CHAPTER II
THEORETICAL BACKGROUND AND LITERATURE REVIEW

2.1 The Atmospheric Distillation Unit

Crude unit is the first unit that processes petroleum in any refinery .The objective is to separate the mixture into several fractions. A schematic diagram of atmospheric crude fractionation unit is shown in Figure 1.Crude distillation unit consists of a desalter, crude furnace an atmospheric tower, pump around side strippers and a debutanizer. Crude oil is preheated by exchanging heat with pump-around reflux streams and then sent to a desalter, where salts, solids and water are removed. The desalted crude oil is further preheated by exchanging heat with products and pump-around reflux stream, and finally heated by a crude furnace to a temperature which provides the required degree of vaporization. The heated crude oil is then introduced to the flash zone of the atmospheric tower. The liquid portion of the flashed crude oil flows down to a bottom stripping section of the atmospheric tower, where distillate fractions dissolved in the liquid are vaporized with steam stripping.

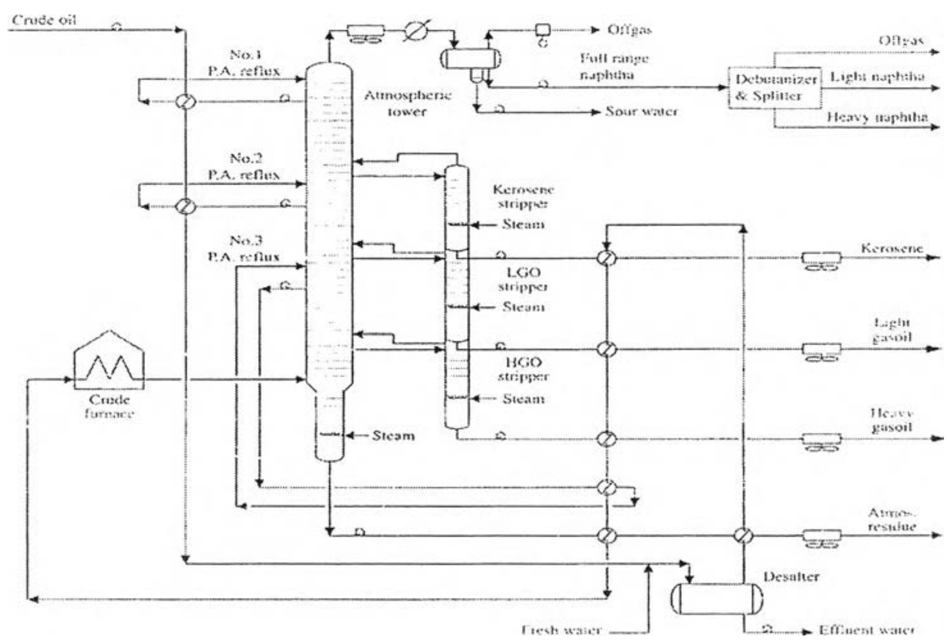


Figure 2.1 Process flow scheme of an atmospheric distillation unit
(Lorenz *et al.*, 1997).

The mixed vapor stream contacts down-flowing internal reflux liquid on the trays, where condensation and fractionation of distillate products take place. The internal reflux liquid is created by condensation of the ascending oil vapor that has contacted cooled pump-around liquid. Use of the several pump-around reflux systems prepares reflux streams of different temperature levels, and enables effective utilization of the reflux heat load for heating the crude oil feed, which improve the amount of energy consumption in distillation unit. The condensed liquid is withdrawn as side-stream products such as kerosene, light gas oil and heavy gas oil. These streams are sent to side strippers, where the lighter gas and oil fractions are removed by steam stripping for adjusting of the flash point. The bottoms of the side strippers are withdrawn as distillate products such as kerosene, light gas oil and heavy gas oil. The overhead vapor of the atmospheric tower is condensed by an overhead condenser(s). The condensed liquid, called full boiling range naphtha, is sent to a debutanizer to remove the butane and lighter hydrocarbons. The debutanizer off gas and gases not condensed in condenser(s) of the atmospheric tower are sent to a gas concentration unit to recover propane and butane (LPG). The debutanizer full range naphtha is separated into light and heavy naphtha by a splitter.

2.2 Heat Exchanger Network Synthesis

Heat exchanger network (HEN) synthesis is one of the most extensively studied problems in industrial process synthesis. This is attributed to the importance of determining the energy costs for a process and improving the energy recovery in industrial sites. The first systematic method was the thermodynamic approach, using the concept of pinch introduced during the 1970s to maximize energy recovery.

The first approaches in the 1960s and early 1970s treated the HEN synthesis problem without applying decomposition into sub-tasks. The limitations of optimization techniques were the bottleneck of the mathematical approaches at that time. For the synthesis problem of the HEN, the thermodynamic approach of pinch analysis was introduced by the work of Hohmann (1971) and Linnhoff and Flower (1978). As a result of the pinch concept, the single task approaches were shifted to procedures introducing techniques for decomposing the problem into three subtasks; minimum utility cost, minimum number of units and minimum investment cost network configurations. The main advantage of decomposing the HEN synthesis problem is that sub-problems can be treated in a much easier fashion than the original single-task problem. The sub-problems are the following

2.2.1 Minimum Utility Cost Target

The maximum energy recovery can be achieved in a feasible HEN for a fixed heat recovery approach temperature (HRAT), allowing for the elimination of several non-energy efficient HEN structures. Minimum utility cost was first introduced by Hohmann (1971) and Linnhoff and Flower (1978) and later as an LP transportation model by Cerda et al. (1983), being an improvement of the LP transshipment model of Papoulias and Grossmann (1983).

2.2.2 Minimum Number of Units Target

The match combination can be determined with the minimum number of units and their load distribution for a fixed utility cost. The MILP transportation model of Cerda and Westerberg (1983) and the MILP transshipment model of Papoulias and Grossmann (1983) are the most common, while the vertical heat transfer formulation of Gundersen and Grossmann (1990) and Gundersen, Duvold and Hashemi-Ahmady (1996) are also used.

2.2.3 Minimum Investment Cost Network Configurations

It is based on the heat load and match information of previous targets. Using the superstructure-based formulation, developed by Floudas et al. (1986), the NLP problem is formulated and optimized for the minimum total cost of the network. The objective function in this model is the investment cost of the heat exchangers that are postulated in a superstructure.

However, limitation of decomposition-based methods is that costs due to energy, units and area cannot be optimized simultaneously, and as a result the trade-offs are not taken into account appropriately. Thus, simultaneous heat exchanger network synthesis methods are taken place. The simultaneous approaches purpose to find the optimal network with or without some decomposed problem. The simultaneous optimization generally results in MINLP formulations, which assumptions exist to simplify these complex models.

In 1986, Floudas and Grossmann introduced a multiperiod MILP model for the minimum utilities cost and minimum number of match of target problems, based on Papoulias and Grossmann's (1983) transshipment model. In this model the changes in the pinch point and utility required at each time period are taken into account. Extensions were presented first by Floudas and Grossmann (1987), and NLP formulation based on a superstructure presentation of possible network topologies to derive automatically network configurations that feature minimum investment cost, minimum number of units, and minimum utility cost for each time period.

Floudas and Ciric (1989) proposed a match-network hyperstructure model to simultaneously optimize all of the capital costs related to the heat exchanger network. This MINLP formulation is based on the combination of the transshipment model of Papoulias and Grossmann (1983) for match selection, and the minimum investment cost network configuration model of Floudas and Grossmann (1986) for determining the heat exchanger areas, temperatures and the flow rate in the network. The proposed simultaneous synthesis may still lead to suboptimal networks, since the value for HRAT must be specified before the design stage.

In 1990, Yee and Grossmann formulated another simultaneous synthesis where within each stage exchanges of heat can occur between each hot and cold stream. This model can simultaneously target for area and energy cost while properly accounting for the differences in heat transfer coefficients between the streams. The match-network hyperstructure model was then further modified by Ciric and Floudas (1991) to treat HRAT as an explicit optimization variable. This MINLP formulation included any decomposition into design targets and simultaneously optimizes trade-offs between energy, units and area. Ciric and Floudas (1991) also demonstrated the benefit of a simultaneous approach versus sequential methods.

