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# APPENDICES

# APPENDIX A

## Computer Programs

The commutator sequences in eq. (4.3.35) have been calculated by programming in Mathematica language. The representation of all input variables in the program is given as follows:

| | |
|---|---|
| A ,B , C | operators |
| X | coordinate operator |
| P | momentum operator |
| H | Hamiltonian |
| hb | $\hbar = h / 2\pi$ = Planck's constant / $2\pi$ |
| a , b | arbitrary constant |
| W | $\omega$ (frequency) |
| U | unit matrix |
| n | integer . |

```
In[1]:= A ** (B ** C) == (A ** B) ** C

Out[1]= True

In[2]:= Unprotect[NonCommutativeMultiply];

In[3]:= A_ ** U := A
        U ** A_ := A

In[5]:= A_ ** (B_ + C_) := A ** B + A ** C
        (A_ + B_) ** C_ := A ** C + B ** C

In[7]:= NumberQ[a] ^= True;
        NumberQ[b] ^= True;

In[8]:= number3Q[x_, y_, n_] := NumberQ[x] && NumberQ[y] && NumberQ[n]

In[9]:= A_ ** (B_ (x_. y_^n_. /; number3Q[x, y, n])) :=
                                              ((x y^n) A ** B)
        (A_ (x_. y_^n_. /; number3Q[x, y, n])) ** B_ :=
                                              ((x y^n) A ** B)

In[11]:= Protect[NonCommutativeMultiply];

In[13]:= commutator[A_, B_] := A ** B - B ** A

In[14]:= commutator[A, B] + commutator[B, A] == 0

Out[14]= True

In[15]:= commutator[a A, b B] - a b commutator[A, B] == 0 //
         ExpandAll

Out[15]= True
```

```
In[16]:= commutator[A, B + C] - commutator[A, B] -
         commutator[A, C] == 0

Out[16]= True

In[17]:= commutator[A + B, C] - commutator[A, C] -
         commutator[B, C] == 0

Out[17]= True

In[18]:= commutator[A, B ** C] - commutator[A, B] ** C -
         B ** commutator[A, C] == 0

Out[18]= True

In[19]:= commutator[A ** B, C] - A ** commutator[B, C] -
         commutator[A, C] ** B == 0

Out[19]= True

In[20]:= commutator[commutator[A, B], C] +
         commutator[commutator[C, A], B] +
         commutator[commutator[B, C], A] == 0

Out[20]= True

In[21]:= xpCommutator[expr_] :=
           ExpandAll[expr //. p ** x :> x ** p - I hb U]

In[22]:= NumberQ[hb] ^= True;

In[23]:= commutator[x, p ** p ** p ** p] // xpCommutator

Out[23]= 4 I hb p ** p ** p
```

In[24]:= commutator[p, x ** x ** p ** p] // xpCommutator

Out[24]= −2 I hb x ** p ** p

In[25]:= commutator[x ** p ** p, p ** x ** x] // xpCommutator

Out[25]= $-6\ hb^2$ x ** p − 3 I hb x ** x ** p ** p

In[26]:= NumberQ[m] ^= True;

NumberQ[w] ^= True;

In[27]:= commutator[p ** p / (2 m) + (m / 2) w^2 x ** x, p] // xpCommutator

Out[27]= $I\ hb\ m\ w^2\ x$

In[28]:= H = p ** p / (2 m) + a x ** x + b x ** x ** x

Out[28]= $\frac{p ** p}{2\ m} + a\ x ** x + b\ x ** x ** x$

In[29]:= c = commutator[H, x] // xpCommutator

Out[29]= $-\frac{I\ hb\ p}{m}$

In[30]:= d = commutator[H, c] // xpCommutator

Out[30]= $\frac{2\ a\ hb^2\ x}{m} + \frac{3\ b\ hb^2\ x ** x}{m}$

In[31]:= e = commutator[H, d] // xpCommutator

Out[31]= $-\frac{2\ I\ a\ hb^3\ p}{m^2} - \frac{3\ b\ hb^4\ U}{m^2} - \frac{6\ I\ b\ hb^3\ x ** p}{m^2}$

In[32]:= f = commutator[H, e] // xpCommutator

Out[32]= $\frac{4\ a^2\ hb^4\ x}{m^2} - \frac{6\ b\ hb^4\ p ** p}{m^3} + \frac{18\ a\ b\ hb^4\ x ** x}{m^2} + \frac{18\ b^2\ hb^4\ x ** x ** x}{m^2}$

In[33]:= g = commutator[H, f] // xpCommutator

Out[33]=
$$-\frac{4\,I\,a^2\,hb^5\,p}{m^3}-\frac{30\,a\,b\,hb^6\,U}{m^3}-\frac{90\,b^2\,hb^6\,x}{m^3}-\frac{60\,I\,a\,b\,hb^5\,x**p}{m^3}-\frac{90\,I\,b^2\,hb^5\,x**x**p}{m^3}$$

In[34]:= h = commutator[H, g] // xpCommutator

Out[34]=
$$\frac{180\,I\,b^2\,hb^7\,p}{m^4}+\frac{8\,a^3\,hb^6\,x}{m^3}-\frac{60\,a\,b\,hb^6\,p**p}{m^4}+\frac{132\,a^2\,b\,hb^6\,x**x}{m^3}-\frac{180\,b^2\,hb^6\,x**p**p}{m^4}+\frac{360\,a\,b^2\,hb^6\,x**x**x}{m^3}+\frac{270\,b^3\,hb^6\,x**x**x**x}{m^3}$$

In[35]:= i = commutator[H, h] // xpCommutator

Out[35]=
$$-\frac{8\,I\,a^3\,hb^7\,p}{m^4}-\frac{252\,a^2\,b\,hb^8\,U}{m^4}-\frac{2160\,a\,b^2\,hb^8\,x}{m^4}-\frac{504\,I\,a^2\,b\,hb^7\,x**p}{m^4}-\frac{3240\,b^3\,hb^8\,x**x}{m^4}+\frac{180\,I\,b^2\,hb^7\,p**p**p}{m^5}-\frac{2160\,I\,a\,b^2\,hb^7\,x**x**p}{m^4}-\frac{2160\,I\,b^3\,hb^7\,x**x**x**p}{m^4}$$

