CHAPTER VI

DISCUSSIONS AND CONCLUSIONS

The problem of Brownian motion was first solved by Einstein. The success of his theory distracted attention from the idealized character and the remoteness of the motion from Newtonian mechanics. Although, his treatment which idealized the motion as random flights is valid only when the time interval considered is much larger than the relaxation time. The generalization for all values of time interval we require to discuss on two equations, namely

(1) Langevin equation, first introduced by Langevin derived from Newton's law of motion, except that, the forces acting on the particle by the liquid molecules are divided into systematic part $-\beta u$ and fluctuating part A(t). The latter part has statistical character.

(2) Fokker-Planck equation, the partial differential equation derived from the integral equation which relates the probability distribution at any given time to the distribution at the earlier time, where the motion is idealized as Markoff process.

Wiener (5) proved that the displacements of this stochastic process are continuous. This is the desirable result which makes the stochastic process acceptable as the mathematical idealization of the motion. Langevin equation was first derived by Ornstein and Uhlenbeck (4). However , mathematician and physicist were unsatisfactory with the derivation (9) ; mathematician, because the solutions obtained are non-differential ; physicist, because the force exerted by the surrounding liquid was still phenomenologically.

The mathematical problem was later solved by Doob (5,6) who showed that the interpretation of Langevin equation as an integral equation made possible the rigorous and correct method. Langevin and Fokker-Plack equations both give the probability distributions of the coordinate and/or velocity of Brownian particle at any given time. The distributions from the two equations are identical and the average displacement and/or velocity obtained from these distributions can be found for all intervals of time. The analysis of Langevin and Fokker-Planck equations was collected, developed and applied to Astronomical problems by Chandrasekhar (6).

At present, physicist's unease goes deeper, that, it should be possible to solve the problem from laws of dynamics alone. Ford, Kac and Mazur try to do this, but in modest manner(9), that, they still use equilibrium statistical mechanics. They showed that a system of coupled harmonic oscillators can be made model of a heat bath, where a Brownian particle, exerted by an external force, of equal mass to a bath particle is coupled harmonically to the bath particles, in the limit that, the interactions between the particle and bath particles are a long range type and the frequencies of the bath particles are within a limit. Regarding Einstein's theory, Langevin and Fokker-Planck equations, Stokes' law used can cause some limitations.

Stokes'law can be used only for the particle moving slowly and is large compared to the size of liquid molecules. This model can, therefore, represents Brownian motion of a particle and makes us understand the behavior of Brownian particle which has the same size as the bath particle. One should extend the present treatment to the case of the particle of mass not equal to the bath particle and to use dynamical laws(9) only in obtaining the physical parameters describing the behavior of Brownian particle.