



CHAPTER III.

NUMERICAL SCHEME

Introduction.

An effective implementation of the nonlinear finite element method in a computer code requires various of numerical methods to solve the resultant system of nonlinear equations. The selection of a scheme for the solution of these equations constitutes an important part of the design of a general nonlinear analysis capability. The purpose of this chapter is to describe the numerical schemes and the computational procedures for the construction of the proposed program. The development of computer program and program verification are also included.

Numerical Methods for the Solutions of Nonlinear Equations.

In the actual analysis of nonlinear response of a structural system, it will lead to systems of nonlinear equations in the unknown nodal displacement components. However, in a majority of nonlinear analyses the linearized forms of the finite element formulation are used in connection with various numerical schemes. Each of these solution schemes seeks to trace the nonlinear load-deformation path of the structure by solving systems of nonlinear equation using one or a combination of the following procedures:

- (a) Incremental loading procedures
- (b) Iterative procedures
- (c) Minimization procedures

The suitable numerical scheme for the computer program development in this research will be the combination of the incremental and iterative procedures. From equation (2.25), a system of n nonlinear equations in n unknown incremental displacement components will be solved. The procedures are as follows:

$$\{K_L + K_G\}.q = {}^2P - {}^1R \quad (3.1)$$

or simply,

$$\bar{K} . q = \bar{P} \quad (3.2)$$

where \bar{K} = the nonlinear (global) stiffness matrix
 q = the incremental nodal displacement vector
 \bar{P} = the generalized force vector

In the incremental loading procedure, the load is applied in increments which are sufficiently small so that during each increment the response of the body is linear. At the end of each increment, a new updated stiffness relation is obtained and another increment of load is applied. By continuing this process, the response of the body is generated as a sequence of linear steps. Naturally, some error will inevitably occur in each step of the process. To improve the accuracy of the incremental loading method, Newton-Raphson iterations with

equilibrium corrections are applied at the end of each load increment. Details of the incremental loading method and the Newton-Raphson iterative scheme can be found in references (10,11).

Computational Procedure.

In the nonlinear finite element method, the path of deformation of a nonlinear elastic body (element) will be divided into a number of equilibrium states corresponding to the discrete load points $0, \Delta P, 2\Delta P, \dots$, where ΔP is an increment in loading. At each step of a load increment, the computational procedures for the proposed method are summarized as follows :

1. Check the number of iteration. If it exceeds the number of iteration limit, run the next load increment; otherwise proceed the next step.
2. Compute the stiffness matrices, K_L and K_G .
3. Compute the consistent nodal load vector in equilibrium with the state of stresses in configuration of the prior cycle of iteration.
4. Calculate the consistent nodal load vector from the evaluation of the body forces and surface tractions.
5. Assemble the master stiffness matrix and the force vector.
6. Apply boundary conditions.
7. Solve for the incremental responses. That is incremental displacements, q , incremental strains, E , incremental stresses, S , respectively.
8. Update the state of motion. That is

$${}^{(p,r)}q = {}^{(p,r-1)}q + {}^{(r)}q$$

in which ${}^{(p,r)}q$ is the displacement vector at r^{th} iteration of p^{th} load step; ${}^{(p,r-1)}q$ is the displacement vector at $r-1^{\text{th}}$ iteration of p^{th} load step; and ${}^{(r)}q$ is the incremental displacement vector calculated at r^{th} iteration.

9. Update the state of stress. That is

$${}^{(p,r)}S = {}^{(p,r-1)}S + {}^{(r)}S$$

in which ${}^{(p,r)}S$ is the stress vector at r^{th} iteration of p^{th} load step; ${}^{(p,r-1)}S$ is the stress vector at $r-1^{\text{th}}$ iteration of p^{th} load step; and ${}^{(r)}S$ is the incremental stress vector calculated at r^{th} iteration.

10. Convergence check. If the ratio of the norm of increment ${}^{(r)}q$ to the norm of the current ${}^{(p,r)}q$, i.e., ${}^{(r)}q / {}^{(p,r)}q <$ Tolerance, run the next load step; otherwise proceed to steps 1 to 10.

11. Print nodal displacements and element stresses for this load step.

12. Repeat step 1 for the next load increment until the final load is reached.

As described in the previous chapter, the stress in the Lagrangian formulation are the second Piola-Kirchhoff stress which is defined with respect to the undeformed state. Thus, it is necessary that these stresses be transformed to the Cauchy stress whenever the stress output is required. The flow sequence of the computation is shown in Figure 3.1.

Computer Program Development.

The nonlinear static three dimensional finite element program in this research is primarily a modification of the linear static three dimensional finite element program developed by Dr. Karoon Chandrangu of Chulalongkorn University. The main feature of the proposed program is to include geometric nonlinear analysis for the element considered, linear isoparametric hexahedral element (8-node).

To carry out program verification, two problems have been solved and the results have been compared with those performed by the well known nonlinear finite element program, NONSAP (15,16). For three dimensional hexahedral analysis in NONSAP program, there is no option for geometric nonlinear analysis. Thus, we have to use two dimensional analysis instead and the element used is the linear isoparmetric quadrilateral element (4-node) which is equivalent to the element considered in the proposed program because both elements use the linear interpolation for their approximations of geometric and displacement field. The first problem which considers a plane stress case, is a large deflection analysis of an elastic cantiliver beam under uniformly distributed load which is shown in Figure 3.2. The relation between load and tip displacement is also shown in Figure 3.3. The second problem which concerns a plane strain case, is the nonlinear analysis of the settlement of uniform soil mass under strip loading which is shown in Figure 3.4. The settlement of the soil surface is also shown in Figure 3.5. For both problems, all displacements and stresses obtained by both programs have been compared and their solutions are in excellent agreement.