CHAPTER V

CONCLUSION AND DISCUSSION

In this thesis, our problem deals with the vortices in superfluid or superconductor which are pinned in the pinning potential. These vortices can be treated as the point particles which are confined in the infinite two dimensional plane ($x-y$ plane) and these two degrees of freedom (x and y degree of freedom) of each vortex couple with each other via their velocity because of the effect from the Magnus force. In the sample of temperature T, if we pay attention to only one vortex, we can treat everything except this one vortex as its environment. From this point of view, this vortex can be treated as a Brownian particle surrounded by the environment at temperature T. Since the quantum mechanical treatment is required in our problem, the standard Gaussian random process methods in classical Brownian motion theory, such as the method of Rice and Fokker-Planck's equation, with the use of Langevin's equation should not be used, instead one should use the quantum Brownian motion theory. As in standard quantum Brownian motion theory, the environment is modeled by the set of uncoupled harmonic oscillator which is linearly coupling with Brownian particle i.e., a vortex in our problem. This model is sometimes called the Caldeira-Leggett model.

From the above discussion, the hamiltonian of our problem can be written as in eq. (4.7). By following P.Ao and D.J. Thouless [19], the vortex potential $V(\bar{r})$ will be written as the sum of pinning potential in x direction and vortex potential $V(y)$. The pinning potential in x direction is taken approximately to be the harmonic potential characterized by frequency $\omega_x = \sqrt{k_x/M}$ and the potential $V(y)$ consists of the contributions from SFVDP Magnus force and pinning potential in the y direction, which has the metastable point at $y=0$. Since the potential $V(y)$ is the metastable potential, a vortex may have some chances to leave the metastable well in y direction by both thermal hopping and quantum tunneling. In our work, we have mainly studied the crossover temperature, the localization of a vortex in the well, and the rate for which a vortex escapes out of the well. The formula that specifies this rate is the escape rate formula. This formula is divided into two regions, $T>T_0$ and $T<T_0$ where T_0 is the crossover temperature. In both regions,

the rate formula of a vortex can be found if one knows the energy of a vortex. Since a vortex can escape in y direction only, the free energy of a vortex which one wants to know in order to find the escape rate, must be related with the effective one-dimensional reduced partition function, Z_d, in y direction by $F = -(1/\beta)\ln Z_d$. Hence, the problem of finding the free energy is equivalent to the problem of finding the reduced partition function Z_d.

The effective one-dimensional reduced partition function has been found by the use of functional integral or path integral method in standard quantum Brownian motion theory. After eliminating all of the environmental coordinates and x degree of freedom of a vortex, we got Z_d in the form $Z_d = \oint Dy(\tau)\exp\left(-S_{\mathrm{eff}}^{\mathrm{E}}[y]/\hbar\right)$. The effective action $S_{\mathrm{eff}}^{\mathrm{E}}[y]$ reflects the nonlocal behavior of a vortex in y direction. This nonlocal behavior is described by both normal and anomalous damping kernels. The normal damping kernel arises because the coordinate y of a vortex couple linearly with the environmental coordinates. The anomalous damping kernel, on the other hand, arises from the fact that the coordinate x of a vortex couple linearly with the environmental coordinates, and also due to the presence of the Magnus force the coordinate y of a vortex couples linearly with the velocity of the x degree of freedom of a vortex. By this reason, the normal damping kernel contains the influence on the y coordinate of a vortex from the environment while the anomalous damping kernel contains both the influence on x coordinate of a vortex from the environment and the influence on y coordinate from the x coordinate of a vortex. Since the anomalous damping kernel contains these influences and the pinning potential in x direction is characterized by ω_x, the anomalous damping kernel depends on the parameters Ω, ω_x, and ξ_n (see eq. (4.39)) which characterize respectively the effect from x degree of freedom of a vortex due to the Magnus force, the pinning potential in x direction, and the dissipation due to the environmental coupling. The appearance of the anomalous damping kernel in the effective action $S_{\mathrm{eff}}^{\mathrm{E}}[y]$ is a crucial point in this thesis. The parameter containing in the anomalous damping kernel that makes the important differences on the physical situation is ω_x. The main results of our study are as follows:

First, the crossover temperature can be obtained via the frequency ω_R where ω_R is the positive root of the equation $x^2 + x\hat{\gamma}_M(x) = \omega_b^2$ (see eq. (4.70)). From theorems 1 and 2 in chapter IV, we have shown that the unique crossover temperature always exists if $\omega_x \neq 0$ or if $\omega_x = 0$ plus the condition $\Omega^2 \Big/ \left[1 + (2/M\pi)\int_0^\infty J(\omega)/\omega^3 d\omega\right] < \omega_b^2$. On the other hand, if $\omega_x = 0$ but $\Omega^2 \Big/ \left[1 + (2/M\pi)\int_0^\infty J(\omega)/\omega^3 d\omega\right] > \omega_b^2$, then there exists no crossover temperature(by this reason, the condition $\Omega^2 \Big/ \left[1 + (2/M\pi)\int_0^\infty J(\omega)/\omega^3 d\omega\right] > \omega_b^2$ is called the criterion for the non-existence of crossover temperature) which implies that the bounce trajectory will not exist so the decay by quantum tunneling will never occur and the functional integral for Z_d is dominated only by the trivial paths $y = 0$ and $y = y_b$ for the whole range of temperature. Moreover, if the criterion for the non-existence of crossover temperature is fulfilled, then $\lambda_0^{(b)}$ is positive so the reduced partition function $Z_d^{(b)}$ dominated by the trivial path $y = y_b$ is finite and real which implies that the reduced partition function $Z_d = Z_d^{(0)} + Z_d^{(b)}$ is also finite and real. Hence the situation, for which the criterion for the non-existence of crossover temperature is fulfilled, implies that a vortex is localized in the well for the whole temperature range. By this reason, the criterion for the non-existence of crossover temperature is identical with the localization criterion which can be used at any temperature. Note that our localization criterion is reduced to the localization criterion in the case of no pinning and dissipation at zero temperature given by P. Ao and D.J. Thouless [19].

Second, the crossover temperature T_0 is always a decreasing function of the dissipation strength when $\omega_x \geq \omega_b$ only. On the other hand, if $\omega_x < \omega_b$, then the relative extrema of T_0 may occur. However, the crossover temperature is always a decreasing function of dissipation strength within the large strength limit. Moreover, being independent of the relation between ω_x and ω_b, the crossover temperature is always a decreasing function of Magnus force strength. If $\omega_x = 0$, then the dissipation strength (with the condition $\Omega \geq \omega_b$) and the Magnus force strength will be restricted by the lower and upper bound respectively because of the effect from the criterion for the non-existence of crossover temperature or equivalently the localization criterion. When looking at the whole trends of these behaviors, they imply that the system has the tendency to decay by thermal activation when the dissipation or Magnus force strength is getting stronger.

Third, the escape rate formula of a vortex for $T > T_0$ has been derived based on Affect's formula [31]. The escape rate formula for $T < T_0$ will not be dealt with since the escape rate depends on the action S_B of the bounce which is very difficult to find because one has to know the bounce solution by solving the nonlinear integro-differential equation with periodic boundary condition. The escape rate formula of a vortex for $T > T_0$ is divided into two cases, k for $\omega_x \neq 0$ and $\widetilde{k}$ for $\omega_x = 0$ provided that the localization criterion is violated i.e., $\Omega^2 \Big/ \left[1 + (2/M\pi)\int_0^\infty J(\omega)/\omega^3 d\omega\right] < \omega_b^2$. The rate formula shows that the escape rate decreases exponentially when the temperature decreases. The factor ρ in k implicitly contains the effect from the environment characterized by $J(\omega)$and x degree of freedom of a vortex characterized by Ω via ω_R. The factor $\widetilde{\rho}$ in $\widetilde{k}$ contains both of these effects implicitly via ω_R and explicitly via ω_{0M} and ω_{bM}. In both cases, the quantum effect on escape process is contained in quantum-mechanical enhancement factor c_{qm}. However, one should be careful to use these escape rate formulae for very low damping region since the influence of the environment is not strong enough to maintain thermal equilibrium in the well and once this nonequilibrium effect occurs within the well, these rate formulae which are based on the thermodynamics method pioneered by Langer [29] may fail. However, this weak coupling region characterized by $\gamma \leq \omega_b k_B T/\nu_b$ [16] is very small particularly for the system with high barrier.

Fourth, we have defined the effective mass of a vortex within the sense of the localization criterion by $M^* = M + (2/\pi)\int_0^\infty J(\omega)/\omega^3 d\omega$. This effective mass has the meaning that, in the presence of dissipation, a damped vortex behaves as if it is an undamped vortex which is effectively uncoupled or freed from its environment but its mass must change to the effective one. In other words, the dissipative effect is put into the original mass in order to make a vortex effectively uncoupled from its environment. Since our definition of effective mass does not come from the dynamical approach, we can not be sure what coordinates are used in describing the effective undamped vortex of mass M^*. However, by introducing new coordinates and new masses, we have shown that the coordinates used in describing this effective undamped vortex is identical with the original coordinates of a damped vortex, which is also identical with the center of mass coordinates of the system, when the system is in the infinite plane. On the other hand, when the system is confined in the finite plane, this conclusion will be true only when the environmental coupling is weak enough. In other words, being independent of the environmental coupling, an original damped vortex can be replaced by an undamped vortex located at the same position which is identical with the center of mass of the system containing the total mass M^* when the system is in the infinite plane while this statement is true only for sufficiently weak environmental coupling when the system is confined in the finite plane.

Although our problem deals with the vortices in superfluid or superconductor, all of the results in this thesis can be used for the electron in magnetic field since the form of the Magnus force is the same with the form of Lorentz force but in the case of electron in magnetic field the potential $V(y)$ consists of the pinning potential in y direction only.