435	1417	1399	1380	1362	1344	1325
7	-1307	1288	1269	1250	1231	1212	1192	1173	1154	1134
8	-1114	1095	1075	1055	1035	1014	0994	0974	0954	0933
9	-0912	0892	0871	0850	0830	0809	0788	0767	0746	0724
13.0	-0703	0682	0661	0639	0618	0596	0575	0553	0532	0510
1	-0489	0467	0445	0423	0402	0380	0358	0336	0314	0293
2	-0271	0249	0227	0205	0183	0161	0139	0117	0096	0074
3	-0052	0030	0008	*0014	*0036	*0057	*0079	*0101	*0123	*0144
4	+0166	0188	0209	0231	0252	0274	0295	0317	0338	0359
5	0380	0402	0423	0444	0465	0486	0507	0528	0548	0569
6	0590	0610	0631	0651	0671	0692	0712	0732	0752	0772
7	0791	0811	0831	0850	0870	0889	0908	0927	0946	0965
8	0984	1003	1021	1040	1058	1076	1094	1112	1130	1148
9	1165	1183	1200	1217	1234	1251	1268	1285	1301	1318
14.0	1334	1350	1366	1382	1397	1413	1428	1443	1458	1473
1	1488	1502	1517	1531	1545	1559	1573	1586	1600	1613
2	1626	1639	1652	1664	1677	1689	1701	1713	1724	1736
3	1747	1758	1769	1780	1791	1801	1811	1821	1831	1841
4	1850	1860	1869	1878	1886	1895	1903	1911	1919	1927
5	1934	1942	1949	1956	1962	1969	1975	1981	1987	1993
6	1999	2004	2009	2014	2019	2023	2027	2031	2035	2039
7	2043	2046	2049	2052	2054	2057	2059	2061	2063	2065
8	2066	2067	2068	2069	2070	2070	2070	2070	2070	2069
9	+2069	2068	2067	2066	2064	2062	2061	2058	2056	2054

For $x > 15$

$$J_1(x) \approx \sqrt{\frac{2}{\pi x}} \left[\sin \left(x - \frac{3\pi}{4} \right) - \frac{3}{8x} \cos \left(x - \frac{3\pi}{4} \right) \right], \text{ error } < 0.0001$$

$$\sqrt{\frac{2}{\pi}} = 0.7979 \quad \frac{3\pi}{4} = 2.3562$$

Bessel Functions: $Y_0(x)$ and $Y_1(x)$
 $Y_0(x)$

x	0	1	2	3	4	5	6	7	8	9
0.0	$-\infty$	3.005	2.564	2.305	2.122	1.979	1.863	1.764	1.678	1.602
1	-1.534	1.473	1.416	1.364	1.316	1.271	1.228	1.189	1.151	1.115
2	-1.081	1.049	1.0175	0.9877	0.9591	0.9316	0.9050	0.8794	0.8546	0.8306
3	-0.8073	7847	7627	7414	7206	7003	6806	6613	6424	6240
4	-6060	5884	5712	5542	5377	5214	5055	4898	4745	4594
5	-4445	4299	4156	4015	3876	3739	3604	3472	3341	3212
6	-3085	2960	2837	2715	2595	2476	2359	2244	2130	2018
7	-1907	1797	1689	1582	1476	1372	1269	1167	1066	0966
8	-0868	0771	0675	0580	0486	0393	0301	0210	0120	0032
9	+0056	0143	0229	0314	0398	0481	0563	0644	0725	0804
1.0	0883	0960	1037	1113	1188	1262	1336	1409	1480	1551
1	1622	1691	1760	1828	1895	1961	2026	2091	2155	2218
2	2281	2343	2404	2464	2523	2582	2640	2698	2754	2810
3	2865	2920	2974	3027	3079	3131	3182	3232	3282	3331
4	3379	3427	3473	3520	3565	3610	3654	3698	3741	3783
5	3824	3865	3906	3945	3984	4022	4060	4097	4133	4169
6	4204	4239	4273	4306	4338	4370	4401	4432	4462	4491
7	4520	4548	4576	4603	4629	4655	4680	4705	4728	4752
8	4774	4796	4818	4839	4859	4879	4898	4916	4934	4951
9	4968	4984	5000	5015	5029	5043	5056	5069	5081	5093
2.0	5104	5114	5124	5133	5142	5150	5158	5165	5172	5177
1	5183	5188	5192	5196	5199	5202	5204	5206	5207	5208
2	5208	5207	5207	5205	5203	5201	5198	5194	5190	5186
3	5181	5175	5169	5163	5156	5148	5141	5132	5123	5114
4	5104	5094	5083	5072	5060	5048	5036	5022	5009	4995
5	4981	4966	4951	4935	4919	4902	4885	4868	4850	4832
6	4813	4794	4775	4755	4735	4714	4693	4672	4650	4628
7	4605	4582	4559	4535	4511	4487	4462	4437	4411	4385
8	4359	4333	4306	4279	4251	4223	4195	4167	4138	4109
9	4079	4049	4019	3989	3958	3927	3896	3865	3833	3801
3.0	3769	3736	3703	3670	3637	3603	3569	3535	3500	3466
1	3431	3396	3361	3325	3289	3253	3217	3181	3144	3108
2	3071	3033	2996	2958	2921	2883	2845	2807	2768	2730
3	2691	2652	2613	2574	2535	2495	2456	2416	2376	2336
4	2296	2256	2216	2175	2135	2094	2054	2013	1972	1931
5	1890	1849	1808	1767	1726	1684	1643	1602	1560	1519
6	1477	1436	1394	1352	1311	1269	1227	1186	1144	1102
7	1061	1019	0977	0936	0894	0853	0811	0769	0728	0686
8	0645	0604	0562	0521	0480	0439	0397	0356	0315	0275
9	+0234	0193	0152	0112	0071	0031	*0009	*0050	*0090	*0130
4.0	-0169	0209	0249	0288	0328	0367	0406	0445	0484	0522
1	-0561	0599	0638	0676	0714	0751	0789	0826	0864	0901
2	-0938	0974	1011	1047	1083	1119	1155	1191	1226	1261
3	-1296	1331	1365	1400	1434	1467	1501	1535	1568	1601
4	-1633	1666	1698	1730	1762	1793	1825	1856	1886	1917
5	-1947	1977	2007	2036	2065	2094	2123	2151	2179	2207
6	-2235	2262	2289	2315	2342	2368	2394	2419	2444	2469
7	-2494	2518	2542	2566	2589	2612	2635	2658	2680	2702
8	-2723	2744	2765	2786	2806	2826	2845	2865	2884	2902
9	-2921	2939	2956	2973	2990	3007	3023	3039	3055	3070
5.0	-3085	3100	3114	3128	3142	3155	3168	3180	3193	3204

Linear interpolation is inaccurate for low values of the argument. For greater accuracy in this range, use the auxiliary functions, Table 13.

$$Y'_0(x) = -Y_1(x)$$

Bessel Functions: $Y_0(x)$ and $Y_1(x)$. (Continued)
 $Y_0(x)$

x	0	1	2	3	4	5	6	7	8	9
5.0	-0.3085	3100	3114	3128	3142	3155	3168	3180	3193	3204
1	-3216	3227	3238	3249	3259	3269	3278	3287	3296	3304
2	-3313	3320	3328	3335	3341	3348	3354	3359	3365	3370
3	-3374	3379	3383	3386	3389	3392	3395	3397	3399	3400
4	-3402	3403	3403	3403	3403	3402	3402	3400	3399	3397
5	-3395	3392	3389	3386	3383	3379	3375	3370	3365	3360
6	-3354	3349	3342	3336	3329	3322	3315	3307	3299	3290
7	-3282	3273	3263	3254	3244	3233	3223	3212	3201	3189
8	-3177	3165	3153	3140	3127	3114	3101	3087	3073	3058
9	-3044	3029	3013	2998	2982	2966	2950	2933	2916	2899
6.0	-2882	2864	2846	2828	2810	2791	2772	2753	2734	2714
1	-2694	2674	2654	2633	2613	2592	2570	2549	2527	2505
2	-2483	2461	2438	2415	2393	2369	2346	2322	2299	2275
3	-2251	2226	2202	2177	2152	2127	2102	2077	2051	2025
4	-1999	1973	1947	1921	1894	1868	1841	1814	1787	1760
5	-1732	1705	1677	1650	1622	1594	1566	1538	1509	1481
6	-1452	1424	1395	1366	1337	1308	1279	1250	1221	1191
7	-1162	1132	1103	1073	1044	1014	984	954	924	894
8	-0864	0834	0804	0774	0744	0714	0684	0653	0623	0593
9	-0563	0532	0502	0472	0441	0411	0381	0350	0320	0290
7.0	-0259	0229	0199	0169	0139	0108	0078	0048	0018	*0012
1	+0042	0072	0102	0131	0161	0191	0221	0250	0280	0309
2	0339	0368	0397	0426	0455	0484	0513	0542	0571	0599
3	0628	0656	0684	0713	0741	0769	0797	0824	0852	0879
4	0907	0934	0961	0988	1015	1042	1068	1095	1121	1147
5	1173	1199	1225	1250	1276	1301	1326	1351	1375	1400
6	1424	1448	1472	1496	1520	1543	1567	1590	1613	1635
7	1658	1680	1702	1724	1746	1768	1789	1810	1831	1852
8	1872	1893	1913	1932	1952	1972	1991	2010	2028	2047
9	2065	2083	2101	2119	2136	2153	2170	2187	2203	2219
8.0	2235	2251	2266	2282	2296	2311	2326	2340	2354	2367
1	2381	2394	2407	2420	2432	2444	2456	2468	2479	2490
2	2501	2512	2522	2532	2542	2551	2561	2570	2578	2587
3	2595	2603	2611	2618	2625	2632	2639	2645	2651	2657
4	2662	2667	2672	2677	2681	2686	2689	2693	2696	2699
5	2702	2705	2707	2709	2710	2712	2713	2714	2714	2715
6	2715	2714	2714	2713	2712	2711	2709	2707	2705	2703
7	2700	2697	2694	2690	2687	2683	2678	2674	2669	2664
8	2659	2653	2647	2641	2635	2628	2621	2614	2607	2599
9	2592	2583	2575	2566	2558	2549	2539	2530	2520	2510
9.0	2499	2489	2478	2467	2456	2444	2433	2421	2408	2396
1	2383	2371	2357	2344	2331	2317	2303	2289	2274	2260
2	2245	2230	2215	2199	2184	2168	2152	2136	2119	2103
3	2086	2069	2052	2034	2017	1999	1981	1963	1945	1926
4	1907	1889	1870	1851	1831	1812	1792	1772	1752	1732
5	1712	1692	1671	1650	1630	1609	1588	1566	1545	1523
6	1502	1480	1458	1436	1414	1392	1369	1347	1324	1302
7	1279	1256	1233	1210	1186	1163	1140	1116	1093	1069
8	1045	1021	0998	0974	0949	0925	0901	0877	0853	0828
9	0804	0779	0755	0730	0705	0681	0656	0631	0606	0582
10.0	0557	0532	0507	0482	0457	0432	0407	0382	0357	0332
Zeros of $Y_0(x)$	1	2	3	4	5	6	7	8		
$x =$	0.8936	3.9577	7.0861	10.2223	13.3611	16.5009	19.6413	22.7820		

