CHAPTER I

SPIN - GLASS

1.1 Physical Description

'Spin - glass'(1-4) refers to magnetic alloys where the spins on the impurities become locked or frozen in. No long range correlation exist between the spins. However there is some short range ordering below a characteristic temperature Tg.

Typical systems of spin - glasses are dilute substitutional magnetic alloys, such as CuMn or AuFe, with magnetic impurity concentrations c roughly between 10^{-3} and 10^{-1}. The first of the two metals, Au in the alloy is the host, and the second, Fe, the impurity.

The explosion of interest in this subject arose from some fascinating and important measurements by Cannella and Mydosh (5) who observed a cusp-like peak in a.c. susceptibility of the AuFe alloy at a well defined temperature Tg, known as the spin - glass freezing temperature. The measurements suggested that some type of phase transition was occuring. This fact was not readily explainable in terms of the accepted ideas at that time on dilute magnetic alloys. Much of the early theoritical work on spin glasses was devoted to trying to understand the a.c. susceptibility data. A calculation which had a considerable influence in this respect was due to Edwards and Anderson (6)

1.2 Edwards and Anderson Model

Recently a theory of spin - glass was proposed by Edwards and Anderson (6) (refer hereafter as EA) in which a classical Heisenberg model with a random distribution of exchange interactions is considered.

The essence of EA theory is that, despite the random arrangement of spins, there is nevertheless a certain configuration which minimizes the potential energy of the system and which corresponds to the ground state. As the spin moves into this ground state there is a well defined magnetic transition giving rise to a cusp like peak in the a.c. susceptibility. EA have introduced an order parameter, q , such that if one observes that a spin has a value $S_i^{(1)}$ at time t_1, then if it is studied a long time later at time t_2, there is a non-vanishing probability that $S_i^{(2)}$ will be pointing in the same direction. Above the freezing temperature Tg the order parameter q is zero. Below Tg one can write

$$q = \langle S_i^{(1)} S_i^{(2)} \rangle \neq 0$$

q increase towards unity as the temperature approaches the absolute zero.

The theory of EA predicted a cusp in the susceptibility although the expect shape of the cusp is somewhat different from that observed experimentally. A considerable amount of theoritical work has stemmed from the original calculation of EA.

1.3 Sherrington and Kirkpatrick Model

Following EA's pioneering work David Sherrington and Scott Kirkpatrick (7) (refer hereafter as S.K.) proposed an infinite-range model of spin - glass in which every spin is coupled with all others pairwise and the distribution of the exchange interaction is assumed to be Gaussian. This is an extension of the infinite-range model of ordering ferromagnets, which we can treat exactly by the mean-field theory.

S.K, however, studied this model using a physically and mathematically unconventional replica method. One calculates for integer n the $n^{\underline{th}}$ power of the partition function but at the end let $n \longrightarrow 0$, i.e. use the identity

$$\ln z = \lim_{n \to 0} \frac{(z^n - 1)}{n}$$

to obtained the various thermodynamic quantities such as internal energy, susceptibility, specific heat and entropy.

1.4 Difficulties Existing in the Solution

Due to the pathology of the replica method, however, S.K. obtained unphysical results, in particular a negative entropy at zero temperature. Otherwise, their results were physically very appealing.

The origin of the difficulties was associated with the interchange of the order of $N \longrightarrow \infty$ (N are Ising spins) limit and the $n \longrightarrow 0$ limit (introduced by the replica method) in the evaluation of the free energy.

1.5 TAP Method

In order to remedy the unphysical result, D.J. Thouless, P.W. Anderson and R.G. Palmer (8) (refer hereafter as TAP) developed a mean-field theory for S.K. model. Making use of the Bethe approximation, they obtained a self-consistent equation. They solved it in two limiting temperature regime, i.e., in the vicinity of the critical temperature Tg and at very low temperatures. The low temperature behavior of the system is described by means of a mean field theory which taking into account the spin fluctuation led among other results to a zero entropy at T = 0 K in contrast to the negative value derived by S.K.

Above the critical temperature the correct S.K. equations were regained, by a high temperature expansion of the free energy in which the contribution of a class of loop diagrams was included.

