# **CHAPTER IV**

# **Empirical Result**



This study estimates the term structure of interest rates in Thailand by using two leading single factor model: Vasicek and CIR model in three aspects. First, the mean of speed of mean reversion is greater than zero since there is the equilibrium borrowing and lending adjustment in economic system. Second, The CIR model outperforms the Vasicek model in term of goodness of fit since the volatility of spot rate is not constant over time. Lastly, the term structure of interest rates from the CIR model can be a better benchmark in bond trading than the Vasicek model since the CIR model has better pricing performance. The results of the research are presented in this chapter as follow:

# 4.1 Properties of the Term Structure of Interest Rates in Thailand over the Period 1999-2003

In this section, the methods from Brown and Dybvig (1986) and Munnik and Schotman (1994) are applied to investigate the properties, especially, the mean reversion property of the term structure of interest rates in the study period.

The result of the 60 monthly estimates are summarized in Table III which presents mean value, maxima and minima of the estimated and implied parameters for the Vasicek model (Panel A) and the CIR model (Panel B).

From Table III, the mean reversion parameter ( $\kappa$ ) behaves erratically; it is usually very different from month to month. The average of mean reversion of the Vasicek model is less than the CIR model and also the standard deviation of this parameter. However, the mean reversion parameter has positive value ( $\kappa > 0$ ) which implies that there is speed of short rate that tend toward a long term rate while an economic shock occurs.

#### Table III

#### **Cross-sectional Estimation of Term Structure of Interest Rates**

This table presents mean value, maxima and minima of the estimated an implied parameters for the Vasicek model (Panel A) and the CIR model (Panel B) over the sample period. The sample period is from January 1999 to December 2003, a total of 60 monthly cross-sections. R is the yield of long term bonds. k is known as the mean reversion. The volatility of change in r (short rate) is  $\sigma$  in the Vasicek model but  $\sigma \sqrt{r}$  in the CIR model. T statistics are in parenthesis.

|             | R            | κ           | σ           |
|-------------|--------------|-------------|-------------|
| (A) Vasicek |              |             |             |
| Mean        | 0.1349       | 0.1317      | 0.0151      |
|             | (13.2829)*** | (7.8883)*** | (6.8415)*** |
| Max         | 0.5910       | 0.5550      | 0.0522      |
| Min         | 0.0230       | 0.0007      | 5.5E-06     |
| SD.         | 0.0786       | 0.1293      | 0.0171      |
| (B) CIR     |              |             |             |
| Mean        | 0.1252       | 0.1725      | 0.1254      |
|             | (21.3521)*** | (9.7014)*** | (8.6078)*** |
| Max         | 0.3146       | 0.5733      | 0.2313      |
| Min         | 0.0757       | 0.0039      | 0.0199      |
| SD.         | 0.0454       | 0.1377      | 0.0677      |

\*\*\* Significant at 1% level

For the yield curve the in long term yield (R) is also estimated. The CIR model produces the mean of long term yield and standard deviation less than the Vasicek model.

The volatility parameter ( $\sigma$  or  $\sigma\sqrt{r}$ ) behaves erratically over time for the CIR model with higher standard deviation than the Vasicek model. The CIR model produces larger scale in term of mean and standard deviation than the Vasicek model. From Table III, the CIR model gives the mean of volatility of the short rate equal to the 12.54% while the mean of the Vasicek model equal to 1.51%. Also, the CIR model produces the larger the standard deviation of the Vasicek model.

This result is differing from the work of De Munnik and Schotman (1994). De Munnik and Schotman (1994) found that the CIR volatility is almost zero while the volatility for the Vasicek model has the higher volatility as the below table. It is due to the lower estimates of mean reversion parameter; with a small mean reversion volatility matters for the shape of term structure.

|             | R      | κ      | σ      |
|-------------|--------|--------|--------|
| (A) Vasicek |        |        |        |
| Mean        | 0.2194 | 0.0012 | 0.0284 |
| Max         | 1.0723 | 0.0049 | 0.1027 |
| Min         | 0.0696 | 0.0001 | 0.0022 |
| SD          | 0.2675 | 0.0013 | 0.0261 |
|             |        |        |        |
| (B) CIR     |        |        |        |
| Mean        | 0.0771 | 0.0019 | 0.0356 |
| Max         | 0.2068 | 0.0126 | 0.4271 |
| Min         | 0.0178 | 0.0001 | 0.0005 |
| SD          | 0.0391 | 0.0032 | 0.1119 |

The scatter diagrams of Figure V in panel A- D show some extreme outlier in the parameter estimates.

