CHAPTER II

THEROY OF THE HETEROSTRUCTURE BIPOLAR TRANSISTOR

2.1 Terminal currents and gains in BJT [11],[12]

For simplicity, p-n-p transistor is used to study due to the hole current flowing in the same direction as the hole flux. The currents in a BJT can be determined from the solution of the continuity equation. The following assumptions are used to calculate the currents in BJTs.

(1) Drift is negligible in the base region. Holes diffuse from the emitter to the collector.

(2) The emitter current is comprised entirely of holes. This implies that the emitter injection efficiency is unity.

(3) The collector saturation current is negligible.

(4) The current flow in the base can be treated as one-dimensional direction from the emitter to the collector.

(5) All current and voltage are in steady state.

Assumed that the BJT is biased in the normal or active mode with the strongly emitter-base forward biased and strongly collector base reverse biased. Generally, the excess hole carrier concentration on the n-side of the EB junction and on the n-side of the CB junction are given as

$$\Delta P_E = P_B\left(e^{qV_{EB}/kT} - 1\right)$$

$$\Delta P_C = P_B\left(e^{qV_{CB}/kT} - 1\right) \tag{2.1}$$

where, P_B is the equilibrium hole concentration within the n-type base region and the value of P_B can be found in terms of the base donor doping concentration N_{dB} as $p_B = \frac{n_i^2}{N_{dB}}$. If the junctions are strongly forward and reverse biased, Eq. (2.1) becomes

$$\begin{aligned} \Delta p_E &\sim p_B\left(e^{qV_{EB}/kT}\right) \\ \Delta p_C &\sim -p_B \end{aligned} \tag{2.2}$$

The one dimension equation with no drift component is given as,

$$\frac{d}{dt}\delta p = D_B\frac{\partial^2}{\partial x^2}\delta p - \frac{\delta p}{\tau_p} + G_L \tag{2.3}$$

where,

D_B =the hole diffusion constant within the n-type base

δ_p = the excess hole concentration

τ_p = the hole life time

G_L =the generation rate

D and *L* are the diffusion coefficient and diffusion length respectively, subscripted by *E*, *B* or *C* to donate the emitter, base or collector. *D* and *L also correspond to the minority carrier within each of these regions.* Under steady state condition with no illumination Eq. (2.3) becomes

$$\frac{d^2\delta_p}{dx^2} = \frac{\delta_p}{L_B^2} \tag{2.4}$$

where L_B is the hole diffusion length within the base. The general solution of Eq. (2.4) is

$$\delta p = C_1 e^{x/L_B} + C_2 e^{-x/L_B} \tag{2.5}$$

The boundary conditions are applied at the edge of the depletion region of the EB junction within the base, called (x=0) and at the end of the quasineutral base region W_B as shown in Fig. 2.1.The end of the quasineutral base occurs at the edge of the CB depletion region within the base. The boundary conditions are then,

$$\begin{aligned} \delta p(x=0) &= \Delta p_E = C_1 + C_2 \\ \delta p(x=W_B) &= \Delta p_C = C_1 e^{W_B/L_B} + C_2 e^{-W_B/L_B} \end{aligned} \tag{2.6}$$

Solving for the excess hole concentration as a function of *x* yields

$$C_1 = \frac{\Delta p_C - \Delta p_E e^{-W_B/L_B}}{e^{W_B/L_B} - e^{-W_B/L_B}}; C_2 = \frac{\Delta p_E e^{W_B/L_B} - \Delta p_C}{e^{W_B/L_B} - e^{-W_B/L_B}} \tag{2.7}$$

The emitter hole current can now be determined from the relation

$$I_p = -qAD_B \frac{d}{dx} \delta p \tag{2.8}$$

By differentiating Eq. (2.6), we can be written as,

$$\frac{d\delta p}{dx} = \frac{C_1}{L_B} e^{W_B/L_B} - \frac{C_2}{L_B} e^{-W_B/L_B} \tag{2.9}$$

Then, the emitter hole current can be found to be

$$I_{Ep} = I_p(x=0) = \frac{qAD_B}{L_p}(C_2 - C_1) \tag{2.10}$$

and, the collector hole current can be found to be

$$I_{Cp} = I_p(x = W_B) = \frac{qAD_B}{L_B}\left(C_2 e^{-W_B/L_B} - C_1 e^{W_B/L_B}\right) \tag{2.11}$$

