CHAPTER 2

Methodology of Value-at-Risk

2.1 Review of the Literature

The earlier studies on the Thai government's debt does not extensively consider interest rate and exchange rate risks. Nevertheless there is a study involved in capability of repaying external debt, i.e., Pranee Thinakorn and Direk Puttamasiriwat's "Developing Countries' external debt and case of Thai government's debt" (1985) by using Debt Service Ratio that is proportion of repaying the debt to foreign income, including export earning and net unilateral transfer, per gross domestic product (GDP). The study concludes that income of export and services is not enough for the need of importing necessary goods and services, so the payment of external debt depends on foreign cash flow during 1976-1983. However if there are problems in foreign investment and external debt. The government might face liquidity problem or foreign currencies might be scarce in Thai's financial system. In spite of these, capability of repaying external debt had many weaknesses that might cause extreme fluctuation of interest rates and exchange rates, impeding the government to determine a practical target.

Then, an issue of Thai's debt management is studied by Wanarat Mingmaneenakin (1985) on the topic of "Management of Thai's Public Debt". She states that, for effective management and maximization of public debts, the government should rearrange every step of operations, such as disbursement and accounting system, etc., in harmony with it external borrowing plans. However, her study is mainly focused on general matter, without showing risk assessment and management for the government.

For the studies about estimating VaR of holding portfolio in a certain period of time, such studies can be applied to estimate maximum present value of Thai government's external debt.

For this thesis, Value-at-Risk, a practical concept, defines the maximum change in payments for government's debts which could lose at a certain level of confidence over a given time horizon when exchange rates and interest rates play greater roles. VaR approaches have several ways such as Historical, Analytical, Stochastic Simulation. Merrill Lynch uses historical method for risk measurement that would be precise when portfolio composition is relatively constant over time[1]. This method uses profits and losses of actual historical data to built a distribution for calculating VaR.

Risk Metrics proposes a significant view, that is, use of cash flow mapping where it standardizes the risk vertices of cross products.There are two ways for searching VaR of total portfolio. The first one is the Analytical approach[2] of Risk Metrics.Secondly, JP Morgan research team uses a stochastic simulation approach known as Monte Carlo Simulation technique[3]. When dealing with correlated variables based on a large covariance matrices under stochastic simulation, the recent data will be randomly picked up in order to simulate a distribution. This approach focuses on the most recent data to show portfolio characteristic. Under simulation processes, we can

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[1] To see Merrill Lynch's 1995 annual report

[2] To see Kenneth Sleong, "The Right Approach", A Risk Supplement, June 1996,page 8-12

[3] For more description, to see John R.Canada, William G.Sullivan and John A.White, Capital Investment Analysis for Engineering and Management ,2nd ed. page 314-335

use smaller amount of data to show mutual relationship and performance for precise prediction of any future losses. Both Analytical and Monte Carlo technique have been proposed in document of JP Morgan known as Risk Metrics.

Initially, Value-at-Risk was released by JP Morgan in 1994 and then again revised to fit optionally in 1995 .It showed an assumption that the returns of all assets and liabilities are normally distributed. The normal distribution is the statistical properties of financial time series[4]. Richardson and Smith (1993) give some example about a large percentage of such properties focus on daily returns which can be concluded that return distributions had fat tails called excess Kurtosis. The peak around the mean of the return distribution is higher than that predicted by the normal distribution and there were more observations in the left-hand tail than in the right-hand tail called negative skewness. This is due to the fact that, on the one hand, we want to use as many time series as possible to cover all possible events of assets in a portfolio, but on the other hand, we want to use relatively short sample periods so that parameter estimates react sufficiently quick to new information. For these reasons, it will be beneficial to apply Monte Carlo Simulations.

A significant key of Monte Carlo technique is to generate new cash flow outcomes with random selection. In dealing with random variables, there is greater uncertainty that a normal distribution is reasonable as approximation of such outcomes. Beder (1995)[5] has tested VaR methodologies in many ways. He studies eight common VaR methodologies applied to three hypothetical portfolios. For each methodology presented, VaR is calculated for both one-day and two-week time horizons. The first

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[4] The studies of Mandelbrot (1963) and Fama (1965)
[5] Tanya Styblo Bedar, "VaR: Seductive but Dangerous", Financial Analysts Journal, September-Octerber 1995 page 12-24

methodology, historical simulation, is performed twice by changing the data base used from the past 100 trading days to the past 250 trading days. The second methodology, Monte Carlo simulation, also is performed twice by changing the correlation estimates from the JP Morgan Risk Metrics data set to those from the BIS/Basle Commit Proposal Differences in correlation. Differences in correlation estimate between Risk Metrics and BIS/Basle are significant. Risk Metrics permits correlation across all asset classes, using exponentially weighted daily historical observations, but the BIS/Basle proposal permits correlation only within asset classes, not across, effectively forcing the correlation between asset classes to be plus or minus 1, whichever produces the higher estimate of VaR.

