Chapter 6

Estimating the Production Function

The purpose of this chapter is to specify and estimate the production function parameters and to test the model for the two interesting economic hypotheses regarding the firm's. The first hypothesis is the asserted limitations on access to capital and because of that capital has had no significant effect on the output growth. The second is the firm's operation on a point of significant increasing returns to scale.

Before starting the estimation process, the data has to be adjusted and transformed into log-linear form in order to use OLS-regression. This is done in appendix 9.

The chapter starts with examining the variable specification. Second, test for the above mentioned model restrictions. Last, test for the underlying OLS-model assumptions and presenting the estimated parameters is carried out.

6.1 The Cobb-Douglas Function

The previous chapter covered the selection of model variables and measurement issues, and left us with the production function:

$$q = A(t) Lh^{\alpha} K^{\beta} \qquad (5.1)$$

6.2 Model Specification

Specifying the Error Term

»Measuring factor inputs and the production output are not without errors. If the only source to the errors is the measurement of output, the errors are likely to be additive rather than multiplicative and exponential. However, the chosen model may be on an inappropriate functional form, or simply has omitted variables. In this case, the errors are likely to be multiplicative or exponential. If we chose to "live with the problem", the error term can be included in the model as $q = A(t)\, Lh^{\alpha}\, K^{\beta}\, e^{\mu}$ «[1].

Specifying the Technical Progress Term

The efficiency plots for the two primary factor inputs (*Lh* and *K*) in *figure 6.1* and *6.2* exhibit strong trend toward increasing efficiency.

Two ways of introducing technical progress -with the normality assumption in mind- could be as either Bt^m or Be^{mt}. There is no economic theory supporting one method from the other. However, most production literature uses the latter.

The choice should nonetheless, depend on the estimated parameters' conformity with the economic theory, the expected results, and a high model fit.

The introduction of technical progress as Bt^m and Be^{mt} into the restricted model $q= A(t)\, Lh^{\alpha}\, K^{\beta}\, e^{\mu}$ produced the following results (see the appendix 1 and 2 for computer printout):

	Bt^m	Be^{mt}
Overall fit (R^2)	0.74	0.81
returns to scale (ε)	1.5	0.94
technical progress over the observed period	176%	232%

In section 5.4 it was stated that the scale of operation is highly correlated with time, and therefore makes it difficult to distinguish empirically between technical progress and returns to scale«[2].

Moreover, as discussed in section 5.3 it is strongly believed that the Danish Steelworks is producing on a significant increasing returns to scale. Hence, ε significantly over unity is expected.

Labour productivity improved by 208 percent (*figure 6.1*) in the observed period and capital productivity by 74 percent (*figure 6.2*).

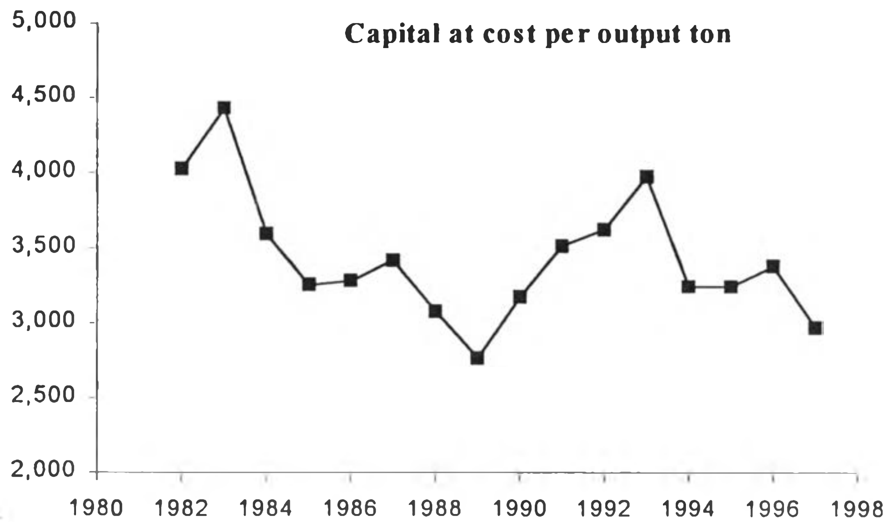
[1] David F. Heathfield and Soeren Wibe, An Introduction to Cost and Production Functions, (Houndmills, Basingstoke, Hampshire RG21 2XS and London: Macmillan Education Ltd., 1987), ch. 9.