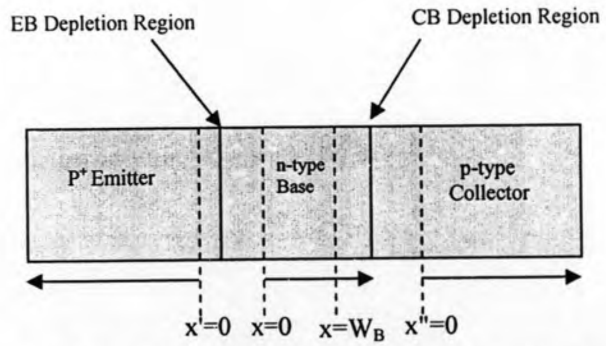

Fig. 2.1 Depletion region and coordinate system used in determining the devices currents

Substituting of the value of C_1 and C_2 in Eq. (2.10) and Eq. (2.11),

$$I_{Ep} = \frac{qAD_B}{L_B}\left(\Delta p_E \coth\frac{W_B}{L_B} - \Delta p_C \operatorname{csch}\frac{W_B}{L_B}\right); I_{Cp} = \frac{qAD_B}{L_B}\left(\Delta p_E \operatorname{csch}\frac{W_B}{L_B} - \Delta p_C \coth\frac{W_B}{L_B}\right) \tag{2.12}$$

And also the base current can be written as,

$$I_B = I_{Ep} - I_{Cp};\ I_B = \frac{qAD_B}{L_B}\left[(\Delta p_E - \Delta p_C)\tanh\frac{W_B}{2L_B}\right] \tag{2.13}$$

It is useful to determine the total currents within the device including the electron component. The total emitter and collector currents are found by adding the electron components to the hole components as

$$I_E = I_{En} + I_{Ep}; \qquad I_C = I_{Cn} + I_{Cp} \tag{2.14}$$

Let us consider the electron component to the emitter current. Generally, the excess electron concentration in the emitter, $\delta n_E\ (x')$ is

$$\delta n_E(x') = A_1 e^{-x'/L_E} \tag{2.15}$$

where, x′ is the coordinate within the emitter as shown in Fig. 2.2 and L_E is the electron diffusion length within the p-type emitter. At (x′=0), the excess electron concentration is given by

$$\Delta n_E = n_E\left(e^{qV_{EB}/kT} - 1\right) \tag{2.16}$$

$$\text{Since, } \delta n_E = \Delta n_E e^{-x'/L_E} = n_E\left(e^{qV_{EB}/kT} - 1\right)e^{-x'/L_E} \tag{2.17}$$

Therefore the electron component of the emitter current is

$$I_{En} = -qAD_E \frac{d}{dx'}\delta n_E\Big|_{x'=0}$$

$$\text{we can be written as, } I_{En} = \frac{qAD_E}{L_E} n_E\left(e^{qV_{EB}/kT} - 1\right) \tag{2.18}$$

$$\text{Similarly, } I_{Cn} = -\frac{qAD_C}{L_C} n_C\left(e^{qV_{CB}/kT} - 1\right) \tag{2.19}$$

where L_C and D_C are the electron diffusion length and electron diffusion coefficient within the collector. The total emitter and collector currents are determined by adding the electron component to the hole components which can be written as

$$I_E = qA\left[\left(\frac{D_E}{L_E}n_E + \frac{D_B}{L_B}p_B \coth\frac{W_B}{L_B}\right)\left(e^{qV_{EB}/kT} - 1\right) - \left(\frac{D_B}{L_B}p_B \csc h\frac{W_B}{L_B}\right)\left(e^{qV_{CB}/kT} - 1\right)\right] \tag{2.20}$$

$$I_C = qA\left[\left(\frac{D_B}{L_B}p_B \csc h\frac{W_B}{L_B}\right)\left(e^{qV_{EB}/kT} - 1\right) - \left(\frac{D_C}{L_C}n_C + \frac{D_B}{L_B}p_B \coth\frac{W_B}{L_B}\right)\left(e^{qV_{CB}/kT} - 1\right)\right] \tag{2.21}$$

To describe the static characteristics of a BJT, four useful quantities are used.

