

## CHAPTER V

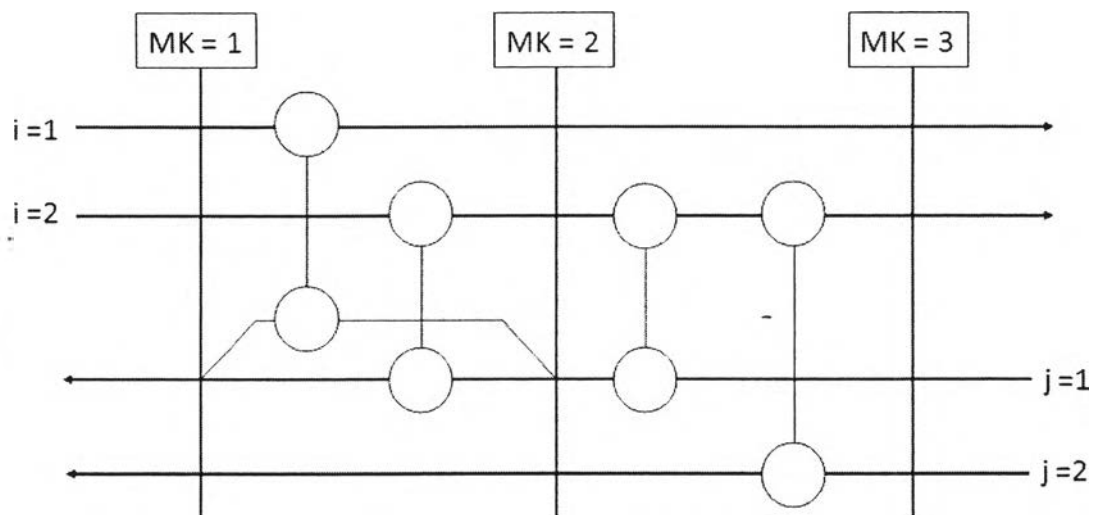
### THE MODIFIED STAGE-WISE SUPERSTRUCTURE

#### 5.1 Modified Stage-wise Superstructure Based on MINLP Model

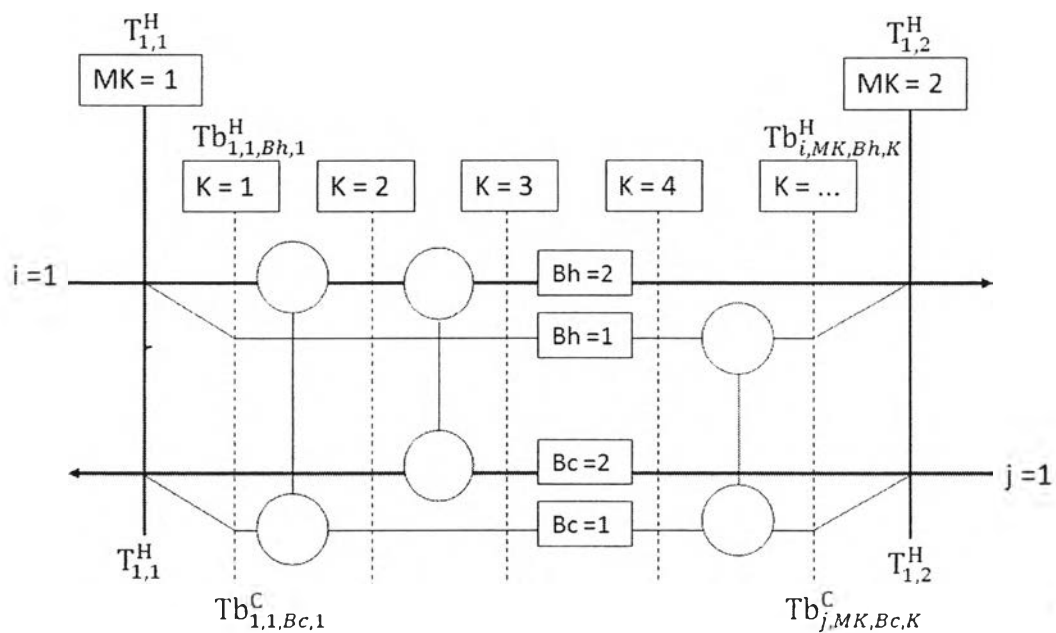
As mentioned earlier in Chapter 2, among the superstructure-based models for HEN design, the most popular one is the stage-wise superstructure approach that was proposed by Yee and Grossmann (1990), namely the SYNHEAT model. One interesting feature of this model is that all constraints are linear due to the assumption of isothermal mixing. However, the model does not address multiple matches in the same branch, a phenomenon that is common in crude units, as depicted in Figure 2.3(a) and Figure 2.3(b), respectively. The absence of these multiple matches may cause better solutions being excluded from the feasible space. Therefore, in this work the modification of the original stage-wise superstructure model is investigated

Huang and Karimi (2012) proposed an extension of the model by Yee and Grossmann (1990) by adding recycle/bypasses and non-isothermal mixing. They also improved the bounds of the branch temperature by not limiting them to be within the initial and final temperatures of their parent streams, hence, upon mixing, the temperatures of the branches can be lower or higher than their parent stream. In our model we add sub-stages and we also allow non-isothermal mixing.