Beder warns the danger of using VaR methodologies and sets longer time horizons to be appropriate for instruments. For example, he shows a surprising result of historical simulation method that VaR of one-day time horizon is higher than two-week time horizon in the result where approach based on two-week time uses the average return within a specific historical period, as another approach uses the actual return within one day. In addition, he points out the differences of VaR result on the same assumptions when he changes from historical simulation to Monte Carlo simulation because difference in VaR is driven by specific historical sample against the relative randomness of key variables. So this article urges us to regard the crucial factors for objectives we wish to analyze over any horizon, or selected methodologies.

Leslie McNew (1996)[6] recommends that JP Morgan's Risk Metrics is a good choice for assets and liabilities which has no optionality such as stocks, bonds and foreign exchange as the stochastic simulation method would be suitable for other

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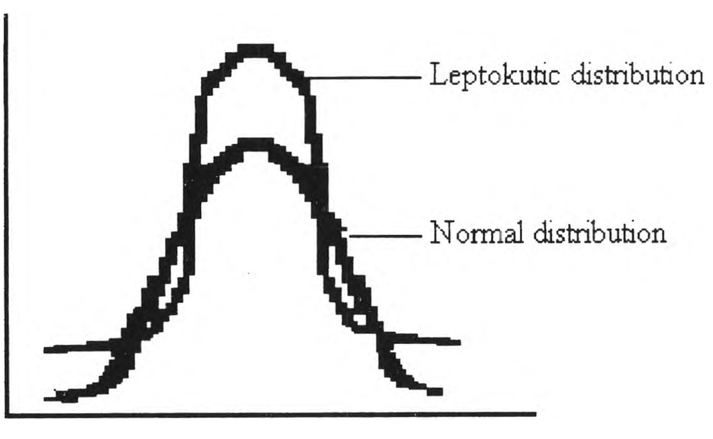
[6]Leslie McNew, "So near ,So VaR" ,risk ,October 1996,page 7-9

optional works. For the historical method, although it is easy to explain to senior management but there might be shortcoming, if portfolio mix changes over time. Since effect of specified historical data would be disappear within short time less than that by using other methods such as analytical Monte Carlo which estimates a whole distribution of observations, she suggested a problem of the main assumption that portfolio returns are normally distributed. In some risky event, such as the devaluation of the Mexican peso, the 1987 crash and the Gulf War, these events could not be forecasted or hedged, so forcing the extreme observations during the periods of event risk to obtain leptokutic distribution which has a higher peak and flatter tail.



On the other hand, we face two sensitive assumptions that portfolios are both marked to market and liquidated easily. In reality, Thai government bonds could not based on these assumptions, but we would instead consider David Shimko's article.[7] He states that "many corporate have expressed unease with traditional VaR for corporate risk management, on the grounds that their future cash flows are not marked to market. Indeed, it could never be traded", therefore, instead of being marked to

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[7] David Shimko, "VaR for cooperates", Risk , June 1996 , page 28

market and liquidated, indeed cash flow mapping is a good alternative for illiquid instruments. Despite the calculation of VaR, to test if those methods are correct, we should have techniques for verifying the accuracy of VaR measurement.

Kupiec (1995)[8] tests VaR model accuracy, but examining might be erroneous unless a large sample of historical data is available. VaR models are different from regression models, as there is no ex ante measure of their goodness of fit similar to the R^2 statistic. Nonetheless, accurate VaR estimates of risk should correspond to the actual profit and loss experience observed for the portfolio over time. Kupiec's tests are based on the binary nature of the outcome, either the actual dollar loss on the portfolio, less than the ex ante estimate, is a success, or greater, is a failure. Kupiec considers the amount of time that elapses until the first inaccurate VaR prediction. Intuitively, the smaller the percentage of time that one would expect a loss of a given size, the longer the time interval one would expect until a loss of that size is observed.

If a loss of that magnitude is observed after only a brief lapse of time, one could infer an inaccurate VaR estimate. In case of Leptokurtic distribution, extreme outcomes occur more frequently than in normal distribution. Kupiec points that such a test is weak because these extreme outcomes in fat-tails of distribution causes inaccuracy by underestimating of the risks from another statistical test. Kupiec uses observed historical data for determining the maximum dollar amount of loss observed; say, 5 percent of time, compared with the VaR model's ex ante estimate of dollar amount at risk over that period where the result of Monte Carlo technique is suffered errors less than that of historical simulation.

