CHAPTER II

REVIEW LOT SIZING PROBLEM

The problem that is investigated in this research has various approaches. In one approach, by looking at the whole flow of materials as an integrated system or a related system, a manufacture can plan and control to make optimal or closely optimal plan. The flow system usually composes of procuring materials or parts from suppliers and conveying them to the manufacturing to establish products, and also, after manufacture, storing parts and products through warehouses and delivering them to customers. In addition, it is well known that existing production planning, procurement or purchasing planning and warehouse suffer from a considerable number of deficiencies. Generally, the traditional systems only possess a simple planning like Material Requirement Planning (MRP), Economic Order Quantity (EOQ) and others, and use them separately. As a consequence, the solution from each of them does not guarantees the optimal solution of the integration.

Moreover, although MRP is one of the most effective concepts for determining the purchasing lot size, MRP only provide feasible solution and deterministic situation. As a consequence, in MRP, cost optimization does not seem to be adequate as lead times and work in process (WIP) has a fundamental impact on important criteria of production planning. For instance, long lead times cause a high level of WIP and hence a high variance of waiting times. According to this variance, safety stocks and safety times have to be increased and the coordination of components (especially in an assembly environment) becomes more difficult (Blackburn et al., 1986; Hon-iden and Nishiyama, 1988; Roll and Karni, 1991; Zapfel and Missbauer, 1993). Furthermore, MRP only decompose requirements to each level requirement, unfortunately in each level or stage of this problem must consider production capacity and warehouse capacity, so that optimal planning needs production plan optimization together with material requirements plan and also procurement plan. Therefore, the integrated optimization model is necessary to find the optimal solution for more realistic problem.

Other approaches, a lot size problem is to determine production lot size, warehouse lot size or procurement lot size. Cost elements are depend on how determined integration of the model. If capacity constraint is considered, then the model is called capacitated lot sizing problem (CLSP). For review the single-item lot sizing, see the works by Brahimi, N., Dauzere-Peres, S., Najid, N.M., Nordi, A. (2006). This great invited review paper focuses all of the cases for single item lot size problem (SILSP) both of the capacitated and uncapacitated versions; they also survey lot size problems (LSPs) for positioning SILSP among all problems. In the review, they mentioned that the capacitated lot sizing problems are NP-hard problem. Besides, for review of the CLSP models, see the works by Karimi, B., Fatemi Ghomi, S.M.T. and Wilson, J.M., (2003). They consider CLSP of single-level lot sizing problems, both of their variants and their solution approach, together with exact and heuristics approaches for the problem. They also mentioned that there is less attention in the literature for the CLSP with complex setup and other variants such as CLSP with backlogging or with setup times and setup carry-over. These problems still need efficient heuristics such as the meta-heuristics. In Pochet, Y. (2001), they focus on reviewing mixed integer programming models and formulations for deterministic production planning problem which include part of the lot size problems, in this article they survey all production lot size problems. Based on these beneficial review papers, these problems can be classified into single level (independent demands) and multiple level (dependent demands), and the multi-level capacitated problems are the most difficult.

Furthermore, all of these approaches can be classified time bucket as big bucket or small bucket. If multiple items are typically produced in each period, the problem is considered as big bucket. If single item is produced in each period, the problem is considered as small bucket. Therefore, big bucket must be considered production sequence with or without setup time or cost. The setup time or cost, which some papers may determined it with maintenance time or cost, and may call it as startup cost or changeover cost in small bucket, will be occurred whenever changeover products on the machine. However, in case one product for one machine this lost consideration may not occurred. The setup cost and maintenance cost can be integrated into one fixed cost which occurs when production occurs.

In this research, the aforementioned model (consideration purchasing planning situation together with production planning situation while feasible capacity constraint) is appropriated and needed to find optimal plan. Such problem can be defined as Multi-level Multi-item Capacitated Lot Sizing Problem with Multi-workstation (MLCLSP-M). Therefore, this chapter is thus organized as follows: first, overview of lot size problem is provided. Second, we review the solution approach.

2.1 Lot size problems

Lot size is one of the most important problems in manufacturing system and supply chain. It can be found in various problem statement aspects and frequently appears in the literatures. The classification and definition of the problems was stated based on the focus of the research topic. The two major base models that common used for developing lot sizing models are based on inventory model and based on production model. In addition, what the base model should be depends on what the research question more focus on. Those new reformulation or invention models are very different.

The one which was developed from inventory model usually look like EOQ model and use differential equation for deriving. Another one, which was developed from production plan models, typically look like combinatorial optimization model and often formulate as Mixed-Integer Programming (MIP) with believing that it's a strong formulation (Armagan and Kingsman, 2004; Constantino, 2000; Gao et al., 2008; Hung and Chien, 2000; Karimi et al., 2003; Miller et al., 2003; Pochet, 2001; Pochet and Wolsey, 1988; Robinson and Lawrence, 2004; Sahinidis and Grossmann, 1992; Schmidt and Wilhelm, 2000; Silvio et al., 2008; Stadtler, 1996; Sung and Maravelias, 2008; Ustun and Demi'rtas, 2008; Wolsey, 1989; Wu and Golbasi, 2004).