Ji and Bagajewicz (2001) introduced the rigorous procedure for the design of conventional atmospheric crude fractionation units. Part I aims to find the best scheme of a multipurpose crude distillation unit which can process the various crude. Heat demand-supply diagrams are used as a guide for optimal scheme instead of grand composite curves. Thus, the total energy consumption from stream, heater and cooler is clearly shown and this leads the process to be easily optimal. In part II, 2001, Soto and Bagajewicz attempted to design a multipurpose heat exchanger network that can handle in variety of crude. In order to overcome the smaller gap between hot and cold composite curves, models that fixed the heat recovery by using the minimum heat recovery approximation temperature (HRAT) and the exchanger minimum approach temperature (EMAT) was performed. In 2003, Part III, Soto and Bagajewicz established a model to determine a heat exchanger network with only two branches above and below desalter. The total annualized costs, operating cost and depreciation of capital, of solution limited to one or two branches are compared with the results of four branches. In this part, the present model is based on a transshipment model and the vertical heat exchange constraints combined with HRAT/EMAT. In addition, investment cost is not directly controlled by this model, but further indirectly controlled by limiting of the minimum unit numbers. The smaller number of units leads to minimal capital cost and energy consumption simultaneously.

In 2001, Grossmann presents review of nonlinear mixed-integer and disjunctive programming techniques. To present a unified overview and derivation of mixed integer nonlinear programming (MILP) techniques as applied to nonlinear discrete optimization problems that are expressed in algebraic form. The solution of MINLP problems with convex functions is presented first, followed by brief discussion on extensions for the no convex case. The solution of logic based representations, known as generalized disjunctive program, is also described, Theoretical properties are presented and numerical comparisons on a small process network problem.

New rigorous one-step MILP formulation for heat exchanger network synthesis was developed by Barbaro and Bagajewicz (2002). This methodology does neither rely on traditional super targeting network design by the pinch technology, nor is a nonlinear model, but further use only one-step to optimize the solution. Cost-optimal networks, cost-effective solutions, can be obtained at once by using this model.

In 2003, Balasubramanian and Grosssmann introduce approximation to multistage stochastic optimization in multi period batch plant scheduling they consider the problem of scheduling under demand uncertainty multi product batch plant represented through the state task network. They present a multistage stochastic mixed integer linear programming (MILP) model and some decisions are take unpon realization of the uncertainty. Computational results indicate that the proposed approximation strategy provides an expected profit within a few percent of the multistage stochastic MILP in a fraction of the computation time, and provides significant improvement in the expected profit over similar deterministic approaches.

In 2005, Grossmann and his teams present an algorithmic framework for convex mixed integer nonlinear programs. This paper is motivated by fact that mixed integer nonlinear programming is an important and difficult area for which there is a need for developing new methods and software for solving large-scale problems. This work represents the first step in an ongoing and ambitious project with in an open-source environment. Coin-Or is our chosen environment for the development of the optimization software. A class of hybrid algorithms, of which branch and bound and polyhedral outer approximation are the two extreme cases, this framework is reported, and a library of mixed integer nonlinear problems that exhibit convex continuous relaxations is made publicly available.

In 2006, Caballero and Grossmann introduce structural considerations and modeling in the synthesis of heat integrated-thermally coupled distillation sequences. Deals with the design of mixed thermally coupled-heat integrated distillation sequences, the approach considers from conventional columns to fully thermally coupled systems. A discussion about superstructure generation and the convenience of using a representation based on separation tasks instead of equipment id presented as well as a set of logical rules in terms of Boolean variables which allow to systematically generating all the feasible structures. The model is base on the Fenske, Underwood Gilliland equations.

2.3 Mathematical Programming Models

Mathematical programming model consists of an objective function and a set of equality constraints as well as inequality constraints.

<u>Classes of Mathematical Programming Models</u>

The mathematical modeling of the systems leads to different types of formulations.

1. Linear Programming (LP)
2. Non-Linear Programming (NLP)
3. Mixed Integer Linear Programming(MILP)
4. Mixed Integer Non-Linear Programming(MINLP)

2.4 Model for Grass-Root synthesis

A rigorous MILP formulation for grass-root design of heat exchanger networks is developed. The methodology does not rely on traditional super targeting followed by network design steps typical of the pinch design method, nor is a non-linear model based on superstructures. It considers splitting ,non-isothermal mixing and it counts shells/unites. The model relies on transportation/transshipment concepts. The model has the following features;

- Count heat exchangers units and shells
- Approximate the area required for each exchanger unit or shell
- Control the total number of units
- Implicitly determine flow rates in splits
- Handle non-isothermal mixing
- Identify bypasses in split situations when convenient
- Control the temperature approximation(HRAT/EMAT of ΔT min)when desired
- Address block-design through the use of zones
- Allow multiple matches between two streams

2.5 Model for Retrofitting Heat Exchanger Network

Not only designing an optimal heat exchanger network, but the problem of heat exchanger network analysis is also play attention in the retrofit part.

In1989, Ciric and Floudas present a two-stage procedure for the optimal redesign problem of existing heat exchanger networks. In the first stage, a mixed-integer linear programming (MILP) model is proposed for the retrofit at the level of matches that is based upon a classification of the proposed for the retrofit at the level of matches that is based upon a classification of the possible structural modifications. The objective function of this optimization model seeks to minimize: (a) the cost of purchasing new heat exchangers;(b) the cost of additional area; and (c) the piping cost, and is subject to a set of constraints that describe:(a) the heat flow model ;(b)the area estimation;(c) the calculation of additional area; and (d) the match-exchanger assignments. The solution of the retrofit model at the level of matched provides information about the process stream matches and their heat loads, the placement/reassignment of new and existing heat exchanger, estimates of the required area of each match and the required increase or decrease of area in each heat exchanger, and estimates of the repiping cost associated with introducing new matches, installing new heat exchangers, moving existing exchanger and repiping streams.

In the second stage, the information generated in the first stage is used to postulate a superstructure containing all possible network configurations. The solution of a nonlinear programming problem based upon this superstructure gives a retrofitted heat exchanger network.

In 2001, Jackson and Grossmann introduce high level optimization model for the retrofit planning of process networks. A strategy is proposed that consists of a high level to analyze the entire network and a low level to analyze a specific process flow sheet in detail. A methodology is presented for the high level to model process flow sheets and retrofit modifications using a multiperiod for generalized disjunctive programming (GDP) model. This problem is reformulated as a mixed-integer linear programming (MILP) using the convex hull formulation.

The MILP model is extended by adding some constraints for being the retrofit configuration. An Existing heat exchanger network is necessarily identified into the model, the location of the presented exchanger units are needed to introduce. A certain reconstruction and financial investment of adding new exchangers or area expanding in an existing process can considerably reduce the total cost of the existing plant. These options are targeted to decrease the total cost by enhancing the heat integration among process streams.

2.5.1 Area Additions for Existing and New Heat Exchanger Units

The number of heat exchanger unit in each match is considered for the additional area. Firstly, for the case where only one heat exchanger unit is allowed for one match, $(i,j) \notin B$, both possibility of adding the exchanger area in the same shell and a new one are proposed. However, when $(i,j) \in B$, there are more than one exchanger exists in the same pair of hot and cold stream matching, the area expansion possibility can be generated by adding area to the existing exchangers and also set up the new units. The following set of constraints is used to identify when a heat exchanger unit is equipped with the existing network.

Area addition to the existing heat exchangers – $(i,j) \notin B$

$$\left.\begin{array}{ll} A_{ij}^{z} \leq A_{ij}^{z^0} + \Delta A_{ij}^{z^0} + A_{ij}^{z^N} & (1) \\ \Delta A_{ij}^{z^0} \leq \Delta A_{ij\max}^{z^0} & (2) \\ A_{ij}^{z^N} \leq A_{ij\max}^{z^N} \cdot \left(U_{ij}^{z} - U_{ij}^{z^0}\right) & (3) \\ U_{ij}^{z} \leq U_{ij\max}^{z} & (4) \end{array}\right\} \quad z \in Z;\ i \in H^z;\ j \in C^z;\ (i,j) \in P;\ (i,j) \notin B;\ U_{ij}^{z,0} \geq 1$$

The area of exchanger per match (i,j) which presented only one exchanger should not be longer thane a summation of the existing area ($A_{ij}^{z^0}$), the area added to the existing shells ($\Delta A_{ij}^{z^0}$) and the area placed into the new shells ($A_{ij}^{z^N}$). The extended area into the existing shells and number of new shell need to be assigned as maximum. Additionally, a new shell is counted whenever the area is increased that shown in constraint (2.119). However, another set of equations is presented for the case in which there is no exchanger unit settled between a pair of hot and cold process streams.