Bessel Functions: $Y_0(x)$ and $Y_1(x)$. (Continued)
 $Y_0(x)$

x	0	1	2	3	4	5	6	7	8	9
10.0	+0.0557	0532	0507	0482	0457	0432	0407	0382	0357	0332
1	0307	0281	0256	0231	0206	0181	0156	0131	0106	0081
2	+0056	0031	0006	*0019	*0044	*0069	*0094	*0119	*0143	*0168
3	-0193	0218	0242	0267	0291	0316	0340	0365	0389	0413
4	-0437	0462	0486	0510	0534	0557	0581	0605	0628	0652
5	-0675	0699	0722	0745	0768	0791	0814	0837	0859	0882
6	-0904	0926	0949	0971	0993	1015	1036	1058	1079	1101
7	-1122	1143	1164	1185	1205	1226	1246	1267	1287	1307
8	-1326	1346	1366	1385	1404	1423	1442	1461	1479	1498
9	-1516	1534	1552	1569	1587	1604	1622	1639	1655	1672
11.0	-1688	1705	1721	1737	1752	1768	1783	1798	1813	1828
1	-1843	1857	1871	1885	1899	1913	1926	1939	1952	1965
2	-1977	1990	2002	2014	2025	2037	2048	2059	2070	2081
3	-2091	2101	2111	2121	2130	2140	2149	2158	2166	2175
4	-2183	2191	2199	2206	2213	2220	2227	2234	2240	2246
5	-2252	2258	2263	2269	2274	2278	2283	2287	2291	2295
6	-2299	2302	2305	2308	2311	2313	2315	2317	2319	2321
7	-2322	2323	2324	2324	2325	2325	2324	2324	2324	2323
8	-2322	2320	2319	2317	2315	2313	2310	2308	2305	2302
9	-2298	2295	2291	2287	2283	2278	2273	2269	2263	2258
12.0	-2252	2247	2241	2234	2228	2221	2214	2207	2200	2192
1	-2184	2176	2168	2160	2151	2142	2133	2124	2115	2105
2	-2095	2085	2075	2064	2054	2043	2032	2021	2009	1998
3	-1986	1974	1962	1949	1937	1924	1911	1898	1885	1871
4	-1858	1844	1830	1816	1802	1787	1772	1758	1743	1727
5	-1712	1697	1681	1665	1649	1633	1617	1601	1584	1567
6	-1551	1534	1517	1499	1482	1464	1447	1429	1411	1393
7	-1375	1357	1338	1320	1301	1282	1264	1245	1226	1206
8	-1187	1168	1148	1129	1109	1089	1069	1049	1029	1009
9	-0989	0968	0948	0927	0907	0886	0866	0845	0824	0803
13.0	-0782	0761	0740	0719	0698	0676	0655	0634	0612	0591
1	-0569	0548	0526	0505	0483	0461	0439	0418	0396	0374
2	-0352	0331	0309	0287	0265	0243	0221	0199	0177	0156
3	-0134	0112	0090	0068	0046	0024	0002	*0019	*0041	*0063
4	+0085	0107	0128	0150	0172	0193	0215	0236	0258	0279
5	0301	0322	0343	0365	0386	0407	0428	0449	0470	0491
6	0512	0533	0554	0574	0595	0615	0636	0656	0677	0697
7	0717	0737	0757	0777	0796	0816	0836	0855	0875	0894
8	0913	0932	0951	0970	0989	1007	1026	1044	1062	1081
9	1099	1117	1134	1152	1169	1187	1204	1221	1238	1255
14.0	1272	1289	1305	1321	1337	1353	1369	1385	1401	1416
1	1431	1446	1461	1476	1491	1505	1520	1534	1548	1562
2	1575	1589	1602	1615	1628	1641	1654	1666	1679	1691
3	1703	1715	1726	1738	1749	1760	1771	1781	1792	1802
4	1812	1822	1832	1842	1851	1860	1869	1878	1886	1895
5	1903	1911	1919	1926	1934	1941	1948	1955	1962	1968
6	1974	1980	1986	1992	1997	2002	2007	2012	2017	2021
7	2025	2029	2033	2036	2040	2043	2046	2049	2051	2054
8	2056	2058	2059	2061	2062	2063	2064	2065	2065	2065
9	+2065	2065	2065	2064	2064	2063	2061	2060	2058	2057

For $x > 15$

$$Y_0(x) = \sqrt{\frac{2}{\pi x}} \left[\sin\left(x - \frac{\pi}{4}\right) - \frac{1}{8x} \cos\left(x - \frac{\pi}{4}\right) \right], \text{ error} < 0.0001$$

$$\sqrt{\frac{2}{\pi}} = 0.7979 \quad \frac{\pi}{4} = 0.7854$$

Bessel Functions: $Y_0(x)$ and $Y_1(x)$. (Continued)
 $Y_1(x)$

x	0	1	2	3	4	5	6	7	8	9
0.0	$-\infty$	63.68	31.86	21.26	15.96	12.79	10.68	9.167	8.038	7.160
1	-6.459	5.886	5.409	5.007	4.662	4.364	4.103	3.873	3.670	3.487
2	-3.324	3.176	3.042	2.919	2.807	2.704	2.609	2.521	2.440	2.364
3	-2.293	2.227	2.165	2.107	2.052	2.000	1.952	1.906	1.862	1.820
4	-1.781	1.743	1.708	1.673	1.641	1.610	1.580	1.551	1.523	1.497
5	-1.471	1.447	1.423	1.401	1.378	1.357	1.337	1.317	1.297	1.279
6	-1.260	1.243	1.226	1.209	1.193	1.177	1.161	1.146	1.132	1.117
7	-1.103	1.090	1.076	1.063	1.050	1.038	1.025	1.013	1.0013	*9896
8	-0.9781	9669	9558	9449	9342	9236	9132	9030	8929	8829
9	-8731	8634	8539	8444	8351	8258	8167	8077	7988	7900
1.0	-7812	7726	7640	7555	7471	7388	7305	7223	7142	7061
1	-6981	6902	6823	6745	6667	6590	6513	6437	6361	6286
2	-6211	6137	6063	5990	5916	5844	5771	5699	5628	5556
3	-5485	5415	5344	5274	5204	5135	5066	4997	4928	4860
4	-4791	4724	4656	4589	4521	4454	4388	4321	4255	4189
5	-4123	4057	3992	3927	3862	3797	3732	3668	3604	3540
6	-3476	3412	3349	3285	3222	3159	3096	3034	2972	2909
7	-2847	2785	2724	2662	2601	2540	2479	2418	2357	2297
8	-2237	2177	2117	2057	1997	1938	1879	1820	1761	1702
9	-1644	1586	1528	1470	1412	1355	1297	1240	1184	1127
2.0	-1070	1014	0958	0902	0846	0791	0736	0681	0626	0571
1	-0517	0463	0409	0355	0301	0248	0195	0142	0090	0037
2	+0015	0067	0118	0170	0221	0272	0323	0373	0423	0473
3	0523	0572	0621	0670	0719	0767	0815	0863	0911	0958
4	1005	1052	1098	1144	1190	1236	1281	1326	1371	1415
5	1459	1503	1547	1590	1633	1675	1718	1760	1801	1843
6	1884	1924	1965	2005	2045	2084	2123	2162	2200	2239
7	2276	2314	2351	2388	2424	2460	2496	2531	2566	2601
8	2635	2669	2703	2736	2769	2802	2834	2866	2897	2929
9	2959	2990	3020	3050	3079	3108	3136	3164	3192	3220
3.0	3247	3273	3300	3326	3351	3376	3401	3425	3449	3473
1	3496	3519	3542	3564	3585	3607	3627	3648	3668	3688
2	3707	3726	3745	3763	3780	3798	3815	3831	3847	3863
3	3879	3893	3908	3922	3936	3949	3962	3975	3987	3999
4	4010	4021	4032	4042	4052	4061	4070	4079	4087	4095
5	4102	4109	4115	4122	4127	4133	4138	4142	4147	4150
6	4154	4157	4160	4162	4164	4165	4166	4167	4167	4167
7	4167	4166	4165	4163	4161	4159	4156	4153	4149	4145
8	4141	4137	4132	4126	4120	4114	4108	4101	4094	4086
9	4078	4070	4061	4052	4043	4033	4023	4013	4002	3991
4.0	3979	3967	3955	3943	3930	3917	3903	3889	3875	3861
1	3846	3831	3815	3800	3783	3767	3750	3733	3716	3698
2	3680	3662	3643	3624	3605	3586	3566	3546	3525	3505
3	3484	3463	3441	3420	3397	3375	3353	3330	3307	3283
4	3260	3236	3212	3187	3163	3138	3113	3087	3062	3036
5	3010	2984	2957	2930	2904	2876	2849	2821	2794	2766
6	2737	2709	2680	2652	2623	2594	2564	2535	2505	2475
7	2445	2415	2384	2354	2323	2292	2261	2230	2199	2167
8	2136	2104	2072	2040	2008	1976	1943	1911	1878	1845
9	1812	1780	1746	1713	1680	1647	1613	1580	1546	1512
5.0	+1479	1445	1411	1377	1343	1309	1275	1240	1206	1172

Linear interpolation is inaccurate for low values of the argument. For greater accuracy in this range, use the auxiliary functions, Table 13.