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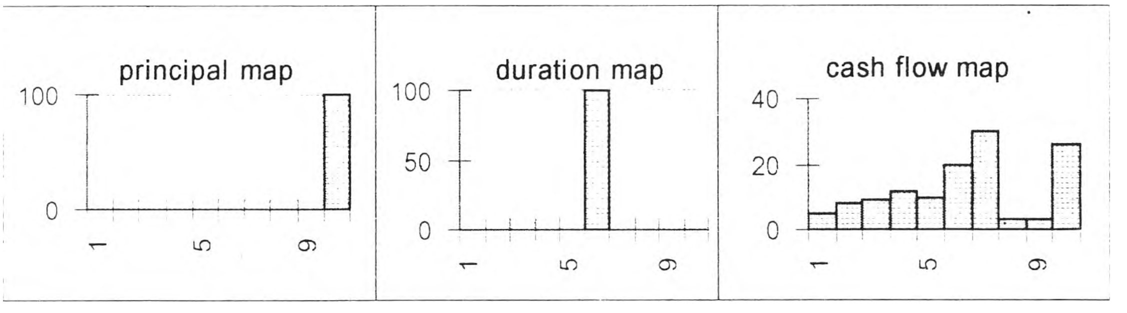
[8]Paul H.Kupiec, "Techniques for Verifying the Accuracy of Risk Management Models", The Journal of Derivatives, Winter 1995 ,page 73-84

2.2 The concepts of Mapping positions of bonds

There are three general concepts to map positions of bonds whose value is principally based on interest rates ,namely principal maps, duration maps and cash flow maps[9].



Principal maps assume that all interest payments occur at the current market rates which is not a correct assumption to longer maturity bonds and to a position of unstable interest rates. Then, for duration mapping, it relates the current market price of the bond to the present value of all cash flow as follows:

$$p = \sum_{t=1}^{n} c_t /(1+r)^t$$

where p = present value of bond

r = yield to maturity on the bond

t = time period in which the coupon or principal payment occurs

c_t =interest or principal payment that occurs in period t

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[9] Morgan Guaranty Trust Company Global Research, Risk Metrics-Technical Document 3rd.ed., May 1995 page 107-116

With duration mapping, we can measure the weighted average time to full recovery of principal and interest payments as follows

$$D = [\sum_{t=1}^{n} (c_t * t)/(1+r)^t] / \sum_{t=1}^{n} c_t /(1+r)^t$$

An adjusted measure of duration called modified duration (Macaulay duration) which could be shown as

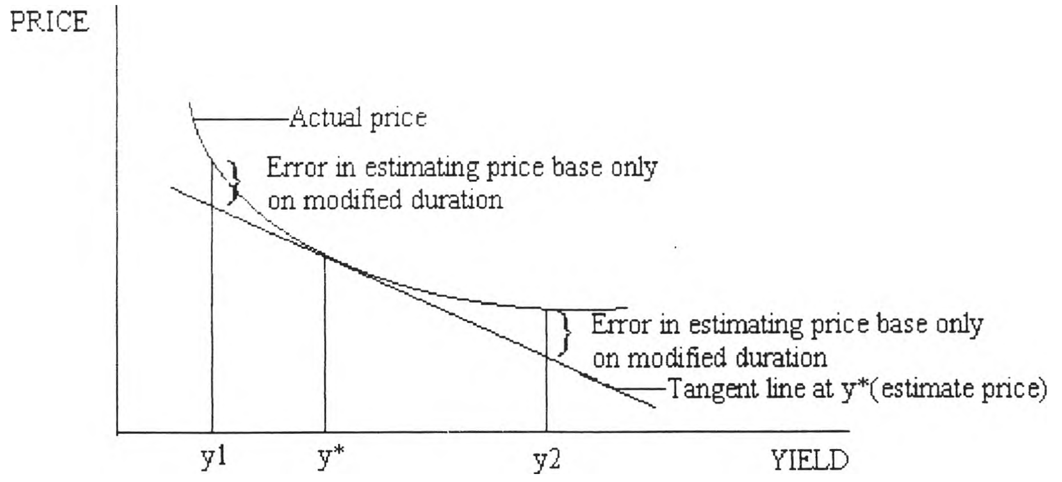
$$D_{mod} = (1/p)*(dp/di)$$

Notice that the dp/di line is tangent to the price yield curve at a given yield, so modified duration is the percentage change in price for a nominal change in yield where we can measure the curvature of the price-yield relationship mathematically with the second derivative of price with respect to yield, as follow:

$$\text{Convexity} = (1/p)*(d^2p/di^2)$$

where $$d^2p/di^2 = (1/(1+i)^2)* \sum_{i=1}^{t-1} (c_t *(t^2+t))/(1+i)^t$$

As duration measure linearly percentage change in price for the change in yield ,convexity is a measure of how much a bond's price-yield curve deviates from the linear approximation of that curve .Based upon this concept, change in a bond price is up to price change due to duration and convexity shown as