Area required for new matches – (i,j) $\notin$ B

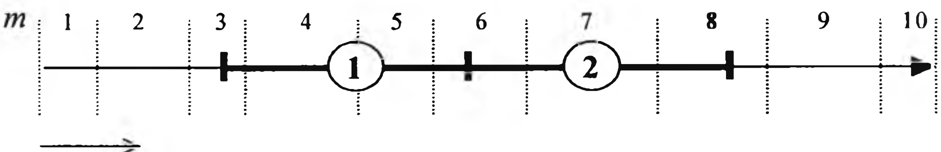
$$A_{ij}^{z^N} \leq A_{ij\max}^{z^N} \cdot U_{ij}^{z} \qquad z \in Z;\ i \in H^z;\ j \in C^z;\ (i,j) \in P;\ (i,j) \notin B;\ U_{ij}^{z,0} = 0 \qquad (5)$$

$$U_{ij}^{z} \leq U_{ij\max}^{z} \qquad z \in Z;\ i \in H^z;\ j \in C^z;\ (i,j) \in P;\ (i,j) \notin B;\ U_{ij}^{z,0} = 0 \qquad (6)$$

On the other hand, when there is more than one exchanger unit presented in the same pair of streams, $(i,j)\in B$, the position and order of each unit is necessary to record. A variable δ_{hk} is used to identify the exchanger location, an example is shown in Figure 2. For example, variable $\delta_{13}=1$ indicate that the exchanger presented in the first location in the original network and it has been equipped in the third position in the retrofitted design network. Definition of variable δ_{hk} is defined below

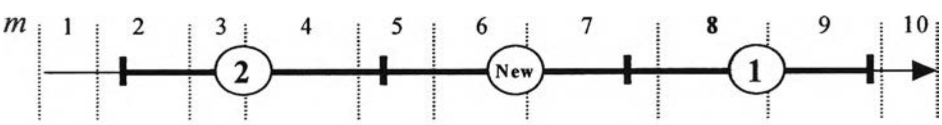
$$\delta_{hk}=\begin{cases}1 & \text{If the } h\text{-th original heat exchanger is placed in the } k\text{-th position in the retrofitted network}\\ 0 & \text{Otherwise}\end{cases}$$

Before Retrofit



Heat Exchanger Counting

After Retrofit



δ_{hk}		k = 1	2	3
h	1	0	0	1
	2	1	0	0

Figure 2.2 Area computation when $(i,j)\in B$.

The area of the k-th existing exchanger between streams i and j after retrofit should smaller or equal to the combination of original area of h-th exchanger ($\sum_{h=1}^{k_e} A_{ij}^{z,h^0}\delta_{ij}^{z,hk}$), the area added to the existing shells ($\Delta A_{ij}^{z,k^0}$) and the area for new shells (A_{ij}^{z,k^N}). Whenever an existing h-th exchanger unit is analyzed to relocate into k-th position, $\sum_{h=1}^{k_e}\delta_{ij}^{z,hk}=1$, there is no new heat exchanger unit for the retrofit network, therefore the retrofit exchanger area will be the original area combine with the addition area. On the contrary, original area term in constraint (2.124) for retrofit match will be canceled where as the new heat exchanger unit is placed, $\sum_{h=1}^{k_e} \delta_{ij}^{z,hk} = 0$.

Area addition to existing and new heat exchangers when $(i,j) \in B$

$$\left.\begin{aligned}
&A_{ij}^{z,k} \le \sum_{h=1}^{k_e} A_{ij}^{z,h^0} \delta_{ij}^{z,hk} + \Delta A_{ij}^{z,k^0} + A_{ij}^{z,k^N} && (7)\\
&\Delta A_{ij}^{z,k^0} \le \sum_{h=1}^{k_e} \left(\Delta A_{ij\max}^{z,h} \delta_{ij}^{z,hk} \right) && (8)\\
&A_{ij}^{z,k^N} \le A_{ij\max}^{z^N} \cdot \left(1 - \sum_{h=1}^{k_e} \delta_{ij}^{z,hk} \right) && (9)\\
&\sum_{h=1}^{k_e} \delta_{ij}^{z,hk} \le 1 && (10)
\end{aligned}\right\} z \in Z;\ i \in H^z;\ j \in C^z;\ (i,j) \in P;\ (i,j) \in B;\ 1 \le k \le k_{\max}$$

$$\sum_{k=1}^{k_{\max}} \delta_{ij}^{z,hk} \le 1 \qquad z \in Z;\ i \in H^z;\ j \in C^z;\ (i,j) \in P;\ (i,j) \in B;\ 1 \le h \le k_e \tag{11}$$

$$\sum_{k=1}^{k_{\max}} \sum_{h=1}^{k_e} \delta_{ij}^{z,hk} = k_e \qquad z \in Z;\ i \in H^z;\ j \in C^z;\ (i,j) \in P;\ (i,j) \in B \tag{12}$$

In addition, the number of new heat exchanger unit placed into the existing network would be specified as the following constraint.

$$\sum_{z} \sum_{i \in H^z} \sum_{\substack{j \in C^z \\ (i,j) \in P}} \left(U_{ij}^{z} - U_{ij}^{z^0} \right) \le U_{\max}^{N} \tag{13}$$

2.5.2 Objective Function

In retrofit situation, the exchanger investment cost-functions are different from the grassroot design. The objective function for the retrofit heat exchanger network structure also subjects to minimize the total annualized cost but the retrofit programming model has complicated functions for the area cost. Not only count for the number of exchanger unit, but there are also the existing units which need to optimize for area addition or new able place an exchanger. Therefore, the exchanger area for the retrofit target is consisted of area addition to the initial structure and the new exchanger area. All other terms, the hot and cold utility cost, seem to be the same as the grassroots design model. However, fixed charge for the exchanger unit is needed to count as the increasing number of unit which corresponds to subtract the number of exchanger unit, U_{ij}^{z}, with the initial unit, $U_{ij}^{z^0}$.

$$
\begin{aligned}
Min\ Cost = & \sum_{z}\sum_{i\in HU^z}\sum_{\substack{j\in C^z\\(i,j)\in P}} c_i^H F_i^H \Delta T_i + \sum_{z}\sum_{j\in CU^z}\sum_{\substack{i\in H^z\\(i,j)\in P}} c_j^C F_j^C \Delta T_j + \sum_{z}\sum_{i\in H^z}\sum_{\substack{j\in C^z\\(i,j)\in P}} c_{ij}^F \left(U_{ij}^z - U_{ij}^{z^0}\right) \\
& + \sum_{z}\sum_{i\in H^z}\sum_{\substack{j\in C^z\\(i,j)\in P\\(i,j)\notin B}} \left(c_{ij}^{A^0} \Delta A_{ij}^{z^0} + c_{ij}^{A^N} A_{ij}^{z^N}\right) + \sum_{z}\sum_{i\in H^z}\sum_{\substack{j\in C^z\\(i,j)\in P\\(i,j)\in B}} \sum_{k=1}^{k_{max}} \left(c_{ij}^{A^0} \Delta A_{ij}^{z,k^0} + c_{ij}^{A^N} A_{ij}^{z,k^N}\right)
\end{aligned}
\qquad (14)
$$

2.5.3 <u>Model for Heat Exchanger Network Included Pump Around</u>

In a crude fractionation unit the pump-around is used to provide high level temperature sources that can help in increasing the energy efficiency of crude units. The MILP grass root model is extended by adding some constraints for including the pump-around into the design the heat exchanger network. The candidate values of each pump-around are necessarily identified into the model. In this model has to define the new set, variables and equations. The new set PA^Z is introduced as pump around streams in zone z, which are the function of i. The new parameter $FPR_{i,r}^{H}$ is the candidate values for pump-around flow rate which are the function of i and r, Q_{PA} is total of pump-around duty.

2.5.3.1 <u>Heat Balance Equations</u>

These groups of equation are almost the same as the equations in the group of heat balance equations in Grass-Root Synthesis ,but some part are different.

Heat balance for hot process streams – $i \notin NI^H$ *:*

$$
\Delta H_{im}^{z,H} = \sum_{\substack{n\in M^z\\T_n^L<T_m^U}} \sum_{\substack{j\in C_n^z\\j\in P_{im}^H\\i\in P_{jn}^C}} q_{im,jn}^{z} \qquad z\in Z; m\in M^z; i\in H_m^z; i\notin HU^z; i\notin NI^H; i\notin PA^z \qquad (15)
$$

$$FP_i^H Cp_{i,m}^H (T_m^U - T_m^L) = \sum_{\substack{n \in M^z \\ T_n^L < T_m^U}} \sum_{\substack{j \in C_n^z \\ j \in P_{im}^H \\ i \in P_{jn}^C}} q_{im,jn}^z \quad z \in Z; m \in M^z; i \in H_m^z; i \notin HU^z; i \notin NI^H; i \in PA^z \tag{16}$$

Equation (16) is used to calculate heat balance of pump-around but equation (15) is not.