$$Y_1(x) = -Y'_0(x)$$

Bessel Functions: $Y_0(x)$ and $Y_1(x)$ (Continued)
 $Y_1(x)$

x	0	1	2	3	4	5	6	7	8	9
5.0	+0.1479	1445	1411	1377	1343	1309	1275	1240	1206	1172
1	1137	1103	1069	1034	1000	0965	0930	0896	0861	0827
2	0792	0757	0723	0688	0653	0619	0584	0549	0515	0480
3	0445	0411	0376	0342	0307	0273	0238	0204	0170	0136
4	+0101	0067	0033	*0001	*0035	*0069	*0103	*0137	*0170	*0204
5	-0238	0271	0304	0338	0371	0404	0437	0470	0503	0535
6	-0568	0601	0633	0665	0697	0729	0761	0793	0824	0856
7	-0887	0918	0949	0980	1011	1042	1072	1102	1133	1163
8	-1192	1222	1251	1281	1310	1339	1368	1396	1425	1453
9	-1481	1509	1536	1564	1591	1618	1645	1671	1698	1724
6.0	-1750	1776	1801	1827	1852	1877	1902	1926	1950	1974
1	-1998	2022	2045	2068	2091	2114	2136	2158	2180	2201
2	-2223	2244	2265	2285	2306	2326	2346	2365	2385	2404
3	-2422	2441	2459	2477	2495	2512	2530	2547	2563	2580
4	-2596	2611	2627	2642	2657	2672	2686	2700	2714	2728
5	-2741	2754	2767	2779	2791	2803	2814	2826	2836	2847
6	-2857	2868	2877	2887	2896	2905	2913	2922	2930	2937
7	-2945	2952	2958	2965	2971	2977	2983	2988	2993	2997
8	-3002	3006	3010	3013	3016	3019	3022	3024	3026	3028
9	-3029	3030	3031	3032	3032	3032	3031	3031	3030	3028
7.0	-3027	3025	3023	3020	3017	3014	3011	3007	3003	2999
1	-2995	2990	2985	2980	2974	2968	2962	2955	2949	2942
2	-2934	2927	2919	2911	2902	2893	2885	2875	2866	2856
3	-2846	2836	2825	2814	2803	2792	2780	2768	2756	2744
4	-2731	2718	2705	2692	2678	2664	2650	2636	2621	2606
5	-2591	2576	2560	2545	2529	2512	2496	2479	2462	2445
6	-2428	2410	2393	2375	2357	2338	2320	2301	2282	2263
7	-2243	2224	2204	2184	2164	2143	2123	2102	2081	2060
8	-2039	2017	1996	1974	1952	1930	1908	1885	1863	1840
9	-1817	1794	1771	1748	1724	1701	1677	1653	1629	1605
8.0	-1581	1556	1532	1507	1482	1457	1432	1407	1382	1357
1	-1331	1306	1280	1255	1229	1203	1177	1151	1125	1099
2	-1072	1046	1020	0993	0967	0940	0913	0887	0860	0833
3	-0806	0779	0752	0725	0698	0671	0644	0617	0589	0562
4	-0535	0508	0480	0453	0426	0398	0371	0344	0316	0289
5	-0262	0234	0207	0180	0152	0125	0098	0071	0043	0016
6	+0011	0038	0065	0092	0119	0146	0173	0200	0227	0253
7	0280	0307	0333	0360	0386	0413	0439	0465	0491	0518
8	0544	0569	0595	0621	0647	0672	0698	0723	0748	0774
9	0799	0824	0849	0873	0898	0922	0947	0971	0995	1019
9.0	1043	1067	1091	1114	1137	1161	1184	1207	1229	1252
1	1275	1297	1319	1341	1363	1385	1406	1428	1449	1470
2	1491	1512	1532	1553	1573	1593	1613	1633	1652	1671
3	1691	1710	1728	1747	1765	1783	1801	1819	1837	1854
4	1871	1888	1905	1922	1938	1954	1970	1986	2001	2017
5	2032	2047	2061	2076	2090	2104	2118	2131	2145	2158
6	2171	2183	2196	2208	2220	2232	2243	2254	2265	2276
7	2287	2297	2307	2317	2326	2336	2345	2354	2362	2371
8	2379	2387	2394	2402	2409	2416	2423	2429	2435	2441
9	2447	2452	2458	2463	2467	2472	2476	2480	2484	2487
10.0	+2490	2493	2496	2498	2500	2502	2504	2506	2507	2508

Zeros of $Y_1(x)$
 $x =$ 1 2 3 4 5 6 7 8
 2.1971 5.4297 8.5960 11.7492 14.8974 18.0434 21.1881 24.3319

Bessel Functions: $Y_0(x)$ and $Y_1(x)$. (Continued)
 $Y_1(x)$

x	0	1	2	3	4	5	6	7	8	9
10.0	+0.2490	2493	2496	2498	2500	2502	2504	2506	2507	2508
1	2508	2509	2509	2509	2509	2508	2507	2506	2505	2504
2	2502	2500	2498	2495	2492	2489	2486	2483	2479	2475
3	2471	2466	2462	2457	2451	2446	2440	2435	2428	2422
4	2416	2409	2402	2394	2387	2379	2371	2363	2355	2346
5	2337	2328	2319	2309	2299	2289	2279	2269	2258	2247
6	2236	2225	2214	2202	2190	2178	2166	2153	2140	2128
7	2114	2101	2088	2074	2060	2046	2032	2017	2003	1988
8	1973	1958	1942	1927	1911	1895	1879	1863	1846	1830
9	1813	1796	1779	1762	1745	1727	1709	1692	1674	1655
11.0	1637	1619	1600	1581	1562	1543	1524	1505	1486	1466
1	1446	1427	1407	1387	1366	1346	1326	1305	1285	1264
2	1243	1222	1201	1180	1159	1137	1116	1095	1073	1051
3	1029	1008	0986	0964	0941	0919	0897	0875	0852	0830
4	0807	0785	0762	0740	0717	0694	0671	0648	0625	0602
5	0579	0556	0533	0510	0487	0464	0441	0417	0394	0371
6	0348	0324	0301	0278	0254	0231	0208	0184	0161	0138
7	0114	0091	0068	0045	0021	*0002	*0025	*0048	*0072	*0095
8	-0118	0141	0164	0187	0210	0233	0256	0279	0302	0324
9	-0347	0370	0392	0415	0437	0460	0482	0505	0527	0549
12.0	-0571	0593	0615	0637	0659	0681	0702	0723	0745	0766
1	-0787	0809	0830	0851	0871	0892	0913	0933	0954	0974
2	-0994	1014	1034	1054	1074	1093	1113	1132	1151	1171
3	-1189	1208	1227	1246	1264	1282	1300	1318	1336	1354
4	-1371	1389	1406	1423	1440	1457	1474	1490	1506	1522
5	-1538	1554	1570	1585	1601	1616	1631	1645	1660	1675
6	-1689	1703	1717	1730	1744	1757	1771	1783	1796	1809
7	-1821	1834	1846	1857	1869	1880	1892	1903	1914	1924
8	-1935	1945	1955	1965	1975	1984	1993	2002	2011	2020
9	-2028	2036	2044	2052	2060	2067	2074	2081	2088	2095
13.0	-2101	2107	2113	2118	2124	2129	2134	2139	2144	2148
1	-2152	2156	2160	2163	2167	2170	2172	2175	2178	2180
2	-2182	2183	2185	2186	2187	2188	2189	2189	2190	2190
3	-2190	2189	2188	2188	2187	2185	2184	2182	2180	2178
4	-2176	2173	2170	2167	2164	2161	2157	2153	2149	2145
5	-2140	2136	2131	2126	2120	2115	2109	2103	2097	2090
6	-2084	2077	2070	2063	2056	2048	2040	2032	2024	2016
7	-2007	1999	1990	1981	1971	1962	1952	1942	1932	1922
8	-1912	1901	1890	1879	1868	1857	1845	1834	1822	1810
9	-1798	1785	1773	1760	1747	1734	1721	1707	1694	1680
14.0	-1666	1652	1638	1624	1610	1595	1580	1565	1550	1535
1	-1520	1504	1489	1473	1457	1441	1425	1409	1392	1376
2	-1359	1342	1325	1308	1291	1274	1257	1239	1222	1204
3	-1186	1168	1150	1132	1114	1096	1077	1059	1040	1021
4	-1003	0984	0965	0946	0927	0907	0888	0869	0849	0830
5	-0810	0791	0771	0751	0732	0712	0692	0672	0652	0632
6	-0612	0591	0571	0551	0531	0510	0490	0469	0449	0428
7	-0408	0387	0367	0346	0326	0305	0284	0264	0243	0222
8	-0202	0181	0160	0140	0119	0098	0077	0057	0036	0015
9	+0005	0026	0047	0067	0088	0108	0129	0149	0170	0190

For $x > 15$

$$Y_1(x) = \sqrt{\frac{2}{\pi x}} \left[\sin \left(x - \frac{3\pi}{4} \right) + \frac{3}{8x} \cos \left(x - \frac{3\pi}{4} \right) \right], \text{ error } < 0.0001$$