In[36]:= j = commutator[H, i] // xpCommutator

Out[36]=
$$\frac{5400\,I\,a\,b^2\,hb^9\,p}{m^5}+\frac{4320\,b^3\,hb^{10}\,U}{m^5}+\frac{16\,a^4\,hb^8\,x}{m^4}-\frac{504\,a^2\,b\,hb^8\,p**p}{m^5}+\frac{16200\,I\,b^3\,hb^9\,x**p}{m^5}+\frac{1032\,a^3\,b\,hb^8\,x**x}{m^4}-\frac{5400\,a\,b^2\,hb^8\,x**p**p}{m^5}+\frac{5832\,a^2\,b^2\,hb^8\,x**x**x}{m^4}-\frac{8100\,b^3\,hb^8\,x**x**p**p}{m^5}+\frac{10800\,a\,b^3\,hb^8\,x**x**x**x}{m^4}+\frac{6480\,b^4\,hb^8\,x**x**x**x**x}{m^4}$$

In[37]:= k = commutator[H, j] // xpCommutator

Out[37]=
$$-\frac{16\,I\,a^4\,hb^9\,p}{m^5}-\frac{2040\,a^3\,b\,hb^{10}\,U}{m^5}-\frac{42120\,a^2\,b^2\,hb^{10}\,x}{m^5}+\frac{24300\,b^3\,hb^{10}\,p**p}{m^6}-\frac{4080\,I\,a^3\,b\,hb^9\,x**p}{m^5}-\frac{162000\,a\,b^3\,hb^{10}\,x**x}{m^5}+\frac{5400\,I\,a\,b^2\,hb^9\,p**p**p}{m^6}-\frac{42120\,I\,a^2\,b^2\,hb^9\,x**x**p}{m^5}-\frac{162000\,b^4\,hb^{10}\,x**x**x}{m^5}+\frac{16200\,I\,b^3\,hb^9\,x**p**p**p}{m^6}-\frac{108000\,I\,a\,b^3\,hb^9\,x**x**x**p}{m^5}-\frac{81000\,I\,b^4\,hb^9\,x**x**x**x**p}{m^5}$$

In[38]:= l = commutator[H, k] // xpCommutator

Out[38]=
$$\frac{116640\,I\,a^2\,b^2\,hb^{11}\,p}{m^6}+\frac{243000\,a\,b^3\,hb^{12}\,U}{m^6}+\frac{729000\,b^4\,hb^{12}\,x}{m^6}+\frac{32\,a^5\,hb^{10}\,x}{m^5}-\frac{4080\,a^3\,b\,hb^{10}\,p**p}{m^6}+\frac{939600\,I\,a\,b^3\,hb^{11}\,x**p}{m^6}+\frac{8208\,a^4\,b\,hb^{10}\,x**x}{m^5}-\frac{116640\,a^2\,b^2\,hb^{10}\,x**p**p}{m^6}+\frac{1409400\,I\,b^4\,hb^{11}\,x**x**p}{m^6}+\frac{96480\,a^3\,b^2\,hb^{10}\,x**x**x}{m^5}+\frac{16200\,b^3\,hb^{10}\,p**p**p**p}{m^7}-\frac{469800\,a\,b^3\,hb^{10}\,x**x**p**p}{m^6}+\frac{342360\,a^2\,b^3\,hb^{10}\,x**x**x**x}{m^5}-\frac{469800\,b^4\,hb^{10}\,x**x**x**p**p}{m^6}+\frac{486000\,a\,b^4\,hb^{10}\,x**x**x**x**x}{m^5}+\frac{243000\,b^5\,hb^{10}\,x**x**x**x**x**x}{m^5}$$

In[39]:= **n = commutator[H, l] // xpCommutator**

Out[39]=

$$-\frac{2527200\,I\,b^4\,hb^{13}\,p}{m^7}-\frac{32\,I\,a^5\,hb^{11}\,p}{m^6}-\frac{16368\,a^4\,b\,hb^{12}\,U}{m^6}-\frac{780480\,a^3\,b^2\,hb^{12}\,x}{m^6}+$$
$$\frac{1603800\,a\,b^3\,hb^{12}\,p**p}{m^7}-\frac{32736\,I\,a^4\,b\,hb^{11}\,x**p}{m^6}-\frac{5922720\,a^2\,b^3\,hb^{12}\,x**x}{m^6}+$$
$$\frac{116640\,I\,a^2\,b^2\,hb^{11}\,p**p**p}{m^7}+\frac{4811400\,b^4\,hb^{12}\,x**p**p}{m^7}-$$
$$\frac{780480\,I\,a^3\,b^2\,hb^{11}\,x**x**p}{m^6}-\frac{14256000\,a\,b^4\,hb^{12}\,x**x**x}{m^6}+$$
$$\frac{1069200\,I\,a\,b^3\,hb^{11}\,x**p**p**p}{m^7}-\frac{3948480\,I\,a^2\,b^3\,hb^{11}\,x**x**x**p}{m^6}-$$
$$\frac{10692000\,b^5\,hb^{12}\,x**x**x**x}{m^6}+\frac{1603800\,I\,b^4\,hb^{11}\,x**x**p**p**p}{m^7}-$$
$$\frac{7128000\,I\,a\,b^4\,hb^{11}\,x**x**x**x**p}{m^6}-\frac{4276800\,I\,b^5\,hb^{11}\,x**x**x**x**x**p}{m^6}$$