Moreover, Comelli et. al. (2008) review the tactical planning models which including Master production and scheduling (MPS), Material Requirement Planning (MRP) and Multi-site planning. They claim that the problems can be represented relatively with the capacitated lot sizing problem (CLSP) and Multi level capacitated lot sizing problem (MLCLSP) and their extensions. Understanding by this, our research objective is to find what purchasing lot size and production lot size should be. Therefore, this research has more concentrations on the production part and also formulates the model with MIP formulation. As a result, in the next, we will review the problems based on MIP classification.

Following by MIP formulation, generally lot sizing problem can be classified by characteristic of item, product structure, capacity limitations or constraints, set up consideration, period time consideration, etc. Backordering assumptions are also important to make assumptions about system reactions under shortage warehouse situations, e.g., when demand exceeds supply(Brahimi et al., 2006; Drexl and Kimms, 1997; Jans and Degraeve, 2008; Karimi et al., 2003; Pochet, 2001; Robinson et al., 2009). For example, the recently review papers (Brahimi et al., 2006), the problem is classified by number of machines, number of production stages (levels), capacity constraints and their nature (fixed or variable), length of production period, etc. However, in this chapter we classify the lot size problem in four groups which are single item lot sizing problem (SILP), Multi-item Single-level capacitated lot sizing problem (CLSP) and Multi-item Multi-level capacitated lot sizing problem (MLCLSP).

2.1.1 Single item Lot sizing problem (SILP)

The single item lot sizing problem is a very basic type in the series of lot sizing problem. The problem considers only one item for lot sizing decision. Commonly it has two types which are uncapacitated problem and capacitated problem.

2.1.1.1 Uncapacitated Single item Lot sizing problem (USILP)

This problem assumes that the capacity always enough to satisfy demands. The standard problem is modeled as linear programming with various parameter situations. With demand constant over period and infinite planning horizon, the problem can be represented easily as economic order quantity (EOQ). For dynamic demand, finite time period and no extension, the model is formulated by MIP and called as standard model. There are a lot of papers that develop model based on EOQ and try to improve its deficiencies. One of them is the dynamic lot-sizing problem (DLSP) models. The great classical exact algorithm for this problem is dynamic programming (DP) by Wagner and Whitin (1958). They examined the DLSP and provided a forward dynamic programming algorithm solve the problem. Their algorithm can use for solving the model in $O(T^2)$ time (Wagner and Whitin, 1958). Moreover, in their scenario, demand, holding cost and production cost are no longer constant over planning horizon. Later, there are many researches initiated by this algorithm (Aarts et al., 2000; Agra and Constantino, 1999; Aucamp, 1985; Bahl and Taj, 1991; Chung and Chyr, 1997; Evans et al., 1989; Fordyce and Webster, 1984; Fordyce and Webster, 1985; Freeland et al., 1990; Grubbstrom, 2005; Gupta et al., 1992; Gutierrez et al., 2008; Hwang and Sohn, 1985; Kazan et al., 2000; Kim, 1996; Loparic et al., 2001; Ma et al., 2001; Miller et al., 1999; Ning Hsu and Lowe, 2001; Pochet and Wolsey, 1994; Prasad and Krishnaiah Chetty, 2001; Richter et al., 2006; Richter and Sombrutzki, 2000; Richter and Weber, 2001; Saydam, 1987; Toy and Berk, 2006; Van Vyve, 2007; Weiss, 1990; Wolsey, 2006; Yilmaz, 1992).

The uncapacitated problem is also formulated with aggregate formulation (AGG), Shortage path formulation (SHP) and Facility location-based formulation (FAL) (Brahimi et al., 2006). For involved more situation, the extended version of the standard problem is formulated, for example, backlogging extension, multi facilities extension, perishable inventory extension, remanufacturing, time windows extension and others (Brahimi et al., 2006). In some researches reformulated the problem for a specific situation or a particular algorithm such as the Wagner-Whitin Costs structure, an $O(T)$ algorithm to separate the (l,S,WW) Inequalities, an approximate shortest path reformulation, etc. (Pochet and Wolsey, 2006).