David Blake and J.Michael Orszag (1996) illustrated the use of duration and convexity together by taking a second-order Taylor expansion of the present value with respect to yield.

$$p = [\sum_{t=1}^{T} d/(1+r)^t] + B/(1+r)^T$$

where p = present value (current price) of the bond

d = coupon payment per period

B = face value of the bond

r = yield to maturity

T= maturity of the bond in number of periods

then, David and Michael got the following:

$$\Delta p = -MD*\Delta r*p + (C/2)*(\Delta r)^2*p$$

where MD is the modified duration defined as:

$$MD = D/(1+r)K$$

where K is the number of coupon payment period per year (e.g.,K=2 for a semiannual bond) . D is the duration defined by:

$$D = \frac{d}{p}\,\frac{[(1+r)^{T+1} - (1+r) - rT]}{r^2(1+r)^T} + \frac{B}{p} * \frac{T}{(1+r)^T}$$

and C is defined by the closed-form formula for bond convexity, as follows:

$$C = -\frac{d}{p}\left[\frac{(T+1)(T+2)(1/1+r)^{T+2}}{r} + \frac{2[(T+2)(1/1+r)^{T+2} - (1+r)]}{r^2} + \frac{2[(1/(1+r)^{T+2} - (1/(1+r)]]}{r^3}\right] + \frac{B}{p} * \frac{T}{r^2(1+r)^T}$$

For a conventional method, duration mapping is a method using Macaulay duration which measures a bond price sensitivity to changes in interest rates, and then the modern portfolio to measure convexity. Although this concept has been widely used by fixed income managers, it has had a short coming for calculating a risk of all cross products to express portfolio duration.

Another concept, cash flow mapping, which could have been used for this thesis, is the decomposition of present cash flows into standard vertices of synthetic zero coupon bonds by discounting future cash flows with interest rate positions of such standard vertices .The approximation of the original cash flow's price volatility can be obtained by multiplying its modified duration with its interpolate yield and interpolate yield volatility. To solve the allocation (a) of cash flow of each standardized risk vertices, we apply variations (σ^2) of them as follow

$$\sigma^2_{m.n} = a^2 * \sigma^2_m + (1-a)^2 * \sigma^2_{m+1} + 2*a*(1-a)*\rho_{m,m+1} * \sigma_m * \sigma_{m+1}$$

where

$\sigma^2_{m.n}$:variation of cash flow's volatility price of bond having m.n years of maturity

$\rho_{m.n}$: covariance of cash flow's volatility price of bonds having m and m+1 years of maturity

therefore, we can get cash flow of m and m+1 standardized risk vertices as

$CF_m = a * CF_{m.n}$ and $CF_{m+1} = (1-a) * CF_{m.n}$

So this cash flow maps will show the distribution of the current market value of future cash flows over time. From such distribution, we can estimate a maximum potential loss over a certain percentage of time within a given period known as Value-at-Risk.

2.3 Value-at-Risk Modeling

We should look back to the concept of VaR methodology which is the possible value of maximum loss at a confident level. "Value-at-Risk was first transformed from a pleasing concept to a working reality" , according to Dennis Weatherson , chairman of JP Morgan[10]. For this thesis, VaR is used in form of payments of debts where another definition, holding on the mathematical meaning of VaR, showed maximum change in payments of debts at a given level of confidence over a given time horizon. We proposed concepts of VaR measurement on three main methods, namely:

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[10] see "Variations on a theme", a risk special supplement ,June 1996 ,page 2

1. Historical method
2. Analytical method
3. Structure Simulation method

Historical method is collection of historical market date of every risk factor that affect value of products. The market data of Thai government's bonds, however, have hardly indicated their actual market prices, due to low liquidity problems, while government' s direct external loans have almost no transaction after launching. The advance statistical approach and modern portfolio theory are used to measure and explain mapping of cash flows into the risk factor's vertices. We can use analytical and Structured simulation method to simulate present values of payment cash flows and their volatility.

