CHAPTER V

MATHEMATICAL MODEL AND SIMULATION

5.1 Dynamic Model of Heat Pipe and Assumptions Used

A mathematical dynamic model that can deal with the start-up of heat pipe has been developed. In the development of a mathematical model of the heat pipe we adopt on approach in which all the elements of the system are characterized as lumped-parameter objects. For the sake of simplification the following assumptions have been used:

1. The following can be disregarded because of their small contributions:

   a) heat transfer via structural elements (wall and wick) of the heat pipe in the axial direction

   b) the transport time of vapor from the evaporator section to the condenser section

   c) thermal resistance at phase interfaces

2. The values of the thermal resistances between the elements of the heat pipe model are constant.

The thermal model of the heat pipe for our calculations is illustrated in Fig. 5.1. The mathematical model is concieved as a system of equations of energy conservation (heat balance) written for the i-th element of the thermal model. The heat balance has the general form as follows

Heat Input - Heat Output = Heat Accumulation

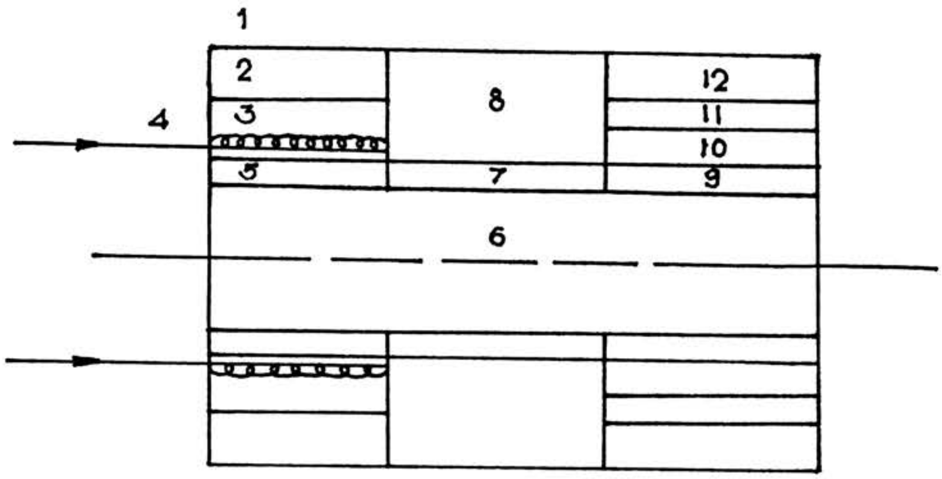

Fig. 5.1 Thermal model of the heat pipe

1) Surrounding medium; 2,8,12) thermal insulation;
3) Refractory; 4) electric heating wire;
5,7,9) walls with capillary structure along evaporator, adiabatic and condenser sections, respectively
6) vapor-flow passage; 10) water; 11) PVC

Under the stated assumptions for the thermal model in Fig. 5.1. The system of equations take the form

$$Q = A_4 \left[(F_{AE1} + F_{AE2}) T_4^4 - F_{AE1} T_{3'}^4 - F_{AE2} T_5^4 \right] \quad (1)$$

$$F_{AE1} A_4 \sigma (T_4^4 - T_{3'}^4) = \sigma_{33} (T_{3'} - T_3) \quad (2)$$

$$\sigma_{33} (T_{3'} - T_3) = C_3 \frac{dT_3}{dt} + \sigma_{32} (T_3 - T_2) \quad (3)$$

$$\sigma_{32}(T_3 - T_2) = C_2\frac{dT_2}{dt} + \sigma_{21}(T_2 - T_1) \quad (4)$$

$$F_{AE2}A_4 \; (T_4^4 - T_5^4) = C_5\frac{dT_5}{dt} + \sigma_{56}(T_5 - T_6) \quad (5)$$

$$\sigma_{56}(T_5 - T_6) = \sigma_{67}(T_6 - T_7) + \sigma_{69}(T_6 - T_9) \quad (6)$$

$$\sigma_{67}(T_6 - T_7) = C_7\frac{dT_7}{dt} + \sigma_{78}(T_7 - T_8) \quad (7)$$