2.1.1.2 Capacitated Single item Lot sizing problem (CSILP)

This problem has a capacity limit and the capacity constraint will be included in the model. The particular cost structure is defined the problem complexity (Bitran and Yanasse, 1982). Although the formulations are based on the uncapacitated problem with including capacity constraint, the solution technique is different. Many researches focus on dynamic programming (DP), branch and bound algorithm (B&B), valid inequalities and branch and cut (B&C), and approximation methods (Brahimi et al., 2006). Besides, there are various extensions for a specific situation such as discrete lot-sizing with constant capacities, discrete lot-sizing with initial stock and constant capacities, lot-sizing with Wagner-Whitin costs and constant capacities, lot-sizing with constant capacities, lot-sizing with varying capacities and lifted (l,S) inequalities for discrete lot-sizing with initial stock and constant capacities (Chen, 1997; Pochet and Wolsey, 2006; Wolsey, 1985). Pochet and Wolsey (2006) show that in the constant capacity cases the problem can be solved in polynomial time, while in case of varying capacity the problem become NP-hard. They also present DP algorithm and some valid inequalities. The backlogging and setup or start-ups also have an important role in the higher complexity of the problem. The analysis and discussion of backlogging and setup or start-ups are also provided in Pochet and Wolsey (2006). In recent paper (Brahimi et al., 2006), they claim that the Lagrangian relaxation is useful for solve this problem with time windows. However, there are many researches study on the approximate algorithm to solve this problem (Brahimi et al., 2006; Chen et al., 1994; Chen et al., 1994; Drexl and Kimms, 1997; Hindi, 1995; Hindi, 1995; Hindi, 1995; Pratsini, 2000; Van den Heuvel and Wagelmans, 2006).

2.1.2 Multi-Item Single-level Capacitated Lot sizing problem (CLSP)

This problem is more practical than single-item problem. It's also classified as uncapacitated model and capacitated model. The single level is mean that no components in the product structures. Likewise, in the bill of material, there is one level. However, the uncapacitated problem is similar to the SILP in mathematical solving. In this section, we provide only the reviews of multi-item capacitated lot sizing problem (CLSP) for big bucket and small bucket. By the setup or start-ups consideration, the time bucket (small or big) is introduced (Comelli et al., 2008; Pochet and Wolsey, 2006). Small bucket is the situation that assumes only one product can be produced during small time windows/periods/ buckets. In the other word, it permits only a single set-up per period. For Big bucket, it is assumed that the several different items can be produced in each time period.

2.1.2.1 Big bucket Model

CLSP is a standard formulation for big bucket lot-sizing problems with a discrete period segmentation and deterministic dynamic demands with capacity constraint (Comelli et al., 2008; Karimi et al., 2003). Comelli et. al. (2008) review the tactical planning models which including Master production and scheduling (MPS), Material Requirement Planning (MRP) and Multi-site planning. They also claim that the problems can be represented relatively with the capacitated lot-sizing problem (CLSP) and Multi level capacitated lot sizing problem (MLCLSP) and their extensions.

These are the notations which we used in the model.

Sets

T represents the set of discrete period time in planning horizon, whereby $T = \{1, \ldots, NT\}$

I represents the set of items, whereby $I = \{1, \ldots, NI\}$

Indices

t, l represents the time period index in planning horizon, whereby $t, l \in T$

i represents the item index, whereby $i \in I$

Decision Variables

x_t^i represents the amount of production item i in period t

s_t^i represents the amount of stock item i at the ending of period t

y_t^i represents the setup decision of the production item i k in period t, whereby $y_t^i \in \{0,1\}$

Costs

p_t^i represents the production unit cost of item i in period t

f_t^i represents the setup cost per unit of production item i in period t

h_t^i represents the holding cost per unit of stock item i at the ending of period t

Parameters

d_t^i represents the independent demand item i in period t

cap_t represents the capacity of a workstation during period t

o_t^i represents production usage resource of item i during period t

The formulation is as follows:

$$Min \sum_{t \in T} \sum_{i \in I} \left(p_t^i \cdot x_t^i + f_t^i \cdot y_t^i + h_t^i \cdot s_t^i \right) \tag{2.1}$$

Subject to

$$s_{t-1}^i + x_t^i = d_t^i + s_t^i, \forall i \in I, \forall t \in T \tag{2.2}$$

$$\sum_{i \in I} \left(o_t^i \cdot x_t^i \right) \le cap_t, \ \forall t \in T \tag{2.3}$$

$$x_t^i \le \min \left\{ \frac{cap_t}{o_t^i}, \sum_{l=t}^{NT} d_l^i \right\} \cdot y_t^i, \ \forall i \in I, \forall t \in T \tag{2.4}$$

$$x_t^i \ge 0, \ \forall i \in I, \forall t \in T \tag{2.5}$$

$$s_t^i \ge 0, \ \forall i \in I, \forall t \in T \tag{2.6}$$

$$y_t^i \in \{0,1\}, \ \forall i \in I, \forall t \in T \tag{2.7}$$

The index of items is introduced in this model. The constraints (2.2) represent balance material equations for each item. The capacity constraints are represented in constraints (2.3). The constraints (2.4) are the setup constraints. Finally, the constraints (2.5) to (2.6) are variable constraints.