Heat balance for hot streams (non-isothermal mixing allowed):

$$\Delta H_{im}^{z,H} = \sum_{\substack{n \in M^z \\ T_n^L < T_m^U}} \sum_{\substack{j \in C_n^z \\ j \in P_{im}^H \\ i \in P_{jn}^C}} q_{im,jn}^z + \sum_{\substack{n \in M^z \\ n > m}} \sum_{i \in H_n^z} \bar{q}_{inm}^{z,H} - \sum_{\substack{n \in M^z \\ n < m}} \sum_{i \in H_n^z} \bar{q}_{imn}^{z,H}$$

$$z \in Z; m \in M^z; i \in H_m^z; i \notin HU^z; i \in NI^H; i \notin PA^z \tag{17}$$

$$FP_i^H Cp_{i,m}^H (T_m^U - T_m^L) = \sum_{\substack{n \in M^z \\ T_n^L < T_m^U}} \sum_{\substack{j \in C_n^z \\ j \in P_{im}^H \\ i \in P_{jn}^C}} q_{im,jn}^z + \sum_{\substack{n \in M^z \\ n > m}} \sum_{i \in H_n^z} \bar{q}_{imn}^{z,H} - \sum_{\substack{n \in M^z \\ n < m}} \sum_{i \in H_n^z} \bar{q}_{imn}^{z,H}$$

$$z \in Z; m \in M^z; i \in H_m^z; i \notin HU^z; i \in NI^H; i \in PA^z \tag{18}$$

Equation (18) is used to calculate heat balance of pump-around but equation (17) is not.

2.5.3.2 Heat Exchanger Definition and Count

This equation is almost the same as the equations in the group of heat exchanger definition and count in Grass-Root synthesis ,but some part are different.

$$q_{ijm}^L Y_{ijm}^{z,H} \le \hat{q}_{ijm}^{z,H} \le \Delta H_{im}^{z,H} Y_{ijm}^{z,H}$$

$$z \in Z; m \in M^z; i \in H_m^z; i \notin HU^z; j \in C^z; j \in P_{im}^H; i \notin PA^z \tag{19}$$

$$q^{L}_{ijm}Y^{z,H}_{ijm} \le \hat{q}^{z,H}_{ijm} \le Cp^{H}_{i,m}(T^{U}_{m} - T^{L}_{m})YFP^{z,H}_{ijm}$$

$z \in Z; m \in M^{z}; i \in H^{z}_{m}; i \notin HU^{z}; j \in C^{z}; j \in P^{H}_{im}; i \in PA^{z}$ (20)

$YFP^{z,H}_{ijm} = \sum_{r} FPR^{H}_{i,r} YW^{z,H}_{ijm}$ $z \in Z; m \in M^{z}; i \in H^{z}_{m}; i \notin HU^{z}; j \in C^{z}; j \in P^{H}_{im}; i \in PA^{z}$ (21)

$YW^{z,H}_{ijm} - Y^{z,H}_{ijm} \le 0$ $z \in Z; m \in M^{z}; i \in H^{z}_{m}; i \notin HU^{z}; j \in C^{z}; j \in P^{H}_{im}; i \in PA^{z}$

(2.138)

$YW^{z,H}_{ijm} \le W_{i,r}$ $z \in Z; m \in M^{z}; i \in H^{z}_{m}; i \notin HU^{z}; j \in C^{z}; j \in P^{H}_{im}; i \in PA^{z}$ (22)

$YW^{z,H}_{ijm} \ge Y^{z,H}_{ijm} + W_{i,r} - 1$ $z \in Z; m \in M^{z}; i \in H^{z}_{m}; i \notin HU^{z}; j \in C^{z}; j \in P^{H}_{im}; i \in PA^{z}$ (23)

Equation (20) to (23) is used for heat exchanger defining heat balance of pump-around but equation (19) is not.

2.5.3.3 Heat Transfer Consistency

These groups of equation are almost same as the equations in group of heat transfer consistency in Grass-Root Synthesis, but some part are different.

Heat transfer consistency for multiple heat exchangers between the same pair of streams.

$$\sum_{\substack{l \in M^{z}_{i} \\ l \le m}} \hat{q}^{z,H}_{ijl} - \tilde{q}^{z,H}_{ijn} \le \sum_{\substack{l \in N^{z}_{j} \\ l \le n}} \hat{q}^{z,C}_{ijl} - \tilde{q}^{z,C}_{ijm} + 4X^{z}_{im,jn} Max\{ \sum_{\substack{l \in M^{z}_{i} \\ l \le m \\ j \in P^{H}_{il}}} \Delta H^{z,H}_{il}; \sum_{\substack{l \in M^{z}_{i} \\ l \le n \\ i \in P^{C}_{jl}}} \Delta H^{z,C}_{jl} \}$$

$z \in Z; m,n \in M^{z}; T^{L}_{n} \le T^{U}_{m}; (i,j) \in B; i \in H^{z}_{m}; j \in C^{z}_{n}; i \in P^{C}_{jn}; j \in P^{H}_{im}; i \notin PA^{z}$ (24)

$$\sum_{\substack{l \in M^{z}_{i} \\ l \le m}} \hat{q}^{z,H}_{ijl} - \tilde{q}^{z,H}_{ijn} \le \sum_{\substack{l \in N^{z}_{j} \\ l \le n}} \hat{q}^{z,C}_{ijl} - \tilde{q}^{z,C}_{ijm} + 4XM^{z}_{im,jn}$$

$z \in Z; m,n \in M^{z}; T^{L}_{n} \le T^{U}_{m}; (i,j) \in B; i \in H^{z}_{m}; j \in C^{z}_{n}; i \in P^{C}_{jn}; j \in P^{H}_{im}; i \in PA^{z}$ (25)

$$XM^{z}_{im,jn} - T^{z}_{im,jn}\Omega^{z}_{im,jn} \le \sum XW^{z}_{im,jn,r} FPR^{H}_{i,r} \sum_{\substack{l \in M^{z}_{i} \\ l \le m \\ j \in P^{H}_{il}}} Cp^{H}_{i,l}(T^{U}_{l} - T^{L}_{l})$$

$$z \in Z; m,n \in M^z; T_n^L \leq T_m^U; (i,j) \in B; i \in H_m^z; j \in C_n^z; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \tag{26}$$

$$XM_{im,jn}^z - T_{im,jn}^z \Omega_{im,jn}^z \geq \sum XW_{im,jn,r}^z FPR_{i,r}^H \sum_{\substack{l \in M_i^z \\ l \leq m \\ j \in P_{il}^H}} Cp_{i,l}^H (T_l^U - T_l^L)$$

$$z \in Z; m,n \in M^z; T_n^L \leq T_m^U; (i,j) \in B; i \in H_m^z; j \in C_n^z; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \tag{27}$$

$$XM_{im,jn}^z - (1 - T_{im,jn}^z)\Omega_{im,jn}^z \leq X_{im,jn}^z \sum_{\substack{l \in M_i^z \\ l \leq n \\ i \in P_{jl}^C}} \Delta H_{jl}^{z,C}$$

$$z \in Z; m,n \in M^z; T_n^L \leq T_m^U; (i,j) \in B; i \in H_m^z; j \in C_n^z; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \tag{28}$$

$$XM_{im,jn}^z \geq X_{im,jn}^z \sum_{\substack{l \in M_o^z \\ l \leq n \\ i \in P_{jl}^C}} \Delta H_{jl}^{z,C}$$

$$z \in Z; m,n \in M^z; T_n^L \leq T_m^U; (i,j) \in B; i \in H_m^z; j \in C_n^z; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \tag{29}$$

$$XW_{im,jn,r}^{z,H} - \Gamma_{im,jn}^{z,H} W_{i,r} \leq 0$$

$$z \in Z; m,n \in M^z; T_n^L \leq T_m^U; (i,j) \in B; i \in H_m^z; j \in C_n^z; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \tag{30}$$

$$(X_{im,jn}^{z,H} - XW_{im,jn,r}^{z,H}) - (1 - W_{i,r})\Gamma_{im,jn}^{z,H} \leq 0$$

$$z \in Z; m,n \in M^z; T_n^L \leq T_m^U; (i,j) \in B; i \in H_m^z; j \in C_n^z; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \tag{31}$$

$$(X_{im,jn}^{z,H} - XW_{im,jn,r}^{z,H}) \geq 0$$

$$z \in Z; m,n \in M^z; T_n^L \leq T_m^U; (i,j) \in B; i \in H_m^z; j \in C_n^z; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \tag{32}$$