In[40] := o = commutator[H, n] // xpCommutator

Out[40]=

$$\frac{2260800\, I\, a^3\, b^2\, hb^{13}\, p}{m^7} + \frac{9830160\, a^2\, b^3\, hb^{14}\, U}{m^7} + \frac{73483200\, a\, b^4\, hb^{14}\, x}{m^7} + \frac{64\, a^6\, hb^{12}\, x}{m^6} - \frac{32736\, a^4\, b\, hb^{12}\, p ** p}{m^7} + \frac{38620800\, I\, a^2\, b^3\, hb^{13}\, x ** p}{m^7} + \frac{110224800\, b^5\, hb^{14}\, x ** x}{m^7} + \frac{65568\, a^5\, b\, hb^{12}\, x ** x}{m^6} - \frac{6415200\, I\, b^4\, hb^{13}\, p ** p ** p}{m^8} - \frac{2260800\, a^3\, b^2\, hb^{12}\, x ** p ** p}{m^7} + \frac{143272800\, I\, a\, b^4\, hb^{13}\, x ** x ** p}{m^7} + \frac{1659168\, a^4\, b^2\, hb^{12}\, x ** x ** x}{m^6} + \frac{1069200\, a\, b^3\, hb^{12}\, p ** p ** p ** p}{m^8} - \frac{19310400\, a^2\, b^3\, hb^{12}\, x ** x ** p ** p}{m^7} + \frac{143272800\, I\, b^5\, hb^{13}\, x ** x ** x ** p}{m^7} + \frac{10238400\, a^3\, b^3\, hb^{12}\, x ** x ** x ** x}{m^6} + \frac{3207600\, b^4\, hb^{12}\, x ** p ** p ** p ** p}{m^8} - \frac{47757600\, a\, b^4\, hb^{12}\, x ** x ** x ** p ** p}{m^7} + \frac{26101440\, a^2\, b^4\, hb^{12}\, x ** x ** x ** x ** x}{m^6} - \frac{35818200\, b^5\, hb^{12}\, x ** x ** x ** x ** p ** p}{m^7} + \frac{29937600\, a\, b^5\, hb^{12}\, x ** x ** x ** x ** x ** x}{m^6} + \frac{12830400\, b^6\, hb^{12}\, x ** x ** x ** x ** x ** x ** x}{m^6}$$

In[41]:= q = commutator[H, o] // xpCommutator

Out[41]=

$$-\frac{280908000\,I\,a\,b^4\,hb^{15}\,p}{m^8}-\frac{64\,I\,a^6\,hb^{13}\,p}{m^7}-\frac{148716000\,b^5\,hb^{16}\,U}{m^8}-\frac{131040\,a^5\,b\,hb^{14}\,U}{m^7}-\frac{14217120\,a^4\,b^2\,hb^{14}\,x}{m^7}+\frac{70761600\,a^2\,b^3\,hb^{14}\,p**p}{m^8}-\frac{842724000\,I\,b^5\,hb^{15}\,x**p}{m^8}-\frac{262080\,I\,a^5\,b\,hb^{13}\,x**p}{m^7}-\frac{197640000\,a^3\,b^3\,hb^{14}\,x**x}{m^7}+\frac{2260800\,I\,a^3\,b^2\,hb^{13}\,p**p**p}{m^8}+\frac{545292000\,a\,b^4\,hb^{14}\,x**p**p}{m^8}-\frac{14217120\,I\,a^4\,b^2\,hb^{13}\,x**x**p}{m^7}-\frac{874800000\,a^2\,b^4\,hb^{14}\,x**x**x}{m^7}+\frac{47174400\,I\,a^2\,b^3\,hb^{13}\,x**p**p**p}{m^8}+\frac{817938000\,b^5\,hb^{14}\,x**x**p**p}{m^8}-\frac{131760000\,I\,a^3\,b^3\,hb^{13}\,x**x**x**p}{m^7}-\frac{1523610000\,a\,b^5\,hb^{14}\,x**x**x**x}{m^7}-\frac{3207600\,I\,b^4\,hb^{13}\,p**p**p**p**p}{m^9}+\frac{181764000\,I\,a\,b^4\,hb^{13}\,x**x**p**p**p}{m^8}-\frac{437400000\,I\,a^2\,b^4\,hb^{13}\,x**x**x**x**p}{m^7}-\frac{914166000\,b^6\,hb^{14}\,x**x**x**x**x}{m^7}+\frac{181764000\,I\,b^5\,hb^{13}\,x**x**x**p**p**p}{m^8}-\frac{609444000\,I\,a\,b^5\,hb^{13}\,x**x**x**x**x**p}{m^7}-\frac{304722000\,I\,b^6\,hb^{13}\,x**x**x**x**x**x**p}{m^7}$$

In[42]:= r = commutator[H, q] // xpCommutator

Out[42]=

$$\frac{41999040\, I\, a^4\, b^2\, hb^{15}\, p}{m^8} + \frac{352728000\, a^3\, b^3\, hb^{16}\, U}{m^8} + \frac{4984416000\, a^2\, b^4\, hb^{16}\, x}{m^8} +$$

$$\frac{128\, a^7\, hb^{14}\, x}{m^7} - \frac{1853118000\, b^5\, hb^{16}\, p \ast\ast p}{m^9} - \frac{262080\, a^5\, b\, hb^{14}\, p \ast\ast p}{m^8} +$$

$$\frac{1397347200\, I\, a^3\, b^3\, hb^{15}\, x \ast\ast p}{m^8} + \frac{17668044000\, a\, b^5\, hb^{16}\, x \ast\ast x}{m^8} +$$

$$\frac{524352\, a^6\, b\, hb^{14}\, x \ast\ast x}{m^7} - \frac{791208000\, I\, a\, b^4\, hb^{15}\, p \ast\ast p \ast\ast p}{m^9} -$$

$$\frac{41999040\, a^4\, b^2\, hb^{14}\, x \ast\ast p \ast\ast p}{m^8} + \frac{9794260800\, I\, a^2\, b^4\, hb^{15}\, x \ast\ast x \ast\ast p}{m^8} +$$

$$\frac{17668044000\, b^6\, hb^{16}\, x \ast\ast x \ast\ast x}{m^8} + \frac{29220480\, a^5\, b^2\, hb^{14}\, x \ast\ast x \ast\ast x}{m^7} +$$

$$\frac{47174400\, a^2\, b^3\, hb^{14}\, p \ast\ast p \ast\ast p \ast\ast p}{m^9} - \frac{2373624000\, I\, b^5\, hb^{15}\, x \ast\ast p \ast\ast p \ast\ast p}{m^9} -$$

$$\frac{698673600\, a^3\, b^3\, hb^{14}\, x \ast\ast x \ast\ast p \ast\ast p}{m^8} + \frac{23094720000\, I\, a\, b^5\, hb^{15}\, x \ast\ast x \ast\ast x \ast\ast p}{m^8} +$$

$$\frac{306171360\, a^4\, b^3\, hb^{14}\, x \ast\ast x \ast\ast x \ast\ast x}{m^7} + \frac{395604000\, a\, b^4\, hb^{14}\, x \ast\ast p \ast\ast p \ast\ast p \ast\ast p}{m^9} -$$