Panel A shows the similarity of the Vasicek and the CIR model. Both models produce the same upward outliers. But the performance of the Vasicek model in term of goodness of fit is found to be strongest when the estimated value around the dash 45 degree line. This result consistence with the average of mean reversion of the Vasicek model is less than the CIR model and also is the standard deviation of this parameter in Table III.

The frequent occurrence of the outliers the mean and the standard deviation of the long term yield (R) over the study period are not very informative as shown in Figure IV in panel B. One interpretation of this result is that the longest bond in our sample only had maturity of 15 years and was inactively traded. This can be too small to be representative of a very long term bond. However, the long term yield that the CIR model estimated has less standard deviation than the Vasicek model.



Figure V Scatter diagram of the monthly estimated structural parameters of the Vasicek and the CIR model. The values for the CIR model are shown on the x-axis, while the corresponding values for the Vasicek model are shown on the Y-axis. Panel A, B, C and D shows the mean reversion, Long Term Yield, Volatility and RMSE respectively.

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In Panel C, the estimated volatility is scattered along the horizontal axis, meaning that the Vasicek volatility is lower value and lower standard deviation than the CIR volatility.

As Figure V in panel D demonstrates that the standard error or RMSE is smaller for the CIR model than the Vasicek model same as Munnik and Schotman (1994) and Sercu and Wu (1997). The outlier in the figure is the first year of the sample, which the bonds were not actively traded and the longest maturity traded bonds is about 10 years. For the next four years, the models are statistically indistinguishable.

Overall the sample period, the CIR model marginally outperforms the Vasicek model in term of goodness of fit. The standard error of the regression is somewhat smaller for the CIR model as shown in Figure IV in panel D.

The estimated parameters in each month provide the information of the yield curve characteristics. The result of different estimated parameters gives the different shape of yield curve in each period. The yield curve comparison between two competing models was shown in Figure VI.

The main difference of the yield curve in each panel is the liquidity of the Thai bond market in each period. Panel A, C and E represent the shape of yield curve in the illiquidity period. Since the trading values in these panels are the minimum monthly trading value in 2001, 2002 and 2003 respectively. On the other hand, Panel B, D and F exhibit the yield curve in liquidity period. The trading values in these panels are the maximum monthly trading value in 2001, 2002 and 2001, 2002 and 2003 respectively.

The figure shows an upward sloping of spot rate curve at the sample month. The upward sloping yield curve means that the direction of market interest rate will increase which indicates that the premium is high and investors are not too happy to hold a long dated bond.



**Figure VI The fitted curve implied by the two estimation methods**. This figure depicts the Thai term structure of interest rates for representative month of the sample. Each panel plots the yield curve estimated by the Vasicek model and the CIR model. Panel A, C and E represent the illiquidity market while panel B, D and F represent the liquidity market in each year

Both the Vasicek and CIR model provides quite a different yield curve in case of the illiquidity period as in panel A, C and E. The reason might be a lack of traded bonds leading to an estimated parameter, which is not accurate. Whereas in panel B, D and F the spot rate curve from the Vasicek model is similar to the CIR model since the number of trading bonds increases and the estimated parameter is more fitted.

From the result it is likely to prefer one term structure model to the other. If the parameters are revised every month, CIR models perform better than. Since the cross sectional fit is so good, more general models with more than one factor will lead to greater estimation difficulties. The simple one factor models already fit, given transaction costs.

### 4.2 Performance of Pricing

One major application of the Vasicek and CIR models is in the valuation of bonds and other financial assets. To compare both models with the bond valuation, the study computed the model price and compare with the real trading price, based on the estimated monthly parameters.