Equation (25) to (32) is used to calculate heat transfer of pump-around but equation (24) is not.

$$\sum_{\substack{l \in M_i^z \\ l \leq m}} \hat{q}_{ijl}^{z,H} - \tilde{q}_{ijn}^{z,H} \geq \sum_{\substack{l \in N_j^z \\ l \leq n}} \hat{q}_{ijl}^{z,C} - \tilde{q}_{ijm}^{z,C} - 4X_{im,jn}^z Max\ \{ \sum_{\substack{l \in M_i^z \\ l \leq m \\ j \in P_{il}^H}} \Delta H_{il}^{z,H}; \sum_{\substack{l \in M_i^z \\ l \leq n \\ i \in P_{jl}^C}} \Delta H_{jl}^{z,C} \}$$

$$z \in Z ; m,n \in M^z ; T_n^L \le T_m^U ; (i,j) \in B, i \in H_m^z ; j \in C_n^z ; i \in P_{jn}^C ; j \in P_{im}^H ; i \notin PA^z \tag{33}$$

$$\sum_{\substack{l \in M_i^z \\ l \le m}} \hat{q}_{ijl}^{z,H} - \tilde{q}_{ijn}^{z,H} \ge \sum_{\substack{l \in N_j^z \\ l \le n}} \hat{q}_{ijl}^{z,C} - \tilde{q}_{ijm}^{z,C} - 4XM_{im,jn}^{z}$$

$$z \in Z ; m,n \in M^z ; T_n^L \le T_m^U ; (i,j) \in B, i \in H_m^z ; j \in C_n^z ; i \in P_{jn}^C ; j \in P_{im}^H ; i \in PA^z \tag{34}$$

Equation (34) is used to calculate heat transfer of pump-around but equation (33) is not.

$$\tilde{q}_{ijm}^{z,H} \le K_{ijm}^{z,H} \Delta H_{im}^{z,H} \qquad z \in Z; m \in M^z; (i,j) \in B; i \in H_m^z; j \in P_{im}^H; i \in PA^z \tag{35}$$

$$\tilde{q}_{ijm}^{z,H} \le KFP_{ijm}^{z,H} Cp_{i,m}^H (T_m^U - T_m^L) \qquad z \in Z; m \in M^z; (i,j) \in B; i \in H_m^z; j \in P_{im}^H; i \in PA^z \tag{36}$$

$$KFP_{ijm}^{z,H} = \sum_r FPR_{i,r}^H \, KW_{ijm,r}^{z,H} \qquad z \in Z; m \in M^z; (i,j) \in B; i \in H_m^z; j \in P_{im}^H; i \in PA^z \tag{37}$$

$$KW_{ijm,r}^{z,H} - K_{ijm}^{z,H} \le 0 \qquad z \in Z; m \in M^z; (i,j) \in B; i \in H_m^z; j \in P_{im}^H; i \in PA^z \tag{38}$$

$$KW_{ijm,r}^{z,H} \le W_{i,r} \qquad z \in Z; m \in M^z; (i,j) \in B; i \in H_m^z; j \in P_{im}^H; i \in PA^z \tag{39}$$

$$KW_{ijm,r}^{z,H} \ge K_{ijm}^{z,H} + W_{i,r} - 1 \qquad z \in Z; m \in M^z; (i,j) \in B; i \in H_m^z; j \in P_{im}^H; i \in PA^z \tag{40}$$

Equation (36) to (40) is used to calculate heat transfer of pump-around but equation (35) is not.

$$\tilde{q}_{ijm}^{z,H} \le \hat{K}_{ijm}^{z,H} \Delta H_{im}^{z,H} \qquad z \in Z; m \in M^z; (i,j) \in B; i \in H_m^z; j \in P_{im}^H; i \notin PA^z \tag{41}$$

$$\tilde{q}_{ijm}^{z,H} \le \widehat{KFP}_{ijm}^{z,H} Cp_{i,m}^H (T_m^U - T_m^L) \qquad z \in Z; m \in M^z; (i,j) \in B; i \in H_m^z; j \in P_{im}^H; i \in PA^z \tag{42}$$

$$\widehat{KFP}_{ijm}^{z,H} = \sum_r FPR_{i,r}^H \, \widehat{KW}_{ijm,r}^{z,H} \qquad z \in Z; m \in M^z; (i,j) \in B; i \in H_m^z; j \in P_{im}^H; i \in PA^z \tag{43}$$

$$\hat{KW}_{ijm,r}^{z,H} - \hat{K}_{ijm}^{z,H} \le 0 \qquad z \in Z; m \in M^z; (i,j) \in B; i \in H_m^z; j \in P_{im}^H; i \in PA^z \qquad (44)$$

$$\hat{KW}_{ijm,r}^{z,H} \le W_{i,r} \qquad z \in Z; m \in M^z; (i,j) \in B; i \in H_m^z; j \in P_{im}^H; i \in PA^z \qquad (45)$$

$$\hat{KW}_{ijm,r}^{z,H} \ge \hat{K}_{ijm}^{z,H} + W_{i,r} - 1 \qquad z \in Z; m \in M^z; (i,j) \in B; i \in H_m^z; j \in P_{im}^H; i \in PA^z \qquad (46)$$

Equations (42) to (46) are used to calculate heat transfer of pump-around but equation (41) is not.

2.5.3.4 <u>Flow Rate Consistency within Heat Exchangers</u>

These groups of equation are almost same as the equations in the group of flow rate consistency in Grass-Root Synthesis but some part are different.

$$\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \le \frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} + \left(1 - \alpha_{ijm}^{z,H}\right) \cdot F_i$$

$$z \in Z; m \in M^z; i \in H_m^z \cap H_{m-1}^z; i \in S^H; j \in C^z; j \in P_{im}^H \cap P_{im-1}^H; i \in PA^z \qquad (47)$$

$$\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \le \frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} + (FP_i^H - FP\alpha_{ijm}^{z,H})$$

$$z \in Z; m \in M^z; i \in H_m^z \cap H_{m-1}^z; i \in S^H; j \in C^z; j \in P_{im}^H \cap P_{im-1}^H; i \in PA^z \qquad (48)$$

$$FP\alpha_{ijm}^{z,H} = \sum FPR_{i,r}^H \, W\alpha_{ijm,r}^{z,H}$$

$$z \in Z; m \in M^z; i \in H_m^z \cap H_{m-1}^z; i \in S^H; j \in C^z; j \in P_{im}^H \cap P_{im-1}^H; i \in PA^z \qquad (49)$$

$$W\alpha_{ijm,r}^{z,H} - \alpha_{ijm}^{z,H} \le 0$$

$$z \in Z; m \in M^z; i \in H_m^z \cap H_{m-1}^z; i \in S^H; j \in C^z; j \in P_{im}^H \cap P_{im-1}^H; i \in PA^z \qquad (50)$$

$$W\alpha_{ijm,r}^{z,H} \le W_{i,r}$$

$$z \in Z; m \in M^z; i \in H_m^z \cap H_{m-1}^z; i \in S^H; j \in C^z; j \in P_{im}^H \cap P_{im-1}^H; i \in PA^z \qquad (51)$$

$$W\alpha_{ijm,r}^{z,H} \ge \alpha_{ijm}^{z,H} + W_{i,r} - 1$$

$$z \in Z; m \in M^z; i \in H_m^z \cap H_{m-1}^z; i \in S^H; j \in C^z; j \in P_{im}^H \cap P_{im-1}^H; i \in PA^z \qquad (52)$$

Equation (48) to (52) is used to calculate flow rate within heat exchanger of pump-around but equation (47) is not.

$$\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \geq \frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} - \left(1 - \alpha_{ijm}^{z,H}\right) \cdot F_i$$

$$z \in Z; m \in M^z; i \in H_m^z \cap H_{m-1}^z; i \in S^H; j \in C^z; j \in P_{im}^H \cap P_{im-1}^H; i \notin PA^z \qquad (53)$$

$$\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \geq \frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} - (FP_i^H - FP\alpha_{ijm}^{z,H})$$

$$z \in Z; m \in M^z; i \in H_m^z \cap H_{m-1}^z; i \in S^H; j \in C^z; j \in P_{im}^H \cap P_{im-1}^H; i \in PA^z \qquad (54)$$