2.1.2.2 Small bucket Model

Although the standard problem of CLSP is formulated for big bucket, there is a formulation for small bucket. The formulation is as follows:

$$Min \ \sum_{t\in T}\sum_{i\in I}\left(q_t^i \cdot z_t^i + p_t^i \cdot x_t^i + f_t^i \cdot y_t^i + h_t^i \cdot s_t^i\right) \tag{2.8}$$

Subject to

$$s_{t-1}^i + x_t^i = d_t^i + s_t^i, \forall i \in I, \forall t \in T \tag{2.9}$$

$$\sum_{i\in I} y_t^i \le 1, \ \forall t \in T \tag{2.10}$$

$$o_t^i \cdot x_t^i \le cap_t \cdot y_t^i, \ \forall i \in I, \forall t \in T \tag{2.11}$$

$$x_t^i \le \min\left\{\frac{cap_t}{o_t^i}, \sum_{l=t}^{NT} d_l^i\right\} \cdot y_t^i, \ \forall i \in I, \forall t \in T \tag{2.12}$$

$$z_t^i \ge y_t^i - y_{t-1}^i, \ \forall i \in I, \forall t \in T \tag{2.13}$$

$$x_t^i \ge 0, \ \forall i \in I, \forall t \in T \tag{2.14}$$

$$s_t^i \ge 0, \ \forall i \in I, \forall t \in T \tag{2.15}$$

$$y_t^i \in \{0,1\}, \ \forall i \in I, \forall t \in T \tag{2.16}$$

The new variables are the start up production usage resource variable (z_t^i) and startup cost parameters (q_t^i). In small bucket, a start up occurs when the machine is set up for an item for which it was not set up in the previous period and the total cost also includes startup cost. The constraint (2.10) is the single node constraint, imposing that at most one type of product can be made in each time period. The startup time variables are represented in constraint (2.13) which force to occur startup only if no setup in previous period. A setup can be carried over to the next period if production of the same product is continued. This problem is also referred to as lot sizing with startup cost.

2.1.2.3 Extensions

Today, the problems have various extensions and various solution methods (Alfieri et al., 2002; Choi and Enns, 2004; Dellaert and Melo, 1996; Drexl and Kimms, 1997; Gopalakrishnan, 2000; Quadt and Kuhn, 2008; Rong et al., 2006). Quadt and Kuhn (2008) focus on studies consider CLSP models and they claim that the extensions of CLSP including back-orders, setup carry-over, sequencing or parallel machines. Back-orders imply that orders can be satisfied by the next or further period. Setup carry-over means that the setup can be cover more than one period and the production under the cover period has no need another setup. Sequencing means that the order of jobs on each machine has to be decided. Parallel machines imply that there is one or more machine to be assigned jobs in each period. The detailed of each extension can be found in the literature.

In recently researches (Gao et al., 2008; Gurgur and Altiok, 2007; Lee and Kumara, 2007; Maiti and Maiti, 2007; Patriksson, 2008; Robinson et al., 2009; Wang and Hu, 2008), the assumption is also to backorder all the excess demand, which is represented by a negative warehouse level. On the other hand, depending on system characteristics, it is also common to assume that all excess demand is lost. A compromise situation where both backorder and lost sales are present is also possible. The review process can be continuous or periodic. Continuous review means that the level of warehouse is known at all times and reorder decisions can be made at any time, while periodic review means that the stock level is known only at discrete points and reorders are only possible at pre-determined points corresponding to the beginning of periods. Changes that occur in the warehouse during storage – Usually warehouse items do not change their characteristics during the stock period but radioactive materials or volatile liquids may experience some losses. Fixed life inventories, such as food, are assumed to have constant utility until the expiration date is reached. Obsolescence may also affect inventories since their useful lifetime may not be predictable in advance.

2.1.3 Multi-Item Multi-level Capacitated Lot sizing problem (MLCLSP)

This model was motivated by problems arising from Material Requirement Planning (MRP) formulation model (Comelli et al., 2008). The main idea of this formulation is that the model considers systems in which two or more items required at least one other items to produce. To represent the relation of each item requirement, Bill of Material (BOM) and "Gozinto" matrix are introduced. The requirement is also defined as internal demand or dependent demand. Besides, the production structures can be represented differently by BOM.