[2] Ibid.

Figure 6.1

source: the Danish Steel Works financial statements

Figure 6.2



source: the Danish Steel Works financial statements

As discussed below a high overall model fit (R^2) is not expected because the model cannot capture the short run adjustments between output and labour hours, and output and capital.

On grounds of the above discussion, the technical progress is introduced as: $A(t)=Bt^m$, B and m are positive constants. Hence, the production function is on the form:

$$q = Bt^m \, Lh^{\alpha} \, K^{\beta} \, e^{\mu} \qquad (6.1)$$

Transforming the function into the log-linear form yields:

$$\ln(q) = C + m\ln(t) + \alpha\ln(Lh) + \beta\ln(K) + \mu$$

Where $C=\ln(B)$.

Output-Labour-Hours and Output-Capital Relationships

Output-labour: The apparent connection or relation between labour-hours and output is, for two reasons, not so apparent as it is at first sight. First, the steelwork has an incentive to stockpile intermediate-products, primarily when scrap prices are low but also when output is low and future sales prospects are good. Producing those intermediate products takes a big fraction of the total labour hours. Second, there is a strong incentive on both employers and the union's side to be reluctant to make quick "adjustments".
For that reason, the model cannot capture all adjustments between output and labour hours. Therefore, a high overall model fit (R^2) is not expected.

Output-capital: As for output-labour hours, the output-capital adjustments is not possible in the short run or medium run because cost of capital is a relatively high burden. Moreover, the European steel industry is not a high profit business. For those reasons, the company has no other alternatives than to adjust capital to the long run market expectations. Hence, a high R^2 is again not expected.

6.3 Estimated Model and Results

Model Estimates

From the regression (see the appendix 1 for computer printout):

$$\ln(q) = C + m\ln(t) + \alpha\ln(Lh) + \beta\ln(K) + \mu, \qquad (6.2)$$

$$\underset{(se)}{\ln(q)} = \underset{(5.970)}{1.174} + \underset{(0.046)}{0.199\ln(t)} + \underset{(0.323)}{0.922\ln(K)} + \underset{(0.509)}{0.625\ln(Lh)}$$

$t=$ (0.196) (4.289) (1.935) (1.812)

$R^2=0.74$ $df=12$

adjusted $R^2=0.68$

The estimated parameters are $B=3.224$, $\alpha=0.92$, $\beta=0.625$, and $m=0.17$.

From *equation 2.34* we know that the elasticity of scale: $\varepsilon=\alpha+\beta=1.545$: The company is operating on a significant increasing returns to scale, which is in conformity with the economic theory and expectations.
$m=0.17$: Technical progress is estimated to have a contribution to the output growth by 176 percent in the observed period.

This gives us the parameter estimates of the restricted Cobb-Douglas production function:

$$q = 3.224 t^{0.17} K^{0.625} Lh^{0.92} \qquad (6.3)$$

6.4 Model Restrictions and Economic Hypothesis Testing

It is, in addition to the standard assumption tests, interesting to draw attention to the tests of hypotheses regarding restrictions and the functional form of the production function.

Strong Limitations on Access to Capital

As discussed earlier: the model developed in chapter 1 and 2 is a theoretical case assuming perfect competitive markets. But there is a possibility that the company has restricted access to capital due to the financial reconstruction in the 1980's; and the high degree of self-financing on new investments.
For this reason, it is believed that the observed behaviour does not represent the long run and the testing of strong limitations on access to capital (H_0: $\beta=0$) is therefore very interesting.

The hypothesis H_0: $\beta=0$ is rejected at the 10% confidence interval. The p-value is 0.077 (see the appendix 4 for computer printout).

The result suggests that capital investments in the observed period have had a significant effect on the output growth. The interesting question is now: how does this coincide with the assertion of restricted access to capital?

The rejection of the hypothesis of strong limitations on access to capital does not mean that the company has not had any restriction on capital. The company's high degree of self-financing and the financial reconstruction in the 1980's still suggest restricted access to capital. Hence, the result can then not be used for a restriction of the model (β=0).

However, the result of ε=1.545, reinforce the hypothesis, because even with small capital investments it has been possible for the Danish Steelworks to increase capital efficiency over the years.

Increasing Returns to Scale

As just discussed: there is a strong case supporting the assumption that the Danish Steel Works, Ltd. operates on a point of significant increasing returns to scale. Hence, the production function must allow for increasing returns to scale.