$$\sqrt{\frac{2}{\pi}} = 0.7979 \quad \frac{3\pi}{4} = 2.3562$$

Bessel Functions: $I_0(x)$ and $I_1(x)$
 $I_0(x)$

x	0	1	2	3	4	5	6	7	8	9
0.0	1.000	1.000	1.000	1.000	1.000	1.001	1.001	1.001	1.002	1.002
1	1.003	1.003	1.004	1.004	1.005	1.006	1.006	1.007	1.008	1.009
2	1.010	1.011	1.012	1.013	1.014	1.016	1.017	1.018	1.020	1.021
3	1.023	1.024	1.026	1.027	1.029	1.031	1.033	1.035	1.036	1.038
4	1.040	1.042	1.045	1.047	1.049	1.051	1.054	1.056	1.058	1.061
5	1.063	1.066	1.069	1.071	1.074	1.077	1.080	1.083	1.086	1.089
6	1.092	1.095	1.098	1.102	1.105	1.108	1.112	1.115	1.119	1.123
7	1.126	1.130	1.134	1.138	1.142	1.146	1.150	1.154	1.158	1.162
8	1.167	1.171	1.175	1.180	1.184	1.189	1.194	1.198	1.203	1.208
9	1.213	1.218	1.223	1.228	1.233	1.239	1.244	1.249	1.255	1.260
1.0	1.266	1.272	1.278	1.283	1.289	1.295	1.301	1.307	1.314	1.320
1	1.326	1.333	1.339	1.346	1.352	1.359	1.366	1.373	1.380	1.387
2	1.394	1.401	1.408	1.416	1.423	1.430	1.438	1.446	1.454	1.461
3	1.469	1.477	1.485	1.494	1.502	1.510	1.519	1.527	1.536	1.545
4	1.553	1.562	1.571	1.580	1.590	1.599	1.608	1.618	1.627	1.637
5	1.647	1.657	1.667	1.677	1.687	1.697	1.707	1.718	1.728	1.739
6	1.750	1.761	1.772	1.783	1.794	1.806	1.817	1.829	1.840	1.852
7	1.864	1.876	1.888	1.900	1.913	1.925	1.938	1.951	1.963	1.976
8	1.990	2.003	2.016	2.030	2.043	2.057	2.071	2.085	2.099	2.113
9	2.128	2.142	2.157	2.172	2.187	2.202	2.217	2.233	2.248	2.264
2.0	2.280	2.296	2.312	2.328	2.344	2.361	2.378	2.395	2.412	2.429
1	2.446	2.464	2.482	2.499	2.517	2.536	2.554	2.573	2.591	2.610
2	2.629	2.648	2.668	2.687	2.707	2.727	2.747	2.768	2.788	2.809
3	2.830	2.851	2.872	2.893	2.915	2.937	2.959	2.981	3.004	3.026
4	3.049	3.072	3.096	3.119	3.143	3.167	3.191	3.215	3.240	3.265
5	3.290	3.315	3.341	3.366	3.392	3.419	3.445	3.472	3.499	3.526
6	3.553	3.581	3.609	3.637	3.666	3.694	3.723	3.752	3.782	3.812
7	3.842	3.872	3.903	3.933	3.965	3.996	4.028	4.060	4.092	4.124
8	4.157	4.190	4.224	4.258	4.292	4.326	4.361	4.396	4.431	4.467
9	4.503	4.539	4.576	4.613	4.650	4.688	4.725	4.764	4.802	4.841
3.0	4.881	4.921	4.961	5.001	5.042	5.083	5.125	5.166	5.209	5.251
1	5.294	5.338	5.382	5.426	5.471	5.516	5.561	5.607	5.653	5.700
2	5.747	5.795	5.843	5.891	5.940	5.989	6.039	6.089	6.140	6.191
3	6.243	6.295	6.347	6.400	6.454	6.508	6.562	6.617	6.672	6.728
4	6.785	6.842	6.899	6.957	7.016	7.075	7.134	7.195	7.255	7.316
5	7.378	7.441	7.503	7.567	7.631	7.696	7.761	7.827	7.893	7.960
6	8.028	8.096	8.165	8.234	8.304	8.375	8.447	8.519	8.591	8.665
7	8.739	8.813	8.889	8.965	9.041	9.119	9.197	9.276	9.356	9.436
8	9.517	9.599	9.681	9.764	9.848	9.933	10.02	10.11	10.19	10.28
9	10.37	10.46	10.55	10.64	10.73	10.82	10.92	11.01	11.11	11.20
4.0	11.30	11.40	11.50	11.60	11.70	11.80	11.90	12.01	12.11	12.22
1	12.32	12.43	12.54	12.65	12.76	12.87	12.98	13.10	13.21	13.33
2	13.44	13.56	13.68	13.80	13.92	14.04	14.16	14.29	14.41	14.54
3	14.67	14.80	14.93	15.06	15.19	15.32	15.46	15.59	15.73	15.87
4	16.01	16.15	16.29	16.44	16.58	16.73	16.88	17.03	17.18	17.33
5	17.48	17.64	17.79	17.95	18.11	18.27	18.43	18.59	18.76	18.92
6	19.09	19.26	19.43	19.61	19.78	19.96	20.13	20.31	20.49	20.67
7	20.86	21.04	21.23	21.42	21.61	21.80	22.00	22.19	22.39	22.59
8	22.79	23.00	23.20	23.41	23.62	23.83	24.04	24.26	24.47	24.69
9	24.91	25.14	25.36	25.59	25.82	26.05	26.28	26.52	26.76	27.00
5.0	27.24	27.48	27.73	27.98	28.23	28.49	28.74	29.00	29.26	29.52

$$I_0(x) = 1 + \frac{x^2}{2^2(1!)^2} + \frac{x^4}{2^4(2!)^2} + \frac{x^6}{2^6(3!)^2} + \dots$$

$$I_0'(x) = I_1(x)$$

Bessel Functions: $I_0(x)$ and $I_1(x)$. (Continued)

x	0	1	2	3	4	5	6	7	8	9
5.0	27.24	27.48	27.73	27.98	28.23	28.49	28.74	29.00	29.26	29.52
1	29.79	30.06	30.33	30.60	30.88	31.15	31.43	31.72	32.00	32.29
2	32.58	32.88	33.17	33.47	33.78	34.08	34.39	34.70	35.01	35.33
3	35.65	35.97	36.30	36.62	36.96	37.29	37.63	37.97	38.31	38.66
4	39.01	39.36	39.72	40.08	40.44	40.81	41.18	41.55	41.93	42.31
5	42.69	43.08	43.47	43.87	44.27	44.67	45.08	45.49	45.90	46.32
6	46.74	47.16	47.59	48.03	48.46	48.90	49.35	49.80	50.25	50.71
7	51.17	51.64	52.11	52.59	53.06	53.55	54.04	54.53	55.03	55.53
8	56.04	56.55	57.07	57.59	58.11	58.65	59.18	59.72	60.27	60.82
9	61.38	61.94	62.51	63.08	63.65	64.24	64.83	65.42	66.02	66.62
6.0	67.23	67.85	68.47	69.10	69.73	70.37	71.02	71.67	72.33	72.99
1	73.66	74.34	75.02	75.71	76.41	77.11	77.82	78.53	79.25	79.98
2	80.72	81.46	82.21	82.97	83.73	84.50	85.28	86.06	86.85	87.65
3	88.46	89.28	90.10	90.93	91.77	92.61	93.47	94.33	95.20	96.08
4	96.96	97.86	98.76	99.67	100.6	101.5	102.5	103.4	104.4	105.3
5	106.3	107.3	108.3	109.3	110.3	111.3	112.3	113.4	114.4	115.5
6	116.5	117.6	118.7	119.8	120.9	122.0	123.2	124.3	125.5	126.6
7	127.8	129.0	130.2	131.4	132.6	133.8	135.1	136.3	137.6	138.8
8	140.1	141.4	142.7	144.1	145.4	146.8	148.1	149.5	150.9	152.3
9	153.7	155.1	156.6	158.0	159.5	161.0	162.5	164.0	165.5	167.0
7.0	168.6	170.2	171.7	173.3	175.0	176.6	178.2	179.9	181.6	183.2
1	185.0	186.7	188.4	190.2	191.9	193.7	195.5	197.4	199.2	201.0
2	202.9	204.8	206.7	208.6	210.6	212.6	214.5	216.5	218.6	220.6
3	222.7	224.7	226.8	229.0	231.1	233.2	235.4	237.6	239.8	242.1
4	244.3	246.6	248.9	251.3	253.6	256.0	258.4	260.8	263.2	265.7
5	268.2	270.7	273.2	275.8	278.3	280.9	283.6	286.2	288.9	291.6
6	294.3	297.1	299.9	302.7	305.5	308.4	311.3	314.2	317.1	320.1
7	323.1	326.1	329.2	332.3	335.4	338.5	341.7	344.9	348.1	351.4
8	354.7	358.0	361.4	364.8	368.2	371.6	375.1	378.6	382.2	385.8
9	389.4	393.1	396.8	400.5	404.2	408.0	411.9	415.7	419.6	423.6
8.0	427.6	431.6	435.6	439.7	443.9	448.0	452.2	456.5	460.8	465.1
1	469.5	473.9	478.4	482.9	487.4	492.0	496.6	501.3	506.0	510.8
2	515.6	520.4	525.3	530.3	535.3	540.3	545.4	550.6	555.7	561.0
3	566.3	571.6	577.0	582.4	587.9	593.4	599.0	604.7	610.4	616.1
4	621.9	627.8	633.7	639.7	645.7	651.8	658.0	664.2	670.5	676.8
5	683.2	689.6	696.1	702.7	709.3	716.0	722.8	729.6	736.5	743.4
6	750.5	757.5	764.7	771.9	779.2	786.6	794.0	801.5	809.1	816.7
7	824.4	832.2	840.1	848.0	856.1	864.2	872.3	880.6	888.9	897.3
8	905.8	914.4	923.0	931.7	940.6	949.5	958.4	967.5	976.7	985.9
9	995.2	1005	1014	1024	1033	1043	1053	1063	1073	1083
9.0	1094	1104	1114	1125	1136	1146	1157	1168	1179	1190
1	1202	1213	1225	1236	1248	1260	1272	1284	1296	1308
2	1321	1333	1346	1359	1371	1384	1398	1411	1424	1438
3	1451	1465	1479	1493	1507	1522	1536	1551	1565	1580
4	1595	1610	1626	1641	1657	1673	1688	1704	1721	1737
5	1753	1770	1787	1804	1821	1838	1856	1874	1891	1909
6	1927	1946	1964	1983	2002	2021	2040	2060	2079	2099
7	2119	2139	2159	2180	2201	2222	2243	2264	2286	2307
8	2329	2352	2374	2397	2419	2442	2466	2489	2513	2537
9	2561	2585	2610	2635	2660	2685	2711	2737	2763	2789
10.0	2816	2843	2870	2897	2925	2952	2981	3009	3038	3067

For larger values of the argument, use the auxiliary functions, Table 13.