Solving the MINLP problem using DICOPT without a good set of initial feasible points does not render a solution, whereas, solving the MINLP using BARON can obtain a solution without providing a feasible initial points. But BARON generally obtains the result slowly (Huang *et al.*, 2013). In this work, we propose a strategy to obtain good initial values for the MINLP by using a heuristic initialization strategy.



a) The original stage-wise superstructure model



b) Modified stage-wise superstructure model

**Figure 5.1** Innovation of stage-wise superstructure model.

In addition, to allow multiple matches per branch stage and stream splitting, the original stage-wise superstructure of Yee and Grossmann (1990) (Figure 5.1a) is modified as depicted in Figure 5.1b. Following a hot (cold) stream, each hot  $i$  (cold  $j$ ) streams is split into a fixed number of branch Bh's (Bc's). After entering the main stage  $MK$ , a fixed number of sub-stage  $K$ 's is added inside each main stage. A

potential heat exchanger match between hot ( $i$ ) and cold ( $j$ ) streams can be taken place in each sub-stage. If no match of hot and cold stream is selected, this sub-stage is bypassed and the next sub-stage  $MK+1$  is entered. By this, we allow multiple matches per branch stream in each main stage  $MK$ . After passing all heat exchangers in the main stage  $MK$ , the hot (cold) branch streams are recombined to their parent stream  $i$  ( $j$ ). The stream then enters the next main stage, splits into branch streams, enters the sub-stage  $K$ 's and recombines until it leaves the last main stage. Finally a cold (hot) utility is used to cooled (heated) the hot (cold) stream at the extreme of the superstructure to adjust the final temperature. The equations of the original model are rearranged as below:

The overall energy balance for each stream is modified from Eq.(2.1) and (2.2) to take into account the all heat exchange in sub-stage  $K$ 's and branch  $Bh$ 's for hot streams and  $Bc$ 's for cold stream.

$$\sum_{MK \in ST} \sum_{j \in CP} \sum_{B \in FB} \sum_{K \in ST} q_{i,j,MK,Bh,Bc,K} + q_{cu_i} = (TIN_i - TOUT_i)CF_i^H, \quad i \in HP \quad (5.1)$$

$$\sum_{MK \in ST} \sum_{i \in HP} \sum_{B \in FB} \sum_{K \in ST} q_{i,j,MK,Bh,Bc,K} + q_{hu_j} = (TOUT_j - TIN_j)CF_j^C, \quad j \in CP \quad (5.2)$$

The energy balance at each stage of Eqs.(2.3) and (2.4) are re-derived at each main-stage and sub-stage as follows:

$$\sum_{i \in HP} QHM_{MK,i} = Ha_{MK}, \quad MK \in ST \quad (5.3)$$

$$QHM_{MK,i} = (T_{i,MK}^H - T_{i,MK+1}^H)CF_i^H, \quad i \in HP, MK \in ST \quad (5.4)$$

$$\sum_{Bh \in HP} QH_{MK,i,Bh} = QHM_{MK,i}, \quad i \in HP, j \in CP, MK \in ST \quad (5.5)$$

$$\sum_{K \in ST} qHK_{i,MK,Bh,K} = QH_{MK,i,Bh},$$

$$i \in HP, MK \in ST, Bh \in HP \quad (5.6)$$

$$qHK_{i,MK,Bh,K} = AH_{i,MK,Bh,K} - AH_{i,MK,Bh,K+1},$$

$$j \in CP, MK \in ST, K \in ST \quad (5.7)$$

$$\sum_{j \in CP} QCM_{MK,j} = Ca_{MK},$$

$$MK \in ST \quad (5.8)$$

$$QCM_{MK,j} = (T_{j,MK}^C - T_{j,MK+1}^C)CF_j^C,$$

$$j \in CP, MK \in ST \quad (5.9)$$

$$\sum_{Bc \in CP} QC_{MK,j,Bc} = QCM_{MK,j},$$

$$j \in CP, MK \in ST, Bc \in CP \quad (5.10)$$

$$\sum_{K \in ST} qCK_{j,MK,Bc,K} = QC_{MK,j,Bc},$$

$$j \in CP, MK \in ST, Bc \in CP \quad (5.11)$$

$$qCK_{j,MK,Bc,K} = AC_{j,MK,Bc,K} - AC_{j,MK,Bc,K+1},$$

$$j \in CP, MK \in ST, K \in ST \quad (5.12)$$

From the above equations, each main stage of hot and cold streams are classified into main stages ( $Ha_{MK}$  and  $Ca_{MK}$ ), branch streams ( $QH_{MK,i}$ ,  $QCM_{MK,j}$ ,  $QH_{MK,i,Bh}$ ,  $QC_{MK,j,Bc}$ ), and sub-stages ( $qHK_{i,MK,Bh,K}$ ,  $qCK_{j,MK,Bc,K}$ ) to reduce the number of dependent variables in each equation as many as possible by introducing intermediate variables.