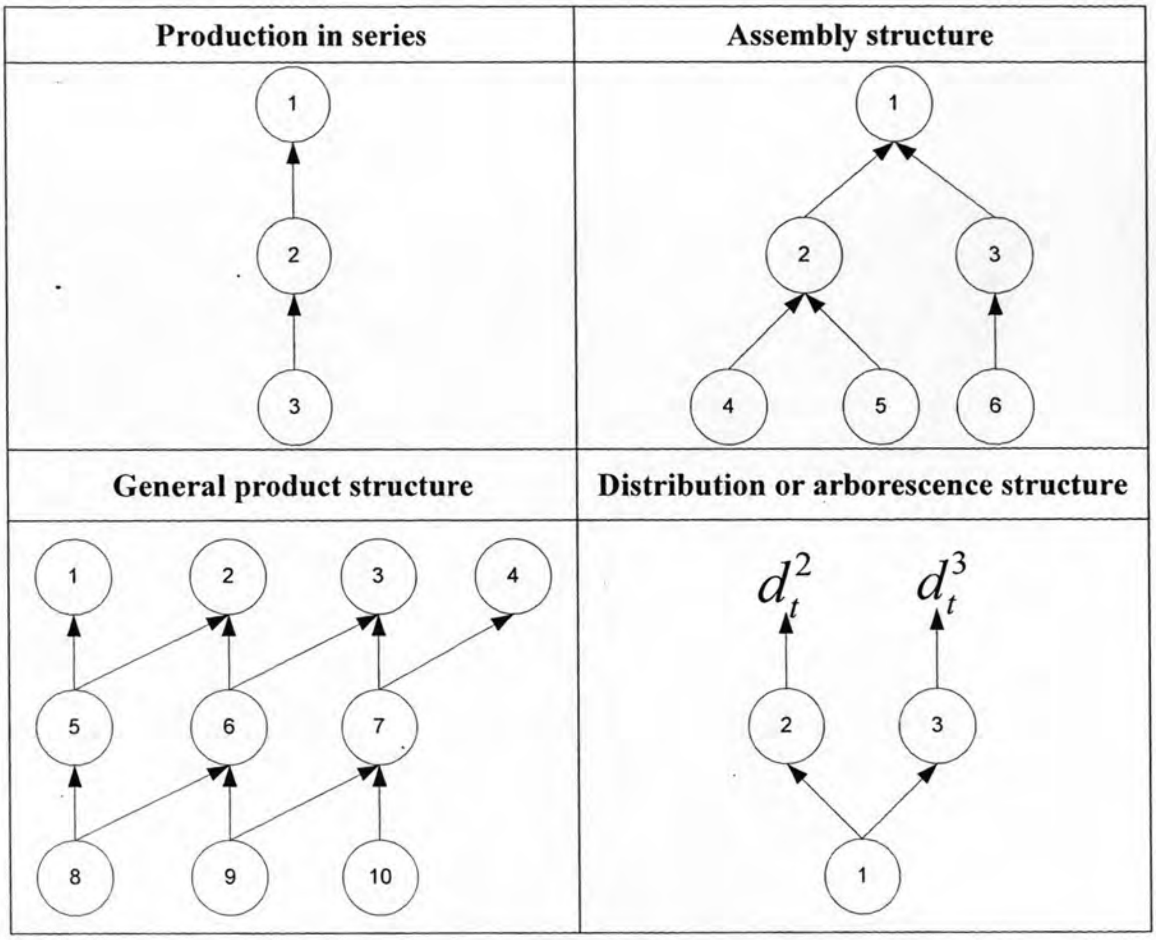

Figure 2.1 Illustration of different types of production structure

Pochet and Wolsey (2006) classify production structures in four types, presented in Figure 2.1, which are production in series, an assembly structure, a general product structure and a distribution or arborescence structure. As in MRP, lead time can be also represented in BOM. However, our research problem focuses on purchasing and production systems with no consideration on distribution. Then, the model formulation will be investigated only for production series, assembly system and general product structure in big bucket situation.

The production system in series can be formulated as follows:

$$Min \sum_{t\in T}\sum_{i\in I}\left(p_t^i \cdot x_t^i + f_t^i \cdot y_t^i + h_t^i \cdot s_t^i\right) \tag{2.17}$$

Subject to

$$s_{t-1}^{i} + x_t^{i} = x_t^{i-1} + s_t^{i}, \forall i \in \{2,..,NI\}, \forall t \in T \tag{2.18}$$

$$s_{t-1}^{1} + x_t^{1} = d_t^{1} + s_t^{1}, \forall t \in T \tag{2.19}$$

$$\sum_{i \in I} \left(o_t^i \cdot x_t^i \right) \le cap_t, \ \forall t \in T \tag{2.20}$$

$$x_t^i \le \min \left\{ \frac{cap_t}{o_t^i}, \sum_{l=t}^{NT} d_l^i \right\} \cdot y_t^i, \ \forall i \in I, \forall t \in T \tag{2.21}$$

$$x_t^i \ge 0, \ \forall i \in I, \forall t \in T \tag{2.22}$$

$$s_t^i \ge 0, \ \forall i \in I, \forall t \in T \tag{2.23}$$

$$y_t^i \in \{0,1\}, \ \forall i \in I, \forall t \in T \tag{2.24}$$

For assembly systems, the two new set are introduced. Let set $S(i)$ denotes the set of direct successors of $i \in I$ and set $Z(i)$ denotes the set of all successors or all items that required item i and other items from item i to produce. In this system, $|S(i)| = 1$ for all intermediate products, and $Z(i)$ is the set of items on the path from $S(i)$ to the unique end-product containing i. Then, the formulation of assembly systems is as follows:

$$Min \ \sum_{t \in T} \sum_{i \in I} \left(p_t^i \cdot x_t^i + f_t^i \cdot y_t^i + h_t^i \cdot s_t^i \right) \tag{2.25}$$