2.3.1 Analytical Method

To decompose bonds and loans into cash equivalent positions, we will create yield curves with swap rates. Such a swap is the trade-off between floating rates of FRN and fix rate which have similarly been FRN prices being equal to par values of bonds.To apply all classes of products suitable to decompose Thai government bonds and debentures, we map cash flow of the products onto standardized risk vertices. Discount factor is present value of one unit which was paid in future. To get the zero coupon rate for the several maturity coupon bond, we can perform bootstrapping for building yield curves with the summations of all the present value of the payments prior to the period that we want to estimate the zero rate and then get zero coupon rate of each period to assume principal equal to one. Hence, we get the following:

$$1 = \sum_{I=1}^{t-1} c_i * e^{-Ri*I} \quad + \quad (c_t + 1)*e^{-Rt*T} \qquad \text{or}$$

$$R_t = \frac{1}{t} * \log\left(\frac{1+c_t}{1-\sum c_I * e^{-Ri*I}}\right)$$

where

$c_{i,t}$: fix rate or price of THB interest rate swaps on year(i,t)

R_i : known year(i) zero coupon rate .

R_t : t-year zero coupon rate

Zero coupon rates of each period were derived accordingly for simulating yield curves of zero coupon bonds. Then we consider risk factors of the government debts comprising two groups, interest rate and exchange rate risk factor, including 1-year to 10-year zero coupon rate and Sch/THB, C$/THB, SFr/THB, DM/THB, DKr/THB, FFr/THB, £/THB, Y/THB, SR/THB, U$THB and ECU/THB exchange rate. These interest rate risk factors are values with zero coupon rates in each periods and the exchange rate rise factors are valued with market values.for zero coupon bond. We can find dp/p (return of prices), as follow:

$$p = \frac{1}{(1+y)^t}$$

where p : price of zero coupon bond

y : yield of zero coupon bond

t : year to maturity

hence

$$dp/dy = \frac{-t}{(1+y)^{t+1}}$$

$$dp/dy = \frac{-t}{(1+y)} * \frac{1}{(1+y)^t}$$

$$dp/dy = \frac{-t*p}{(1+y)}$$

$$dp/p = \frac{-t*dy}{(1+y)}$$

and for exchange rate ,we can find dp/p from

$$dp/p = \frac{\ln(ex_{t+1})}{\ln(ex_t)}$$

where

ex : exchange rates of baht against the foreign currencies

we can estimate daily volatility by risk factors follow as

$$\ln(p_t) = \delta_t + \ln(p_{t-1}) + \varepsilon_t \quad \text{where } t=1,2,\ldots,T$$

δ_t : a non-random drift parameter

ε_{t-1} : an independent and identically distributed normal random variable $N \sim (0,\sigma^2)$

or rewrite

$$\ln(p_t) = \delta_t + \delta_{t-1} + \text{---} + \delta_1 + \ln(p_0) + \varepsilon_1 + \varepsilon_2 + \text{---} + \varepsilon_t$$

$$= \sum_{t=1}^{T} \delta_1 + \ln(p_0) + \sum_{t=1}^{T} \varepsilon_t$$

we let $\alpha = \sum_{t=1}^{T} \delta_1 + \ln(p_0)$ and $\mu_t = \sum_{t=1}^{T} \varepsilon_t$

where the variance of $\ln(p_t)$-$\ln(p_{t-1})$ is

$$E[((\alpha+\mu_T)-E(\alpha+\mu_T))^2] = E[(\sum_{t=0}^{T-1} \varepsilon_{T-t})^2] = T*\sigma^2 \text{ ,by ignoring } \delta_t$$

Since $\ln(p_T)$ - $\ln(p_0)$ is a T-period return , if σ_t is the time t standard deviation of daily returns, the standard deviation of T-period return is

$$\sigma_{t,T} = T^{1/2} * \sigma_t$$

to derive covariance between a time series of data X and that of Y , we let

$$\ln(p_{1t}) = \ln(p_{1t-1}) + \varepsilon_{1t}$$

$$\ln(p_{2t}) = \ln(p_{2t-1}) + \varepsilon_{2t}$$

where t=1,2,...,T and ε_{it} are independent normal $(0,\sigma^2)$,then X,Y are T-period payment of p_{1t} and p_{2t} respectively, therefore

$$X = \ln(p_{1T}/ p_{10}) = \sum_{t=1}^{T} \varepsilon_{1t}$$

$$Y = \ln(p_{2T}/ p_{20}) = \sum_{t=1}^{T} \varepsilon_{2t}$$

Accordingly, we can express the covariance between X and Y as

$$\sigma^2_{m,xy} = E(XY)-E(X)*E(Y)$$

$$= E(\sum_{t=1}^{T}\varepsilon_{1t} * \sum_{t=1}^{T}\varepsilon_{2t}) - 0$$; ε_{1t} and ε_{2t} are serially uncorrected so, E(X) = E(Y) =0