(1) The emitter injection efficiency γ, which is defined as the ratio of the emitter hole current to the total emitter current.

$$\gamma = \frac{I_{Ep}}{I_{Ep} + I_{En}} \tag{2.22}$$

(2) The base transport factor α_T, which is defined as the ratio of the collector hole current to the injected emitter hole current.

$$\alpha_T = \frac{I_{Cp}}{I_{Ep}} \tag{2.23}$$

(3) The common base current gain α_{dc},

$$\alpha_{dc} = \gamma\alpha_T \tag{2.24}$$

(4) The common emitter current gain β_{dc},

$$\beta_{dc} = \frac{\alpha_{dc}}{1 - \alpha_{dc}} \tag{2.25}$$

By assuming that the EB voltage is sufficiently large, the term $e^{qV_{EB}/kT} \gg 1$ and additionally, the CB junction is sufficiently reverse biased such that the second term of Eq. (2.12) can be neglected. This can be written as for I_{Ep}, I_{Cp} and I_{En}

$$I_{Ep} \sim \frac{qAD_B}{L_B}p_B e^{qV_{EB}/kT} \coth\frac{W_B}{L_B} \tag{2.26}$$

$$I_{Cp} = \frac{qAD_B}{L_B}p_B e^{qV_{EB}/kT} \csc h\frac{W_B}{L_B} \tag{2.27}$$

$$I_{En} = \frac{qAD_E}{L_E}n_E e^{qV_{EB}/kT} \tag{2.28}$$

The emitter injection efficiency is then

$$\gamma = \frac{I_{Ep}}{I_{Ep} + I_{En}} = \frac{1}{1 + \frac{D_E n_E L_B}{L_E p_B D_B} \tanh\left(\frac{W_B}{L_B}\right)} \tag{2.29}$$

The base transport factor is $$\alpha_T = \frac{1}{\cosh\left(\frac{W_B}{L_B}\right)} \tag{2.30}$$

The common base current gain is

$$\alpha_{dc} = \gamma\alpha_T = \frac{1}{\cosh\left(\frac{W_B}{L_B}\right) + \frac{D_E L_B n_E}{D_B L_E p_B} \sinh\left(\frac{W_B}{L_B}\right)} \tag{2.31}$$

The common emitter current gain is

$$\beta_{dc} = \frac{1}{\cosh\left(\frac{W_B}{L_B}\right) + \frac{D_E L_B n_E}{D_B L_E p_B} \sinh\left(\frac{W_B}{L_B}\right) - 1} \tag{2.32}$$

If it is assumed that $W_B << L_B$ and can be written as

$$\beta_{dc} = \frac{1}{\left(\frac{D_E n_E W_B}{D_B p_B W_E}\right) + \frac{1}{2}\left(\frac{W_B}{L_B}\right)^2} \tag{2.33}$$

Since, $n_E = \frac{n_{iE}^2}{N_{aE}}$; $\qquad p_B = \frac{n_{iB}^2}{N_{dB}}$

The common-emitter dc current gain can be written in term of the emitter and base doping concentration N_{aE} and N_{dB}.

$$\beta_{dc} = \frac{1}{\frac{D_E N_{dB} W_B}{D_B N_{aE} L_E} + \frac{1}{2}\left(\frac{W_B}{L_B}\right)^2} \tag{2.34}$$

Since, $W_B << L_B$, we can be neglected the last term in the denominator of Eq. (2.34) and which becomes the common-emitter current gain in p-n-p transistor as

$$\beta_{dc} \sim \frac{1}{\left(\dfrac{D_E N_{dB} W_B}{D_B N_{aE} L_E}\right)} \tag{2.35}$$

According to above equation, β_{dc} depends on the ratio of the base to the emitter diffusion constant, emitter diffusion length to base width and the emitter to the base doping concentrations. To the first order of approximation, the ratios of the diffusion constants and emitter diffusion length to base width are about one. The factors thus most influence for β_{dc} are the doping concentrations within the emitter and base regions. In order to get high gain, emitter doping must be higher than the base doping. Higher emitter doping leads to higher emitter capacitance whereas the low base doping will result in increasing the base resistance. Therefore, a high ratio of emitter-to-base doping concentration resulting in a reduction in frequency performance.