Subject to

$$s_{t-1}^{i} + x_t^{i} = x_t^{S(i)} + s_t^{i}, \forall i \in \{2,..,NI\}, \forall t \in T \tag{2.26}$$

$$s_{t-1}^{1} + x_t^{1} = d_t^{1} + s_t^{1}, \forall t \in T \tag{2.27}$$

$$\sum_{i \in I} \left(o_t^i \cdot x_t^i \right) \le cap_t, \ \forall t \in T \tag{2.28}$$

$$x_t^i \le \min \left\{ \frac{cap_t}{o_t^i}, \sum_{l=t}^{NT} d_l^i \right\} \cdot y_t^i, \ \forall i \in I, \forall t \in T \tag{2.29}$$

$$x_t^i \ge 0, \ \forall i \in I, \forall t \in T \tag{2.30}$$

$s_t^i \geq 0$, $\forall i \in I, \forall t \in T$ (2.31)

$y_t^i \in \{0,1\}$, $\forall i \in I, \forall t \in T$ (2.32)

For general systems, the new parameter is the usage item i or producing item j, whereby $j \in S(i)$ represented by u^{ij}. This parameter is needed for representation requirement relations between items. The formulation can be represented as follows:

$$Min \sum_{t \in T} \sum_{i \in I} \left(p_t^i \cdot x_t^i + f_t^i \cdot y_t^i + h_t^i \cdot s_t^i \right) \quad (2.33)$$

Subject to

$$s_{t-1}^i + x_t^i = \sum_{j \in S(i)} u^{ij} \cdot x_t^j + s_t^i, \forall i \in \{2,..,NI\}, \forall t \in T \quad (2.34)$$

$$s_{t-1}^1 + x_t^1 = d_t^1 + s_t^1, \forall t \in T \quad (2.35)$$

$$\sum_{i \in I} \left(o_t^i \cdot x_t^i \right) \leq cap_t, \ \forall t \in T \quad (2.36)$$

$$x_t^i \leq \min \left\{ \frac{cap_t}{o_t^i}, \sum_{l=t}^{NT} d_l^i \right\} \cdot y_t^i, \ \forall i \in I, \forall t \in T \quad (2.37)$$

$$x_t^i \geq 0, \ \forall i \in I, \forall t \in T \quad (2.38)$$

$$s_t^i \geq 0, \ \forall i \in I, \forall t \in T \quad (2.39)$$

$$y_t^i \in \{0,1\}, \ \forall i \in I, \forall t \in T \quad (2.40)$$

The complexity of this problem is known as NP-hard and adding setup time constraint can make it to be NP-complete (Comelli et al., 2008; Tempelmeier and Derstroff, 1996).Moreover, the problem with capacity relaxation is called Multi-level lot sizing problem (MLLP). It's well known that most MLLP result is a very good lower bound of MLCLSP (Comelli et al., 2008). In small bucket situation, the MLCLSP can be extended to time Continuity model. The model formulation and enhancements can be found in Suerie (2005). Next section we will investigate on the solution approach of the lot size problem.

2.2 Solution Approaches for MLCLSP

Although considerable research has been devoted to traditional methods of search, optimization using such methods is not that efficient, particularly in finding a solution for very complex search space. Significant less attention has been paid to stochastic search and optimization techniques like heuristics or meta-heuristics (Akartunali and Miller, 2007; Bahl, 1983; Berretta et al., 2005; Bomberger, 1966; Chaudhry and Luo, 2005; Clark, 2003; Comelli et al., 2007; Gaafar, 2006; Gaafar and Alỳ, 2006; Grubbstrom and Thu Thuy Huynh, 2006; Hindi, 1995; Jans and Degraeve, 2007; Karimi et al., 2003; Karimi et al., 2006; Khouja et al., 1998; Kimms, 1999; Lambrecht and Vanderveken, 1979; Megala and Jawahar, 2006; Ornek and Cengiz, 2006; Ozdamar and Barbarosoglu, 1999; Ozdamar and Barbarosoglu, 2000; Roach and Nagi, 1996; Silver, 2004; Tempelmeier and Derstroff, 1996; Xie and Dong, 2002). In recently, the problem has been extended to another level of complication in real world problem based. The trend of the solution approach is trying to decompose the complex problem to simpler problem and solve with a heuristic procedure. For more inclusively understanding the solution approaches, this section will present the investigation in three parts. First, the mathematical based heuristic or meta-heuristic will be presented. Second, the decomposition based approached are reviewed. Finally, the summary of all approaches is presented.