Bessel Functions: $I_0(x)$ and $I_1(x)$. (Continued)
 $I_1(x)$

x	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0050	0100	0150	0200	0250	0300	0350	0400	0450
1	0501	0551	0601	0651	0702	0752	0803	0853	0904	0954
2	1005	1056	1107	1158	1209	1260	1311	1362	1414	1465
3	1517	1569	1621	1673	1725	1777	1829	1882	1935	1987
4	2040	2093	2147	2200	2254	2307	2361	2415	2470	2524
5	2579	2634	2689	2744	2800	2855	2911	2967	3024	3080
6	3137	3194	3251	3309	3367	3425	3483	3542	3600	3659
7	3719	3778	3838	3899	3959	4020	4081	4142	4204	4266
8	4329	4391	4454	4518	4581	4646	4710	4775	4840	4905
9	4971	5038	5104	5171	5239	5306	5375	5443	5512	5582
1.0	5652	5722	5793	5864	5935	6008	6080	6153	6227	6300
1	6375	6450	6525	6601	6677	6754	6832	6910	6988	7067
2	7147	7227	7308	7389	7470	7553	7636	7719	7803	7888
3	7973	8059	8146	8233	8321	8409	8498	8588	8678	8769
4	8861	8953	9046	9140	9235	9330	9426	9522	9620	9718
5	9817	9916	1.002	1.012	1.022	1.032	1.043	1.053	1.064	1.074
6	1.085	1.096	1.106	1.117	1.128	1.139	1.151	1.162	1.173	1.185
7	1.196	1.208	1.220	1.232	1.244	1.256	1.268	1.280	1.292	1.305
8	1.317	1.330	1.343	1.355	1.368	1.381	1.395	1.408	1.421	1.435
9	1.448	1.462	1.476	1.490	1.504	1.518	1.532	1.547	1.561	1.576
2.0	1.591	1.606	1.621	1.636	1.651	1.666	1.682	1.698	1.713	1.729
1	1.745	1.762	1.778	1.795	1.811	1.828	1.845	1.862	1.879	1.897
2	1.914	1.932	1.950	1.968	1.986	2.004	2.022	2.041	2.060	2.079
3	2.098	2.117	2.136	2.156	2.176	2.196	2.216	2.236	2.257	2.277
4	2.298	2.319	2.340	2.362	2.383	2.405	2.427	2.449	2.471	2.494
5	2.517	2.540	2.563	2.586	2.610	2.633	2.657	2.682	2.706	2.731
6	2.755	2.780	2.806	2.831	2.857	2.883	2.909	2.935	2.962	2.989
7	3.016	3.043	3.071	3.099	3.127	3.155	3.184	3.213	3.242	3.271
8	3.301	3.331	3.361	3.392	3.422	3.453	3.485	3.516	3.548	3.580
9	3.613	3.645	3.678	3.712	3.745	3.779	3.813	3.848	3.883	3.918
3.0	3.953	3.989	4.025	4.062	4.098	4.136	4.173	4.211	4.249	4.287
1	4.326	4.365	4.405	4.445	4.485	4.526	4.567	4.608	4.650	4.692
2	4.734	4.777	4.820	4.864	4.908	4.953	4.997	5.043	5.088	5.134
3	5.181	5.228	5.275	5.323	5.371	5.420	5.469	5.519	5.569	5.619
4	5.670	5.722	5.773	5.826	5.879	5.932	5.986	6.040	6.095	6.150
5	6.206	6.262	6.319	6.376	6.434	6.493	6.552	6.611	6.671	6.732
6	6.793	6.854	6.917	6.979	7.043	7.107	7.171	7.237	7.302	7.369
7	7.436	7.503	7.572	7.640	7.710	7.780	7.851	7.922	7.994	8.067
8	8.140	8.215	8.289	8.365	8.441	8.518	8.595	8.674	8.753	8.832
9	8.913	8.994	9.076	9.159	9.242	9.326	9.411	9.497	9.584	9.671
4.0	9.759	9.848	9.938	10.03	10.12	10.21	10.31	10.40	10.50	10.59
1	10.69	10.79	10.88	10.98	11.08	11.18	11.29	11.39	11.49	11.60
2	11.71	11.81	11.92	12.03	12.14	12.25	12.36	12.48	12.59	12.71
3	12.82	12.94	13.06	13.18	13.30	13.42	13.54	13.67	13.79	13.92
4	14.05	14.17	14.30	14.44	14.57	14.70	14.84	14.97	15.11	15.25
5	15.39	15.53	15.67	15.82	15.96	16.11	16.26	16.41	16.56	16.71
6	16.86	17.02	17.17	17.33	17.49	17.65	17.81	17.98	18.14	18.31
7	18.48	18.65	18.82	18.99	19.17	19.35	19.52	19.70	19.88	20.07
8	20.25	20.44	20.63	20.82	21.01	21.20	21.40	21.60	21.80	22.00
9	22.20	22.40	22.61	22.82	23.03	23.24	23.46	23.67	23.89	24.11
5.0	24.34	24.56	24.79	25.02	25.25	25.48	25.72	25.95	26.19	26.44

$$I_1(x) = \frac{x}{2 \cdot 0!1!} + \frac{x^3}{2^2 \cdot 1!2!} + \frac{x^5}{2^3 \cdot 2!3!} + \dots$$

$$I_1(x) = I_0'(x)$$

Bessel Functions: $I_0(x)$ and $I_1(x)$. (Continued)
 $I_1(x)$

x	0	1	2	3	4	5	6	7	8	9
5.0	24.34	24.56	24.79	25.02	25.25	25.48	25.72	25.95	26.19	26.44
1	26.68	26.93	27.18	27.43	27.68	27.94	28.20	28.46	28.72	28.99
2	29.25	29.53	29.80	30.07	30.35	30.63	30.92	31.20	31.49	31.79
3	32.08	32.38	32.68	32.98	33.29	33.59	33.91	34.22	34.54	34.86
4	35.18	35.51	35.84	36.17	36.51	36.85	37.19	37.53	37.88	38.23
5	38.59	38.95	39.31	39.67	40.04	40.41	40.79	41.17	41.55	41.94
6	42.33	42.72	43.12	43.52	43.93	44.33	44.75	45.16	45.58	46.01
7	46.44	46.87	47.30	47.74	48.19	48.64	49.09	49.55	50.01	50.48
8	50.95	51.42	51.90	52.38	52.87	53.37	53.86	54.36	54.87	55.38
9	55.90	56.42	56.95	57.48	58.02	58.56	59.10	59.66	60.21	60.77
6.0	61.34	61.91	62.49	63.08	63.67	64.26	64.86	65.47	66.08	66.70
1	67.32	67.95	68.58	69.22	69.87	70.53	71.18	71.85	72.52	73.20
2	73.89	74.58	75.27	75.98	76.69	77.41	78.13	78.86	79.60	80.35
3	81.10	81.86	82.63	83.40	84.18	84.97	85.77	86.57	87.38	88.20
4	89.03	89.86	90.70	91.55	92.41	93.28	94.15	95.04	95.93	96.83
5	97.74	98.65	99.58	100.5	101.5	102.4	103.4	104.3	105.3	106.3
6	107.3	108.3	109.3	110.4	111.4	112.4	113.5	114.6	115.6	116.7
7	117.8	118.9	120.0	121.2	122.3	123.5	124.6	125.8	127.0	128.2
8	129.4	130.6	131.8	133.1	134.3	135.6	136.9	138.1	139.4	140.8
9	142.1	143.4	144.8	146.1	147.5	148.9	150.3	151.7	153.1	154.6
7.0	156.0	157.5	159.0	160.5	162.0	163.5	165.1	166.6	168.2	169.8
1	171.4	173.0	174.6	176.3	177.9	179.6	181.3	183.0	184.7	186.5
2	188.3	190.0	191.8	193.6	195.5	197.3	199.2	201.0	202.9	204.9
3	206.8	208.7	210.7	212.7	214.7	216.7	218.8	220.9	222.9	225.0
4	227.2	229.3	231.5	233.7	235.9	238.1	240.4	242.6	244.9	247.2
5	249.6	251.9	254.3	256.7	259.2	261.6	264.1	266.6	269.1	271.7
6	274.2	276.8	279.4	282.1	284.8	287.4	290.2	292.9	295.7	298.5
7	301.3	304.2	307.0	310.0	312.9	315.9	318.8	321.9	324.9	328.0
8	331.1	334.2	337.4	340.6	343.8	347.1	350.4	353.7	357.1	360.4
9	363.9	367.3	370.8	374.3	377.9	381.4	385.1	388.7	392.4	396.1
8.0	399.9	403.7	407.5	411.4	415.3	419.2	423.2	427.2	431.3	435.4
1	439.5	443.7	447.9	452.1	456.4	460.7	465.1	469.5	474.0	478.5
2	483.0	487.6	492.3	496.9	501.7	506.4	511.2	516.1	521.0	526.0
3	531.0	536.0	541.1	546.2	551.4	556.7	562.0	567.3	572.7	578.2
4	583.7	589.2	594.8	600.5	606.2	611.9	617.8	623.6	629.6	635.6
5	641.6	647.7	653.9	660.1	666.4	672.7	679.1	685.6	692.1	698.7
6	705.4	712.1	718.9	725.7	732.6	739.6	746.7	753.8	760.9	768.2
7	775.5	782.9	790.4	797.9	805.5	813.2	820.9	828.7	836.6	844.6
8	852.7	860.8	869.0	877.3	885.6	894.1	902.6	911.2	919.9	928.7
9	937.5	946.5	955.5	964.6	973.8	983.1	992.5	1002	1012	1021
9.0	1031	1041	1051	1061	1071	1081	1091	1102	1112	1123
1	1134	1144	1155	1166	1178	1189	1200	1212	1223	1235
2	1247	1259	1271	1283	1295	1307	1320	1332	1345	1358
3	1371	1384	1397	1411	1424	1438	1452	1465	1479	1494
4	1508	1522	1537	1552	1566	1581	1596	1612	1627	1643
5	1658	1674	1690	1707	1723	1739	1756	1773	1790	1807
6	1824	1842	1859	1877	1895	1913	1931	1950	1969	1987
7	2006	2026	2045	2065	2084	2104	2125	2145	2165	2186
8	2207	2228	2250	2271	2293	2315	2337	2359	2382	2405
9	2428	2451	2475	2498	2522	2547	2571	2596	2621	2646
10.0	2671	2697	2722	2749	2775	2802	2828	2856	2883	2911

For larger values of the argument, use the auxiliary functions, Table 13.