$$\frac{3264753600\, a^2\, b^4\, hb^{14}\, x \ast\ast x \ast\ast x \ast\ast p \ast\ast p}{m^8} +$$

$$\frac{17321040000\, I\, b^6\, hb^{15}\, x \ast\ast x \ast\ast x \ast\ast x \ast\ast p}{m^8} +$$

$$\frac{1270080000\, a^3\, b^4\, hb^{14}\, x \ast\ast x \ast\ast x \ast\ast x \ast\ast x}{m^7} +$$

$$\frac{593406000\, b^5\, hb^{14}\, x \ast\ast x \ast\ast p \ast\ast p \ast\ast p \ast\ast p}{m^9} -$$

$$\frac{5773680000\, a\, b^5\, hb^{14}\, x \ast\ast x \ast\ast x \ast\ast x \ast\ast p \ast\ast p}{m^8} +$$

$$\frac{2531088000\, a^2\, b^5\, hb^{14}\, x \ast\ast x \ast\ast x \ast\ast x \ast\ast x \ast\ast x}{m^7} -$$

$$\frac{3464208000\, b^6\, hb^{14}\, x \ast\ast x \ast\ast x \ast\ast x \ast\ast x \ast\ast p \ast\ast p}{m^8} +$$

$$\frac{2437776000\, a\, b^6\, hb^{14}\, x \ast\ast x \ast\ast x \ast\ast x \ast\ast x \ast\ast x \ast\ast x}{m^7} +$$

$$\frac{914166000\, b^7\, hb^{14}\, x \ast\ast x \ast\ast x \ast\ast x \ast\ast x \ast\ast x \ast\ast x \ast\ast x}{m^7}$$

In[43]:= s = commutator[H, r] // xpCommutator

Out[43]=
$$-\frac{20658110400\, I\, a^2\, b^4\, hb^{17}\, p}{m^9} - \frac{128\, I\, a^7\, hb^{15}\, p}{m^8} - \frac{26121528000\, a\, b^5\, hb^{18}\, U}{m^9} - \frac{1048512\, a^6\, b\, hb^{16}\, U}{m^8} - \frac{78364584000\, b^6\, hb^{18}\, x}{m^9} - \frac{257230080\, a^5\, b^2\, hb^{16}\, x}{m^8} + \frac{2662113600\, a^3\, b^3\, hb^{16}\, p ** p}{m^9} - \frac{150010704000\, I\, a\, b^5\, hb^{17}\, x ** p}{m^9} - \frac{2097024\, I\, a^6\, b\, hb^{15}\, x ** p}{m^8} - \frac{6407061120\, a^4\, b^3\, hb^{16}\, x ** x}{m^8} + \frac{41999040\, I\, a^4\, b^2\, hb^{15}\, p ** p ** p}{m^9} + \frac{40575556800\, a^2\, b^4\, hb^{16}\, x ** p ** p}{m^9} - \frac{225016056000\, I\, b^6\, hb^{17}\, x ** x ** p}{m^9} - \frac{257230080\, I\, a^5\, b^2\, hb^{15}\, x ** x ** p}{m^8} - \frac{47202912000\, a^3\, b^4\, hb^{16}\, x ** x ** x}{m^8} - \frac{2967030000\, b^5\, hb^{16}\, p ** p ** p ** p}{m^{10}} + \frac{1774742400\, I\, a^3\, b^3\, hb^{15}\, x ** p ** p ** p}{m^9} + \frac{146651472000\, a\, b^5\, hb^{16}\, x ** x ** p ** p}{m^9} - \frac{4271374080\, I\, a^4\, b^3\, hb^{15}\, x ** x ** x ** p}{m^8} - \frac{144674424000\, a^2\, b^5\, hb^{16}\, x ** x ** x ** x}{m^8} - \frac{395604000\, I\, a\, b^4\, hb^{15}\, p ** p ** p ** p ** p}{m^{10}} + \frac{13525185600\, I\, a^2\, b^4\, hb^{15}\, x ** x ** p ** p ** p}{m^9} + \frac{146651472000\, b^6\, hb^{16}\, x ** x ** x ** p ** p}{m^9} - \frac{23601456000\, I\, a^3\, b^4\, hb^{15}\, x ** x ** x ** x ** p}{m^8} - \frac{196690032000\, a\, b^6\, hb^{16}\, x ** x ** x ** x ** x}{m^8} - \frac{1186812000\, I\, b^5\, hb^{15}\, x ** p ** p ** p ** p ** p}{m^{10}} + \frac{32589216000\, I\, a\, b^5\, hb^{15}\, x ** x ** x ** p ** p ** p}{m^9} - \frac{57869769600\, I\, a^2\, b^5\, hb^{15}\, x ** x ** x ** x ** x ** p}{m^8} - \frac{98345016000\, b^7\, hb^{16}\, x ** x ** x ** x ** x ** x}{m^8} +$$

$$\frac{24441912000\ I\ b^6\ hb^{15}\ x**x**x**x**p**p**p}{m^9} -$$

$$\frac{65563344000\ I\ a\ b^6\ hb^{15}\ x**x**x**x**x**x**p}{m^8} -$$

$$\frac{28098576000\ I\ b^7\ hb^{15}\ x**x**x**x**x**x**x**p}{m^8}$$

## APPENDIX B

### Computer Programs

From equation (4.3.43c), we use programming in mathematica to calculate to find propagator $K(x,\xi,t)$ with quadratic and cubic potential terms. In the last step, we normalize $G4$ for finding normalization constant $C$ by using integration formula

$$\int_{-\infty}^{\infty} e^{-(ax^2+bx+c)}dx = \sqrt{\frac{\pi}{a}}e^{(b^2-4ac)/4a} \tag{B.1}$$

and condition

$$1 = \int_{-\infty}^{\infty} K(x,\xi,0)dx = \int_{-\infty}^{\infty} G4 \ dx. \tag{B.2}$$

The representation of all input variables in the program is given as follows:

| | |
|---|---|
| C | normalization constant |
| b | arbitrary constant |
| $G_n(t)$ | $K_n(t)$ (time - dependent integration constant) |
| W | $\omega$ (frequency) |
| Z | $\xi$ |