Deviations between actual price and model price can be analyzed longitudinally, i.e. per asset rather than per cross section, so as to verify whether or not the model consistently misprices some individual bond. Table IV reports the mean absolute percentage error (MAPE) in panel A. For panel B the errors are described by the root mean squared error (RMSE).

There is no surprising result since the MAPE of the CIR model is smaller than Vasicek model by across function. The average of MAPE of all samples for the CIR model is 0.62 % and for the Vasicek model are 0.67%. The standard deviation of CIR model is 0.90% which is also less than the standard deviation of the Vasicek model 1.16 %.

#### Table IV

#### Cross-sectional analysis of term structure of in sample

This table reports the mean absolute percentage error (MAPE) which is useful in examining the error magnitude without regard to the distribution of error in panel A. For panel B, the errors are described by the root mean squared error (RMSE). This table reports the mean, maximum, minimum and standard deviation values across both functions. All reported values are expressed in terms of percentage. The smallest of average MAPE and RMSE reveals the best function in term of goodness of fit. T statistics represent that the average MAPE and RMSE of the CIR model is less than the Vasicek model.

| Panel A: Mea | an Absolute Percentage Err | ror (MAPE) |              |
|--------------|----------------------------|------------|--------------|
|              | Vasicek                    | CIR        | t-statistics |
| Mean         | 0.6783                     | 0.6247     | 3.68***      |
| Max          | 23.4353                    | 17.4294    |              |
| Min          | 7.6E-11                    | 1.4E-09    |              |
| SD.          | 1.1612                     | 0.9045     |              |
|              |                            |            |              |
| Panel B: Roo | t Mean Squared Error (RN   | 1SE)       |              |
|              | Vasicek                    | CIR        | t-statistics |
| Mean         | 150.6323                   | 125.0174   | 15.62***     |
| Max          | 2594.6323                  | 1992.0012  |              |
| Min          | 8.3E-09                    | 1.3E-07    |              |
| SD.          | 129.7802                   | 103.1381   |              |

\*\*\* Significan at 1% level

Similarly, the root mean squared error (RMSE) provides the best fit for the CIR model by cross function and also per data point. The mean of RMSE of all samples for the CIR model is 125.01 %. Also the standard deviation of the CIR model equals to 103.13 % which less than the standard deviation of the Vasicek model (129.78 %).

This result is similar to the result of Sercu and Wu(1997). During the sample period, the CIR model is marginally outperforms the Vasicek model in term of goodness of fit. Since the RMSE is smaller for CIR model (0.12%) than the Vasicek model (0.13%). Also, the residual RMSE produced by both model is already low relative to De Munnik and Schotman (1994) and result in this study. However, the average of RMSE in this result still high when compare to both previous work. The one reason of difference between their and our result is the shape of yield curve. The yield curves obtained by De Munnik and Schotman are almost flat, Sercu and Wu have steeply declining and hump curve. While the yield curves in this study have upward sloping.

While Table V gives the pricing performance of competing yield curve estimation techniques for different maturity range, this table classifies each bond in the sample into categories depending on their maturity.

#### Table V

**Cross-sectional analysis of term structure of in sample classified by time to maturity** This table gives the pricing performance of competing yield curve estimation techniques in terms of explaining variation in the market price. This table classifies each bond in the sample into categories depending upon their time to maturity. Panel A presents mean absolute percentage error while panel B presents root mean squared error for both model. All reported values are expressed in term of percentage. The smaller magnitude of average error is the superior model. T statistics represent that the average MAPE and RMSE of the CIR model is less than the Vasicek model.