$$= T^*\sigma^2_{XY} \qquad \text{where } \sigma^2_{XY} \text{ is the daily covariance}$$

and the formula for the correlation is given by

$$\rho_{XY,t} = \frac{\sigma_{XY,t}}{\sigma_{X,t}^*\sigma_{Y,t}}$$

Then, a cash equivalent amount of each interest rate risk factor is given by

$$\alpha_t = \sum_{k=1}^{11} \frac{[A_{kt} * e_k]}{(1 + R_t)^t} \qquad \text{baht}$$

and a cash equivalent amount of each exchange rate risk factor is given by

$$\alpha_k = \sum_{t=1}^{10} \left[\frac{A_{kt} * e_k}{(1 + R_t)^t} \right] \qquad \text{baht}$$

α_t :weighted value or cash equivalent amount of t-year zero coupon rate risk factor.

α_k :weighted value or cash equivalent amount of exchange rate risk factor

e_k :value of Sch/THB, C$/THB, SFr/THB, DM/THB, BFr/THB, £/THB, Y/THB,SFr/THB, US/THB and ECU/THB exchange rate at settlement date

R_t : value of t-year zero coupon rate at settlement date

A_{kt} : future annual payment denominated in k^{th} currency at period t

Cash flows of standardized vertices have been looked like value of a security. With 95% confidence level, we have had covariance matrixes of such securities, so total risk or volatility of overall payment denominated in baht currency of government's external direct debts have been given as:

$$\sigma_p^2 = \sum_{i=1}^{n}\alpha_i^2\sigma_i^2 + 2*\sum_{i=1}^{n}\sum_{j\#i}^{n}\alpha_i\alpha_j\rho_{ij}\sigma_i\sigma_j$$

or

$$\sigma_p^2 = \sum_{i=1}^{n}\alpha_i^2\sigma_i^2 + 2*\sum_{i=1}^{n}\sum_{j\#i}^{n}\alpha_i\alpha_j\sigma_{ij}$$

where α_i : present cash flows of standardized vertices(i).

σ_i : variance of standardized vertices(i)

σ_{ij} : covariance of standardized vertices(i,j).

ρ_{ij} : the correlation between return of asset i and asset j

α_i : cash equivalent amount of risk factors

,then $VaR = z . \sigma_p$

where z = a standard score of the normal distribution at a given level of confidence, for example, z = 1.96 at 95% level of confidence.

2.3.2 Structured Simulation Method of Monte Carlo[11]

The method has dealt with market risk as a probabilistic phenomenon. It is also called as, the concept step of structured simulation method. Generally, we can put our view into this step and obtain approximation of characteristics of the desired answer, such as mean, variance, distribution shape to explain more clearly as follows:

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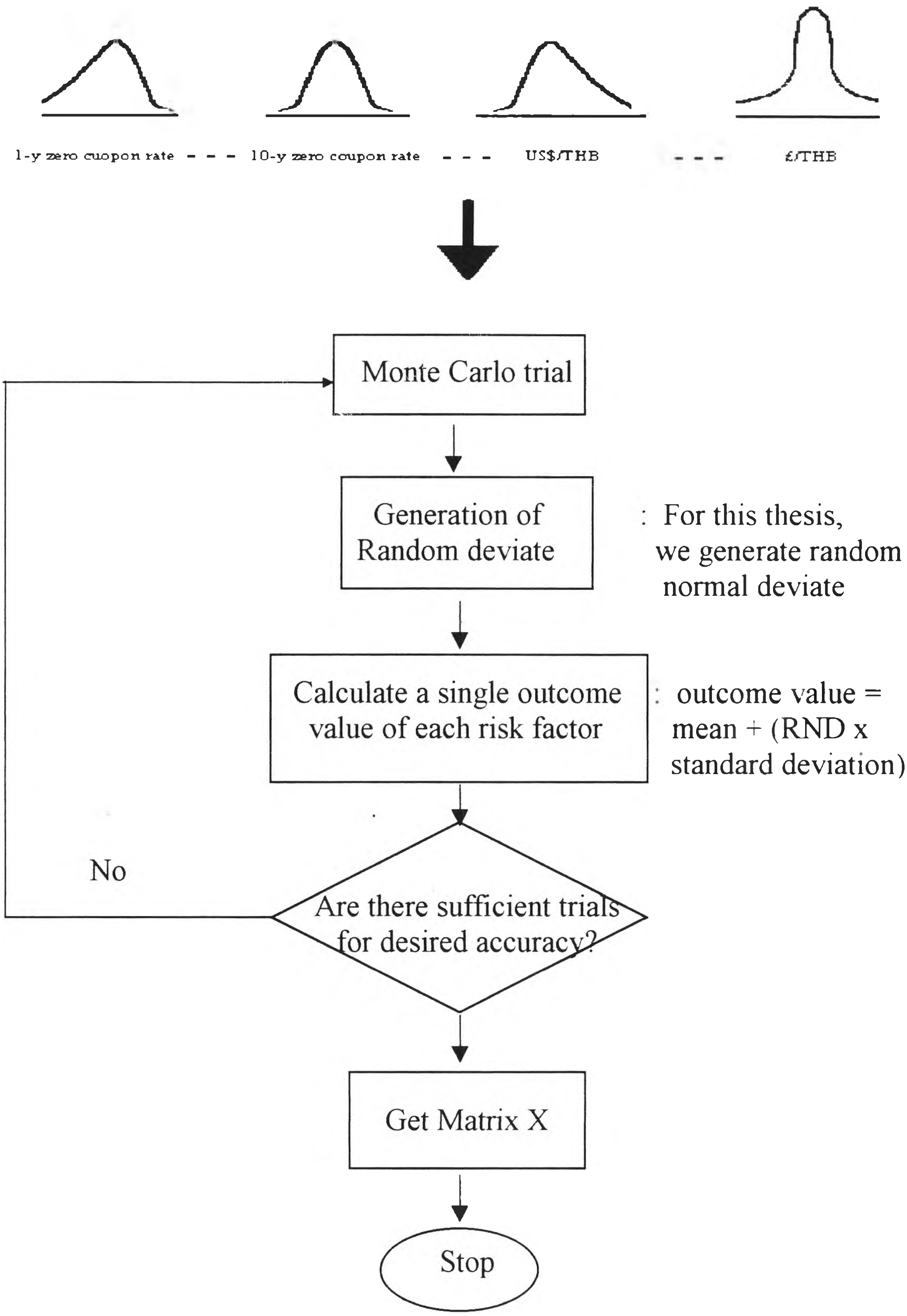
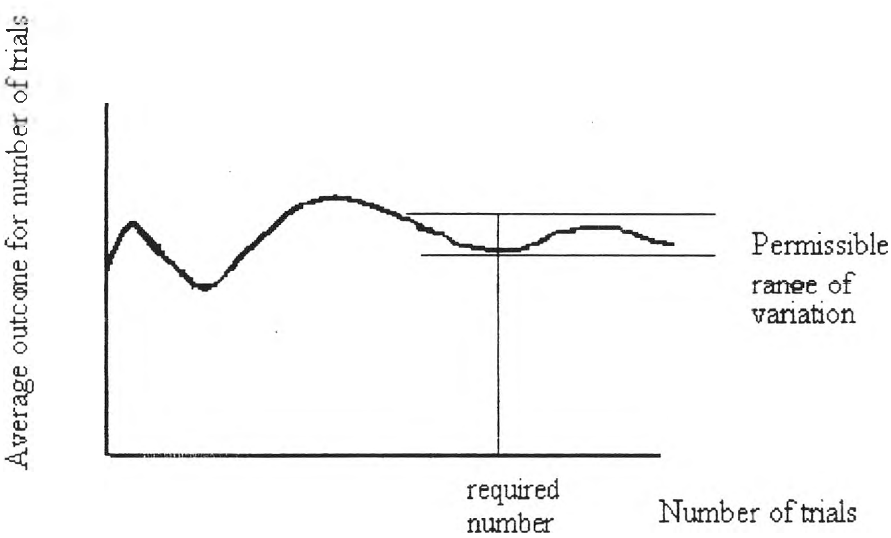
[11] To see Risk Matrices , page 98-106 and Capital Analysis for Engineering and Management textbook, page314-335 ε appendix D.



1-y zero cuopon rate - - - 10-y zero coupon rate - - - US$/THB - - - £/THB
Monte Carlo trial
Generation of Random deviate
: For this thesis, we generate random normal deviate
Calculate a single outcome value of each risk factor
: outcome value = mean + (RND x standard deviation)
No
Are there sufficient trials for desired accuracy?
Get Matrix X
Stop

To determine the approximate number of Monte Carlo trials required to obtain sufficiently accurate answers, we should keep a running plot on the average answers of interest for increasing numbers of trails and judge the number of trails at which those answers have become stable enough to be within the accuracy required .We can illustrate a figure as follows



Of this thesis, the annual payment in baht currency of the government foe external debts will be assumed as random number and statistically independent cash flows. The central limit theorem, from probability theory, establishes that the sum of independently distributed random variable trends to be normally distributed as the number of terms in the summation increase. Hence, basically we will desire any risk factors' distribute shape as a set of multivariate normal with mean 0 and variance 1.