Bessel Functions: $K_0(x)$ and $K_1(x)$
 $K_0(x)$

x	0	1	2	3	4	5	6	7	8	9
0.0	∞	4.721	4.028	3.624	3.337	3.114	2.933	2.780	2.647	2.531
1	2.427	2.333	2.248	2.170	2.097	2.030	1.967	1.909	1.854	1.802
2	1.753	1.706	1.662	1.620	1.580	1.542	1.505	1.470	1.436	1.404
3	1.372	1.342	1.314	1.286	1.259	1.233	1.208	1.183	1.160	1.137
4	1.115	1.093	1.072	1.052	1.032	1.013	0.9943	9761	9584	9412
5	0.9244	9081	8921	8766	8614	8466	8321	8180	8042	7907
6	7775	7646	7520	7397	7277	7159	7043	6930	6820	6711
7	6605	6501	6399	6300	6202	6106	6012	5920	5829	5740
8	5653	5568	5484	5402	5321	5242	5165	5088	5013	4940
9	4867	4796	4727	4658	4591	4524	4459	4396	4333	4271
1.0	4210	4151	4092	4034	3977	3922	3867	3813	3760	3707
1	3656	3605	3556	3507	3459	3411	3365	3319	3273	3229
2	3185	3142	3100	3058	3017	2976	2936	2897	2858	2820
3	2782	2746	2709	2673	2638	2603	2569	2535	2502	2469
4	2437	2405	2373	2342	2312	2282	2252	2223	2194	2166
5	2138	2111	2083	2057	2030	2004	1979	1953	1928	1904
6	1880	1856	1832	1809	1786	1763	1741	1719	1697	1676
7	1655	1634	1614	1593	1573	1554	1534	1515	1496	1478
8	1459	1441	1423	1406	1388	1371	1354	1337	1321	1305
9	1288	1273	1257	1242	1226	1211	1196	1182	1167	1153
2.0	1139	1125	1111	1098	1084	1071	1058	1045	1033	1020
1	1008	*9956	*9836	*9717	*9600	*9484	*9370	*9257	*9145	*9035
2	0.08927	8820	8714	8609	8506	8404	8304	8204	8106	8010
3	7914	7820	7726	7634	7544	7454	7365	7278	7191	7106
4	7022	6939	6856	6775	6695	6616	6538	6461	6384	6309
5	6235	6161	6089	6017	5946	5877	5808	5739	5672	5606
6	5540	5475	5411	5348	5285	5223	5162	5102	5042	4984
7	4926	4868	4811	4755	4700	4645	4592	4538	4485	4433
8	4382	4331	4281	4231	4182	4134	4086	4039	3992	3946
9	3901	3856	3811	3767	3724	3681	3638	3597	3555	3514
3.0	3474	3434	3395	3356	3317	3279	3241	3204	3168	3131
1	3095	3060	3025	2990	2956	2922	2889	2856	2824	2791
2	2759	2728	2697	2666	2636	2606	2576	2547	2518	2489
3	2461	2433	2405	2378	2351	2325	2298	2272	2246	2221
4	2196	2171	2146	2122	2098	2074	2051	2028	2005	1982
5	1960	1938	1916	1894	1873	1852	1831	1810	1790	1770
6	1750	1730	1711	1692	1673	1654	1635	1617	1599	1581
7	1563	1546	1528	1511	1494	1477	1461	1445	1428	1412
8	1397	1381	1366	1350	1335	1320	1306	1291	1277	1262
9	1248	1234	1221	1207	1194	1180	1167	1154	1141	1129
4.0	1116	1104	1091	1079	1067	1055	1044	1032	1021	1009
1	0.009980	9869	9760	9652	9545	9439	9334	9231	9128	9027
2	8927	8829	8731	8634	8539	8444	8351	8259	8167	8077
3	7988	7900	7813	7726	7641	7557	7473	7391	7309	7229
4	7149	7070	6992	6915	6839	6764	6689	6616	6543	6471
5	6400	6329	6260	6191	6123	6056	5989	5923	5858	5794
6	5730	5668	5605	5544	5483	5423	5363	5305	5246	5189
7	5132	5076	5020	4965	4911	4857	4804	4751	4699	4648
8	4597	4547	4497	4448	4399	4351	4304	4257	4210	4164
9	4119	4074	4030	3986	3942	3899	3857	3814	3773	3732
5.0	3691	3651	3611	3572	3533	3494	3456	3419	3382	3345

$K_0'(x) = -K_1(x)$

Bessel Functions: $K_0(x)$ and $K_1(x)$. (Continued)

x	0	1	2	3	4	5	6	7	8	9
5.0	0.003691	3651	3611	3572	3533	3494	3456	3419	3382	3345
1	3308	3272	3237	3202	3167	3132	3098	3065	3031	2998
2	2966	2934	2902	2870	2839	2808	2778	2748	2718	2688
3	2659	2630	2602	2574	2546	2518	2491	2464	2437	2411
4	2385	2359	2333	2308	2283	2258	2234	2210	2186	2162
5	2139	2116	2093	2070	2048	2026	2004	1982	1961	1939
6	1918	1898	1877	1857	1837	1817	1798	1778	1759	1740
7	1721	1703	1684	1666	1648	1630	1613	1595	1578	1561
8	1544	1528	1511	1495	1479	1463	1447	1432	1416	1401
9	1386	1371	1356	1342	1327	1313	1299	1285	1271	1258
6.0	1244	1231	1217	1204	1191	1179	1166	1153	1141	1129
1	1117	1105	1093	1081	1070	1058	1047	1035	1024	1013
2	1002	*9918	*9811	*9706	*9602	*9499	*9398	*9297	*9197	*9099
3	0.0009001	8905	8810	8715	8622	8530	8438	8348	8259	8171
4	8083	7997	7911	7827	7743	7660	7578	7497	7417	7338
5	7259	7182	7105	7029	6954	6880	6806	6734	6662	6591
6	6520	6451	6382	6314	6246	6180	6114	6048	5984	5920
7	5857	5795	5733	5672	5611	5551	5492	5434	5376	5318
8	5262	5206	5150	5095	5041	4987	4934	4882	4830	4778
9	4728	4677	4627	4578	4529	4481	4434	4386	4340	4294
7.0	4248	4203	4158	4114	4070	4027	3984	3942	3900	3858
1	3817	3777	3737	3697	3658	3619	3580	3542	3505	3468
2	3431	3394	3358	3323	3287	3253	3218	3184	3150	3117
3	3084	3051	3019	2987	2955	2924	2893	2862	2832	2802
4	2772	2742	2713	2685	2656	2628	2600	2573	2545	2518
5	2492	2465	2439	2413	2388	2363	2338	2313	2288	2264
6	2240	2216	2193	2170	2147	2124	2102	2079	2057	2036
7	2014	1993	1972	1951	1930	1910	1890	1870	1850	1830
8	1811	1792	1773	1754	1736	1717	1699	1681	1664	1646
9	1629	1611	1594	1578	1561	1545	1528	1512	1496	1480
8.0	1465	1449	1434	1419	1404	1389	1374	1360	1346	1331
1	1317	1303	1290	1276	1263	1249	1236	1223	1210	1198
2	1185	1172	1160	1148	1136	1124	1112	1100	1089	1077
3	1066	1055	1043	1032	1022	1011	10002	*9897	*9793	*9690
4	0.0009588	9487	9387	9288	9191	9094	8998	8904	8810	8717
5	8626	8535	8445	8356	8269	8182	8096	8011	7926	7843
6	7761	7679	7598	7519	7439	7361	7284	7208	7132	7057
7	6983	6909	6837	6765	6694	6624	6554	6485	6417	6350
8	6283	6217	6152	6088	6024	5961	5898	5836	5775	5714
9	5654	5595	5536	5478	5420	5364	5307	5252	5197	5142
9.0	5088	5035	4982	4930	4878	4827	4776	4726	4677	4628
1	4579	4531	4484	4437	4390	4344	4299	4254	4209	4165
2	4121	4078	4036	3993	3951	3910	3869	3829	3789	3749
3	3710	3671	3632	3594	3557	3519	3483	3446	3410	3374
4	3339	3304	3270	3235	3202	3168	3135	3102	3070	3038
5	3006	2974	2943	2912	2882	2852	2822	2793	2763	2734
6	2706	2678	2650	2622	2595	2567	2541	2514	2488	2462
7	2436	2411	2385	2360	2336	2311	2287	2263	2240	2216
8	2193	2170	2148	2125	2103	2081	2059	2038	2017	1995
9	1975	1954	1934	1913	1894	1874	1854	1835	1816	1797
10.0	1778	1759	1741	1723	1705	1687	1670	1652	1635	1618

For larger values of the argument, use the auxiliary functions, Table 13.

Bessel Functions: $K_0(x)$ and $K_1(x)$. (Continued)
 $K_1(x)$

x	0	1	2	3	4	5	6	7	8	9
0.0	∞	99.97	49.95	33.27	24.92	19.91	16.56	14.17	12.37	10.97
1	9.854	8.935	8.169	7.519	6.962	6.477	6.053	5.678	5.345	5.046
2	4.776	4.532	4.309	4.106	3.919	3.747	3.588	3.440	3.303	3.175
3	3.056	2.944	2.839	2.740	2.647	2.559	2.476	2.397	2.323	2.252
4	2.184	2.120	2.059	2.001	1.945	1.892	1.840	1.792	1.745	1.700
5	1.656	1.615	1.575	1.536	1.499	1.464	1.429	1.396	1.364	1.333
6	1.303	1.274	1.246	1.219	1.192	1.167	1.142	1.118	1.095	1.072
7	1.050	1.029	1.008	9882	9686	9496	9311	9130	8955	8784
8	0.8618	8456	8298	8144	7993	7847	7704	7564	7428	7295
9	7165	7039	6915	6794	6675	6560	6447	6336	6228	6122
1.0	6019	5918	5819	5722	5627	5534	5443	5354	5267	5181
1	5098	5016	4935	4856	4779	4703	4629	4556	4485	4415
2	4346	4279	4212	4147	4084	4021	3960	3900	3841	3782
3	3725	3670	3615	3561	3508	3455	3404	3354	3305	3256
4	3208	3161	3115	3070	3026	2982	2939	2897	2855	2814
5	2774	2734	2695	2657	2620	2583	2546	2510	2475	2440
6	2406	2373	2340	2307	2275	2244	2213	2182	2152	2123
7	2094	2065	2037	2009	1982	1955	1928	1902	1876	1851
8	1826	1802	1777	1754	1730	1707	1684	1662	1640	1618
9	1597	1575	1555	1534	1514	1494	1474	1455	1436	1417
2.0	1399	1380	1362	1345	1327	1310	1293	1276	1260	1244
1	1227	1212	1196	1181	1166	1151	1136	1121	1107	1093
2	1079	1065	1052	1038	1025	1012	*9993	*9867	*9742	*9620
3	0.09498	9379	9261	9144	9029	8916	8804	8694	8586	8478
4	8372	8268	8165	8063	7963	7864	7767	7670	7575	7482
5	7389	7298	7208	7119	7031	6945	6859	6775	6692	6609
6	6528	6448	6369	6292	6215	6139	6064	5990	5917	5845
7	5774	5704	5634	5566	5498	5432	5366	5301	5237	5174
8	5111	5050	4989	4929	4869	4811	4753	4696	4639	4584
9	4529	4474	4421	4368	4316	4264	4213	4163	4113	4064
3.0	4016	3968	3921	3874	3828	3782	3738	3693	3649	3606
1	3563	3521	3480	3438	3398	3358	3318	3279	3240	3202
2	3164	3127	3090	3054	3018	2983	2948	2913	2879	2845
3	2812	2779	2746	2714	2682	2651	2620	2589	2559	2529
4	2500	2471	2442	2414	2385	2358	2330	2303	2276	2250
5	2224	2198	2173	2147	2123	2098	2074	2050	2026	2003
6	1979	1957	1934	1912	1890	1868	1846	1825	1804	1783
7	1763	1743	1722	1703	1683	1664	1645	1626	1607	1589
8	1571	1553	1535	1517	1500	1483	1466	1449	1432	1416
9	1400	1384	1368	1353	1337	1322	1307	1292	1277	1263
4.0	1248	1234	1220	1206	1193	1179	1166	1152	1139	1126
1	1114	1101	1089	1076	1064	1052	1040	1028	1017	10052
2	0.009938	9826	9715	9605	9497	9390	9284	9179	9076	8973
3	8872	8772	8674	8576	8479	8384	8290	8196	8104	8013
4	7923	7834	7746	7659	7573	7488	7404	7321	7239	7158
5	7078	6999	6920	6843	6766	6691	6616	6542	6469	6397
6	6325	6254	6185	6116	6047	5980	5913	5847	5782	5717
7	5654	5591	5529	5467	5406	5346	5286	5228	5169	5112
8	5055	4999	4943	4889	4834	4781	4727	4675	4623	4572
9	4521	4471	4421	4372	4324	4276	4229	4182	4136	4090
5.0	4045	4000	3956	3912	3869	3826	3784	3742	3700	3660