Panel A: Mean Absolute Percentage Error (MAPE) for each time to maturity range

|              | <lyr< th=""><th>1Yr-5Yr</th><th>5Yr-10Yr</th><th>11-15Yr</th><th>&gt;15Yr</th></lyr<> | 1Yr-5Yr | 5Yr-10Yr | 11-15Yr | >15Yr   |
|--------------|---|---------|----------|---------|---------|
| (A) Vasicek  |   |         |          |         |         |
| Mean         | 0.0803  | 0.7769  | 1.0646   | 1.6695  | 2.4953  |
| Max          | 5.0807  | 23.4353 | 19.3363  | 19.3363 | 17.4294 |
| Min          | 3.9E-05   | 4.5E-10 | 1.2E-09  | 7.6E-11 | 4.4E-10 |
| SD.          | 0.2087  | 0.7446  | 1.3366   | 2.3120  | 2.3767  |
|              |   |         |          |         |         |
| (b) CIR      |   |         |          |         |         |
| Mean         | 0.0791  | 0.7564  | 0.9274   | 1.5126  | 2.3414  |
| Max          | 5.1729  | 7.8756  | 15.2606  | 13.9198 | 17.4294 |
| Min          | 3.1E-05   | 2.1E-07 | 1.4E-09  | 3.2E-09 | 2.4E-06 |
| SD.          | 0.2059  | 0.6227  | 0.8806   | 1.5757  | 2.4462  |
| t-statistics | 0.38  | 1.65*   | 6.11***  | 1.99**  | 0.83    |

Panel B: Root Mean Squared Error (RMSE) for each time to maturity range

|              | <1Yr     | lYr-5Yr   | 5Yr-10Yr  | 11-15Yr   | >15Yr     |
|--------------|----------|-----------|-----------|-----------|-----------|
| (A) Vasicek  |          |           |           |           |           |
| Mean         | 22.6970  | 120.9280  | 193.8818  | 312.3225  | 389.4038  |
| Max          | 528.3484 | 2594.6323 | 1959.6945 | 2019.8285 | 1992.0027 |
| Min          | 0.0039   | 5.1E-08   | 1.3E-07   | 8.3E-09   | 4.7E-08   |
| SD.          | 21.1853  | 83.7243   | 151.2831  | 247.6948  | 270.7607  |
|              |          |           |           |           |           |
| (b) CIR      |          |           |           |           |           |
| Mean         | 22.3652  | 111.1697  | 146.5451  | 246.4268  | 388.1682  |
| Max          | 537.9374 | 765.8837  | 1950.1064 | 1782.0308 | 1992.0012 |
| Min          | 0.0030   | 2.3E-05   | 1.3E-07   | 3.6E-07   | 0.0003    |
| SD.          | 20.8811  | 71.4812   | 101.2127  | 174.7662  | 283.5260  |
| t-statistics | 0.57     | 1.88**    | 6.91**    | 1.76**    | 0.03      |

\*\*\* Significant at 1% level

\*\* Significant at 5% level

\* Significant at 10% level

The magnitude of errors and standard deviation is generally smaller at the short end and increase at the long end. The result observes that the CIR model is superior to the Vasicek model both in MAPE and RMSE value. However, Ioannides (2001) recommended that the result of long maturity issues should be interpreted with caution. There is a danger of overfitting the long end of the term structure due to lack of data at the long end.

For the out of sample result, Table VI presents results from pricing bonds that are excluded from the estimation procedure.

## Table VI Cross-sectional analysis of term structure of out sample

This table reports the mean absolute percentage error (MAPE) in panel A and the root mean squared error (RMSE) in panel B. The data covers period form February of 1999 to January of 2003. This table reports the mean, maximum, minimum and standard deviation values across both functions. All reported values are expressed in terms of percentage. T statistics represent that the average MAPE and RMSE of the CIR model is less than the Vasicek model.