However, when testing the hypothesis of constant returns to scale (H_0: $\alpha+\beta=1$), the model does not reject the hypothesis. The estimated p-value is 0.5 (see the appendix 5 for computer printout). This result suggests a possibility of constant returns to scale. On the other hand, as mentioned above, restricting the model to constant returns to scale will be to deny what is most obvious.

Therefore the final parameter estimates of the Cobb-Douglas production function is as estimated before:

$$q = 3.224 t^{0.17} K^{0.625} Lh^{0.92} \qquad (6.3)$$

6.5 Problems Concerning the OLS- Assumptions

The OLS-model is built on two groups of simplifying assumptions: (1) assumptions about the specification of the model and about the disturbances μ_i, and (2) assumptions about the data. Assumptions 1, 2, 3, 4, 5, 9, and 11 belong to the first category. Those in the second category are 6, 7, 8, and 10. In addition, data problems such as outliers and errors of measurement in the data also fall in the second category[3].

(1) The model is linear in its parameters.

(2) Fixed values taken by the regressors in repeated samples.

(3) Zero mean value of disturbances, $E(\mu_i \mid X_i)=0$.

(4) Homoscedasticity, $var(\mu_i \mid X_i)=\sigma^2$.

(5) No autocorrelation between disturbances, $cov(\mu_i, \mu_j \mid X_i, X_j)=0$.

(6) Zero covariance between disturbances and regressors, $cov(\mu_i \mid X_i)=0$.

(7) The number of observation *n* must be greater than the number of estimated parameters.

(8) Variability in regressors' values.

(9) Correctly specified regression model.

(10) There is no perfect multicollinearity.

(11) Normality of the disturbance term, $\mu_i=0$.

Assumption 1: The Cobb-Douglas function is made linear in its parameters by transforming it into a log-linear regression model.
Assumption 2 and 6: The data applied in the model are secondary data collected from the annual statements. Hence, the regressors are fixed in repeated samples, and there is zero covariance between disturbances, and repressors are strongly assumed to be fulfilled.
Assumption 7:the number of observation *n* is greater than the number of parameters.
Assumption 8: all regressors exhibit variability (see *figure 6.3 and 6.4*).
Assumption 9: the regression model is considered correctly specified and to have a strong underlying economic foundation.

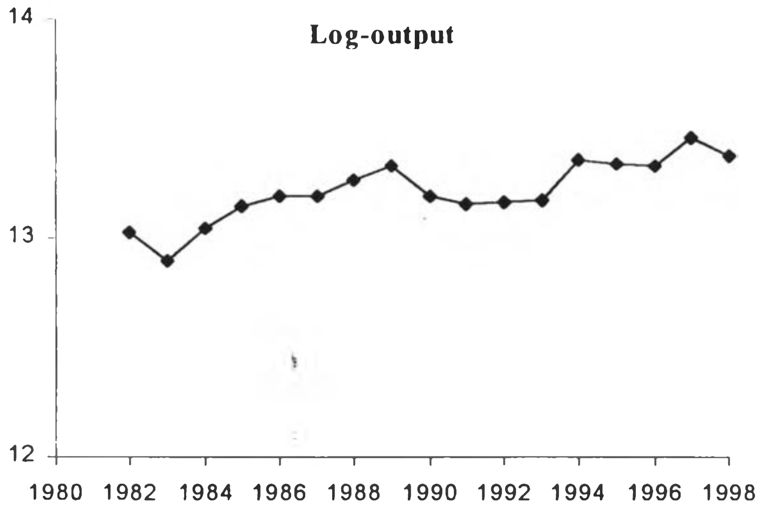
This leaves only the assumptions 3,4,5,10, and 11.

Data Problems

Outliers: A visual analysis of the model log-linear variables (see *figure 6.3* and *6.4*) suggests no reason to concern about outliers.