Sub-stage heat balance:

$$\sum_{j \in CP} \sum_{Bc \in CP} q_{i,j,MK,Bh,Bc,K} = qHK_{i,MK,Bh,K},$$

$$i \in HP, MK \in ST, Bh \in HP, K \in ST \quad (5.13)$$

$$\sum_{i \in \text{HP}} \sum_{\text{Bh} \in \text{HP}} q_{i,j,\text{MK},\text{Bh},\text{Bc},\text{K}} = q^{\text{CK}}_{j,\text{MK},\text{Bc},\text{K}},$$

$$j \in \text{CP}, \text{MK} \in \text{ST}, \text{Bc} \in \text{CP}, \text{K} \in \text{ST} \quad (5.14)$$

Multiple of Temperature and Heat capacity flow:

$$\text{AH}_{i,\text{MK},\text{Bh},\text{K}} = \text{Tb}_{i,\text{MK},\text{Bh},\text{K}}^{\text{H}} \text{CFb}_{i,\text{MK},\text{Bh}}^{\text{H}}$$

$$i \in \text{HP}, \text{MK} \in \text{ST}, \text{Bh} \in \text{HP}, \text{K} \in \text{ST} \quad (5.15)$$

$$\text{AC}_{j,\text{MK},\text{Bc},\text{K}} = \text{Tb}_{j,\text{MK},\text{Bc},\text{K}}^{\text{C}} \text{CFb}_{j,\text{MK},\text{Bc}}^{\text{C}}$$

$$j \in \text{CP}, \text{MK} \in \text{ST}, \text{Bc} \in \text{CP}, \text{K} \in \text{ST} \quad (5.16)$$

Superstructure inlet temperature:

$$\text{T}_{i,1}^{\text{H}} = \text{TIN}_i,$$

$$i \in \text{HP} \quad (5.17)$$

$$\sum_{\text{Bh} \in \text{HP}} \text{AH}_{i,\text{MK},\text{Bh},1} = \text{CF}_i^{\text{H}} \text{T}_{i,\text{MK}}^{\text{H}},$$

$$i \in \text{HP}, \text{MK} \in \text{ST}, \text{K} \in \text{ST} \quad (5.18)$$

$$\sum_{\text{Bh} \in \text{HP}} \text{AH}_{i,\text{MK},\text{Bh},\text{NOK}+1} = \text{CF}_i^{\text{H}} \text{T}_{i,\text{MK}+1}^{\text{H}},$$

$$i \in \text{HP}, \text{MK} \in \text{ST}, \text{K} \in \text{ST} \quad (5.19)$$

$$\text{T}_{i,\text{MK}}^{\text{H}} = \text{Tb}_{i,\text{MK},\text{Bh},1}^{\text{H}},$$

$$i \in \text{HP}, \text{MK} \in \text{ST}, \text{Bh} \in \text{HP} \quad (5.20)$$

$$\text{T}_{j,\text{NOK}+1}^{\text{C}} = \text{TIN}_j,$$

$$j \in \text{CP} \quad (5.21)$$

$$\sum_{\text{Bc} \in \text{CP}} \text{AC}_{j,\text{MK},\text{Bc},1} = \text{CF}_j^{\text{C}} \text{T}_{j,\text{MK}}^{\text{C}},$$

$$j \in \text{CP}, \text{MK} \in \text{ST} \quad (5.22)$$

$$\sum_{Bc \in CP} AC_{j,MK,Bc,NOK+1} = CF_j^C T_{j,MK+1}^C, \\ j \in CP, MK \in ST \quad (5.23)$$

$$T_{j,MK+1}^C = Tb_{j,MK,Bc,NOK+1}^C, \\ j \in CP, MK \in ST, Bc \in CP \quad (5.24)$$