The maximum frequency of operation $f_{\max}$ is one of the important figures of merit in the RF performance and is defined as the frequency at the unilateral power gain of the transistors goes to one. The most common formula for $f_{\max}$ is given as

$$f_{\max} = \sqrt{\frac{f_t}{8\pi 4 r_{bb} C_C}} \tag{2.36}$$

where, f_t is the cut-off frequency, r_{bb} is the base resistance and C_C is the collector base junction capacitance. Since $f_{\max}$ is inversely proportional to the square root of the base resistance. To get the high frequency, the base resistance must be reduced with the high base doping which contradict to the requirement mentioned above to get higher gain.

2.2 Emitter-Base Heterojunction Bipolar Transistors (HBTs)

The insertion of wide- bandgap semiconductor materials for the emitter enables the use of a higher base doping concentration without compromising the dc common-emitter current gain. Thus high gain and high frequency performance can be obtained simultaneously. Bipolar junction transistors that utilize a wider-bandgap material to form the emitter region are called heterostructure bipolar transistors (HBTs).The energy band diagram of GaAlAs/GaAs HBT in equilibrium and under bias are shown in Fig. 2.2.The heterojunction is assumed to be abrupt and the device type is n-p-n, implying that the emitter is doped n-type, the base p-type and the collector n-type. An n-p-n transistor is used here instead of the p-n-p in section 2.1, since n-p-n HBTs are far more commonly employed. This is due to the fact that n-p-n HBTs offer far superior frequency performance than p-n-p devices due to the much higher velocity of the electrons.

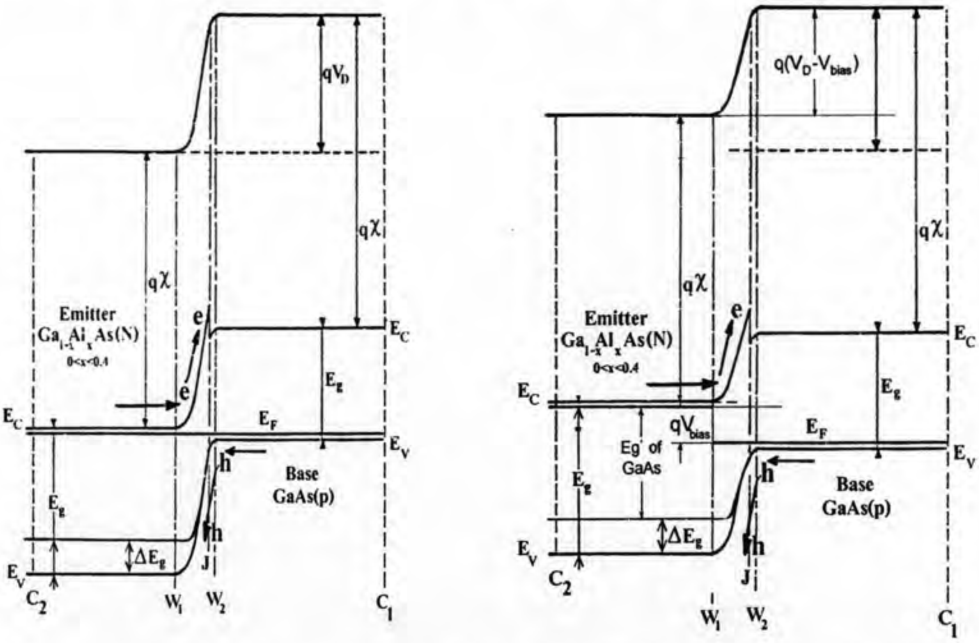

Fig. 2.2 Emitter-Base Heterojunction (left) in equilibrium (Right) under active biasing

Examination of Fig. 2.2 shows that the energy barrier for electron injection from the emitter into the base is significantly less than the corresponding energy barrier for hole injection from the base into the emitter. Notice that the band bending is the same for both the conduction and valance bands within the emitter, but the valance band edge discontinuity adds to the energy barrier for the holes. Thus hole injection from the base into the emitter requires a higher energy to surmount the potential barrier at the emitter-base junction than electron injection from the emitter into the base. Under active biasing conditions, electrons are more readily injected from the emitter into the base than holes from the base into the emitter. The base current due to hole injection is reduced significantly without compromising the emitter current. Therefore, the base doping concentration can be increased without altering the common-emitter current gain, so high frequency performance can be maintained at high β_{dc}.