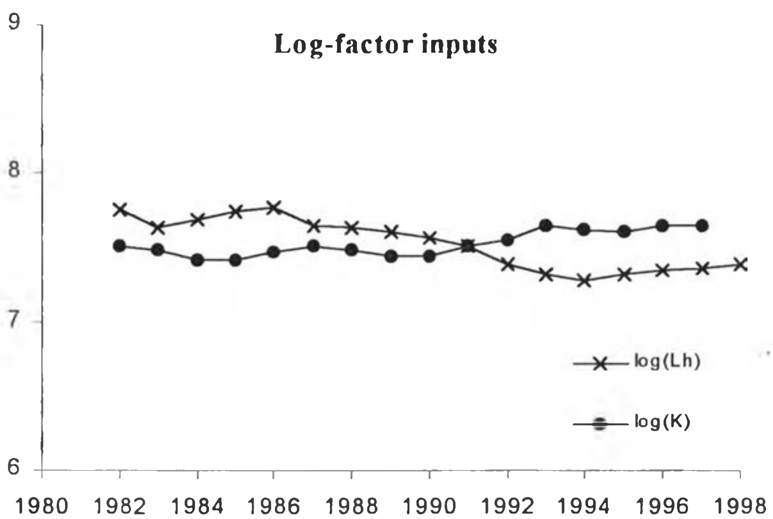
[3] Damodar N. Gujarrati, Basic Econometric, (McGraw-Hill International Edition, 1995).

Figure 6.3



source: the Danish Steel Works financial statements

Figure 6.4



source: the Danish Steel Works financial statements

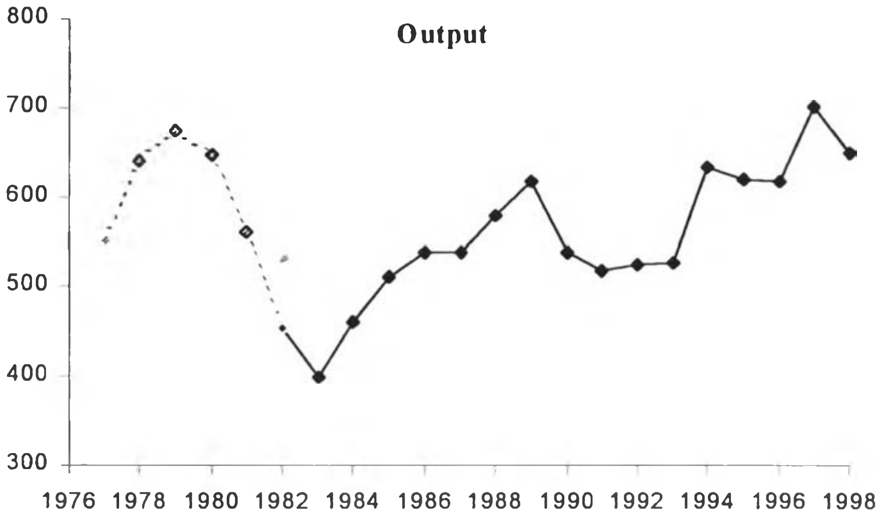
Stationarity of the time series data: »Regression analysis based on time series implicitly assumes that the underlying time series are stationary. The *t*-test, *F*-test, etc. are based on this assumption.

If the variables in question exhibit strong trend, the time series often obtain high R^2, although there are no meaningful relation. Therefore, in regression involving time series data the trend variable is often included as one of the regressors to avoid the problem of spurious correlation«[4].

In all literature of production-functions, the precedence is to introduce the time trend. The rationale is that nonstationarity is assumed deterministic and related to technical progress. Moreover, the production literature is not concerned about nonstationarity for production functions (with the trend term introduced).

Structural stability of the model: As depicted in *figure 6.5* the output was reduced drastically in the period 1978 to 1981. Closing one of the older production units in 1981 did this.

Figure 6.5 Output ton 1976 to 1998



source: the Danish Steel Works financial statements

Because, it is not possible to explain the period before 1981 and after using the same function the estimated production function will attempt to explain the period after 1981.

The company has not invested significantly in capital in the period after 1982. Therefore, it is asserted that the estimated production function is representative for the whole period and it makes therefore, no economic sense to test for structural stability.

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[4] Gujarrati, Basic Econometric.

Assumption Tests and Their Results

Multicollinearity: is a question of degree and not of kind. Therefore, we wish to measure the degree in the sample (see computer printout in appendix 6 for the result).

The signs of multicollinearity are:

(1) »High par-wise correlation among regressors (zero order correlation), in excess of 0.8«[5].

Correlation matrix estimates: r_{12}=-0.86, r_{14}=-0.81, and r_{24}=0.62. Where (*1*=ln*(K)*; *2*=ln*(Lh)*; *3*=ln*(q)*; *4*=ln*(t)*)

(2) R^2 of the auxiliary regressions exceeds the models R^2=0.74.