$$\sigma_{78}(T_7 - T_8) = C_8\frac{dT_8}{dt} + \sigma_{81}(T_8 - T_1) \quad (8)$$

$$\sigma_{69}(T_6 - T_9) = C_9\frac{dT_9}{dt} + \sigma_{9,10}(T_9 - T_{10}) \quad (9)$$

$$\sigma_{9,10}(T_9 - T_{10}) = C_{10}\frac{dT_{10}}{dt} + \sigma_{10,11}(T_{10} - T_{11}) - G(CW)(TW-T_{10}) \quad (10)$$

$$\sigma_{10,11}(T_{10} - T_{11}) = C_{11}\frac{dT_{11}}{dt} + \sigma_{11,12}(T_{11} - T_{12}) \quad (11)$$

$$\sigma_{11,12}(T_{11} - T_{12}) = C_{12}\frac{dT_{12}}{dt} + \sigma_{12,1}(T_{12} - T_1) \quad (12)$$

Notation: Q = heat input

T = temperature

t = time

σ_{ij} = thermal conductance

C = total heat capacitance

F_{AE1}, F_{AE2} = effective view factor for heat radiation from heating wire

σ = Stefan-Boltzmann constant

5.2 Estimation of Parameters in Model

Total heat capacity (C) in the thermal model can be estimated from the products of volume, density and specific heat of each element in thermal model base on dimensions of the thermal model in Fig. 5.2 to 5.4 and they are shown in Table 5.1.

Based on dimensions in Fig. 5.2 to 5.4, thermal conductances (σ) can be determined and are summarized in Table 5.2.

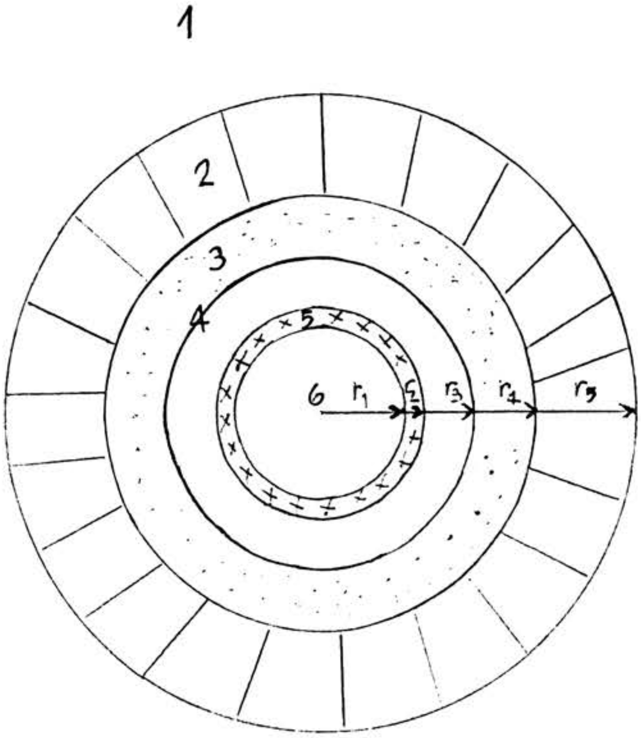

Fig. 5.2 Diagram of Heating section

1.	AMBIENT AIR	r_1 = 2.8 mm
2.	MICROFIBER INSULATION	r_2 = 4 mm
3.	REFRACTORY	r_3 = 1.3 cm.
4.	ELECTRIC HEATING WIRE	r_4 = 3.6 cm.
5.	EVAPORATION SECTION	r_5 = 8.6 cm.
6.	VAPOR CORE	