Feasibility of temperatures (monotonic decrease in temperature):

$$T_{i,MK}^H \geq T_{i,MK+1}^H, \\ i \in HP, MK \in ST \quad (5.25)$$

$$Tb_{i,MK,Bh,K}^H \geq Tb_{i,MK,Bh,K+1}^H, \\ i \in HP, MK \in ST, K \in ST, Bh \in HP \quad (5.26)$$

$$TOUT_i \leq T_{i,NOK+1}^H, \\ i \in HP, \quad (5.27)$$

$$T_{j,MK}^C \geq T_{j,MK+1}^C, \\ j \in CP, MK \in ST \quad (5.28)$$

$$Tb_{j,MK,Bc,K}^C \geq Tb_{j,MK,Bc,K+1}^C, \\ j \in CP, MK \in ST, K \in ST, Bc \in CP \quad (5.29)$$

$$TOUT_j \leq T_{j,1}^C, \\ j \in CP, \quad (5.30)$$

Hot and cold utility load:

$$(T_{i,NOK+1}^H - TOUT_i)CF_i^H = qcu_i, \\ i \in HP, \quad (5.31)$$

$$(TOUT_j - T_{j,1}^C)CF_j^C = qhu_j, \\ j \in CP, \quad (5.32)$$

Logical constraints:

$$Q_{i,j,MK,Bh,Bc,K} - \Omega Z_{i,j,MK,Bh,Bc,K} \leq 0$$

$$i \in HP, j \in CP, MK \in ST, Bc \in CP, Bh \in HP, K \in ST \quad (5.33)$$

$$qcu_i - \Omega zcu_i \leq 0,$$

$$i \in HP \quad (5.34)$$

$$qhu_j - \Omega zhu_j \leq 0,$$

$$j \in CP \quad (5.35)$$

Maximum matching:

$$\sum_{i \in CP} \sum_{Bh \in BF} Z_{i,j,MK,Bh,Bc,K} \leq 1,$$

$$j \in CP, MK \in ST, K \in ST, Bc \in CP \quad (5.36)$$

$$\sum_{j \in CP} \sum_{Bc \in BF} Z_{i,j,MK,Bh,Bc,K} \leq 1,$$

$$i \in HP, MK \in ST, K \in ST, Bh \in HP \quad (5.37)$$

Mass balance at each stage MK of cold stream:

$$\sum_{Bh \in HP} CFb_{i,MK,Bh}^H = CF_i^H,$$

$$i \in HP, MK \in ST \quad (5.38)$$

$$\sum_{Bc \in CP} CFb_{i,MK,Bc}^C = CF_j^C,$$

$$j \in CP, MK \in ST \quad (5.39)$$

Energy constraints:

$$\sum_{i \in \text{HP}} q_{\text{cu}_i} \leq \text{Maximum Cold Utility}, \quad (5.40)$$

$$\sum_{i \in \text{HP}} q_{\text{cu}_i} \geq \text{Minimum Cold Utility}, \quad (5.41)$$

$$q_{\text{cu}_i} \leq Z_{\text{cu}_i} (\text{TIN}_i - \text{TOUT}_i) \text{CF}_i^{\text{H}}, \quad (5.42)$$

$$\text{Total cold utility} = \sum_{i \in \text{HP}} q_{\text{cu}_i}, \quad (5.43)$$

$$\sum_{j \in \text{CP}} q_{\text{hu}_j} \leq \text{Maximum Hot Utility}, \quad (5.44)$$

$$\sum_{j \in \text{CP}} q_{\text{hu}_j} \geq \text{Minimum Hot Utility}, \quad (5.45)$$

$$q_{\text{hu}_j} \leq Z_{\text{hu}_j} (\text{TOUT}_j - \text{TIN}_j) \text{CF}_j^{\text{C}}, \quad (5.46)$$

$$\text{Total hot utility} = \sum_{j \in \text{CP}} q_{\text{hu}_j}, \quad (5.47)$$

$$\sum_{\text{MK} \in \text{ST}} \sum_{i \in \text{HP}} \sum_{j \in \text{CP}} \sum_{\text{Bc} \in \text{FB}} \sum_{\text{K} \in \text{ST}} Q_{i,j,\text{MK},\text{Bh},\text{Bc},\text{K}} = \text{Total Heat exchange}, \quad (5.48)$$

$$\text{Total Heat exchange} = \text{Max hot Utility} - \text{Total hot Utility}, \quad (5.49)$$

$$Q_{i,j,\text{MK},\text{Bh},\text{Bc},\text{K}} \leq Z_{i,j,\text{MK},\text{Bh},\text{Bc},\text{K}} \text{QUP}_{i,j,\text{MK},\text{Bh},\text{Bc},\text{K}}, \quad (5.50)$$

$$\text{QUP}_{i,j,\text{MK},\text{Bh},\text{Bc},\text{K}} = \text{Min}[(\text{TIN}_i - \text{TOUT}_i) \text{CF}_i^{\text{H}}, (\text{TOUT}_j - \text{TIN}_j) \text{CF}_j^{\text{C}}, \text{Max}[0, \text{TIN}_i - \text{TIN}_j - \text{EMAT}] * \text{Min}[\text{CF}_i^{\text{H}}, \text{CF}_j^{\text{C}}]], \quad (5.51)$$

To minimize the amount of utility usage, the heat exchange between the hot and cold process streams is to be maximized. Once a better heat exchange constraint is obtained (Eq.5.49), it is set as a new bound. The heat exchange constraint is one of the key strategies employed to converge the model.