The actual common-emitter current gains for a HBT and a BJT can be compared as follows. For n-p-n BJT the simplified value of β_{dc} is given as

$$\beta_{dc} \sim \frac{D_B L_E N_{dE}}{D_E W_B N_{aB}} \tag{2.37}$$

Where N_{dE} is the emitter donor concentration and N_{aB} is the base acceptor concentration. Eq. (2.37) is not mainly applied to BJTs but also graded HBTs which the emitter-base junction is graded, diffusion dominates the current flow. But, in abrupt HBTs, the current flows with a strong thermionic emission, hence Eq. (2.37) can not be applied. For n-p-n transistor, β_{dc} can be written in terms of the intrinsic carrier concentration in the emitter and base regions as follow

$$\beta_{dc} = \frac{1}{\left(\frac{D_E W_B p_E}{D_B L_E n_B}\right) + \frac{1}{2}\left(\frac{W_B}{L_B}\right)^2}$$

If W_B is much less than L_B, as before.

$$\beta_{dc} \sim \frac{D_B L_E n_B}{D_E W_B p_E} \tag{2.38}$$

Since $n_B = \frac{n_{iB}^2}{N_{aB}}; p_E = \frac{n_{iE}^2}{N_{dE}}$, Eq. (2.38) becomes

$$\beta_{dc} = \frac{D_B L_E N_{dE}}{D_E W_B N_{aB}} \frac{n_{iB}^2}{n_{iE}^2} \tag{2.39}$$

From Eq. (2.39), the base doping concentration can be larger than the emitter doping concentration and the gain can still be relatively large. The typical doping concentration of GaAlAs/GaAs HBTs is shown in Fig. 2.3.

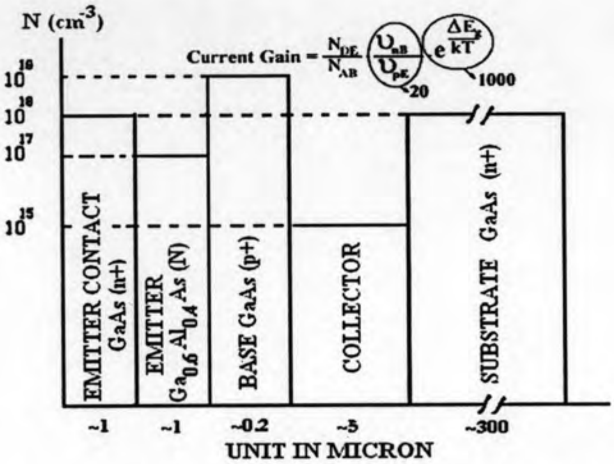

Fig. 2.3 Typical doping concentration of GaAlAs/GaAs HBTs

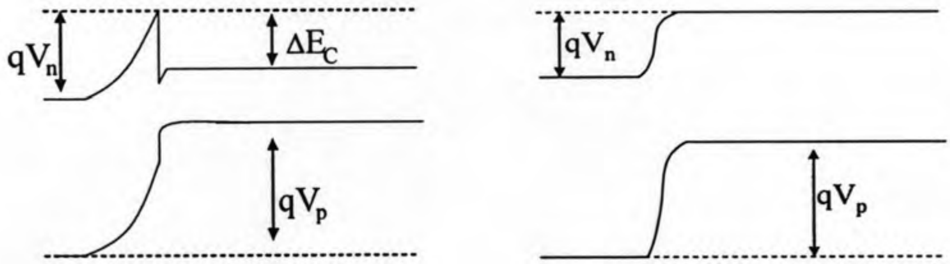

Fig. 2.4 Abrupt (Left)and graded (Right) junction of n-p-n BJT band diagram

Moreover, HBT devices can be made either an abrupt or graded heterojunctions to form the emitter-base junction as seen in Fig. 2.4. In an abrupt HBT, the wider band gap emitter is directly contact to the narrow band-gap base but in the case of a graded HBT, the wider band gap is gradually contact to the base with compositional grading. The energy differences between on each side of the abrupt junction is given as

$$qV_P + Eg_2 + \Delta E_C = qV_n + Eg_1$$

$$q(V_p - V_n) = \Delta E_g - \Delta E_C$$

$$q\left(V_p - V_n\right) = \Delta E_g - \Delta E_C = \Delta E_v \quad \text{(For abrupt HBT)}$$