A=I m w/(ħ Sin[w t]) Integrate[x Cos[w t] - z + b x^2/(m w^2) Cos[w t]
+ b x^2/(2 m w^2) Cos[2 w t] -
I ħ b/(m^2 w^3) Sin[w t] + I ħ b/(2 m^2 w^3) Sin[2 w t] -
3 b x^2/(2 m w^2) +
4 b x Sin[w t/2]^2/(m w^2) (x Cos[w t] - z),x]

```
(I m w Csc[t w] (

     2                2
    x  (-2 b z + m w  Cos[t w] + 2 b z Cos[t w])
    -------------------------------------------- +
                                 2
                            2 m w

          3                        t w 2
    2 b x  (-2 + Cos[t w]) Sin[---]
                                2
    -------------------------------- +
                               2
                          3 m w

              2  3
    x (-2 m  w  z - 2 I b ħ Sin[t w] + I b ħ Sin[2 t w])
    ---------------------------------------------------)) / ħ
                               2  3
                           2 m  w
```

G= G0[t] Exp[A]

```
                                        2
Power[E, (I m w Csc[t w] ((x

                        2                                     2
         (-2 b z + m w  Cos[t w] + 2 b z Cos[t w]))/(2 m w ) \

                3                            2          2
       + (2 b x  (-2 + Cos[t w]) Sin[(t w)/2] )/(3 m w ) +

              2  3
      (x (-2 m  w  z - 2 I b ħ Sin[t w] + I b ħ Sin[2 t w]))/

          2  3
       (2 m  w ))))/ħ] G0[t]
```

solg0=DSolve[D[G,t] - I ħ /(2 m) D[D[G,x],x] + I m w^2 x^2/(2 ħ) G + I b x^3/ħ G==0,G0[t],t]

```
                   2  4             2           2  2
{{G0[t] -> C[1] (-((m  w  + 16 b m w  x + 24 b  x  +

              2            2              t w            2  4
      8 b m w  z + 16 b  x z) Log[Cos[---]]) / (2 m  w )\
                                           2

        t w
     Log[Sin[---]]
              2
   - ------------- +
           2

     t w      t w            2  2               3  4  3
  (Csc[---] Sec[---] (-24 I b  h  - 128 I b m  w  x  -
        2        2

           2  2  2  4             3  4  2
     504 I b  m  w  x  - 192 I b m  w  x  z -

           2  2  2  3            3  4    2
     624 I b  m  w  x  z - 96 I b m  w  x z  -

           2  2  2  2  2          2  2
     192 I b  m  w  x  z  + 3 I b  ħ  Cos[t w] +

               3  4  3
     96 I b m  w  x  Cos[t w] +

           2  2  2  4
     435 I b  m  w  x  Cos[t w] +

                3  4  2
     192 I b m  w  x  z Cos[t w] +

           2  2  2  3
     576 I b  m  w  x  z Cos[t w] +

            4  6  2
     24 I m  w  z  Cos[t w] +

              3  4    2
     96 I b m  w  x z  Cos[t w] +

           2  2  2  2  2
     192 I b  m  w  x  z  Cos[t w] +

          2  2
     24 I b  ħ  Cos[2 t w] +

              3  4  3
     32 I b m  w  x  Cos[2 t w] +
```

$$72\ I\ b^{2}\ m^{2}\ w^{2}\ x^{4}\ \mathrm{Cos}[2\ t\ w] +$$

$$48\ I\ b^{2}\ m^{2}\ w^{2}\ x^{3}\ z\ \mathrm{Cos}[2\ t\ w] -$$

$$3\ I\ b^{2}\ \hbar^{2}\ \mathrm{Cos}[3\ t\ w] - 3\ I\ b^{2}\ m^{2}\ w^{2}\ x^{4}\ \mathrm{Cos}[3\ t\ w] +$$

$$36\ I\ b^{2}\ \hbar^{2}\ t\ w\ \mathrm{Sin}[t\ w] + 6\ b^{2}\ \hbar\ m\ w\ x^{2}\ \mathrm{Sin}[t\ w] +$$

$$96\ I\ b\ m^{3}\ t\ w^{5}\ x^{3}\ \mathrm{Sin}[t\ w] +$$

$$348\ I\ b^{2}\ m^{2}\ t\ w^{3}\ x^{4}\ \mathrm{Sin}[t\ w] +$$

$$144\ I\ b\ m^{3}\ t\ w^{5}\ x^{2}\ z\ \mathrm{Sin}[t\ w] +$$

$$384\ I\ b^{2}\ m^{2}\ t\ w^{3}\ x^{3}\ z\ \mathrm{Sin}[t\ w] +$$

$$96\ I\ b^{2}\ m^{2}\ t\ w^{3}\ x^{2}\ z^{2}\ \mathrm{Sin}[t\ w] +$$

$$72\ b\ \hbar\ m^{2}\ w^{3}\ x\ \mathrm{Sin}[2\ t\ w] +$$

$$96\ b^{2}\ \hbar\ m\ w\ x^{2}\ \mathrm{Sin}[2\ t\ w] +$$

$$48\ b^{2}\ \hbar\ m\ w\ x\ z\ \mathrm{Sin}[2\ t\ w] -$$

$$6\ b^{2}\ \hbar\ m\ w\ x^{2}\ \mathrm{Sin}[3\ t\ w])) / (96\ \hbar\ m^{3}\ w^{5}))))$$