After we had already decomposed the net cash flows of any risk factor vertices, we calculate covariance matrix $\Sigma = A^T * A$ using the Cholesky factorization. With covariance matrix Σ, we can decompose matrix A^T from $\Sigma = A^T * A$ by using the Cholesky decomposition. As the dimension of covariance matrix Σ is 21 x 21, we can derive the elements of matrix A^T from Σ. From definition, $\Sigma = A^T * A$

$$\begin{pmatrix} s_{1x1} & --- & s_{1x11} & --- & s_{1x21} \\ s_{11x1} & --- & s_{11x11} & --- & s_{11x21} \\ s_{21x1} & --- & s_{21x11} & --- & s_{21x21} \end{pmatrix} = \begin{pmatrix} a_{1x1} & --- & 0 & --- & 0 \\ a_{11x1} & --- & a_{11x11} & --- & 0 \\ a_{21x1} & --- & a_{21x11} & --- & a_{21x21} \end{pmatrix} * \begin{pmatrix} a_{1x1} & --- & a_{11x1} & --- & a_{21x1} \\ 0 & --- & a_{11x11} & --- & a_{21x11} \\ 0 & --- & 0 & --- & a_{21x21} \end{pmatrix}$$

$$\begin{pmatrix} s_{1x1} & --- & s_{1x11} & --- & s_{1x21} \\ s_{11x1} & --- & s_{11x11} & --- & s_{11x21} \\ s_{21x1} & --- & s_{21x11} & --- & s_{21x21} \end{pmatrix} =$$

$$\begin{pmatrix} a^2_{1x1} & --- & a_{1x1} * a_{11x1} & --- & a_{1x1} * a_{21x1} \\ a_{11x1} * a_{1x1} & --- & a^2_{11x1} + a^2_{11x2} + --- + a^2_{11x11} & --- & a_{11x1} * a_{21x1} + a_{11x2} * a_{21x2} + -- + a_{11x11} * a_{21x1} \\ a_{21x1} * a_{1x1} & --- & a_{21x1} * a_{11x1} + a_{21x2} * a_{11x2} + --- + a_{21x11} * a_{11x11} & --- & a^2_{21x1} + a^2_{21x2} + -- + a^2_{21x2} \end{pmatrix}$$

or each element of A^T can be solved by using

$$a_{i1} = (1/a_{11})*s_{i1}$$

$$a_{ii} = \left[s_{ii} - \sum_{k=1}^{i-1} a_{ik} \right]^{1/2}$$

$$a_{ij} = \frac{1}{a_{jj}} * \left[s_{ij} - \sum_{k=1}^{j-1} (a_{ik} * a_{jk}) \right] \quad \text{where } i=3,4,..21 \text{ and } j=i-1$$

We need random selections of each risk factor vertices to generate matrix X. a set of multivariate normal MVN (0,1) to multiply Transport of matrix A and matrix X to generate yield matrix, $y = A^T*X$, where y is MVN $(0,\Sigma)$ to obtain the combination of statistical trail values with covariance characteristics of risk factors in term of the desired answer and to generate multivariate log normal price $Z= F*e^Y$ where F is vector of the expected price .Then we revalue the positions of values to distribute present values of payments of each risk vertice with histogram of the distribution of simulated changes in values and the cumulative histogram for analyzing VaR. Finally, we can conclude all processes as

1.To generate zero coupon rates in several lengths of Times or yield curves of zero coupon bonds by using THB interest rate swap.

2.To map each cash flow of any bonds and debentures into risk factor vertices.

3.To calculate present values (current values) for any risk vertices over a series of time observations.

4. Analytical and Monte Carlo method.

4.1 Analytical Method

4.1.1To analyze the optimal decay factor for such present values

4.1.2To estimate variance and covariance of any risk factor vertices.

4.1.3To calculate total variance and VaR.

4.2 Monte Carlo Method

4.2.1To analyze the optimal decay factor for the returns, $p_{tT}- p_{t(T-1)}$

4.2.2To calculate covariance matrix $= A^T*A$ of logarithm values of returns for any risk factor vertices, we can find A^T

4.2.3To randomly select a set of multivariate normal matrix, x composed with any risk factor vertices by using Table of Random Normal Deviates or Excel program.[12]

4.2.4 To generate yield matrix $y = A^T * X$ and then

4.2.5To revalue $Z = F * e^Y$ and to distribute their values into a normal histogram and a cumulative histogram for analyzing VaR

[12] Please see appendix D of Capital Investment Analysis for Engineering and Management, 2nd.ed., 1996