1.6 Other Techniques to Solve Spin - Glass Problem

1.6.1 Nakanishi Method

TAP found that in the infinite-range model there exist an extra field besides the conventional mean field. This extra field is due to the two-spin cluster effect, indicating the importance of the cluster effect in spin - glasses.

In order to make a further investigation, Nakanishi (9) formulated a theory in which the effect of the two and three spin cluster is taken into account exactly and apply it mainly to the infinite-range model.

Incidentally it is not hard to show that the approximation made in the two - spin cluster theory is generally equivalent to the Bethe approximation, which TAP used, therefore it is quite natural that Nakanishi method reproduces TAP's result.

1.6.2 Differential Operators Technique

Takahito Kaneyoshi (10) proposed a method for evaluating the spin - glass ordering temperature beyond its mean field value.

By introduced the method of differential operators (10), a new type of series expansion for the parameter q is derived under the restriction of $P(J_{ij}) = P(-J_{ij})$

He evaluated the series two ways; one using a decoupling approximation, another calculated term by term using diagram method.

The series depicted by diagrams reduces to a polynomial equation. After finding a solution of a n-order polynomial equation, where n is the highest power in it, the Tg can be determined from the extrapolation of each order Tg to $\frac{1}{n} \longrightarrow 0$

1.6.3 The work of M.W. Klein, L.J. Schowalter and P. Shukla

Since the replica method gives a negative entropy, some mean field method which avoid the replica trick are the Bethe-Peierls-Weiss (BPW) method (11,14,16), the self-consistent mean random field approximation (MRF) and the method present by TAP (8).

Klein et al (14) use a modified BPW method couple with the probability distribution of internal fields compare the predictions of various mean field approximation with each other. When the effective number of neighbors z approaches infinity, they show that all the magnetic properties arising from the BPW, MRF and S.K. method are identical. Also, the internal energy in the BPW method is identical to that obtained by S.K. while the MRF gives a different result.

By integrating the BPW internal energy and adding a plausible phenomenological constant of integration they obtained TAP free energy, using this free energy they find that any probability distribution of fields H which does not go to zero at zero field is unphysical in that it gives a negative entropy.

1.7 Prototype Model of a Spin Glass

A model in which the bonds between Ising spins can take on only two values J_1 and J_2 with the distribution of bonds being totally random has been proposed as being a prototype model of a spin glass.

Syozi et al (15) has applied the n-replica method of Edwards and Anderson to this model. Three phases, ferromagnetic, antiferromagnetic and Mattis phases are seen to arise for different concentrations. They found that for the quenched system, the lower bound of the critical concentration is 0.853 (which is the critical concentration of the annealed system). They point out that this was due to the frustration effect in the quenched system disturbing the ferromagnetic order.

1.8 Difference between Quenched and Annealed system

In both the quenched and annealed systems (17-22), the magnetic impurities are distributed completely randomly throughout the crystal. In the quenched system, the spins are frozen at the (approximate) distribution existing before their rapid quenching down to a final temperature is done. The system is lowered to the final temperature so fast that the spins in the system do not have enough time to relax to the equilibrium distributions existing at the intermediate temperatures between the initial and final temperatures. Therefore the spins of the system can assume the configuration expected of a system making a reversible transition from T_i to T_f.

In the annealed system, however, the temperature is lowered slowly enough so that the spins can come into equilibrium existing at each of the intermediate temperatures. At each step, the system assumes the lowest ground state energy configuration possible and so the final configuration assumed at T_f will have an energy lowered than that of the quenched system.

To calculate the free energy of a quenched system (17), one has to compute the free energy for a given distribution of impurities, and then average all their possible configurations. In order to get the free energy of an annealed system (17), we have to sum up simultaneously over spins and impurities configurations.

Mathematically, the quenched free energy can be written as(21)

$$F_q = - kT \langle \ln z \rangle$$

and the annealed free energy

$$F_a = - kT \ln \langle z \rangle$$

As a consequence of Jensen's inequality

$$\langle f(x) \rangle \geqslant f(\langle x \rangle)$$

for the mean value of any convex function f (x) of a random variable x, the quenched free energy is bounded from below by the annealed free energy: $F_q \geqslant F_a$

In our calculation we deal with quenched system of prototype model of a spin glass.