g01=solg0[[1,1,2]]/C[1]

```
   2  4             2          2  2            2          2
-((m   w  + 16 b m w  x + 24 b  x  + 8 b m w  z + 16 b  x z)

                                         t w
                                  Log[Sin[---]]
         t w        2  4                  2
   Log[Cos[---]]) / (2 m  w ) - ------------- +
          2                             2

   t w      t w          2  2          3  4  3
 (Csc[---] Sec[---] (-24 I b  h  - 128 I b m  w  x  -
       2        2

          2   2   2   4            3   4   2
    504 I b  m   w   x  - 192 I b m  w  x  z -

          2   2   2   3            3   4    2
    624 I b  m   w   x  z - 96 I b m  w  x z  -

          2   2   2   2   2          2  2
    192 I b  m   w   x   z  + 3 I b  h  Cos[t w] +

            3   4   3                   2   2   2   4
    96 I b m  w   x  Cos[t w] + 435 I b  m   w   x  Cos[t w] +

             3   4   2
    192 I b m  w  x  z Cos[t w] +

          2   2   2   3                  4   6   2
    576 I b  m   w   x  z Cos[t w] + 24 I m  w  z  Cos[t w] +

            3   4     2
    96 I b m  w  x z  Cos[t w] +

          2   2   2   2   2                   2   2
    192 I b  m   w   x   z  Cos[t w] + 24 I b  h  Cos[2 t w] +

            3   4   3
    32 I b m  w  x  Cos[2 t w] +

         2   2   2   4
    72 I b  m   w   x  Cos[2 t w] +

         2   2   2   3                      2   2
    48 I b  m   w   x  z Cos[2 t w] - 3 I b  h  Cos[3 t w] -

        2   2   2   4                   2   2
    3 I b  m   w   x  Cos[3 t w] + 36 I b  h  t w Sin[t w] +

      2          2                   3     5   3
    6 b  h m w x  Sin[t w] + 96 I b m  t w  x  Sin[t w] +
```

```
          2  2     3  4
348 I b  m   t w   x   Sin[t w] +

             3     5  2
144 I b m   t w   x   z Sin[t w] +

          2  2     3  3
384 I b  m   t w   x   z Sin[t w] +

         2  2     3  2  2
96 I b  m   t w   x  z   Sin[t w] +

           2  3                        2         2
72 b ħ m   w   x Sin[2 t w] + 96 b  ħ m w x   Sin[2 t w] +

    2                                  2         2
48 b   h m w x z Sin[2 t w] - 6 b   ħ m w x   Sin[3 t w])) \

          3  5
/ (96 ħ m   w )
```

Unprotect[Exp]

{Exp}

Exp[x_]:=x

Protect[Exp]

{Exp}

g02=Normal[Series[g01,{b,0,1}]]

```
            t w               t w
-Log[Cos[---]]    Log[Sin[---]]
             2                 2
-------------- - -------------- +
      2                 2

I        2                 t w        t w
- m w z   Cos[t w] Csc[---] Sec[---]
4                           2          2
------------------------------------ +
                    ħ

             2            2               t w
   (-16 m w   x - 8 m w   z) Log[Cos[---]]
                                          2
b (-------------------------------------- +
                         2  4
                      2 m   w

      t w        t w              3  4  3           3  4  2
   (Csc[---] Sec[---] (-128 I m   w   x  - 192 I m   w   x   z -
          2          2

              3  4     2           3  4  3
         96 I m   w   x z   + 96 I m   w   x   Cos[t w] +

               3  4  2
         192 I m   w   x   z Cos[t w] +

              3  4     2                       3  4  3
         96 I m   w   x z   Cos[t w] + 32 I m   w   x   Cos[2 t w] +

              3     5  3
         96 I m   t w   x   Sin[t w] +
```

```
                3     5   2                               2    3
     144 I m   t w   x   z Sin[t w] + 72 ħ m   w   x Sin[2 t w])

                 3   5
    ) / (96 ħ m   w ))
```

Unprotect[Exp]

{Exp}

Exp[x_]:=HoldForm[Exp[x]]

g03=Exp[g02]

```
              t w           t w
   -Log[Cos[---]]   Log[Sin[---]]
               2             2
Exp[-------------- - ------------- +
          2               2

  I      2              t w       t w
  - m w z   Cos[t w] Csc[---] Sec[---]
  4                       2         2
  ----------------------------------- +
                  ħ

          2           2           t w
    (-16 m w   x - 8 m w   z) Log[Cos[---]]
                                        2
  b (------------------------------------- +
                           2   4
                        2 m   w

          t w       t w             3   4   3              3   4   2
    (Csc[---] Sec[---] (-128 I m   w   x   - 192 I m   w   x   z -
           2         2

             3   4      2            3   4   3
         96 I m   w   x z   + 96 I m   w   x   Cos[t w] +

              3   4   2
         192 I m   w   x   z Cos[t w] +

             3   4      2
         96 I m   w   x z   Cos[t w] +

             3   4   3
         32 I m   w   x   Cos[2 t w] +

             3     5   3
         96 I m   t w   x   Sin[t w] +

              3     5   2                               2   3
         144 I m   t w   x   z Sin[t w] + 72 ħ m   w   x Sin[2 t w]

                     3   5
         )) / (96 ħ m   w ))]
```

g04=Simplify[g03]

```
                t w               t w
    -Log[Cos[---]]    Log[Sin[---]]
                 2                 2
Exp[-------------- - ------------- +
          2                 2

   I      2                 t w      t w
   - m w z  Cos[t w] Csc[---] Sec[---]
   4                         2        2
   ----------------------------------- +
                  ħ

                             t w
     -4 (2 x + z) Log[Cos[---]]
                              2
   b (-------------------------- +
                  2
               m w

      I        t w                t w             2  2     t w
     (- x Sec[---] (-9 I ħ Cos[---] + 12 m t w  x  Cos[---] +
      6         2                  2                          2

                 2         t w              3 t w
          18 m t w  x z Cos[---] - 9 I ħ Cos[-----] -
                            2                  2

                 2     t w                    t w
          16 m w x  Sin[---] - 24 m w x z Sin[---] -
                        2                     2

                 2     t w           2     3 t w            2
          12 m w z  Sin[---] - 4 m w x  Sin[-----])) / (ħ m w )
                        2                   2

     )]
```

G1= g04 Exp[A]

```
Exp[(I m w Csc[t w] (

        2               2
       x  (-2 b z + m w  Cos[t w] + 2 b z Cos[t w])
       --------------------------------------------- +
                                   2
                             2 m w

           3                        t w 2
      2 b x  (-2 + Cos[t w]) Sin[---]
                                     2
      -------------------------------- +
                                2
                   3 m w

             2   3
      x (-2 m   w  z - 2 I b ħ Sin[t w] + I b ħ Sin[2 t w])
      ---------------------------------------------------)) \
                                    2   3
                                2 m   w

                      t w              t w
            -Log[Cos[---]]    Log[Sin[---]]
                       2                2
    / ħ] Exp[-------------- - ------------- +
                   2                2

   I      2          t w       t w
   - m w z  Cos[t w] Csc[---] Sec[---]
   4                    2         2
   ---------------------------------- +
                   ħ

                          t w
     -4 (2 x + z) Log[Cos[---]]
                           2
   b (------------------------- +
                   2
                m w

      I        t w            t w           2  2      t w
     (- x Sec[---] (-9 I ħ Cos[---] + 12 m t w   x   Cos[---] +
      6         2              2                          2

              2          t w              3 t w
         18 m t w  x z Cos[---] - 9 I ħ Cos[-----] -
                            2                 2

              2      t w                      t w
         16 m w x  Sin[---] - 24 m w x z Sin[---] -
                      2                        2

              2      t w             2     3 t w
         12 m w z  Sin[---] - 4 m w x  Sin[-----])) /
                      2                     2

            2
       (ħ m w ))]
```

