CHAPTER I

Introduction

The mathematical models of many problems and phenomena in the real world can be described by impulsive differential equations. They have been studied quite extensively because they have advantage over the traditional initial value problems. They can be used to model other phenomena that can not be modeled by the traditional initial value, such as the dynamics of the systemic arterial pressure[1], the dynamics of populations subjected to abrupt changes (harvesting, diseases, etc.).

Recently, impulsive systems on an infinite dimensional Banach space have been considered in several papers. X.Xiang and Wei Wei[13] proved mild solution for a class of nonlinear impulsive evolution inclusion on a Banach space, Jame H.Liu [8] proved the mild and classical solution for nonlinear impulsive differential equations and Chonwerayote[4] proved the existence and uniqueness of a classical solution of an integro-differential equation by using fractional powers of operators and analytic semigroups.

In this paper, we consider the nonlinear impulsive differential system:

$$\begin{cases} x'(t) + Ax(t) = f(t, x(t)), \quad t \neq t_i \ ; \\ \Delta x(t_i) = J_i(x(t_i)), \quad t = t_i, \ i \in \mathbb{N} \ ; \\ x(t_0) = x_0 \end{cases} \tag{1.1}$$

where $t_{i-1} < t_i$, $i \in \mathbb{N}$, $-A$ is the infinitesimal generator of an analytic semigroup $\{S(t)\}_{t\geq 0}$ satisfying the exponential stability i.e., $\|S(t)\| \leq Me^{-\beta t}$, $t \geq 0$ for some positive constant M, $\beta > 0$. Let $f : \Re \times \Omega \to X$, $\Omega \subseteq X$ be a uniformly piecewise continuous almost periodic (UPCAP) function and $J_i : X \to X$ be uniformly almost periodic in i, $\Delta x(t_i) = x(t_i^+) - x(t_i^-) = x(t_i^+) - x(t_i)$ presents the jump in

the state x at $t = t_i$ with $J_i(i \in \mathbb{N})$ determining the size of the jump at $t = t_i$. Traditional initial value problems are replaced by the impulsive conditions.
We will prove the existence and uniqueness of solution by using a tool called fractional powers of operators. More precisely, we assume that the function $f : \Re \times X_\alpha \to X$ and $J_i(i \in \mathbb{N}) : X_\alpha \to X$ are satisfying the condition:
(JF) There are constants $L > 0$ and $0 < \theta < 1$ such that

$$|f(t_1, x_1) - f(t_2, x_2)| \leq L(|t_1 - t_2|^\theta + |x_1 - x_2|_\alpha)$$

and

$$|J_i(x_1) - J_i(x_2)| \leq L\,|x_1 - x_2|_\alpha$$

for all $x_1, x_2 \in X_\alpha$ and $t_1, t_2 \in \Re$
where X is a real or complex Banach space with norm $|.|$, A^α is the fractional power and X_α is the Banach space $D(A^\alpha)$ endowed with the norm $|x|_\alpha = |A^\alpha x|$.

1.1 Scope

Firstly, we consider a class of impulsive system:

$$\begin{cases} u'(t) + Au(t) = g(t), & t \neq t_i \ ; \\ \Delta u(t_i) = h_i(t_i), & t = t_i, \ i \in \mathbb{N} \ ; \end{cases} \tag{1.2}$$

Assume that $g : \Re \to X$ is piecewise continuous almost periodic (PCAP) and $h_i(i \in \mathbb{N}) : \Re \to X$ is almost periodic in i. We show that if g and $h_i(i \in \mathbb{N})$ are both locally Hölder continuous on $\Re$ then there exists a unique classical PCAP solution over $\Re$ of the system (1.2) and moreover, the solution is

$$u(t) = \int_{-\infty}^{t} S(t-s)g(s)ds + \sum_{t_i < t} S(t - t_i)h_i(t_i).$$

Secondly, we consider system(1.1) as $t_0 \to -\infty$ and prove that the map

$$Fx(t) = \int_{-\infty}^{t} S(t-s)f(s, A^{-\alpha}x(s))ds + \sum_{t_i < t} S(t - t_i)J_i(A^{-\alpha}x(t_i))$$

for each $x \in PCAP(X)$ where $PCAP(X)$ is a space containing all PCAP functions in X, is a strictly contraction. Then we prove the existence and uniqueness of a classical PCAP solution of system (1.1). Our main theorem, we consider the PCAP solution instead of PCAP mild solution.

Our work is organized as follows: Chapter 2 is devoted to a review of preliminary; semigroups, analytic semigroups, some results on the fractional powers of operator, basic knowledge of almost periodic and PCAP, impulsive differential equations and definitions of stability. In Chapter 3, we state and prove our main results. Chapter 4, we give some examples satisfying our main results and the last Chapter is devoted to concluding our thesis and give an idea to develop and extend this work to the other problems.

1.2 Research objectives

1) To prove existence and uniqueness of piecewise almost periodic mild solution of (1.1).

2) To prove regularity and also continuous dependence of the solution.

3) To investigate asymptotic stability of PCAP solution.