Equation (54) is used to calculate flow rate within heat exchanger of pump-around but equation (53) is not.

$$\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \geq \frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} - \left(1 + \hat{K}_{ijm-1}^{z,H} + \hat{K}_{ijm}^{z,H} - K_{ijm-1}^{z,H}\right) \cdot F_i$$

$$z \in Z; m \in M^z; i \in H_m^z \cap H_{m-1}^z; j \in P_{im}^H \cap P_{im-1}^H; i \in S^H; j \in C^z; (i,j) \notin B; i \notin PA^z \qquad (55)$$

$$\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \geq \frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} - (FP_i^H + \hat{K}FP_{ijm-1}^{z,H} + \hat{K}FP_{ijm}^{z,H} - KFP_{ijm-1}^{z,H})$$

$$z \in Z; m \in M^z; i \in H_m^z \cap H_{m-1}^z; j \in P_{im}^H \cap P_{im-1}^H; i \in S^H; j \in C^z; (i,j) \notin B; i \in PA^z \qquad (56)$$

Equation (56) is used to calculate flow rate within heat exchanger of pump-around but equation (55) is not.

$$\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \leq \frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} + \left(1 + K_{ijm-1}^{z,H} + K_{ijm}^{z,H} - \hat{K}_{ijm}^{z,H}\right) \cdot F_i$$

$$z \in Z; m \in M^z; i \in H_m^z \cap H_{m-1}^z; j \in P_{im}^H \cap P_{im-1}^H; i \in S^H; j \in C^z; (i,j) \notin B; i \notin PA^z \qquad (57)$$

$$\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \leq \frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} + (FP_i^H + KFP_{ijm-1}^{z,H} + KFP_{ijm}^{z,H} - \hat{K}FP_{ijm}^{z,H})$$

$$z \in Z; m \in M^z; i \in H_m^z \cap H_{m-1}^z; j \in P_{im}^H \cap P_{im-1}^H; i \in S^H; j \in C^z; (i,j) \notin B; i \in PA^z \qquad (58)$$

Equation (58) is used to calculate flow rate within heat exchanger of pump-around but equation (57) is not.

$$\frac{\hat{q}^{z,H}_{ijm}}{Cp_{im}(T^U_m - T^L_m)} \geq \frac{\hat{q}^{z,H}_{ijm-1}}{Cp_{im-1}(T^U_{m-1} - T^L_{m-1})} - \left(1 + \hat{K}^{z,H}_{ijm-1} + \hat{K}^{z,H}_{ijm} - K^{z,H}_{ijm-1}\right) \cdot F_i$$

$$z \in Z; m \in M^z; i \in H^z_m \cap H^z_{m-1}; j \in P^H_{im} \cap P^H_{im-1}; i \in S^H; j \in C^z; (i,j) \in B; i \notin PA^z \quad (59)$$

$$\frac{\hat{q}^{z,H}_{ijm}}{Cp_{im}(T^U_m - T^L_m)} \geq \frac{\hat{q}^{z,H}_{ijm-1}}{Cp_{im-1}(T^U_{m-1} - T^L_{m-1})} - (FP^H_i + \hat{K}FP^{z,H}_{ijm-1} + \hat{K}FP^{z,H}_{ijm} - KFP^{z,H}_{ijm-1})$$

$$z \in Z; m \in M^z; i \in H^z_m \cap H^z_{m-1}; j \in P^H_{im} \cap P^H_{im-1}; i \in S^H; j \in C^z; (i,j) \in B; i \in PA^z \quad (60)$$

Equation (60) is used to calculate flow rate within heat exchanger of pump-around but equation (59) is not.

$$\frac{\hat{q}^{z,H}_{ijm}}{Cp_{im}(T^U_m - T^L_m)} \geq \frac{\tilde{q}^{z,H}_{ijm-1}}{Cp_{im-1}(T^U_{m-1} - T^L_{m-1})} - \left(2 + \hat{K}^{z,H}_{ijm} - K^{z,H}_{ijm-1} - Y^{z,H}_{ijm-1}\right) \cdot F_i$$

$$z \in Z; m \in M^z; i \in H^z_m \cap H^z_{m-1}; j \in P^H_{im} \cap P^H_{im-1}; i \in S^H; j \in C^z; (i,j) \in B; i \notin PA^z \quad (61)$$

$$\frac{\hat{q}^{z,H}_{ijm}}{Cp_{im}(T^U_m - T^L_m)} \geq \frac{\tilde{q}^{z,H}_{ijm-1}}{Cp_{im-1}(T^U_{m-1} - T^L_{m-1})} - (2FP^H_i + \hat{K}FP^{z,H}_{ijm} - KFP^{z,H}_{ijm-1} - YFP^{z,H}_{ijm-1})$$

$$z \in Z; m \in M^z; i \in H^z_m \cap H^z_{m-1}; j \in P^H_{im} \cap P^H_{im-1}; i \in S^H; j \in C^z; (i,j) \in B; i \in PA^z \quad (62)$$

Equation (62) is used to calculate flow rate within heat exchanger of pump-around but equation (61) is not.

$$\frac{\hat{q}^{z,H}_{ijm} - \tilde{q}^{z,H}_{ijm}}{Cp_{im}(T^U_m - T^L_m)} \leq \frac{\hat{q}^{z,H}_{ijm-1}}{Cp_{im-1}(T^U_{m-1} - T^L_{m-1})} + \left(2 + K^{z,H}_{ijm-1} - \hat{K}^{z,H}_{ijm} - Y^{z,H}_{ijm}\right) \cdot F_i$$

$$z \in Z; m \in M^z; i \in H^z_m \cap H^z_{m-1}; j \in P^H_{im} \cap P^H_{im-1}; i \in S^H; j \in C^z; (i,j) \in B; i \notin PA^z \quad (63)$$

$$\frac{\hat{q}_{ijm}^{z,H}-\tilde{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U-T_m^L)}\leq\frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U-T_{m-1}^L)}+(2FP_i^H+KFP_{ijm-1}^{z,H}-\hat{K}FP_{ijm}^{z,H}-YFP_{ijm}^{z,H})$$

$$z\in Z\,;m\in M^z;i\in H_m^z\cap H_{m-1}^z;j\in P_{im}^H\cap P_{im-1}^H;i\in S^H\,;\,j\in C^z;(\,i,j)\in B;i\in PA^z \quad (64)$$

Equation (64) is used to calculate flow rate within heat exchanger of pump-around but equation (63) is not

$$\hat{q}_{ijm}^{z,H}\geq\left(Y_{ijm}^{z,H}-K_{ijm}^{z,H}-\hat{K}_{ijm}^{z,H}\right)\cdot\Delta H_{im}^{z,H} \qquad \begin{matrix} z\in Z\,;m\in M^z;\;i\in H_{m-1}^z\cap H_m^z\cap H_{m+1}^z\,;i\notin S^H \\ j\in C^z\,;\,j\in P_{im-1}^H\cap P_{im}^H\cap P_{im+1}^H;i\notin PA^z \end{matrix} \quad (65)$$

$$\hat{q}_{ijm}^{z,H}\geq\left(YFP_{ijm}^{z,H}-KFP_{ijm}^{z,H}-\hat{K}FP_{ijm}^{z,H}\right)\cdot Cp_{i,m}^H(T_m^U-T_m^L) \qquad \begin{matrix} z\in Z\,;m\in M^z;\;i\in H_{m-1}^z\cap H_m^z\cap H_{m+1}^z\,;i\notin S^H \\ j\in C^z\,;\,j\in P_{im-1}^H\cap P_{im}^H\cap P_{im+1}^H;i\in PA^z \end{matrix} \quad (66)$$

Equation (66) is used to calculate flow rate within heat exchanger of pump-around but equation (65) is not

2.5.3.5 Temperature Difference Enforcing

These groups of equation are almost same as the equations in the group of temperature difference enforcing in Grass-Root Synthesis but some part are different.