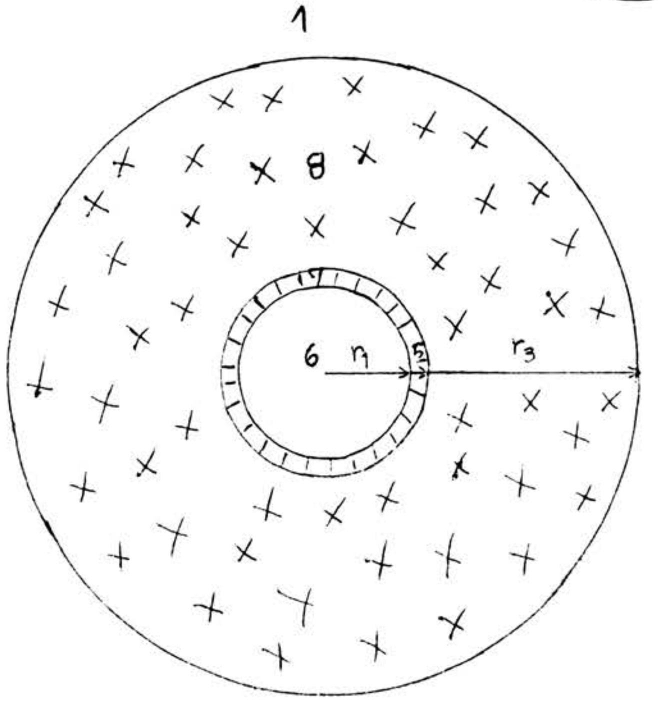

Fig. 5.3 Diagram of Adiabatic Section

1. AMBIENT AIR $r_1 = 2.8$ mm
6. VAPOR CORE $r_2 = 4$ mm
7. ADIABATIC SECTION $r_3 = 8.6$ cm
8. MICROFIBER INSULATION

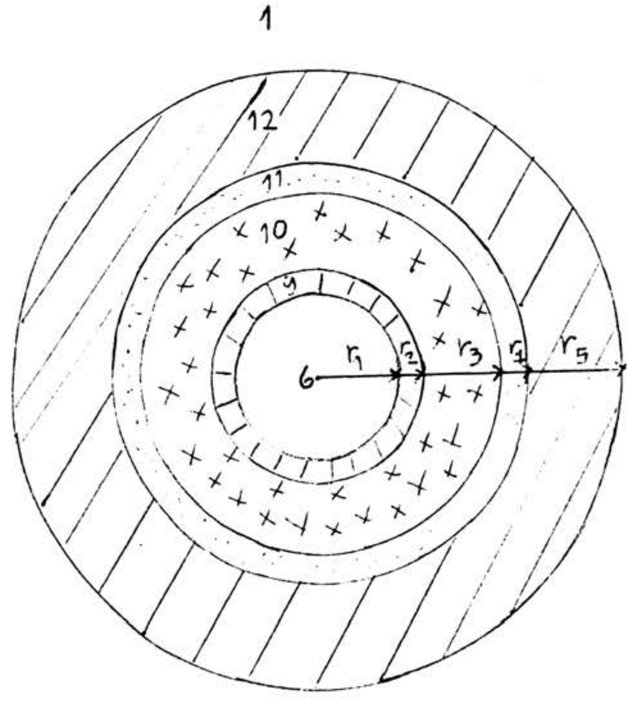

Fig. 5.4 Diagram of Cooling Section

1.	AMBIENT AIR	r_1 = 2.8 mm
9.	COOLING SECTION	r_2 = 4 mm
10.	COOLING WATER	r_3 = 3.6 cm.
11.	PVC CONTAINER	r_4 = 3.8 cm.
12.	MICROFIBER INSULATION	r_5 = 8.6 cm.

TABLE 5.1 Total Heat Capacitance of Elements Used in Thermal Model

ELEMENTS	HEAT CAPACITANCE (Whr./K)
Microfiber insulation (heating section)	0.04
Furnace wall	0.2
Microfiber insulation (adiabatic section)	0.033
Cooling water	0.63
PVC container (water cooler)	0.052
Microfiber insulation (cooling section)	0.040

TABLE 5.2 Thermal Conductance Used in Thermal Model

ELEMENTS	THERMAL CONDUCTANCE (W/K)
Refractory	0.043
Refractory-microfiber (heating section)	0.026
Microfiber (heating section)-air film	0.039

TABLE 5.2 (continued)

ELEMENTS	THERMAL CONDUCTANCE (W/K)
Wall/wick of heat pipe-vapor core	1.966
(heating, adiabatic and cooling section)	
Microfiber (adiabatic section)	0.013
Microfiber (adiabatic section)-air film	0.012
Cooling water	1.001
Microfiber (cooling section)	0.076
PVC container	0.348
Microfiber (cooling section)-air film	0.070

5.3 Development of Computer Code for Dynamic Model

A simplified block diagram of the algorithm is shown in Fig. 5.5. A fourth-order Runge-Kutta method is used to integrate simultaneously the set of differential equations representing the dynamic model.