Similarly, we can be described for the energy differences between on each side of the graded junction as follow

$$qV_n + E_{g1} = qV_p + E_{g2}$$

$$q\left(V_p - V_n\right) = \Delta E_g \qquad \text{(For graded HBT)}$$

By using electron and hole current density equation proposed by Kroemer (1982) which is expressed in terms of potential energy barrier as

$$J_n = N_{dE} v_n e^{-qV_n/kT}; J_p = N_{aB} v_p e^{-qV_p/kT}$$

Where, v_n and v_p *are the mean electron and hole velocity,* if we assumed the base transport factor is one, the collector and emitter currents are equal. The maximum current gain can be obtained as

$$\beta_{\max} = \frac{J_n}{J_p} = \frac{N_{dE} v_n}{N_{aB} v_p} e^{\left(q/kT\right)\left(V_p - V_n\right)}$$

$$\beta_{\max} = \frac{N_{dE} v_n}{N_{aB} v_p} e^{\Delta E_V/kT} \qquad \textbf{(Abrupt)}$$

$$\beta_{\max} = \frac{N_{dE} v_n}{N_{aB} v_p} e^{\Delta E_g/kT} \qquad \textbf{(Graded)}$$

Notice that, β_{max} of graded HBT is greater than that of abrupt due to ΔE_g - ΔE_C = ΔE_V. Hence, ΔE_g is always larger than ΔE_V for every pair of heterojunction devices.

2.3 Determination of offset voltage ($V_{CE,\,offset}$)

Generally, the simple diode current equations of the EB junction and CB junction can be written as

$$I_E = I_S\left(e^{qV_{BE}/kT} - 1\right) \quad \text{(For E-B junction)}$$

$$I_C = -I_S\left(e^{qV_{BC}/kT} - 1\right) \quad \text{(for C-B junction)}$$

Where current direction and voltage polarity are defined in Fig. 2.5, "I_S" is the reverse saturation current. In case of normal mode operation as shown in Fig. 2.6-a, the normal mode emitter current (I_{EN}) can be written as

$$I_{EN} = I_{ES}\left(e^{qV_{BE}/kT} - 1\right) \tag{2.40}$$

where I_{ES} is the magnitude of the emitter saturation current in the normal mode with the collector junction short circuited. Similarly, the collector current in the inverted mode as shown in Fig. 2.5-b, can be written as

$$I_{CI} = -I_{CS}\left(e^{qV_{BC}/kT} - 1\right) \tag{2.41}$$

where I_{CS} is the magnitude of the collector saturation current with $V_{EB} = 0$.

The corresponding collected currents for each mode of operation can be written by defining a new "α" for each case:

$$I_{CN} = \alpha_F I_{EN} = \alpha_F I_{ES}\left(e^{qV_{BE}/kT} - 1\right) \tag{2.42}$$

$$I_{EI} = \alpha_R I_{CI} = -\alpha_R I_{CS}\left(e^{qV_{BC}/kT} - 1\right) \tag{2.43}$$

Where α_F and α_R are the ratio of collected current to injected current in each mode. Notice that in the inverted mode, the injected current is I_{CI} and the collected current is I_{EI}.