2.2.1 Mathematical based heuristic or Meta-heuristic

As the literature review, the optimization algorithms solving lot-sizing problems usually make use of techniques of integer programming and/or combinatorial optimization, such as dynamic programming, branch and bound or other related methods (Afentakis and Gavish, 1986; Berretta et al., 2005; Brahimi et al., 2006; Chaudhry and Luo, 2005; Chen et al., 1994; Chen et al., 1994; Schwarz and Schrage, 1975; Stadtler, 2005; Zangwill, 1969).

Unfortunately, all these optimization algorithms for multi-level lot-sizing problems are only applicable to unrealistically small scale problems because of their time consuming enumeration nature. The altered approach on lot sizing solving is a heuristic approach. The meta-heuristics algorithms such as the simulated annealing algorithm and genetic algorithm are also suggested to tackle the multi-level lot-sizing problems (Berretta et al., 2005; Brahimi et al., 2002; Comelli et al., 2007; Dellaert and Jeunet, 2003; Han et al., 2008; Hindi, 1995; Karimi et al., 2003; Karimi et al., 2006; Tang, 2004). For example, Tang (2004) applied the simulated annealing algorithm to solve the lot-sizing problems. Hindi (1995) proposed Tabu Search (TS) for the single item, capacitated dynamic lot-sizing problem with start up and reservation costs for the LP relaxation lot sizing model.

However, these approached cannot be applied in the more complicate problem or the specific extensions of the MLCLSP. Therefore, the next section we will present the recent approach which is the decomposition heuristic.

2.2.2 Decomposition heuristic

The recently researches propose the decomposition based heuristic and then solve the sub problem with the variety of the ordinary or meta-heuristic. (Berretta et al., 2005; Han et al., 2008; Nie et al., 2008; Nie et al., 2007; Ozdamar and Barbarosoglu, 2000; Pitakaso et al., 2007; Shin, 2007) and some presented an improved heuristics by modifying the cost parameters between different stages (Pitakaso et al., 2006; Pratsini, 2000; Zheng et al., 2006).

For example, Ozdamar and Barbarosoglu (2000) propose a heuristic combining the capability of the Lagrangean relaxation to decompose the hard-to-solve problems into smaller subproblems and the intensive search capability of the simulated annealing. As the first attempt, two Lagrangean relaxation schemes are designed and different versions of simulated annealing are incorporated into relaxation designs as the Lagrangean heuristic. Then in order to improve the performance of the heuristic, a Phase-1 procedure is developed as a recursive algorithm to restore capacity feasibility. It is observed that the best results are obtained by executing first Phase-1 procedure and then simulated annealing approach with only improving moves in each Lagrangean cycle. The performance of these approaches is compared by using the benchmark problems available in literature.

Berretta (2005) proposes the use of meta-heuristics for the resource-capacitated multilevel lot-sizing problem with general product structures, setup costs, setup times, and lead times. Initially, they develop a heuristic which moves production in time in order to obtain feasible solutions with good quality. Strategies for the short-term memory and long-term memory of Tabu search are then included to guide the search of the subordinate heuristic for new, feasible, and better solutions. Simulated annealing components are embedded into Tabu search in order to improve its performance. For small problems, the solutions provided by Tabu search and the hybrid meta-heuristic are compared to optimal solutions and for larger problems, the quality of the solutions is evaluated against a lower bound generated by Lagrangean relaxation.

Pitakaso et al. (2006) present an ant-based algorithm to solve multi-level capacitated lot-sizing problems. They apply a hybrid approach where we use the ant system to optimize the decomposition of the problem into smaller sub problems. These sub problems, containing only a few items and periods, are solved using CPLEX. Then the overall solution is derived by consolidating the partial solutions. This hybrid approach provides superior results with respect to solution quality in comparison with the existing approaches in the literature.

Nie, Xu and Zhan (2007, 2008) develop a collaborative planning framework combining the Lagrangean Relaxation method and Genetic Algorithms to coordinate and optimize the production planning of the independent partners linked by material flows in multiple tier supply chains. They also propose linking constraints and dependent demand constraints were added to the monolithic Multi-Level, multi-item Capacitated Lot Sizing Problem (MLCLSP) for supply chains. Model MLCLSP was Lagrangean relaxed and decomposed into facility-separable sub-problems based on the separability of it. Genetic Algorithms was incorporated into Lagrangean Relaxation method to update Lagrangean multipliers, which coordinated decentralized decisions of the facilities in supply chains.