$$K_1(x) = -K'_0(x)$$

Bessel Functions: $K_0(x)$ and $K_1(x)$. (Continued)
 $K_1(x)$

x	0	1	2	3	4	5	6	7	8	9
5.0	0.004045	4000	3956	3912	3869	3826	3784	3742	3700	3660
1		3619	3579	3540	3501	3462	3424	3386	3349	3312
2		3239	3204	3168	3133	3099	3065	3031	2998	2965
3		2900	2868	2836	2805	2774	2744	2714	2684	2655
4		2597	2568	2540	2512	2485	2457	2430	2404	2377
5		2326	2300	2275	2250	2225	2201	2177	2153	2130
6		2083	2060	2038	2016	1994	1972	1950	1929	1908
7		1866	1846	1826	1806	1786	1767	1748	1729	1710
8		1673	1654	1636	1619	1601	1584	1566	1549	1532
9		1499	1483	1467	1451	1435	1419	1404	1389	1374
6.0		1344	1329	1315	1301	1286	1273	1259	1245	1232
1		1205	1192	1179	1166	1154	1141	1129	1116	1104
2		1081	1069	1057	1046	1034	1023	1012	1001	*9904
3	0.0009691	9586	9483	9380	9279	9178	9079	8981	8884	*9797
4		8693	8599	8506	8414	8324	8234	8145	8057	7970
5		7799	7715	7632	7549	7468	7387	7308	7229	7151
6		6998	6922	6848	6774	6701	6629	6558	6487	6417
7		6280	6212	6145	6079	6014	5949	5885	5822	5759
8		5636	5576	5516	5456	5398	5340	5282	5226	5170
9		5059	5005	4951	4898	4845	4793	4742	4691	4641
7.0		4542	4493	4445	4397	4350	4304	4257	4212	4167
1		4078	4034	3991	3948	3906	3864	3823	3782	3741
2		3662	3623	3584	3545	3508	3470	3433	3396	3360
3		3288	3253	3219	3184	3150	3116	3083	3050	3018
4		2953	2922	2891	2860	2829	2799	2769	2740	2710
5		2653	2625	2597	2569	2542	2514	2488	2461	2435
6		2383	2358	2333	2308	2283	2259	2235	2211	2188
7		2141	2118	2096	2074	2051	2030	2008	1987	1966
8		1924	1903	1883	1863	1843	1824	1804	1785	1766
9		1729	1710	1692	1674	1656	1639	1621	1604	1587
8.0		1554	1537	1521	1505	1489	1473	1457	1442	1427
1		1396	1382	1367	1352	1338	1324	1310	1296	1282
2		1255	1242	1229	1216	1203	1190	1177	1165	1153
3		1128	1116	1105	1093	1081	1070	1058	1047	1036
4		1014	10036	*9930	*9825	*9721	*9618	*9516	*9415	*9316
5	0.0009120	9023	8928	8833	8740	8648	8556	8466	8376	8288
6		8200	8113	8028	7943	7859	7776	7694	7612	7532
7		7374	7296	7219	7142	7067	6992	6918	6845	6773
8		6631	6561	6492	6423	6355	6288	6222	6156	6091
9		5964	5901	5838	5777	5716	5656	5596	5537	5479
9.0		5364	5307	5251	5196	5141	5087	5033	4980	4928
1		4825	4774	4723	4674	4624	4576	4528	4480	4433
2		4340	4294	4249	4204	4160	4116	4073	4030	3988
3		3904	3863	3822	3782	3742	3703	3664	3626	3587
4		3512	3476	3439	3403	3367	3332	3297	3262	3228
5		3160	3127	3094	3062	3029	2998	2966	2935	2904
6		2843	2814	2784	2755	2726	2697	2669	2641	2613
7		2559	2532	2505	2479	2453	2427	2402	2377	2352
8		2302	2278	2254	2231	2207	2184	2161	2139	2116
9		2072	2050	2029	2008	1987	1966	1945	1925	1905
10.0		1865	1845	1826	1807	1788	1769	1751	1732	1714

For larger values of the argument, use the auxiliary functions, Table 13.

Bessel Functions: Auxiliary Functions

Auxiliary Functions $Y_0(x)$ and $Y_1(x)$ for Small Values of Argument

For small values of the argument, $Y_0(x)$ and $Y_1(x)$ are rapidly changing functions and linear interpolation is inaccurate. These tables of auxiliary functions can be used to give accurate interpolated values. For values of the argument above 0.1 the main tables are satisfactory if interpolation formulas are used.

$$Y_0(x) = C_0 + D_0 \log x$$

$$Y_1(x) = (C_1/x) + D_1 \log x$$

C_0

	0	1	2	3	4	5	6	7	8	9
0.0	-0.0738	0738	0737	0736	0735	0734	0732	0729	0727	0724
1	-0720	0717	0713	0708	0703	0698	0693	0687	0681	0674
2	-0667	0660	0652	0645	0636	0628	0619	0609	0600	0590
3	-0579	0569	0558	0547	0535	0523	0511	0498	0485	0472
4	-0458	0444	0430	0415	0400	0385	0369	0353	0337	0321

D_0

	0	1	2	3	4	5	6	7	8	9
0.0	1.4659	4658	4657	4655	4653	4650	4646	4641	4635	4629
1	1.4622	4614	4606	4597	4587	4576	4565	4553	4540	4527
2	1.4512	4498	4482	4465	4448	4431	4412	4393	4373	4352
3	1.4331	4309	4286	4262	4238	4213	4188	4161	4134	4107
4	1.4078	4049	4019	3989	3958	3926	3893	3860	3826	3792

C_1

	0	1	2	3	4	5	6	7	8	9
0.0	-0.6366	6366	6367	6368	6369	6371	6373	6376	6379	6382
1	-6386	6390	6394	6399	6404	6410	6416	6422	6429	6436
2	-6444	6452	6460	6468	6477	6487	6496	6506	6517	6527
3	-6538	6550	6561	6573	6586	6598	6611	6624	6638	6652
4	-6666	6681	6695	6710	6726	6741	6757	6773	6789	6806

D_1

	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0073	0146	0220	0293	0366	0440	0513	0586	0659
1	0732	0805	0878	0951	1024	1096	1169	1241	1314	1386
2	1459	1531	1603	1675	1746	1818	1890	1961	2032	2103
3	2174	2245	2316	2386	2456	2526	2596	2666	2735	2804
4	2873	2942	3011	3079	3148	3215	3283	3351	3418	3485

Auxiliary Functions $K_0(x)$ and $K_1(x)$ for Small Values of Argument

For small values of the argument, $K_0(x)$ and $K_1(x)$ are rapidly changing functions and linear interpolation is inaccurate. These tables of auxiliary functions can be used to give accurate interpolated values. For values of the argument above 0.1 the main tables are satisfactory if interpolation formulas are used.

$$K_0(x) = E_0 + F_0 \log x$$

$$K_1(x) = (E_1/x) + F_1 \log x$$

$E_0(x)$

	0	1	2	3	4	5	6	7	8	9
0.0	0.1159	1160	1160	1162	1164	1166	1169	1173	1177	1182
1	1187	1193	1200	1207	1214	1222	1231	1240	1250	1260
2	1271	1283	1295	1308	1321	1335	1349	1364	1380	1396
3	1412	1430	1448	1466	1485	1505	1525	1546	1567	1590
4	1612	1635	1659	1684	1709	1735	1761	1788	1816	1844

Bessel Functions: Auxiliary Functions. (Continued)

$E_1(x)$										
	0	1	2	3	4	5	6	7	8	9
0.0	1.0000	1.0000	9999	9997	9995	9992	9989	9985	9980	9975
1	0.9969	9963	9955	9948	9939	9930	9921	9910	9899	9888
2	9875	9863	9849	9835	9820	9804	9788	9771	9753	9735
3	9716	9696	9676	9654	9633	9610	9586	9562	9537	9512
4	9485	9458	9430	9401	9371	9341	9310	9278	9245	9211

$F_0(x)$										
	0	1	2	3	4	5	6	7	8	9
0.0	-2.3026	3026	3028	3031	3035	3040	3047	3054	3063	3073
1	-2.3083	3096	3109	3123	3139	3156	3173	3193	3213	3234
2	-2.3257	3280	3305	3331	3359	3387	3417	3447	3479	3513
3	-2.3547	3582	3619	3657	3696	3736	3778	3821	3865	3910
4	-2.3956	4004	4053	4103	4154	4206	4260	4315	4371	4429

$F_1(x)$										
	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0115	0230	0345	0461	0576	0691	0806	0922	1037
1	1153	1268	1384	1500	1616	1732	1848	1964	2081	2197
2	2314	2431	2548	2666	2783	2901	3019	3137	3255	3374
3	3493	3612	3731	3851	3971	4092	4212	4333	4454	4576
4	4698	4820	4943	5066	5189	5313	5437	5562	5687	5812

Examples of use of auxiliary functions for small values of argument:

Example 1. $Y_0(0.115) = -0.0715 + 1.4610 \times 1.0607 = -0.0715 - 1.4610 + 0.0887 = -1.4438$.
Linear interpolation from the direct-reading table of Y_0 would give the less accurate value

$$Y_0(0.115) = -1.4444$$

Example 2.

$$Y_1(0.115) = \frac{-0.6392}{0.115} + 0.08415 \times 1.0607 = -5.558 - 0.084 + 0.005 = -5.637$$

compared with the less accurate value of -5.648 obtained by linear interpolation of the table for $Y_1(x)$.