$$T_m^L+\frac{\hat{q}_{ijm}^{z,H}}{F_iCp_{im}}\geq T_n^L+\frac{\hat{q}_{ijn}^{z,C}}{F_jCp_{jn}}-\left(2-K_{ijm}^{z,H}-K_{ijn}^{z,C}\right)\cdot T_n^U$$

$$z\in Z;m,n\in M^z;T_n^L\leq T_m^U;T_n^U\geq T_m^L;i\in H_m^z;j\in C_n^z;i\notin S^H;j\notin S^C;i\in P_{jn}^C;j\in P_{im}^H;i\notin PA^z \quad (67)$$

$$T_m^L+\frac{FP\hat{q}_{ijm}^{z,H}}{Cp_{im}}\geq T_n^L+\frac{\hat{q}_{ijn}^{z,C}}{F_jCp_{jn}}-(2-K_{ijm}^{z,H}-K_{ijn}^{z,C})\cdot T_n^U$$

$$z\in Z;m,n\in M^z;T_n^L\leq T_m^U;T_n^U\geq T_m^L;i\in H_m^z;j\in C_n^z;i\notin S^H;j\notin S^C;i\in P_{jn}^C;j\in P_{im}^H;i\in PA^z \quad (68)$$

$$FP\hat{q}_{ijm}^{z,H}=\sum_r\frac{W\hat{q}_{ijm,r}^{z,H}}{FPR_{i,r}^H}$$

$$z \in Z; m,n \in M^z; T_n^L \le T_m^U; T_n^U \ge T_m^L; i \in H_m^z; j \in C_n^z; i \notin S^H; j \notin S^C; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \quad (69)$$

$$W\hat{q}_{ijm,r}^{z,H} - \Gamma_{im,jn}^{z,H} W_{i,r} \le 0$$

$$z \in Z; m,n \in M^z; T_n^L \le T_m^U; T_n^U \ge T_m^L; i \in H_m^z; j \in C_n^z; i \notin S^H; j \notin S^C; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \quad (70)$$

$$(\hat{q}_{ijm,r}^{z,H} - W\hat{q}_{ijm,r}^{z,H}) - (1 - W_{i,r})\Gamma_{im,jn}^{z,H} \le 0$$

$$z \in Z; m,n \in M^z; T_n^L \le T_m^U; T_n^U \ge T_m^L; i \in H_m^z; j \in C_n^z; i \notin S^H; j \notin S^C; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \quad (71)$$

$$(\hat{q}_{ijm,r}^{z,H} - W\hat{q}_{ijm,r}^{z,H}) \ge 0$$

$$z \in Z; m,n \in M^z; T_n^L \le T_m^U; T_n^U \ge T_m^L; i \in H_m^z; j \in C_n^z; i \notin S^H; j \notin S^C; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \quad (72)$$

Equation (68) to (72) is used to control temperature for pump-around heat exchanger but equation (67) is not.

$$T_m^U - \frac{\hat{q}_{ijm}^{z,H}}{F_i\, Cp_{im}} \ge T_n^U - \frac{\hat{q}_{ijn}^{z,C}}{F_j Cp_{jn}} - (2 - \hat{K}_{ijm}^{z,H} - \hat{K}_{ijn}^{z,C}).T_n^U$$

$$z \in Z; m,n \in M^z; T_n^L \le T_m^U; T_n^U \ge T_m^L; i \in H_m^z; j \in C_n^z; i \notin S^H; j \notin S^C; i \in P_{jn}^C; j \in P_{im}^H; i \notin PA^z \quad (73)$$

$$T_m^U - \frac{FP\hat{q}_{ijm}^{z,H}}{Cp_{im}} \ge T_n^U - \frac{\hat{q}_{ijn}^{z,C}}{F_j Cp_{jn}} - (2 - \hat{K}_{ijm}^{z,H} - \hat{K}_{ijn}^{z,C}).T_n^U$$

$$z \in Z; m,n \in M^z; T_n^L \le T_m^U; T_n^U \ge T_m^L; i \in H_m^z; j \in C_n^z; i \notin S^H; j \notin S^C; i \in P_{jn}^C; j \in P_{im}^H; i \in PA^z \quad (74)$$

Equation (74) used to control temperature for pump-around heat exchanger but equation (73) is not.

$$\frac{\hat{q}_{ijm}^{z,H}}{Min\{T_m^U;T_n^U\}-T_m^L} \geq \frac{\hat{q}_{ijm+1}^{z,H}}{T_{m+1}^U-T_{m+1}^L}\frac{Cp_{im}}{Cp_{im+1}}-\left(2-K_{ijm}^{z,H}-K_{ijn}^{z,C}\right)\frac{\Delta H_{im+1}^{z,H}}{T_{m+1}^U-T_{m+1}^L}$$

$$z\in Z;m,n\in M^z\ ;i\in S^H;j\in S^C;T_n^L<T_m^U;T_n^U>T_m^L;i\in H_m^z\cap H_{m+1}^z;j\in C_n^z\cap C_{n+1}^z;i\in P_{jn}^C\cap P_{jn+1}^C;j\in P_{im}^H\cap P_{im+1}^H;i\notin PA^z \tag{75}$$

$$\frac{\hat{q}_{ijm}^{z,H}}{Min\{T_m^U;T_n^U\}-T_m^L} \geq \frac{\hat{q}_{ijm+1}^{z,H}}{T_{m+1}^U-T_{m+1}^L}\frac{Cp_{im}}{Cp_{im+1}}-\left(2FP_i^H-KFP_{ijm}^{z,H}-KFP_{ijn}^{z,C}\right)\frac{Cp_{i,m+1}^H(T_{m+1}^U-T_{m+1}^L)}{T_{m+1}^U-T_{m+1}^L}$$

$$z\in Z;m,n\in M^z\ ;i\in S^H;j\in S^C;T_n^L<T_m^U;T_n^U>T_m^L;i\in H_m^z\cap H_{m+1}^z;j\in C_n^z\cap C_{n+1}^z;i\in P_{jn}^C\cap P_{jn+1}^C;j\in P_{im}^H\cap P_{im+1}^H;i\in PA^z \tag{76}$$

Equation (76) used to control temperature for pump-around heat exchanger but equation (75) is not.

$$\frac{\hat{q}_{ijm}^{z,H}}{T_m^U-T_n^L} \leq \frac{\hat{q}_{ijm-1}^{z,H}}{T_{m-1}^U-T_{m-1}^L}\frac{Cp_{im}}{Cp_{im-1}}+\left(2-\hat{K}_{ijm}^{z,H}-\hat{K}_{ijn}^{z,C}\right)\frac{\Delta H_{im}^{z,H}}{T_m^U-T_n^L}$$

$$z\in Z;m,n\in M^z\ ;i\in S^H;j\in S^C;T_n^L<T_m^U;T_n^U>T_m^L;i\in H_m^z\cap H_{m-1}^z;j\in C_n^z\cap C_{n-1}^z;i\in P_{jn}^C\cap P_{jn-1}^C;j\in P_{im}^H\cap P_{im-1}^H;i\notin PA^z \tag{77}$$

$$\frac{\hat{q}_{ijm}^{z,H}}{T_m^U-T_n^L} \leq \frac{\hat{q}_{ijm-1}^{z,H}}{T_{m-1}^U-T_{m-1}^L}\frac{Cp_{im}}{Cp_{im-1}}+\left(2FP_i^H-\hat{K}FP_{ijm}^{z,H}-\hat{K}FP_{ijn}^{z,C}\right)\frac{Cp_{i,m}^H(T_m^U-T_m^L)}{T_m^U-T_n^L}$$

$$z\in Z;m,n\in M^z\ ;i\in S^H;j\in S^C;T_n^L<T_m^U;T_n^U>T_m^L;i\in H_m^z\cap H_{m-1}^z;j\in C_n^z\cap C_{n-1}^z;i\in P_{jn}^C\cap P_{jn-1}^C;j\in P_{im}^H\cap P_{im-1}^H;i\in PA^z \tag{78}$$

Equation (78) used to control temperature for pump-around heat exchanger but equation (77) is not.

$$\frac{\hat{q}_{ijm}^{z,H}}{Min\{T_m^U;T_n^U\}-T_m^L} \geq \frac{\hat{q}_{ijm+1}^{z,H}}{T_{m+1}^U-T_{m+1}^L}\frac{Cp_{im}}{Cp_{im+1}}-\left(2-K_{ijm}^{z,H}-K_{ijn}^{z,C}\right)\frac{\Delta H_{im+1}^{z,H}}{T_{m+1}^U-T_{m+1}^L} \tag{79}$$

$$z\in Z;m,n\in M^z;i\in S^H;j\in S^C;\ T_n^L<T_m^U;T_n^U>T_m^L;i\in H_m^z\cap H_{m+1}^z;$$
$$j\in C_n^z\cap C_{n+1}^z;i\in P_{jn}^C\cap P_{jn+1}^C;j\in P_{im}^H\cap P_{im+1}^H;i\notin PA^z$$

$$\frac{\hat{q}_{ijm}^{z,H}}{Min\{T_m^U;T_n^U\}-T_m^L} \geq \frac{\hat{q}_{ijm+1}^{z,H}}{T_{m+1}^U - T_{m+1}^L}\frac{Cp_{im}}{Cp_{im+1}} - \left(2FP_i^H - KFP_{ijm}^{z,H} - KFP_{ijn}^{z,C}\right).\frac{Cp_{i,m+1}^H(T_{m+1}^U - T_{m+1}^L)}{T_{m+1}^U - T_{m+1}^L} \tag{80}$$