Calculation of approach temperature:

$$dT_{i,j,\text{MK},\text{Bh},\text{Bc},\text{K}}^{\text{HS}} \leq T_{i,\text{MK},\text{Bh},\text{K}}^{\text{H}} - T_{i,\text{MK},\text{Bc},\text{K}}^{\text{C}} + \Gamma(1 - Z_{i,j,\text{MK},\text{Bh},\text{Bc},\text{K}}) \\ i \in \text{HP}, j \in \text{CP}, \text{MK} \in \text{ST}, \text{Bc} \in \text{CP}, \text{Bh} \in \text{HP}, \text{K} \in \text{ST} \quad (5.52)$$



$$dT_{i,j,MK,Bh,Bc,K}^{CS} \leq T_{i,MK,Bh,K+1}^H - T_{i,MK,Bc,K+1}^C + \gamma(1 - z_{i,j,MK,Bh,Bc,K})$$

$$i \in HP, j \in CP, MK \in ST, Bc \in CP, Bh \in HP, K \in ST \quad (5.53)$$

$$dT_{cu_i} \leq T_{i,NOK+1}^H - TOUT_i + \gamma(1 - z_{cu_i})$$

$$i \in HP \quad (5.54)$$

$$dTh_{uj} \leq TOUT_j - T_{j,1}^C + \gamma(1 - z_{hu_j})$$

$$j \in CP \quad (5.55)$$

Minimum approach temperature (lower bounds):

$$dT_{i,j,MK,Bh,Bc,K}^{HS} \geq EMAT$$

$$i \in HP, j \in CP, MK \in ST, Bc \in CP, Bh \in HP, K \in ST \quad (5.56)$$

$$dT_{i,j,MK,Bh,Bc,K}^{CS} \geq EMAT$$

$$i \in HP, j \in CP, MK \in ST, Bc \in CP, Bh \in HP, K \in ST \quad (5.57)$$

$$dT_{cu_i} \geq EMAT$$

$$i \in HP \quad (5.58)$$

$$dTh_{uj} \geq EMAT$$

$$j \in CP \quad (5.59)$$

Logarithmic-mean temperature difference (LMTD) (Chen, 1987):

$$LMTD_{i,j,MK,Bh,Bc,K} = \left[ dT_{i,j,MK,Bh,Bc,K}^{HS} dT_{i,j,MK,Bh,Bc,K}^{CS} \frac{dT_{i,j,MK,Bh,Bc,K}^{HS} + dT_{i,j,MK,Bh,Bc,K}^{CS}}{2} \right]^{1/3}$$

$$i \in HP, j \in CP, MK \in ST, Bc \in CP, Bh \in HP, K \in ST \quad (5.60)$$

$$LMTD_{cu_i} = \left[ dT_{cu_i} (TMH_{i,NOK+1} - TOUT_i) \frac{dT_{cu_i} + (TMH_{i,NOK+1} - TOUT_i)}{2} \right]^{1/3}$$

$$i \in HP, j \in CP, MK \in ST, Bc \in CP, Bh \in HP, K \in ST \quad (5.61)$$

$$LMTD_{hu_j} = \left[ dTh_{uj} (TOUT_j - TMC_{j,1}) \frac{dTh_{uj} + (TOUT_j - TMC_{j,1})}{2} \right]^{1/3}$$

$$i \in HP, j \in CP, MK \in ST, Bc \in CP, Bh \in HP, K \in ST \quad (5.62)$$

$$LMTD_{i,j,MK,Bh,Bc,K} \leq \frac{dT_{i,j,MK,Bh,Bc,K}^{HS} + dT_{i,j,MK,Bh,Bc,K}^{CS}}{2},$$

$$i \in HP, j \in CP, MK \in ST, Bc \in CP, Bh \in HP, K \in ST \quad (5.63)$$

Area calculation:

$$q_{i,j,MK,Bh,Bc,K} - A_{i,j,MK,Bh,Bc,K} U_{ij} LMTD_{i,j,MK,Bh,Bc,K} \leq 0,$$

$$i \in HP, j \in CP, MK \in ST, Bh \in HP, Bc \in CP, K \in ST \quad (5.64)$$

$$q_{cu_i} - A_{cu_i} U_{cu_i} LMTD_{cu_i} \leq 0,$$

$$i \in HP, \quad (5.65)$$

$$q_{hu_j} - A_{hu_j} U_{hu_j} LMTD_{hu_j} \leq 0,$$

$$j \in CP, \quad (5.66)$$

Additional bound variables:

$$TOUT_i \leq T_{i,MK}^H \leq TIN_i,$$

$$i \in HP, MK \in ST \quad (5.67)$$

$$\min\{TOUT_i, TIN_j\} \leq Tb_{i,MK,Bh,K}^H \leq \max\{TIN_i, TOUT_j\},$$

$$i \in HP, MK \in ST, Bh \in HP, K \in ST \quad (5.68)$$

$$TOUT_j \leq T_{j,MK}^C \leq TIN_j,$$

$$j \in CP, MK \in ST \quad (5.69)$$

$$\min\{TOUT_i, TIN_j\} \leq Tb_{j,MK,Bc,K}^C \leq \max\{TIN_i, TOUT_j\},$$

$$j \in CP, MK \in ST, Bc \in CP, K \in ST \quad (5.70)$$

$$0 \leq q_{i,j,MK,Bh,Bc,K} \leq \max\{CF_i^H(TIN_i - TOUT_i), CF_j^C(TOUT_j - TIN_j)\},$$

$$i \in HP, j \in CP, MK \in ST, Bh \in HP, Bc \in CP, K \in ST \quad (5.71)$$

$$0 \leq q_{HK_{i,MK,Bh,K}} \leq CF_i^H(TIN_i - TOUT_i),$$

$$i \in HP, MK \in ST, Bh \in HP, K \in ST \quad (5.72)$$

$$0 \leq qCK_{j,MK,Bc,K} \leq CF_j^C(TOUT_j - TIN_j),$$

$$j \in CP, MK \in ST, Bc \in CP, K \in ST \quad (5.73)$$

$$0 \leq qcu_i \leq CF_i^H(TIN_i - TOUT_i),$$

$$i \in HP \quad (5.74)$$

$$0 \leq qhu_j \leq CF_j^C(TOUT_j - TIN_j),$$

$$j \in CP \quad (5.75)$$

$$0 \leq QHM_{MK,i} \leq CF_i^H(TIN_i - TOUT_i),$$

$$i \in HP, MK \in ST \quad (5.76)$$

$$0 \leq QH_{MK,i,Bh} \leq CF_i^H(TIN_i - TOUT_i),$$

$$i \in HP, Bh \in HP, MK \in ST \quad (5.77)$$

$$0 \leq Ha_{MK} \leq \sum_{i \in HP} CF_i^H(TIN_i - TOUT_i),$$

$$MK \in ST \quad (5.78)$$

$$0 \leq QCM_{MK,j} \leq CF_j^C(TOUT_j - TIN_j),$$

$$j \in CP, MK \in ST \quad (5.79)$$

$$0 \leq QC_{MK,j,Bc} \leq CF_j^C(TOUT_j - TIN_j),$$

$$j \in CP, Bc \in CP, MK \in ST \quad (5.80)$$

$$0 \leq Ca_{MK} \leq \sum_{j \in CP} CF_j^C(TOUT_j - TIN_j),$$

$$MK \in ST \quad (5.81)$$

$$0 \leq CFb_{i,MK,Bh}^H \leq CF_i^H$$

$$i \in HP, MK \in ST, Bh \in HP, K \in ST \quad (5.82)$$

$$0 \leq CFb_{j,MK,Bc}^C \leq CF_j^C$$

$$j \in CP, MK \in ST, Bc \in CP, K \in ST \quad (5.83)$$

Because the model for formulating optimal HEN configuration is very complex, highly non-linear and non-convex as well as, long computational time requirement, it is necessary to develop an efficient approach to obtain an optimal solution by using the General Algebraic Modeling System (GAMS). GAMS programming composes of three main substantial organizational flexibility including DATA, MODEL, and SOLUTION. In general, the program arrangement of these groups is settled first on DATA, which is a group of set, definition parameters, and assignments. Next, MODEL will be supplied by the user in the forms of variables and compatible mathematical equations. Finally, the model is compiled using a selected solver and the results are displayed (Brooke *et al.*, 1998).

## 5.2 Solution Strategy

In the original formulation by Yee and Grossmann (1990), the feasible is defined by strict linear constraints. Nonetheless, the stage-wise model in this work is non-convex due to the presence of non-linear, non-convex terms in the objective function related to the area costs. In our work we make the following changes:

- The areas of heat exchangers are used explicitly in the objective function (The original SYNHEAT model used the ratio of the heat transferred to the log mean temperature difference).
- The area costs are assumed to be linearly dependent on the areas, thus making the objective function linear.
- Because the areas of heat exchangers are explicitly defined in the objective function, new constraints to calculate them are incorporated.

Apart from the area costs, the assumption for developing our model is mainly based on non-isothermal mixing by adding heat balance equation on both entrance and final of each main-stage and the number of matching between hot and cold steam that can have more than one in series by applying sub-stage into the main-stage. We also simplify our formulation by assuming  $C_p$  to be constant.