|                                    | Vasicek  | CIR   | t-statistics             |  |
|------------------------------------|--|---|--------------------------|--|
| Mean                               | 1.2824   | 0.9859  | 9.50***                  |  |
| Max                                | 51.9384  | 20.4034   |                          |  |
| Min                                | 3.6E-05  | 3.8E-08   |                          |  |
| SD                                 | 2 6611   | 1 8058  |                          |  |
| Panel B: Roc                       | t Mean Squared Error (RN   | 1.8058<br>(ASE)                                 |                          |  |
| Panel B: Roc                       | ot Mean Squared Error (RN<br>Vasicek   | MSE)<br>CIR                                     | t-statistics             |  |
| Panel B: Roc<br>Mean               | ot Mean Squared Error (RM<br>Vasicek<br>327.0708                                   | ASE)<br>CIR<br>222.5667                         | t-statistics<br>30.62*** |  |
| Panel B: Roc<br>Mean<br>Max        | t Mean Squared Error (RN<br>Vasicek<br>327.0708<br>5378.3584                       | ASE)<br>CIR<br>222.5667<br>2598.4383            | t-statistics<br>30.62*** |  |
| Panel B: Roc<br>Mean<br>Max<br>Min | 2.0011<br>ot Mean Squared Error (RN<br>Vasicek<br>327.0708<br>5378.3584<br>3.6E-03 | ASE)<br>CIR<br>222.5667<br>2598.4383<br>4.3E-06 | t-statistics<br>30.62*** |  |

\*\*\* Significant at 1% level

Table VI shows the performance of forecasting bond price. The Vasicek model produces a much higher pricing error (1.28%) than the corresponding CIR model (0.98%) because the volatility and standard error of the CIR model is very small compared to the Vasicek model as shown in Figure IV.

If bonds are arranged by maturity for each of the competing model, Table VII exhibits the pricing performance of competing yield curve estimation technique in term of variation in market price.

#### Table VII

**Cross-sectional analysis of term structure of out of sample classified by time to maturity** This table classifies each bond in the sample into categories depending upon their time to maturity. Panel A presents the mean absolute percentage error while panel B presents root mean squared error for both functions. The smaller magnitude of average error is the superior model.

|              | <lyr< th=""><th>lYr-5Yr</th><th>5Yr-10Yr</th><th>11-15Yr</th><th>&gt;15Yr</th></lyr<> | lYr-5Yr | 5Yr-10Yr | 11-15Yr  | >15Yr   |
|--------------|---|---------|----------|----------|---------|
| (A) Vasicek  |   |         |          |          |         |
| Mean         | 0.0792  | 1.1755  | 2.3133   | 3.7649   | 4.9103  |
| Max          | 5.1026  | 17.9135 | 42.7232  | 51.9384  | 25.7857 |
| Min          | 3.6E-05   | 0.0001  | 0.0018   | 0.0068   | 0.0157  |
| SD.          | 0.2066  | 1.6042  | 3.6487   | 4.5584   | 3.6921  |
| (b) CIR      |   |         |          |          |         |
| Mean         | 0.0644  | 1.0012  | 1.9222   | 2.1220   | 2.7208  |
| Max          | 2.8987  | 12.3451 | 17.3110  | 20.4034  | 14.7161 |
| Min          | 7.0E-06   | 3.8E-08 | 0.0001   | 0.0017   | 0.0040  |
| SD.          | 0.0849  | 1.2961  | 2.2051   | 3.3248   | 3.1993  |
| t-statistics | 5.78***   | 6.63*** | 6.57***  | 10.49*** | 8.28*** |

Panel A: Mean Absolute Percentage Error (MAPE) for each time to maturity range

Panel B: Root Mean Squared Error (RMSE) for each time to maturity range

|              | <1Yr     | lYr-5Yr   | 5Yr-10Yr  | 11-15Yr   | >15Yr     |
|--------------|----------|-----------|-----------|-----------|-----------|
| (A) Vasicek  |          |           |           |           |           |
| Mean         | 22.4337  | 222.6339  | 475.4776  | 656.1094  | 690.9876  |
| Max          | 530.6243 | 1973.9403 | 4742.6088 | 5378.3584 | 2449.3312 |
| Min          | 0.0036   | 0.0082    | 2.1E-01   | 7.3E-01   | 1.6648    |
| SD.          | 20.95035 | 179.40390 | 398.28415 | 497.30312 | 411.83659 |
|              |          |           |           |           |           |
| (b) CIR      |          |           |           |           |           |
| Mean         | 10.8647  | 183.0934  | 316.5086  | 420.1654  | 433.4495  |
| Max          | 291.5448 | 1372.9487 | 1962.5024 | 2598.4383 | 1464.0511 |
| Min          | 7.0E-04  | 4.3E-06   | 0.0101    | 0.1757    | 0.4386    |
| SD.          | 8.7038   | 145.6342  | 238.2479  | 353.2313  | 325.5429  |
| t-statistics | 4.86***  | 4.77***   | 5.70***   | 7.86***   | 6.56***   |

\*\*\* Significant at 1% level

The CIR model produces better MAPE and RMSE for all maturities respectively as shown in Table VII. The standard deviation is smaller at the short end and increases at the long end.