Auxiliary functions $I_0(x)$, $I_1(x)$, $K_0(x)$, $K_1(x)$ for large values of argument

$e^{-x}I_0(x)$										
	0	1	2	3	4	5	6	7	8	9
10.0	0.1278	1272	1265	1259	1253	1247	1241	1235	1229	1223
11.0	1217	1212	1206	1201	1195	1190	1185	1180	1174	1170
12.0	1164	1159	1154	1150	1145	1140	1136	1131	1126	1122
13.0	1118	1113	1109	1105	1100	1096	1092	1088	1084	1080
14.0	1076	1072	1068	1065	1061	1057	1053	1050	1046	1043
15.0	1039	1035	1032	1029	1025	1022	1018	1015	1012	1009
16.0	1005	1002	0999	0996	0993	0990	0987	0984	0981	0978
17.0	0975	0972	0969	0966	0963	0961	0958	0955	0952	0950
18.0	0950	0944	0942	0940	0937	0934	0931	0929	0926	0924
19.0	0921	0919	0917	0914	0912	0909	0907	0905	0902	0900
20	0898	0876	0856	0836	0819	0802	0786	0771	0757	0744
30	0731	0719	0708	0697	0687	0677	0667	0658	0649	0641

$I_0(x) = \text{tabulated number} \times e^x$.

For greater values of x , $e^{-x}I_0(x) \approx \left(1 + \frac{1}{8x}\right) / \sqrt{2\pi x}$.

Bessel Functions: Auxiliary Functions. (Continued)

$e^{-x}I_1(x)$

	0	1	2	3	4	5	6	7	8	9
10.0	0.1213	1207	1202	1196	1191	1186	1181	1175	1170	1165
11.0	1161	1156	1151	1146	1142	1137	1132	1128	1123	1119
12.0	1115	1110	1106	1102	1098	1094	1090	1086	1082	1078
13.0	1074	1070	1066	1062	1059	1055	1051	1048	1044	1040
14.0	1037	1034	1030	1027	1023	1020	1017	1013	1010	1007
15.0	1004	1001	0997	0994	0991	0988	0985	0982	0979	0976
16.0	0973	0971	0968	0965	0962	0959	0957	0954	0951	0948
17.0	0946	0943	0941	0938	0935	0933	0930	0928	0925	0923
18.0	0920	0918	0915	0913	0911	0908	0906	0904	0901	0899
19.0	0897	0895	0892	0890	0888	0886	0884	0881	0879	0877
20	0875	0855	0836	0818	0801	0786	0771	0757	0744	0731
30	0719	0708	0697	0687	0677	0667	0658	0649	0641	0633

$I_1(x)$ = tabulated number $\times e^x$.

For greater values of x , $e^{-x}I_1(x) \approx \left(1 - \frac{3}{8x}\right) / \sqrt{2\pi x}$.

$e^{-x}K_0(x)$

	0	1	2	3	4	5	6	7	8	9
10.0	0.3916	3897	3879	3860	3842	3824	3806	3789	3772	3755
11.0	3738	3721	3705	3689	3673	3657	3642	3627	3612	3597
12.0	3582	3567	3553	3539	3525	3511	3497	3484	3470	3457
13.0	3444	3431	3418	3406	3393	3381	3368	3356	3344	3333
14.0	3321	3309	3298	3286	3275	3264	3253	3242	3231	3221
15.0	3210	3200	3189	3179	3169	3159	3149	3139	3129	3119
16.0	3110	3100	3091	3081	3072	3063	3054	3045	3036	3027
17.0	3018	3009	3001	2992	2984	2975	2967	2959	2950	2942
18.0	2934	2926	2918	2910	2903	2895	2887	2879	2872	2864
19.0	2857	2850	2842	2835	2828	2821	2813	2806	2799	2792
20	2785	2719	2658	2599	2545	2494	2446	2401	2358	2318
30	2279	2242	2207	2174	2142	2111	2082	2054	2027	2001

$K_0(x)$ = tabulated number $\times e^{-x}$.

For greater values of x , $e^{-x}K_0(x) \approx \sqrt{\frac{\pi}{2x}} \left(1 - \frac{1}{8x}\right)$.

$e^{-x}K_1(x)$

	0	1	2	3	4	5	6	7	8	9
10.0	0.4108	4086	4064	4043	4023	4002	3982	3962	3943	3923
11.0	3904	3886	3867	3849	3831	3813	3796	3779	3762	3745
12.0	3728	3712	3696	3680	3664	3649	3633	3618	3603	3589
13.0	3574	3560	3545	3531	3518	3504	3490	3477	3464	3450
14.0	3437	3425	3412	3399	3387	3375	3363	3351	3339	3327
15.0	3315	3304	3292	3281	3270	3259	3248	3237	3226	3216
16.0	3205	3195	3185	3174	3164	3154	3144	3135	3125	3115
17.0	3106	3096	3087	3077	3068	3059	3050	3041	3032	3023
18.0	3015	3006	2997	2989	2980	2972	2964	2955	2947	2939
19.0	2931	2923	2915	2907	2900	2892	2884	2877	2869	2862
20	2854	2783	2717	2655	2598	2544	2493	2445	2400	2357
30	2317	2278	2241	2206	2173	2141	2110	2081	2053	2026

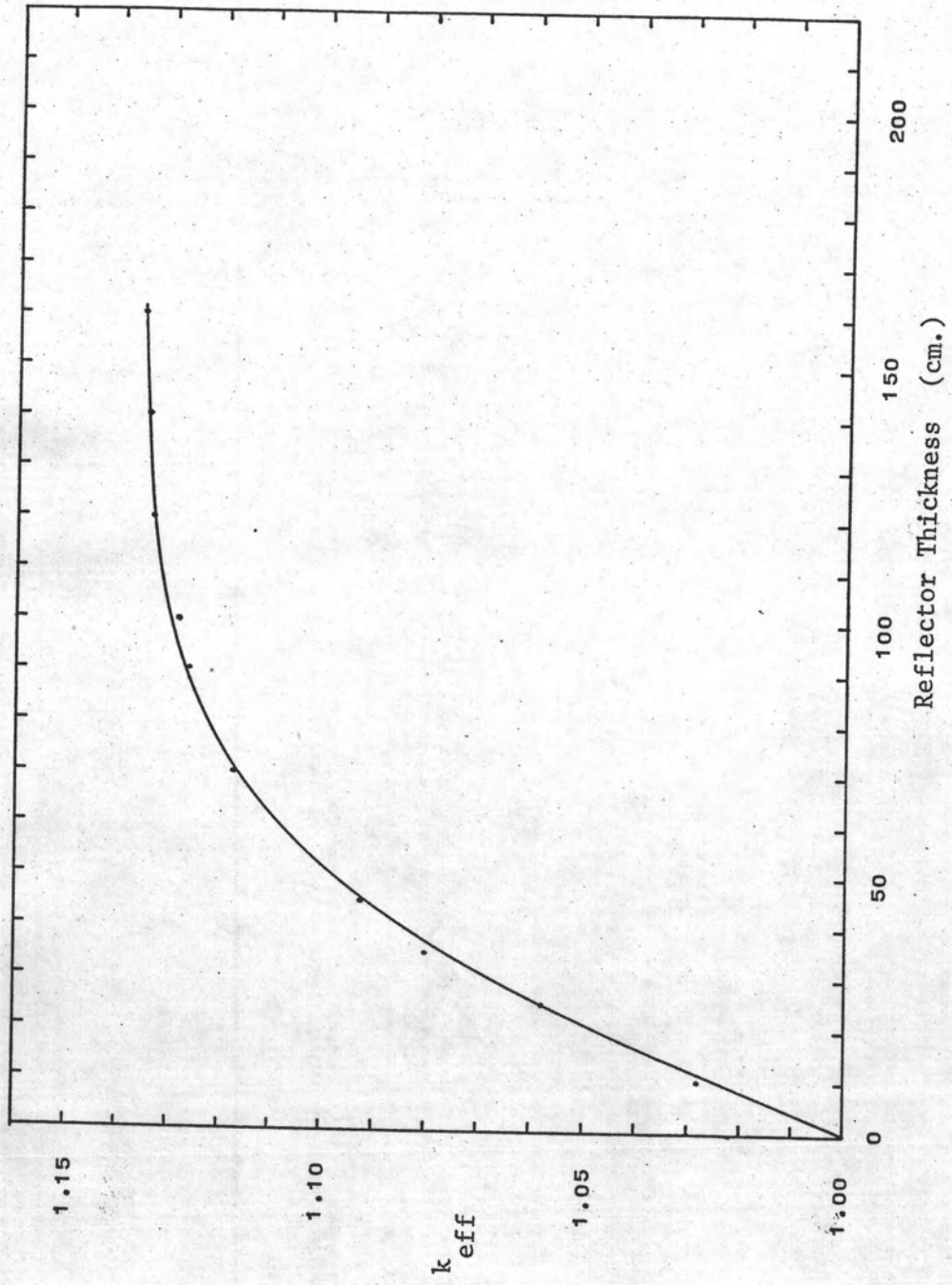
$K_1(x)$ = tabulated number $\times e^{-x}$.

For greater values of x , $e^{-x}K_1(x) \approx \sqrt{\frac{\pi}{2x}} \left(1 + \frac{3}{8x}\right)$.

Example of use of auxiliary functions for large values of argument:

$$I_0(25) = 7.202 \times 10^{10} \times 0.0802 = 5.776 \times 10^9$$

ภาคผนวก จ. รูปแสดงค่า k_{eff} ที่เพิ่มขึ้นเมื่อเพิ่มความหนาของตัวสะท้อนนิวตรอนซึ่งเป็นน้ำชนิดหนัก



ประวัติผู้เขียน

นายสมยศ ศรีสถิตย์ เกิดเมื่อวันที่ 27 มิถุนายน พ.ศ. 2501 จังหวัดมหาสารคาม
สำเร็จปริญญาวิทยาศาสตรบัณฑิต สาขาฟิสิกส์ จากมหาวิทยาลัยขอนแก่น ปีการศึกษา 2525
ปัจจุบันรับราชการในตำแหน่งนักฟิสิกส์รังสี 3 กองป้องกันอันตรายจากรังสี กรมวิทยาศาสตร์-
การแพทย์ กระทรวงสาธารณสุข