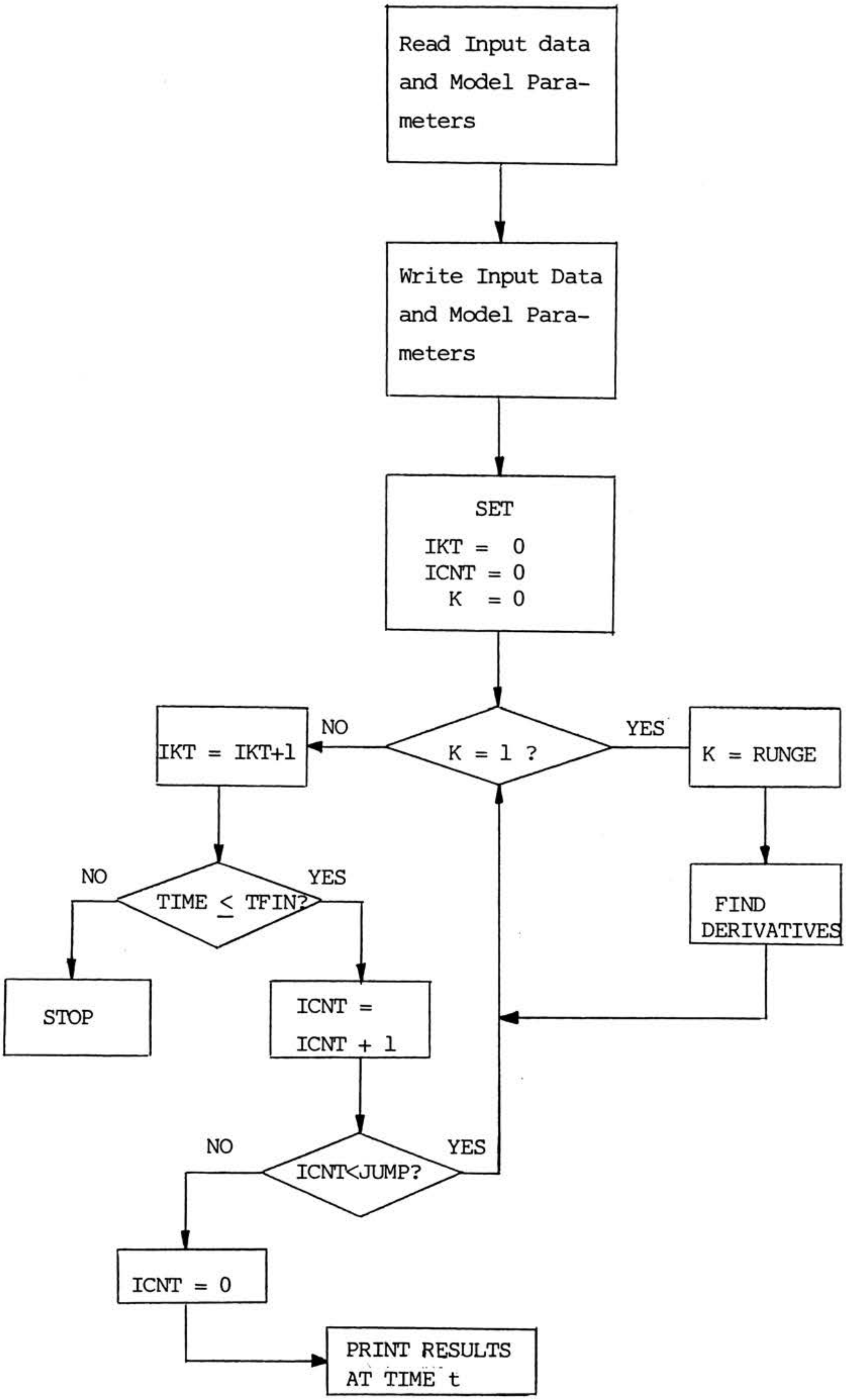

Fig. 5.5 Block Diagram of Computer Simulation Program.