The implication of these results implies that the functional form of model is important. The functional form of the Vasicek model assumes that the volatility of the short rate is independent of the level of the short rate, while the CIR model specifies that the volatility of the short rate as an increasing function of the short rate. This means that as the short term interest rate increases, standard deviation of the CIR model increases. This performance may be attributed to pricing errors.

## 4.3Trading Strategy based on Estimated Yield Curve

Sercu and Wu (1997) investigate the degree of mispricing by taking positions in bonds that are mispriced according to a given yield curve estimation method, buying underpriced bonds and selling overpriced bonds. Since one conceptual weakness of equilibrium model, like the Vasicek or the CIR model is that a model does not take the current term structure as input thus, it is likely that all bonds are mispriced.

The pricing error from the previous section will provide the raw material for the analysis in this section. This test uses the model residual and verifies whether the estimated model has any economic significance by constructing a portfolio based on the estimated yield curve.

This study provides return of an equally weighted portfolio as benchmark in order to compare return of portfolio in each model. The return of equally weighted portfolio consists of all government bonds listed in the market at the beginning day of the rebalance period.

The return of the contrarian strategy is shown in Table VIII. The results are classified according to the model employed.

Compare the mean of returns across model reveals that the CIR model is superiority the Vasicek model at 1999, 2000 and 2003, while the Vasicek model produce better average returns at the remaining sub-interval.

# Table VIIIAbnormal return based on yield curve

This table reports the results of the trading strategy for the Vasicek and CIR model, which are based on contrarian strategy. This study provides return of equally weighted portfolio as benchmark in order to compare return of portfolio in each model. Model which has the higher abnormal return can be a better benchmark in bond trading. All values are expressed in terms of percentages. All values are expressed in terms of percentages. The trading takes place from February 1999 to December 2003, a total of 59 trading months. The rules for trading tests consist of purchasing all underpriced bonds and short selling for all overpriced bonds in given trading month. The weight of each bond is computed by proportional factor to size of misprice.

|               |       | Abnormal Return |       |      |      |           |  |
|---------------|-------|-----------------|-------|------|------|-----------|--|
|               | 1999  | 2000            | 2001  | 2002 | 2003 | 1999-2003 |  |
| Vasicek       | -0.55 | -1.25           | 0.58  | 1.75 | 0.58 | 0.23      |  |
| CIR           | 0.03  | -0.77           | -0.24 | 1.50 | 0.87 | 0.28      |  |
| t- statistics | 0.70  | 0.72            | 0.77  | 0.29 | 0.27 | 0.10      |  |

Notice that both competing yield curves cannot generate profits significantly in the first three periods if taking a position by value weighted. The cause might be due to the illiquidity of traded bonds. Specifically, the average of traded bonds in the first three years is 203 bonds per month while the average of traded bonds in the last two year is 499 bonds per month. However, the CIR model provides higher average of return and less standard deviation of return than the Vasicek model in the 1999-2003.

According to the above results, the abnormal return from the CIR model is higher than that of the Vasicek model for all the periods when compared with the return of the equally weighted portfolio since the CIR model provides the better pricing. Hence, the term structure of interest rates from the CIR model can be a better benchmark in bond trading than the Vasicek model.

Moreover, this result reflects the efficiency of the market. If the Thai bonds market is efficient, all information will be included in bonds prices and investors cannot make the abnormal return from mispriced bonds. But from above results, investors can make an abnormal return from trading on a given yield curve. It implies that the Thai bond market is inefficient.