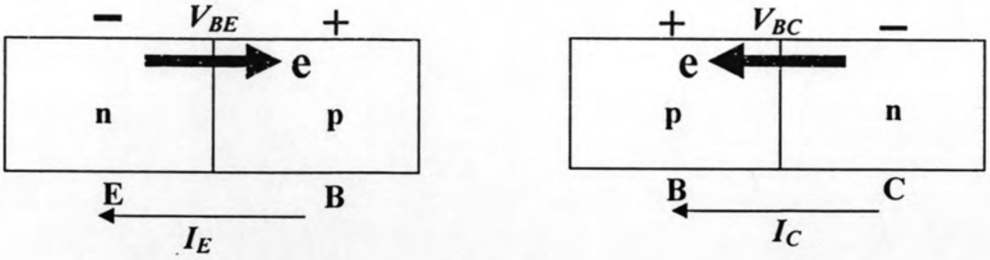

Fig. 2.5 Current direction and voltage polarity definitions

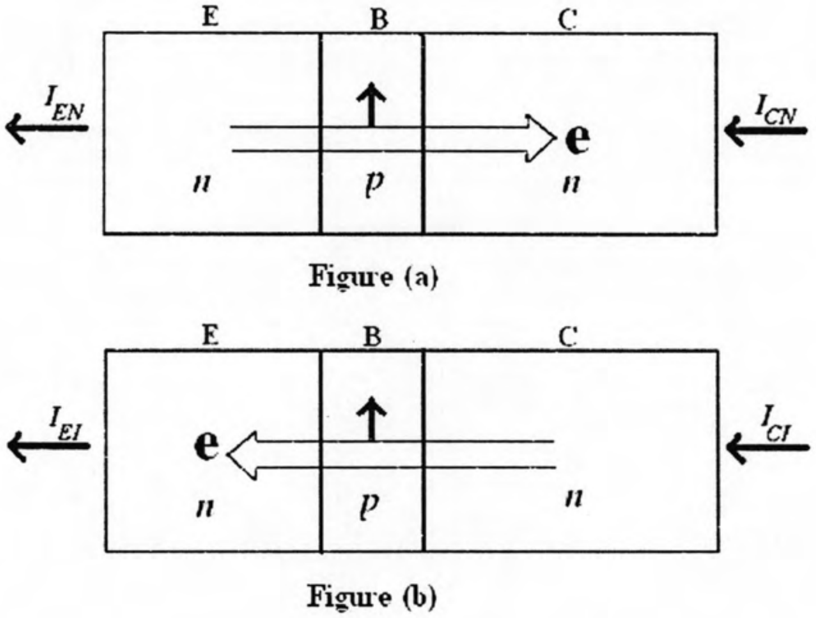

Fig. 2.6 Carrier injection and collection in each mode (a) normal mode (b) inverted mode

Therefore, the total current for each mode can be obtained by superposition of the components. [11],[12]

$$I_E = I_{EN} + I_{EI} = I_{ES}\left(e^{qV_{BE}/kT} - 1\right) - \alpha_R I_{CS}\left(e^{qV_{BC}/kT} - 1\right) \tag{2.44}$$

$$I_C = I_{CN} + I_{CI} = \alpha_F I_{ES}\left(e^{qV_{BE}/kT} - 1\right) - I_{CS}\left(e^{qV_{BC}/kT} - 1\right) \tag{2.45}$$

In diode equations (2.44) and (2.45), we assumed that the voltage V_{BE} and V_{BC} are entirely across the junction. However, practical devices do exhibit ohmic effect which in turn causes the deviation of I_E and I_C to be:

$$\begin{gathered} I_E = A_E J_{ES} \exp[q(V_{BE} - I_E R_E - I_B R_B)/k_B T] - \\ \alpha_R A_C J_{CS} \exp[q(V_{BC} - I_B R_B - I_C R_C)/k_B T] \end{gathered} \tag{2.46}$$

$$\begin{gathered} I_C = \alpha_F A_E J_{ES} \exp[q(V_{BE} - I_E R_E - I_B R_B)/k_B T] - \\ A_C J_{CS} \exp[q(V_{BC} - I_B R_B - I_C R_C)/k_B T] \end{gathered} \tag{2.47}$$

Where, J_{ES} and J_{CS} are current densities, R_E, R_B, *and* R_C are emitter, base and collector series resistances, and A_E and A_C are emitter and collector areas respectively. The expression for the offset voltage therefore can be determined from Eq. (2.46) and Eq. (2.47) by setting $I_C = 0$ [13], as follow,

$$V_{CE,offset} = R_E I_B + \frac{k_B T}{q}\ln\left(\frac{A_C}{A_E}\right) + \frac{k_B T}{q}\ln\left(\frac{J_{CS}}{\alpha_F J_{ES}}\right) \tag{2.48-a}$$

According to $\alpha_N J_{ES} = \alpha_R J_{CS}$ for every transistor, $V_{CE,\,offset}$ can then be written in term of common base inverted mode gain, α_R :

$$V_{CE,offset} = R_E I_B + \frac{k_B T}{q}\ln\left(\frac{A_C}{A_E}\right) + \frac{k_B T}{q}\ln\left(\frac{1}{\alpha_R}\right) \tag{2.48-b}$$