G2=Together[G1]

```
C Exp[(I m w Csc[t w] (

          2                 2
         x   (-2 b z + m w   Cos[t w] + 2 b z Cos[t w])
         ------------------------------------------------ +
                                 2
                            2 m w

            3                          t w 2
        2 b x   (-2 + Cos[t w]) Sin[---]
                                        2
        ------------------------------- +
                            2
                       3 m w

              2   3
        x (-2 m   w   z - 2 I b ħ Sin[t w] + I b ħ Sin[2 t w])
        --------------------------------------------------------)) \
                                    2   3
                                 2 m   w

                        t w              t w
               -Log[Cos[---]]    Log[Sin[---]]
                         2                2
    / ħ] Exp[-------------- - ------------- +
                    2                2

      I      2            t w       t w
      - m w z  Cos[t w] Csc[---] Sec[---]
      4                    2         2
      ----------------------------------- +
                        ħ

                                  t w
         -4 (2 x + z) Log[Cos[---]]
                                   2
      b (-------------------------- +
                         2
                      m w

          I       t w               t w               2   2       t w
         (- x Sec[---] (-9 I ħ Cos[---] + 12 m t w   x   Cos[---] +
          6        2                  2                            2

                     2        t w                  3 t w
             18 m t w   x z Cos[---] - 9 I ħ Cos[-----] -
                                 2                   2

                   2      t w                       t w
             16 m w x   Sin[---] - 24 m w x z Sin[---] -
                             2                        2

                   2      t w               2      3 t w
             12 m w z   Sin[---] - 4 m w x   Sin[-----])) /
                             2                     2

              2
         (ħ m w ))]
```

G1= g02 + A

$$
-\frac{\mathrm{Log}[\mathrm{Cos}[\frac{t\,w}{2}]]}{2}-\frac{\mathrm{Log}[\mathrm{Sin}[\frac{t\,w}{2}]]}{2}+\frac{\frac{I}{4}\,m\,w\,z^{2}\,\mathrm{Cos}[t\,w]\,\mathrm{Csc}[\frac{t\,w}{2}]\,\mathrm{Sec}[\frac{t\,w}{2}]}{\hbar}+
$$

$$
(I\,m\,w\,\mathrm{Csc}[t\,w]\,\Big(\frac{x^{2}\,(-2\,b\,z+m\,w^{2}\,\mathrm{Cos}[t\,w]+2\,b\,z\,\mathrm{Cos}[t\,w])}{2\,m\,w^{2}}+\frac{2\,b\,x^{3}\,(-2+\mathrm{Cos}[t\,w])\,\mathrm{Sin}[\frac{t\,w}{2}]^{2}}{3\,m\,w^{2}}+
$$

$$
\frac{x\,(-2\,m^{2}\,w^{3}\,z-2\,I\,b\,\hbar\,\mathrm{Sin}[t\,w]+I\,b\,\hbar\,\mathrm{Sin}[2\,t\,w])}{2\,m^{2}\,w^{3}}\Big))\;/
$$

$$
\hbar+b\,\Big(\frac{(-16\,m\,w^{2}\,x-8\,m\,w^{2}\,z)\,\mathrm{Log}[\mathrm{Cos}[\frac{t\,w}{2}]]}{2\,m^{2}\,w^{4}}+
$$

$$
(\mathrm{Csc}[\frac{t\,w}{2}]\,\mathrm{Sec}[\frac{t\,w}{2}]\,(-128\,I\,m^{3}\,w^{4}\,x^{3}-192\,I\,m^{3}\,w^{4}\,x^{2}\,z-
$$

$$
96\,I\,m^{3}\,w^{4}\,x\,z^{2}+96\,I\,m^{3}\,w^{4}\,x^{3}\,\mathrm{Cos}[t\,w]+
$$

$$
192\,I\,m^{3}\,w^{4}\,x^{2}\,z\,\mathrm{Cos}[t\,w]+
$$

$$
96\,I\,m^{3}\,w^{4}\,x\,z^{2}\,\mathrm{Cos}[t\,w]+32\,I\,m^{3}\,w^{4}\,x^{3}\,\mathrm{Cos}[2\,t\,w]+
$$

$$
96\,I\,m^{3}\,t\,w^{5}\,x^{3}\,\mathrm{Sin}[t\,w]+
$$

$$
144\,I\,m^{3}\,t\,w^{5}\,x^{2}\,z\,\mathrm{Sin}[t\,w]+72\,\hbar\,m^{2}\,w^{3}\,x\,\mathrm{Sin}[2\,t\,w])
$$

$$
)\;/\;(96\,\hbar\,m^{3}\,w^{5})\Big)
$$

G2=Normal[Series[G1,{b,0,1}]]

```
                  2
                 x  Cos[t w]                          t w
I m w (-(x z) + -----------) Csc[t w]   Log[Cos[---]]
                      2                               2
------------------------------------- - ------------- -
                 ħ                             2

   t w       I       2              t w      t w
 Log[Sin[---]]   - m w z  Cos[t w] Csc[---] Sec[---]
        2        4                          2        2
 ------------- + ----------------------------------- +
       2                          ħ

         2           2              t w
   (-16 m w  x - 8 m w  z) Log[Cos[---]]
                                     2
 b (------------------------------------ +
                         2  4
                      2 m  w

       t w       t w            3  4  3            3  4  2
   (Csc[---] Sec[---] (-128 I m  w  x  - 192 I m  w  x  z -
        2         2

            3  4    2            3  4  3
       96 I m  w  x z  + 96 I m  w  x  Cos[t w] +

             3  4  2
       192 I m  w  x  z Cos[t w] +

            3  4    2                        3  4  3
       96 I m  w  x z  Cos[t w] + 32 I m  w  x  Cos[2 t w] +

            3    5  3
       96 I m  t w  x  Sin[t w] +

             3    5  2                        2  3
       144 I m  t w  x  z Sin[t w] + 72 ħ m  w  x Sin[2 t w])

                3  5
      ) / (96 ħ m  w ) +

                           2
   (I m w Csc[t w] (x  (-2 z + 2 z Cos[t w])
                    ----------------------- +
                                   2
                              2 m w

        3                       t w 2
     2 x  (-2 + Cos[t w]) Sin[---]
                                 2
     ----------------------------- +
                   2
              3 m w

     x (-2 I ħ Sin[t w] + I ħ Sin[2 t w])
     ------------------------------------)) / ħ)
                      2  3
                   2 m  w
```