Finding good initial values is a challenge to solve the non-linear problem, especially for solving DICOPT in GAMS, whereas, solving the MINLP by BARON in GAMS can obtain a solution without providing good feasible initial point. However, BARON generally obtains the result slowly (Huang *et al.*, 2013). In this work, the solving strategy includes four decomposition models as presented in Figure 5.2 to help tuning the initial values to a proper direction towards the optimal solution. The initial values are added manually to the first model and the results after solving each model are used as the initial values of the next model. Hence, the initial values are gradually tuned from each model to become a suitable set of initial values for the final MINLP model. The role of each model is presented below.

**First MILP** is used to find a minimum number of matches and utility consumption by fixing value of the branch flow variables. The objective function of this model is as follow:

$$\begin{aligned} \min \quad & C_{cu} \sum_{i \in HHP} q_{cu_i} + C_{hu} \sum_{j \in ECP} q_{hu_j} \\ & + C_{HE} \sum_{i \in HHP} \sum_{j \in ECP} \sum_{MK \in ST} \sum_{Bh \in HHP} \sum_{Bc \in ECP} \sum_{K \in ST} z_{i,j,MK,Bh,Bc,K} \\ & + C_{HE} \sum_{i \in HHP} Z_{cu_i} + C_{HE} \sum_{j \in ECP} Z_{hu_j} \end{aligned} \quad (5.85)$$

**First NLP** is used to find the minimum utility cost by adjusting the branch flow variables. The Heat exchanger location (Z) result that obtained from the First MILP is fixed while solving the First NLP under the objective function of:

$$\min \quad C_{cu} \sum_{i \in HHP} q_{cu_i} + C_{hu} \sum_{j \in ECP} q_{hu_j} \quad (5.86)$$

**Second MILP** This model is used to synthesize the network by minimizing the number of matches and the utility consumption. Again, the branch flow variables (CFb) are fixed while solving this step. This second MILP provides a solution with a better set of branch flows under the objective function of:

$$\begin{aligned} \min \quad & C_{cu} \sum_{i \in HHP} q_{cu_i} + C_{hu} \sum_{j \in ECP} q_{hu_j} \\ & + C_{HE} \sum_{i \in HHP} \sum_{j \in ECP} \sum_{MK \in ST} \sum_{Bh \in HHP} \sum_{Bc \in ECP} \sum_{K \in ST} z_{i,j,MK,Bh,Bc,K} \\ & + C_{HE} \sum_{i \in HHP} Z_{cu_i} + C_{HE} \sum_{j \in ECP} Z_{hu_j} \end{aligned} \quad (5.87)$$

**Second NLP** We again adjust the branch flows minimizing utility and investment cost by fixing the heat exchanger location ( $Z$ ) obtained from the Second MILP.

$$\begin{aligned}
\min \quad & C_{cu} \sum_{i \in HHP} q_{cu_i} + C_{hu} \sum_{j \in ECP} q_{hu_j} \\
& + C_{Aij} \sum_{i \in HHP} \sum_{j \in ECP} \sum_{MK \in ST} \sum_{Bh \in HHP} \sum_{Bc \in ECP} \sum_{K \in ST} A_{i,j,MK,Bh,Bc,K} \\
& + C_{Ai} \sum_{i \in HHP} A_{cu_i} + C_{Aj} \sum_{j \in ECP} A_{hu_j} \quad (5.88)
\end{aligned}$$

**MINLP** The initial values for this model are those obtained by the Second NLP and the Second MILP. If the Second NLP returns an infeasible result, the results are still useful since the moving direction towards convergence of the results is in the desired direction. However, if all results from the Second NLP are used as the initial values for MINLP model, these initial values can cause the MINLP to fail. Therefore, all of the initial values for the MINLP are obtained from the Second NLP model except the branch flow variable which are obtained from Second MILP, as illustrated in Figure. 5.2.

$$\begin{aligned}
\min \quad & C_{cu} \sum_{i \in HHP} q_{cu_i} + C_{hu} \sum_{j \in ECP} q_{hu_j} \\
& + C_{HE} \sum_{i \in HHP} \sum_{j \in ECP} \sum_{MK \in ST} \sum_{Bh \in HHP} \sum_{Bc \in ECP} \sum_{K \in ST} z_{i,j,MK,Bh,Bc,K} \\
& + C_{HE} \sum_{i \in HHP} z_{cu_i} + C_{HE} \sum_{j \in ECP} z_{hu_j} \\
& + C_{Aij} \sum_{i \in HHP} \sum_{j \in ECP} \sum_{MK \in ST} \sum_{Bh \in HHP} \sum_{Bc \in ECP} \sum_{K \in ST} A_{i,j,MK,Bh,Bc,K} \\
& + C_{Ai} \sum_{i \in HHP} A_{cu_i} + C_{Aj} \sum_{j \in ECP} A_{hu_j} \quad (5.89)
\end{aligned}$$

All the models are solved sequentially as follows: First MILP  $\rightarrow$  First NLP  $\rightarrow$  Second MILP  $\rightarrow$  Second NLP  $\rightarrow$  MINLP.