$z\in Z;m,n\in M^z;i\in S^H;j\in S^C;\ T_n^L<T_m^U;T_n^U>T_m^L;i\in H_m^z\cap H_{m+1}^z;$
$j\in C_n^z\cap C_{n+1}^z;i\in P_{jn}^C\cap P_{jn+1}^C;j\in P_{im}^H\cap P_{im+1}^H;i\in PA^z$

Equation (80) used to control temperature for pump-around heat exchanger but equation (79) is not.

$$\frac{\hat{q}_{ijm}^{z,H}-\tilde{q}_{ijm}^{z,H}}{T_m^U-T_n^L} \leq \frac{\hat{q}_{ijm-1}^{z,H}}{T_{m-1}^U-T_{m-1}^L}\frac{Cp_{im}}{Cp_{im-1}} + \left(2-\hat{K}_{ijm}^{z,H}-\hat{K}_{ijn}^{z,C}\right).\frac{\Delta H_{im}^{z,H}}{T_m^U-T_n^L}$$

$z\in Z;m,n\in M^z;i\in S^H;j\in S^C;\ T_n^L<T_m^U;T_n^U>T_m^L;i\in H_m^z\cap H_{m-1}^z;$ (81)
$j\in C_n^z\cap C_{n-1}^z;i\in P_{jn}^C\cap P_{jn-1}^C;j\in P_{im}^H\cap P_{im-1}^H;i\notin PA^z$

$$\frac{\hat{q}_{ijm}^{z,H}-\tilde{q}_{ijm}^{z,H}}{T_m^U-T_n^L} \leq \frac{\hat{q}_{ijm-1}^{z,H}}{T_{m-1}^U-T_{m-1}^L}\frac{Cp_{im}}{Cp_{im-1}} + \left(2FP_i^H-\hat{K}FP_{ijm}^{z,H}-\hat{K}FP_{ijn}^{z,C}\right).\frac{Cp_{i,m}^H(T_m^U-T_m^L)}{T_m^U-T_n^L}$$

$z\in Z;m,n\in M^z;i\in S^H;j\in S^C;\ T_n^L<T_m^U;T_n^U>T_m^L;i\in H_m^z\cap H_{m-1}^z;$ (82)
$j\in C_n^z\cap C_{n-1}^z;i\in P_{jn}^C\cap P_{jn-1}^C;j\in P_{im}^H\cap P_{im-1}^H;i\in PA^z$

Equation (82) used to control temperature for pump-around heat exchanger but equation (81) is not.

2.6 Model for Retrofit with Relocation of Existing Heat Exchangers

The MILP model, when considering for retrofitting network, an existing exchanger located in the same pair of hot and cold stream. Considering retrofit with relocation the location of match *(i',j')*of existing heat exchanger units is changed from original match *(i,j)*. Relocation possibility can be calculated by the binary variables, such as 1000 binary variables are used to define the possible relocations for the network composed of 10 hot, 10 cold streams and 10 original heat exchanger units. A very large number of integers will effect to the model performance. Thus, this algorithm is considered the exchanger relocation for the case where highly reducing cost occurs. So, the designer should define which exchanger is relocated and the following constraints are used to figure out the exchanger area after repositioning.

2.6.1 Area requirement for existing

Relocated and new heat exchangers – $(i,j) \notin$ B

$$\left.\begin{aligned}
&A_{ij}^{z} \le A_{ij}^{z^0} + \Delta A_{ij}^{z^0} + A_{ij}^{z^N} && (83)\\
&A_{ij}^{z^0} = \sum_{k=1}^{k_e^t} A^k \delta_{ij}^{z,k} && (84)\\
&\Delta A_{ij}^{z^0} \le \sum_{k=1}^{k_e^t} \Delta A_{\max}^k \delta_{ij}^{z,k} && (85)\\
&A_{ij}^{z^N} \le A_{ij\max}^{z^N} \cdot \left(U_{ij}^{z} - U_{ij}^{z^0} \cdot \sum_{h=1}^{k_e^t} \delta_{ij}^{z,k} \right) && (86)\\
&U_{ij}^{z} \le U_{ij\max}^{z} \sum_{h=1}^{k_e} \phi_{ij}^{z,k} \le 1 && (87)\\
&\sum_{h=1}^{k_e} \delta_{ij}^{z,k} \le 1 && (88)
\end{aligned}\right\} \quad z \in Z;\ i \in H^z;\ j \in C^z;\ (i,j) \in P;\ (i,j) \notin B$$

Where A^k is the area of original exchanger that has been relocated to the new match (i',j'). Whenever the original k-th exchanger is utilized to serve in a new match, $\sum_{k=1}^{k_e} \delta_{ij}^{z,k}$ is equal to one, and then relocation constraint (2.130) forces that the existing exchanger area at the new match (i',j'), $A_{ij}^{z^0}$, also equals to the unit area of the original match, A^k . Maximum area addition for the existing unit which served to relocate is also required.

Area requirement for existing, relocated and new heat exchangers – $(i,j) \in$ B

$$\left.\begin{aligned}
&A_{ij}^{z,k} \le A_{ij}^{z,k^0} + \Delta A_{ij}^{z,k^0} + A_{ij}^{z,k^N} && (89)\\
&A_{ij}^{z,k^0} = \sum_{h=1}^{k_e^t} A^h \delta_{ij}^{z,hk} && (90)\\
&\Delta A_{ij}^{z,k^0} \le \sum_{h=1}^{k_e^t} \Delta A_{\max}^h \delta_{ij}^{z,hk} && (91)\\
&A_{ij}^{z,k^N} \le A_{ij\,\max}^{z^N} \cdot \left(1 - \sum_{h=1}^{k_e^t} \delta_{ij}^{z,hk}\right) && (92)\\
&\sum_{h=1}^{k_e^t} \delta_{ij}^{z,hk} \le 1 && (93)\\
&\sum_{i,j} \delta_{ij}^{z,hk} \le 1 && (94)
\end{aligned}\right\} z \in Z;\ i \in H^z;\ j \in C^z;\ (i,j) \in P;\ (i,j) \in B;\ 1 \le k \le k_{\max}$$

2.6.2 <u>Objective Function for Retrofit with Relocation</u>

In retrofit with relocation situation, the exchanger investment cost-functions are different from the grassroots and retrofit design. The objective function for the retrofit with relocation heat exchanger network structure also subjects to minimize the total annualized cost but the retrofit programming model has complicated functions for the area cost. Not only count for the number of exchanger unit, but there are also switching the existing units which need to optimize for area addition or new able place an exchanger. For the exchanger relocation, the objective function would be

$$\begin{aligned}
Min\ Cost = &\sum_z \sum_{i \in HU^z} \sum_{\substack{j \in C^z \\ (i,j) \in P}} c_i^H F_i^H \Delta T_i + \sum_z \sum_{j \in CU^z} \sum_{\substack{i \in H^z \\ (i,j) \in P}} c_j^C F_j^C \Delta T_j \\
&+ \sum_z \sum_{i \in H^z} \sum_{\substack{j \in C^z \\ (i,j) \in P \\ (i,j) \notin B}} \left(c_{ij}^F \left(U_{ij}^z - U_{ij}^{z^0} \cdot \sum_{k=1}^{k_e^t} \phi_{ij}^{z,k} \right) + c_{ij}^{A^0} \Delta A_{ij}^{z^0} + c_{ij}^{A^N} A_{ij}^{z^N} \right) \\
&+ \sum_z \sum_{i \in H^z} \sum_{\substack{j \in C^z \\ (i,j) \in P \\ (i,j) \in B}} \left(c_{ij}^F \left(U_{ij}^z - U_{ij}^{z^0} \sum_{k=1}^{k_{\max}} \sum_{h=1}^{k_e^t} \delta_{ij}^{z,hk} \right) + \sum_{k=1}^{k_{\max}} \left(c_{ij}^{A^0} \Delta A_{ij}^{z,k^0} + c_{ij}^{A^N} A_{ij}^{z,k^N} \right) \right)
\end{aligned} \qquad (95)$$