G3=C Exp[G2]

$$
\begin{aligned}
&C\,\mathrm{Exp}\Bigg[\frac{I\, m\, w\left(-(x\, z)+\frac{x^{2}\,\mathrm{Cos}[t\, w]}{2}\right)\mathrm{Csc}[t\, w]}{\hbar}-\frac{\mathrm{Log}[\mathrm{Cos}[\frac{t\, w}{2}]]}{2}-\\
&\frac{\mathrm{Log}[\mathrm{Sin}[\frac{t\, w}{2}]]}{2}+\frac{\frac{I}{4}\, m\, w\, z^{2}\,\mathrm{Cos}[t\, w]\,\mathrm{Csc}[\frac{t\, w}{2}]\,\mathrm{Sec}[\frac{t\, w}{2}]}{\hbar}+\\
&b\Bigg(\frac{(-16\, m\, w^{2}\, x-8\, m\, w^{2}\, z)\,\mathrm{Log}[\mathrm{Cos}[\frac{t\, w}{2}]]}{2\, m^{2}\, w^{4}}+\\
&\quad(\mathrm{Csc}[\tfrac{t\, w}{2}]\,\mathrm{Sec}[\tfrac{t\, w}{2}]\,(-128\, I\, m^{3}\, w^{4}\, x^{3}-192\, I\, m^{3}\, w^{4}\, x^{2}\, z-\\
&\qquad 96\, I\, m^{3}\, w^{4}\, x\, z^{2}+96\, I\, m^{3}\, w^{4}\, x^{3}\,\mathrm{Cos}[t\, w]+\\
&\qquad 192\, I\, m^{3}\, w^{4}\, x^{2}\, z\,\mathrm{Cos}[t\, w]+\\
&\qquad 96\, I\, m^{3}\, w^{4}\, x\, z^{2}\,\mathrm{Cos}[t\, w]+\\
&\qquad 32\, I\, m^{3}\, w^{4}\, x^{3}\,\mathrm{Cos}[2\, t\, w]+\\
&\qquad 96\, I\, m^{3}\, t\, w^{5}\, x^{3}\,\mathrm{Sin}[t\, w]+\\
&\qquad 144\, I\, m^{3}\, t\, w^{5}\, x^{2}\, z\,\mathrm{Sin}[t\, w]+\\
&\qquad 72\,\hbar\, m^{2}\, w^{3}\, x\,\mathrm{Sin}[2\, t\, w]))\,/\,(96\,\hbar\, m^{3}\, w^{5})+\\
&\quad\Bigg(I\, m\, w\,\mathrm{Csc}[t\, w]\,\Bigg(\frac{x^{2}\,(-2\, z+2\, z\,\mathrm{Cos}[t\, w])}{2\, m\, w^{2}}+\\
&\qquad\frac{2\, x^{3}\,(-2+\mathrm{Cos}[t\, w])\,\mathrm{Sin}[\frac{t\, w}{2}]^{2}}{3\, m\, w^{2}}+\\
&\qquad\frac{x\,(-2\, I\, \hbar\,\mathrm{Sin}[t\, w]+I\, \hbar\,\mathrm{Sin}[2\, t\, w])}{2\, m^{2}\, w^{3}}\Bigg)\Bigg)\,/\,\hbar\Bigg)\Bigg]
\end{aligned}
$$

G4=Simplify[G3]

$$
\begin{aligned}
C\,\mathrm{Exp}\Big[ & \frac{\frac{I}{2}\, m\, w\, x\, (-2 z + x \,\mathrm{Cos}[t\, w])\, \mathrm{Csc}[t\, w]}{\hbar} - \frac{\mathrm{Log}[\mathrm{Cos}[\frac{t\, w}{2}]]}{2} - \\
& \frac{\mathrm{Log}[\mathrm{Sin}[\frac{t\, w}{2}]]}{2} + \frac{\frac{I}{4}\, m\, w\, z^2\, \mathrm{Cos}[t\, w]\, \mathrm{Csc}[\frac{t\, w}{2}]\, \mathrm{Sec}[\frac{t\, w}{2}]}{\hbar} + \\
& b\, \Big(\frac{x}{m\, w^2} + \frac{2\, I\, t\, x^3}{\hbar} + \frac{3\, I\, t\, x^2\, z}{\hbar} + \frac{2\, x\, \mathrm{Cos}[t\, w]}{m\, w^2} - \\
& \frac{8\, x\, \mathrm{Log}[\mathrm{Cos}[\frac{t\, w}{2}]]}{m\, w^2} - \frac{4\, z\, \mathrm{Log}[\mathrm{Cos}[\frac{t\, w}{2}]]}{m\, w^2} - \\
& \frac{\frac{I}{2}\, x^3\, \mathrm{Sec}[\frac{t\, w}{2}]\, \mathrm{Sin}[\frac{3\, t\, w}{2}]}{\hbar\, w} - \frac{\frac{7\, I}{2}\, x^3\, \mathrm{Tan}[\frac{t\, w}{2}]}{\hbar\, w} - \\
& \frac{5\, I\, x^2\, z\, \mathrm{Tan}[\frac{t\, w}{2}]}{\hbar\, w} - \frac{2\, I\, x\, z^2\, \mathrm{Tan}[\frac{t\, w}{2}]}{\hbar\, w}\Big)\Big]
\end{aligned}
$$

# VITA

Mr. Thera Thara was born on Oetober 5, 1961 in Prachuapkirikhan province. He recieved a B.S. degree in physucs from Ramkhamhaeng University in 1989. He is now a teacher in Muthayuomtrakanputphon School, Ubon Ratchathani