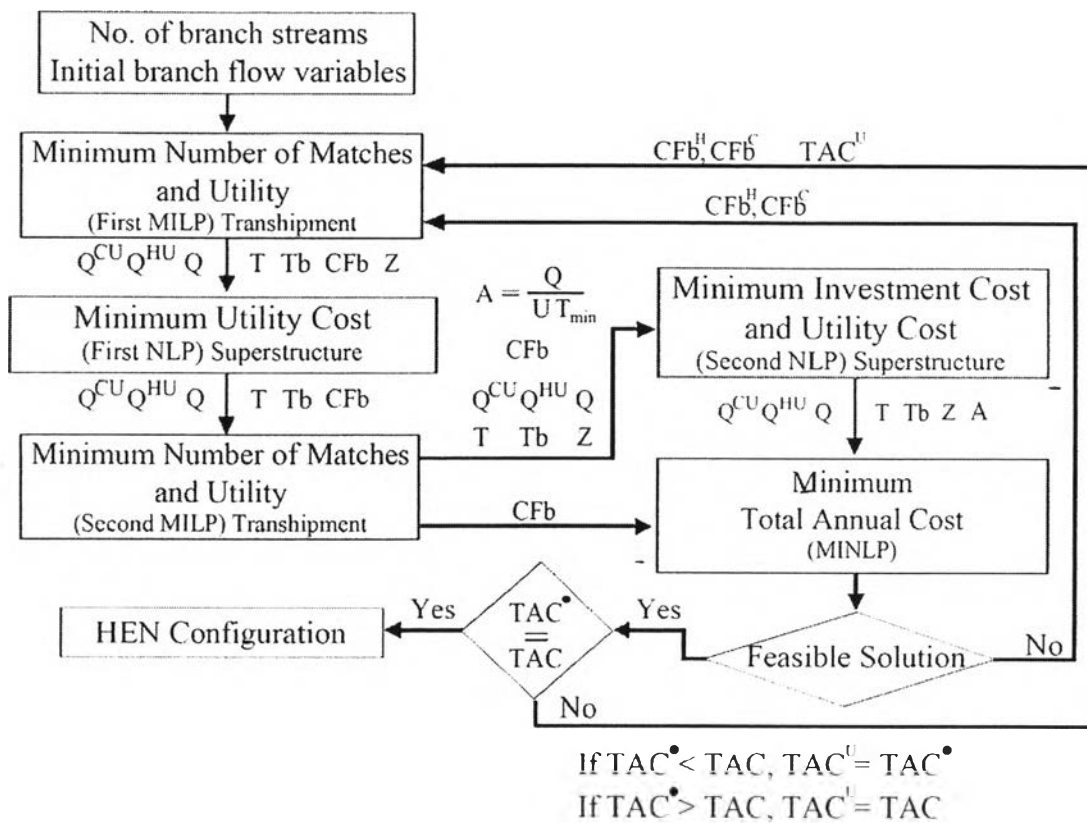
The solving strategy is as follow:

1. Guessing the initial branch flow is an important step. The best initial branch flow for the First MILP model should have a potential to give HEN configuration with its heat exchange value close to the maximum heat exchange. Therefore, for the first iteration, the initial point of each branch flow are manually set by giving one branch flow a lot higher than the other (For example, if

its parent stream value is 8, first branch stream will be 6 while second branch stream is 2). However, the lower branch flow value should be higher than one.

2. One key strategy to minimize the total annualized cost (TAC) is to set the upper bound TAC ( $TAC^U$ ) constraint. After the first feasible result, if the new TAC result is lower than the previous TAC from last iteration ( $TAC^*$ ), the new TAC will be set as a new bound  $TAC^U$ . On the other hand, if the new TAC result larger than  $TAC^*$  or if infeasible solution was obtained from that iteration, the  $TAC^*$  will still be set as a  $TAC^U$ .

3. After solving the first iteration of all models (First MILP  $\rightarrow$  First NLP  $\rightarrow$  Second MILP  $\rightarrow$  Second NLP  $\rightarrow$  MINLP), the results obtained from MINLP are used as the initial point of the next iteration in the First MILP model and re-run all models. There are some exceptions of using the initial values from the MINLP: If the value of branch flow variables is zero, a non-zero number has to be reassigned as initial value. At least five percent of its parent stream is to be assigned to that zero branch flow variable. If any value of branch flow stream in First MILP is zero, that branch flow stream will be eliminated in the next solving model and this can cause missing other plausible network configuration.



**Figure 5.2** Step of decomposition based on HEN synthesis approach by sequential technique.