Pitakaso, Almeder, Doerner and Hartl (2007) present an ant-based algorithm for solving unconstrained multi-level lot-sizing problems called ant system for multi-level lot-sizing algorithm (ASMLLS). They apply a hybrid approach where we use ant colony optimization in order to find a good lot-sizing sequence, i.e. a sequence of the different items in the product structure in which they apply a modified Wagner-Whitin algorithm for each item separately. Based on the setup costs each ant generates a sequence of items. Afterwards a simple single-stage lot-sizing rule is applied with modified setup costs. This modification of the setup costs depends on the position of the item in the lot-sizing sequence, on the items which have been lot-sized before, and on two further parameters, which are tried to be improved by a systematic search. For small-sized problems ASMLLS is among the best algorithms, but for most medium- and large-sized problems it outperforms all other approaches regarding solution quality as well as computational time.

Shin (2007) proposes a hybrid heuristic approach for the Multi-level, multi-item Capacitated Lot Sizing Problem (MLCLSP) in supply chain network. They use the proposed heuristic search to relax MIP to Linear programming (LP), then repeat solving with a heuristic search method in a hybrid manner. They claim that the proposed approach allocates finite manufacturing resources for each distributed facilities and generates feasible production plans. Two heuristic search algorithms, tabu search and simulated annealing are presented to solve the MIP problems. They also present an experimental test to evaluate the computational performance under a variety of problem scenarios.

Han et al. (2008) study on the multi-level lot-sizing (MLLS) problem in discrete manufacturing industry is not only theoretically but computationally hard. To avoid the decrease in search efficiency caused by prematurity, the repulsion operator was integrated into GA (RGA) to solve uncapacitated MLLS problem with assembly structure. They also conduct simulation testes introducing both GA and RGA for 6 groups of MLLS problems of different sizes, and the computational results showed that RGA is obviously superior to GA and that RGA is an effective method to solve MLLS problem.

This trend is upon believing that this complex problem is really hard to solve with the ordinary approach and it is obvious that the decomposition approach is useful for solving efficiently MLCLSP with extension in real world. Therefore, there is a need to find the right decomposition that still represents the full problem.

2.2.3 Summary of the solution approaches

As the reviews, it's clear that the lot sizing problem is very extensive interest for a tons of researches. Therefore, in this section, the summary of the solution approaches will be shown only for MLCLSP problem and its extensions. The summary of the researches is shown in Table 2.2.

Table 2.2 Summary of the MLCLSP research

Authors	Extension	Structure	Solution Approach
Billington et al. (1983)	Setup time Overtime	General	-
Billington et al. (1986)	Setup time	General	Branch and Bound
Maes et al. (1991)	Setup time Overtime	Serial	Linear Relaxation
Roll et al. (1991)	No lead time and Mono resource	Assembly	Greedy Algorithm
Kuik et al. (1993)	No lead time and Mono resource	Assembly	Simulated Annealing
Tempelmeier et al. (1994)		General	Lagrangean Relaxation
Clark et al. (1995)	Setup time	General	Branch and Cut
Stadler et al. (1996)	Setup time/ Overtime/ No lead time	General	Lagrangean Relaxation
Tempelmeir et al. (1996)	Setup time	General	Lagrangean Relaxation
Franca et al. (1997)	Setup time/ Overtime	General	Improve heuristic
Katok et al. (1998)	Setup time/ Overtime	Assembly	Solver Heuristic
Kimms, A. (1999)	Multi-machine proportional lot sizing/ Scheduling	General	Genetic Algorithm
Barbarosoglu et al. (2000)	Setup time/ Overtime	General	Simulated Annealing
Ozdama et al. (2000)	Setup time/ Overtime	General	Simulated Annealing/Lagrangean Relaxation
Clark (2002)	Setup time/ Backlogging	General	Solver Heuristic
Xie et al. (2002)	Setup time/ Overtime/ No lead time	General	Genetic Algorithm
Chen et al. (2003)	No lead time	General	Lagrangean Relaxation
Beretta et al. (2004)	Setup time/ No lead time	General	Memetic Algorithm

Table 2.2 Summary of the MLCLSP research (cont.)

Authors	Extension	Structure	Solution Approach
Beretta (2005)	Setup Lead time	General	Decomposition, Simulated Annealing/ Tabu Search
Stadtler (2005)	Resource-constrained project-scheduling problem (RCPSP)	General	-
Ornek and Cengiz (2006)	Alternative routings and overtime decisions	General	-
Grubbstrom and Thu Thuy Huynh (2006)	Lead time/ Net present value	General	Dynamic Programming
Shin (2007)	-	Assembly	Decomposition, Relaxation MIP to LP and solved with Tabu search/ Simulated Annealing
Pitakaso, Almeder, Doerner and Hartl (2007)	Ant system	General	Decomposition, Ant colony and Lagrangean Relaxation
Nie, L.,Xu, X. and Zhan, D. (2008)	-	General	Decomposition, Lagrangean Relaxation/Genetic Algorithms
Han et al. (2008)	MLLS	Assembly	Decomposition, Repulsion operator integrated into Genetic Algorithm (RGA)