The model results:

$\ln(Lh) = c + \ln(K) + \ln(t);\ R^2_{\ln(Lh)} = 0.76$

$\ln(K) = c + \ln(Lh) + \ln(t);\ R^2_{\ln(K)} = 0.86$

$\ln(t) = c + \ln(Lh) + \ln(K);\ R^2_{\ln(t)} = 0.67$

Both $R^2_{\ln(Lh)}$ and $R^2_{\ln(K)}$ exceeds the overall R^2 suggesting serious multicollinearity.

The relevant implications (for this study) of multicollinearity are shortly:

(i) Estimators have large variances and covariances making precise estimation difficult.

(ii) Because of (i), the confidence intervals tend to be much wider leading to acceptance of null-hypothesis (including low *t*-ratios) more readily.

This may be a reason to explain the insignificant intersection *t*-ratio, and to explain why the hypothesis of constant returns to scale (H_0: $\alpha+\beta=1$) is not rejected.

How serious is the multicollinearity problem in the sample regression? The degree with which variances and covariances is increased can be estimated using the variance-

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[5] Gujarrati, Basic Econometric.

inflation factor (*VIF*). For the *k*-variable regression model, the variance of a partial coefficient can be expressed as:

$$se(\beta_j) = \sqrt{\operatorname{var}(\beta_j)} \qquad (6.4)$$

$$\operatorname{var}(\beta_j) = \sigma^2 / \sum x_j^2 \cdot VIF \text{, where } VIF = \left(\frac{1}{1-R_j^2}\right) \qquad (6.5)$$

Hence, with a R_j^2=0 the *VIF*=1 and the standard error (*se*) will not increase. If R_j^2=0.9 the *VIF*=5.26 and the standard error will increase 2.3 times.

Regarding whether multicollinearity is a problem or not. The rule of thumb is that if *VIF* exceeds 10 (that is R_j^2=0.95*)* that variable is said to be highly collinear, which none of them are.

The alternative to "living with the problem" is (in this study) to:

(1) Transforming the data into ratios e.g. $(q/K) = A(t)K^{\gamma}(Lh/K)^{\alpha}e^{\mu}$, where $\gamma=\alpha+\beta-1$. However, this transformed numbers does not make economic sense and we may lose valuable information in the transformation.

(2) Estimating the Cobb-Douglas production function from its dynamic version. Differentiating *equation* 5.*1* with respect to time (*t*) yields the model $q = a + \alpha Lh + \beta k$. Where the lower case letters denote the growth rates of the relevant variables and $a = (\partial \ln A(t)/\partial t)$ is the rate of technical progress.

The transformation does not make economic sense because (as mentioned in section 6.2.3) the factor inputs adjustments are *not* solely determined by the production output. Moreover, as in (1) we may lose valuable information in the transformation.

There is more to lose if solving of the multicollinearity problem is tried. Hence, the *remedy is worse than the disease*.

Serial correlation: The Durbin-Watson statistics (d) is 1.81, hence, there is no evidence of positive correlation in the estimated residuals: for n=16, *and* k'=3 the 5% critical values are d_L=0.857 and d_H=1.728 (see appendix 1 for computer printout).

The Q-statistics result: that all correlations are zero (H_0: $E(\rho_k)=0$) is accepted. That is, all autocorrelation coefficient (ρ_k)'are simultaneously equal to zero (see the appendix 7 for computer printout).

Examining the "correlogram of residuals" shows that, at all lags, ρ_k is not statistically, significantly, different from zero, and hence, there are no signs of significant correlation (see the appendix 7 for computer printout).

If the estimated ρ_k is statistically significant, an introduction of it into the model means to incorporate a false remedy that is contrary to economic facts. Because it contradicts to the facts that factor inputs adjustments are not solely done due to output -as already discussed above in section 6.2.3.

If it contradicts the economic facts to correct for the serial correlation, what cause the (in this case, rather insignificant) serial correlation? The answer is that many factors are attributable; first, factor-input adjustments are not solely determined by output -as already discussed; market shocks (drastically falling or rising prices, etc.); excluded model variables or incorrect functional form.

Heteroscedasticity: The *Whites Heteroscedasticity Test* which in addition to testing for heteroscedasticity also tests for assumption 6: $E(\mu_i X_i)=0$, and assumption 9: that "linear specification of the model is correct". The output is an F-statistic and any failure of any of these three conditions lead to a significant test statistic.
The test F-statistic is insignificant suggesting that none of the conditions is violated (se appendix 8 for computer printout).